Appendix B

Integrals

Here we present the explicit calculations leading to the results discussed in Chapter 4. Due to the great number of indices appearing in the calculation of the integrals, we shall use Greek indices instead of Latin ones. In all calculations the Greek indices take the values 1 and 2.

B.1 Tensor structure

Typical integrals appearing in our calculations are of the form

$$\int \frac{\mathrm{d}^2 k}{(2\pi)^2} \frac{k_\mu k_\nu}{[r_0 k^2 + \kappa_0 k^4 - (r_0 + 2\kappa_0 k^2)(q\cdot k)^2 + \kappa_0 (q\cdot k)^4]^n} = F_2^n(q^2)\delta_{\mu\nu} + G_2^n(q^2)\frac{q_\mu q_\nu}{q^2} \tag{B.1}$$

$$\int \frac{\mathrm{d}^2 k}{(2\pi)^2} \frac{k_\mu k_\nu k_\alpha k_\beta}{[r_0 k^2 + \kappa_0 k^4 - (r_0 + 2\kappa_0 k^2)(q\cdot k)^2 + \kappa_0 (q\cdot k)^4]^n} = F_4^n(q^2)(\delta_{\mu\nu}\delta_{\alpha\beta} + \text{perm.}) + G_4^n(q^2)(\frac{q_\mu q_\nu}{q^2}\delta_{\alpha\beta} + \text{perm.}) + H_4^n(q^2)\frac{q_\mu q_\nu q_\alpha q_\beta}{q^4} \tag{B.2}$$

$$\int \frac{\mathrm{d}^2 k}{(2\pi)^2} \frac{k_\mu k_\nu k_\alpha k_\beta k_\sigma k_\rho}{[r_0 k^2 + \kappa_0 k^4 - (r_0 + 2\kappa_0 k^2)(q\cdot k)^2 + \kappa_0 (q\cdot k)^4]^n} = F_6^n(q^2)(\delta_{\mu\nu}\delta_{\alpha\beta}\delta_{\sigma\rho} + \text{perm.}) + G_6^n(q^2)(\frac{q_\mu q_\nu}{q^2}\delta_{\alpha\beta}\delta_{\sigma\rho} + \text{perm.})$$

$$+H_6^n(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta}{q^4}\delta_{\sigma\rho} + \text{perm.}) + J_6^n(q^2)\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho}{q^6} \tag{B.3}$$

$$\int \frac{\mathrm{d}^2 k}{(2\pi)^2}\frac{k_\mu k_\nu k_\alpha k_\beta k_\sigma k_\rho k_{\mu_1} k_{\nu_1}}{[r_0 k^2 + \kappa_0 k^4 - (r_0 + 2\kappa_0 k^2)(q\cdot k)^2 + \kappa_0 (q\cdot k)^4]^n} =$$
$$F_8^n(q^2)(\delta_{\mu\nu}\delta_{\alpha\beta}\delta_{\sigma\rho}\delta_{\mu_1\nu_1} + \text{perm.}) + G_8^n(q^2)(\frac{q_\mu q_\nu}{q^2}\delta_{\alpha\beta}\delta_{\sigma\rho}\delta_{\mu_1\nu_1} + \text{perm.})$$
$$+H_8^n(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta}{q^4}\delta_{\sigma\rho}\delta_{\mu_1\nu_1} + \text{perm.}) + J_8^n(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho}{q^6}\delta_{\mu_1\nu_1} + \text{perm.})$$
$$+L_8^n(q^2)\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho q_{\mu_1} q_{\nu_1}}{q^8} \tag{B.4}$$

$$\int \frac{\mathrm{d}^2 k}{(2\pi)^2}\frac{k_\mu k_\nu k_\alpha k_\beta k_\sigma k_\rho k_{\mu_1} k_{\nu_1} k_{\alpha_1} k_{\beta_1}}{[r_0 k^2 + \kappa_0 k^4 - (r_0 + 2\kappa_0 k^2)(q\cdot k)^2 + \kappa_0 (q\cdot k)^4]^n} =$$
$$F_{10}^n(q^2)(\delta_{\mu\nu}\delta_{\alpha\beta}\delta_{\sigma\rho}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1} + \text{perm.})$$
$$+G_{10}^n(q^2)(\frac{q_\mu q_\nu}{q^2}\delta_{\alpha\beta}\delta_{\sigma\rho}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1} + \text{perm.})$$
$$+H_{10}^n(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta}{q^4}\delta_{\sigma\rho}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1} + \text{perm.})$$
$$+J_{10}^n(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho}{q^6}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1} + \text{perm.})$$
$$+L_{10}^n(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho q_{\mu_1} q_{\nu_1}}{q^8}\delta_{\alpha_1\beta_1} + \text{perm.})$$
$$+M_{10}^n(q^2)\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho q_{\mu_1} q_{\nu_1} q_{\alpha_1} q_{\beta_1}}{q^{10}} \tag{B.5}$$

