

Chapter 4

Optimal Hierarchies with Diverse Decision-Makers

4.1 Introduction

A large number of start-up firms in the New Economy sector are characterized - among other attributes - by very flat hierarchies implying decentralized decision-making, while more traditionally structured firms often have a very small number of decision-making superiors and a large number of subordinates. The chapter provides a possible explanation for the occurrence of these different structures based upon the alignment of the firm members' preferences and the character of the decisions to be made. In either hierarchy, decision-makers exert externalities on the other members of the organization. If, for example, a software firm changes the features of a certain product, this decision affects the work of the software developer as well as the marketing specialist, whoever has made the decision. As a matter of taste, the potential decision-makers have different preferences with respect to the design of the product that may conflict with the firm's objective. If contracts are incomplete, these preferences influence the optimal allocation of decision rights and therefore the optimal hierarchy of the firm.

In this chapter, we develop a simple model of the hierarchical structure of an organization. We consider a principal who hires two agents to undertake a project. The agents are protected by limited liability. Two noncontractible project-oriented decisions have to be made, which may be viewed as two sub-projects to be chosen. The decision rights are contractually assigned to the agents, the decisions themselves remain unverifiable *ex post*. The allocation of decision rights determines the hierarchy. In a vertical (or steep) hierarchy, one person is in charge of all decisions and might be viewed as the other persons' superior. In a horizontal (or flat) hierarchy, every decision is made by a different person. Independent of the hierarchy, the decisions influence the expected project output and both agents' private costs, which reflect their preferences. The agents have different cost functions, but both agents' preferences are not aligned with the principal's goal of maximizing the expected project output. The further the implemented decisions are away from an agent's favorite decisions, the larger are his costs. The costs are separable with respect to the decisions, any interaction among the subprojects is covered by the expected project output. We assume a binary output which is zero in case of failure and positive in case of success. The project output is verifiable so that payments can condition on it. To align the decision-makers' interests with the objective of maximizing expected project output, the principal has to provide incentives.

If transfers are unrestricted, the hierarchy has no impact. The principal can always extract the whole surplus so that she implements the surplus-maximizing decisions, first best efficiency is attained. Under limited liability, the principal faces a trade off between surplus maximization and rent extraction. To compensate the decision-makers for a surplus-maximizing decision behavior forces the principal to give up a share of the surplus. This trade off is present under any hierarchy, but the hierarchy impacts how it is solved. In a vertical hierarchy, there is only one decision-maker who has to be incentivized. This advantage might be outweighed if the agents' preferences are different so that one of the agents has larger marginal costs for subproject 1 than for subproject 2 while for the other agent, it is the other way around.

In this case, switching from the vertical to the horizontal hierarchy reduces the expected wage payment of the former superior but increases the expected wage payment of the former subordinate. This pays out if, for every agent, his marginal costs for the two subprojects are sufficiently different. That might be viewed as a result of the decisions being very dissimilar. Even in the optimal hierarchy, the principal does not necessarily implement a first best efficient project since she may be better off with a larger share of a smaller surplus.

The cost functions cover (dis)similarities between the agents, but also between the decisions. If the agents have identical preferences with respect to a subproject, their marginal costs for this subproject are the same. If, on the other hand, the subprojects are very similar in the sense that their impact on the agents is identical, an agent's marginal costs are the same for the two subprojects. For example, consider the assembly-line workers in a car factory. They are all strongly concerned about a decision that affects their task but hardly concerned about the design of the cars. The agents are similar, but the decisions dissimilar. All agents have large marginal costs for the first decision and small marginal costs for the second decision. Differently, in an advertising agency, the graphic artist might suffer a lot from a decision for a dull layout while the ad writer is hardly affected, but the ad writer may be annoyed at a decision concerning the text which hardly affects the artist. In this example, the decisions as well as the agents are very dissimilar. To interpret our result, we can say that a horizontal hierarchy is optimal if the agents as well as the decisions are diverse.

In our model, we assume the decisions to be noncontractible so that the contracts remain incomplete. Instead of specifying the decisions themselves, the principal can contractually assign the right to make (one or both of) them, that is, she can allocate authority. Such a concept is also used in, for example, Grossman and Hart (1986, Aghion and Bolton (1992, Bester (2005) and Schmitz (2005). To enforce a certain allocation of authority, the principal can use, for example, asset ownership so that a transfer of decision

rights is in fact a transfer of property rights. This property rights approach is based on Grossman and Hart (1986) and usually assumes decisions which are not describable at the contracting stage but verifiable at the bargaining stage.¹ Different from that, we assume our decisions to remain unverifiable ex post and therefore follow Aghion and Bolton (1992, Schmitz (2005) and Bester (2005). Aghion and Rey (2002) show that the optimality of this approach is often robust to the introduction of message games, which justifies our modeling choice. In our model, there is no information transmission from the subordinate to the superior. Different from Aghion and Tirole (1997), our superior cannot rubberstamp his subordinate. The subordinate in our model simply does not choose any action.

The agents' costs from undertaking the project might represent disutility from work, but it is rather the agents' taste which determines the costs. We do not model effort² choices or task³ assignment since the agents' costs are determined by all decisions no matter who has made them. Therefore, we differ from Schmitz (2005) who uses effort decisions in a model that describes the allocation of control rights. Similar to our model, private benefits in Aghion and Bolton (1992) do not depend on the allocation of decision rights. In our model, an agent in charge of a decision exerts an externality on the other agent, which is similar to Bester (2005). He models cost complementarities or substitutabilities which affect the efficiency of decisions, but since he does not allow for incentive payments, they do not influence the agents' decision behavior. In difference, the cost functions in our model are assumed to be separable with respect to the two decisions. An agent's costs with respect to one decision are independent of his costs with respect to the other decision. The interaction between the two decisions is covered by the overall project's success probability. Combined with incentive payments, complementarity

¹In Rajan and Zingales (2001), the hierarchy affects the agent's bargaining power, while bargaining does not occur at all in our model.

²Aghion and Tirole (1997) introduce the effort choice as a decision that influences the decision-maker's costs only.

³In task assignment models like Holmstrom and Milgrom (1991), the agent incurs costs from the task he is in charge of but not from any other task.

resp. substitutability does have an impact on the agents' decision behavior in our model.

The agents' decision behavior is subject to moral hazard (hidden action problem). The principal needs to provide incentives to overcome their preferences, that is, to induce them to make decisions different from their favorite ones. Both agents' liability is limited.⁴ While Aghion and Bolton (1992) and Aghion and Rey (2002) investigate parties with different wealth, we follow Schmitz (2005) and assume both agents to be protected by completely limited liability. The agents are wealth constrained or ex post payments are not enforceable so that an agent could break up the contract and walk away instead of paying.

