Chapter 6

Adaptive Control

6.1 Introduction

Computerized process control has advanced by leaps and bounds over the last ten years in hardware and methods. Conventional PID controllers [79], whether analog or digital, are only efficient where the system to be controlled (the plant) or rather the model of that system represented within the controller is characterized by constant parameters applicable at all operating points. And yet, most complex systems are characterized by parameters that vary with the system operating point, thus failing to meet the basic assumption just stated. In such cases, a control signal generated by a conventional PID controller (i.e. one for which the parameters are computed once and for all on the basis of a constant-parameter system model) will inevitably give rise to progressively more degraded operation of the overall control loop as the errors between controller and actual process parameters increase. This can only be corrected by modifying the controller coefficients. Which brings us to adaptive control.

6.2 Theory of Adaptive Control

Adaptive control represents an advanced level of controller design. It is recommended for systems operating in variable environments and/or featuring variable parameters. Adaptive control is a set of techniques for the automatic, on-line, real-time adjustment of control-loop regulators designed to attain or maintain a given level of system performance where the controlled process parameters are unknown and/or time-varying. Adaptive control is based entirely on the following hypothesis: the process to be controlled can be mathematically modelled and the structure of this model (delay and order) is known in advance. The determination of the structure of a parametric system model is thus a vital step before going on to design an adaptive control algorithm. The capabilities of the adaptive control algorithm depends, to a large extent, on the faithfulness with which the model represents the system and its behavior. The chief advantage, in practical terms, of adaptive control appears to be the capability to ensure quasi-optimal system performance in the presence of a model with time-varying parameters.

Once the model and its structure have been identified, the next step is to select a control strategy. This choice depends in part on the nature of the problem (regulation or tracking) and on the system characteristics (minimum phase or not). The number of options available depends on the extent of our advance knowledge of these characteristics.

The adaptive control algorithm is then designed in accordance with the structure of the system model and the selected control strategy. As a rule, the adaptive control algorithm can be seen as a combination of two algorithms. An identification algorithm uses measurements made on the system and generates information (a succession of estimates) for input to a control law computation algorithm. This second algorithm determines, at each instant, the adaptive controller parameters and the control to be applied to the system. This type of adaptive control is termed **indirect**. For example, no control law computation algorithm is required at all if the parameters characterizing the adaptive controller are directly identified. This is known as **direct adaptive control**.

In this thesis the two approaches of adaptive control previously mentioned are treated. We will look first at adaptive control based on a **direct scheme using a reference model**. There are two main reasons for this choice: first, this type of control is relatively easy to implement; second, it has already found practical applications in systems. A discussion follows on an adaptive control system based on an indirect scheme which offers the best system response. This type of control was introduced by Clarke [80,81]. It produces optimal control over any system, with or without time delays and irrespective of whether the inverse is stable or unstable. This scheme is known as **generalized predictive control**.

The basic principle underlying adaptive control systems is relatively simple (Figure 6.1). An adaptive control system measures a certain performance rating of the system (or plant) to be controlled. Starting with the difference between the desired and measured performance ratings, the adjustment system modifies the parameters of the adaptive controller (or regulator) and the control law in order to maintain the system performance rating close to the desired value(s).

Note that, in order to design and correctly adjust (or tune) a good controller, we must specify the desired performance of the regulation loop and determine the dynamic process model describing the relation between variations in control signals and output. This means we must determine the representation model which, in turn, means that we must establish the system's order and time delay. The literature on adaptive control includes hundreds of papers on different approaches to the problem. As a result, computer vision engineers who are not specialists in adaptive control theory often find it very difficult to determine which approach they should use to solve a given problem. The aim of this chapter is to introduce to the two main principles of adaptive control identified to date and to use these techniques in the control of the robotics

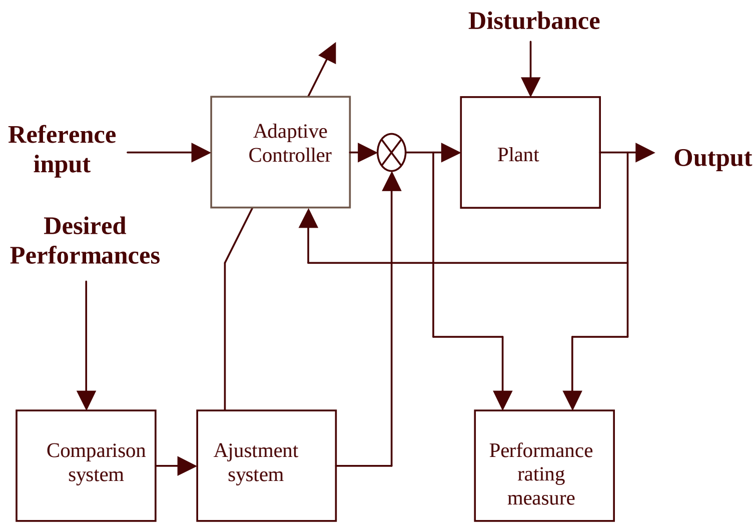

Figure 6.1: Basic principles of adaptive control

head for the object tracking. The two principles selected for discussion were chosen on the basis of the goal of any design project, namely the determination of a real-time control law applicable to a given process and the total number of operations required to parameterize the control law is assumed to be one of the criteria most important to the design engineer.

6.3 Numerical Domain Representation (Parametric Models)

For any continuous, mono or multi-variable physical system, the search for a suitable parametric model (whether by empirical methods or on the basis of experimental data) leads to the use of linear differential equations to represent the process to be identified. These equations are of the form:

$$\frac{d^n\mathrm{Y(t)}}{dt^n} + \alpha_1 \frac{d^{n-1}\mathrm{Y(t)}}{dt^{n-1}} + ... + \alpha_n \mathrm{Y(t)} = \beta_0 \frac{d^m\mathrm{U(t)}}{dt^m} + ... + \beta_m \mathrm{U(t)} \tag{6.1}$$

In nature, no system is rigorously linear in the mathematical sense. However, most processes approach linear behavior over a limited operating range. Contrary to non-parametric models (finite impulse response), parametric models depend on a specific structure. The parametric model characterizes the dynamic behavior of a physical system in terms of its transfer function. This may be deduced using a z-transform. Applying such a transform to Eqn. 6.1, we obtain:

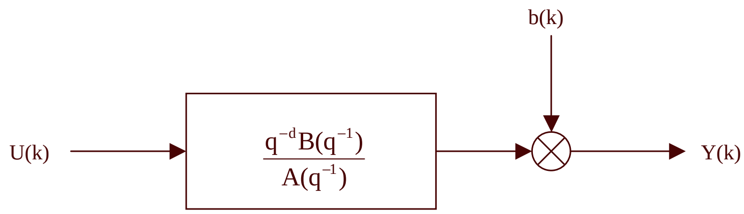

Figure 6.2: Parametric model description in terms of process input and output.

$$\mathrm{G(z)} = \frac{\mathrm{Y(z)}}{\mathrm{U(z)}} = \frac{z^{-d}(b_0 + ... + b_m z^{-m})}{1 + a_1 z^{-1} + ... + a_n z^{-n}} = \frac{z^{-d}\mathrm{B(z^{-1})}}{\mathrm{A(z^{-1})}} \tag{6.2}$$

where (a_1,...,a_n) and (b_0,...,b_m) represent the parameters of the sampled model, d represent the time delay, n determines the order of the model, U(z) is the model input and Y(z) is the output model. The most widely used parametric model is illustrated in Figure 6.2.

With

- q^{-1}, is the time delay operator.
- $A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_n q^{-1}$
- $B(q^{-q}) = b_0 + ... + b_m q^{-1}$
- $b(k)$, represents all noise sources expressed in terms of their equivalent effect on output.

The model described by Eqn. 6.2 is known in the literature as the polynomial parametric model. Expression Eqn. 6.2 is solely in terms of the process input and output. The model can also be represented as a first-order differential equation by converting expression Eqn. 6.1. This representation is known as the parametric state model and is defined in accordance with equation Eqn. 6.3. Throughout the remainder of this chapter we will assume that polynomial $B(q^{-1})$ is of the same degree as polynomial $A(q^{-1})$.

$$\mathbf{X}_{k+1} = \mathbf{P}\mathbf{X}_k + \mathbf{Q}\mathrm{U}_k$$

$$\mathrm{Y}_k = \mathbf{C}\mathbf{X}_k \tag{6.3}$$

Where
$\mathbf{X}_k$ is the state vector pf dimension ((*n*+*d*) x 1).
P is the state matrix of dimension ((*n*+*d*) x (*n*+*d*)).
Q is the input vector of dimension ((*n*+*d*) x 1).
C is the output vector of dimension (1 x (*n*+*d*)).

n is the order of the system.