$$\int \frac{\mathrm{d}^2 k}{(2\pi)^2}\frac{k_\mu k_\nu k_\alpha k_\beta k_\sigma k_\rho k_{\mu_1} k_{\nu_1} k_{\alpha_1} k_{\beta_1} k_{\sigma_1} k_{\rho_1}}{[r_0 k^2 + \kappa_0 k^4 - (r_0 + 2\kappa_0 k^2)(q\cdot k)^2 + \kappa_0 (q\cdot k)^4]^n} =$$
$$F_{12}^n(q^2)(\delta_{\mu\nu}\delta_{\alpha\beta}\delta_{\sigma\rho}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1}\delta_{\sigma_1\rho_1} + \text{perm.})$$
$$+G_{12}^n(q^2)(\frac{q_\mu q_\nu}{q^2}\delta_{\alpha\beta}\delta_{\sigma\rho}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1}\delta_{\sigma_1\rho_1} + \text{perm.})$$
$$+H_{12}^n(q^2)(\frac{q_\mu q_\nu q_\alpha q\beta}{q^4}\delta_{\sigma\rho}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1}\delta_{\sigma_1\rho_1} + \text{perm.})$$
$$+J_{12}^n(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho}{q^6}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1}\delta_{\sigma_1\rho_1} + \text{perm.})$$
$$+L_{12}^n(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho q_{\mu_1} q_{\nu_1}}{q^8}\delta_{\alpha_1\beta_1}\delta_{\sigma_1\rho_1} + \text{perm.})$$

$$+M^n_{12}(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho q_{\mu_1} q_{\nu_1} q_{\alpha_1} q_{\beta_1}}{q^{10}}\delta_{\sigma_1\rho_1} + \text{perm.})$$
$$+N^n_{12}(q^2)\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho q_{\mu_1} q_{\nu_1} q_{\alpha_1} q_{\beta_1} q_{\sigma_1} q_{\rho_1}}{q^{12}} \tag{B.6}$$

$$\int\frac{\mathrm{d}^2k}{(2\pi)^2}\frac{k_\mu k_\nu k_\alpha k_\beta k_\sigma k_\rho k_{\mu_1} k_{\nu_1} k_{\alpha_1} k_{\beta_1} k_{\sigma_1} k_{\rho_1} k_{\sigma_2} k_{\rho_2}}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} =$$
$$F^n_{14}(q^2)(\delta_{\mu\nu}\delta_{\alpha\beta}\delta_{\sigma\rho}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1}\delta_{\sigma_1\rho_1}\delta_{\sigma_2\rho_2} + \text{perm.})$$
$$+G^n_{14}(q^2)(\frac{q_\mu q_\nu}{q^2}\delta_{\alpha\beta}\delta_{\sigma\rho}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1}\delta_{\sigma_1\rho_1}\delta_{\sigma_2\rho_2} + \text{perm.})$$
$$+H^n_{14}(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta}{q^4}\delta_{\sigma\rho}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1}\delta_{\sigma_1\rho_1}\delta_{\sigma_2\rho_2} + \text{perm.})$$
$$+J^n_{14}(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho}{q^6}\delta_{\mu_1\nu_1}\delta_{\alpha_1\beta_1}\delta_{\sigma_1\rho_1}\delta_{\sigma_2\rho_2} + \text{perm.})$$
$$+L^n_{14}(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho q_{\mu_1} q_{\nu_1}}{q^8}\delta_{\alpha_1\beta_1}\delta_{\sigma_1\rho_1}\delta_{\sigma_2\rho_2} + \text{perm.})$$
$$+M^n_{14}(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho q_{\mu_1} q_{\nu_1} q_{\alpha_1} q_{\beta_1}}{q^{10}}\delta_{\sigma_1\rho_1}\delta_{\sigma_2\rho_2} + \text{perm.})$$
$$+N^n_{14}(q^2)(\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho q_{\mu_1} q_{\nu_1} q_{\alpha_1} q_{\beta_1} q_{\sigma_1} q_{\rho_1}}{q^{12}}\delta_{\sigma_2\rho_2} + \text{perm.})$$
$$+P^n_{14}(q^2)\frac{q_\mu q_\nu q_\alpha q_\beta q_\sigma q_\rho q_{\mu_1} q_{\nu_1} q_{\alpha_1} q_{\beta_1} q_{\sigma_1} q_{\rho_1} q_{\sigma_2} q_{\rho_2}}{q^{14}}. \tag{B.7}$$

