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Fairness and competition in a bilateral matching market ☆

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ABSTRACT

This paper analyzes fairness and bargaining in a dynamic bilateral matching market. Traders from both sides of the market are pairwise matched to share the gains from trade. The bargaining outcome depends on the traders' fairness attitudes. In equilibrium fairness matters because of market frictions. But, when these frictions become negligible, the equilibrium approaches the Walrasian competitive equilibrium, independently of the traders' inequity aversion. Fairness may yield a Pareto improvement; but also the contrary is possible. Overall, the market implications of fairness are very different from its effects in isolated bilateral bargaining.

1. Introduction

How does welfare in a bilateral matching market depend on whether traders are fair or egoistic? We address this question by embedding the ultimatum bargaining game in a dynamic bilateral matching market. In the ultimatum game, the equilibrium played by selfish players yields an outcome that is substantially different from a fair division of the available surplus. Therefore, it is well suited to explore how fairness and selfishness differ in their implications for the matching market equilibrium.

In ultimatum bargaining, one of the parties in a pairwise match makes a take–it–or–leave–it offer on how to share the gains from trade; the other party can either accept or reject the offer. If the offer is rejected, both parties have to wait for a new match with another trader. The game theoretic prediction is that, when all traders are selfish and rational, the party making the offer appropriates the entire gains from the match. Since the influential first experiment by Güth et al. (1982), observations from numerous experimental studies refute this prediction.[1] This has sparked the idea in behavioral economics that individuals have social rather than selfish preferences, exhibiting considerations of fairness and inequity aversion in bargaining. We use this approach by applying the well–established formalization of inequity aversion by Fehr and Schmidt (1999) to the traders' preferences. In our model, fairness motives play a role only in bilateral bargaining. But, the bargaining payoffs determine the traders' incentives to enter the market and

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[1] See Güth and Kocher (2014) for a survey.

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to search for a trading partner. Therefore, by varying the parameters of their Fehr–Schmidt utilities, we can investigate how changes in the degree of inequity aversion affect the matching market equilibrium.

We analyze the steady state equilibrium of the pairwise matching market: At each date the outflow of agents, who have concluded a transaction, is equal to the inflow of new agents, who decide to enter the market. There are two types of agents, e.g., sellers and buyers or workers and employers. If two agents of opposite types meet, they bargain about sharing the gains from trade. All traders of the same type have the same Fehr–Schmidt utility function. This allows us to study how the fairness attitudes on either side of the market are reflected in the steady state equilibrium. When an active trader fails to find a partner in the current period, he has to wait until the next period to search again. The same happens to the parties in a match if they do not reach an agreement in the ultimatum game. We refer to these waiting costs as market frictions in the matching process.

Our analysis shows that because of market frictions the matching market equilibrium depends on the traders' fairness concerns. But, when these frictions vanish, the decentralized matching market equilibrium tends towards the Walrasian competitive equilibrium of a centralized market, independently of whether the traders are fair or egoistic in a match. Thus, in the frictionless limit fairness attitudes play no role. The reason is that in this limit the delay costs of disagreement in bargaining vanish. This implies that the *net* surplus that two traders can share in a match is negligible. Therefore, also considerations of fairness in bargaining about the net surplus become insignificant in a frictionless market.

Further, we can compare welfare in the matching market equilibrium and the Walrasian equilibrium. It turns out that the Walrasian outcome generically Pareto dominates the outcome of the matching market. It is not possible, therefore, that one side of the market is better off than in the competitive equilibrium because, e.g., traders on the other side make very fair offers. Indeed, it is true more generally that any variation in the parameters of the traders' Fehr–Schmidt utilities always affects welfare on both sides of the matching market in the same way. The distributional impact of inequity aversion is therefore very different from isolated bilateral bargaining. The reason is that in the matching environment any change in expected payoffs has repercussions on market entry. To keep entry balanced on both sides of the market, the matching probabilities adjust and move the market entry payoffs of all traders always in the same direction.

By the above insight, welfare comparisons of fairness and selfishness can always be expressed in terms of the Pareto criterion. Our analysis identifies situations in which all traders are better off if they all split the gains from trade in a match fairly, instead of making selfish proposals. But, for other parameter combinations also the contrary can happen: Two–sided selfishness can yield an equilibrium outcome that is Pareto superior to the outcome with two–sided fairness. This is the case when the outcome with selfish agents is closer to the competitive equilibrium than the outcome with fair agents. Another interesting comparison is possible for the constellation where traders on the short side of the market are fair, whereas on the long side they are selfish. In this case, both sides of the market would be better off if also the traders on the long side were fair instead of selfish. The intuition is that this would make market entry more attractive on the short side, thereby also increasing the matching probability on the long side.

