Chapter 5

Economies of Scale and the Intra-Household Distribution of Income

"Two live as cheaply as one." This proverb is often cited in the context of equivalence scales measurement and theory.[1] Two could indeed live as cheaply as one, if they could share everything and would share it evenly. Some goods are public and can be used jointly and thus be shared without giving up anything. Other, private goods have to be divided, but what is lost by sharing might be made up in love and kindness.

Maybe even a romantic soul would doubt that love and kindness is a sufficient compensation. In any case, with respect to purely material living standards, one is not interested in love and kindness but in the purely economic level of well-being in the sense of purchasing power. Even if two are not living as cheaply as one, there are certainly great opportunities for joint consumption. By cooking together, living in the same house, maybe jointly using a car and other services, two live more cheaply together than they would if they were living separately.

Even though this is not stated explicitly in the proverb, the romantic would imagine that in the perfect couple resources would be pooled and shared unanimously between partners according to their needs. But in the real world couples are not perfect and marriage is a contract; ressources may be consumed jointly, but their distribution might depend as much on the partners' bargaining positions as on affection. Pooling has indeed been rejected many times in the literature (see below for a review). Consequentially a more complex model of the household decision process is needed for the estimation of equivalence scales. This is the topic of this chapter.

[1] (e.g. Nelson, 1993; Kakwani and Son, 2005)

The cost of living of a single relative to a couple depends both on the possible economies of scale from joint consumption, and on intra-household distribution. If one partner gets a smaller share of household consumption, then he or she also needs a lower income to attain the same standard of living, when living alone. Combining joint consumption and sharing makes the task of calculating equivalence scales for couples and singles more complex, but there is also an important benefit. Unitary equivalence scales that take households as the centre of scrutiny usually suffer from an identification problem, because they encompass interpersonal or rather inter-household utility comparisons. With an individualistic view of the household, equivalence scales can be calculated using a situation comparison, comparing for the same type of person, a man or a woman, the situation of living alone with the situation of living with a partner (Lewbel, 2002).[2] While the matching of indifference curves between households is not unique, it is unique in a situation comparison, because the indifference curves belong to the same person.

Preferences of men and women can be recovered from data of male and female single households. Taking these estimated preferences as an input, the economies of scale from joint consumption and the level of sharing between partners can be identified from data of couples. Some caution has to be applied, though, because in the presented model, changes in preferences that occur when a person moves in with a partner cannot be separated from economies of scale.

There are two central results of the enquiry. *One:* Under equal sharing, two living together have about 1.4 times the cost of one. *Two:* Pooling (i.e. equal sharing of all resources) is strongly rejected for unmarried partners, while married couples share their ressources almost evenly: marriage matters. While relative personal income has no significant effect on distribution in married couples, it has a strong effect among unmarried couples. Total income, however, has a negative effect on the woman's share in both groups.

The chapter is structured as follows: The first part gives a review of the literature, the second deals with the technology of joint consumption. In the third part the collective model of equivalence scales is developed. A general identification result is given in the fourth part which is the centre of the chapter together with a specific parametric result for the QES, given in the appendix. The model is applied and estimates are discussed in the last part of the chapter.

[2] In the Rothbarth, Barten and Gorman models of chapters 3 and 4, the problem was solved by taking the couple as a reference point.

5.1 Relevant Literature

The model that is presented in this chapter can rely on a small body of work on economies of scale in joint consumption. Lau (1985) develops a theory of joint consumption based on axiomatic assumptions on the properties of a joint consumption function. The Barten (1964) model of household utility and composition, where composition enters the utility function as an effect on prices, can be interpreted as a special case of Lau's consumption technology framework. This model has been frequently used to estimate economies of scale for individual commodities as well as to estimate equivalence scales (Nelson, 1988; Lazear and Michael, 1980). Gorman's (1976) linear consumption technology is a generalization of the Barten model, but it is not consistent with Lau's framework, because it violates a homogeneity condition: While Lau's joint consumption technology is independent of the total quantity consumed, the Gorman technology is not. Lau's model as well as its relation to the Barten and Gorman models is discussed in Section 5.2.

Considerably more work has been done on the perhaps more interesting and complex intra-household decision process. According to traditional theory, the household can be modelled as a unitary entity, where pooled resources are spent as if the household had a single utility function (Samuelson, 1956; Becker, 1981). In applied work, however, the unitary model is frequently rejected (see below). Thus, following the seminal works on Nash bargained household equilibria by Manser and Brown (1980) and McElroy and Horney (1981), a wide variety of models describing the intra-household decision process has been developed. Most of these models are derived using cooperative game theory.[3] Cooperative models assume an efficient bargaining solution that depends on the bargaining power of partners, where either dissolution of the marriage or a non-cooperative behaviour is used as a threat point in the bargain.

The collective model developed by Chiappori (1988, 1992) simply assumes a Pareto efficient solution to the bargaining problem. The particularities of the bargaining process are of no interest in the collective model, the focus rather lies on a quantification of the sharing rule and tests on pooling and on the validity of the Pareto optimality assumption. Chiappori et al. (2002) derive testable restrictions on labour supply from the collective model. The model is not rejected.

Following the theoretical foundations of the non-unitary view of the household, pooling has been tested by many authors. One group of tests relies on the effect of different bargaining positions of partners on children. In a study of fertility and female labour supply in Thailand, Schultz (1990) rejects the

[3]However, there are non-cooperative models as well. Supporting empirical evidence to non-cooperative behaviour is given by Udry (1996) who finds that the allocation of household resources to male- and female-controlled agricultural plots in Burkina Faso is inefficient.

neoclassical restriction that income is pooled and partners exhibit the same behavioural preferences. Thomas (1990) examines the effect of individual income on children's health with Brazilian dataand rejects income pooling. Lundberg et al. (1997) use a policy change in the UK child benefit, where a substantial amount of income was transferred from the male earner to the mother, to test for pooling. They reject pooling for families with two or more children as well.

Another group of tests observes the direct distribution between partners within (usually childless) couples. Phipps and Burton (1997) test pooling on Engel curves for 14 categories of goods. They find pooling for some categories but not for others: income spent on restaurant food, household food, wife's and men's clothing, child care and transportation flows is not pooled, while the pooling hypothesis cannot be rejected for housing and household operations, recreation flows and stocks, donations and tobacco and alcohol. The collective model is tested by Bourguignon et al. (1993). Income pooling is rejected for a sample of French data. Browning et al. (1994) test pooling in a collective model with Canadian data and reject it. The effect of personal income in married couples is significant but not strong. Going from 25% of earned income to 75% increases the share of a partner by just about 2.3 percent. On the other hand, the effect of total expenditure on distribution is quite substantial. A 60 percent increase in total expenditure increases the wife's share by about 12 percent.

Despite the long list of works on bargaining models only few authors combine bargaining and joint consumption. Apps and Rees (1997) extend the collective model by adding economies of scale in household production. For an identification of the model, however, knowledge of individual consumption of household members is needed. This is a serious drawback, because data on individual members' consumption are usually not collected in household surveys.

Bradbury (1997) also develops a collective model with joint consumption. He uses assumed scale factors instead of estimates and performs a sensitivity analysis of the effect of the size of the assumed scale factors on estimated equivalence scales. Bradbury finds that among retired people in Australia income is shared relatively evenly. This work is one of the sparse examples where equal sharing cannot be rejected.

Browning et al. (2004) and Lewbel (2002) show that a collective setting, which is augmented by a linear (Barten or Gorman) joint consumption technology, can be fully identified using data of singles and couples that contains price variation. These authors call the resulting equivalence scales a "collective model based equivalence scale". The model presented in this chapter is largely based on their work. The difference lies in the estimation process. In contrast to Browning et al.'s model, the model presented here is estimated in a parametric framework using a quadratic expenditure system, constant prices and restrictions on the joint consumption parameters.

5.2 The Technology of Joint Consumption

When a good is consumed jointly by several persons in a household, there are possibly *economies of scale* in consumption: The sum of the services flowing from the joint use of a good can be larger than the flow of services would be if a person used the same amount of the good alone. E.g. the flow of services from a TV set is doubled when two persons are sitting in front of it. There is a limit to the multiplication of services because of congestion: if there are so many people in front of the TV that not everyone has a good view on the screen, or if not everyone wants to watch the same program. Another example are the services of a telephone, the joint use of which is limited as well. Also parts of a dwelling can be shared. The kitchen of a family of five does not have to be five times as large as the kitchen of a single person to have the same "personal kitchen space". All these economies of scale result from the fact that goods in a household of several persons are partly or completely public and that the public part is used jointly.

Additional savings can arise from the fact that larger quantities can be bought more cheaply. This applies to bulk rebates in supermarkets as well as to the housing market, where the average rent per square meter for smaller apartments is often higher than for larger ones. As opposed to economies of scale that result from a shared flow of services from a certain good, these savings arise from a lower price. Both effects can be combined. In fact, both effects cannot be distinguished if unit prices are not observed.

The living standard of a household's members is determined by the effective flow of services they receive from the use of goods. Yet, in most budget surveys only the total household expenditure on a good is recorded. To separate the consumption by household members it is necessary to build a theory of joint consumption.

In the following section the theory of joint consumption is developed for the case of two persons (m and f). The model is easily extended to more than two persons. The exposition follows closely the work of Lau (1985). The theory has also been used by Bradbury (1997) in the context of family equivalence scales.

If the two members of the household consume the individual quantities q_i^m and q_i^f, then the total observed household consumption q_i is a function of the individual consumptions q_i^m and q_i^f:

$$q_i = F_i(q_i^m, q_i^f) \quad , \tag{5.1}$$

where F_i is assumed to have the following properties (a graphic illustration will be given in Figure 5.1 below):

Single-consumer equivalence. If only one person consumes a good then the total quantity consumed is equal to this person's consumption:

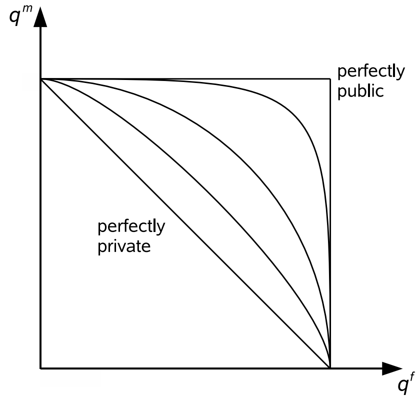

Figure 5.1: *Economies of scale from joint consumption: different degrees of economies of scale from the perfectly private to the perfectly public good.*

$F_i(q_i^m, 0) = q_i^m$ and $F_i(0, q_i^f) = q_i^f$.