Multiplying (B.1)-(B.7) with adequate combinations $(q_{\mu_1}q_{\mu_2}\cdots q_{\mu_k})/q^k$ and contracting indices, we obtain the following systems of equations:

$$\begin{cases} A^n_2 = 2F^n_2 + G^n_2 \\ B^n_2 = F^n_2 + G^n_2 \end{cases} \tag{B.8}$$

$$\begin{cases} A^n_4 = 8F^n_4 + 8G^n_4 + H^n_4 \\ B^n_4 = 4F^n_4 + 7G^n_4 + H^n_4 \\ C^n_4 = 3F^n_4 + 6G^n_4 + H^n_4 \end{cases} \tag{B.9}$$

$$\begin{cases} A^n_6 = 48F^n_6 + 72G^n_6 + 18H^n_6 + J^n_6 \\ B^n_6 = 24F^n_6 + 60G^n_6 + 17H^n_6 + J^n_6 \\ C^n_6 = 18F^n_6 + 51G^n_6 + 16H^n_6 + J^n_6 \\ D^n_6 = 15F^n_6 + 45G^n_6 + 15H^n_6 + J^n_6 \end{cases} \tag{B.10}$$

$$
\left\{
\begin{aligned}
A_8^n &= 384F_8^n + 768G_8^n + 288H_8^n + 32J_8^n + L_8^n \\
B_8^n &= 192F_8^n + 624G_8^n + 264H_8^n + 31J_8^n + L_8^n \\
C_8^n &= 144F_8^n + 528G_8^n + 243H_8^n + 30J_8^n + L_8^n \\
D_8^n &= 120F_8^n + 465G_8^n + 225H_8^n + 29J_8^n + L_8^n \\
E_8^n &= 105F_8^n + 420G_8^n + 210H_8^n + 28J_8^n + L_8^n
\end{aligned}
\right. \tag{B.11}
$$

$$
\left\{
\begin{aligned}
A_{10}^n &= 3840F_{10}^n + 9600G_{10}^n + 4800H_{10}^n + 800J_{10}^n \\
&\quad +50L_{10}^n + M_{10}^n \\
B_{10}^n &= 1920F_{10}^n + 7680G_{10}^n + 4320H_{10}^n + 760J_{10}^n \\
&\quad +49L_{10}^n + M_{10}^n \\
C_{10}^n &= 1440F_{10}^n + 6480G_{10}^n + 3930H_{10}^n + 723J_{10}^n \\
&\quad +48L_{10}^n + M_{10}^n \\
D_{10}^n &= 1200F_{10}^n + 5700G_{10}^n + 3615H_{10}^n + 689J_{10}^n \\
&\quad +47L_{10}^n + M_{10}^n \\
E_{10}^n &= 1050F_{10}^n + 5145G_{10}^n + 3360H_{10}^n + 658J_{10}^n \\
&\quad +46L_{10}^n + M_{10}^n \\
\bar{F}_{10}^n &= 945F_{10}^n + 4725G_{10}^n + 3150H_{10}^n + 630J_{10}^n \\
&\quad +45L_{10}^n + M_{10}^n
\end{aligned}
\right. \tag{B.12}
$$

$$
\left\{
\begin{aligned}
A_{12}^n &= 46080F_{12}^n + 138240G_{12}^n + 86400H_{12}^n + 19200J_{12}^n \\
&\quad +1800L_{12}^n + 72M_{12}^n + N_{12}^n \\
B_{12}^n &= 23040F_{12}^n + 109440G_{12}^n + 76800H_{12}^n + 18000J_{12}^n \\
&\quad +1740L_{12}^n + 71M_{12}^n + N_{12}^n \\
C_{12}^n &= 17280F_{12}^n + 92160G_{12}^n + 69360H_{12}^n + 16944J_{12}^n \\
&\quad +1683L_{12}^n + 70M_{12}^n + N_{12}^n \\
D_{12}^n &= 14400F_{12}^n + 81000G_{12}^n + 63540H_{12}^n + 16017J_{12}^n \\
&\quad +1629L_{12}^n + 69M_{12}^n + N_{12}^n \\
E_{12}^n &= 12600F_{12}^n + 73080G_{12}^n + 58905H_{12}^n + 15204J_{12}^n \\
&\quad +1578L_{12}^n + 68M_{12}^n + N_{12}^n \\
\bar{F}_{12}^n &= 11340F_{12}^n + 67095G_{12}^n + 55125H_{12}^n + 14490J_{12}^n \\
&\quad +1530L_{12}^n + 67M_{12}^n + N_{12}^n \\
\bar{G}_{12}^n &= 10395F_{12}^n + 62370G_{12}^n + 51975H_{12}^n + 13860J_{12}^n \\
&\quad +1485L_{12}^n + 66M_{12}^n + N_{12}^n
\end{aligned}
\right. \tag{B.13}
$$