Our stylized model may be helpful to contemplate the role of fairness in decentralized markets. Bilateral negotiations are important not only in bazaars but also in the markets for professionals and professional services, used cars, real estate, and inputs for manufacturing firms. Our findings may be relevant also for fair trade arrangements that support buying products from producers in developing countries at a fair price.[2] The analysis of this paper indicates that the impact on market participation decisions can be critical for the welfare implications of such arrangements. Our model can also be applied to wage bargaining in a labor market with matching frictions. Differences in bargaining attitudes between men and women have been brought forward in labor economics as a factor contributing to the gender wage gap.[3] The argument is that women negotiate worse wages than men because they tend to be less egoistic. Our results suggest that this argument applies especially when market frictions are important. The removal of frictions should not only increase welfare but also reduce the gender wage gap.[4]

Related literature This paper combines fairness preferences with bargaining in a dynamic matching environment. The experimental evidence from ultimatum bargaining and other games has motivated the integration of concerns for fairness, reciprocity, and altruism in individual decision making.[5] The pioneering model of Rabin (1993) incorporates fairness by the idea that individuals respond non–selfishly to fair intentions of others. In Bolton and Ockenfels (1999) and Fehr and Schmidt (1999), individuals are inequity averse and care not only about their own payoff but also about its relation to other agents' payoffs. This paper uses the utility specification proposed by Fehr and Schmidt (1999), which is linear in the inequity terms. This simplifies the analysis of the steady state equilibrium.

The non–cooperative approach to the bargaining problem in a dynamic matching market goes back to Rubinstein and Wolinsky (1985).[6] This approach has the advantage that we can explicitly include the bargaining attitudes of traders in a match. By not restricting traders to be egoistic, we extend the literature on decentralized trade by behavioral aspects.

2 For a survey, see Dragusanu et al. (2014).

3 See, e.g., Andreoni and Vesterlund (2001) for experimental evidence and Card et al. (2016) for an empirical study.

4 To address the gender wage gap more explicitly, our model can be extended to allow for gender differences in bargaining preferences on the workers' side of the market.

5 For surveys, see Fehr and Schmidt (2006) and Sobel (2005).

6 The literature on dynamic matching markets is too vast to review here in detail. Instead we briefly point out those articles that are most relevant for our contribution. For a detailed overview of dynamic matching and bargaining models, see Osborne and Rubinstein (1990).

As pointed out by Gale (1987), in a dynamic matching environment the distinction between flows and stocks of traders is important for comparing the matching market equilibrium with the competitive equilibrium.[7] Our analysis follows Gale (1987) and Binmore and Herrero (1988) by adding a market entry stage to the matching process. Thereby, the Walrasian competitive equilibrium in terms of flows into the market becomes well–defined as a reference point. While Binmore and Herrero (1988) consider the case where traders enter the market only at some initial date, our steady state analysis considers a constant flow of potential entrants as in Gale (1987). However, while in Gale (1987) the entry costs are identical for all agents, we consider a distribution of entry costs that generates continuous supply and demand schedules at the entry stage. In Gale (1987) supply and demand are step functions because agents on both sides of the market are distinguished by finitely many valuations. In contrast, in our model agents on each side of the market are identical after their entry costs are sunk, and so in a match they bargain over a unit surplus as in Rubinstein and Wolinsky (1985). These features make the present matching market model easy to analyze and facilitate the derivation of comparative static results.

Whereas this paper analyzes the partial equilibrium of a single market with decentralized trade, Dufwenberg et al. (2011) study other–regarding preferences in a general equilibrium model of centralized trade. In their model, individual preferences depend not only on own consumption but also on the consumption and budget sets of the other traders. But, if utilities are separable between own consumption and the consumption and budget sets of others, then other–regarding preferences do not affect demand decisions and the equilibrium *allocation*. This may look a bit like our result that in the frictionless limit the matching market equilibrium is independent of the agents' fairness preferences. But, in Dufwenberg et al. (2011) the agents' equilibrium *utilities* do depend on the externalities generated by other–regarding preferences.[8] In the frictionless limit of our model, however, the equilibrium utilities do not depend on the parameters of the traders' Fehr–Schmidt utilities. The intuition is that – in line with the experimental setting of the ultimatum game – in our model fairness preferences are defined over the split of the gains from trade in the personal interaction between a pair of traders. As the matching market becomes more competitive, the available gains from trade in a bilateral match become smaller and this reduces the impact of inequity aversion on the traders' equilibrium utilities. Fairness does not matter for the equilibrium utilities in the competitive limit without market frictions.

Whereas in Dufwenberg et al. (2011) individuals are competitive price takers, Sobel (2015) establishes conditions on other–regarding preferences that lead to competitive outcomes in a centralized double–auction market. Under these conditions, the market participants' behavior looks selfish, even though they are not selfish. Furthermore, these conditions become weaker in a large market. Fairness is probably more relevant for individual behavior in small groups rather than in centralized environments with many participants. Indeed, for extensions of the ultimatum game with multiple responders or proposers, the erosion of fairness by competition is also theoretically predicted by the Fehr and Schmidt (1999) model.[9] Similarly, Bolton and Ockenfels (1999) show that Bertrand and Cournot games may induce competitive self–interested behavior, even though firms care not only about their own profit.[10] This is so because a single player cannot induce a fair outcome in centralized competitive environments. This paper shows that something similar happens not only in centralized interactions but also when the *bilateral* ultimatum game is embedded in a matching market with negligible search frictions. But, here the reason is that competition shrinks the available net surplus in a match and so in the limit it becomes irrelevant whether traders share this surplus fairly or not.

