# Chapter 2

## Magnetic domain theory in static

Magnetic domains in ferromagnetic materials are generated in order to minimize the sum of energy terms, e.g., the magnetostatic, the exchange, the anisotropy, and the Zeeman energies. In the case the magnetic film has infinite lateral extension and uniformly magnetized in the film plane, the magnetostatic energy is zero. However, if the film has a finite size, the surface charging has to be taken into account, which leads to a demagnetizing field. This demagnetizing field separates the film into domains with different orientations of magnetization to reduce the magnetostatic energy (long range magnetic interaction). The boundary between these domains is called *magnetic domain wall*. Inside the wall, the spins rotate gradually, leading to a certain width of the wall . The wall width is mainly determined by the competition between two energy terms, the exchange energy and the anisotropy energy. The exchange energy between neighboring spins tends to increase the wall width. A larger rotation of spins between two neighbors causes a higher exchange energy. However, a wider wall induces a higher anisotropy energy, because inside the wall the direction of spins is away from the easy axis of magnetization. These two energies lead to short range interactions (nm range).

In the reminder of this chapter, first, the magnetic energy terms, magnetostatic, exchange, anisotropy and Zeeman energies, are introduced. Then the formation of magnetic domains by a competition of all the energy terms is mentioned. At the end of this chapter, the different types of magnetic domain walls, Bloch and Néel walls, will be expressed. Throughout, SI units are used.

## 2.1 Magnetostatic energy

The magnetostatic (dipole) energy depends on the magnetization M, the magnetic-dipole moment per volume, that arises from the alignment of atomic magnetic dipoles. In a solid, the dipoles arise primarily from electron spins. Although the orbital motion of electrons usually contributes less to the dipole strength, it plays a significant role for the magnetic anisotropy. The existence of magnetic domains is a consequence of energy minimization [12, 24]. A single domain finite specimen has associated with it a large magnetostatic energy, but the breakup of the magnetization into localized regions (domains), e.g., providing for flux closure at the ends of the specimen, can reduce the magnetostatic energy. If the decrease in magnetostatic energy is greater than the energy needed to form magnetic domain walls, then multi-domain specimens will arise.

One example is the case of an infinitely extended magnetic film magnetized uniformly normal to the surface, for which the magnetostatic energy can be easily derived. The magnetic dipoles in the film,  $\vec{M}$ , create the stray field,  $H_{stray}$ . However, due to the surface charging, inside the film, there exists the field which has the same amplitude as  $H_{stray}$  but directs opposite to  $\vec{M}$ , the so called demagnetizing field,  $H_d$ . So the magnetostatic energy, which has the same amplitude to the demagnetizing energy in this case, will be described with  $\vec{M}$  and  $\vec{H}_d$  as

$$K_d = -\mu_0 \int_V \vec{M} \cdot \vec{H}_d dV = -\mu_0 \int_V \vec{M} \cdot \frac{\vec{H}}{2} dV = \frac{\mu_0}{2} M_S^2, \qquad (2.1)$$

where  $\vec{M}$  is exchanged to the saturation magnetization,  $M_S$ , and  $\vec{H}$  is a sum of  $H_{stray}$  and  $H_d$ , will have the same field as  $M_S$ .

## 2.2 Exchange energy

The basic interaction which causes cooperative magnetic ordering is the exchange interaction. The exchange interaction energy,  $E_{ex}$ , between two spins,  $\vec{S}_i$  and  $\vec{S}_j$ , scales with the exchange integral, J (dimension; J),

$$E_{ex} = -J_{ij}\vec{S}_i \cdot \vec{S}_j, \qquad (2.2)$$

where  $\vec{S}_i$  and  $\vec{S}_j$  are the unit vectors of interacting spins on two atoms. The exchange interaction is a manifestation of the Coulomb interaction between electron charges and the Pauli principle.