Symmetry. The technology is independent of the final consumer of the service, that is: $F_i(q_i^m, q_i^f) = F_i(q_i^f, q_i^m)$

Monotonicity. If the quantity of services for one individual is increased, while the quantity for the other individual is held constant, then the total quantity will not be reduced: $F_i(q_i^m + \epsilon, q_i^f) \geq F_i(q_i^m, q_i^f)$ for $\epsilon > 0$.

Quasiconvexity. For a given total amount of a good, the set of possible pairs of services q_i^m, q_i^f is a convex set. This assumption implies that, given the quantity of a good purchased by the household, the total of individual consumption will not decrease, if personal consumption levels are made more equal.

Homogeneity. If the quantity of services for both individuals is increased by the same factor, then the increase of the total amount is proportional: $F_i(\lambda q_i^m, \lambda q_i^f) = \lambda F_i(q_i^m, q_i^f)$

The following properties are a consequence of the assumptions:

Origin. If the consumed individual quantities of good i are zero, then the total quantity of the good is zero: $F_i(0, 0) = 0$.

Positivity. If the consumed individual quantities are positive, then the total quantity is positive: $F_i(q_i^m, q_i^f) > 0$ unless $q_i^m = q_i^f = 0$.

Subadditivity The sum of individual quantities is always greater than or equal to the total quantity consumed: $q_i^m + q_i^f \geq F_i(q_i^m, q_i^f)$

Figure 5.1 shows possible joint consumption functions with different degrees of economies of scale from the perfectly private to the perfectly public good. In the case of the private good, the joint consumption function is simply the sum of private consumptions $F_i(q_i^m, q_i^f) = q_i^m + q_i^f$. For the public good, the joint consumption function is the maximum of private consumptions $F_i(q_i^m, q_i^f) = \max(q_i^m, q_i^f)$, as one partner can freely use the good up to her partner's consumption without adding to the total amount consumed.

What is the relation between the joint consumption function and individual household members' demand functions? Of course the possibility of joint consumption affects the effective price that each member has to pay for a certain consumed quantity. Because of the subadditivity property, instead of paying the amount of $x_i = p_i(q_i^m + q_i^f)$ when they are consuming separately, a couple has to pay only the amount of $x_i = p_i F_i(q_i^m, q_i^f)$, which is smaller than the first amount if the good is not perfectly private and consumed by both partners. The couple as a unit has to pay an effective price of $\pi_i = p_i F_i(q_i^m, q_i^f)/(q_i^m + q_i^f)$.

The effective price can change with the individual consumption quantities because of the quasiconvexity axiom: Assume that the individual consumption quantities of both partners for a good i are different, say $q_i^f > q_i^m$. Quasiconvexity implies that a redistribution of individual consumption between partners by an amount ϵ, that is small enough as preserve the relation between individual consumption quantities ($q_i^f > q_i^m$), does not increase total household consumption: $F_i(q_i^m + \epsilon, q_i^f - \epsilon) \leq F_i(q_i^m, q_i^m)$. In the general case of a non-linear joint consumption technology, the effective price will be reduced, when individual consumption quantities are more equal, and prices prices will vary in a complicated way with the relative consumption of the good between partners.[4] The effective price of the good will be constant only for a particular type of linear joint consumption function.

Consequentially, with a non-linear joint consumption function, the additional amount that has to be paid for a marginal consumption unit of a good i is not independent of which partner is consuming it. The relative amount that partners have to pay for a marginal consumption unit is given by the slope of the joint consumption function. This is in general equal to one only at the point of equal sharing, due to the symmetry property. The marginal gain in personal consumption per additional unit purchased is higher for the partner who consumes less of the good than for the one who consumes more.

This relationship between relative consumption and price renders the solution of any household maximization problem that relies on the joint consumption function intractable, more so in applied econometric work. Bradbury (1994) points out that a linear joint consumption technology also satisfies all properties of a joint consumption function, without the complication of prices that are changing with changing relative consumption quantities. A graphic

[4] This is also emphasized by Bradbury (1994) and Browning et al. (2004).

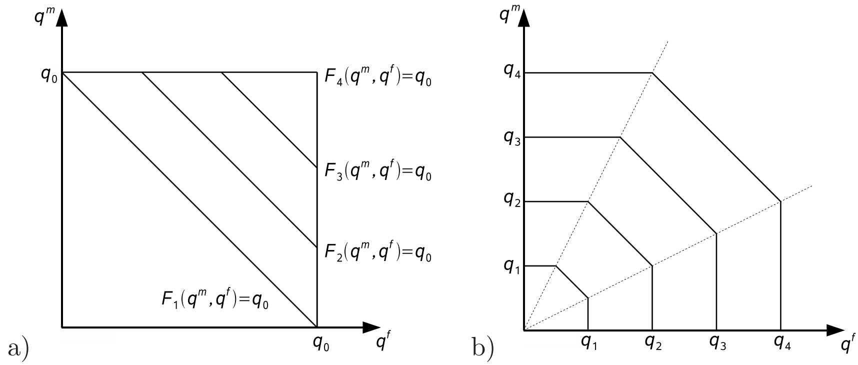

Figure 5.2: *Linear technology of joint consumption: a) different degrees of economies of scale from the perfectly private to the perfectly public good. b) Barten technology: Same economies of scale for different consumption levels.*

representation of the proposed linear joint consumption function is given in Figure 5.2: a) shows linear joint consumption functions with different degrees of scale economies but for the same total household consumption quantity q_0. $F_1(q^m, q^f)$ is the joint consumption function for a perfectly private good, while $F_4(q^m, q^f)$ represents a fully public good. b) shows a linear joint consumption function for a good that is half public/half private for different total consumption quantities (q_1 to q_4).

The linear consumption technology can be represented by Barten like scale factors m_i, where the joint consumption function is equal to:

$$F_i(q_i^m, q_i^f) = \begin{cases} m_i(q_i^m + q_i^f) & \text{for } \frac{m_i}{1-m_i} q_i^f > q_i^m > \frac{1-m_i}{m_i} q_i^f \\ \max(q_i^m, q_i^f) & \text{otherwise} \end{cases} . \tag{5.2}$$

The interpretation of the scale factors here is somewhat different from the interpretation for the case of families with children in chapter 4. While factors for families include the additional cost for children (and a wife) ("A penny bun costs threepence if you have a wife and a child"), factors here describe the individual savings from sharing ("Two live as cheaply as one"). In both cases, however, the childless couple is the reference.

The factor m_i for one person relative to a couple is equal to the distance from the origin to the point of equal sharing relative to the same distance for the fully public good. The possible range for the m_i parameter is therefore 0.5 for a private good to 1.0 for a public good. For equal personal consumption, a single adult needs half as much as two adults of an evenly shared private good and the same amount as a couple for a fully public good.

The linear joint consumption function implies that there is a range of values for q^m and q^f where the good in question is partly private and the scaled price for both partners is the same. If, however, the demands for the good are too different, the behaviour of the consumption function switches and the total consumption is determined by the consumption of the partner who consumes more of the good, while the other partner can free-ride on the public part of his partner's consumption. Even though it is not ruled out a priori, an outcome that lies outside the kink points of the joint consumption function – on the horizontal and vertical parts of the function – will not be observed. Because here the marginal cost of consumption to the other partner is zero, he or she can increase consumption up to the kink at no additional cost.

If consumption patterns of both partners are similar enough, their consumption can be approximated by the shared part of the linear joint consumption function and the consumption function does not depend on individual consumption quantities (q_i^f, q_i^m) but on their sum $(q_i^f + q_i^m)$:

$$F_i(q_i^m, q_i^f) = \tilde{F}_i(q_i^m + q_i^f) = m_i(q_i^m + q_i^f). \tag{5.3}$$

For this approximation to work, consumptions have to be more similar for more public goods.[5]

The linear (Barten) joint consumption function can be generalized to a Gorman technology (Gorman, 1976) by adding overheads β_i:

$$F_i(q_i^m, q_i^f) = \begin{cases} \max(q_i^m, q_i^f) & \text{for } m_i(q_i^m + q_i^f) + \beta_i < \max(q_i^m, q_i^f) \\ q_i^m + q_i^f & \text{for } q_i^m + q_i^f > m_i(q_i^m + q_i^f) + \beta_i \\ m_i(q_i^m + q_i^f) + \beta_i & \text{otherwise} \end{cases} . \tag{5.4}$$

With this type of technology, economies of scale can increase or decrease with the amount of total consumption (Figure 5.3 a) and b), respectively). The case differentiation is necessary to prevent the function from violating the subadditivity and single-consumer equivalence. The overhead β_i can be positive or negative depending on the good getting more public or less public with higher total consumption. The scaling parameter can take any positive value, even outside the range of 0.5 to 1.0, depending on the sign of the overhead. With a positive overhead it has to be lower than one, while with a negative overhead its value has to be higher than 0.5.

[5]Equation 5.3 can also be interpreted as a first order approximation of any joint consumption function around the point of equal sharing. An approximation of a non-linear joint consumption function around any other point would have very different properties, namely the slope of the function would not be minus one: both partners would "pay" different effective prices for the good. In addition, the symmetry property would be violated by the approximation (but not by the approximated joint consumption function).

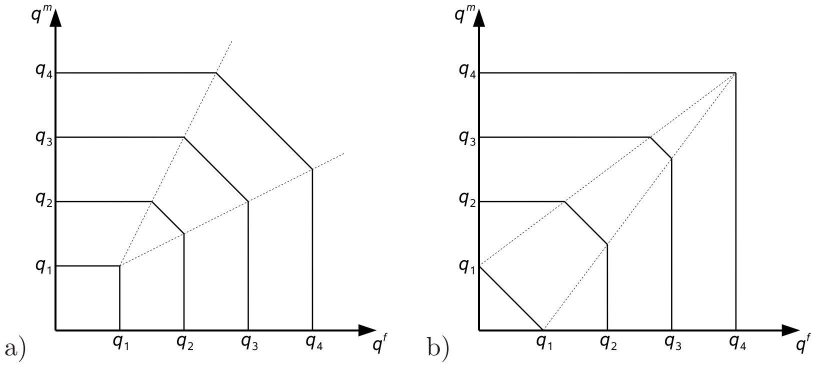

Figure 5.3: *Gorman technology: a) good gets* less *public with increasing total consumption. b) good gets* more *public with increasing total consumption.*

In contrast to the pure Barten technology, for the inner part of the formula to apply $(m_i(x_i^m + x_i^f) + \beta_i)$, the consumption of the couple is not only restricted to sufficiently similar individual consumptions, but also to a limited range of total consumption.

The Gorman technology does violate the homogeneity condition. However, it is quite conceivable that for larger aggregates the degree of sharing can vary with total consumption. E.g. for a poor household that does not own a car, public transportation is entirely private, even if both partners travel together, while a middle income household can share a car. Sharing can even decrease again for a rich household: for a couple that owns two cars and goes on vacation by plane, transportation is perfectly private again. Another example is housing. If there are some private rooms in the house and some public rooms, and the amount of public rooms increases disproportionately with the size of the house, the possibility of sharing is increased and homogeneity is violated.