Overview of contents The remainder of this paper is organized as follows: Section 2 describes market entry and the matching process in our model. As a reference point, we specify the Walrasian competitive equilibrium in Section 3. Section 4 explains the role of fairness preferences in the ultimatum game. In Section 5 it is shown that the matching market has a unique steady state equilibrium. Section 6 relates the matching market outcome to the Walrasian equilibrium and analyzes the welfare implications of fairness. In Section 7 we discuss some limitations of applying fairness considerations in bilateral matching markets with endogenous entry. Concluding remarks are contained in Section 8. All formal proofs are relegated to Appendix A.

2. The model

Market entry We study the steady state of a market with two types of traders (or agents) denoted by $i \in \{a, b\}$. When two agents of type a and b meet, they can share a total surplus that is normalized to unity. For example, the two types can be sellers and buyers who can trade one unit of an indivisible good. Another example is a labor market where each employer can hire one worker. All agents are risk–neutral and discount future payoffs by the common discount factor $\delta \in (0, 1)$.

In each period t, a mass of $\bar{M}_i > 0$ of new agents of type i appears. These decide whether to enter the matching market or not. If an agent of type i refrains from entering, he disappears and receives the outside option payoff r_i. Alternatively, r_i can be interpreted as agent i's cost of entering the market. For example, r_i could be the seller's cost of producing the good before entering the market. Among the agents of type i the value of r_i is distributed on $[0, \bar{r}_i]$, with $\bar{r}_i \geq 1$, according to the continuous distribution function $F_i(r_i)$, with $F_i'(r_i) > 0$ for all $r_i \in (0, \bar{r}_i)$.

[7] The relationship between the matching market equilibrium and the competitive Walrasian equilibrium is discussed in Binmore and Herrero (1988), De Fraja and Sákovics (2001), Gale (1986, 1987), Lauermann (2013), Moreno and Wooders (2002), Mortensen and Wright (2002), and Rubinstein and Wolinsky (1985, 1990).

[8] These externalities are the reason for why the first welfare theorem does not hold in their model.

[9] See their Propositions 2 and 3. The role of competition in market experiments with multiple responders or proposers is studied in Roth et al. (1991) and Fischbacher et al. (2009).

[10] See Section 5 of their article.

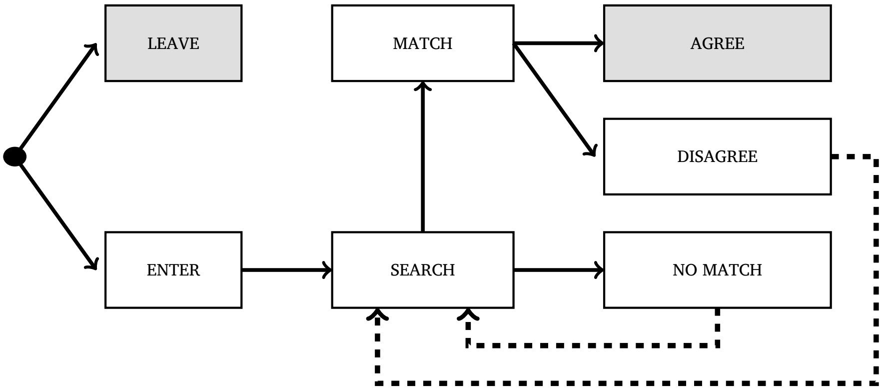

Fig. 1. The sequence of events.

We denote by V_i type i's expected utility from entering the market. In Section 3 we derive V_a and V_b in a Walrasian competitive market. This serves as reference point for the analysis in Section 5, where V_a and V_b are determined by bilateral bargaining in a pairwise matching market. Agent i enters the market only if this gives him a higher payoff than his outside option r_i. Therefore, the masses of agents of type a and b who enter the market at each date are given by

$$F_a(V_a)\bar{M}_a, \quad F_b(V_b)\bar{M}_b. \tag{1}$$

Note that V_i is equal to the market entry cost of the marginal trader of type i.

To perform a partial equilibrium welfare analysis, we abstract from income effects and measure all utilities in terms of some numeraire good.[11] Thus, V_i represents the type i traders' willingness to pay for entering the market. This includes not only the material payoffs that they expect from trade but also the monetary equivalent of potential psychological utility losses from inequity aversion. By expressing utilities in monetary units, we can measure social welfare and compare welfare for different degrees of fairness. The social welfare surplus on side i of the market equals

$$W_i(V_i) \equiv \int_0^{V_i} \left[V_i - r_i\right] \bar{M}_i \mathrm{d}F_i(r_i), \tag{2}$$

and is increasing in V_i. If $V_a' > V_a$ *and* $V_b' > V_b$, then (V_a', V_b') constitutes a Pareto improvement over (V_a, V_b): For each type i, the utility difference $\max[V_i', r_i] - \max[V_i, r_i]$ is positive for all agents with $r_i < V_i'$ and zero for all others.