The relation between these two representations of the parametric model is given by:

$$\mathbf{P} = \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ -a_n & \vdots & \ddots & \ddots & 0 \\ 0 & \vdots & & \ddots & 1 \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_0 \\ \vdots \\ \vdots \\ b_n \end{bmatrix} \mathbf{C} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$$

While it is true that the parametric model approximates the behavior of the physical system, one must be cautious when it comes to the physical interpretation of the parameters contributing to the model's structure. The purpose of the parametric model is to approximate as closely as possible the behavior of the system by ensuring the closest possible match between predicted and observed output. This is done, moreover, within the limits of an accuracy vs. simplicity trade-off that the automatic control specialist defines when choosing the parametric model to generate the control law.

The advantages of the parametric model approach lie in its structure:

1. It enables us to describe, sufficiently accurately, the dynamics of an arbitrary physical process using fewer parameters that are required by the non-parametric model (finite impulse response).

2. It is relatively simple to implement on the controller. Using a well-known property of the z transform (time delay theorem), we can proceed from the polynomial parametric model to the difference equation of the following form:

$$\mathrm{Y}(k) = b_0\mathrm{U}(k-d) + \ldots + b_n\mathrm{U}(k-n-d) - a_1\mathrm{Y}(k) - \ldots - a_n\mathrm{Y}(k-n) + \mathrm{e}(k) \quad (6.4)$$

with

- *e(k)* representing the generalized or residual noise
- $e(k) = b(k)(1 + a_1q^{-1} + \ldots + a_nq^{-1})$

Given that we now have the time-history of the input and output signals, we can readily predict the model output values. This important point is widely used in modern regulation theory. The state parametric model is useful for describing multivariable systems. The main drawback of the parametric model is the difficulty of determining the order of the system. If the designer underestimates the process order, model predictions will not match actual system behavior. On the other hand, if the designer overestimates the order, the increased complexity of the model will mean longer computation times. This same comment also applies to the estimation of pure time delays. With most industrial systems, we do not have access to the states values, which is a major handicap for the state parametric model. There are state observer techniques allowing the state estimation, but having a heavy penalty in terms of computation time.

6.4 Adaptive Control Using Reference Models

An adaptive controller may be of conventional design or it may be more complex in structure [84], including adjustable coefficients such that their tuning, using a suitable algorithm, either optimizes or extends the operating range of the process to be regulated. The different methods of adaptive control differ as to the method chosen to adjust (or tune) the control coefficients.

This section discusses adaptive control using parallel-serial reference models which, along with self-tuning control, are the only control schemes to have found practical applications to date. The adaptive control scheme using parallel reference models (i.e. located in parallel on the closed-loop system) was originally proposed by Whitaker in 1958. The version proposed at the time offered a solution to the tracking problem, but not the regulation problem. Taking the figure 6.3 as reference note that a tracking problem is defined when the reference value (**r(k)**) varies and when no disturbances (**d(k)**) are present in the output (**y(k)**). A regulation problem is defined when the reference value is zero or steady and when there is a disturbance in the output such that its effect must be reduced by the control (**u(k)**).

The parallel model structure is suitable for solving the tracking problem and is demonstrated by the fact that the model requires reasonable control signals; the structure is not suitable for solving regulation problems and is demonstrated by the fact that, in this case, the model requires unreasonable control signals. We obtain unreasonable control signals because the estimated error (differences between the output of the parallel reference model and that of the system) converges to zero during a single sampling interval. To attenuate the control signal, a serial reference model (i.e. in series with the estimated error) can be added to the general structure. This imposes a converge-to-zero requirement, with a chosen dynamic response, that is less severe than in the previous case. Let us now look at this adaptive control method using parallel-serial reference models more closely.

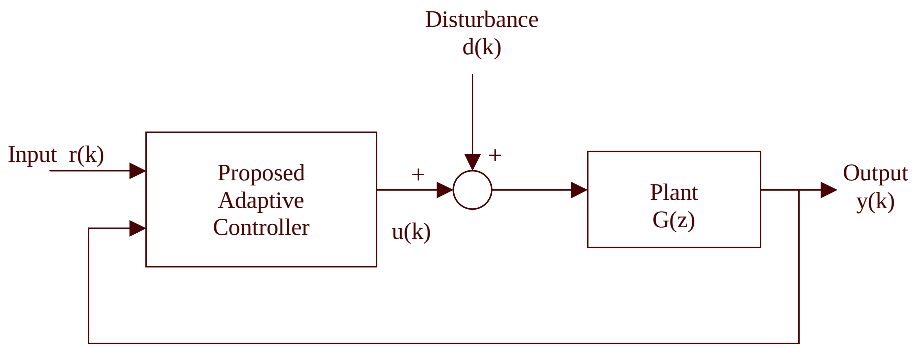

Figure 6.3: Adaptive controller in closed loop.

6.4.1 Closed-Loop System

An adaptive control system comprises not only a feedback-type control loop (or inner loop) including an adaptive controller, but also an additional, or outer, loop acting on the controller parameters in order to maintain system performance in the presence of variations in the process parameters. This second loop also has a feedback-loop-type structure, the controlled variable being the performance of the control system itself. The arrangement is schematically shown in Figure 6.4.

where:

- $\mathbf{e}_p$ represents the parallel estimated error
- $\mathbf{e}_s$ represents the serial estimated error.

This type of adaptive scheme offers the advantage of being able to accommodate separately both tracking and regulation problems. This is because the desired performance of the controlled system are defined by a parallel model for a tracking problem and by a serial model for a regulation problem.

6.4.2 Control Law

The dynamic behavior of the simulated system is defined by a parametric model. We recall that its general structure is given by the relation:

$$\mathrm{A}(q^{-1})\mathrm{Y}(k) = q^{-d}\mathrm{B}(q^{-1})\mathrm{U}(k) \tag{6.5}$$

where

- $A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_n q^{-1}$
- $B(q^{-q}) = b_0 + ... + b_m q^{-1}$

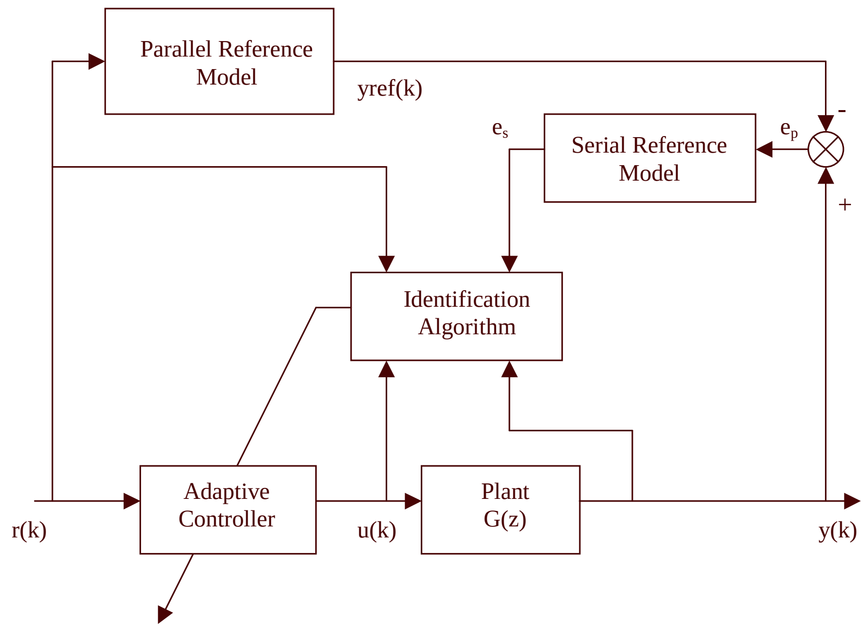

Figure 6.4: Adaptive control using reference models in closed loop.

- d, time delay.

The order of polynomials $A(q^{-1})$ and $B(q^{-1})$ and also the time delay of the parametric model enable us to correctly dimension the control law. To bring us nearer to the formulation of the adaptive control law, we first consider the case where the system parameters are known.

6.4.3 Known System Parameters

With the objectives of tracking and regulation being independent, we can formalize their respective equations as: A → Regulation. (r(k) = 0). The problem here is to determine a control (u(k)) that will eliminate an initial disturbance (d(k)) with a dynamic response defined by the relation:

$$\mathrm{A_r}(q^{-1})\mathrm{Y}(k+d) = 0 \tag{6.6}$$

with

- $\mathrm{A_r}(q^{-1}) = 1 + a_{r1}q^{-1} + ... + a_{rn}q^{-n}$
- d, time delay.
- n, system order.

