

## Chapter 10

# Camera calibration for stereo vision

### 10.1 Introduction

The camera calibration problem is to relate the locations of pixels in the image array to points in the scene. Since each pixel is imaged through perspective projection, it corresponds to a ray of points in the scene. The camera calibration problem is to determine the equation for this ray in the absolute coordinate system of the scene. The camera calibration problem includes both the exterior and interior orientation problems, since the position and orientation of the camera and the camera constant must be determined to relate image plane coordinates to absolute coordinates, and the location of the principal point, the aspect ratio, and lens distortions must be determined to relate image array locations (pixels coordinates) to positions in the image plane. The camera calibration problem involves determining two sets of parameters [60]: the extrinsic parameters for rigid body transformation (exterior orientation) and the intrinsic parameters for the camera itself (interior orientation).

We can use an initial approximation for the intrinsic parameters to get a mapping from image array (pixel) coordinates to image plane coordinates. Suppose that there are  $n$  rows and  $m$  columns in the image array and assume that the principal point is located at the center of the image array:

$$c_x = \frac{m-1}{2} \quad (10.1)$$

$$c_y = \frac{n-1}{2} \quad (10.2)$$

The image plane coordinates for the pixel at grid location  $[i,j]$  are

$$\tilde{x} = \tau_x d_x (j - c_x) \quad (10.3)$$

$$\tilde{y} = -d_y(i - c_y) \quad (10.4)$$

where  $d_x$  and  $d_y$  are the center-to-center distances between pixels in the  $x$  and  $y$  directions, respectively, and  $\tau_x$  is a scale factor that accounts for distortions in the aspect ratio caused by timing problems in the digitizer electronics. The row and column distances,  $d_x$  and  $d_y$ , are available from the specifications for the CCD camera and are very accurate, but the scale factor  $\tau_x$  must be added to the list of intrinsic parameters for the camera and determined through calibration. Note that these are uncorrected image coordinates, marked with a tilde to emphasize that the effects of lens distortions have not been removed. The coordinates are also affected by errors in the estimates for the location of the principal point  $(c_x, c_y)$  and the scale factor  $\tau_x$ .

We must solve the exterior orientation problem before attempting to solve the interior orientation problem, since we must know how the camera is positioned and oriented in order to know where the calibration points project into the image plane. Once we know where the projected points should be, we can use the projected locations  $p'_i$  and the measured locations  $\tilde{p}_i$  to determine the lens distortions and correct the location of the principal point and the image aspect ratio. The solution to the exterior orientation problem must be based on constraints that are invariant to the lens distortions and camera constant, which will not be known at the time that the problem is solved.

## 10.2 Simple Method for Camera Calibration

This section explains the widely used camera calibration method published by Tsai [61]. Let  $\mathbf{p}'_0$  be the location of the origin in the image plane,  $\mathbf{r}'_i$  be the vector from  $\mathbf{p}'_0$  to the image point  $\mathbf{p}'_i = (x'_i, y'_i)$ ,  $\mathbf{p}_i = (x_i, y_i, z_i)$  be a calibration point, and  $\mathbf{r}_i$  be the vector from the point  $(0, 0, z_i)$  on the optical axis to the point  $\mathbf{p}_i$ . If the difference between the uncorrected image coordinates  $(\tilde{x}_i, \tilde{y}_i)$  and the true image coordinates  $(x'_i, y'_i)$  is due only to radial lens distortion, then  $\mathbf{r}'_i$  is parallel to  $\mathbf{r}_i$ . The camera constant and translation in  $z$  do not affect the direction of  $\mathbf{r}_i$ , since both image coordinates will be scaled by the same amount. These constraints are sufficient to solve the exterior orientation problem [61]. The figure 10.1 shows graphically the problem.

Assume that the calibration points lie in a plane with  $z = 0$  and assume that the camera is placed relative to this plane to satisfy the following two crucial conditions:

1. The origin in absolute coordinates is not in the field of view.
2. The origin in absolute coordinates does not project to a point in the image that is close to the  $y$  axis of the image plane coordinate system.