Matching The mass of active agents in the matching process is endogenously determined by the flows of agents who enter and exit the market. Let M_i denote the steady state mass of traders of type i who are actively searching for a match. In each period, each active agent of type i meets at most one agent of the other type $j \neq i$. We denote by $\alpha \in [0,1]$ the probability that a trader of type a is matched with a trader of type b; analogously, a trader of type b is matched with a trader of type b with probability $\beta \in [0,1]$. The probabilities α and β are functions of the meeting technology and the numbers of active traders, M_a and M_b.

For our analysis, we assume that the matching technology is efficient in the sense that all feasible matches are exhausted[12]: If $M_i \leq M_j$ then all traders of type i on the short side of the market are randomly matched with a trader of type $j \neq i$. The assumption of efficient matching minimizes the frictions generated by the matching process and allows us to focus on the equilibrium implications of the traders' bargaining attitudes as specified in Section 4. Also, it facilitates the comparison between decentralized trade and the Walrasian competitive equilibrium, which we derive in Section 3. With efficient matching, the probabilities α and β are given by

$$\alpha \equiv \min\left[\frac{M_b}{M_a}, 1\right], \quad \beta \equiv \min\left[\frac{M_a}{M_b}, 1\right]. \tag{3}$$

Thus, $\alpha = 1$ and $\beta < 1$ if $M_a < M_b$, and $\alpha \leq 1$ and $\beta = 1$ otherwise. Further, $\alpha M_a = \beta M_b$ because with bilateral matching the same mass of agents is matched on both sides of the market.

When matched in period t, agent a and b bargain about sharing the gains from trade. The bargaining game and the role of the agents' inequity aversion for the bargaining outcome are described in Section 4. If both parties reach an agreement, they leave the market. Otherwise, in the event of disagreement, they enter the matching process again in period $t + 1$.

The flowchart in Fig. 1 illustrates the sequence of events from the individual trader's perspective. The gray shaded boxes indicate terminal states. The dashed arrows indicate a delay of one period, which matters because traders discount future payoffs by the factor

[11] Cf. chapter 10 in Mas-Colell et al. (1995). In our analysis all agents of the same type get the same payoffs, conditional on entering the market. Therefore, we restrict welfare comparisons to outcomes of this type.

[12] To capture inefficiencies in matching, one could replace α and β in (3) by $\alpha\rho$ and $\beta\rho$. The exogenous parameter $\rho \in (0,1]$ then measures the efficiency of the matching process because a fraction ρ of traders on the short side is matched.

$\delta \in (0, 1)$. Upon arrival at date t, the trader chooses whether to enter the market or to leave. If he enters, he searches for a trading partner. After not finding a trading partner at date t, he re–enters the matching process again at date $t+1$. If his search is successful and he reaches an agreement with the other party, he leaves the market. Otherwise, in the event of disagreement he re–enters the matching process again at date $t+1$.

3. Competitive equilibrium

As a benchmark, we first consider the Walrasian competitive equilibrium of the market in the absence of personal interactions and fairness considerations. In this equilibrium, *centralized* anonymous competition rather than decentralized bilateral bargaining determines the traders' payoffs. In a Walrasian market, the split of the unit surplus equates demand and supply at the entry stage. Thus, type a agents get V_a from entering the market and type b agents get $V_b = 1 - V_a$. The Walrasian auctioneer adjusts V_a and V_b so that the same masses of both types enter the market.[13] Thus, the market is cleared at each date t and all agents leave the market after trading successfully. By (1), the masses of agents entering the market are

$$M_a = F_a(V_a)\bar{M}_a, \quad M_b = F_b(V_b)\bar{M}_b. \tag{4}$$

The market is in equilibrium if $M_a = M_b$. Therefore, V_a and V_b have to satisfy the *market clearing* condition and the feasibility constraint:

$$F_a(V_a)\bar{M}_a = F_b(V_b)\bar{M}_b, \quad V_a + V_b = 1. \tag{5}$$

Definition 1. $\mathscr{C} = (\hat{V}_a, \hat{V}_b, \hat{M}_a, \hat{M}_b)$ is a *competitive equilibrium* if (4) and (5) hold.

It is easy to see that $\hat{V}_a$ and $\hat{V}_b$ are uniquely determined by (5).[14] Therefore, by (4) also $\hat{M}_a$ and $\hat{M}_b$ are unique.

4. Ultimatum bargaining and fairness

We adopt the ultimatum bargaining game to describe negotiations between two traders, a and b, after being matched.[15] Following the famous study of Güth et al. (1982), the evidence from a huge number of laboratory experiments fails to support the idea that players act egoistically in the ultimatum game.[16] This makes the ultimatum game an attractive starting point to investigate the implications of non–selfish behavior for bilateral bargaining in a market context.