$$\left\{\begin{array}{lcl} A^n_{14} & = & 645120F^n_{14} + 2257920G^n_{14} + 1693440H^n_{14} + 470400J^n_{14} \\ & & +58800L^n_{14} + 3528M^n_{14} + 98N^n_{14} + P^n_{14} \\ B^n_{14} & = & 322560F^n_{14} + 1774080G^n_{14} + 1491840H^n_{14} + 436800J^n_{14} \\ & & +56280L^n_{14} + 3444M^n_{14} + 97N^n_{14} + P^n_{14} \\ C^n_{14} & = & 241920F^n_{14} + 1491840G^n_{14} + 1340640H^n_{14} + 408240J^n_{14} \\ & & +53970L^n_{14} + 3363M^n_{14} + 96N^n_{14} + P^n_{14} \\ D^n_{14} & = & 201600F^n_{14} + 1310400G^n_{14} + 1224720H^n_{14} + 383880J^n_{14} \\ & & +51855L^n_{14} + 3285M^n_{14} + 95N^n_{14} + P^n_{14} \\ E^n_{14} & = & 176400F^n_{14} + 1181880G^n_{14} + 1133370H^n_{14} + 362985J^n_{14} \\ & & +49920L^n_{14} + 3210M^n_{14} + 94N^n_{14} + P^n_{14} \\ \bar{F}^n_{14} & = & 158760F^n_{14} + 1084860G^n_{14} + 1059345H^n_{14} + 344925J^n_{14} \\ & & +48150L^n_{14} + 3138M^n_{14} + 93N^n_{14} + P^n_{14} \\ \bar{G}^n_{14} & = & 145530F^n_{14} + 1008315G^n_{14} + 997920H^n_{14} + 329175J^n_{14} \\ & & +46530L^n_{14} + 3069M^n_{14} + 92N^n_{14} + P^n_{14} \\ \bar{H}^n_{14} & = & 135135F^n_{14} + 945945G^n_{14} + 945945H^n_{14} + 315315J^n_{14} \\ & & +45045L^n_{14} + 3003M^n_{14} + 91N^n_{14} + P^n_{14}, \end{array}\right. \tag{B.14}$$

where

$$A^n_2 = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^2}{[r_0k^2 + \kappa_0k^4 - (r_0 + 2\kappa_0k^2)(q\cdot k)^2 + \kappa_0(q\cdot k)^4]^n} \tag{B.15}$$

$$B^n_2 = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{(k\cdot q)^2/q^2}{[r_0k^2 + \kappa_0k^4 - (r_0 + 2\kappa_0k^2)(q\cdot k)^2 + \kappa_0(q\cdot k)^4]^n} \tag{B.16}$$

$$A^n_4 = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^4}{[r_0k^2 + \kappa_0k^4 - (r_0 + 2\kappa_0k^2)(q\cdot k)^2 + \kappa_0(q\cdot k)^4]^n} \tag{B.17}$$

$$B^n_4 = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^2(k\cdot q)^2/q^2}{[r_0k^2 + \kappa_0k^4 - (r_0 + 2\kappa_0k^2)(q\cdot k)^2 + \kappa_0(q\cdot k)^4]^n} \tag{B.18}$$

$$C^n_4 = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{(k\cdot q)^4/q^4}{[r_0k^2 + \kappa_0k^4 - (r_0 + 2\kappa_0k^2)(q\cdot k)^2 + \kappa_0(q\cdot k)^4]^n} \tag{B.19}$$

$$A^n_6 = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^6}{[r_0k^2 + \kappa_0k^4 - (r_0 + 2\kappa_0k^2)(q\cdot k)^2 + \kappa_0(q\cdot k)^4]^n} \tag{B.20}$$

$$B^n_6 = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^4(k\cdot q)^2/q^2}{[r_0k^2 + \kappa_0k^4 - (r_0 + 2\kappa_0k^2)(q\cdot k)^2 + \kappa_0(q\cdot k)^4]^n} \tag{B.21}$$

$$C_6^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^2(k\cdot q)^4/q^4}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.22)$$

$$D_6^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{(k\cdot q)^6/q^6}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.23)$$

$$A_8^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^8}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.24)$$

$$B_8^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^6(k\cdot q)^2/q^2}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.25)$$

$$C_8^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^4(k\cdot q)^4/q^4}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.26)$$

$$D_8^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^2(k\cdot q)^6/q^6}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.27)$$

$$E_8^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{(k\cdot q)^8/q^8}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.28)$$

$$A_{10}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{10}}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.29)$$

$$B_{10}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^8(k\cdot q)^2/q^2}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.30)$$

$$C_{10}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^6(k\cdot q)^4/q^4}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.31)$$

$$D_{10}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^4(k\cdot q)^6/q^6}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.32)$$

$$E_{10}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^2(k\cdot q)^8/q^8}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.33)$$

$$\bar{F}_{10}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{(k\cdot q)^{10}/q^{10}}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.34)$$

$$A_{12}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{12}}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad (B.35)$$

$$B_{12}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{10}(k\cdot q)^2/q^2}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.36)}$$

$$C_{12}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{8}(k\cdot q)^4/q^4}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.37)}$$

$$D_{12}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{6}(k\cdot q)^6/q^6}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.38)}$$