Both Barten and Gorman joint consumption functions can be written in matrix form with $\boldsymbol{F}(\boldsymbol{q}^m, \boldsymbol{q}^f)$ being the vector of household consumption quantities:

$$\boldsymbol{F}(\boldsymbol{q}^m, \boldsymbol{q}^f) = \boldsymbol{M}(\boldsymbol{q}^m + \boldsymbol{q}^f) \tag{5.5}$$

and

$$\boldsymbol{F}(\boldsymbol{q}^m, \boldsymbol{q}^f) = \boldsymbol{M}(\boldsymbol{q}^m + \boldsymbol{q}^f) + \boldsymbol{\beta}, \tag{5.6}$$

respectively. $\boldsymbol{M}$ is a diagonal matrix with the m_i as its diagonal elements and $\boldsymbol{\beta}$ is the vector of the overheads β_i.

5.3 A Bargaining Model of the Household

In a couple, both partners have individual utility functions $U^f(\boldsymbol{q}^f)$ and $U^m(\boldsymbol{q}^m)$ with the usual properties of transitivity and completeness. The distribution within the household is described by the maximization of a social welfare or bargaining function $\tilde{U}$. Assuming there are only private goods, $\tilde{U}$ is maximized at the household optimum according to:

$$\max_{\boldsymbol{q}^f,\boldsymbol{q}^m} \left\{ \tilde{U}\left[U^f(\boldsymbol{q}^f), U^m(\boldsymbol{q}^m), \boldsymbol{p}, \mu\right] | \boldsymbol{p}'(\boldsymbol{q}^f + \boldsymbol{q}^m) = \mu \right\} \tag{5.7}$$

The bargaining function has very general properties. It is not necessary to make any assumptions on exactly how a bargaining solution is achieved. It is merely assumed that household decisions are Pareto efficient and that $\tilde{U}$ is separable in the individual utility functions U^f and U^m which take only the personal consumption quantities of household members as their arguments.

The assumption of a Pareto efficient bargaining outcome is plausible, despite occasional anecdotal evidence to the contrary. Partners engage in repeated bargains and supposedly know each other's preferences well enough, making a Pareto efficient solution very likely. Less plausible is the assumption that there are no external effects, i.e. that the consumption of one partner does not influence the other's well-being. Drinking and smoking are obvious examples. Clothing could also be affected: partners may be not indifferent to their partner's looks.

The Pareto efficient solution of the maximization problem can be decentralized via a sharing function ϱ. With private goods, the solution is:

$$\boldsymbol{q} = \boldsymbol{q}^f(\boldsymbol{p}, \varrho\mu) + \boldsymbol{q}^m(\boldsymbol{p}, (1-\varrho)\mu) \ . \tag{5.8}$$

Following Chiappori (1988) a suitable functional form of the sharing function is:

$$\varrho = \varrho_0 + \varrho_1\mu + \boldsymbol{\varrho}_z{}'\boldsymbol{z} \ , \tag{5.9}$$

where μ is total household income and $\boldsymbol{z}$ is the vector of variables that influence the distribution within the household. The most important of these is a measure of relative personal income, either expressed as the relation of partners' incomes or as the share in personal income of one partner. The distribution parameters can also include differences in age or education.

To model economies of scale that are possible inside the household, a joint consumption function $\boldsymbol{F}(\boldsymbol{q}^f, \boldsymbol{q}^m)$ is introduced. To make the maximization problem tractable, the linear case of the joint consumption function, $\tilde{\boldsymbol{F}}(\boldsymbol{q}^f + \boldsymbol{q}^m)$, rather than the general case is used, as discussed above. The maximization problem becomes

$$\max_{\boldsymbol{q}^f,\boldsymbol{q}^m} \left\{ \tilde{U}\left[U^f(\boldsymbol{q}^f), U^m(\boldsymbol{q}^m), \boldsymbol{p}, \mu\right] | \boldsymbol{p}'\tilde{\boldsymbol{F}}(\boldsymbol{q}^f + \boldsymbol{q}^m) = \mu \right\} \tag{5.10}$$

and the decentralized solution becomes

$$\boldsymbol{q} = \tilde{\boldsymbol{F}}(\boldsymbol{q}^f(\boldsymbol{\pi}, \varrho\mu) + \boldsymbol{q}^m(\boldsymbol{\pi}, (1-\varrho)\mu)) \ , \tag{5.11}$$

where $\boldsymbol{q} = \tilde{\boldsymbol{F}}(\boldsymbol{q}^f + \boldsymbol{q}^m)$ is the bundle of goods that the household is observed purchasing and $\boldsymbol{\pi}$ is the vector of effective prices. The bundle $\boldsymbol{q} = \tilde{\boldsymbol{F}}(\boldsymbol{q}^f + \boldsymbol{q}^m)$ with sharing is equivalent to consuming the bundle $\bar{\boldsymbol{q}} = \boldsymbol{q}^f + \boldsymbol{q}^m$ without sharing. The individual utility functions can be derived from the observation of men and women living alone.

For a Barten joint consumption function, the vector of scaled prices takes a particularly simple form with:

$$\boldsymbol{\pi}(\boldsymbol{p}) = \boldsymbol{M}\boldsymbol{p} \ . \tag{5.12}$$

while with Gorman technology it becomes[6]

$$\boldsymbol{\pi}(\boldsymbol{p}, \mu) = \frac{\mu}{\mu - \boldsymbol{\beta}'\boldsymbol{p}} \cdot \boldsymbol{M}\boldsymbol{p} \ . \tag{5.13}$$

Using the cost function, equivalence scales for men and women relative to a couple can be derived at any given utility level $\tilde{u}$:

$$\begin{aligned} m_0^f &= \frac{c^f(\tilde{u}, \boldsymbol{p})}{c^f(\tilde{u}, \boldsymbol{\pi})/\varrho} \\ m_0^m &= \frac{c^m(\tilde{u}, \boldsymbol{p})}{c^m(\tilde{u}, \boldsymbol{\pi})/(1-\varrho)} \ . \end{aligned} \tag{5.14}$$

These equivalence scales depend on the sharing rule. Reflecting the different shares in household income and expenditure, they do not have the same value for men and women. In general, scales differ even under equal sharing, because of different preferences of men and women: First, different substitution elasticities could allow one gender to adapt better to changing effective prices than the other. Second, if one partner has a higher preference for goods with higher economies of scale than the other, then this partner will have higher equivalence scales, because he loses more after separation.

[6]Equation 5.13 is related to the Gorman joint consumption function 5.6 via $\boldsymbol{\pi}'(\boldsymbol{q}^f + \boldsymbol{q}^m) = \mu$ and $\boldsymbol{p}'\boldsymbol{F}(\boldsymbol{q}^m, \boldsymbol{q}^f) = \mu$. Of these two equations, the latter can be transformed to $\boldsymbol{p}'\boldsymbol{M}(\boldsymbol{q}^f + \boldsymbol{q}^m)\frac{1}{\mu - \boldsymbol{p}'\boldsymbol{\beta}} = 1$ (using 5.6), which together with the former (5.13).

5.4 General Identification

In this section it will be shown how the linear consumption function (Equation 5.5) can be identified, if single male, single female and couple households are observed and it is assumed that men and women living as singles and as couples share the same preferences.

Using the decentralized solution of the optimization problem (Equ. 5.11) and the Barten joint consumption function (Equation 5.5), the Marshallian demand equations can be written in a unified form for all household types:

$$q_i = g_i(\mu, \boldsymbol{\pi}, s^f, s^m) = m_i s^f g_i^f(\varrho^f \mu, \boldsymbol{\pi}) + m_i s^m g_i^m(\varrho^m \mu, \boldsymbol{\pi}) \ , \tag{5.15}$$

where s^f and s^m are the number of females and males in the household, $\boldsymbol{\pi}$ is the vector of scaled prices $(m_1 p_1, m_2 p_2, \ldots, m_n p_n)$, ϱ^f is the share in total expenditures of a woman and ϱ^m is the share of a man. In a single household all income goes to the single person in the household and for any household the equation holds: $s^f \varrho^f + s^m \varrho^m = 1$. The m_i are normalized to one for the single household and depend only on the number of persons in the household $s = s^f + s^m$.

For a single female or male household, equation 5.15 reduces to $q_i^t = g_i^t(u^t, \boldsymbol{p})$ with $t \in \{f, m\}$. In general, s^f and s^m can take any value, but because in the current application attention is restricted to singles and couples, s^f and s^m take only the values zero and one. This is sufficient to calculate any derivatives that are needed for identification. Even though s, s^f and s^m take only discrete values, it is convenient to calculate derivatives of the linear function that is defined by the two values (0 and 1) that the parameters s^f and s^m can take, as if the parameters were continuous.

Taking the derivative of Marshallian demands with respect to household composition reveals the effect of the change in household composition on distribution and economies of scale:

$$\begin{aligned} \frac{\partial q_i}{\partial s^t} = & \frac{\partial m_i}{\partial s} \frac{\partial s}{\partial s^t} \frac{q_i}{m_i} + m_i g_i^t + \\ & s^f m_i \left(\frac{\partial g_i^f}{\partial \varrho^f \mu} \mu \frac{\partial \varrho^f}{\partial s^t} + \sum_{j=1}^{n} \frac{\partial g_i^f}{\partial \pi_j} p_j \frac{\partial m_j}{\partial s} \frac{\partial s}{\partial s^t} \right) + \\ & s^m m_i \left(\frac{\partial g_i^m}{\partial \varrho^m \mu} \mu \frac{\partial \varrho^m}{\partial s^t} + \sum_{j=1}^{n} \frac{\partial g_i^m}{\partial \pi_j} p_j \frac{\partial m_j}{\partial s} \frac{\partial s}{\partial s^t} \right) \end{aligned} \tag{5.16}$$

Multiplication of (5.16) with $1/q_i$ and noting that $\partial s / \partial s^t = 1$ gives the semi-elasticity of demands with respect to household composition:

$$\frac{\partial q_i}{\partial s^t}\frac{1}{q_i} = \frac{\partial m_i}{\partial s}\frac{1}{m_i} + \frac{m_i g_i^t}{q_i} + s^f\frac{m_i g_i^f}{q_i}\left(\frac{\partial g_i^f}{\partial \varrho^f \mu}\frac{\varrho^f \mu}{g_i^f}\cdot\frac{\partial \varrho^f}{\partial s^t}\frac{1}{\varrho^f} + \sum_{j=1}^{n}\frac{\partial g_i^f}{\partial \pi_j}\frac{\pi_j}{g_i^f}\cdot\frac{\partial m_j}{\partial s}\frac{1}{m_j}\right) + s^m\frac{m_i g_i^m}{q_i}\left(\frac{\partial g_i^m}{\partial \varrho^m \mu}\frac{\varrho^m \mu}{g_i^m}\cdot\frac{\partial \varrho^m}{\partial s^t}\frac{1}{\varrho^m} + \sum_{j=1}^{n}\frac{\partial g_i^m}{\partial \pi_j}\frac{\pi_j}{g_i^m}\cdot\frac{\partial m_j}{\partial s}\frac{1}{m_j}\right) \quad . \tag{5.17}$$