Condition 1 decouples the effects of radial lens distortion from the camera constant and distance to the calibration plane. Condition 2 guarantees that the  $y$

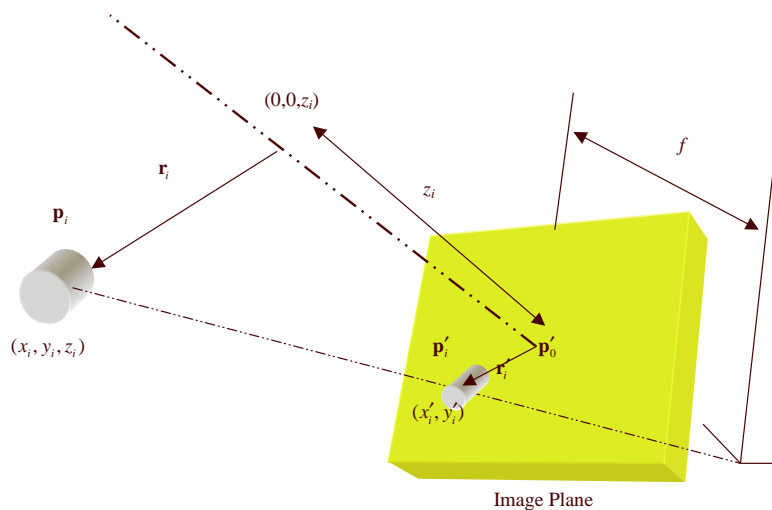


Figure 10.1: Representation of the calibration problem

component of the rigid body translation, which occurs in the denominator of many equations below, will not be close to zero. These conditions are easy to satisfy in many imaging situations. For example, suppose that the camera is placed above a table, looking down at the middle of the table. The absolute coordinate system can be defined with  $z = 0$  corresponding to the plane of the table, with the  $x$  and  $y$  axes running along the edges of the table, and with the corner of the table that is the origin in absolute coordinates outside of the field of view [62].

Suppose that there are  $n$  calibration points. For each calibration point, we have the absolute coordinates of the point  $(x_i, y_i, z_i)$  and the uncorrected image coordinates  $(\tilde{x}_i, \tilde{y}_i)$ . Use these observations to form a matrix  $A$  with rows  $a_i$ .

$$a_i = (\tilde{y}_i x_i, \tilde{y}_i y_i, -\tilde{x}_i x_i, -\tilde{x}_i y_i, \tilde{y}_i) \quad (10.5)$$

Let  $u = (u_1, u_2, u_3, u_4, u_5)$  be a vector of unknown parameters that are related to the parameters of the rigid body transformation:

$$u_1 = \frac{r_{xx}}{p_y} \quad (10.6)$$

$$u_2 = \frac{r_{xy}}{p_y} \quad (10.7)$$

$$u_3 = \frac{r_{yx}}{p_y} \quad (10.8)$$

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$$u_4 = \frac{r_{yy}}{p_y} \quad (10.9)$$

$$u_5 = \frac{p_x}{p_y} \quad (10.10)$$

Form a vector  $\mathbf{b} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)$  from the  $n$  observations of the calibration points. With more than five calibration points, we have an overdetermined system of linear equations,

$$\mathbf{A}\mathbf{u} = \mathbf{b} \quad (10.11)$$

for the parameter vector  $\mathbf{u}$ . Solve this linear system using singular value decomposition, and use the solution parameters,  $u_1, u_2, u_3, u_4$  and  $u_5$ , to compute the rigid body transformation, except for  $p_z$ , which scales with the camera constant  $f$  and will be determined later.

