Summary

In this thesis we discuss the equation of one-dimensional space-time fractional diffusion with drift

$$\underset{t\ *}{D}^{\beta}\, u(x,t) = \underset{x\ 0}{D}^{\alpha}\, u(x,t) - \frac{\partial}{\partial x}(F(x)u(x,t))\,,\; u(x,0) = \delta(x)\ ,$$

where $0 < \beta \leq 1$, $0 < \alpha \leq 2$. Here $\underset{t\ *}{D}^{\beta}$ is the Caputo time-fractional derivative and $\underset{x\ 0}{D}^{\alpha}$ is the spatial Riesz fractional differential operator with symbol $-|\kappa|^{\alpha}$.

In Chapter 3, discrete approximations to time-fractional diffusion processes, ($\alpha = 2$) with drift towards the origin are obtained as explicit and implicit difference schemes and as a random walk models. We have simulated these random walk models and given numerical results for the discrete approximations. Then we discuss the convergence of the discrete solutions to the stationary solutions of the model. Numerical solutions are displayed for central linear drift and for cubic central drift. Furthermore we discuss in detail the relations to the classical Ehrenfest model which is described carefully in Chapter 2.

In Chapter 4, we give a survey of the theory of continuous time random walk. We show how the above space-time-time-fractional diffusion equation, with $F(x) = 0$, can be obtained from the integral equation for a continuous time random walk or from that describing a cumulative renewal process, through well-scaled limits of vanishing waiting times and jumps. We simulate the random walk models for different values of fractional orders α and β. Then we use a transformation of the independent variables x and t to simulate the random walk for space-fractional diffusion with central linear drift (i. e. $F(x) = -x$ and $\beta = 1$). The simulation shows how jumps and waiting times are somehow compressed with respect to the case of no drift. We generalize the transformation theorem to the case of non-symmetric spatial operators.

Finally in Chapter 5, we give mathematical proofs for convergence of discrete solutions of space-time-fractional diffusion without and with central linear drift to the solution of the above equation for integer and fractional values of α and β in the Fourier-Laplace domain. The numerical solutions are obtained by explicit and implicit difference schemes, and the simulations of random walks are made by the Monte-Carlo method.