The polynomial $\mathrm{A_r}(q^{-1})$ is determined by the designer to be asymptotically stable for order *n*. The polynomial represents the serial (or regulation) model.

B →Tracking. (d(k) = 0). The problem here is to determine a control (u(k)) such that the system output (y(k)) satisfies a relation of the form:

$$\mathrm{A_p}(q^{-1})\mathrm{Y}(k+d) = \mathrm{B_p}(q^{-1})\mathrm{R}(k) \tag{6.7}$$

where
$\mathrm{A_p}(q^{-1}) = 1 + a_{p1}q^{-1} + ... + a_{pn}q^{-n}$
$\mathrm{B_p}(q^{-1}) = b_{p0} + b_{p1}q^{-1} + ...b_{p(n-d)}q^{-(n-d)}$
n, order system.
d, time delay.

This corresponds to tracking a trajectory defined by the following reference model:

$$\mathrm{G_p}(q^{-1}) = \frac{q^{-d}\mathrm{B_p}(q^{-1})}{\mathrm{A_p}(q^{-1})} \tag{6.8}$$

In general, one may assume that there is some link between the tracking dynamic response $\mathrm{A_p}(q^{-1})$ and the regulation dynamic response $\mathrm{A_r}(q^{-1})$. However, in this thesis, and for the sake of simplicity, we shall assume identical dynamic response to a variation in either load or reference value, i.e. we shall assume $\mathrm{A_p}(q^{-1})$ = $\mathrm{A_r}(q^{-1})$. We shall further assume a reference model such that the output is described by the relation:

$$\mathrm{A_p}(q^{-1})\mathrm{Yref}(k+d) = \mathrm{B_p}(q^{-1})\mathrm{R}(k) \tag{6.9}$$

Under these conditions, the aims to be achieved by the control signal can be expressed in the form:

$$\mathrm{e_s}(k+d) = \mathrm{A_p}(q^{-1})\left[\mathrm{Y}(k+d) - \mathrm{Yref}(k+d)\right] = 0 \tag{6.10}$$

The control law, with a parallel-serial reference model, can be deduced by minimizing the following quadratic criterion:

$$\mathrm{J}(k+d) = \mathrm{e_s^2}(k+d) = \left[\mathrm{A_p}(q^{-1})\left[\mathrm{Y}(k+d) - \mathrm{Yref}(k+d)\right]\right]^2 \tag{6.11}$$

In the case of unit time delay (*d*=1), we can determine the control law directly by minimizing criterion Eqn. 6.11 relative to U(*k*). The problem may be different, however, if the pure time delay of the controlled system is equal to or greater than twice the sampling period. In order to obtain a causal regulator, i.e. one such that U(*k*) is of the form:

$$\mathrm{U}(k) = \mathrm{F_U}(\mathrm{Y}(k), \mathrm{Y}(k-1), ..., \mathrm{U}(k-1), ...) \tag{6.12}$$

We must first rewrite the process output prediction in terms of the quantities measurable at time *k* and prior to time *k*. The prediction can be expressed in the form:

$$\mathrm{A_p}(q^{-1})\mathrm{Y}(k+d) = \mathrm{F_y}(\mathrm{Y}(k), \mathrm{Y}(k-1), ..., \mathrm{U}(k), \mathrm{U}(k-1), ...) \quad (6.13)$$

In the literature, an expression of this form is known as a *d-step-ahead predictive model*. An expression such as Eqn. 6.13 can be obtained directly using the general polynomial identity:

$$\mathrm{A_p}(q^{-1}) = \mathrm{A}(q^{-1})\mathrm{S}(q^{-1}) + q^{-d}\mathrm{R}(q^{-1}) \quad (6.14)$$

where

- $\mathrm{S}(q^{-1}) = 1 + s_1 q^{-1} + ... + s_{d-1} q^{-(d+1)}$
- $\mathrm{R}(q^{-1}) = r_0 + r_1 q^{-1} + ... + r_{n-1} q^{-(n+1)}$

This relation yields a unique solution for polynomials $\mathrm{S}(q^{-1})$ and $\mathrm{R}(q^{-1})$ when the degree of $\mathrm{S}(q^{-1})$ is *d*-1. Polynomials $\mathrm{S}(q^{-1})$ and $\mathrm{R}(q^{-1})$ can be obtained either recursively or by dividing polynomial $\mathrm{A_p}(q^{-1})$ by polynomial $\mathrm{A}(q^{-1})$. Polynomial $\mathrm{S}(q^{-1})$ then corresponds to the quotient while $q^{-d}\mathrm{R}(q^{-1})$ corresponds to the remainder. Multiplying both sides of Eqn. 6.14 by y(*k*+*d*) and taking into account expression Eqn. 6.6, we obtain:

$$\mathrm{A_p}(q^{-1})\mathrm{Y}(k+d) = \mathrm{R}(q^{-1})Y(k) + \mathrm{B}(q^{-1})\mathrm{S}(q^{-1})\mathrm{U}(k) \quad (6.15)$$

This can be rewritten in the form:

$$\mathrm{A_p}(q^{-1})\mathrm{Y}(k+d) = \mathrm{R}(q^{-1})\mathrm{Y}(k) + b_0\mathrm{U}(k) + \mathrm{B_s}(q^{-1})\mathrm{U}(k-1) \quad (6.16)$$

where

- $\mathrm{B}(q^{-1})\mathrm{S}(q^{-1}) = b_0 + q^{-1}\mathrm{B_s}(q^{-1})$

Substituting Eqn. 6.16 into criterion expression Eqn. 6.11, we obtain:

$$\mathrm{J}(k+d) = \left[\mathrm{R}(q^{-1})\mathrm{Y}(k) + b_0\mathrm{U}(k) + \mathrm{B_s}(q^{-1})\mathrm{U}(k-1) - A_p(q^{-1})\mathrm{Yref}(k+d)\right]^2 \quad (6.17)$$

The criterion can now be minimized by determining the control u(k) for which:

$$\frac{\delta \mathrm{J}(k+d)}{\delta \mathrm{U}(k)} = 0 \quad (6.18)$$

Combining this with expression Eqn. 6.17, we obtain:

$$b_0\left[\mathrm{R}(q^{-1})\mathrm{Y}(k) + b_0\mathrm{U}(k) + \mathrm{B_s}(q^{-1})\mathrm{U}(k-1) - \mathrm{A_p}(q^{-1})\mathrm{Yref}(k+d)\right] = 0 \quad (6.19)$$

Now, using expression Eqn. 6.9, we obtain the required control output in the form:

$$\mathrm{U}(k) = \frac{1}{b_0}\left[\mathrm{B_p}(q^{-1})\mathrm{R}(k) - \mathrm{R}(q^{-1})\mathrm{Y}(k) - \mathrm{B_s}(q^{-1})\mathrm{U}(k-1)\right] \tag{6.20}$$

where polynomials $\mathrm{B_p}(q^{-1})$ and $\mathrm{R}(q^{-1})$ are defined by:
$\mathrm{B_p}(q^{-1}) = b_{p0} + b_{p1}q^{-1} + ... + b_{p(n-d)}q^{-(n-d)}$
$\mathrm{R}(q^{-1}) = (a_{p1} - a_1) + (a_{p2} - a_2)q^{-1} + ... + (a_{p(n-d+1)} - a_{n-d+1})q^{-(n-d)} = r_0 + r_1q^{-1} + ... + r_{n-d+1}q^{-(n-d)}$

The control expressed in relation Eqn. 6.20 thus has the property of reducing criterion Eqn. 6.11 to zero while independently meeting the requirements of both tracking and regulation. In other words, in the case of regulation (r(t) = 0), criterion expression Eqn. 6.11 represents a minimum-variance condition on the process output. Physically, this criterion implies minimizing the mean energy of the filtered error expression in relation Eqn. 6.10. The equations presented in this section were made possible by the fact that we knew the parameters of the controlled process. Let us now look at the case where these process parameters are unknown.

6.4.4 Unknown System Parameters

In the adaptive case, the structure of the controller is the same as for known system parameters, except that we replace the fixed parameters by variable ones. With the role of the adaptive, or outer, loop being to determine the correct values of these parameters, the self-tuning controller equation can be derived from Eqn. 6.20 and written as:

$$\mathrm{U}(k) = \frac{1}{b_0}\left[\mathrm{B_p}(q^{-1})\mathrm{R}(k) - \hat{\mathrm{R}}(k, q^{-1})\mathrm{Y}(k) - \hat{\mathrm{B}}_\mathrm{s}(k, q^{-1})\mathrm{U}(k-1)\right] \tag{6.21}$$

where
$\hat{\mathrm{R}}(k, q^{-1})$and $\hat{\mathrm{B}}_\mathrm{s}(k, q^{-1})$ are de controller parameter estimates at time k.