This can be written as:

$$\phi_{it} = \gamma_i + \omega_i^t + s^f\omega_i^f\left(\eta_i^f\tau_t^f + \sum_{j=1}^{n}\varepsilon_{ij}^f\gamma_j\right) + s^m\omega_i^m\left(\eta_i^m\tau_t^m + \sum_{j=1}^{n}\varepsilon_{ij}^m\gamma_j\right) \quad . \tag{5.18}$$

$\gamma_i = \frac{\partial m_i}{\partial s}\frac{1}{m_i}$ is the elasticity of the scale factor and the direct effect of increased economies of scale in a larger household. ω_i^t is the direct effect of the additional person's demand for good i, where $\omega_i^t = \frac{m_i g_i^t}{q_i}$ is the share of one person of type $t \in \{m, f\}$ in the total consumption of the good. The other terms in Equation 5.18 are the income and price reactions of all household members, male and female. The income reaction is due to the redistribution within the household, where $\tau_t^f = \frac{\partial \varrho^f}{\partial s^t}\frac{1}{\varrho^f}$ and $\tau_t^m = \frac{\partial \varrho^m}{\partial s^t}\frac{1}{\varrho^m}$ are the elasticities of a woman's and a man's income share with respect to a change in household composition and η_i^f and η_i^m are the respective income elasticities of their demands for good i. The price reaction follows from the change in scaled prices which is caused by the changes in scale factors, where ε_{ij}^f and ε_{ij}^m are the uncompensated price elasticities of a single woman's or man's demands.

Identification of Expenditure Shares and Sharing Rules

In all household types the shares of all persons in the total consumption of each good and the shares in total expenditures add up to unity:

$$s^f\omega_i^f + s^m\omega_i^m = 1 \quad \text{and} \tag{5.19}$$

$$s^f\varrho^f + s^m\varrho^m = 1 \quad . \tag{5.20}$$

The aim of the exercise is to estimate the sharing rules for the couple $\tilde{\varrho}^f$ and $\tilde{\varrho}^m$. For singles only one sharing rule is determined, ϱ^m is not determined when $s^m = 0$, the same applies to ϱ^f when $s^f = 0$. To determine the elasticities of the sharing rule, a general form for ϱ^f and ϱ^m has to be found. It can be assumed that the relation of ϱ^f and ϱ^m is constant for all household types with $\varrho^f/\varrho^m = \tilde{\varrho}^f/\tilde{\varrho}^m$. This is no loss of generality in the present setting, because only couples and singles are observed. Then the sharing rule can be written as:

$$\varrho^t = \frac{\tilde{\varrho}^t}{s^m \tilde{\varrho}^m + s^f \tilde{\varrho}^f} \quad \text{for } t \in \{f, m\} \ . \tag{5.21}$$

Equation 5.21 can be used to determine the elasticities τ of the sharing rule with:

$$\tau_f^m = \tau_f^f = -\varrho^f \quad \text{and} \tag{5.22}$$

$$\tau_m^m = \tau_m^f = -\varrho^m \quad . \tag{5.23}$$

The share in total income ϱ^t of a person $t \in \{m, f\}$ is the weighted sum of her shares in in each good, where $\mathrm{w}_i = q_i p_i/\mu$ is the total expenditure share of good i in the household:

$$\varrho^t = \frac{\sum_{i=1}^n \omega_i^t p_i q_i}{\mu} = \sum_{i=1}^n \omega_i^t \mathrm{w}_i \tag{5.24}$$

Because there are only singles and couples in the sample, identification of expenditure shares and the sharing rule can be simplified. In a couple, $s^f = 1$ and $s^m = 1$. It follows that:

$$\tilde{\omega}_i^m = 1 - \tilde{\omega}_i^f \quad \text{and} \tag{5.25}$$

$$\tilde{\varrho}^m = 1 - \tilde{\varrho}^f \quad . \tag{5.26}$$

Using equations 5.22–5.24, the shares of men and women can be derived from the difference of the household composition elasticities, even without observing any price variation in the data:

$$\phi_{if} - \phi_{im} = \omega_i^f - \omega_i^m + (\varrho_f - \varrho_m)(\omega_i^f \eta_i^f + \omega_i^m \eta_i^m) \ . \tag{5.27}$$

For a couple Equation 5.27 can be written as:

$$\begin{aligned} \phi_{if} - \phi_{im} &= \tilde{\omega}_i^f - \tilde{\omega}_i^m + (\tilde{\varrho}_f - \tilde{\varrho}_m)(\tilde{\omega}_i^f \eta_i^f + \tilde{\omega}_i^m \eta_i^m) \\ &= 1 - 2\tilde{\omega}_i^f + (2\tilde{\varrho}_f - 1)(\tilde{\omega}_i^f \eta_i^f + (1 - \tilde{\omega}_i^f)\eta_i^m) \\ &= 1 + 2\tilde{\omega}_i^f + \left(-1 + 2\sum_{i=1}^n \omega_i^f \mathrm{w}_i\right)\left(\tilde{\omega}_i^f \eta_i^f + (1 - \tilde{\omega}_i^f)\eta_i^m\right) \end{aligned} \tag{5.28}$$

Income elasticities η_i^t for men and women can be recovered from data on single households, and expenditure shares w_i can be recovered from data on couples, while the household composition elasticities have to be calculated from combined data on couples and single households. Only the ω_i^f cannot be observed directly. There is a system of n generally independent equations which can be solved for the n unknowns, the ω_i^f. Then $\tilde{\varrho}^f$, $\tilde{\varrho}^m$ and ω_i^m can be determined from Equations 5.24–5.26.

The observation of an exclusive good considerably facilitates the identification of the sharing rule $\tilde{\varrho}^f$. For an exclusive women's good k, $\omega_k^f = 1$ and $\eta_k^m = 0$. ϕ_{kf} and ϕ_{km} can be calculated from the expenditure shares of the good for women's households w_k^f, and for couples w_k^c. By definition, the share of k in men's households is zero, and one gets:

$$\begin{aligned} \phi_{kf} &= 1 \quad \text{and} \\ \phi_{km} &= 1 - \frac{w_k^f}{w_k^c}. \end{aligned} \tag{5.29}$$

Then $\tilde{\varrho}$ can be calculated directly from Equation 5.27 with:

$$\tilde{\varrho} = 1 + \frac{1 - w_k^f / w_k^c}{2\eta_k^f} \ . \tag{5.30}$$

However, event though male and female clothing is observed separately and could be used as an assignable good, this restriction was not imposed in the estimation process.

Identification of scale factor elasticities γ_i

Once the shares and the sharing rule are identified, identification of the scale factor elasticities is straightforward, provided the matrix of individual uncompensated price elasticities $\boldsymbol{E}^t$, $t \in \{f, m\}$ can be recovered from data on single households. Let $\boldsymbol{\Omega}^t$ be a diagonal matrix with the ω_i^t as its diagonal elements. Then Equation 5.18 can be written for a couple in vector form, which can be solved for $\boldsymbol{\gamma}$:

$$\boldsymbol{\phi}_t = \boldsymbol{\gamma} + \boldsymbol{\omega}^t + \boldsymbol{\Omega}^f \boldsymbol{\eta}^f \tau_t^f + \boldsymbol{\Omega}^f \boldsymbol{E}^f \boldsymbol{\gamma} + \boldsymbol{\Omega}^m \boldsymbol{\eta}^m \tau_t^m + \boldsymbol{\Omega}^m \boldsymbol{E}^m \boldsymbol{\gamma} \tag{5.31}$$

Analogous to the result in chapter 4, identification is also possible in the framework of the QES, when price elasticities are not observed. Again, results can be sharply improved, if at least on scale factor elasticity is fixed. A specific identification result for the QES is given in Appendix 5.A to this chapter.

Identification of Derivatives of the Sharing Rule

According to Equation 5.9, the sharing rule can depend on household characteristics that influence the distribution between household members, but up to now only the absolute value of ϱ at mean household characteristics, $\tilde{\varrho}$, was identified. ϱ can depend on any distribution influencing household characteristics z, including income μ. For a couple, $\varrho^f = \varrho$ and $\varrho^m = 1 - \varrho$. Derive Equation 5.15 with respect to z:

$$\frac{\partial q_i}{\partial z} = m_i \frac{\partial g_i^f}{\partial \varrho\mu} \mu \frac{\partial \varrho}{\partial z} - m_i \frac{\partial g_i^m}{\partial (1-\varrho)\mu} \mu \frac{\partial \varrho}{\partial z} \tag{5.32}$$

This can be written in elasticity form:

$$\frac{\partial q_i}{\partial z}\frac{z}{q_i} = \frac{m_i g_i^f}{q_i}\frac{\partial g_i^f}{\partial \varrho\mu}\frac{\varrho\mu}{g_i^f}\frac{\partial \varrho}{\partial z}\frac{z}{\varrho} - \frac{m_i g_i^m}{q_i}\frac{\varrho}{1-\varrho}\frac{\partial g_i^m}{\partial (1-\varrho)\mu}\frac{(1-\varrho)\mu}{g_i^m}\frac{\partial \varrho}{\partial z}\frac{z}{\varrho} \tag{5.33}$$

$$\Leftrightarrow \qquad \phi_{iz} = \omega_i^f \eta_i^f \tau_z - \frac{\varrho}{1-\varrho}\omega_i^m \eta_i^m \tau_z \quad , \tag{5.34}$$

where τ_z is the elasticity of ϱ with respect to z. The characteristics elasticities of ϱ can be calculated directly from Equation 5.34.[7]

5.5 Estimation

The collective model is estimated in the framework of a quadratic expenditure system using data from a single cross section. A specific identification result for the collective model using the QES specification is given in appendix 5.A to this chapter.

For estimation, data on couples and single men (M) and women (F) from the EVS 93 were used. To reduce computational cost, and to reduce the effect of possible preference changes associated with age, the sample was limited to persons aged 30 to 50 living in rented housing, with men and women working at least part time. Sample size, net household income and total expenditures on the modelled goods basket[8] are given in Table 5.1.