While Schmitz (2005) compares integration (comparable to a vertical hierarchy) and separation (comparable to a horizontal hierarchy) in case of sequential actions, our decisions are made simultaneously. In Bester (2005) and Dessein, Garicano, and Gertner (2007), asymmetric information affects the allocation of authority. Similarly, Dessein (2002) considers an agent who has private information not available to the principal. While these models allow for communication or information aggregation, Athey and Roberts (2001) consider asymmetric information without communication. They focus on the linkage between the allocation of decision rights and the design of incentive schemes. In their model, incentivizing an agent to exert effort also influences his investment decision and vice versa. We differ from these models and assume information to be completely symmetric. Further, we take the information as given independent of the hierarchical structure.⁵

In a vertical hierarchy, a multitask problem⁶ might occur. A decision-maker undertaking several noncontractible decisions optimizes the signal payments base upon. If this is not perfectly aligned with the principal's

⁴Sappington (1983) introduced this moral hazard problem for a single agent.

⁵Different from us, Qian, Roland, and Xu (2006) and Maskin, Qian, and Xu (2000) compare several organizational forms that affect the informational structure.

⁶For multitask problems, see Holmstrom and Milgrom (1991).

objective, the resulting choice is not optimal. In a horizontal hierarchy, each agent is in charge of one decision only so that the principal can provide incentives that target a specific decision directly instead of effecting the overall project only. Nevertheless, this is no comparative advantage of the horizontal hierarchy in our model since in all relevant cases, the multitask problem is no issue even in the vertical hierarchy.

A team problem⁷ can arise in a horizontal hierarchy. Even if the project output reveals that someone made a decision different from a stipulated choice, the principal does not know who has deviated. A kind of free riding is possible, making the moral hazard problem more severe compared to the vertical hierarchy. This moral hazard in teams⁸ can be solved by breaking the budget balance condition so that each agent's marginal reward equals his marginal costs for the efficient decision. Such a payment scheme is feasible in a horizontal hierarchy so that any project can be implemented, but limited liability prevents the principal from extracting the whole surplus.

Several trade offs between different hierarchical structures have been analyzed in the literature. Hart and Moore (2005) study the impact of gains to coordination on the optimal hierarchy. Dessein, Garicano, and Gertner (2007) deal with the potential synergies from coordination in a centralized structure, which in turn might weaken incentives. In Corts (2005), solving the multitask problem conflicts with the efficient allocation of risk. In our model, all parties are risk neutral so that this is not an issue. We also do not have any cost savings due to synergies. The impact of the hierarchical structure in our model is to change the incentive payments the principal has to provide. The trade off between the hierarchies in our model is between the number of decision-makers to be incentivized and the size of the payments needed to incentivize each decision-maker.

⁷For team problems, see Alchian and Demsetz (1972).

⁸For moral hazard in teams, see Holmstrom (1982).

The chapter is structured as follows: Section 4.2 provides a simple model of the allocation of authority among diverse agents. In section 4.3, vertical hierarchies are analyzed, while in section 4.4, we investigate horizontal hierarchies. Section 4.5 endogenizes the design of hierarchies, that is, the allocation of authority, in order to find the optimal ones. In section 4.6, we give a brief summary along with the conclusions and open research questions. Several proofs are deferred to section 4.7.

4.2 The Model

This section describes a simple formal model of the allocation of authority among different agents. The timing is as follows: The principal offers a contract to the agents. The contract specifies a payment scheme and allocates the decision rights to the agents. The agents accept if their participation constraints are fulfilled. The decision-maker(s) choose(s) the subprojects' characteristics after the contract is signed. If each agent is in charge of one decision (instead of one agent being in charge of both decisions), the agents make their choices simultaneously. The project is undertaken, private costs occur and the project output is realized. The payment scheme is executed. The details are given in what follows.

A principal P hires two agents $i = 1, 2$ to jointly undertake a project. For a given agent $i \in \{1, 2\}$, we denote the other agent with i^- . The project is characterized by the two parameters d_1, d_2 with $d_j \in \mathcal{D}_j \subseteq \mathbb{R}$ so that the set of feasible projects is $\mathcal{D}_1 \times \mathcal{D}_2$. One can view a project to consist of two subprojects $j = 1, 2$. Again, for a given subproject $j \in \{1, 2\}$, we denote with $j^- \in \{1, 2\}$ the subproject different from j . Throughout the chapter, we shortly speak of subproject d_j instead of subproject j with characteristic d_j . To keep the analysis tractable, we assume binary decisions so that $\mathcal{D}_j = \{a_j, b_j\}$. The subprojects d_1, d_2 are noncontractible, but the principal can contractually allocate the rights to choose them to the agents. An agent who has the decision right over a subproject's characteristic is said to have authority over this subproject. We denote the allocation of authority with

$\delta = (\delta_1, \delta_2)$ where $\delta_j = i$ means that agent i gets the decision right over subproject j . If one agent receives authority over both subprojects, he is viewed as the other agent's superior. We speak of a vertical hierarchy. If each agent receives authority over one subproject, this is called a horizontal hierarchy.

Project output is random. In case of success, the principal receives an exogenously given output $X > 0$. In case of failure, no output is generated. The success probability depends on the subprojects chosen and is denoted as $p(d_1, d_2)$. We assume

$$p(a_1, a_2) < p(a_1, b_2) = p(b_1, a_2) < p(b_1, b_2) < 1 \quad (4.1)$$

and define

$$p_{bb} := p(b_1, b_2), \quad p_{ab} := p(a_1, b_2) = p(b_1, a_2), \quad p_{aa} := p(a_1, a_2) \quad . \quad (4.2)$$

Further, we assume⁹

$$p_{aa} = 0 \quad . \quad (4.3)$$

The principal cares about the project's success only and therefore always prefers b_j over a_j . The success probability also incorporates the interaction between the subprojects. If a switch from a_j to b_j increases the marginal returns of a switch from a_{j-} to b_{j-} so that $p_{bb} - p_{ab} \geq p_{ab} - p_{aa}$ and therefore

$$p_{ab} \leq \frac{p_{bb}}{2} \quad , \quad (4.4)$$

the subprojects are complements. If $p_{ab} \geq p_{bb}/2$, they are substitutes. Given p_{bb} and p_{aa} , a large p_{ab} is interpreted as a high degree of substitutability resp. a low degree of complementarity between the subprojects.

The agents do not care about the project output but favor certain projects. These preferences are reflected by the private costs the agents incur. Each agents' costs depend on both subprojects. Independent of who has chosen

⁹The case $p_{aa} > 0$ is briefly discussed in the conclusion.