$$E_{12}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{4}(k\cdot q)^8/q^8}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.39)}$$

$$\bar{F}_{12}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{2}(k\cdot q)^{10}/q^{10}}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.40)}$$

$$\bar{G}_{12}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{(k\cdot q)^{12}/q^{12}}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.41)}$$

$$A_{14}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{14}}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.42)}$$

$$B_{14}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{12}(k\cdot q)^2/q^2}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.43)}$$

$$C_{14}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{10}(k\cdot q)^4/q^4}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.44)}$$

$$D_{14}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{8}(k\cdot q)^6/q^6}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.45)}$$

$$E_{14}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{6}(k\cdot q)^8/q^8}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.46)}$$

$$\bar{F}_{14}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{4}(k\cdot q)^{10}/q^{10}}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.47)}$$

$$\bar{G}_{14}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{k^{2}(k\cdot q)^{12}/q^{12}}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n} \quad \text{(B.48)}$$

$$\bar{H}_{14}^n = \int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{(k\cdot q)^{14}/q^{14}}{[r_0k^2+\kappa_0k^4-(r_0+2\kappa_0k^2)(q\cdot k)^2+\kappa_0(q\cdot k)^4]^n}. \quad \text{(B.49)}$$

The systems of equations (B.8)-(B.14) can be inverted, yielding:

$$\begin{cases} F_2^n &= A_2^n - B_2^n \\ G_2^n &= -A_2^n + 2B_2^n \end{cases} \tag{B.50}$$

$$\begin{cases} F_4^n &= \frac{1}{3}(A_4^n - 2B_4^n + C_4^n) \\ G_4^n &= \frac{1}{3}(-A_4^n + 5B_4^n - 4C_4^n) \\ H_4^n &= A_4^n - 8B_4^n + 8C_4^n \end{cases} \tag{B.51}$$

$$\begin{cases} F_6^n &= \frac{1}{15}(A_6^n - 3B_6^n + 3C_6^n - D_6^n) \\ G_6^n &= \frac{1}{15}(-A_6^n + 8B_6^n - 13C_6^n + 6D_6^n) \\ H_6^n &= \frac{1}{5}(A_6^n - 13B_6^n + 28C_6^n - 16D_6^n) \\ J_6^n &= -A_6^n + 18B_6^n - 48C_6^n + 32D_6^n \end{cases} \tag{B.52}$$

$$\begin{cases} F_8^n &= \frac{1}{105}(A_8^n - 4B_8^n + 6C_8^n - 4D_8^n + E_8^n) \\ G_8^n &= \frac{1}{105}(-A_8^n + 11B_8^n - 27C_8^n + 25D_8^n - 8E_8^n) \\ H_8^n &= \frac{1}{105}(3A_8^n - 54B_8^n + 179C_8^n - 208D_8^n + 80E_8^n) \\ J_8^n &= \frac{1}{7}(-A_8^n + 25B_8^n - 104C_8^n + 144D_8^n - 64E_8^n) \\ L_8^n &= A_8^n - 32B_8^n + 160C_8^n - 256D_8^n + 128E_8^n \end{cases} \tag{B.53}$$

$$\begin{cases} F_{10}^n &= \frac{1}{945}(A_{10}^n - 5B_{10}^n + 10C_{10}^n - 10D_{10}^n \\ & \quad +5E_{10}^n - \bar{F}_{10}^n) \\ G_{10}^n &= \frac{1}{945}(-A_{10}^n + 14B_{10}^n - 46C_{10}^n + 64D_{10}^n \\ & \quad -41E_{10}^n + 10\bar{F}_{10}^n) \\ H_{10}^n &= \frac{1}{315}(A_{10}^n - 23B_{10}^n + 103C_{10}^n - 181D_{10}^n \\ & \quad +140E_{10}^n - 40\bar{F}_{10}^n) \\ J_{10}^n &= \frac{1}{63}(-A_{10}^n + 32B_{10}^n - 181C_{10}^n + 382D_{10}^n \\ & \quad -344E_{10}^n + 112\bar{F}_{10}^n) \\ L_{10}^n &= \frac{1}{9}(A_{10}^n - 41B_{10}^n + 280C_{10}^n - 688D_{10}^n \\ & \quad +704E_{10}^n - 256\bar{F}_{10}^n) \\ M_{10}^n &= -A_{10}^n + 50B_{10}^n - 400C_{10}^n + 1120D_{10}^n \\ & \quad -1280E_{10}^n + 512\bar{F}_{10}^n \end{cases} \tag{B.54}$$