The model was estimated for eleven commodity groups, which are identical to those used in chapter 4, except that clothing was separated into male and female clothing. The groups are: *food, female clothing, male clothing,*

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[7]Equation 5.34 also offers a test of the model. Because a change in distribution acts as an income effect in the same way on all demands, estimates of τ_z have to be identical among goods. However, in the estimation of the model in this chapter, equality of the τ_z among goods has been imposed and no such test has been carried out.

[8]Not all expenditures are included in the basket, such as insurance or health expenditures.

Household type	F	M	Couple	Of all couples: married	unmarried
# of cases	859	785	608	494	114
Net household income					
Minimum	5078	6553	19840	19840	20680
Median	37250	41410	68520	67620	70600
Maximum	108700	247700	264200	264200	167200
Total expenditures (μ)					
Minimum	8184	8214	14690	14690	19770
Median	29650	28750	47760	47760	47440
Maximum	83000	107700	154900	154900	97920

Table 5.1: *Case numbers of household types, net household income and total expenditures on the modelled basket of goods.*

housing, home & furniture, transportation, recreation, personal care, vacation, tobacco, and alcohol.

It is theoretically possible to determine all parameters of the QES from only one cross section, provided demand curves are sufficiently non-linear. Apart from the questionable practice of identifying price elasticities from non-linearities of income elasticities, estimation results show that in practice some parameters are not identified, because demand curves are too linear.

As a solution the value of the scaling factor of the joint consumption function for at least one good has to be fixed by assumption. Given the set of goods available, several choices are possible. To test the sensitivity of the method to the specific assumptions, four different models are tested: Model *CTA* where the m_i for both men's and women's clothing, as well as tobacco and alcohol are fixed; model *C* where the values for men's and women's clothing are fixed; model *Cf*, where only the scale factor for women's clothing is fixed; and model *T* where the factor for tobacco was fixed. Model *C* is intended for testing the assumption of *tobacco* and *alcohol* being private goods, models *Cf* and *T* are intended for testing the private-goods assumption for *male* and *female clothing*. A reference model (*Base*) was estimated with no restrictions. In all cases the respective goods were assumed to be perfectly private and thus their respective scale factors were fixed at a value of one.[9]

A linear version of the sharing rule was used (Equation 5.9), with five distribution influencing variables: net household income (instead of total consumption μ), the woman's share in gross assignable income, a dummy for the couple being not married and interactions between marriage status and net household income as well as between marriage status and the woman's share

[9]Provided the availability of time series data with sufficient price variation, these assumptions can easily be replaced with parameters which are estimated in the conventional way.

in gross assignable income. Assignable income comprises earned income and a variety of transfers: pensions, social benefits etc. Over 98% of observed couples have at least some income that is not assignable, and which on average makes up less than 5% of gross income. Instead of using a three-way differentiation of income into the woman's share, the man's share and non-assignable income, non-assignable income is ignored. Preliminary tests have shown, that its effect is negligible. The woman's income share is simply her share of assignable income. However, non-assignable income is included in net household income.

As in the previous chapter the model was estimated using full information maximum likelihood. Because of endogeneity of total expenditures, total expenditures are instrumented by net household income, household type dummies, the marriage status dummy for couples and interactions between dummies and net household income.

Public or Private: Commodity Group Specific Scale Factors

The goods specific scale factors give an initial indication of the model's validity. Factors should lie within the admissible range of 0.5 for a perfectly public good to 1.0 for a perfectly private good. Scale factors are shown in Table 5.2 for all four models plus the *Base* model without any restrictions on the scale factors. For all models with at least one fixed factor, none of the scale factors lies significantly outside the possible range. Indeed, the actual estimates are all inside the range (up to a percentage point), with the only notable exception of *home and furniture*, the estimates of which are consistently higher than one and *female clothing* and *personal care* in model *T*.

The picture is somewhat different for the unconstrained model (*Base*), where the factors for *female* and *male clothing*, and *home & furniture* are not identified, with very high values and large standard errors. The value for *personal care* is extremely low, with a low standard error. This is the only scale factor that significantly lies outside the range of private to public goods. The very low scale factor is the result of extreme estimated substitution elasticities. More strongly than the unidentified factors, this indicates the limits of the chosen estimation procedure. Given sufficient price variation, price elasticities are clearly better identified from time series data. However, often enough, such data are not available. The effect of this extreme result on the estimated values of equivalence scales is rather small, though, as *personal care* is the smallest of all groups in terms of expenditure share after *tobacco* and *alcohol*.

It is reassuring that for the five largest commodity groups, *food*, *housing*, *transportation*, *recreation* and *vacation* all models, including *Base*, show similar scale factors, leaving the results quite independent of model specification. The size of scales is plausible for all groups. *Food* has lower, but significant economies of scale, while *housing* and *transportation* have the same

	Model				
Good	CTA	C	Cf	T	Base
Food	0.81	0.81	0.79	0.79	0.78
	(0.002)	(0.002)	(0.003)	(0.003)	(0.011)
Female clothing	1.00	1.00	1.00	1.86	13.61
				(2.104)	(3499.876)
Male clothing	1.00	1.00	0.79	0.91	1.82
			(0.033)	(0.087)	(14.059)
Housing	0.59	0.59	0.62	0.61	0.61
	(0.001)	(0.002)	(0.001)	(0.002)	(0.004)
Home & furniture	1.13	1.15	1.03	1.18	4.78
	(0.136)	(0.139)	(0.040)	(0.227)	(114.222)
Transportation	0.59	0.61	0.60	0.59	0.58
	(0.005)	(0.005)	(0.007)	(0.008)	(0.015)
Recreation	0.49	0.50	0.55	0.50	0.45
	(0.012)	(0.012)	(0.011)	(0.017)	(0.065)
Personal care	0.49	0.54	0.61	0.25	0.004
	(0.100)	(0.076)	(0.049)	(0.175)	(0.002)
Vacation	0.66	0.68	0.70	0.68	0.83
	(0.022)	(0.020)	(0.015)	(0.025)	(0.176)
Tobacco	1.00	0.92	1.01	1.00	0.75
		(0.017)	(0.070)		(0.042)
Alcohol	1.00	0.96	0.87	0.92	0.88
		(0.080)	(0.014)	(0.048)	(0.077)

Table 5.2: *Goods specific scale factors for eleven goods. Except for one model (Base) some scale factors are fixed at a value of one, assuming these goods are private: clothing, alcohol and tobacco (CTA); clothing (C); female clothing only (Cf) and tobacco only (T). West German households, age 30–50, all persons working, rented housing. Standard errors are given in parentheses.*

high economies of scale in joint consumption. *Recreation*, which contains also many expensive durables like TV sets and computers that can be shared is almost a perfectly public good with estimated scales close to or even slightly below 0.5.

A scale factor higher than one in category *home & furniture* can be plausibly explained by a change in preferences of couples who start some kind of "home making". This is a reminder that scale factors have to be interpreted with caution because of possible preference changes that are not separable from the scale factors. Yet, because of the high standard error in this category, scale factors are not significantly different from one.

It is not clear if the private good assumption is more appropriate for tobacco than for alcohol. For alcohol there are not only possible economies of scale, because of the occasionally shared bottle of wine, there are also stronger substitution effects, because only alcohol consumed at home is observed in the alcohol category which could be substituted for alcohol consumed away from home. Conversely, negative external effects of tobacco consumption on a non smoking partner are not covered by the model. These could lead to a reduction of tobacco consumption in couples that would also appear as economies of scale. However, all models in which the scale factor for tobacco and alcohol were not fixed show that both goods are almost private. Evidence on complete privateness is mixed: where estimated, scale factors can be significantly lower than one, for alcohol (model *Cf*) as well as for tobacco (model *C* and model *Base*).

Clothing is not expected to be shared in mixed couples. There is limited scope for economies of scale in washing clothes, as couples can fill a washing machine more often. Thus fewer clothes are needed to have a fresh shirt every morning, provided clothes are generally replaced before they are worn out. There might also be some positive or negative external effects in clothing as the partner enjoys a good looking vis-a-vis. In a couple, someone might spend more on clothing just to please his or her partner. Conversely it could be argued that someone might spend more on clothing when being a single to increase the chance of finding a partner. To test this, two models were estimated where scale parameters for *men's clothing* were not fixed relative to *women's clothing* (*Cf* and *T*, respectively). In both models the value for *men's clothing* is smaller than the value for *women's clothing*, however, the result is significant only in model *Cf*, were the estimate is smaller than one, indicating that economies of scale and the reduced need to attract women outweigh external effects. The result is somewhat flawed, though, because by fixing the women's scale at a value of one it is assumed that such effects do not exist for *women's clothing*.

Overall, scale factors are quite plausible. The results show, that for identification it is necessary to fix at least one scale factor, but that for the four largest commodity groups estimates are independent of the choice of the fixed factor.

The Household Consumption Technology

The consumption technology is an idealization and describes the "true" consumption technology best, when consumption quantities of partners are similar. The linear consumption technology also has kink points, and actual consumption quantities should lie between these. However, this is not implemented as a restriction in the estimation process. Therefore, it is enlightening to plot the estimated consumption technologies for all categories and the range of the individual consumption quantities that have been estimated with the model. The results for Model *C* are shown in Figure 5.4. Results for other models show a similar pattern.

The individual consumption quantities of partners relative to total household consumption (q_i^f/q_i and q_i^m/q_i) are shown as a box plot inside the consumption technology plot. The extremes indicate those households with the lowest relative quantity of the woman to those with the highest relative quantity of the woman, while the box shows the 5^{th} to the 95^{th} percentile of households in this ordering. Relative quantities of the inner 90% of households are within the admissible range (between the kink points) for all but two goods categories. Only recreation and personal care show values that are outside the range. Recreation is estimated to be a perfectly public good. Therefore all quantities should be equal to one. The actual estimates are slightly tilted towards male consumption, but they are still close to the equal distribution required by a perfectly public good. Personal care is far more off the mark, but this category is also one of the least well determined, and one of the smallest. Probably the scale factor estimate is too low. Apart from clothing, which is assumed to be private, personal care is also the category with the most unequal consumption. It could be that the model shows here its limits, but overall the model seems to represent the joint consumption of a couple rather well.

One Number: The Equivalence Scale with Equal Sharing

Equivalence scales in the collective model are generally different for men and women. To be as well off materially when living alone as when living with a partner, not only depends on possible economies of scale in joint consumption. It also depends on the share of total consumption received when living with a partner, and on differences in preferences between men and women. Men and women might enjoy different possibilities to substitute goods with higher scale factors, and they might spend different income shares on goods with different scale factors.

To separate the effects of unequal sharing, first equivalence scales with equal sharing are calculated (Table 5.3). Scales are shown for all estimated models. As one would expect from the results for the goods specific scales, results for all models are very similar, if not identical. Even the scale for the *Base* model is, although lower, statistically not different from the others.