the subproject's characteristic, the costs increase in the distance between the implemented characteristic and the agent's favorite one. We assume the agents' preferences not to be aligned with the principle's objective so that the agents always prefer a_j over b_j . The cost functions are

$$\begin{aligned} c_1(d_1, d_2) &= l_{11}\ell_1 + l_{12}\ell_2 \quad , \\ c_2(d_1, d_2) &= l_{21}\ell_1 + l_{22}\ell_2 \end{aligned} \quad (4.5)$$

with $l_{ij} > 0$ ¹⁰, $\ell_j = 1$ if $d_j = b_j$ and $\ell_j = 0$ if $d_j = a_j$. If $d_j = b_j$, agent i incurs a loss of l_{ij} . We speak of l_{ij} as agent i 's marginal costs on subproject j . With respect to the costs, there is no interaction among subprojects. An agent's costs on one subproject are independent of his costs on the other subproject. We say that agent i *mainly cares* or is *more concerned* about subproject j if $l_{ij} \geq l_{ij^-}$. There are two main cases to distinguish. If both agents have larger marginal costs on subproject j compared to the other subproject so that $l_{1j} \geq l_{1j^-}$ but $l_{2j} \geq l_{2j^-}$, we say the agents *mainly care about the same subproject*. If one agent is more concerned about subproject j while the other agent is more concerned about j^- so that $l_{1j} \geq l_{1j^-}$ but $l_{2j} < l_{2j^-}$, we say that the agents *mainly care about different subprojects*. Further, the ratio $l_i := \min\{l_{i1}, l_{i2}\} / \max\{l_{i1}, l_{i2}\}$ turns out to play a decisive role. If l_i is close to one, we say agent i has *similar marginal costs on the subprojects*. Both decisions have a similar effect on agent i . If this is true for both agents, we can interpret it as a similarity of decisions. If l_i is small instead, we say agent i has *different marginal costs on the subproject*. If, for example, $l_{11} = l_{21} \approx 0$ and $l_{12} = l_{22} > 0$, the agents' interests are perfectly aligned. Not only that they both always prefer a_j over b_j , they do not really care about d_1 but do care about d_2 to the same extent.

Payments condition on project output. The principal pays agent i a basic wage v_i independent of the project outcome and a success premium w_i paid in case of success only.¹¹ We assume that the agents are protected by limited

¹⁰We exclude the case $l_{ij} = 0$ to avoid a situation in which agent i , if not receiving any incentives, is indifferent between $d_j = a_j$ and $d_j = b_j$.

¹¹This is equivalent to paying a wage w_h in case of success and w_l in case of failure.

liability so that all payments have to be nonnegative.

The principal and the agents are assumed to be risk neutral so that their payoff functions are composed of the expected output, the expected payments and the private costs. We have the payoffs

$$\begin{aligned} U_P &= p(d_1, d_2) (X - w_1 - w_2) - v_1 - v_2 \quad , \\ U_1 &= p(d_1, d_2)w_1 + v_1 - c_1(d_1, d_2) \quad , \\ U_2 &= p(d_1, d_2)w_2 + v_2 - c_2(d_1, d_2) \quad . \end{aligned} \tag{4.6}$$

The agents' outside options are set to zero. The principal offers a contract which is accepted by the agents if and only if their participation constraints $U_1, U_2 \geq 0$ are fulfilled. A contract consists of a payment scheme W and an allocation of authority (δ_1, δ_2) . In case of a vertical hierarchy, we have $\delta_1 = \delta_2$ and the decision-maker opportunistically chooses a project (d_1^*, d_2^*) which fulfills his incentive constraint

$$(d_1^*, d_2^*) \in \operatorname{argmax}_{(d_1, d_2)} U_{\delta_1}(d_1, d_2) \quad . \tag{4.7}$$

In a horizontal hierarchy, we have $\delta_1 \neq \delta_2$ and the agents choose the sub-projects simultaneously. This is a noncooperative game and we assume the agents to play a Nash equilibrium. The equilibrium conditions are

$$\begin{aligned} d_1^* &\in \operatorname{argmax}_{d_1} U_{\delta_1}(d_1, d_2^*) \quad , \\ d_2^* &\in \operatorname{argmax}_{d_2} U_{\delta_2}(d_1^*, d_2) \quad . \end{aligned} \tag{4.8}$$

For notational simplicity, we widely omit the asterisk. In case of multiple equilibria,¹² the principal decides which equilibrium is played.¹³ We do not consider mixed equilibria. The principal offers a contract which maximizes her own payoff subject to the agents' participation constraints, the limited

More advanced payment schemes turn out not to improve the results.

¹²To ensure the uniqueness of the Nash equilibrium, it might be necessary to discriminate between the agents, as shown in Winter (2004).

¹³Think of the principal announcing her favorite equilibrium and the agents following her recommendation since they cannot gain from deviating unilaterally.

liability constraints and the decision-maker's incentive constraint resp. the equilibrium conditions. Such a contract is called optimal. Overall expected surplus is

$$S(d_1, d_2) = p(d_1, d_2)X - c_1(d_1, d_2) - c_2(d_1, d_2) \quad . \quad (4.9)$$

A project (d_1, d_2) which maximizes the surplus is called first best efficient.

4.3 Vertical Hierarchy

This section analyzes the vertical hierarchy in order to later on compare it with the horizontal hierarchy. The following lemma states the principal's payoff dependent on the project she implements, taking as given that agent i is the decision-maker in a vertical hierarchy.

Lemma 4.1 *Let agent i be the decision-maker so that agent i^- is his subordinate. A principal who implements (a_1, a_2) can extract the whole surplus and receives a payoff $U_P = 0$. She can implement (a_1, b_2) if and only if*

$$l_{i1} \geq l_{i2} \quad \text{and} \quad \frac{p_{ab}}{p_{bb} - p_{ab}} l_{i1} - l_{i2} \geq 0 \quad . \quad (4.10)$$

Again, she extracts the whole surplus and her resulting payoff is

$$U_P = p_{ab}X - l_{i2} - l_{i-2} \quad . \quad (4.11)$$

She can implement (b_1, a_2) if and only if

$$l_{i2} \geq l_{i1} \quad \text{and} \quad \frac{p_{ab}}{p_{bb} - p_{ab}} l_{i2} - l_{i1} \geq 0 \quad . \quad (4.12)$$

She extracts the whole surplus and her resulting payoff is

$$U_P = p_{ab}X - l_{i1} - l_{i-1} \quad . \quad (4.13)$$

If the principal implements (b_1, b_2) , she receives a payoff

$$U_P = p_{bb}X - l_{i-1} - l_{i-2} - \max \left\{ \frac{p_{bb} \max\{l_{i1}, l_{i2}\}}{p_{bb} - p_{ab}}, l_{i1} + l_{i2} \right\} \quad (4.14)$$

and she extracts the whole surplus if and only if

$$\frac{p_{ab}}{p_{bb} - p_{ab}} \max\{l_{i1}, l_{i2}\} - \min\{l_{i1}, l_{i2}\} \leq 0 \quad . \quad (4.15)$$

If a project is first best efficient, there is always a vertical hierarchy that allows the principal to implement this project.

Proof: See section 4.7.

To implement a project different from the decision-maker's favorite (a_1, a_2) , the principal has to provide incentives. If the subprojects are highly complementary so that (4.10) and (4.12) are not fulfilled, any success premium that incentivizes the decision-maker to choose b_j instead of a_j for one subproject also induces him to choose b_{j-} instead of a_{j-} for the other subproject. It is impossible to implement (a_1, b_2) or (b_1, a_2) , but a principal who implements (b_1, b_2) can extract the whole surplus. Now consider the case where (4.10) or (4.12) is fulfilled, that is, the subprojects are sufficiently substitutable and the decision-maker's marginal costs l_{i1}, l_{i2} are sufficiently different so that he cares much more about one of the subprojects than about the other one. If the principal implements (b_1, b_2) , she has to share the surplus with the decision-maker. The left hand side in (4.15) reflects the share of the surplus the principal has to give up. If this is large, we say that the decision-maker is *hard to incentivize*. This is the case if there is a high degree of substitutability and the decision-maker's marginal costs differ a lot for the two subprojects so that $l_i = \min\{l_{i1}, l_{i2}\} / \max\{l_{i1}, l_{i2}\}$ is small.