If one estimates the exchange energy in a magnetic domain wall, it will be convenient to take all spins together, i.e., a continuous model of spin rotation in a one-dimensional domain wall, the total  $E_{ex}$  inside the wall is,

$$E_{ex} = A(\frac{d\theta}{dx})^2, \qquad (2.3)$$

where  $A = s^2 a_L^2 J N_V / 2$  is the exchange stiffness constant (J/m), and is temperature dependent. s is the spin quantum number (= 1/2), and  $a_L$  is the lattice constant.  $N_V$  indicates the number of nearestneighbor atoms per unit volume.  $\theta$  is the angle with respect to the easy axis of magnetization. For simple cubic (SC) and body centered cubic (BCC), J will be

$$J = 0.54k_B T_C \quad \text{for SC and}, \tag{2.4}$$

$$J = 0.34k_B T_C \quad \text{for BCC}, \tag{2.5}$$

where  $k_B$  and  $T_C$  are the Boltzmann constant and the Curie temperature, respectively.

It should be noted that the exchange energy tends to make a magnetic domain wall as wide as possible. Since the exchange energy decreases with decreasing angle between spins on neighboring atoms inside the wall, the spins rotate gradually, leading to a certain width of the magnetic domain wall.

## 2.3 Anisotropy energy

The energy also depends on the orientation of the magnetization with respect to the crystallographic axes of the material. This energy term is called the magnetic anisotropy energy. It basically results from spin-orbit interaction. Many kinds of FM films have a uniaxial anisotropy, whether they are polycrystalline or single crystal, elements or alloys. In undisturbed crystals, the anisotropy energy will be minimized along certain crystal axes. However, anisotropy can be induced by symmetry breaking of the crystal structure at the interface and surface, by anisotropic modulation of atoms, or by alignment of surface/interface defects.

#### 2.3.1 Cubic anisotropy

The cubic anisotropy is magnetocrystalline in nature, and arises from the interaction of the atomic magnetic moments with the intrinsic symmetry of their crystalline environment via a spin-orbit interaction. The cubic anisotropy energy density is basically expressed by

$$E_C = K_1(m_x^2 m_y^2 + m_x^2 m_z^2 + m_y^2 m_z^2) + K_2 m_x^2 m_y^2 m_z^2 + \dots,$$
(2.6)

where  $m_x$ ,  $m_y$  and  $m_z$  are the magnetization along x, y and z crystal axes, respectively.  $K_i$  is the *i*th order anisotropy. At room temperature, the second order term and other higher order terms can be mostly neglected, but they become important at low temperature.

#### 2.3.2 Volume and surface/interface anisotropies

When working with magnetic ultrathin films, surface and interface anisotropies have to be considered, because of missing of neighbor atoms [25]. Surprisingly, a much stronger anisotropy was found for mono atomic transition metal films compared with bulk materials [26]. In the case the magnetic film with out-of-plane uniaxial anisotropy (along the z axis), the energy is symmetric in the film plane. Then the surface anisotropy energy density is

$$E_S = K_S sin^2 \theta, \tag{2.7}$$

where  $\theta$  is the angle between the magnetization and the z axis, and  $K_S$  the surface anisotropy.

The volume anisotropy is also not negligible for magnetic thin films. Hexagonal or tetragonal crystals (for example, a (0001)-hcp Co [27] or an epitaxially grown Ni film on a Cu(001) surface [28], respectively) show a uniaxial volume anisotropy ( $K_u$ ) normal to the film plane for a certain thickness range. The volume anisotropy energy density can be written as

$$E_V = K_{u1}sin^2\theta + K_{u2}sin^4\theta, \qquad (2.8)$$

where  $\theta$  is the angle between anisotropy axis and magnetization. Because of time inversion symmetry, the odd powers do not appear.

The total anisotropy energy density in a magnetic thin film is the following (with d the film thickness),

$$E = E_V + \frac{E_S}{d}.$$
(2.9)

#### 2.3.3 Field- and morphology-induced anisotropies

Most FM materials exhibit uniaxial anisotropy when they are heat-treated or grown in a magnetic field. The preferable magnetization is parallel to the field direction, so called field induced anisotropy. Even single crystal films can have this induced anisotropy [29, 30].

The magnetic annealing effect can explain this kind of induced anisotropy. When the film is deposited at some temperature in a magnetic field, it is structurally strained in the field direction. After it is cooled down to the room temperature, the film is tightly bound to the substrate, and consequently the atoms cannot diffuse to relieve the strain caused by the different magnetostriction. This anisotropic distortion of crystal causes the magnetic anisotropy energy along its direction.