To assume equal sharing is also a sensible approach for those social policy applications, where the intra-household distribution is of no interest. For some applications, social benefits for example, it would even be inappropriate to apply different scales to men and to women, because of gender equality considerations. Fortunately, scales are almost identical for both men and women when income is equally shared, with an average estimate of between 0.70 and 0.72. Thus, a specific solution for finding a unique equivalence scale is not a serious issue here.

The equal sharing equivalence scale summarizes the result of the model with respect to different needs of different household types. The problem of

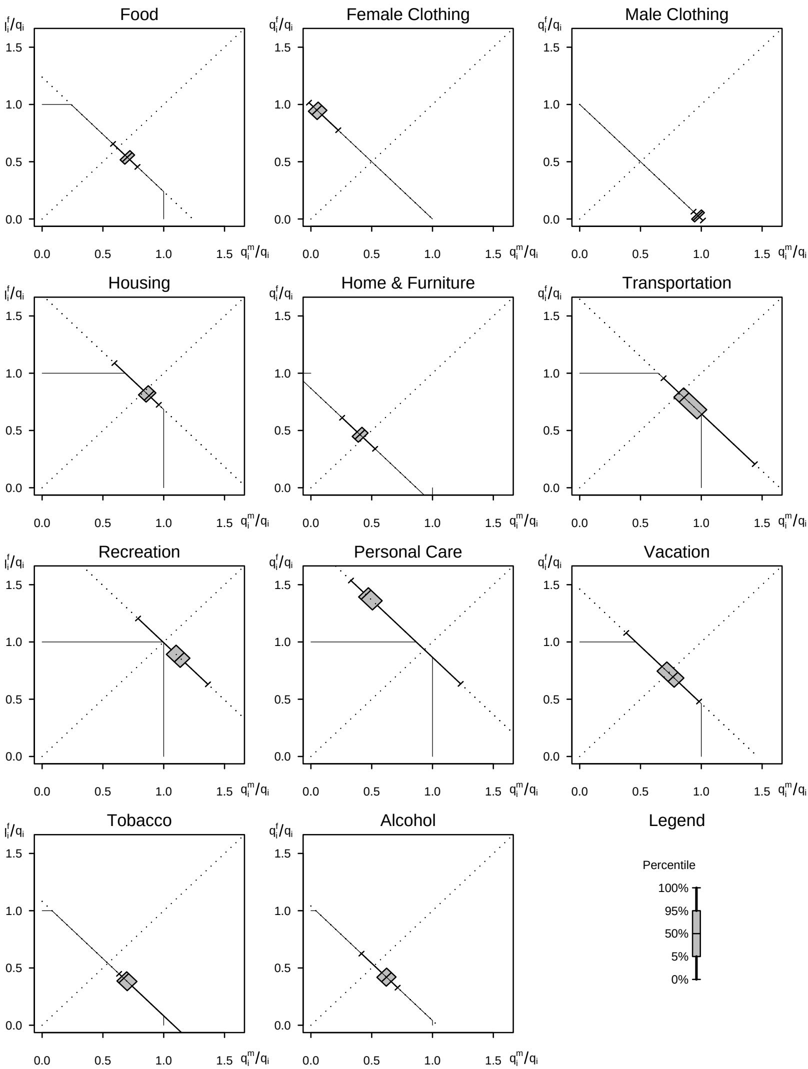

Figure 5.4: *Household technology: the joint consumption function and the distribution of estimated consumption points. Model C. Total household quantities are normalized to one. The box of the boxplot shows the 5^{th} to 95^{th} percentile of consumption points. Minimum, maximum and median are also indicated. Compare Figure 5.2.*

	Model				
Scale	CTA	C	Cf	T	Base
m_0^f	0.71	0.70	0.70	0.69	0.61
	(0.060)	(0.052)	(0.054)	(0.073)	(0.196)
m_0^m	0.73	0.72	0.72	0.72	0.64
	(0.054)	(0.051)	(0.060)	(0.070)	(0.127)

Table 5.3: *Equivalence scales for women (m_0^f) and men (m_0^m) at median net household income with equal sharing. West German households, age 30–50, all persons working, rented housing. Standard errors are given in parentheses.*

uneven distribution within the household is best assessed in the framework of the sharing rule.

The Sharing Rule: Marriage Matters

Again, I find that all models give quite similar results on the sharing rule (see Table 5.4)[10]. The results suggest that income for married couples is shared almost evenly and does not depend on income shares. This finding stands in stark contrast to other work in this field on French, Australian and British data by Bourguignon et al. (1993), Bradbury (1989), and Phipps and Burton (1997), that does not find equal sharing.

Unlike the result for married couples, the sharing rule for non-married couples depends strongly on relative income shares. While married couples share their resources almost equally independent of whose share in earnings is higher, this is very different for non-married couples. These share approximately equally if both partners have the same income, but retain a stronger control over their own income if the income distribution is not equal. In non-married couples, the woman can control up to two thirds of expenditures when she is the sole earner, and vice versa.

Even though not surprising in itself, the contrast to the even sharing in married couples is striking. This strong result supports the finding for married couples. As the number of non-married couples is rather small, one can be confident that the finding that pooling is not rejected for the larger number of married couples is neither accidental nor does it result from a lack of data.

For a graphical analysis, model C is chosen, because it shows the lowest standard errors. Figure 5.5 shows the dependence of a woman's expenditure share on her share of assignable household income and on total household

[10] The sharing rule depends on net household income (NET_INC, in DM), the woman's share in assignable income (SHARE) and the marriage status of the couple (STATUS), which can interact with all other variables. The STATUS variable is defined to be zero when the couple is married and one if it is not married. The sharing rule can be written as: $\varrho = \text{constant} + \text{netinc} \cdot \text{NET\_INC} + \text{share} \cdot \text{SHARE} + \text{STATUS} \cdot (\text{unmarried} + \text{unmarried\_netinc} \cdot \text{NET\_INC} + \text{unmarried\_share} \cdot \text{SHARE})$.

	Model				
Parameter	CTA	C	Cf	T	Base
constant	0.52	0.51	0.53	0.57	0.62
	[159.04]	[210.27]	[179.92]	[112.53]	[16.14]
netinc	$-4.31 \cdot 10^{-7}$	$-3.88 \cdot 10^{-7}$	$-7.11 \cdot 10^{-7}$	$-8.04 \cdot 10^{-7}$	$-1.43 \cdot 10^{-6}$
	[−7.14]	[−6.05]	[−9.25]	[−10.64]	[−4.33]
share	−0.010	−0.008	−0.003	0.007	0.025
	[−1.61]	[−1.43]	[−0.73]	[1.44]	[6.43]
unmarried	0.021	0.023	0.022	0.015	0.010
	[3.47]	[3.77]	[3.84]	[5.90]	[1.88]
unmarried_netinc	$-1.99 \cdot 10^{-7}$	$-2.03 \cdot 10^{-7}$	$-1.56 \cdot 10^{-7}$	$-1.59 \cdot 10^{-8}$	$1.72 \cdot 10^{-7}$
	[−2.25]	[−2.31]	[−1.90]	[−0.52]	[2.00]
unmarried_share	0.248	0.243	0.196	0.192	0.074
	[8.00]	[8.01]	[8.08]	[7.15]	[4.01]

Table 5.4: *Parameters of the sharing rule. West German households, age 30–50, all persons working, rented housing. t-values are given in square brackets.*

income. For married couples, the share is shown at median income in Figure 5.5 a). Expenditure is almost equally shared, with no effect of personal income. For non-married couples the effect of the intra-household income distribution is shown as well as the effect of different total household income levels. The shown graph covers the full range of observed income shares for both household types.

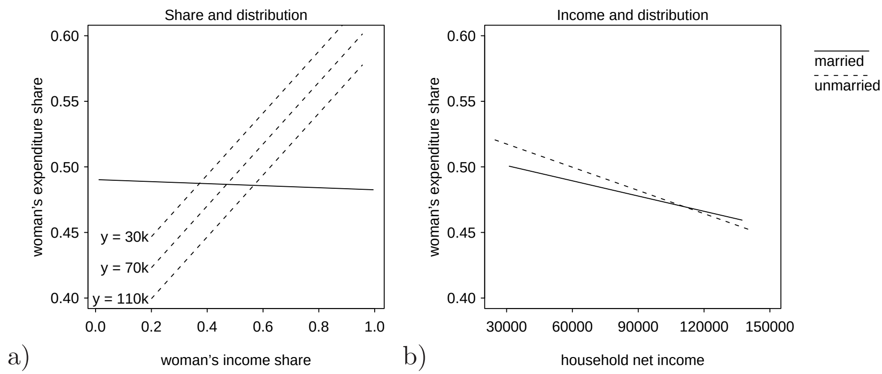

Figure 5.5: *The sharing rule:* a) *Dependence of woman's expenditure share on her share of assignable household income with three different income levels for non-married couples, median income for married couples.* b) *Dependence of woman's expenditure share on total household income with equal income shares. Model* C.

Figure 5.5 b) shows the effect of net household income on expenditure shares. There is a significant effect, with the woman's share falling in total income, but the effect is smaller than the effect of the income share. One can only speculate over why there is an effect of net income on the sharing rule. One possibility would be that because men generally have higher incomes than women, a share for men that is increasing in income would be a second order effect of an increased income share of the male partner, e.g. if the shared amount is reduced with increasing income because basic needs are supplied first, but beyond the supply of basic needs redistribution is more limited. However, this hypothesis cannot be tested in the current framework, because a test would require a more complex sharing rule.

Equivalence Scales with Unequal Sharing and the Cost of Separation

When partners do not share household income evenly, equivalence scales depend on the sharing rule, because a partner who has a smaller share in a couple's household consumption needs less income when living alone to be as well off materially as before. Respective equivalence scales without imposed equal sharing are shown in Table 5.5 for model *C*. Two dimensions, the income share and total household income, influence the scales, which are also differentiated between men and women and married and unmarried couples. As

		Net household income				
Woman's/man's income share		30000 f/m	50000 f/m	70000 f/m	90000 f/m	110000 f/m
10%/90%	*married*	73/72	71/72	69/74	68/75	66/76
	unmarried	62/83	59/85	57/86	55/88	53/90
30%/70%	*married*	73/72	70/73	69/74	67/75	66/76
	unmarried	68/76	66/78	63/80	61/81	60/83
50%/50%	*married*	73/72	70/73	69/74	67/75	66/76
	unmarried	74/70	72/71	70/73	68/74	66/76
70%/30%	*married*	72/72	70/73	68/74	67/75	66/76
	unmarried	81/63	78/64	76/66	74/68	72/69
90%/10%	*married*	72/72	70/73	68/74	67/76	65/77
	unmarried	87/56	85/58	82/59	81/61	79/63

Table 5.5: *Equivalence scales for women and men in couples depending on net household income and the woman's income share. Each cell of the table contains four numbers: for the woman and the man in a couple, that is married or unmarried. Equivalence scale values times 100. Incomes cover the 2^{nd} to 92^{nd} percentile of the income range, indicated shares represent the 4^{th} to the 100^{th} percentile of all covered households. Model* C.

with expenditure shares, equivalence scales hardly depend on income shares for married couples, but scales between men and women do diverge with increasing income due to the change in intra-household distribution, with the man's share increasing and the woman's share decreasing in income. This is reflected in the reported equivalence scales. The case is different for non-married couples, where equivalence scales, like expenditure shares, depend strongly on the respective income share, adding to the influence of income.A graphic representation of the effect of income shares on equivalence scales is given in Figure 5.6.