In the ultimatum game one of the traders, called the proposer, makes a proposal on how to divide the available surplus. The other trader, called the responder, can either accept or reject the offer. If the responder rejects the offer, the bargaining game ends: Both trader a and trader b re–enter the matching market in the next period. Thus their expected payoffs in case of disagreement are δV_a and δV_b, respectively. These payoffs are computed in the next section. If trader i is inequity averse, then his payoff V_i also reflects the utility loss that he expects from an unfair bargaining outcome in a future match. However, while traders' utilities may include inequity aversion, they are rational in the sense that they maximize expected utility. This immediately implies that no trader i will accept a proposal that gives him less than δV_i in a match. Thus, the "cake" which matched pairs of agents effectively bargain over is the gains from trade $1 - \delta(V_a + V_b)$. Furthermore, because the gains from trade are positive, rational bargainers will always reach an agreement, which is a standard feature of bargaining under perfect information.[17]

A selfish trader i who gets a share $s_i \in [0, 1]$ of the gains from trade gains $s_i[1 - \delta(V_a + V_b)]$ and leaves market with the payoff $s_i[1 - \delta(V_a + V_b)] + \delta V_i$. As is well–known from the logic of the ultimatum game, if both agents in a match are selfish, then the proposer i will demand $s_i = 1$, which makes the responder j indifferent between accepting and rejecting the proposal. In the subgame perfect equilibrium of the ultimatum game he accepts and leaves the match without benefiting from trade.

Experimental evidence from isolated ultimatum games, however, shows that proposers frequently do not demand the entire cake and responders often do not accept a proposal that gives them a only a rather small share. A theoretical explanation for this behavior is that fairness motives influence the behavior of many people. It seems reasonable that this also applies to our setting, where the ultimatum game is embedded in a matching market. To model the idea that a proposer may make a more generous offer to the responder than pure selfishness of the bargainers would predict, we apply the idea of inequity aversion to the division of the net surplus $1 - \delta(V_a + V_b)$, which represents the joint gains from trade.

Specifically, we adopt the widely popular formulation of inequity aversion by Fehr and Schmidt (1999). To avoid complications from imperfect information, we assume that all traders of type i have the same preferences for fairness or inequity aversion and that these are commonly known. Suppose trader a and b agree that a gets the share s_a and trader b the share $s_b = 1 - s_a$ of the net surplus. Then trader i's utility gain is given by

[13] In the Walrasian auction, the agents submit their entry decisions for every possible $V_i, i = a, b$. The auctioneer then sets (V_a, V_b) so that the market is cleared.

[14] By the intermediate value theorem and our continuity assumptions on $F_a(\cdot)$ and $F_b(\cdot)$, the equation $F_a(V_a)\bar{M}_a - F_b(1 - V_a)\bar{M}_b = 0$ has a solution $\hat{V}_a \in (0, 1)$ because $F_a(0)\bar{M}_a - F_b(1)\bar{M}_b < 0$ and $F_a(1)\bar{M}_a - F_b(0)\bar{M}_b > 0$. Moreover, the solution is unique because $F_a(V_a)\bar{M}_a - F_b(1 - V_a)\bar{M}_b$ is strictly increasing in V_a.

[15] Implicitly we assume that each trader is uninformed about the past interactions of the other trader. This rules out history dependent bargaining strategies, cf. Rubinstein and Wolinsky (1990).

[16] For a survey see Güth and Kocher (2014).

[17] As the gross surplus is normalized to unity, $V_a + V_b \le 1$. Therefore, $1 - \delta(V_a + V_b)$ is always positive.

$$U_i(s_a, s_b)\left[1 - \delta(V_a + V_b)\right], \tag{6}$$

with

$$U_i(s_a, s_b) \equiv s_i - k_{i1} \max\left[s_j - s_i, 0\right] - k_{i2} \max\left[s_i - s_j, 0\right], \quad j \neq i. \tag{7}$$

The second term in the definition of $U_i(s_a, s_b)$ represents the utility loss of agent i from disadvantageous inequality if $s_i < s_j$; the third term is the loss from advantageous inequality if $s_i > s_j$. As Fehr and Schmidt (1999), we assume that $k_{i2} \leq k_{i1}$ and $0 \leq k_{i2} < 1$. In addition, we ignore the non–generic borderline case $k_{i2} = 0.5$ by assuming that $k_{i2} \neq 0.5$.[18]

Utilities are piecewise linear functions in (7). As Fehr and Schmidt (1999) point out, this keeps their model as simple as possible, while still being consistent with the most important facts in ultimatum games. Simplicity is also the main reason why we adopt their formulation. As Lemma 1 below shows, the subgame perfect bargaining equilibrium can be computed explicitly and depends on the parameters (k_{i1}, k_{i2}), $i \in \{a, b\}$, in an intuitive and simple way. This is useful for deriving comparative static results in Section 6. However, qualitatively similar results should be expected, e.g., for a utility function that is concave in the amount of advantageous inequality.