By defining the tuning vector $\theta(k)$ and the measurement vector $\psi(k)$ by the following expressions:

$$\hat{\theta}^{\mathrm{T}}(k) = \left[\hat{b}_0 \ \hat{b}_{s1} \ \hat{b}_{s2} \ .... \ \hat{b}_{s(n-d)} \ \hat{r}_0 \ \hat{r}_1 \ .... \ \hat{r}_{n-d+1}\right] \tag{6.22}$$

$$\psi(k) = [\mathrm{U}(k)\mathrm{U}(k-1).....\mathrm{U}(k-d)\mathrm{Y}(k)\mathrm{Y}(k-1).....\mathrm{Y}(k-d)] \tag{6.23}$$

The controller equation can be rewritten in the form:

$$\mathrm{B_p}(q^{-1})\mathrm{R}(k) = \hat{\theta}^{\mathrm{T}}(k)\psi(k) \tag{6.24}$$

The next step is to determine the recursive parameter-vector self-tuning algorithm.

6.4.5 Determination of Controller Parameters

The self-tuning controller parameters are determined by recursive minimization of a least-squares type criterion starting from asymptotic stability conditions dictated by the model-process error. The aim then is to estimate the parameter vector at time k in such a way that it minimizes the sum of the squares of the filtered errors between the process and the model over a time-horizon of k measurements. This is expressed by the relation:

$$\mathrm{J}_1(k) = \sum_{i=1}^{k} \mathrm{e}_\mathrm{s}^2(i) = \sum_{i=1}^{k} \left[\mathrm{A}_\mathrm{p}(q^{-1})\left(\mathrm{Y}(i) - \mathrm{Yref}(i)\right)\right]^2 \tag{6.25}$$

This same condition can also be expressed in the form:

$$\mathrm{J}_1(k) = \sum_{i=1}^{k} \left[\mathrm{B}_\mathrm{p}(q^{-1})\mathrm{R}(i) - \hat{\theta}^{\mathrm{T}}(i)\psi(i)\right]^2 \tag{6.26}$$

The values of $\hat{\theta}(k)$ which minimize criterion Eqn. 6.26 are obtained by determining the value of $\hat{\theta}(k)$ which cancels in the expression:

$$\frac{\delta \mathrm{J}_1(k)}{\delta \hat{\theta}(k)} = 0 \tag{6.27}$$

Applying relation Eqn. 6.27 to relation Eqn. 6.26, we obtain:

$$\frac{\delta \mathrm{J}_1(k)}{\delta \hat{\theta}(k)} = -\sum_{i=1}^{k} \left[\psi(i)(\mathrm{B}_\mathrm{p}(q^{-1})\mathrm{R}(i) - \hat{\theta}^{\mathrm{T}}(i)\psi(i))\right] = 0 \tag{6.28}$$

From equation Eqn. 6.29 we have:

$$\hat{\theta}(k) = \frac{\sum_{i=1}^{k} \mathrm{B}_\mathrm{p}(q^{-1})\mathrm{R}(i)\psi(i)}{\sum_{i=1}^{k} \psi(i)\psi^{\mathrm{T}}(i)} \tag{6.29}$$

In the previous expression, we now let:

$$\hat{\theta}(k) = \frac{\sum_{i=1}^{k} \mathrm{B}_\mathrm{p}(q^{-1})\mathrm{R}(i)\psi(i)}{\mathrm{F}(k)} \tag{6.30}$$

where

- $\mathrm{F}(k) = \sum_{i=1}^{k} \psi(i)\psi^{\mathrm{T}}(i)$

Expression Eqn. 6.29 corresponds to the non-recursive least-squares algorithm. To obtain a recursive algorithm, we recompute the optimal value of $\hat{\theta}(k+1)$ for the minimization condition J(k+1) and express $\hat{\theta}(k+1)$ as a function of $\hat{\theta}(k)$. This yields:

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \mathrm{F}(k+1)\psi(k)\mathrm{e_s}(k+1) \tag{6.31}$$

where

- $\mathrm{F}(k+1) = \mathrm{F}(k) + \psi(k+1)\psi^{\mathrm{T}}(k+1)$

Here, F(k+1) represents the estimator tuning gain. This is an important variable since it gives us an indication of the quality of estimation (covariance of parameter estimates).

It has been shown elsewhere [82] that if k (experiment time) increases, the $\hat{\theta}(k)$ estimates tend towards constants. In this case, the variance of the estimates tends towards zero (F(k+1) = 0). The least-squares algorithm briefly presented here has progressively less effect on new measurement values. This is acceptable if the process is unvarying in time. However, this is not the case in this thesis since the system parameters are explicitly assumed variable. This problem can be resolved by modifying the $\mathrm{J}_1(k)$ criterion. We need to arrange for the criterion to forget earlier measurement values by adding a suitable weighting factor. When this is done, the criterion to be minimized becomes:

$$\mathrm{J}_2(k) = \sum_{i=1}^{k} \lambda^{\mathrm{k-i}}\mathrm{e_s^2}(i) \tag{6.32}$$

where

- λ represents the weighting, or forgetting factor $(0 < \lambda < 1)$.

The thus modified least-squares algorithm is detailed in APPENDIX A. The main difference between algorithms is which variables are contained in vector ψ. In the literature, this quantity is referred to as the **measurement vector** while $\mathrm{e}_s(k)$ is termed the **post-prediction tuning error**. In order to ensure the stability of the overall system, the recursive least-squares identification algorithm must meet the following three conditions:

1. The rapid decrease in the prediction error ($\mathrm{e}_s(k)$) must occur during the periods when $\hat{\theta}(k)$, the unknown parameter of the system to be identified, is constant.

2. Irrespective of any variations in the domain bounded by $\hat{\theta}(k)$, the adjusted parameter $\hat{\theta}(k)$ of the identifier must remain within the appropriate bounded domain.

3. The variation $\hat{\theta}(k)$ - $\hat{\theta}(k+1)$ in the estimated parameter must decrease at the same time as the prediction error $e_s(k)$. If $e_s(k)$ is below a certain threshold, then $\hat{\theta}(k)$ - $\hat{\theta}(k+1)$ must be zero.

These conditions can only be met by making further changes to the recursive least-squares algorithm. Several authors have already tackled this problem. We have used their results to improve the robustness of the controller parameter estimation algorithm.

6.4.6 Comment

The use of a control strategy based on an output signal minimum-variance criterion theoretically requires that the system to be regulated (the plant) have a stable inverse (i.e. $b_{n-1} > b_n$). It is therefore important to have some prior knowledge of the nature of the plant, and its behavior over its entire operating range. Parametric identification is used to determine not only the structure of the representation model (order and time delays), but also the nature of the system to be regulated (i.e. whether it is a minimum-phase system or not). Note also that this type of controller can be used to define tracking and regulation performance totally independently.

The main disadvantage of a control strategy using a minimum-variance criterion applied to the variable to be regulated is that it always leads to a direct adaptive scheme. This presents a problem for the other control strategies where the control law parametering is broken down into two distinct steps, namely:

- Estimation of parameters of the system representative model, and
- Adjustment of controller parameters using system parameters.

This method of breaking down control law parametering leads to an indirect adaptive scheme. Note, however, that it can be an advantage to have a means of monitoring system dynamic response in real time. Thus, the estimation of process parameters can be used for diagnostics, monitoring, etc. Let us now look at this indirect adaptive scheme more closely.

6.5 Generalized Predictive Control

6.5.1 Introduction

The adaptive control scheme presented in the previous section is useful when the system to be controlled has a stable inverse. This leads to investigations to see if other control schemes, associated with the least-squares identification method, can generate stable control signals irrespective of the nature of the system to be controlled. Given that the minimization of the mean tracking error energy (Eqn. 6.25) is not sufficient to ensure control stability in the case of a so-called non-minimum phase system, it seems fairly natural to investigate what happens if one introduces a control weighting term into the expression for the criterion to be minimized. This is expressed by the relation:

$$J_3(K) = \sum_{i=1}^{k} \left[A_p(q^{-1})(Y(i) - \text{Yref}(i))\right]^2 + \alpha U(k)^2 \qquad (6.33)$$

where

- α is the strictly positive weighting term.