First, compute the magnitude of the  $y$  component of translation. If  $u_1$  and  $u_2$  are not both zero and  $u_3$  and  $u_4$  are not both zero, then

$$p_y^2 = \frac{U - [U^2 - 4(u_1u_4 - u_2u_3)^2]^{\frac{1}{2}}}{2(u_1u_4 - u_2u_3)} \quad (10.12)$$

where  $U = u_1^2 + u_2^2 + u_3^2 + u_4^2$ ; otherwise, if  $u_1$  and  $u_2$  are both zero, then

$$p_y^2 = \frac{1}{u_3^2 + u_4^2} \quad (10.13)$$

otherwise, using  $u_1$  and  $u_2$ ,

$$p_y^2 = \frac{1}{u_1^2 + u_2^2} \quad (10.14)$$

Second, determine the sign of  $p_y$ . Pick the calibration point  $\mathbf{p} = (x, y, z)$  that projects to an image point that is farthest from the center of the image (the scene point and corresponding image point that are farthest in the periphery of the field of view). Compute  $r_{xx}, r_{xy}, r_{yx}, r_{yy}$  and  $p_x$  from the solution vector obtained above:

$$r_{xx} = u_1p_y \quad (10.15)$$

$$r_{xy} = u_2p_y \quad (10.16)$$

$$r_{yx} = u_3 p_y \quad (10.17)$$

$$r_{yy} = u_4 p_y \quad (10.18)$$

$$p_x = u_5 p_y \quad (10.19)$$

Let  $\xi_x = r_{xx}x + r_{xy}y + p_x$  and  $\xi_y = r_{yx}x + r_{yy}y + p_y$ . If  $\xi_x$  and  $\tilde{x}$  have the same sign and  $\xi_y$  and  $\tilde{y}$  have the same sign, then  $p_y$  has the correct sign (positive); otherwise negate  $p_y$ . Note that the parameters of the rigid body transformation computed above are correct, regardless of the sign of  $p_y$ , and do not need to be changed.

Third, compute the remaining parameters of the rigid body transformation:

$$r_{xz} = \sqrt{1 - r_{xx}^2 - r_{xy}^2} \quad (10.20)$$

$$r_{yz} = \sqrt{1 - r_{yx}^2 - r_{yy}^2} \quad (10.21)$$

Since the rotation matrix must be orthonormal, it must be true that  $R^T R = \mathbf{I}$ . Use this fact to compute the elements in the last row of the rotation matrix:

$$r_{zx} = \frac{1 - r_{xx}^2 - r_{xy}r_{yx}}{r_{xz}} \quad (10.22)$$

$$r_{zy} = \frac{1 - r_{yx}r_{xy} - r_{yy}^2}{r_{yz}} \quad (10.23)$$

$$r_{zz} = \sqrt{1 - r_{zx}r_{xz} - r_{zy}r_{yz}} \quad (10.24)$$

If the sign of  $r_{xx}r_{yx} + r_{xy}r_{yy}$  is positive, negate  $r_{yz}$ . The signs of  $r_{zx}$  and  $r_{zy}$  may need to be adjusted after computing the camera constant in the following step.

Fourth, compute the camera constant  $f$  and  $p_z$ , the  $z$  component of translation. Use all of the calibration points to form a system of linear equations,

$$A\mathbf{v} = \mathbf{b} \quad (10.25)$$

for estimating  $f$  and  $p_z$ . Use each calibration point to compute the corresponding row of the matrix,

$$a_i = (r_{yx}x_i + r_{yy}y_i + p_y, -d_y\tilde{y}_i) \quad (10.26)$$

and the corresponding element of the vector on the right side of Equation 10.25,

$$b_i = (r_{zx}x_i + r_{zy}y_i)d_y\tilde{y}_i \quad (10.27)$$

The vector  $\mathbf{v}$  contains the parameters to be estimated:

$$\mathbf{v} = (f, p_z)^T \quad (10.28)$$

Use singular value decomposition to solve this system of equations. If the camera constant  $f < 0$ , then negate  $r_{zy}$  and  $r_{zy}$  in the rotation matrix for the rigid body transformation.