$$\left\{\begin{array}{lcl}
F_{12}^n &=& \frac{1}{10395}(A_{12}^n - 6B_{12}^n + 15C_{12}^n - 20D_{12}^n \\
&& +15E_{12}^n - 6\bar{F}_{12}^n + \bar{G}_{12}^n) \\
G_{12}^n &=& \frac{1}{10395}(-A_{12}^n + 17B_{12}^n - 70C_{12}^n + 130D_{12}^n \\
&& -125E_{12}^n + 61\bar{F}_{12}^n - 12\bar{G}_{12}^n) \\
H_{12}^n &=& \frac{1}{3465}(A_{12}^n - 28B_{12}^n + 158C_{12}^n - 372D_{12}^n \\
&& +433E_{12}^n - 248\bar{F}_{12}^n + 56\bar{G}_{12}^n) \\
J_{12}^n &=& \frac{1}{3465}(-5A_{12}^n + 195B_{12}^n - 1395C_{12}^n + 3961D_{12}^n \\
&& -5388E_{12}^n + 3528\bar{F}_{12}^n - 896\bar{G}_{12}^n) \\
L_{12}^n &=& \frac{1}{495}(5A_{12}^n - 250B_{12}^n + 2165C_{12}^n - 7184D_{12}^n \\
&& +11152E_{12}^n - 8192\bar{F}_{12}^n + 2304\bar{G}_{12}^n) \\
M_{12}^n &=& \frac{1}{11}(-A_{12}^n + 61B_{12}^n - 620C_{12}^n + 2352D_{12}^n \\
&& -4096E_{12}^n + 3328\bar{F}_{12}^n - 1024\bar{G}_{12}^n) \\
N_{12}^n &=& A_{12}^n - 72B_{12}^n + 840C_{12}^n - 3584D_{12}^n \\
&& +6912E_{12}^n - 6144\bar{F}_{12}^n + 2048\bar{G}_{12}^n
\end{array}\right. \tag{B.55}$$

$$\left\{\begin{array}{lcl}
F_{14}^n &=& \frac{1}{135135}(A_{14}^n - 7B_{14}^n + 21C_{14}^n - 35D_{14}^n \\
&& +35E_{14}^n - 21\bar{F}_{14}^n + 7\bar{G}_{14}^n - \bar{H}_{14}^n) \\
G_{14}^n &=& \frac{1}{135135}(-A_{14}^n + 20B_{14}^n - 99C_{14}^n + 230D_{14}^n \\
&& -295E_{14}^n + 216\bar{F}_{14}^n - 85\bar{G}_{14}^n + 14\bar{H}_{14}^n) \\
H_{14}^n &=& \frac{1}{135135}(3A_{14}^n - 99B_{14}^n + 674C_{14}^n - 1990D_{14}^n \\
&& +3095E_{14}^n - 2663\bar{F}_{14}^n + 1024\bar{G}_{14}^n \\
&& -224\bar{H}_{14}^n) \\
J_{14}^n &=& \frac{1}{45045}(-5A_{14}^n + 230B_{14}^n - 1990C_{14}^n + 7104D_{14}^n \\
&& -12941E_{14}^n + 12754\bar{F}_{14}^n - 6496\bar{G}_{14}^n \\
&& +1344\bar{H}_{14}^n) \\
L_{14}^n &=& \frac{1}{6435}(5A_{14}^n - 295B_{14}^n + 3095C_{14}^n - 12941D_{14}^n \\
&& +26968E_{14}^n - 29888\bar{F}_{14}^n + 16896\bar{G}_{14}^n \\
&& -3840\bar{H}_{14}^n) \\
M_{14}^n &=& \frac{1}{429}(-3A_{14}^n + 216B_{14}^n - 2663C_{14}^n + 12754D_{14}^n \\
&& -29888E_{14}^n + 36736\bar{F}_{14}^n - 22784\bar{G}_{14}^n \\
&& +5632\bar{H}_{14}^n) \\
N_{14}^n &=& \frac{1}{13}(A_{14}^n - 85B_{14}^n + 1204C_{14}^n - 6496D_{14}^n \\
&& +16896E_{14}^n - 22784\bar{F}_{14}^n + 15360\bar{G}_{14}^n \\
&& -4096\bar{H}_{14}^n) \\
P_{14}^n &=& -A_{14}^n + 98B_{14}^n - 1568C_{14}^n + 9408D_{14}^n \\
&& -26880E_{14}^n + 39424\bar{F}_{14}^n - 28672\bar{G}_{14}^n \\
&& +8192\bar{H}_{14}^n
\end{array}\right. . \tag{B.56}$$

Inserting the results (B.50)-(B.56) in the expressions (B.1)-(B.7), we are able to determine those integrals. The evaluation of the (B.15)-(B.49) for $n = 1, 2, 3, 4$ in dimensional regularization is sketched in the next section.