The principal can incentivize the decision-maker to choose (a_1, b_2) or (b_1, a_2) , but she cannot influence which of these is chosen so that a multitask problem occurs. The principal cannot provide incentives which target a specific subproject. The following lemma endogenizes the allocation of

authority, that is, it analyzes who is the optimal superior.

Lemma 4.2 *Take as given that the principal implements project (d_1, d_2) . If $(d_1, d_2) \neq (b_1, b_2)$, every allocation of authority that allows the principal to implement (d_1, d_2) is optimal. If $(d_1, d_2) = (b_1, b_2)$, the optimal vertical hierarchy allocates authority to agent i if and only if*

$$\max\left\{0, \frac{p_{ab}}{p_{bb}-p_{ab}} \max\{l_{i1}, l_{i2}\} - \min\{l_{i1}, l_{i2}\}\right\} \leq \frac{p_{ab}}{p_{bb}-p_{ab}} \max\{l_{i-1}, l_{i-2}\} - \min\{l_{i-1}, l_{i-2}\} \quad . \quad (4.16)$$

Proof: The results follow directly from Lemma 4.1. ■

If the left hand side in (4.16) is nonpositive, the principal can implement (b_1, b_2) and extract the whole surplus when she allocates authority to agent i . This allocation is clearly optimal. If the left hand side is nonpositive for both $i = 1, 2$, the optimal allocation is not unique. If the left hand side is positive for both $i = 1, 2$, both allocations of authority do not allow the principal to extract the whole surplus. The right hand side is also positive and (4.16) reflects the share of the surplus the principal has to give up under either allocation, which determines the optimal allocation. Since the principal cannot extract the whole surplus, she faces a trade off between surplus maximization and rent extraction due to limited liability.¹⁴ The following lemma gives the conditions for first best efficiency (not) being reached.

Lemma 4.3 *The principal does not implement a first best efficient project if and only if the following conditions hold at the same time:*

1. *The unique first best efficient project is (b_1, b_2) .*
2. *The subprojects are sufficiently substitutable or each agents' marginal costs differ sufficiently among subprojects¹⁵ so that*

$$\frac{p_{ab}}{p_{bb} - p_{ab}} \max\{l_{i1}, l_{i2}\} - \min\{l_{i1}, l_{i2}\} \quad (4.17)$$

¹⁴Under unlimited liability, the principal could always extract the whole surplus by choosing, if necessary, a negative v_i .

¹⁵That is, each agent cares sufficiently more about one subproject than the other subproject so that l_1, l_2 are large, but the agents might mainly care about different subprojects.

is large enough for $i = 1, 2$.¹⁶

3. For at least one subproject, the marginal costs are large enough so that $l_{11} + l_{21}$ or $l_{12} + l_{22}$ is large enough.

Proof: See section 4.7.

If a project different from (b_1, b_2) is first best efficient, the principal can implement it and extract the whole surplus. Condition 2 ensures that each agent, if in charge, is so hard to incentivize that a principal who implements (b_1, b_2) has to give up a large share of the surplus. The third condition ensures that choosing the most profiting project different from (b_1, b_2) does not decrease the surplus too much. The principal is better off by choosing this project which leaves her with a (much) larger share¹⁷ of a (slightly) smaller surplus. Note that condition 1 on the one hand and condition 2 and 3 on the other hand might conflict. For example, increasing p_{ab} in order to fulfill condition 2 might in turn lead to a violation of condition 1.

Further, the multitask problem turns out to be irrelevant. A principal desires to implement (b_1, a_2) (resp. (a_1, b_2)) if this project is first best efficient or Lemma 4.3 applies. As can be seen from the proof of the lemma, condition 2 implies that (4.10) or (4.12) is fulfilled. In either case, the principal can implement the project. To provide a better intuition for our results, we consider some examples.

Example 1 Assume that agent 1 mainly cares about subproject d_1 but hardly cares about subproject d_2 , while for agent 2 it is the other way around. Mathematically, this is $l_{12}, l_{21} \rightarrow 0$ and without loss of generality we set $l_{22} > l_{11}$. The principal can implement (a_1, a_2) no matter who receives authority. To implement (a_1, b_2) , she has to allocate authority to agent 1 and for (b_1, a_2) to agent 2. In these three cases she extracts the whole surplus. If the principal

¹⁶In condition 2 and 3, we do not explicitly mention the critical values because they do not provide further insights. Further note that *large enough* does not necessarily mean large in absolute terms.

¹⁷In fact, this share is 100% here.

implements (b_1, b_2) , she has to share the surplus with the decision-maker. According to (4.16), she allocates authority to agent 1. The principal does not implement a first best efficient project if and only if

$$\begin{aligned} \max \left\{ \frac{l_{11} + l_{22}}{p_{bb}}, \frac{l_{22}}{p_{bb} - p_{ab}} \right\} < X < \\ \max \left\{ \frac{l_{22}}{p_{bb} - p_{ab}} + \frac{p_{ab}l_{11}}{[p_{bb} - p_{ab}]^2}, \frac{l_{22}}{p_{bb}} + \frac{l_{11}}{p_{bb} - p_{ab}} \right\} . \end{aligned} \quad (4.18)$$

The left hand side of the inequality ensures that (b_1, b_2) is the unique first best efficient project, while the right hand side ensures that the principal prefers to implement a different project. To illustrate the conditions 2 and 3 of Lemma 4.3, note that the right hand side is increasing in $l_{11} = l_{11} + l_{21}$, $l_{22} = l_{21} + l_{22}$ and $l_{11}p_{ab}/[p_{bb} - p_{ab}] = l_{11}p_{ab}/[p_{bb} - p_{ab}] - l_{12}$, while a large $l_{11}p_{ab}/[p_{bb} - p_{ab}]$ implies a large $l_{22}p_{ab}/[p_{bb} - p_{ab}]$.

Example 2 Now assume both agents to be concerned about subproject d_1 but disinterested in subproject d_2 . Mathematically, we assume $l_{12}, l_{22} \rightarrow 0$ and without loss of generality $l_{11} < l_{21}$. It is impossible to implement (b_1, a_2) in a vertical hierarchy, and (b_1, a_2) cannot be first best efficient. Both allocations of authority allow the principal to implement (a_1, b_2) or (a_1, a_2) and to extract the whole surplus. If the principal implements (b_1, b_2) , she has to share the surplus with the decision-maker. She allocates authority to agent 1. The principal does not implement a first best efficient project if and only if

$$\frac{l_{11} + l_{21}}{p_{bb} - p_{ab}} < X < \frac{p_{bb}l_{11}}{[p_{bb} - p_{ab}]^2} + \frac{l_{21}}{p_{bb} - p_{ab}} . \quad (4.19)$$

Examples 1 and 2 have in common that, even under the optimal vertical hierarchy, the decision-maker cares much more about one subproject than about the other one. Substitutability has no impact on the optimal allocation of authority.