Fifth, use the estimates for  $f$  and  $p_z$  obtained in the previous step as the initial conditions for nonlinear regression to compute the first-order lens distortion  $\kappa_1$  and better estimates for  $f$  and  $p_z$ . The true (corrected) image plane coordinates  $(x', y')$  are related to the calibration points in camera coordinates  $(x_c, y_c, z_c)$  through perspective projection:

$$x' = f \frac{x_c}{z_c} \quad (10.29)$$

$$y' = f \frac{y_c}{z_c} \quad (10.30)$$

Assume that the true (corrected) image plane coordinates are related to the measured (uncorrected) image plane coordinates using the first term in the model for radial lens distortion:

$$x' = \tilde{x}(1 + \kappa_1 r^2) \quad (10.31)$$

$$y' = \tilde{y}(1 + \kappa_1 r^2) \quad (10.32)$$

where the radius  $r$  is given by

$$r = \sqrt{\tilde{x}^2 + \tilde{y}^2} \quad (10.33)$$

Note that the uncorrected (measured) image plane coordinates  $(\tilde{x}, \tilde{y})$  are not the same as the pixel coordinates  $[i, j]$  since the location of the image center

$(c_x, c_y)$ , the row and column spacing  $d_x$  and  $d_y$  and the estimated scale factor  $\tau_x$  have already been applied.

Use the  $y$  components of the equations for perspective projection, lens distortion, and the rigid body transformation from absolute coordinates to camera coordinates to get a constraint on the camera constant  $f$ ,  $z$  translation, and lens distortion:

$$\tilde{y}_i(1 + \kappa_1 r^2) = f \frac{r_{yx}x_{a,i} + r_{yy}y_{a,i} + r_{yz}z_{a,i} + p_y}{r_{zx}x_{a,i} + r_{zy}y_{a,i} + r_{zz}z_{a,i} + p_z} \quad (10.34)$$

This leads to a nonlinear regression problem for the parameters  $p_z$ ,  $f$ , and  $\kappa_1$ . We use the measurements for  $y$ , rather than  $r$ , because the  $x$  measurements are affected by the scale parameter  $\tau_x$ . The spacing between image rows  $d_y$  is very accurate and readily available from the camera specifications and is not affected by problems in the digitizing electronics.

Since the calibration points were in a plane, the scale factor  $\tau_x$  cannot be determined. Also, the location of the image center,  $c_x$  and  $c_y$ , has not been calibrated.

### 10.3 Affine Method for Camera Calibration

The interior orientation problem can be combined with the exterior orientation problem to obtain an overall transformation that relates (uncalibrated) image coordinates to the position and orientation of rays in the absolute coordinate system. Assume that the transformation from uncorrected image coordinates to true image coordinates can be modeled by an affine transformation within the image plane [63]. This transformation accounts for several sources of camera error:

**Scale error** due to an inaccurate value for the camera constant.

**Translation error** due to an inaccurate estimate for the image origin (principal point).

**Rotation** of the image sensor about the optical axis.

**Skew error** due to nonorthogonal camera axes.

**Differential scaling** caused by unequal spacing between rows and columns in the image sensor (nonsquare pixels).

However, an affine transformation cannot model the errors due to lens distortions.

In the development of the exterior orientation problem, we formulated equations for the transformation from absolute coordinates to image coordinates. Now we will add an affine transformation from true image coordinates to measured (uncorrected) image coordinates to get the overall transformation from absolute coordinates to measured image coordinates.

The affine transformation in the image plane that models the distortions due to errors and unknowns in the intrinsic parameters is

$$\tilde{x} = a_{xx}x' + a_{xy}y' + b_x \quad (10.35)$$

$$\tilde{y} = a_{yx}x' + a_{yy}y' + b_y \quad (10.36)$$

where we are mapping from true image plane coordinates  $(x', y')$  to uncorrected (measured) image coordinates  $(\tilde{x}, \tilde{y})$ . Use the equations for perspective projection,