B.2 Evaluation of integrals in dimensional regularization

Consider the integral

$$\begin{aligned} A_2^1 &= \int \frac{\mathrm{d}^2 k}{(2\pi)^2} \frac{k^2}{r_0 k^2 + \kappa_0 k^4 - (r_0 + 2\kappa_0 k^2)(q \cdot k)^2 + \kappa_0 (q \cdot k)^4} \\ &= \int \frac{\mathrm{d}^2 k}{(2\pi)^2} \frac{k^2}{r_0 k^2 + \kappa_0 k^4 - (r_0 + 2\kappa_0 k^2) k^2 q^2 \cos^2\theta + \kappa_0 k^4 q^4 \cos^4\theta} \\ &= \int \frac{\mathrm{d}^2 k}{(2\pi)^2} \frac{k^2}{r_0 k^2 (1 - q^2\cos^2\theta)[1 + \frac{\kappa_0}{r_0} k^2 (1 - q^2 \cos^2\theta)]}, \end{aligned} \tag{B.57}$$

where θ is the angle between the two vectors k and q. Making the change of variables

$$k^2 \longrightarrow \frac{r_0}{\kappa_0} \frac{1}{1 - q^2\cos^2\theta} k^2, \tag{B.58}$$

and choosing the coordinate system such that the constant vector q is parallel to the x-axis, we obtain

$$A_2^1 = \frac{1}{4\pi^2 \kappa_0} \int_0^{2\pi} \mathrm{d}\theta \frac{1}{(1 - q^2\cos^2\theta)^2} \int_0^\infty \mathrm{d}k \frac{k}{1 + k^2}. \tag{B.59}$$

The integral in θ is finite and can be evaluate immediately to give

$$\int_0^{2\pi} \mathrm{d}\theta \frac{1}{(1 - q^2\cos^2\theta)^2} = \frac{\pi(2 - q^2)}{(1 - q^2)^{3/2}}. \tag{B.60}$$

The k-integral is divergent. To deal with it, we first consider its counterpart in D dimensions, namely, the integral

$$\int_0^\infty \mathrm{d}k \frac{k^{D-1}}{1 + k^2} = \frac{\pi}{2} \csc\left(D\frac{\pi}{2}\right), \tag{B.61}$$

which diverges in the ultraviolet for $D = 2$. Writing $D = 2 - \epsilon$, expanding (B.61) in ϵ, and inserting the resulting expression together with (B.60) in (B.59), we finally obtain

$$A_2^1 = \frac{1}{4\pi\kappa_0} \frac{2 - q^2}{(1 - q^2)^{3/2}} \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0). \tag{B.62}$$

All integrals (B.15)-(B.49) can be evaluated in a similar way. The results are shown in the following tables. Infrared divergent integrals are also treated in the same way, and we distinguish them from the UV-divergent ones by adding the subscript ir to the expansion parameter ϵ. Finite integrals are not listed.

$n = 1$	
A_2^1	$\frac{1}{4\pi\kappa_0} \frac{2-q^2}{(1-q^2)^{3/2}} \frac{1}{\epsilon}$
B_2^1	$\frac{1}{4\pi\kappa_0} \frac{1}{(1-q^2)^{3/2}} \frac{1}{\epsilon}$
A_4^1	$-\frac{r_0}{16\pi\kappa_0^2} \frac{8-8q^2+3q^4}{(1-q^2)^{5/2}} \frac{1}{\epsilon}$
B_4^1	$-\frac{r_0}{16\pi\kappa_0^2} \frac{4-q^2}{(1-q^2)^{5/2}} \frac{1}{\epsilon}$
C_4^1	$-\frac{3r_0}{16\pi\kappa_0^2} \frac{1}{(1-q^2)^{5/2}} \frac{1}{\epsilon}$

$n = 2$	
A_2^2	$\frac{1}{4\pi r_0^2} \frac{2-q^2}{(1-q^2)^{3/2}} \frac{1}{\epsilon_{\text{ir}}}$
B_2^2	$\frac{1}{4\pi r_0^2} \frac{1}{(1-q^2)^{3/2}} \frac{1}{\epsilon_{\text{ir}}}$
A_6^2	$\frac{1}{32\pi\kappa_0^2} \frac{16-24q^2+18q^4-5q^6}{(1-q^2)^{7/2}} \frac{1}{\epsilon}$
B_6^2	$\frac{1}{32\pi\kappa_0^2} \frac{8-4q^2+q^4}{(1-q^2)^{7/2}} \frac{1}{\epsilon}$
C_6^2	$\frac{1}{32\pi\kappa_0^2} \frac{6-q^2}{(1-q^2)^{7/2}} \frac{1}{\epsilon}$
D_6^2	$\frac{5}{32\pi\kappa_0^2} \frac{1}{(1-q^2)^{7/2}} \frac{1}{\epsilon}$
A_8^2	$-\frac{r_0}{256\pi\kappa_0^3} \frac{128-256q^2+288q^4-160q^6+35q^8}{(1-q^2)^{9/2}} \frac{2}{\epsilon}$
B_8^2	$-\frac{r_0}{256\pi\kappa_0^3} \frac{64-48q^2+24q^4-5q^6}{(1-q^2)^{9/2}} \frac{2}{\epsilon}$
C_8^2	$-\frac{r_0}{256\pi\kappa_0^3} \frac{48-16q^2+3q^4}{(1-q^2)^{9/2}} \frac{2}{\epsilon}$
D_8^2	$-\frac{5r_0}{256\pi\kappa_0^3} \frac{8-q^2}{(1-q^2)^{9/2}} \frac{2}{\epsilon}$
E_8^2	$-\frac{35r_0}{256\pi\kappa_0^3} \frac{1}{(1-q^2)^{9/2}} \frac{2}{\epsilon}$