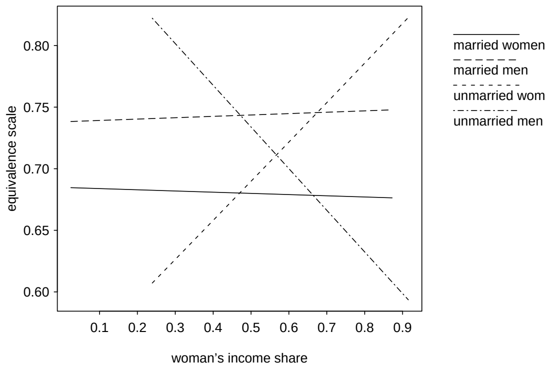

Figure 5.6: *Equivalence scales without equal sharing: dependence of men's and women's equivalence scale share on on the woman's share of assignable household income. Model* C.

The relation of the equivalence scale to the personal share in assignable income gives a measure of the material gains from living together, plotted in Figure 5.7. The relation indicates, by how much the personal income is multiplied when a person moves in with a partner. With equal sharing, the relation reflects only economies of scale, while with unequal sharing, it also reflects the redistribution of income within the household. A value of one indicates that savings from economies of scale and redistribution effects cancel each other out. A value higher than one indicates that personal effective consumption is increased, and a person with a value below one would – materially – be better off living alone, provided separation is possible without cost. For example, a partner who earns 50% of household income and who has an equivalence scale of 0.7 would need 70% of household income to be as well of when living alone. Personal income is increased by a factor of $0.7/50\% = 1.4$. A partner with an income share of 80% and an equivalence scale of 0.76, the relation would be only 0.95: the partner would consume less than she/he earns.

For childless, unmarried couples, separation usually carries no cost beyond the pure transaction costs of finding a new apartment and moving out. Therefore it is more likely that a partner moves out (or does not move in

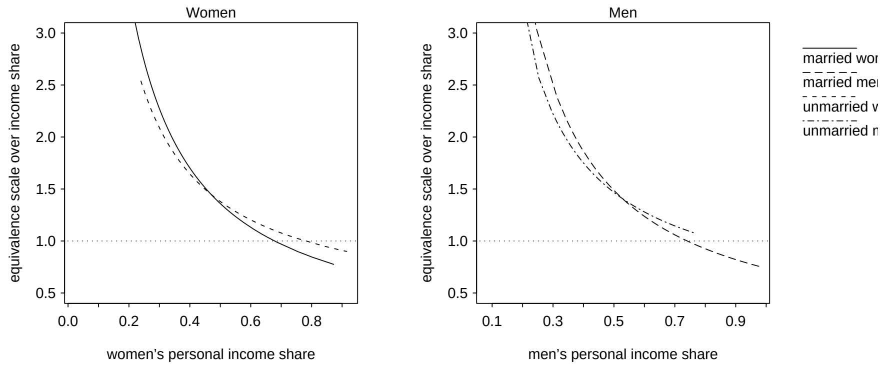

Figure 5.7: *Increase of personal income through joint consumption and redistribution: personal consumption after sharing and joint consumption in relation to personal income. A number higher than one indicates an improvement. Model* C.

in the first place), when living together means less personal consumption. Consequentially, only in 4% of cohabiting couples has one partner a personal consumption that is below her personal income, while the corresponding number for married couples is 25%. Part of the difference can be explained by the different sharing rule: Because unmarried partners retain a higher part of their personal income, it is less likely that they give so much that they would be better off alone. The sharing rule again is influenced by bargaining power: easier separation weakens the bargaining position of the lower earning partner leading to less income redistribution. But the different sharing rule can explain only a small part of the difference between married and unmarried couples: If unmarried couples had a sharing rule that were identical to the sharing rule of married couples, the percentage of partners who would be better off alone would increase only to 9%.

The remaining difference can be explained by the higher separation cost of married couples. A spouse who earns 80% of a couple's total income cannot expect to separate without paying some alimony or losing tax privileges[11] leaving her effectively with less after separation. Therefore the values calculated above are somewhat misleading as they tend to understate the multiplier for a partner's consumption. Assuming that the personal income of a spouse with a high share would be reduced by a merer 10% after separation, reduces the percentage of spouses who would be materially better off alone by ten percentage points (to 15%).

[11] In Germany, the income of husband and wife can be pooled for tax assessment and be taxed on the basis of equal halves (the so called *"Ehegattensplitting"*). This brings great savings for couples with unequal incomes, while not affecting couples with equal incomes. When alimony is paid, not all of this tax privilege is lost, because alimony is added to the reciving partner's income.

Certainly there are other influences as well. There could be a selection bias, for example: Partners who evaluate each other's material well-being more highly will share more evenly, because every euro spent that makes the partner more happy will also increase one's own utility.[12] If such partners are more likely to marry, distribution will be more equal in married couples.

In summary, the discussion about married and non married couples, sharing and the advantages of living together can be positively condensed into a single question: "What do I get if I marry you?" The answer is: "A ring and about half of everything."

5.6 Conclusion

The collective model estimated in this chapter avoids many problems of other equivalence scale models. By making comparisons on an individual basis, the identification problem of equivalence scales can be overcome: The necessary comparisons are situation comparisons, taking the same person in different situations: living alone, or living as a couple. The person retains her utility map in both situations, thus allowing for the direct matching of ordinal utility levels. Regarding household consumption technology, the model incorporates qualities of both child-cost models employed in Chapters 3 and 4: separability of preferences and economies of scale from joint consumption that can affect the partner's consumption choices. With respect to the identification of model parameters, there are fewer degrees of freedom in the model, because the preferences of all members of the all-adult household are defined by the observation of single households, whereas in the parents-children-models the consumption of children can never be observed directly.

The collective equivalence scale model was the only model used for the comparison of couples and singles in this work, and for a reason. Singles could also be included in a Barten-Gorman model, but this should not be done, on theoretical and empirical grounds. The Barten-Gorman model interprets additional household members as an addendum to the reference unit. If necessary, this might be an acceptable interpretation for children, but not for a husband or wife, especially when they neither share the same preferences nor pool their income.

Like the Barten-Gorman model, the collective model had to be integrated into an empirical demand system. The QES framework that was developed for the Barten-Gorman model could be used here as well. Again, restrictions on the effective prices of some goods were necessary to make estimation possible. Private goods are best suited for this as they should show no economies of scale and no change in effective prices. Using different restrictions on clothing, tobacco and alcohol, the model could be successfully estimated.

[12] This argument is similar to the public/private goods argument for the Rothbarth model, Section 3.4, p. 55.

Apart from the identification of equivalence scales, the model also allowed for a closer look at intra-household distribution. It was found that married couples share household income almost evenly, while partners in non-married couples retain a higher control over their personal income. While the latter result is not surprising, the former is in contrast to findings for other countries that show a stronger dependence of each spouse's income share on bargaining power.

Equivalence scales were influenced as much by the economies of scale from joint consumption as by distribution between partners. For practical applications, equivalence scales were calculated for the case of equal sharing, which is a good approximation for married couples, as well as for different sharing situations, resulting in Equivalence scales that depend on distribution characteristics.

Using the estimated equivalence scales it was also possible to evaluate the individual economic gains from living together. It was found that these gains usually outweigh the redistribution between partners, especially in non-married couples. This is an innovative finding, because standard household bargaining literature does not estimate equivalence scales and therefore cannot assess the gains from joint consumption.

A linear joint consumption technology was used in the model. Future research should assess the effect of the linearity assumption and find solutions for the bargaining problem with more general joint consumption technologies.

5.A Identification in the Quadratic Expenditure System

Individual demands in the QES are:

$$x_i^s = b_i^s + \sum \beta_{ik}^s Z_k^s + a_i^s \left(\mu - \sum_{j=1}^n b_j^s\right) + \left(c_i^s - a_i^s \sum_{j=1}^n c_j^s\right) \left(\mu - \sum_{j=1}^n b_j^s\right)^2 \tag{5.35}$$

with $s \in \{m, f\}$. Using the linear joint consumption function (5.3)

$$F_i(x_i^f, x_i^m) = m_i(x_i^f + x_i^m) \ ,$$

these combine to the joint demand function:

$$\begin{aligned} z_i =& m_i(b_i^m + b_i^f) + a_i^m \left((1-\varrho)\mu - \sum_{j=1}^n m_j b_j^m\right) + a_i^f \left(\varrho\mu - \sum_{j=1}^n m_j b_j^f\right) \\ &+ \left(m_i c_i^m - a_i^m \sum_{j=1}^n m_j c_j^m\right) \prod_{j=1}^n m_j^{-2a_j^m} \left((1-\varrho)\mu - \sum_{j=1}^n m_j b_j^m\right)^2 \\ &+ \left(m_i c_i^f - a_i^f \sum_{j=1}^n m_j c_j^f\right) \prod_{j=1}^n m_j^{-2a_j^f} \left(\varrho\mu - \sum_{j=1}^n m_j b_j^f\right)^2 \end{aligned} \tag{5.36}$$