$$\frac{x'}{f} = \frac{x_c}{z_c} \quad (10.37)$$

$$\frac{y'}{f} = \frac{y_c}{z_c} \quad (10.38)$$

to replace  $x'$  and  $y'$  with ratios of the camera coordinates:

$$\frac{\tilde{x}}{f} = a_{xx} \left( \frac{x_c}{z_c} \right) + a_{xy} \left( \frac{y_c}{z_c} \right) + \frac{b_x}{f} \quad (10.39)$$

$$\frac{\tilde{y}}{f} = a_{yx} \left( \frac{x_c}{z_c} \right) + a_{yy} \left( \frac{y_c}{z_c} \right) + \frac{b_y}{f} \quad (10.40)$$

Camera coordinates are related to absolute coordinates by a rigid body transformation:

$$x_c = r_{xx}x_a + r_{xy}y_a + r_{xz}z_a + p_x \quad (10.41)$$

$$y_c = r_{yx}x_a + r_{yy}y_a + r_{yz}z_a + p_y \quad (10.42)$$

$$z_c = r_{zx}x_a + r_{zy}y_a + r_{zz}z_a + p_z \quad (10.43)$$

We can use these equations to replace the ratios of camera coordinates in the affine transformation with expressions for the absolute coordinates,

$$\frac{\tilde{x} - b_x}{f} = \frac{s_{xx}x_a + s_{xy}y_a + s_{xz}z_a + t_x}{s_{zx}x_a + s_{zy}y_a + s_{zz}z_a + t_z} \quad (10.44)$$



$$\frac{\tilde{y} - b_y}{f} = \frac{s_{yx}x_a + s_{yy}y_a + s_{yz}z_a + t_y}{s_{zx}x_a + s_{zy}y_a + s_{zz}z_a + t_z} \quad (10.45)$$

where the coefficient are sums of products of the coefficients in the affine transformation and the rigid body transformation. What we have is a pair of equations, that relate absolute coordinates to uncorrected image coordinates. The affine model for camera errors has been absorbed into the transformation from absolute to camera coordinates. Equations 10.44 and 10.45 can be written as

$$\frac{\tilde{x} - b_x}{f} = \frac{\tilde{x}_c}{\tilde{z}_c} = \frac{s_{xx}x_a + s_{xy}y_a + s_{xz}z_a + t_x}{s_{zx}x_a + s_{zy}y_a + s_{zz}z_a + t_z} \quad (10.46)$$

$$\frac{\tilde{y} - b_y}{f} = \frac{\tilde{x}_c}{\tilde{z}_c} = \frac{s_{yx}x_a + s_{yy}y_a + s_{yz}z_a + t_y}{s_{zx}x_a + s_{zy}y_a + s_{zz}z_a + t_z} \quad (10.47)$$

to show that the (uncorrected) image coordinates are related to the camera coordinates by perspective projection, but the space of camera coordinates has been warped to account for the camera errors.

Returning to Equations 10.44 and 10.45, we can absorb the corrections to the location of the principal point,  $b_x$  and  $b_y$ , into the affine transformation to get

$$\begin{aligned} & \tilde{x}_i(s_{zx}x_{a,i} + s_{zy}y_{a,i} + s_{zz}z_{a,i} + t_z) \\ & - f(s_{xx}x_{a,i} + s_{xy}y_{a,i} + s_{xz}z_{a,i} + t_x) = 0 \end{aligned} \quad (10.48)$$

$$\begin{aligned} & \tilde{y}_i(s_{zx}x_{a,i} + s_{zy}y_{a,i} + s_{zz}z_{a,i} + t_z) \\ & - f(s_{yx}x_{a,i} + s_{yy}y_{a,i} + s_{yz}z_{a,i} + t_y) = 0 \end{aligned} \quad (10.49)$$

which shows that each calibration point and its corresponding measured location in the image plane provides two linear equations for the parameters of the transformation. The nominal value  $f$  for the camera constant is not absorbed into the affine transformation since it is needed for constructing the ray in camera coordinates.