The anisotropy energy can be induced in another way, by growing films onto a modulated surface [31–33]. If the FM material was deposited on the surface, in which atomic steps or stepbunches are aligned in one direction, the film shows a uniaxial anisotropy along or perpendicular to the steps. The details of this morphology induced anisotropy will be found in Section 5.2.1 and 5.5.1.

## 2.4 Zeeman energy

Zeeman energy is the interaction energy of the magnetization vector field  $\vec{M}$  with an external magnetic field  $\vec{H}_{Ext}$ . Then the Zeeman energy is

$$E_Z = \mu_0 \int \vec{M} \cdot \vec{H}_{Ext} dV.$$
 (2.10)

#### **2.5 Formation of domains**

To reduce the total magnetic energy  $\varepsilon$ , FM films create domains of a certain size. The size of domains is defined by minimization of the sum of all the above energy terms, exchange, anisotropy, Zeeman, and demagnetizing energies,

$$\varepsilon = E_{ex} + E_A + E_Z + E_d. \tag{2.11}$$

Ultrathin films with out-of-plane anisotropy often form stripe- or labyrinth-shaped domains, separated by Bloch-type walls, will be explained in subsequent section. Let's assume that the x axis is perpendicular to the wall plane and that the y axis is parallel to the wall in the film plane (as in Fig. 2.1). Then the total energy in the stripe-shaped domains becomes

$$\varepsilon = L_y d \int_0^{P/2} \left( A\left(\frac{d\theta}{dx}\right)^2 + K_1 \sin^2(\theta(x)) - \mu_0 M_S H_{ext} \sin(\theta(x)) \right) dx + E_d, \tag{2.12}$$

where P,  $L_y$ , d and  $H_{ext}$  are the period of stripe domains, the domain length along the stripe direction, the film thickness, and the external field in the direction normal to the surface (z axis). The last term, demagnetizing energy, for the stripe domains has a complicated formula derived by Yafet and Gyorgy [34], which will be written as

$$E_d = \frac{P}{2} M_S^2 \sum_{m=1,3,\dots} \frac{1}{m} b_m^2(\delta) (1 - \exp(-\frac{4\pi m d}{P})), \qquad (2.13)$$

where

$$b_m = (-1)^{(m-1)/2} \frac{4}{\pi m} \frac{1}{1 - m^2 \delta^2} \cos(\pi m \delta/2).$$
(2.14)

δ is the wall width, *w*, divided by a half period of the stripe (2w/*P*). By minimizing ε, the period of the stripe domains can be derived.

The determination of domain size by magnetic energy minimization [24,34–38] has been widely discussed, one example experimentally obtained by myself is introduced in Section 5.1.1.

### 2.6 Bloch wall and Néel wall

As mentioned above the magnetic domain boundaries have a certain width. There are two main types of spin structures inside the domain walls, Bloch and Néel types. Besides many other kinds of domain walls exist. One of them is called the cross-tie wall, which is an intermediate state between Bloch and Néel walls, and it is composed of a mixture of Bloch and Néel walls [24, 39]. In Fig. 2.1, the spin structures of Bloch and Néel walls are shown. The Bloch wall is usually

preferable in bulk materials. Spins rotate in the plane parallel to the wall plane. The wall width of a 180° Bloch wall is most commonly defined by  $\pi \sqrt{A/K}$ , where *A* and *K* are the exchange constant and anisotropy energy, respectively [24]. Then the wall profile basically follows a sine law. In thin films, however, a Bloch wall induces surface charges by its stray field. Then the Néel wall become more favorable when the film thickness becomes smaller than the wall width. In a Néel wall, spins rotate in the film plane (Fig. 2.1 (b)). The width and profile of the Néel wall are difficult to define. The Néel wall has a narrow core and  $\mu$ m-long tails on both sides. The core width is of the order of the exchange length [=  $\sqrt{A/K_d}$  (nm scale)], where  $K_d$  is the demagnetizing energy. The Néel wall profile is well explained in [24, 40].



Figure 2.1: The rotation of the magnetization vector in the Bloch wall (a) and in the Néel wall (b).