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Unbiased Beta Despite Biased Data: Some Elementary Results

In many valuations, the cost of equity is determined on the basis of the CAPM. The dominating instrument for estimating the beta factor within this framework is the so-called Market Model. As far as the own beta of the valuation object is concerned, the reliability of the data available for this purpose is often questioned and an alternative tool, in particular a peer group beta, is used. The following article identifies types of data bias that do not affect the outcome of beta estimation – both for an individual company and for a peer group. The results urge caution against a hasty rejection of the object beta and an almost automatic use of peer group betas.

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I. Introduction

Having passed its academic peak, the CAPM is still of outstanding importance in practice for determining the cost of capital in company valuations. Most of the issues and discussions there concern metric problems, especially for small and medium-sized corporations (SMC).

As the CAPM is a single-factor model, attempts to "modify" the CAPM by introducing further factors[1] do not appear to be helpful. Beyond such and similar attempts to explain higher capital costs by other factors, however, serious conceptual problems and questions remain with regard to the classical measurement procedure. Looking at beta, this procedure is still coined by Sharpe´s original "Market Model",[2] a simple linear regression using historical return data.

A classic problem with this approach, to which the following sections of this article are devoted, arises when the valuer expects the prices of the valuation object to be biased in the relevant reference period.[3] Of course, such a judgement is problematic, because the "true" prices or rates of return are unknown stochastic variables, and we just see their historic realizations. Nonetheless, we have to decide whether we are comfortable with the measured data or whether we suspect biases of varying quality. Thus, it is no surprise that the consequences of measurement errors or other data problems on the reliability of beta estimations have been examined under different perspectives. Lagged disturbances and serial correlations,[4] the inconsistent use of market indices[5] or a time varying equity premium[6] are prominent examples for conditions that do not fulfil the data requirements of the Market Model or the logic of the CAPM itself. While previous studies start with the definition of the respective problem and look for the resulting impact on the beta estimation, our contribution starts by the question how the measured value of share prices or returns may deviate from the respective "true value" without disturbing the resulting beta: The following sections II and III identify conditions under which data biases will not distort conventional beta estimates. Thus, if one of these conditions is fulfilled, beta estimates are still unbiased and there is no need to fall back to other alternatives to obtain (supposedly) less biased betas e.g. derived from peer groups.

In practice, however, the usual procedure for suspectedly distorted prices or returns is generally to use a peer group beta. This results in two problems: Firstly, an obvious representation problem and secondly, the distribution of returns/prices of the peer group could deviate from the valuation object's distribution. Section IV therefore applies the approach used in the two previous chapters to this special form of problem handling.

Summarized in section V, the results reveal some important misunderstandings about the nature of biased data and their consequences for beta determination.

II. Deterministic (Functional) Bias

One of the factors responsible for the success of the CAPM is the structural conformity of the beta with the slope factor of a simple regression called the Market Model. Looking at the definition:

$$\beta_k = \frac{\mathrm{Cov}(r_k, r_M)}{\mathrm{Var}(r_M)}, \tag{1}$$

r_k and r_m are the stochastic rates of return of the share k (our valuation object) and the market portfolio, respectively. The only difference between the theoretical parameter within the CAPM and its empirical estimator within the Market Model is that the latter uses historical realizations of the two returns.

If the share prices are not distorted in relation to the theoretical specifications of the CAPM, an unbiased estimate will be obtained using this method.[7] In other words: Unbiased prices are a sufficient condition for an unbiased estimation of beta:

unbiased price ⇒ unbiased beta

At the same time, this implication does not mean that every form of price bias necessarily leads to a biased beta estimate. Even in the deterministic domain, the opposite holds true e.g. if the following, very simple condition is satisfied. Let a be a deterministic parameter that biases the "true" price of share k proportionally like a scaling factor:

$$\hat{P}_{k(a),t} = aP_{k,t} \tag{2}$$

In general, distorted parameters are denoted by a hat.

Then, there is no effect on k´s rate of return because:

$$\hat{r}_{k(a),t} = \frac{aP_{k,t}}{aP_{k,t-1}} - 1 = \frac{P_{k,t}}{P_{k,t-1}} - 1 = r_{k,t} \tag{3}$$

Consequently, such a strictly linear bias of the price of k or the market portfolio or of both will not influence the measurement of k´s beta because it leaves the argument of the covariance unchanged.