Demand functions for individuals as well as for couples are quadratic in expenditure. Provided sufficiently nonlinear demands, estimation of the parameters of the demand equations leads to $3 \cdot 3 \cdot (n-1)$ linearly independent equations:

$$\theta_{1i}^s = b_i - a_i \sum_{j=1}^n b_j^s + \theta_{3i}^s \left(\sum_{j=1}^n b_j^s\right)^2 \tag{5.37}$$

$$\theta_{2i}^s = a_i^s - 2\theta_{3i}^s \sum_{j=1}^n b_j^s \tag{5.38}$$

$$\theta_{3i}^s = c_i^s - a_i^s \sum_{j=1}^n c_j^s \tag{5.39}$$

$$\theta^c_{1i} = m_i b^f_i - a^f_i \sum_{j=1}^n m_j b^f_j + \left(m_i c^f_i - a^f_i \sum_{j=1}^n m_j c^f_j \right) \prod_{j=1}^n m_j^{-2a^f_j} \left(\sum_{j=1}^n m_j b^f_j \right)^2 + \\ m_i b^m_i - a^m_i \sum_{j=1}^n m_j b^m_j + \left(m_i c^m_i - a^m_i \sum_{j=1}^n m_j c^m_j \right) \prod_{j=1}^n m_j^{-2a^m_j} \left(\sum_{j=1}^n m_j b^m_j \right)^2 \tag{5.40}$$

$$\theta^c_{2i} = a^f_i - 2 \left(m_i c^f_i - a^f_i \sum_{j=1}^n m_j c^f_j \right) \prod_{j=1}^n m_j^{-2a^f_j} \sum_{j=1}^n m_j b^f_j \varrho + \\ a^m_i - 2 \left(m_i c^m_i - a^m_i \sum_{j=1}^n m_j c^m_j \right) \prod_{j=1}^n m_j^{-2a^m_j} \sum_{j=1}^n m_j b^m_j (1-\varrho) \tag{5.41}$$

$$\theta^c_{3i} = \left(m_i c^f_i - a^f_i \sum_{j=1}^n m_j c^f_j \right) \prod_{j=1}^n m_j^{-2a^f_j} \varrho^2 + \\ \left(m_i c^m_i - a^m_i \sum_{j=1}^n m_j c^m_j \right) \prod_{j=1}^n m_j^{-2a^m_j} (1-\varrho)^2 \tag{5.42}$$

The two demand systems for men and women have $3n-1$ parameters each plus n scaling parameters and one ϱ. This are overall $7n-1$ parameters that have to be estimated. There are $9(n-1)$ equations. Therefore the parameters are identified if the number of commodities is at least 4. Estimation is carried out using full information maximum likelihood.

Even though all parameters are identified, identification relies, as in the Barten case, on nonlinearities of the demand equations. Indeed, not all parameters, foremost the scaling parameters which are so important for the determination of equivalence scales, are not well identified. It is therefore sensible to fix at least one scaling parameter. The resulting Hicksian demand elasticity then allows for the estimation of the overall equivalence scale and all other Hicksian demand elasticities.

Identification of the absolute value of the sharing function ϱ depends on either different preferences of the partners or a nonlinear demand function. In a linear expenditure system, ϱ is only identified if partners have different preferences.

5.B Quadratic Engel curves and their parameters

Table 5.6 (next page) gives an overview of parameter estimates of quadratic Engel curves for the respective household types. The Engel curves are estimated in a square equation using two stage least squares with net income, net income squared, the average age of household members and its square as instruments, analogous to the estimation of parameters reported in section 4.7.

Estimated parameters are not significant for most goods due to the small number of households in the selected sample. As has been discussed earlier, Missong (2004) shows in a comparison of parametric forms and non-parametric Engel curves, that a quadratic form is nevertheless preferred over a linear model for most goods. Therefore the quadratic expenditure system is a valid choice for the estimation.

Engel curves are shown in Figure 5.8. Engel curves show quite different pictures for men and women for many goods, most notably for transportation and personal care and of course clothing.

Household type		Food	Female clothing	Male clothing	Housing	Home & furniture	Transport	Recreation	Personal care	Vacation	Tobacco	Alcohol
F	θ^f_{1i}	−538.6	−1736.7	53.6	6535.2	−249.2	−1850.4	−454.1	−717.4	−1764.1	−237.2	−118.3
		[0.65]	[−2.36]*	[0.49]	[5.74]***	[−0.26]	[−2.64]**	[−0.54]	[−1.80]	[−1.81]	[−0.67]	[−0.52]
	θ^f_{2i}	0.150	0.202	0.0004	−0.060	0.024	0.286	0.081	0.091	0.173	0.037	0.016
		[2.27]*	[3.43]***	[0.05]	[−0.66]	[0.30]	[5.10]***	[1.21]	[2.87]**	[2.21]*	[1.32]	[0.86]
	θ^f_{3i}	$-4.17\cdot10^{-8}$	$-1.33\cdot10^{-6}$	$-9.04\cdot10^{-9}$	$3.58\cdot10^{-6}$	$1.75\cdot10^{-6}$	$-3.58\cdot10^{-6}$	$1.23\cdot10^{-6}$	$-7.30\cdot10^{-7}$	$-4.79\cdot10^{-7}$	$-4.62\cdot10^{-7}$	$7.93\cdot10^{-8}$
		[−0.04]	[−1.28]	[−0.06]	[2.23]*	[1.28]	[−3.62]***	[1.04]	[−1.30]	[−0.35]	[−0.93]	[0.25]
M	θ^m_{1i}	3083.0	−55.4	−1854.0	865.2	−1389.7	1069.3	−473.5	490.5	−4150.5	2134.1	280.8
		[1.87]	[−0.20]	[−2.24]*	[0.51]	[−1.26]	[0.88]	[−0.31]	[1.49]	[−2.72]**	[3.13]**	[0.63]
	θ^m_{2i}	−0.029	0.005	0.184	0.372	0.122	0.055	0.093	−0.023	0.375	−0.148	−0.005
		[−0.22]	[0.20]	[2.68]**	[2.66]**	[1.33]	[0.55]	[0.72]	[−0.85]	[2.97]**	[−2.62]**	[−0.13]
	θ^m_{3i}	$4.08\cdot10^{-6}$	$2.18\cdot10^{-8}$	$-1.94\cdot10^{-6}$	$-3.97\cdot10^{-6}$	$-5.61\cdot10^{-7}$	$1.08\cdot10^{-6}$	$1.40\cdot10^{-6}$	$7.69\cdot10^{-7}$	$-4.10\cdot10^{-6}$	$2.80\cdot10^{-6}$	$4.32\cdot10^{-7}$
		[1.70]	[0.05]	[−1.61]	[−1.61]	[−0.35]	[0.61]	[0.62]	[1.60]	[−1.84]	[2.81]**	[0.67]
Couple	θ^{fm}_{1i}	2968.6	1051.9	863.6	5411.0	−634.1	−1714.0	−8264.0	453.0	195.9	−1952.1	1620.3
		[1.32]	[0.86]	[1.08]	[2.30]*	[−0.24]	[−0.96]	[−3.84]***	[0.63]	[0.08]	[−2.01]*	[1.85]
	θ^{fm}_{2i}	0.101	−0.010	−0.020	0.131	0.046	0.202	0.433	0.008	0.043	0.117	−0.053
		[0.98]	[−0.17]	[−0.54]	[1.21]	[0.37]	[2.47]*	[4.37]***	[0.23]	[0.37]	[2.62]**	[−1.31]
	θ^{fm}_{3i}	$7.45\cdot10^{-7}$	$8.90\cdot10^{-7}$	$6.97\cdot10^{-7}$	$-3.32\cdot10^{-7}$	$1.01\cdot10^{-6}$	$-1.06\cdot10^{-6}$	$-3.06\cdot10^{-6}$	$2.68\cdot10^{-7}$	$1.22\cdot10^{-6}$	$-1.10\cdot10^{-6}$	$7.16\cdot10^{-7}$
		[0.69]	[1.52]	[1.82]	[−0.30]	[0.79]	[−1.24]	[−2.98]**	[0.78]	[1.01]	[−2.38]*	[1.71]

Table 5.6: *Parameters for unpooled quadratic Engel curves for eleven categories of goods. West German households. t-values are given in square brackets.*

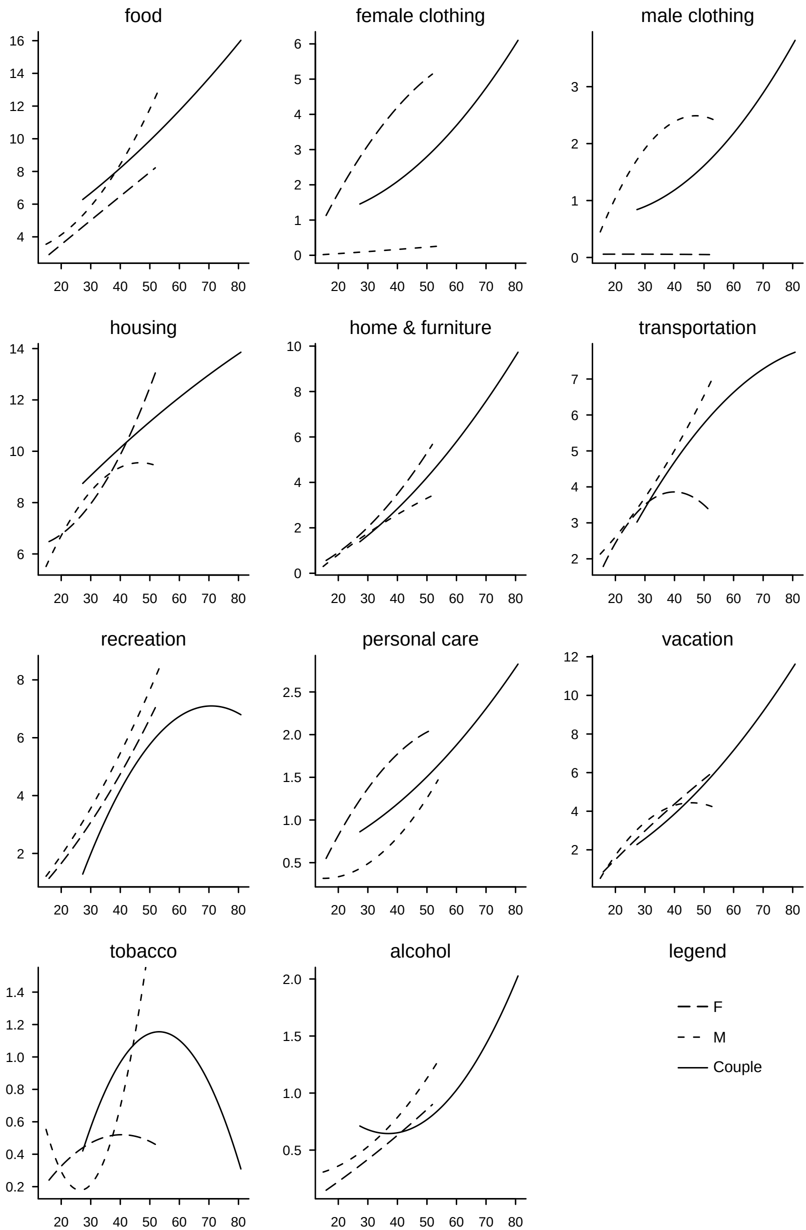

Figure 5.8: *Quadratic Engel curves for all expenditure categories. 5^{th} to 95^{th} percentile of expenditure range. x-axis shows total expenditure, y-axis shows expenditures on the respective good, both in 1000 DM.*