The set of calibration points yields a set of homogeneous linear equations that can be solved for the coefficients of the transformation. At least six points

are needed to get 12 equations for the 12 unknowns, but more calibration points should be used to increase accuracy. To avoid the trivial solution with all coefficients equal to zero, fix the value of one of the parameters, such as  $t_x$  or  $t_y$ , and move it to the right side of the equation. Form a system of linear equations,

$$A\mathbf{u} = \mathbf{b} \quad (10.50)$$

where  $\mathbf{u}$  is the vector of transformation coefficients; row  $i$  of the  $A$  matrix is filled with absolute coordinates for calibration point  $i$  and products of the absolute coordinates and  $\tilde{x}_i$ ,  $\tilde{y}_i$  or  $f$ ; and element  $i$  of the  $\mathbf{b}$  vector is the constant chosen for  $t_x$  or  $t_y$ . Since the affine transformation within the image plane is combined with the rotation matrix for exterior orientation, the transformation matrix is no longer orthonormal. The system of linear equations can be solved, without the orthonormality constraints, using common numerical methods such as singular value decomposition.

The transformation maps absolute coordinates to measured image coordinates. Applications require the inverse transformation, given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = S^{-1} \left[ \begin{pmatrix} \tilde{x}_i \\ \tilde{y}_i \\ f \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} \right] \quad (10.51)$$

which can be used to determine the equation of a ray in absolute coordinates from the measured coordinates in the image. Note that the camera constant  $f$  has been carried unchanged through the formulation of the calibration algorithm. Since corrections to the camera constant are included in the affine transformation (Equations 10.35 and 10.36), the focal length of the lens can be used for  $f$ . Finally, the transformation from pixel coordinates  $[i,j]$  to image coordinates,

$$\tilde{x} = s_x(j - c_x) \quad (10.52)$$

$$\tilde{y} = -s_y(i - c_y) \quad (10.53)$$

is an affine transformation that can be combined with the model for camera errors (Equations 10.35 and 10.36) to develop a transformation between absolute coordinates and pixel coordinates.

## 10.4 Nonlinear Method for Camera Calibration

Given a set of calibration points, determine the projections of the calibration points in the image plane, calculate the errors in the projected positions, and

use these errors to solve for the camera calibration parameters. Since it is necessary to know where the calibration points should project to in the image plane, the exterior orientation problem is solved simultaneously [64]. The method presented in this section is different from the procedure explained in Section 10.3, where the interior and exterior orientation problems were combined into a single affine transformation, in that the actual camera calibration parameters are obtained and can be used regardless of where the camera is later located in scene.

The principle behind the solution to the camera calibration problem is to measure the locations  $(x'_i, y'_i)$  of the projections of the calibration points onto the image plane, calculate the deviations  $(\delta x_i, \delta y_i)$  of the points from the correct positions, and plug these measurements into the equations that model the camera parameters. Each calibration point yields two equations. The solution requires at least enough equations to cover the unknowns, but for increased accuracy more equations than unknowns are used and the overde-termined set of equations is solved using nonlinear regression.

Assume that the approximate position and orientation of the camera in absolute coordinates is known. Since we have initial estimates for the rotation angles, we can formulate the exterior orientation problem in terms of the Euler angles in the rotation matrix. The parameters of the regression problem are the rotation angles  $\omega$ ,  $\phi$ , and  $\kappa$ ; the position of the camera in absolute coordinates  $p_x, p_y$  and  $p_z$ ; the camera constant  $f$ ; the corrections to the location of the principal point  $(x_p, y_p)$ ; and the polynomial coefficients for radial lens distortion  $\kappa_1$ ,  $\kappa_2$ , and  $\kappa_3$ . The equations for the exterior orientation problem are

$$\frac{x'}{f} = \frac{r_{xx}x_a + r_{xy}y_a + r_{xz}z_a + p_x}{r_{zx}x_a + r_{zy}y_a + r_{zz}z_a + p_z} \quad (10.54)$$

$$\frac{y'}{f} = \frac{r_{yx}x_a + r_{yy}y_a + r_{yz}z_a + p_y}{r_{zx}x_a + r_{zy}y_a + r_{zz}z_a + p_z} \quad (10.55)$$