1 Cf. e.g. Barr/Lohrey, Working with Data: Public Company and Private Company Size Premia, The Value Examiner, March-April (2018): 18, for the size premium within the "modified CAPM Method" and the "Build-Up Method". For critique on an additional country risk premium, see Kruschwitz/Löffler/Mandel, Damodaran's Country Risk Premium: A Serious Critique, Business Valuation Review, vol. 31 (2-3), (2012): 75.

2 Cf. Sharpe, A Simplified Model for Portfolio Analysis, Management Science, vol. 9 (2), (1963): 277.

3 Most discussions regarding potential biases relate to the lack of liquidity of the share concerned, cf. e.g. Ehrhardt/Koerstein/Möller, Betaschätzung bei unregelmäßig gehandelten Aktien, Die Wirtschaftsprüfung, vol. 73 (13), (2020): 768, and Grbenic, Beta Estimation under Thin Trading Conditions, The European Business Valuation Magazine, vol. 2 (2), (2023): 18.

4 Cf. Scott/Brown, Biased Estimators and Unstable Betas, The Journal of Finance, vol. 35 (1), (1980): 49-55.

5 Cf. Bartholdy/Peare, Unbiased Estimation of Expected Return Using CAPM, International Review of Financial Analysis, vol. 12 (1), (2003): 69-81.

6 Cf. Lewellen/Nagel, The Conditional CAPM Does not Explain Asset-pricing Anomalies, Journal of Financial Economics, 82 (2), (2006): 289-314.

7 Cf. Knoll, Value, Price and Beta: Some Clarifications, Corporate Finance, vol. 8 (11-12), (2018): 350 (351-353), for the following analysis.

This simple insight has not yet found its way into the understanding of many players in the valuation scene. As an example, here is a passage from a legal essay, written by a prominent corporate lawyer:

“However, it is correct not to take into account the downward distortion of the company's original beta factor corresponding to an upwardly distorted share price in the context of the capitalized earnings value valuation and to reject the original beta factor as not meaningful … For unlike the upwardly distorted share price, the downwardly distorted beta factor and the significant increase in the capitalized earnings value necessarily associated with it do not correspond to the actual operating earning power.”[8]

Is that true? If speculators or others have driven up the price level by $D\%$ above the “true” value, what is the consequence for the measured return r of the prices P used between two measurement points t and $t\text{-}1$? Compared to unbiased prices, we always get $D\%$ higher measurement results. Thus, the increase by $D\%$ causes a scaling factor $a = 1{+}D\%$ and reproduces the correct beta as shown in (3). “The downward distortion of the company's original beta factor corresponding to an upwardly distorted share price” does not exist, so all conclusions based on it collapse! Like any formulation, the quote may also show specific traits, but in its invocation of price distortions in order to reject the use of the beta of the valuation object, it is quite characteristic of parts of the legally influenced company valuation in German-speaking countries.

III. Stochastic Bias

So far, so well, but deterministic functional biases are very strong forms of bias. Looking at the relevant data, you are more likely to think about stochastic distortions. Therefore, we will regard this case now.

To further simplify this case, we directly regard the return level and model the stochastic bias by adding a stochastic disturbance term to the rate of return of share k only.[9] The formal presentation of this procedure is:

$$\hat{r}_{k(\omega)} = r_k + \omega_k \tag{4}$$

$\hat{r}_{k(\omega)}$ and ω_k are the biased rate of return and the disturbance term for share k, respectively. Contrary to the deterministic proportional bias in chapter II, now the argument of the covariance is no longer unchanged. Therefore, we must look for a condition under which this change causes no other result for the covariance than the use of the “true” rates of return.