Example 3 Assume $l_{11} = l_{21} = l_{12} = l_{22} =: l$. Both vertical hierarchies generate the same results. If the subprojects are complements, it is impossible

to implement (b_1, a_2) or (a_1, b_2) . Given the complementarity, the decision-maker's marginal costs for the two subprojects are sufficiently similar so that the principal can implement (b_1, b_2) and extract the whole surplus. First best efficiency is always reached. If the subprojects are substitutes, the principal can implement (a_1, b_2) or (b_1, a_2) . But for the substitutability present, the decision-maker's marginal costs for the subprojects are sufficiently different¹⁸ to force a principal who implements (b_1, b_2) to share the surplus with the decision-maker. If the subprojects are substitutes, the principal does not implement a first best efficient project if and only if

$$\frac{2l}{p_{bb} - p_{ab}} < X < \frac{p_{bb}l}{[p_{bb} - p_{ab}]^2} \quad . \quad (4.20)$$

4.4 Horizontal Hierarchy

This section analyzes the horizontal hierarchy. The following lemma describes the principal's payoff dependent on the implemented project, taking the allocation of authority as given.

Lemma 4.4 *Take the horizontal hierarchy (δ_1, δ_2) with $\delta_1 \neq \delta_2$ as given so that agent δ_j is in charge of subproject j . A principal who implements (a_1, a_2) can extract the whole surplus and receives a payoff $U_P = 0$. If she implements (b_1, a_2) , her payoff is*

$$U_P = p_{ab}X - l_{11} - l_{21} \quad (4.21)$$

and she extracts the whole surplus. For (a_1, b_2) , she receives

$$U_P = p_{ab}X - l_{12} - l_{22} \quad (4.22)$$

and again she extracts the whole surplus. A principal who implements (b_1, b_2)

¹⁸Again, sufficiently different does not mean different in absolute terms. In this case, the marginal costs are in fact identical, but for the given substitutability, the condition is fulfilled.

gets the payoff

$$\begin{aligned} U_P = p_{bb}X & - \max \left\{ \frac{p_{bb}l_{\delta_1 1}}{p_{bb} - p_{ab}}, l_{11} + l_{12} \right\} \\ & - \max \left\{ \frac{p_{bb}l_{\delta_2 2}}{p_{bb} - p_{ab}}, l_{21} + l_{22} \right\} . \end{aligned} \quad (4.23)$$

She extracts the whole surplus if and only if

$$\frac{p_{ab}}{p_{bb} - p_{ab}} l_{\delta_1 1} - l_{\delta_1 2} \leq 0 \quad \text{and} \quad (4.24)$$

$$\frac{p_{ab}}{p_{bb} - p_{ab}} l_{\delta_2 2} - l_{\delta_2 1} \leq 0 \quad . \quad (4.25)$$

Proof: See section 4.7.

Different from the vertical hierarchy, the principal can implement any project (d_1, d_2) in both horizontal hierarchies. She can use a success premium to target directly the decision behavior on a certain subproject. We do not have a multitask problem in a horizontal hierarchy.

Again, a principal who implements (b_1, b_2) might have to give up a share of the surplus. The larger the left hand side in (4.24) or (4.25), the harder it is to incentivize the respective agent and the larger is the share of the surplus this agent receives. If agent i is in charge of the subproject he cares more about, he is hard to incentivize if

$$\frac{p_{ab}}{p_{bb} - p_{ab}} \max\{l_{i1}, l_{i2}\} - \min\{l_{i1}, l_{i2}\} \quad (4.26)$$

is large, that is, if his marginal costs differ a lot among subprojects so that l_i is small. If agent i is in charge of the subproject he cares less about, he is hard to incentivize if

$$\frac{p_{ab}}{p_{bb} - p_{ab}} \min\{l_{i1}, l_{i2}\} - \max\{l_{i1}, l_{i2}\} \quad (4.27)$$

is large, that is, if his marginal costs differ little among the subprojects so that l_i is large. In general, the larger the marginal costs for the subproject he is in charge of and the smaller the marginal costs for the other subproject,

the harder it is to incentivize the agent. Therefore, it is easier to incentivize an agent who is in charge of the subproject he cares less about as if he were in charge of the subproject he cares more about. The following lemma endogenizes the allocation of authority.

Lemma 4.5 *If the principal implements (a_1, a_2) , (a_1, b_2) or (b_1, a_2) , the optimal horizontal hierarchy is not unique. If the principal implements (b_1, b_2) , the allocation $(\delta_1 = 1, \delta_2 = 2)$ is an optimal horizontal hierarchy if and only if*

$$\begin{aligned} & \max \left\{ \frac{p_{bb}l_{11}}{p_{bb} - p_{ab}}, l_{11} + l_{12} \right\} + \max \left\{ \frac{p_{bb}l_{22}}{p_{bb} - p_{ab}}, l_{21} + l_{22} \right\} \leq \\ & \max \left\{ \frac{p_{bb}l_{12}}{p_{bb} - p_{ab}}, l_{11} + l_{12} \right\} + \max \left\{ \frac{p_{bb}l_{21}}{p_{bb} - p_{ab}}, l_{21} + l_{22} \right\}. \end{aligned} \quad (4.28)$$

The allocation $(\delta_1 = 2, \delta_2 = 1)$ is optimal if and only if \geq holds in (4.28).

Proof: The proof is a direct comparison of the principal's payoffs under both allocations. ■

If the principal has to share the surplus under both allocations, she prefers an allocation that makes it easy to incentivize the agents with respect to the subprojects they are in charge of. There are two general cases. If the agents mainly care about different subprojects, the principal optimally puts each agent in charge of the project he cares less about. Both agents are easier to incentivize under this allocation compared to the alternative allocation. If both agents mainly care about the same subproject, say d_1 , and the principal implements $(\delta_1 = 1, \delta_2 = 2)$, agent 1 is harder and agent 2 is easier to incentivize compared to the alternative horizontal hierarchy. The optimal allocation depends on the details of the parameter constellation. One agent ends up in charge of the subproject he cares more about, while the other agent gets authority over the subproject he is less interested in. Again, a principal who cannot extract the whole surplus might face a trade off between surplus maximization and rent extraction due to limited liability. The following lemma gives the conditions for first best efficiency (not) being reached.