An improvement in this criterion [83] has been suggested on the basis of the following observation. A car driver does not need to have a complex mathematical model in mind in order to be able to drive. All he needs is the ability to recall a set of images of possible trajectories produced by a corresponding set of control actions on the car steering wheel. Given the driver's view of the road to be followed, the human control algorithm chooses the control action (or signal) that will produce the vehicle trajectory closest to the desired trajectory.

From this we conclude, in other words, that to obtain a robust control scheme, we can use the predictions obtained from the identification of the system to be controlled and minimize a least squares criterion involving the difference between the predicted desired trajectory and the predicted trajectories in response to the control signals. This criterion has been formulated by Clarke and is expressed in the form:

$$J_4(k) = \sum_{i=0}^{N_y-1} \left[\text{Yref}(k+i+d) - \hat{Y}(k+i+d)\right]^2 + \sum_{i=0}^{N_u-1} \alpha \Delta U(k+1)^2 \qquad (6.34)$$

where:

- $\hat{Y}(k+i+d)$ is the output prediction with N_y.
- Yref is the predicted output of the reference model over horizon N_y.
- U(*k*+i) represents the predicted control over N_u.
- N_y determines the horizon on the outputs.
- N_u determines the horizon on the control
- α is the control weighting factor.
- Δ represents the differentiation operator (Δ = 1-q^{-1}).

Thus, the control weighting term (α) ensures control stability in all cases where the system has an unstable inverse, provided the time delay is greater than unity. The differentiation operator (Δ) enables us to obtain a control that is free of static error in the variable to be controlled (Y) relative to the reference trajectory (Yref).

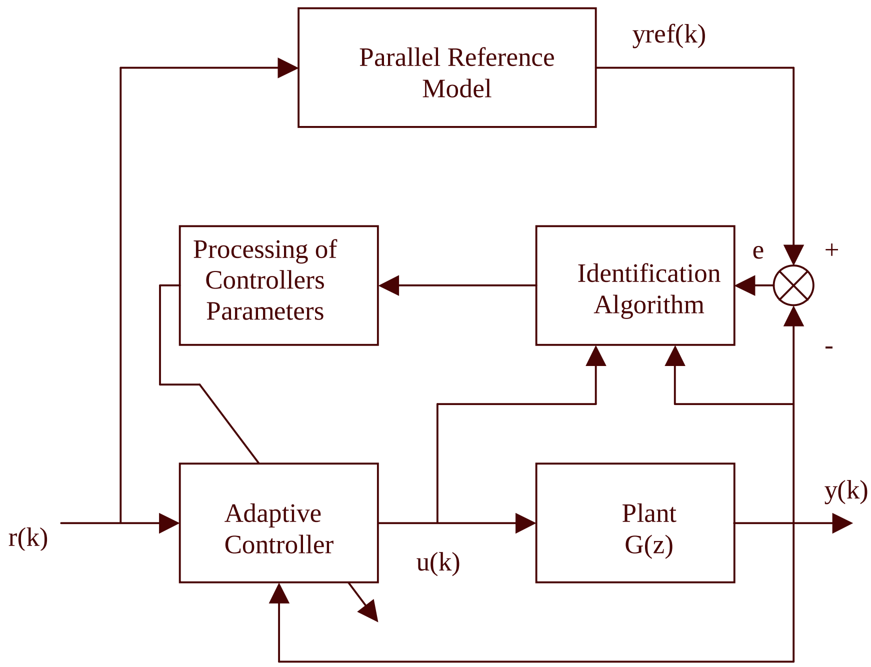

Figure 6.5: Generalized predictive control using closed loop.

6.5.2 Closed-Loop System

As with all adaptive control systems, the system discussed in this section features not only a conventional servo-type feedback loop, but also an additional loop designed to identify the on-line process and determine the parameters to be adjusted on the basis of the process parameters. The arrangement is schematically shown in figure 6.5.

The main advantage of this indirect adaptive scheme is that it gives access to the process parameters, which is important for monitoring or diagnostics. The main disadvantage is the increased computation time required to parameter the control law.

6.5.3 Control Law (Definition of Parametric Model)

The parametric model required to formulate the control law was defined earlier on. Recall that the mathematical structure is of the form:

$$\hat{\mathrm{A}}(k, q^{-1})Y(k) = q^{-d}\,\hat{\mathrm{B}}(k, q^{-1})\mathrm{U}(k) \tag{6.35}$$

where

- $\hat{A}(k, q^{-1})$ and $\hat{B}(k, q^{-1})$ are the polynomials estimated by the identifier at each sampling interval.

In the remainder of this thesis, we will simplify the mathematical notation by omitting the ^ symbols (indicating estimated variables) and the (*k*) portion of the different terms indicating that the variable is estimated at each sampling interval *k* x *Ts*.

6.5.4 Definition of System Output Prediction

The prediction of the parametric model output which is to say the probable behavior of the process output between time k_1 and some future time k_j is deduced using a *j*-step-ahead prediction model.

The general expression for such a model is:

$$\mathrm{E}_j(q^{-1})\mathrm{A}(q^{-1})\Delta + q^{-d}\mathrm{F}_\mathrm{j}(q^{-1}) = 1 \tag{6.36}$$

where:

- j = 1...k_j
- k_j is the given future time-horizon
- $\mathrm{F}_\mathrm{j}(q^{-1}) = \mathrm{f}_{0j} + ... + \mathrm{f}_{(kj-1)j}q^{-(kj+1)}$
- $\mathrm{E}_j(q^{-1}) = 1 + ... + e_{j-1}q^{-(j+1)}$

This expression is known as a Diophantine equation. The expression for the predicted model output can be deduced by multiplying the two sides of equation Eqn. 6.35 by $\mathrm{E}_j(q^{-1})\ \Delta$, then substituting the expression for $\mathrm{E}_j(q^{-1})\mathrm{A}(q^{-1})\Delta$ from equation Eqn. 6.36. This gives:

$$\mathrm{Y}(k+j) = \mathrm{F}_\mathrm{j}(q^{-1})\mathrm{Y}(k) + \mathrm{G}_\mathrm{j}(q^{-1})\Delta\mathrm{U}(k-d+j) \tag{6.37}$$

where

- $\mathrm{G}_\mathrm{j}(q^{-1}) = \mathrm{E}_\mathrm{j}(q^{-1})\mathrm{B}(q^{-1})$.

The sequence of predicted parametric model outputs can now be represented by vector Y. Note that for all future sampling intervals smaller than or equal to the system time delay (i.e. for $j \leq d$), the Y(*k*+*j*) values can be computed using the input and output data available up to time *k*. For sampling intervals greater than the system time delay (i.e. for *j* > *d*), we need to know the future control U(*k*+*j*). These assumptions form the basis of generalized predictive control.

6.5.5 Determination of Polynomials $\mathrm{F}_j(q^{-1})$ and $\mathrm{G}_j(q^{-1})$

In section 3.4.3 we showed that we could obtain the polynomials $\mathrm{S}(q^{-1})$ and $\mathrm{R}(q^{-1})$ by simple division. The disadvantage, however, of this technique is that it is very time consuming. Clarke [3] proposes a recursive method for the determination of polynomials $\mathrm{F}_j(q^{-1})$ and $\mathrm{E}_j(q^{-1})$. This is the solution that we have adopted.

6.5.6 Determination of Control Law

Above, we derived an expression (Eqn. 6.37) for predicting the behavior of the process output signal. The behavior of the reference model output signal, on the other hand, is predicted by expression (Eqn. 6.38). We must now solve this equation from sampling time (*k*+*d*) to the chosen time-horizon (*k*+*d*+N-1).

$$\mathrm{Yref}(k+d) = -\mathrm{A}_{\mathrm{p}}^{*}(q^{-1})\mathrm{Yref}(k+d-1) + \mathrm{B}_{\mathrm{p}}(q^{-1})\mathrm{R}(k) \tag{6.38}$$

where

- $\mathrm{A}_{\mathrm{p}}^{*}(q^{-1}) = a_1 q^{-1} + ... + a_n q^{-n}$

Determining the output of the parallel reference model at a future time is not difficult since the polynomials $\mathrm{A}_{\mathrm{p}}(q^{-1})$ and $\mathrm{B}_{\mathrm{p}}(q^{-1})$ are known and are constant at each sampling time. The sequence of reference model outputs from sampling time (*k*+*d*), can be expressed in vector form as follows:

$$\mathrm{Yref} = [\mathrm{Yref}(k+d) ... \mathrm{Y}(k+d+N-1)] \tag{6.39}$$

Recall that the parallel reference model plays the same role as that defined for the adaptive controller of a parallel-serial model (Section 1.4).