Inserting (4) in (1), we get the measured beta in the case of a stochastic bias[10]

$$\hat{\beta}_{k(\omega)} = \frac{\mathrm{Cov}\left[r_k + \omega_k, r_M\right]}{\mathrm{Var}\left[r_M\right]} = \frac{\mathrm{Cov}\left[r_k, r_M\right] + \mathrm{Cov}\left[\omega_k, r_M\right]}{\mathrm{Var}\left[r_M\right]}$$

$$= \frac{\mathrm{Cov}\left[r_k, r_M\right]}{\mathrm{Var}\left[r_M\right]} + \frac{\mathrm{Cov}\left[\omega_k, r_M\right]}{\mathrm{Var}\left[r_M\right]} \tag{5}$$

and, consequently, a remarkably simple result:

$$\hat{\beta}_{k(\omega)} = \beta_k \Leftrightarrow \mathrm{Cov}\left[\omega_k, r_M\right] = 0 \tag{6}$$

As long as there is no linear dependence between the disturbance term and the market return,[11] we can use the biased returns for estimating share k´s beta with the Market Model.

To demonstrate this effect, we present a strongly simplified example. We regard 12 measurement periods with the following parameter values:

Table 1: Numbers of the example

t	$r_{M,t}$	ω_t	$r_{k,t}$	$\hat{r}_{k(\omega),t} = r_{k,t} + \omega_t$
1	7	0.8	6	6.8
2	3	1.8	3	4.8
3	4	-0.2	5	4.8
4	-3	-0.2	-1	-1.2
5	2	-1.2	1	-0.2
6	-4	1.8	-2	-0.2
7	6	0.8	5	5.8
8	7	-0.8	3	2.2
9	5	0.0	3	3.0
10	-3	-0.8	-1	-1.8
11	0	-0.8	0	-0.8
12	4	0.0	2	2.0
$\mathrm{Cov}[r_m,*]$		0	9.5455	9.5455
$\mathrm{Var}[r_m]$	15.6970	Beta	0.6081	0.6081

8 Decher, Die Bedeutung des Börsenkurses für die Unternehmensbewertung bei Strukturmaßnahmen: Eine Reise über vier Stationen, Die Aktiengesellschaft, vol. 68 (4), (2023): 106 (114), translation of the German original.

9 Scott/Brown, Biased Estimators and Unstable Betas, The Journal of Finance, vol. 35 (1), (1980): 49 (50) use an additional stochastic disturbance term (with a zero mean) to the market return as the starting point for their incorporation of econometric problems into the Market Model. In valuation practice, the argumentation is completely different: Most criticism based on distorted prices/returns is formulated only in relation to the share in question and, therefore, corresponds to an otherwise unbiased pricing – perhaps because the critics assume that a biased return in one company within the market portfolio does not distort the whole market return in a measurable dimension. Thus, we focus directly on k and forego severe restrictions on the additional term (like the zero mean in Scott/Brown).

10 Cf. for the calculation e.g. Henze, Stochastik für Einsteiger, 11th ed. 2017: 166.

11 If we can assume that the parameters are jointly normally distributed, they must be independent without any restriction, cf. e.g. Wooldridge, Introductory Econometrics, 6th ed. 2016: 668.

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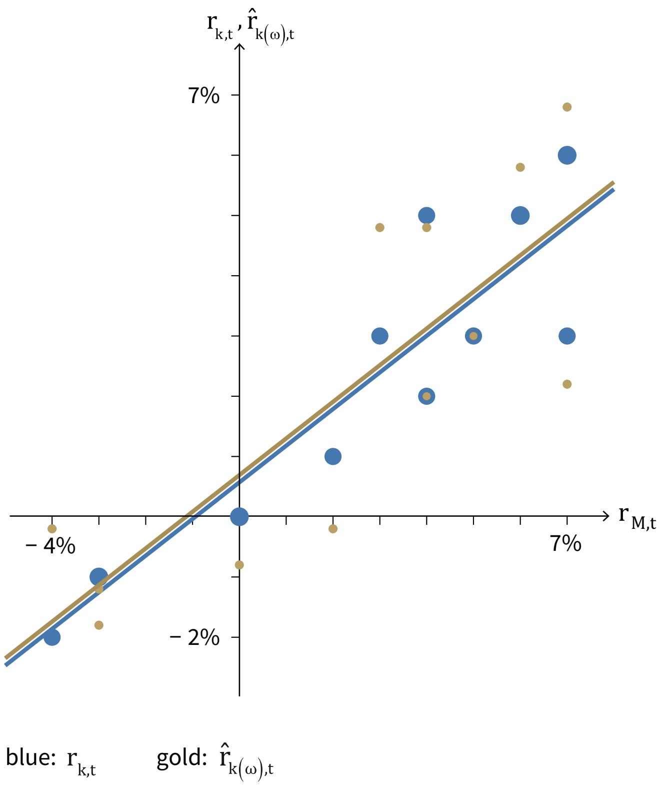
You can interpret the figures as monthly values rounded (for the sake of simplicity) to the nearest thousandth for all variables independent of ω and to the nearest basis point for the others.[12]