Replace  $x'$  and  $y'$  with the corrected positions from the camera model,

$$\begin{aligned} & \frac{(\tilde{x} - x_p)(1 + \kappa_1 r^2 + \kappa_2 r^4 + \kappa_3 r^6)}{f} \\ &= \frac{r_{xx}x_a + r_{xy}y_a + r_{xz}z_a + p_x}{r_{zx}x_a + r_{zy}y_a + r_{zz}z_a + p_z} \end{aligned} \quad (10.56)$$

$$\begin{aligned} & \frac{(\tilde{y} - y_p)(1 + \kappa_1 r^2 + \kappa_2 r^4 + \kappa_3 r^6)}{f} \\ &= \frac{r_{yx}x_a + r_{yy}y_a + r_{yz}z_a + p_y}{r_{zx}x_a + r_{zy}y_a + r_{zz}z_a + p_z} \end{aligned} \quad (10.57)$$

and replace the elements of the rotation matrix with the formulas for the rotation matrix entries in terms of the Euler angles, provided in Equation 10.13. Solve for the camera parameters and exterior orientation using nonlinear regression. The regression algorithm will require good initial conditions. If the target is a plane, the camera axis is normal to the plane, and the image is roughly centered on the target, then the initial conditions are easy to obtain. Assume that the absolute coordinate system is set up so that the  $x$  and  $y$  axes are parallel to the camera axes. The initial conditions are:

$$\omega = \phi = \kappa = 0.$$

$x$  = translation in  $x$  from origin.

$y$  = translation in  $y$  from origin.

$z$  = distance of the camera from the calibration plane.

$f$  = focal length of the lens.

$$x_p = y_p = 0.$$

$$\kappa_1 = \kappa_2 = \kappa_3 = 0.$$

It is easy to build a target of dots using a laser printer. The uncorrected positions of the dots in the image can be found by computing the first moments of the connected components.

The disadvantage to nonlinear regression is that good initial values for the parameters are needed, but the advantage is that there is a body of literature on nonlinear regression with advice on solving nonlinear problems and methods for estimating errors in the parameter estimates.

## 10.5 Binocular Stereo Calibration

In this section we will discuss how the techniques presented in this chapter can be combined in a practical system for calibrating stereo cameras and using the stereo measurements. This provides a forum for reviewing the relationships between the various calibration problems.

There are several tasks in developing a practical system for binocular stereo:

1. Calibrate the intrinsic parameters for each camera.
2. Solve the relative orientation problem.
3. Resample the images so that the epipolar lines correspond to image rows.
4. Compute conjugate pairs by feature matching or correlation.
5. Solve the stereo intersection problem for each conjugate pair.
6. Determine baseline distance.
7. Solve the absolute orientation problem to transform point measurements from the coordinate system of the stereo cameras to an absolute coordinate system for the scene.

There are several ways to calibrate a binocular stereo system, corresponding to various paths through the diagram in Figure 10.2. To start, each camera must be calibrated to determine the camera constant, location of the principal point, correction table for lens distortions, and other intrinsic parameters. Once the left and right stereo cameras have been calibrated, there are basically three approaches to using the cameras in a stereo system.

The first approach is to solve the relative orientation problem and determine the baseline by other means, such as using the stereo cameras to measure points that are a known distance apart. This fully calibrates the rigid body transformation between the two cameras. Point measurements can be gathered in the local coordinate system of the stereo cameras. Since the baseline has been calibrated, the point measurements will be in real units and the stereo system can be used to measure the relationships between points on objects in the scene. It is not necessary to solve the absolute orientation problem, unless the point measurements must be transformed into another coordinate system.