The prediction error at future time *k*+*j* is given by:

$$\mathrm{e}(k+j) = \mathrm{Yref}(k+j) - \overset{\wedge}{\mathrm{Y}}(k+j) \tag{6.40}$$

Thus:

$$\mathbf{e} = \mathbf{Yref} - \mathbf{Y}$$

We saw earlier that at time k, certain elements of vector Y are functions of known and unknown data. Among the unknowns, we can define the predicted control vector U as follows:

$$\mathrm{U} = [\Delta\mathrm{U}(k) ..... \Delta\mathrm{U}(k+N-1)] \tag{6.41}$$

A clever decomposition of expression Eqn. 6.44 enables us to separate the terms that depend on known data at time *k* and those that are unknown at time *k*, such as vector **U**. We thus obtain:

e=Yref-GU-f

where

- **G** is a triangular matrix of dimension N x N

The elements of **G** are generated by reformulating the Diophantine equation in recursive form.

$$\mathbf{G} = \begin{bmatrix} g_0 & 0 & \bullet & \bullet & \bullet & \bullet & 0 \\ g_1 & g_0 & \bullet & & & & \bullet \\ \bullet & \bullet & \bullet & \bullet & & & \bullet \\ \bullet & \bullet & & \bullet & \bullet & & \bullet \\ \bullet & \bullet & & & \bullet & \bullet & \bullet \\ \bullet & \bullet & & & & \bullet & 0 \\ g_N & g_{N-1} & \bullet & \bullet & \bullet & \bullet & g_0 \end{bmatrix} \tag{6.42}$$

The elements of vector **f** are the components of the part of the prediction depending on known data at time k. This vector can be written in the form:

$$\mathbf{f} = \begin{bmatrix} \mathrm{F_d}(q^{-1})\mathrm{Y}(k) + q\left[\mathrm{G_d}(q^{-1}) - g_0\right]\Delta\mathrm{U}(k-1) \\ \mathrm{F_{d+1}}(q^{-1})\mathrm{Y}(k) + q^2\left[\mathrm{G_{d+1}}(q^{-1}) - g_0 - g_1q^{-1}\right]\Delta\mathrm{U}(k-1) \\ \vdots \\ \mathrm{F_{d+N}}(q^{-1})\mathrm{Y}(k) + q^N\left[\mathrm{G_{d+N-1}}(q^{-1}) - g_0 - g_1q^{-1}.....g_{N-1}q^{-(N-1)}\right]\Delta\mathrm{U}(k-1) \end{bmatrix} \tag{6.43}$$

The control vector, **U**, can be determined by minimizing criterion $\mathrm{J}_4(k)$ as expressed in equation Eqn. 6.45. In vector form, this criterion is given by:

$$\mathrm{J}_4 = \mathbf{e}^{\mathrm{T}}\mathbf{e} + \alpha\mathbf{U}^{\mathrm{T}}\mathbf{U} \tag{6.44}$$

It can be shown that this criterion has a simple optimal solution for:

$$\mathbf{U} = \left[\mathbf{G}^T\mathbf{G} + \alpha\mathbf{I}\right]^{-1}\mathbf{G}^T\left[\mathbf{Yref} - \mathbf{f})\right] \tag{6.45}$$

Control u(k) is computed from $\Delta\mathrm{U}(k)$ using the following expression:

$$\mathrm{U}(k) = \mathrm{U}(k-1) + \Delta\mathrm{U}(k-1) \tag{6.46}$$

The power of generalized predictive control can be gauged from the fact that it allows us to reduce the control prediction time-horizon. The only difficulties are the mathematical problems posed by the inversion of matrix ($\mathbf{G}^T\mathbf{G}+\alpha\mathbf{I}$) of dimension N x N and the associated computation times. But this can be overcome by defining a control prediction time-horizon such that $\mathrm{N}_u<$ N and an output prediction time-horizon such that $\mathrm{N}_y<$ N . This reduces the number of columns of matrix ($\mathbf{G}^T\mathbf{G}$) so that we can now write:

$$\Delta\mathrm{U}(k + N_u) = \Delta\mathrm{U}(k + N_u + 1) = \cdots\cdots = 0 \tag{6.47}$$

6.5.7 Comment

The main advantage of generalized predictive control is that the control is always stable irrespective of the nature of the system to be regulated (the plant). Thus, without making any changes to the control law obtained by minimizing criterion Eqn. 6.41, generalized predictive control can readily control systems with an unstable inverse matrix.

The approach suffers, however, from one serious drawback. The weighting factor a plays a determining role in system dynamic response, enabling us to obtain reasonable control signals for trajectory tracking or for attenuating the effects of disturbance. This is not, however, very satisfactory since a defines the dynamic response of the loop system in a fashion that is difficult to determine in advance. In our study, the only way we found of approaching the desired performance through the adjustment of a was by iterative trial and error. This is a step backwards compared to the asymptotic performance of adaptive control using parallel-serial reference models where the desired dynamic response was explicitly contained in polynomials $\mathrm{A_p}(q^{-1})$ and $\mathrm{A_r}(q^{-1})$. Worse still, when the parameters of an industrial system vary from one operating point to another, weighting factor a must be modified to match the variation in the process dynamic response.

6.6 Monocular and stereo head description

In this section it is presented the description of the active vision systems used in this thesis. First the monocular mechanical system is described and later the stereo system. In this section it is also described the firmware used inside the microcontroller to assure the electronic control of both vision systems.

6.6.1 Mechanical description

The monocular head system (figure 6.6) consists of two aluminium links coupled to two motors in such a way that the complete system has two degrees of freedom. The first link is coupled to the motor 1 allowing to have an angular movement on the z axis of the monocular head, while the second link is coupled to the motor 2 and the camera; allowing to have a movement around the x axis of the monocular head. With this configuration controlling appropiately the movement of the motor 1 can be accomplished the object tracking in the x coordinate of the image, while if it is controlled appropiately the motor 2 can be accomplished the object tracking in the y coordinate of the image.

For the angular movement of the system was chosen by their relationship of speed and control easiness servomotors. For the Motor 2 a servomotor J/R S-3251 was chosen, while for the motor 1 which supports the total system weight was chosen the servomotor FUTABA (conrad ES-030), that is lightly bigger in relation to the motor 2. The images of the monocular head are captured by a USB camera that gives 30 frames per second, where each frame has the

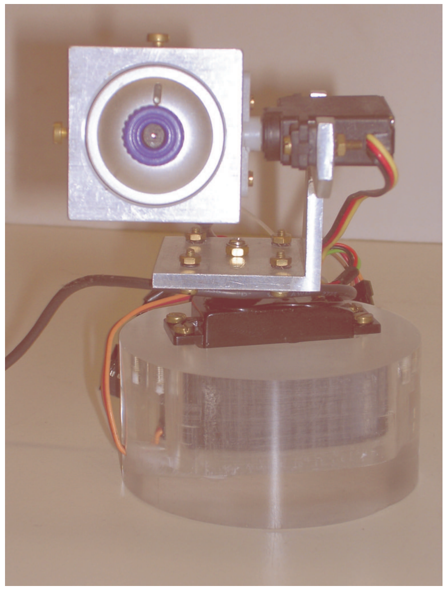

Figure 6.6: Physical monocular head system.

dimensions of 352 x 288. The figure 6.7 shows the complete system as well as the lateral and top views.

The stereo head was built following the same model that the monocular, this is with two degrees of freedom generated by two motors acting in different ortogonal axes (figure 6.8). The motors of the stereo system are two FUTABA (conrad ES-030) servomotors whose torque allows to support bigger loads in comparison to the monocular system.

The stereo head possesses 2 cameras mounted in the superior part that allow to follow objects and to measure its respective distance. The cameras have an interface USB and work to a speed of 15 frames per second, each camera gives an image of dimensions 240 x 176 . The figure 6.9 shows the complete system as well as the lateral and top views.

6.6.2 Electronic description

Because the speed of the object to track is uncertain, the motors that in this case determine the tracking speed should be controlled to react swiftly to the changes that experiences the object to follow. To fulfil these objectives a control system based on PIC microcontroler to regulate the behavior of both servomotors was designed, as well as to communicate to high speed by means of control commands with the PC.