Figure 1 shows the different relationships graphically. The horizontal axis scales the market return, the vertical axis $r_{k,t}$ and $\hat{r}_{k(\omega),t}$. The dots for the different pairings between the market return and the other parameters are completed by corresponding linear regression lines. As expected from the calculated betas, both regression lines are parallel – the line of the measured returns runs slightly higher than the line of the unbiased returns, since the mean of ω_t is slightly greater than zero. This or any other vertical difference between the parallel lines is irrelevant for the calculated beta value that is mathematically the slope factor of the regression line.

Figure 1: Data points and linear regressions



IV. Peers

Although there are other methods that are closer to the theoretical core of the CAPM,[13] in practice beta is mainly determined using the Market Model for a peer group if distorted returns for the share of the valuation object are suspected. The main problem of this procedure is certainly the identification of suitable peers. We do not want to present all the criteria discussed in practice and literature, but just refer to a requirement formulated by *Ziemer*.[14] As the CAPM itself defines assets just by their stochastic properties, *Ziemer* demands that a peer´s share should have the same distribution function for its rate of return as the valuation object. Looking at a peer group, this condition must be fulfilled by all its members. The correspondence to the formal implication in section II is straightforward, but we have seen that this is a sufficient condition and there are harmless injuries of the requirement.

Consequently, one can try to transfer the effects described in sections II and III on peer betas by interpreting "biased" not as a current divergence of share $\mathbf{k}$´s return distribution from its "true" value, but as the normal divergence between the rate of return of the peer corporation and the "true" rate of return of share $\mathbf{k}$.

In order to find suitable peers in practice, comparability criteria generally apply that are based on assumed drivers of the operating business rather than a direct estimate of risk measures or distribution functions. While it is already daring to assume a sufficient operational similarity, it is almost utopian to find sufficient statements on econometric details like covariance matrices in practice![15] Nevertheless, the importance of peer group betas in practice justifies a closer look at how the described fault tolerance can be applied to them.

1. Pure-Play Beta

Seen from a technical perspective, the transfer is quite simple if there is just one peer used to estimate the desired beta. More than 40 years ago, *Fuller* and *Kerr* introduced this "Pure-Play Technique" under the special perspective on divisional cost of capital.[16] At the same time, the identification problem described is becoming more important. Already before *Fuller/Kerr's* contribution, the rarity of perfect matches had been noted and subsequently confirmed time and again.[17] Even the greatest optimists today will not claim that two companies are identical in reality. For this reason in particular, "Pure-Play Beta" has not gained any major significance beyond its use for divisions. The positive aspect of easy technical transferability of the approach is therefore more than compensated for by the fact that it is largely irrelevant in practice. Nevertheless, it is helpful for the following presentation to write the corresponding equations for the share of one single peer $\mathbf{p}$ by adding $\mathbf{p}$ to the equation number.

$$P_{p,t} = aP_{k,t} \tag{2p}$$

which again causes irrelevance results like in (**3**). Furthermore,

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12 The different rounding levels help to fulfil condition (6) exactly. For practical purposes, a small deviation from zero does not matter as long as its effect on the calculated beta is beyond the regularly displayed two decimal places.

13 Cf. Knoll/Kruschwitz, Anmerkungen zum Problem der Geld-Brief-Spanne für Betaermittlung und Renditeerwartung, Corporate Finance, vol. 13 (7-8), (2022): 221.

14 Cf. Ziemer, Quantitative Bestimmung der Verzerrung des Beta-Faktors durch Rückgriff auf gefilterte Peer-Group, Journal of Banking Law and Banking, vol. 24 (1), (2012): 50.

15 In Germany, valuation reports mainly show the t-value of the estimated beta and the coefficient of determination of the regression, although both are not very helpful and can be converted into each other, cf. already Knoll/Ehrhardt/Bohnet, Kleines Beta – kleines Bestimmtheitsmaß: großes Problem?, CFO aktuell, vol. 1 (5), (2007): 210.