The second approach is to solve the relative orientation problem and obtain point measurements in the arbitrary system of measurement that results from assuming unit baseline distance. The point measurements will be correct, except for the unknown scale factor. Distance ratios and angles will be correct, even though the distances are in unknown units. If the baseline distance is obtained later, then the point coordinates can be multiplied by the baseline distance to get point measurements in known units. If it is necessary to transform the point measurements into another coordinate system, then solve the absolute orientation problem with scale (Section 12.7), since this will accomplish the calibration of the baseline distance and the conversion of point coordinates into known units without additional computation.

The third approach is to solve the exterior orientation problem for each stereo camera. This provides the transformation from the coordinate systems of the left and right camera into absolute coordinates. The point measurements obtained by intersecting rays using the methods of Section 12.6 will automatically be in absolute coordinates with known units, and no further transformations are necessary.

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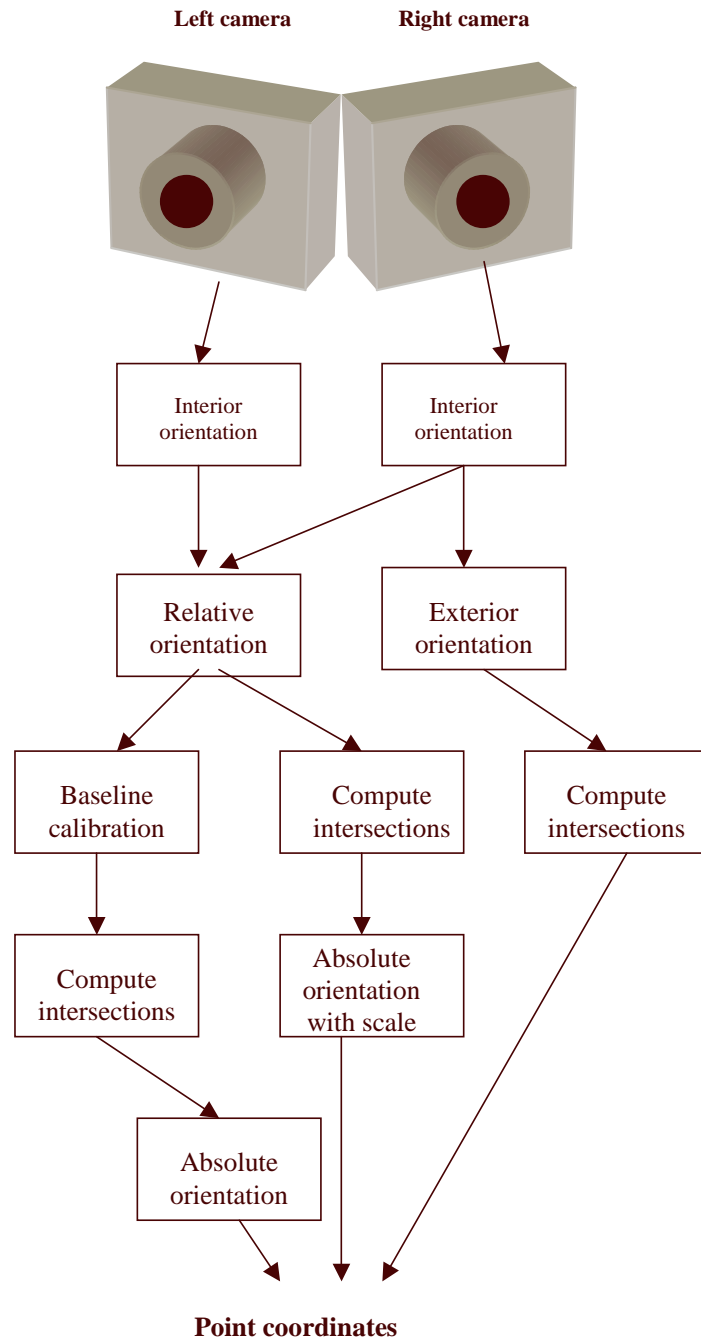


Figure 10.2: A diagram of the steps in various procedures for calibrating a binocular stereo system.