Chapter 7

Conclusions and Perspectives

Several aspects of systems consisting of fluctuating fluid membranes have been considered. Firstly, the issue of the renormalizability of the Canham-Helfrich model for fluid membranes has been investigated. In contrast to former studies, where the model had been expanded in a power series in the displacement field and treated perturbatively with help of Feynman diagrams, we kept the full Canham-Helfrich energy throughout the whole calculation. With help of the derivative expansion method, we were able to determine the renormalization of the full surface and bending energy terms, and proved the renormalizability of the model at one-loop order.

Following the investigation of the classical Canham-Helfrich membrane, we extended the model to account for quantum fluctuations. While inadequate for biomembranes, which typically exist at high temperatures, our model provides a good description of quantum interfaces, such as the Helium liquid-vapor interface at very low temperatures, or the He^3 film on top of the He^4 bulk in He^3-He^4 mixtures. Both perturbative one-loop calculations and a non-perturbative approach in a large number of dimensions of the embedding space showed that the model displays a crumpling transition, and the critical temperature was determined as a function of the bending rigidity of the membrane. An investigation of the crumpled phase in large embedding space dimension revealed the existence of a persistence length determining the minimum lateral size a membrane must have to appear crumpled, even above the critical temperature. This persistence length, derived in the non-perturbative approach, was shown to agree with the perturbatively obtained de Gennes-Taupin persistence length for fluid membranes.

In addition, we studied a system consisting of a parallel arrangement of

membranes separated by thin layers of solvent. The lamellar structure of such a stack of membranes was shown in a perturbative one-loop calculation to undergo a melting-like transition into a vertically disordered phase. This transition does not take place in the simple Helfrich model for a stack, which contains only a harmonic curvature energy, being equivalent to de Gennes' theory of smectic-A liquid crystals. In our perturbative calculations, we extended the Helfrich model to contain curvature energy terms of up to fourth order in the displacement field. The phase transition we found is caused between a competition between the compressibility energy of the stack and the softening of the bending rigidity due to thermal fluctuations, an inherently anharmonic effect. The critical temperature for the transition was determined as a function of the bending rigidity and the critical exponents for the compressibility and specific heat of the stack were calculated. To characterize the disordered phase, not accessible by perturbative methods, we resorted again to a non-perturbative calculation in a large number of dimensions of the embedding space. In this way, we were able to calculate the full phase diagram of the model as a function of temperature and interlayer separation. While the melting transition predicted by our model may not be observed in stacks of lipid membranes separated by water, due to their large bending rigidity, our results are still applicable to diluted smectic systems, which are found experimentally to have very low rigidities, depending on the solvent. Another application field of our model also may be systems of oil and water separated by soap films, where the transition carries lamellar phases into microemulsions. Also, our system is very similar to a smectic-A liquid crystal. The vertical melting transition we predict is equivalent to the smectic-to-nematic transition in these systems.

The non-perturbative approach used both for the quantum membrane and for the stack of membranes revealed that the common assumption of in-plane incompressibility of fluid membranes cannot be maintained for systems of very small rigidities. In this case, the in-plane elasticity energy of the membranes should be taken into account. It would be interesting to investigate the effect of such terms on the critical exponents of the melting transition. This implies including the effect of the in-plane flow of molecules within the membrane, thus allowing for more degrees of freedom in the energies of both the quantum membrane and of the stack. Another important point, which was left out of our previous discussions, is the possibility of topology changes of the membrane. Although rendering practical calculations more difficult, the inclusion of the Gaussian curvature, which characterizes the topology of

the membrane, would render fluid membrane models more realistic, and thus applicable to experimental situations.