Lemma 4.6 *Take as given that the principal has to allocate authority according to a horizontal hierarchy. The principal does not implement a first best efficient project if and only if the following conditions hold at the same time:*

1. *The unique first best efficient project is (b_1, b_2) .*
2. *The subprojects are sufficiently substitutable or*
 - *both agents mainly care about different subprojects and l_1, l_2 are large so that their marginal costs differ little among subprojects, which ensures that (4.27) is large enough for both agents, or*
 - *the agents mainly care about the same subproject and l_1, l_2 are similar so that both agents' marginal costs show a similar amount of variation among subprojects, which ensures that (4.26) is large enough for both agents or (4.27) is large enough for both agents*

so that the expressions on both sides in (4.28) are large enough.
3. *For at least one subproject, the marginal costs are large enough so that $l_{11} + l_{21}$ or $l_{12} + l_{22}$ is large enough.*

Proof: The proof is analog to Lemma 4.3. ■

The first two conditions guarantee that, under any horizontal allocation, a principal who implements a first best efficient project has to give up a large share of the surplus, that is, there is at least one agent who is significantly hard to incentivize. If the agents mainly care about different subprojects, this is the case if their marginal costs do not differ much among subprojects. If both agents mainly care about the same subproject, the variation of marginal costs among subprojects has to be similar for both agents. If, for example, both agents' marginal costs differ a lot among subprojects, the one in charge of the subproject they care more about is hard to incentivize under both allocation. If both agents' marginal costs vary little among subprojects, the one in charge of the subproject they care less about is hard to incentivize under both allocations. On the other hand, if agent 1's marginal costs differ

much more among subprojects than agent 2's marginal costs do, the principal can put agent 1 in charge of the subproject they care less about so that both agents are easy to incentivize. Condition 3 ensures that there is a project which is not first best efficient but generates a surplus not much smaller than the first best surplus. We continue the examples from the previous section.

Example 1 (cont.) Let $l_{12}, l_{21} \rightarrow 0$ and $l_{22} > l_{11}$ so that the agents mainly care about different subprojects. If the principal implements (b_1, b_2) , she optimally chooses $(\delta_1 = 2, \delta_2 = 1)$ so that each agent is in charge of the subproject for which he has (approximately) no marginal costs. Independent of the substitutability, each agent's marginal costs differ so much among subprojects (that is, l_1, l_2 are so small) that the principal can extract the whole surplus and a first best efficient project is always implemented.

Example 2 (cont.) Let $l_{12}, l_{22} \rightarrow 0$ and $l_{11} < l_{21}$ so that the agents mainly care about the same subproject. A principal who implements (b_1, b_2) optimally sets $(\delta_1 = 1, \delta_2 = 2)$. With respect to subproject 2, no incentives are needed no matter who is in charge of it. With respect to subproject 1, agent 1 is at least as easy to incentivize as agent 2 since (4.26) is smaller for him. The remaining results are the same as if he were the decision-maker in a vertical hierarchy.

Example 3 (cont.) Let $l_{11} = l_{21} = l_{12} = l_{22} =: l$. Both horizontal allocations generate the same result. If the principal implements (b_1, b_2) , she can extract the whole surplus if and only if the subprojects are complements. If the subprojects are substitutes instead, the principal has to share the surplus with the agents. She implements a project different from first best if and only if

$$\frac{2l}{p_{bb} - p_{ab}} < X < \frac{p_{bb} 2l}{[p_{bb} - p_{ab}]^2} - \frac{2l}{p_{bb} - p_{ab}} . \quad (4.29)$$

4.5 The Optimal Hierarchy

In this section, we completely endogenize the design of the hierarchy. If the principal implements a given project (d_1, d_2) , every allocation that allows the principal to extract the whole surplus is optimal. Therefore, a principal who implements (a_1, a_2) , (b_1, a_2) or (a_1, b_2) is indifferent between a vertical and a horizontal hierarchy if and only if both hierarchies enable her to implement the desired project. If it is impossible to implement the project in a vertical hierarchy, the principal necessarily chooses a horizontal one. The following proposition analyzes the optimal hierarchy to implement (b_1, b_2) .

Proposition 4.1 *Consider a principal who implements (b_1, b_2) . If the agents mainly care about the same subproject, a vertical hierarchy is optimal. If the agents mainly care about different subprojects, a horizontal hierarchy is optimal if and only if, for each agent, the marginal costs are sufficiently different among the two subprojects so that (4.26) is large enough and (4.27) is small enough for both agents.*

Proof: See section 4.7.

To interpret this result, we might say that a horizontal hierarchy is optimal if the agents as well as the decisions are dissimilar. If the agents mainly care about the same subproject and the principal chooses a horizontal hierarchy, the agent in charge of the project they care more about receives the same expected wage payment as if he were the decision-maker in a vertical hierarchy. The other agent receives an expected wage at least as high as if he were the subordinate in a vertical hierarchy. If the agents mainly care about different subprojects, the optimal horizontal hierarchy puts each agent in charge of the subproject he is less concerned about. Switching from the vertical to the horizontal hierarchy saves the principal part of the decision-maker's expected wage payment but might increase the other agent's expected wage. If there is an agent who has similar marginal costs for both subprojects, this agent is quite easy to incentivize if he is the decision-maker in a vertical hierarchy. It is only a little bit easier to incentivize him in the optimal horizontal hierarchy, which is outweighed by the fact that the other agent's participation

constraint is binding in the vertical hierarchy. The principal can use a horizontal hierarchy in order to gain from the differences between the agents' costs. Note that there is no monotonicity in the sense that, for example, the horizontal hierarchy is optimal if there is enough substitutability or vice versa. Increasing p_{ab} increases both (4.26) and (4.27). As can be seen from (4.45) in the proof, it depends on the marginal costs if increasing substitutability works in favor of horizontal or vertical hierarchies. To provide a better intuition, we continue our examples.

Example 1 (cont.) Let $l_{12}, l_{21} \rightarrow 0$ and $l_{22} > l_{11}$. If the principal implements (b_1, b_2) , the horizontal hierarchy ($\delta_1 = 2, \delta_2 = 1$) is optimal.

Example 2 (cont.) Let $l_{12}, l_{22} \rightarrow 0$ and $l_{11} < l_{21}$. In the limit, a principal who implements (b_1, b_2) is indifferent between agent 1 being the decision-maker in a vertical hierarchy or implementing a horizontal hierarchy with $(\delta_1 = 1, \delta_2 = 2)$. But as long as l_{22} is positive, the vertical hierarchy is strictly better.

Example 3 (cont.) Let $l_{11} = l_{21} = l_{12} = l_{22} =: l$. If the projects are complements, we have already seen that a principal who implements (b_1, b_2) can extract the whole surplus under any allocation. If the projects are substitutes, a vertical hierarchy is optimal. Each agent has the same marginal costs for each subproject. Agents as well as subprojects are identical.

The following lemma shows that even a principal who can freely choose the allocation of authority still does not necessarily implement a first best efficient project.