16 Cf. Fuller/Kerr, Estimating the Divisional Cost of Capital: An Analysis of the Pure-Play Technique, Journal of Finance, vol. 36 (5), (1981): 997.

17 Cf. Bower/Jenks, Divisional Screening Rates, Financial Management, vol. 4 (3), (1975): 42 (47), and Herget, Strategic Resource Allocation Using Divisional Hurdle Rates, Planning Review, vol. 15 (1), (1987): 28 (30) as examples before and after Fuller/Kerr´s proposal.

the stochastic bias is presented in

$$r_p = r_k + \omega_p \tag{4p}$$

with the same consequences as (4).

If we look at the minor application of Pure-Play Beta, it must be assumed that obviously nobody believes in (2p) and (4p). We will soon see that this has an unpleasant consequence.

2. Peer Group

The missing 1:1-identity is probably the most important reason for the victory march of peer group betas in practice, although it is of course completely unclear why the average of a set of biased data should result an (approximately) unbiased estimate. Not surprisingly, we will reveal other real and supposed problems of this procedure in the following.

a) 1/Π-Calculations

These problems are ultimately related to the fact that instead of a single stochastic variable, the average of a group of such variables must be considered. For the sake of simplicity, we assume the use of an unweighted average of all peer group betas, which is at least common in practice.[18]

Let us begin with the Ziemer-criterion. Its transformation for a group of Π peers is straightforward: The distribution function of an equally weighted linear combination of the peer returns must be identical to that of share k. If $F(x)$ is the distribution function with the argument x, and r represents the rates of return, we should get:

$$F\left(r_k\right) = F\left(\frac{1}{\Pi}\sum_{p=1}^{\Pi} r_p\right) \tag{7}$$

That is, for expected value and variance:[19]

$$E\left[r_k\right] = E\left[\frac{1}{\Pi}\sum_{p=1}^{\Pi} r_p\right] = \frac{1}{\Pi}\sum_{p=1}^{\Pi} E\left[r_p\right] \tag{8}$$

$$Var\left[r_k\right] = Var\left[\frac{1}{\Pi}\sum_{p=1}^{\Pi} r_p\right] = \frac{1}{\Pi^2}\sum_{p=1}^{\Pi}\sum_{q=1}^{\Pi} Cov\left[r_p, r_q\right] \tag{9}$$

Looking at this, we must remember that the return distributions of the different peers will not be identical – otherwise each peer must have the same return distribution as the valuation object and a “grouping” would lose its main sense. Beside the mean-condition (8), condition (9) will only be fulfilled by chance, if at all, since we are not aware of any case in which a short list of possible peers was created with a view to a covariance matrix.

18 Sometimes the median is used together with or instead of the unweighted mean.

19 Cf. for the general formulas concerning the calculations of mean and variance for a linear combination of stochastic variables Auer/Rottmann, Statistik und Ökonometrie für Wirtschaftswissenschaftler, 4th ed. (2020): 241. The simple notation is based on the fact that we use an unweighted average of the peers.

Do things look better for not sufficient, but necessary conditions?

Looking at the deterministic bias, we go from (2p) to:

$$\frac{1}{\Pi}\sum_{p=1}^{\Pi} P_{p,t} = aP_{k,t} \tag{2Π}$$

What seems to be comparatively harmless here represents an essential requirement for possible changes to a peer group that fulfills (2Π). You may only add or exclude companies whose price is itself the multiple of P_k. In the case of an exclusion of an arbitrary chosen peer Π the average of the remaining peers must itself be equal to $aP_{k,t}$ or formally

$$\frac{1}{\Pi - 1}\sum_{p=1}^{\Pi-1} P_{p,t} = aP_{k,t} \tag{10}$$

and then the same must hold for $P_{\Pi,t}$, because otherwise (2Π) would be false.[20] The proof for an extended peer group is analogous.

This is the unpleasant consequence announced in section IV.1: If you identify such a company, you can use the Pure-Play Technique and no longer need a group. Conversely, if you are convinced that no such company exists, and looking at the end of section IV.1 most people seem to be convinced, 2Π is a really demanding condition!