$n=3$	
A_4^3	$\frac{1}{16\pi r_0^3}\frac{8-8q^2+3q^4}{(1-q^2)^{5/2}}\frac{1}{\epsilon_{\rm ir}}$
B_4^3	$\frac{1}{16\pi r_0^3}\frac{4-q^2}{(1-q^2)^{5/2}}\frac{1}{\epsilon_{\rm ir}}$
C_4^3	$\frac{3}{16\pi r_0^3}\frac{1}{(1-q^2)^{5/2}}\frac{1}{\epsilon_{\rm ir}}$
A_{10}^3	$\frac{1}{512\pi\kappa_0^3}\frac{256-640q^2+960q^4-800q^6+350q^8-63q^{10}}{(1-q^2)^{11/2}}\frac{1}{\epsilon}$
B_{10}^3	$\frac{1}{512\pi\kappa_0^3}\frac{128-128q^2+96q^4-40q^6+7q^8}{(1-q^2)^{11/2}}\frac{1}{\epsilon}$
C_{10}^3	$\frac{3}{512\pi\kappa_0^3}\frac{32-16q^2+6q^4-q^6}{(1-q^2)^{11/2}}\frac{1}{\epsilon}$
D_{10}^3	$\frac{1}{512\pi\kappa_0^3}\frac{80-20q^2+3q^4}{(1-q^2)^{11/2}}\frac{1}{\epsilon}$
E_{10}^3	$\frac{7}{512\pi\kappa_0^3}\frac{10-q^2}{(1-q^2)^{11/2}}\frac{1}{\epsilon}$
$\bar{F}_{10}^3$	$\frac{63}{512\pi\kappa_0^3}\frac{1}{(1-q^2)^{11/2}}\frac{1}{\epsilon}$

$n=4$	
A_6^4	$\frac{1}{32\pi r_0^4}\frac{16-24q^2+18q^4-5q^6}{(1-q^2)^{7/2}}\frac{1}{\epsilon_{\rm ir}}$
B_6^4	$\frac{1}{32\pi r_0^4}\frac{8-4q^2+q^4}{(1-q^2)^{7/2}}\frac{1}{\epsilon_{\rm ir}}$
C_6^4	$\frac{1}{32\pi r_0^4}\frac{6-q^2}{(1-q^2)^{7/2}}\frac{1}{\epsilon_{\rm ir}}$
D_6^4	$\frac{5}{32\pi r_0^4}\frac{1}{(1-q^2)^{7/2}}\frac{1}{\epsilon_{\rm ir}}$
A_{14}^4	$\frac{1}{4096\pi\kappa_0^4}\frac{2048-7168q^2+16128q^4-22400q^6+19600q^8-10584q^{10}+3234q^{12}-429q^{14}}{(1-q^2)^{15/2}}\frac{1}{\epsilon}$
B_{14}^4	$\frac{1}{4096\pi\kappa_0^4}\frac{1024-1536q^2+1920q^4-1600q^6+840q^8-252q^{10}+33q^{12}}{(1-q^2)^{15/2}}\frac{1}{\epsilon}$
C_{14}^4	$\frac{1}{4096\pi\kappa_0^4}\frac{768-640q^2+480q^4-240q^6+70q^8-9q^{10}}{(1-q^2)^{15/2}}\frac{1}{\epsilon}$
D_{14}^4	$\frac{1}{4096\pi\kappa_0^4}\frac{640-320q^2+144q^4-40q^6+5q^8}{(1-q^2)^{15/2}}\frac{1}{\epsilon}$
E_{14}^4	$\frac{1}{4096\pi\kappa_0^4}\frac{560-168q^2+42q^4-5q^6}{(1-q^2)^{15/2}}\frac{1}{\epsilon}$
$\bar{F}_{14}^4$	$\frac{3}{4096\pi\kappa_0^4}\frac{168-28q^2+3q^4}{(1-q^2)^{15/2}}\frac{1}{\epsilon}$
$\bar{G}_{14}^4$	$\frac{33}{4096\pi\kappa_0^4}\frac{14-q^2}{(1-q^2)^{15/2}}\frac{1}{\epsilon}$
$\bar{H}_{14}^4$	$\frac{429}{4096\pi\kappa_0^4}\frac{1}{(1-q^2)^{15/2}}\frac{1}{\epsilon}$