We denote by $s_{ji} \in [0, 1]$ the share that agent i in the role of the proposer offers the responder j and by $s_{ii} = 1 - s_{ji}$ the share that he demands for himself. The following lemma employs the characterization of the equilibrium outcome of the ultimatum game in Fehr and Schmidt (1999).[19]

Lemma 1. *In the subgame perfect equilibrium of the ultimatum game, responder j accepts an offer s_{ji} by proposer i if and only if*

$$s_{ji} \geq \bar{s}_j \equiv k_{j1}/(1 + 2k_{j1}) < 0.5.$$

Proposer i offers

$$s^*_{ji} \equiv \begin{cases} 0.5 & \text{if } k_{i2} > 0.5 \\ \bar{s}_j & \text{if } k_{i2} < 0.5 \end{cases}$$

and gets the share $s^*_{ii} \equiv 1 - s^*_{ji}$.

As is standard in bargaining with perfect information, the traders always agree on a division of the net surplus. If the proposer is *strongly fair*, $k_{i2} > 0.5$, the net surplus is split evenly.[20] Otherwise, if $k_{i2} < 0.5$, the proposer's offer $s^*_{ji} = \bar{s}_j$ makes the responder indifferent between accepting and rejecting, and in equilibrium he accepts. In particular, if $k_{a1} = k_{a2} = k_{b1} = k_{b2} = 0$, both parties are purely egoistic and the proposer gets the entire net surplus as $s^*_{ii} = 1$.

Let

$$U^*_{ai} \equiv U_a\left(s^*_{ai}, s^*_{bi}\right), \quad U^*_{bi} \equiv U_b\left(s^*_{ai}, s^*_{bi}\right), \quad i = a, b, \tag{8}$$

where (s^*_{ai}, s^*_{bi}) is the equilibrium outcome described by Lemma 1 when trader i is the proposer.[21] Then after bargaining, trader $j \in \{a, b\}$ leaves the market with the payoff

$$U^*_{ji}\left[1 - \delta(V_a + V_b)\right] + \delta V_j \tag{9}$$

if type $i \in \{a, b\}$ has been the proposer in the match. The discount factor δ can be thought of as a parameter that indicates the importance of market interactions. In the limit $\delta \to 0$ trader j receives the same payoff as in an isolated ultimatum game. As δ increases, his payoff becomes more and more dependent on the utilities from re–entering the matching process.

It remains to specify the assignment of the roles of proposer and responder to the two parties in a match. As there is no straightforward argument as to who should act naturally in which role, we resort to the random proposer approach (cf. Binmore, 1987)[22]: In a match, trader a is selected with the exogenous probability λ to become the proposer, and with probability $1 - \lambda$ trader b is chosen to make a take–it–or–leave–it offer to type a. We assume that $0 < \lambda < 1$, which allows us to study also the limiting extremes $\lambda \to 0$ and $\lambda \to 1$. As long as the proposer is not strongly fair, by Lemma 1 he gets a larger share of the net surplus than the responder. We, therefore, interpret λ as a measure of *bargaining power* of the type a agents relative to the type b agents.

[18] We thus sidestep the problem that for $k_{i2} = 0.5$ the equilibrium outcome of the ultimatum game is not unique (cf. Proposition 1 in Fehr and Schmidt (1999)): In Lemma 1 any proposal $s^*_{ji} \in [\bar{s}_j, 0.5]$ would be optimal for proposer i if $k_{i2} = 0.5$. For the implications for the matching market equilibrium, see footnote 32.

[19] The lemma is part of their Proposition 1 on p. 826f. For a proof we refer to their argument on p. 828.

[20] Notice that this is equivalent to applying the cooperative Nash (1950) bargaining solution, which maximizes the product of the (selfish) traders' utility gains over the disagreement outcome. The symmetry axiom underlying this bargaining solution reflects a kind of equal treatment.

[21] Lemma 2 in Appendix A shows how $(U^*_{ai}, U^*_{bi}), i = a, b$, depend on the fairness parameters k_{a1}, k_{a2}, k_{b1}, and k_{b2}.

[22] In different settings Bester (1993, 1994) investigates whether the sellers can profit from avoiding haggling by committing to a posted price offer.

5. Matching market equilibrium

We now consider *decentralized* trade in the steady state of the matching market described in Section 2. In the steady state equilibrium, all matches lead to agreement and the utility gains of both traders are determined by the bargaining solution derived in Section 4. The matching market equilibrium depends on the traders' inequity aversion because it affects the bargaining outcome in a match.