Finally, the possibility that the average price of the reduced or extended peer group is also proportional to $P_{k,t}$ but by another scaling factor than the average price of the original peer group is not really a convincing argument either: The joint occurrence of (2Π) and (10) is extremely unlikely and has never been addressed in valuation reports.

Transferring the stochastic bias (4) of section III to a peer group is a little tedious. First, we have to transfer equation (4p) on the whole peer group:

$$r_\Pi = \frac{1}{\Pi}\sum_{p=1}^{\Pi} r_p = \frac{1}{\Pi}\sum_{p=1}^{\Pi}\left(r_k + \omega_p\right) = r_k + \frac{1}{\Pi}\sum_{p=1}^{\Pi}\omega_p \tag{4Π}$$

and a corresponding beta:

$$\beta_\Pi = \frac{Cov\left[r_k + \frac{1}{\Pi}\sum_{p=1}^{\Pi}\omega_p, r_M\right]}{Var\left[r_M\right]} = \frac{Cov\left[r_k, r_M\right] + Cov\left[\frac{1}{\Pi}\sum_{p=1}^{\Pi}\omega_p, r_M\right]}{Var\left[r_M\right]}$$
$$= \frac{Cov\left[r_k, r_M\right]}{Var\left[r_M\right]} + \frac{\frac{1}{\Pi}\sum_{p=1}^{\Pi} Cov\left[\omega_p, r_M\right]}{Var\left[r_M\right]} \tag{5Π}$$

Obviously, we have an analogue independence condition for the average of the disturbance terms within the peer group as for share k in (6). We are convinced that this argument has never been used in practice as the main criterion for a peer group composition to estimate beta.

20 Equating (2Π) and (10) and solving for $P_{\Pi,t}$ gives $aP_{k,t}$.

b) Sequential irrelevance

The previous sections do not provide good news for the use of peer groups. The problems mentioned are real, but surprisingly they are discussed almost nowhere. Another irritating aspect is also barely presented.

According to the elementary beta calculation, a regression between the linear combination of the equally weighted peer returns and the market return $\mathbf{r}_M$ would have to be carried out and the slope coefficient determined in this way used as an estimator of the beta sought:

$$\frac{\text{Cov}\left[\frac{1}{\Pi}\sum_{p=1}^{\Pi} r_p, r_M\right]}{\text{Var}\left[r_M\right]} = \frac{1}{\Pi} \cdot \frac{\text{Cov}\left[\sum_{p=1}^{\Pi} r_p, r_M\right]}{\text{Var}\left[r_M\right]} = \frac{1}{\Pi} \cdot \frac{\sum_{p=1}^{\Pi} \text{Cov}\left[r_p, r_M\right]}{\text{Var}\left[r_M\right]} \tag{11}$$

In fact, the regressions are performed for the individual peers and then an equally weighted mean of the different betas is determined as the estimator.

This procedure leads to the following estimate:

$$\frac{1}{\Pi}\sum_{p=1}^{\Pi} \frac{\text{Cov}\left[r_p, r_M\right]}{\text{Var}\left[r_M\right]} = \frac{1}{\Pi} \cdot \frac{\sum_{p=1}^{\Pi} \text{Cov}\left[r_p, r_M\right]}{\text{Var}\left[r_M\right]} \tag{12}$$

As the results are identical, this time we are dealing with a just supposed problem. It remains to be seen how many users are aware of this fact, which is ultimately based on the linearity property of the mean-operator.

V. So what?

The starting point of our presentation was quite simple: Is it possible to get unbiased beta estimates with the common Market Model although the data used are biased? The answer is clear: yes!

This conclusion only applies to certain types of biases and therefore valuers must always check what type of bias may be affecting their data. If the basic version of the Market Model no longer seems appropriate, there are a number of alternative methods. Since peer group betas are then used most frequently, we examined them more closely with regard to the previously identified irrelevant data biases. The results are sobering, because regardless of the fact that the derived requirements can hardly be fulfilled, peer groups are never formed in practice with them in mind.

Therefore, the message for listed valuation objects is that alternative methods should be considered before using a peer group for beta estimation, and sometimes even using the basic version of the Market Model may be the lesser of two evils.[21] ◆

21 Cf. Knoll, op. cit. (fn. No. 7): 353. The literature on methods and problems of beta estimation has grown considerably in recent years. An overview of this would go beyond the narrow scope of this article.