Lemma 4.7 *The principal implements a project different from first best if and only if the following conditions hold at the same time:*

1. *The unique first best efficient project is (b_1, b_2) .*
2. *The subprojects are sufficiently substitutable or*

- *the agents mainly care about the same subproject and l_1, l_2 are small so that each agent's marginal costs are sufficiently different for the two subprojects, which ensures that (4.26) is large enough for both agents, or*
 - *the agents mainly care about different subprojects and l_1, l_2 are not too small and not too large so that each agent's marginal costs for the two subprojects are neither too similar nor too different, which ensures that (4.26) and (4.27) are large enough for both agents.*
3. *For at least one subproject, the marginal costs are large enough so that $l_{11} + l_{21}$ or $l_{12} + l_{22}$ is large enough.*

Proof: Condition 1 and 3 follow directly from Lemmata 4.3 and 4.6. Condition 2 follows from Lemmata 4.3, 4.6 and 4.16 as follows: If there is a high degree of substitutability, the agents are hard to incentivize under any allocation. First, assume that the agents mainly care about the same subproject so that a vertical hierarchy is optimal. To ensure that under both vertical allocations the decision-maker is hard to incentivize we need both agents' marginal costs to be very different for the two subprojects so that (4.26) is large for both agents. Now consider the case where the agents mainly care about different subprojects. To ensure that incentives are hard to provide under any allocation, we need (4.26) and (4.27) to be large enough. Keeping (4.27) constant while increasing (4.26) might have two effects: First, in the vertical hierarchy, the principal has to give up a larger share of the surplus. Second, the horizontal hierarchy might become better than the vertical one. But given that (4.27) is large, the principal again has to give up a large share of the surplus. Analog arguments apply for increasing (4.27) while keeping (4.26) constant. ■

In general, the set of parameters that lead to an outcome different from first best is not empty. If the agents mainly care about the same subproject, we have already seen in Example 2 that the principal does not necessarily implement a first best efficient project even if she can freely choose the hierarchy. In the following example, the agents mainly care about different subprojects, but the principal does not always implement a first best efficient project.

Example 4 Let $l_{12} = l_{21} =: l, l_{11} + l_{22} = 2l$ and $p_{ab} = 3/4p_{bb}$. If the principal implements (b_1, b_2) , she cannot extract the whole surplus under any allocation of authority. The optimal hierarchy is $(\delta_1 = 2, \delta_2 = 1)$ so that each agent is in charge of the project he cares less about. This horizontal hierarchy results in an overall expected wage payment of $8l$, while expected wages in a vertical hierarchy were $11l$. The principal does not implement (b_1, b_2) if and only if

$$\frac{12l}{p_{bb}} < X < \frac{20l}{p_{bb}} \quad , \quad (4.30)$$

while (b_1, b_2) is the unique first best efficient project if and only if

$$\frac{8l}{3p_{bb}} < X < \frac{16l}{p_{bb}} \quad (4.31)$$

so that first best efficiency is not reached if and only if

$$\frac{12l}{p_{bb}} < X < \frac{16l}{p_{bb}} \quad (4.32)$$

4.6 Conclusion

This chapter provides a simple model of the hierarchical structure of a firm or organization. The advantage of the vertical hierarchy is that there is only one decision-maker to be incentivized. But the principal might need large incentive payments to overcome the decision-maker's preferences, especially on the decision he is more concerned about. In a horizontal hierarchy, both agents need to be incentivized. If the agents mainly care about different decisions, the principal might gain from these differences by putting each agent in charge of the decision he cares less about. Compared to the vertical hierarchy, there are lower expected wage payments to the superior but higher expected wage payments to the subordinate. This pays out if the agents' marginal costs are very different for the two subprojects, that is, each agent is much more concerned about one subproject than about the other one. The horizontal hierarchy is optimal if agents as well as decisions are dissimilar. Due to limited liability, the principal might face a trade off between sur-

plus maximization and rent extraction. She implements a project different from first best if, even under the optimal hierarchy, it is hard to provide the necessary incentives. This is the case if there is a decision both agents care much more about than the other decision or if the agents mainly care about different decisions, but their marginal costs for the subprojects are neither too different nor too similar.

Our main results hold true for more general success probability functions. The binary character of the subproject's choice and the project output have no substantial impact. Further, if we allow for $p_{aa} > 0$, the principal cannot extract the whole surplus if she implements (b_1, b_2) , (b_1, a_2) or (a_1, b_2) , but the results do not change qualitatively. Now we relax the assumption that the conflict of interest between the principal and the agents is as large as possible and consider the alternative settings. If

$$p_{aa} > \max\{p_{bb}, p_{ab}\} \quad , \quad (4.33)$$

there is no conflict of interest at all and the hierarchy has no impact. Now consider

$$p_{aa} < p_{bb} < p_{ab} \quad . \quad (4.34)$$

In this setting, the project (b_1, b_2) is effectively ruled out. It cannot be first best efficient since it results in higher overall costs and lower success probability than (a_1, b_2) and (b_1, a_2) . It is impossible to implement (b_1, b_2) under any allocation of authority. Analog, in case of $p_{ab} < p_{aa} < p_{bb}$, the projects (b_1, a_2) and (a_1, b_2) cannot be first best efficient and it is impossible to implement them in either hierarchy. Apart from that, the results from the previous sections are mainly replicated.

Changing the agents' favorite subprojects does not provide new insights. Assume we have $p_{aa} < p_{ab} < p_{bb}$ as in the previous sections, but agent 1 now prefers $d_1 = b_1$. The analysis simplifies since the principal does not need to provide incentives with respect to subproject 1.

Further, our main results hold true in a setting with more than two decisions or more than two agents. This enables additional hierarchies so that, for example, it might be optimal to have one agent in charge of two decisions, another agent in charge of one decisions and a third agent in charge of no decision, which is intermediate between horizontal and vertical hierarchies. But the forces identified in our simplified model are still at work in the broader setting.

In our model, a vertical hierarchy incorporates what Schmitz (2005) calls the *rent saving effect* known from task assignment problems. The rent (or the incentive payment) used to induce a certain decision on one subproject also positively effects the decision on the other subproject. But if this rent is too high, the vertical hierarchy is suboptimal. In Schmitz (2005), this situation can occur due the complementarity between the two sequentially undertaken decisions. The decision-maker might shirk in the first stage since failure on the first stage reduces the effect of effort in the second stage, increasing the agent's rent on the second stage. It is so expensive to incentivize the agent to exert high effort on both stages that separated control rights (comparable to a horizontal hierarchy) might be optimal since they reduce the effect of the complementarity. In our model, decisions are made simultaneously and there is only one signal payments base upon so that this effect is ruled out. While the agents in Schmitz (2005) are identical, the potential advantage of the horizontal hierarchy in our model is to gain from differences between the agents' cost functions.

In Bester (2005), complementarity of decisions implies the optimality of centralized decisions, while substitutability might lead to decentralized decisions being more efficient. Since there are no incentive payments in his model, each decision-maker always chooses his favorite project, which is independent of substitutability or complementarity. This is different in our model. Complementarity resp. substitutability influence the payments necessary to induce a certain decision. As we have seen in the previous sections,

substitutability might work in favor of a vertical or horizontal hierarchy, dependent on the agents' costs.

Our approach does not only apply to firms but also to other institutions and organizations, for example, administrations. Possible extensions of our model include the interactions with task assignment or effort incentives in the spirit of Athey and Roberts (2001) or the integration of asymmetric information. These are left for future research.