Most of the existent circuits in the market for servomotor control have answers intervals from 0.5 to 0.25 seconds, this speeds can be considered as inad-

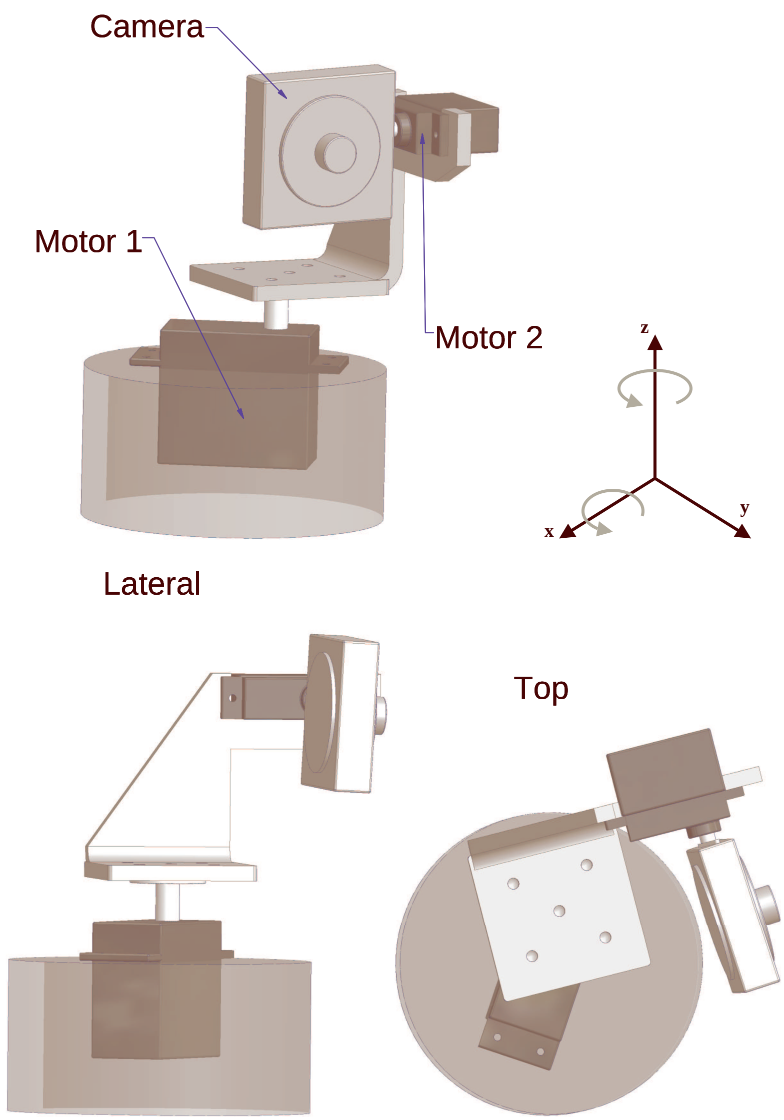

Figure 6.7: Monocular head system.

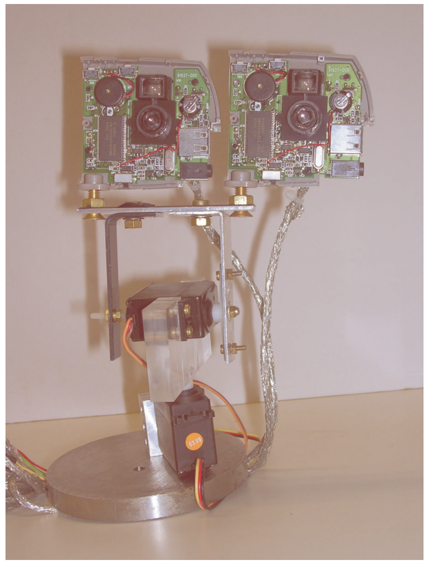

Figure 6.8: Physical stereo head system.

missible for a tracking application in real time. Another additional problem is the communication protocol with the electronic circuit which is based usually on the send and verification of data, this makes slower the positions processing that the motors should carry out. Also most of the circuits are not distributed as circuit integrated but as monolithic electronic target with several components that which complicates the flaw detection. For all this in this thesis was preferred to develop a servomotor controller that does not have the previous problems. The developed controller is based on the microcontroller PIC16F84, which being based on a recursiv program allows to improve the answer speeds to the minimum as well as to assure the simplicity of the communication protocol. The programmed device will be in the successive identified as servo-DRIVER.

6.6.3 Program description

The servoDRIVER program can be divided in three parts. In the first part all the internal resources of the microcontroller are configured, in the second part the initial positions of the motors are configured, finally the third part has an infinite cycle that consists of two phases that act jointly; in the first one the position of the servo motors it is refreshed one by one while in the other one is proven if exist data in the communication port to change the position of the motors. The program description can be summarized in the flow diagram of the figure 6.10.

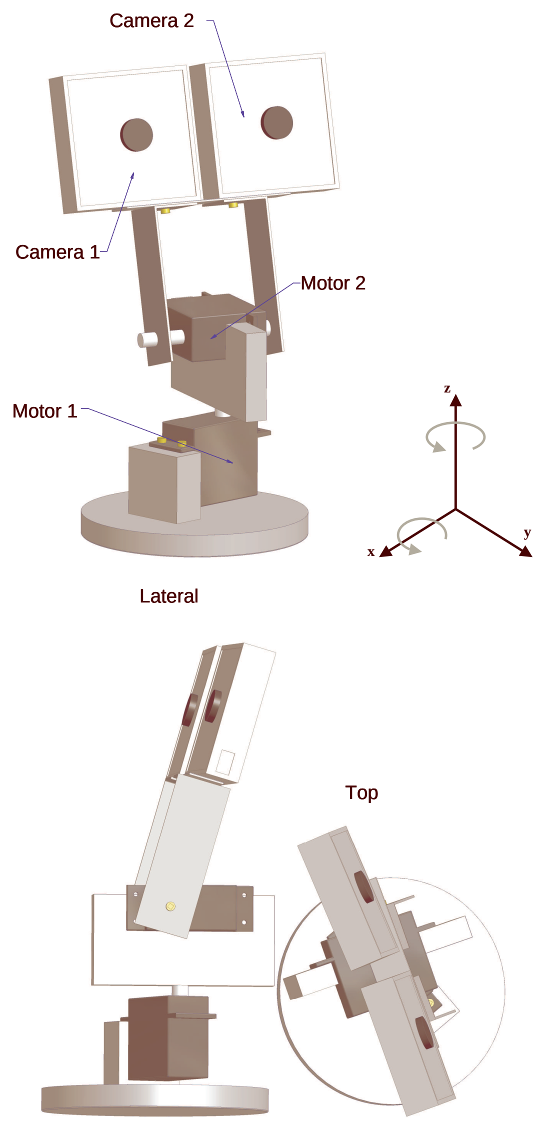

Figure 6.9: Stereo head system.

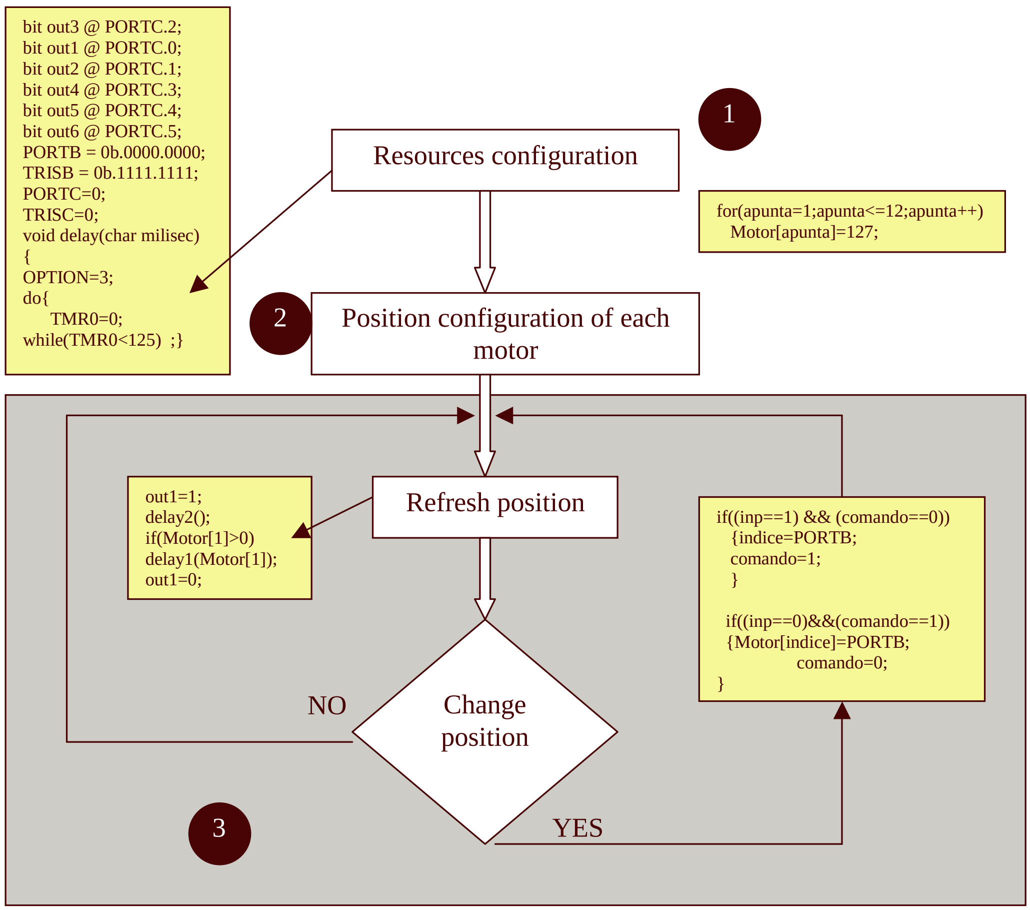

Figure 6.10: Flow diagram of the servoDRIVER program.