First, we derive the agents' expected payoffs, V_a and V_b, from entering the matching process. When joining the matching process, trader a finds a trading partner b with probability α. In this event, he is selected as the proposer with probability λ and as the responder with probability $1-\lambda$. Thus, we can use formula (9) to determine the expected payoff that trader a gets in the role of the proposer or the responder, respectively. With probability $1-\alpha$ trader a remains unmatched and re–enters the matching process again in the subsequent period. Therefore, his expected payoff from entering the market is given by

$$V_a = \alpha\left[\left(\lambda U^*_{aa} + (1-\lambda)U^*_{ab}\right)\left[1-\delta(V_a+V_b)\right] + \delta V_a\right] + (1-\alpha)\,\delta V_a. \tag{10}$$

Analogously, we obtain for traders of type b that

$$V_b = \beta\left[\left(\lambda U^*_{ba} + (1-\lambda)U^*_{bb}\right)\left[1-\delta(V_a+V_b)\right] + \delta V_b\right] + (1-\beta)\,\delta V_b. \tag{11}$$

Implicitly in (10) and (11), each trader takes into account that in any match the outcome depends not only on his own but also on the fairness attitudes of all other traders in the market.

In the steady state, the numbers of active agents, M_a and M_b, in the matching market have to be constant over time. Therefore, also the matching probabilities α and β in (3) are time independent. Thus at each date, the mass of agents entering the market has to be equal to the mass of agents that leave the market after being matched and reaching an agreement: The inflows of new traders are given by (1). The masses of matched traders, who leave the market in each period after trading, are αM_a for type a and βM_b for type b. Therefore, a steady state requires that

$$\alpha M_a = F_a(V_a)\bar{M}_a, \quad \beta M_b = F_b(V_b)\bar{M}_b. \tag{12}$$

Recall that $\alpha M_a = \beta M_b$ by the bilateral matching property of (3). Therefore, (12) implies that the matching market equilibrium satisfies the *balance condition*

$$F_a(V_a)\bar{M}_a = F_b(V_b)\bar{M}_b, \tag{13}$$

analogously to the market clearing condition of competitive equilibrium in (5).

Definition 2. $\mathscr{M} = (V^*_a, V^*_b, M^*_a, M^*_b)$, with $M^*_a > 0, M^*_b > 0$, is a *matching market equilibrium* if (10) – (12) hold, with α and β defined by (3) and $(U^*_{aa}, U^*_{ab}, U^*_{ba}, U^*_{bb})$ defined by (8).

We include the requirement that M^*_a and M^*_b are positive to exclude the trivial equilibrium where no agents of type i enter the market because there are no agents of the other type $j \neq i$ to trade with. This also ensures that the matching probabilities α and β in (3) are well–defined and positive. The following result lays the basis for the analysis of inequity aversion in the matching market and for its comparison with the competitive equilibrium[23]:

Proposition 1. *There exists a unique matching market equilibrium* $\mathscr{M}$.

In the competitive equilibrium the payoffs V_a and V_b adjust directly to equilibrate the market. In contrast, in the matching market equilibrium the adjustment process can be thought of in terms of the matching probabilities: If there is excessive entry on side i of the market, this reduces the likelihood of active traders of type i to find a trading partner. Therefore, they have to search longer and face higher delay costs due to discounting. This in turn lowers their payoff V_i from entering the market until entry is reduced to its equilibrium level.[24]

6. Fairness and competition

Even with efficient matching, as defined by (3), there are search or matching frictions if the masses of active traders, M_a and M_b, are not the same on both sides of the market. All traders on the short side are matched, but some fraction of traders on the long side remains unmatched and re–enters the matching process again in the next period. As traders discount future payoffs, this generates an inefficiency due to delay costs. Yet, when the discount factor δ is close to 1, then these costs become negligible. Following Rubinstein and Wolinsky (1985), the next result considers the *frictionless* matching market in the limit $\delta \to 1$.

[23] Lauermann and Nöldeke (2015) prove existence of steady-state equilibrium in a class of matching models with search frictions. Our model does not fall into this class because the inflow of new traders is endogenously determined, whereas in their model it is exogenous.

[24] A formal analysis of the stability of adjustment dynamics is beyond the scope of this paper.

Proposition 2. *The utilities (V_a^*, V_b^*) in the matching market equilibrium $\mathscr{M}$ converge to the utilities $(\hat{V}_a, \hat{V}_b)$ in the competitive equilibrium $\mathscr{C}$ in the limit $\delta \to 1$:*

$$\lim_{\delta\to 1}(V_a^*, V_b^*) = (\hat{V}_a, \hat{V}_b).$$

In the frictionless limit traders get the same payoffs as in the Walrasian equilibrium in Definition 1. This holds independently of their fairness attitudes in bilateral bargaining. Also the level of trade in the frictionless market is identical to the competitive equilibrium: In the limit $\delta \to 1$, by (1) the number of agents who enter the matching process market is the same as in the competitive equilibrium. As the inflow of agents is equal to the outflow of agents after trade, in each period the number of successful matches coincides therefore with the level of trade in the competitive outcome.

Proposition 2 accords with the "folk wisdom in behavioral economics that social preferences do not matter in competitive markets" (Schmidt, 2011, p. 207). The usual reasoning is that fairness concerns seem unimportant in *centralized settings* where many agents interact anonymously with each other.[25] But, Proposition 2 goes beyond this argument by showing that fairness also does not matter in a frictionless *decentralized* matching market with bilateral interactions.