4.7 Proofs

Proof of Lemma 4.1:

A principal who implements a project (d_1, d_2) optimally chooses $w_{i-} = 0$, $v_{i-} = c_{i-}(d_1, d_2)$, $v_i = \max\{0, -p(d_1, d_2)w_i + c_i(d_1, d_2)\}$ and the smallest $w_i \geq 0$ that fulfills the incentive constraint (4.7). A success premium w_i which enables the principal to implement (a_1, b_2) exists if and only if (4.10) is fulfilled, analog for (b_1, a_2) . To implement (b_1, b_2) , the principal optimally sets

$$w_i = \max \left\{ \frac{\max\{l_{i1}, l_{i2}\}}{p_{bb} - p_{ab}}, \frac{l_{i1} + l_{i2}}{p_{bb}} \right\}, \quad (4.35)$$

$v_i = 0$, $w_{i-} = 0$, $v_{i-} = l_{i-1} + l_{i-2}$ and extracts the whole surplus if and only if

$$\frac{\max\{l_{i1}, l_{i2}\}}{p_{bb} - p_{ab}} \leq \frac{l_{i1} + l_{i2}}{p_{bb}}, \quad (4.36)$$

which is equivalent to (4.15).

It remains to be shown that a first best efficient project can always be implemented. This is obvious for (a_1, a_2) or (b_1, b_2) . Now assume (a_1, b_2) to be first best efficient. We have to show that there is an $i \in \{1, 2\}$ fulfilling (4.10). Assume that this is not the case. If $l_{i1} < l_{i2}$ for $i = 1, 2$, we have the contradiction $S(b_1, a_2) > S(a_1, b_2)$. If $l_{i2} \leq l_{i1} < [p_{bb} - p_{ab}]l_{i2}/p_{ab}$ for $i = 1, 2$,

we have

$$p_{ab}(l_{11} + l_{21}) < [p_{bb} - p_{ab}](l_{12} + l_{22}) \quad (4.37)$$

which is again a contradiction since first best efficiency of (a_1, b_2) requires

$$\frac{l_{12} + l_{22}}{p_{ab}} \leq X \leq \frac{l_{11} + l_{21}}{p_{bb} - p_{ab}} \quad (4.38)$$

If

$$\begin{aligned} l_{i2} \leq l_{i1} < \frac{p_{bb} - p_{ab}}{p_{ab}} l_{i2} \quad \text{and} \\ \frac{p_{bb} - p_{ab}}{p_{ab}} l_{i-2} \leq l_{i-1} < l_{i-2} \end{aligned} \quad (4.39)$$

hold, $p_{bb} - p_{ab} > p_{ab}$ and $p_{bb} - p_{ab} < p_{ab}$ are both implied which is again a contradiction. In any other case, there is an $i \in \{1, 2\}$ for which (4.10) holds. Analog results hold if (b_1, a_2) is first best efficient. ■

Proof of Lemma 4.3:

If there is a first best efficient project different from (b_1, b_2) , the principal can implement it and extract the whole surplus so that condition 1 is necessary. Condition 2 ensures that (4.15) does not hold under both possible allocations so that the principal cannot implement (b_1, b_2) and extract the whole surplus. Further, according to Lemma 4.1, the more profiting of the two projects (b_1, a_2) and (a_1, b_2) can be implemented.

Let $l_{11} \geq l_{12}$ and (4.16) be fulfilled (the proof is analog for any other case). A principal who implements (b_1, b_2) optimally allocates authority to agent 1. She is better off by implementing (a_1, a_2) instead of (b_1, b_2) if and only if her payoff from (b_1, b_2) is negative, that is,

$$p_{bb}X \leq l_{21} + l_{22} + \frac{p_{bb}l_{11}}{p_{bb} - p_{ab}} \quad (4.40)$$

Implementing the more profiting of the two projects (a_1, b_2) and (b_1, a_2) leaves

the principal better off than (b_1, b_2) if and only if

$$[p_{bb} - p_{ab}]X < \frac{p_{bb}}{p_{bb} - p_{ab}}l_{11} + \max\{l_{21} - l_{12}, l_{22} - l_{11}\} \quad . \quad (4.41)$$

Using

$$\frac{p_{bb}l_{11}}{p_{bb} - p_{ab}} = \frac{p_{ab}l_{11}}{p_{bb} - p_{ab}} + l_{11} + l_{12} - l_{12} \quad (4.42)$$

we get that increasing (4.17) while keeping $l_{11} + l_{21}$ and $l_{12} + l_{22}$ constant as well as increasing $l_{11} + l_{21}$ or $l_{12} + l_{22}$ and keeping (4.17) constant increases the right hand sides of (4.40) and (4.41). ■

Proof of Lemma 4.4:

To implement a project (d_1, d_2) , the principal optimally chooses the smallest $w_1, w_2 \geq 0$ that fulfill the equilibrium conditions (4.8) and

$$v_i = \max\{0, -p(d_1, d_2)w_i + c_i(d_1, d_2)\} \quad (4.43)$$

for $i = 1, 2$. Agent i 's participation constraint is binding if and only if $p(d_1, d_2)w_i - c_i(d_1, d_2) \leq 0$. The principal extracts the whole surplus if and only if both agents' participation constraints are binding. ■

Proof of Proposition 4.1:

If both agents mainly care about subproject d_j , the one in charge of subproject d_j in a horizontal hierarchy receives the same expected wage as if he were the decision-maker in a vertical hierarchy. The other agents' participation constraint may or may not bind in the horizontal hierarchy, but it is binding in the vertical one. For each horizontal hierarchy, there is a vertical one which offers the principal at least the same payoff.

Now assume that the agents mainly care about different projects. Without loss of generality, assume $l_{11} \geq l_{12}$ and $l_{22} \geq l_{21}$ so that the principal either implements a vertical hierarchy or puts agent i in charge of subproject

d_i . Assume that, in a vertical hierarchy, agent 1 optimally is the decision-maker. Comparing the principal's payoffs shows that the horizontal hierarchy is optimal if and only if

$$\begin{aligned} & \max \left\{ \frac{p_{bb}l_{11}}{p_{bb}-p_{ab}}, l_{11} + l_{12} \right\} - \max \left\{ \frac{p_{bb}l_{12}}{p_{bb}-p_{ab}}, l_{11} + l_{12} \right\} \\ & \geq -(l_{21} + l_{22}) + \max \left\{ \frac{p_{bb}l_{21}}{p_{bb}-p_{ab}}, l_{21} + l_{22} \right\} \quad , \end{aligned} \quad (4.44)$$

which is equivalent to

$$\begin{aligned} & \max \left\{ 0, \frac{p_{ab}l_{11}}{p_{bb}-p_{ab}} - l_{12} \right\} - \max \left\{ 0, \frac{p_{ab}l_{12}}{p_{bb}-p_{ab}} - l_{11} \right\} \\ & \geq \max \left\{ 0, \frac{p_{ab}l_{21}}{p_{bb}-p_{ab}} - l_{22} \right\} \quad . \end{aligned} \quad (4.45)$$

The right hand side is small if (4.27) is small for agent 2, while the left hand side is large if (4.26) is large and (4.27) is small for agent 1. Further, (4.26) is large for agent 2 since otherwise, it would not be optimal to have agent 1 as the decision-maker in a vertical hierarchy. ■