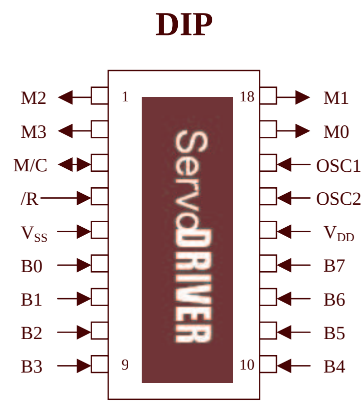

Figure 6.11: Integrated circuit ServoDRIVER

The program was coded completely in C language using the compiler HI-THEC®.

6.6.4 Operation

The servoDRIVER allows to control 4 motors simultaneously, its pins distribution is shown in the figure 6.11. The pin description can be seen in the table 6.1.

When the Chip is connected or reseted places all the motors halfway its value taking a position of 127. The chip operation is made in two phases in the first it is sent the number of motor that is wanted to control and later the position for it.

6.6.4.1 Phase 1

To select the motor only should be placed first in the data bus (B0 ...B7) the motor number (a number from 0 to 3), if in any way a superior value is sent (that does not correspond to any motor), the chip simply ignores it. After it is placed the pin M/C at 1 logical maintaining the state as minimum **1 ms**.

6.6.4.2 Phase 2

To send the position, it is carried out a similar process. First it is placed in the data bus (B0 ...B7) the position coded as minimum 0 and as maximum 255. Assuring this we have only to put the pin M/C at 0 and to maintain it as minimum **1 ms.**

Pin	Description
M0,M1,M2 y M3	Ouputs (Servomotors)
B0..........B7	if M/C=0 Angular position of the motor from 0 to 255 if M/C=1 Motor number (0,1,2,3)
V_{SS}	Ground
V_{DD}	3 to 5 Volts
OSC1 y OSC2	Oscillator 8 MHz
R	Reset
M/C	Selection of internal register 1 Motor number 0 Position

Table 6.1: Pin description

6.6.5 Connection

For the chip operation should be connect two very simple auxiliary circuits one for the operation of the oscillator and another for the chip reset.

For the oscillator operation can be considered two different options; to use an quartz oscillator (figure 6.12 Top) or a resonator (quartz and capacitors are in the same package, figure 6.12 Down).

In both cases the quartz oscillator or resonator should be of 8 MHz and in the case of using quartz oscillator the capacitors C1 and C2 are of 22pF.

In the chip operation is important to have the possibility to begin the control of the motors in an initial possition, this is specially desirable in those cases in which the motors take harmful positions for the mechanical system and we want to place the motors in the initial position without necessity of energy desconection. The circuit that is proposed for this objective is schematized in the figure 6.13.

6.6.6 Connection with the PC

The servoDriver to control the position of the motors receives the position commands of the PC parallel port actively. The connection is made connecting the data bus from the parallel port to the pins of the servoDRIVER labelled as B0,B1,B2,B3,B4,B5,B6 and B7, as well as to the pin M/C. The figure 6.14 shows the details of the interface.

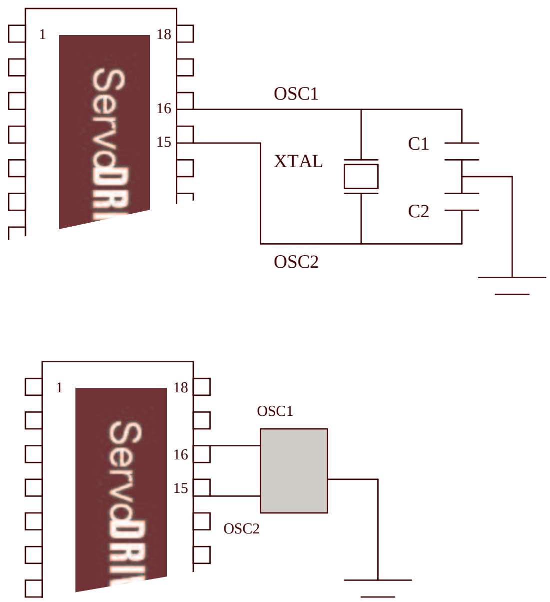

Figure 6.12: *Top:* Quartz oscillator, *Down*: resonator.

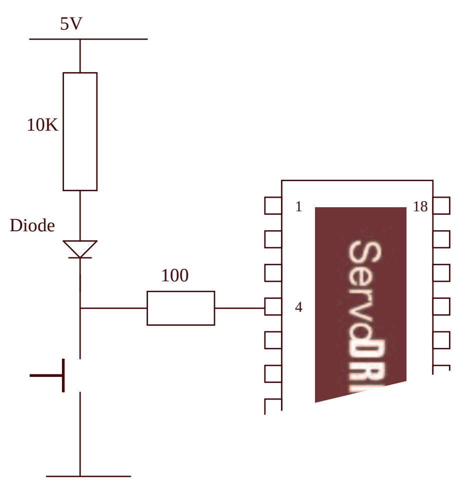

Figure 6.13: Reset circuit.

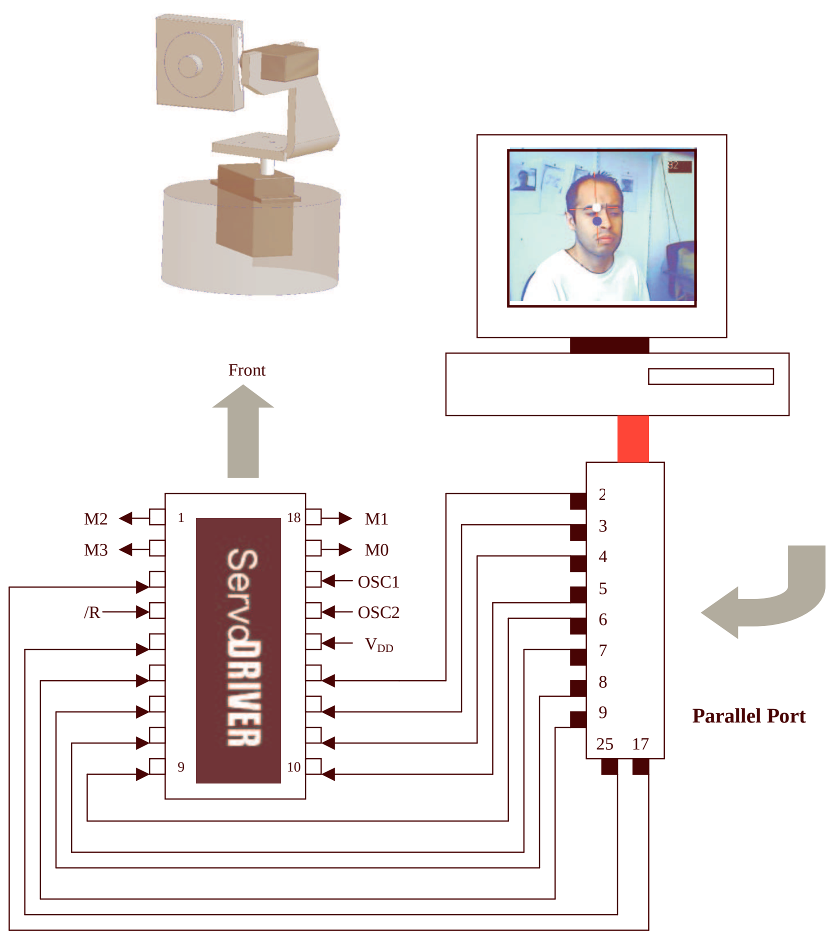

Figure 6.14: Connection with the PC