The logic behind Proposition 2 is similar to other convergence results in the literature[26]: In the limit $\delta \to 1$ the market approaches the no–surplus characterization of perfect competition (cf. Ostroy, 1982) because the gains from bilateral trade $1 - \delta(V_a^* + V_b^*)$ tend to zero. Together with the balance condition (13) this implies that V_a^* and V_b^* approach the competitive equilibrium defined in (5). Since the gains from trade in a match become negligible, also the agents' preferences over the sharing of the net gains become insignificant for the equilibrium in the frictionless matching market.

While most traditional models of bilateral matching assume selfish traders, Proposition 2 shows that the convergence result does not depend on this assumption. In fact, the exact split of the gains from trade in a match is less important. What is important for convergence to the competitive equilibrium is that for $\delta \in (0, 1)$ in expectation each trader gains from being matched and is strictly better off than remaining unmatched.[27] This ensures that $V_a^* > 0$ and $V_b^* > 0$ so that entry occurs on both sides of the market. As long as this is the case, Proposition 2 is not restricted to the specific form of preferences in (7). Other types of fairness preferences would yield the same result as the net surplus of a match vanishes in the frictionless limit.

We next compare the welfare properties of the matching market equilibrium with the competitive equilibrium. There are two potential sources of inefficiencies in the matching market: First, finding a trading partner may involve waiting costs; second, there may exist utility losses from inequity aversion. Recall that in equilibrium there is no disagreement in bargaining. Therefore, waiting costs occur only if the matching probability is less than unity on one side of the market. Further, there are no inefficiencies from inequity aversion if either both sides of the market are purely selfish or strongly fair.[28]

It turns out that, generically, the competitive equilibrium Pareto dominates the matching market equilibrium, because in the latter some of the active traders remain unmatched.

Proposition 3. *There exists a $\mu > 0$ such that the matching market equilibrium $\mathscr{M}$ has the following properties*[29]*:*

(i) $V_a^* < \hat{V}_a$ *and* $V_b^* < \hat{V}_b$ *whenever* $\bar{M}_a/\bar{M}_b \neq \mu$,
(ii) $M_a^* > M_b^*$ *if* $\bar{M}_a/\bar{M}_b > \mu$; *and* $M_a^* < M_b^*$ *if* $\bar{M}_a/\bar{M}_b < \mu$.

As one would expect from the first welfare theorem, it is not possible that welfare is higher in the matching market than in the Walrasian equilibrium. But, part (i) of Proposition 3 makes the stronger statement that generically *both* sides of the matching market are worse off than in the competitive equilibrium. For example, it cannot happen that one side of the market gains from the fairness of traders on the other side so that it is better off than in the competitive equilibrium.

Part (ii) of Proposition 3 indicates why the condition $\bar{M}_a/\bar{M}_b \neq \mu$ is important in part (i). If for instance $\bar{M}_a/\bar{M}_b > \mu$, then $M_a^* > M_b^*$ and so by (3) the matching probability α for type a traders is less than one. This means that delay costs generate a welfare loss in comparison with the Walrasian equilibrium. This reduces the market entry payoffs for all agents below the Walrasian level.[30] All active traders are matched, i.e., $M_a^* = M_b^*$, only if by coincidence $\bar{M}_a/\bar{M}_b = \mu$. In this case, there are no matching frictions. If, in addition, all traders are either purely selfish or strongly fair, then there are also no welfare losses from inequity aversion. The matching market equilibrium then satisfies $V_a^* = \hat{V}_a$ and $V_b^* = \hat{V}_b$ and coincides with the competitive equilibrium.

How does the matching market outcome depend on the traders' fairness preferences and their bargaining power? To address this question we consider the comparative statics effects of the parameters

$$k \equiv (k_{a1}, k_{a2}, k_{b1}, k_{b2}) \tag{14}$$

25 See Section 2 in Schmidt (2011) for an overview of some experimental evidence.

26 See footnote 7 for some references discussing the convergence to the competitive equilibrium.

27 Our assumption that $\lambda \in (0, 1)$ prevents that bargaining power is concentrated on one market side. This is important if traders are not strongly fair, see Proposition 5 (ii) and (iii).

28 By (7), there are no losses from inequity aversion if all traders are selfish, i.e., $k_{a1} = k_{a2} = k_{b1} = k_{b2} = 0$. If all traders are strongly fair, i.e., $k_{a2} > 0.5$ and $k_{b2} > 0.5$, then by Lemma 1, the net surplus is shared equally in a match and so (7) implies that there are no utility losses from inequity aversion.

29 The parameter μ depends on the preference parameters in (7). For the exact definition see equation (23) in Appendix A.

30 See the argument for Proposition 4 for why both sides of the market are affected in the same way.

of the agents' utilities in (7) and the parameter λ, which represents the probability of type a becoming the proposer in a match. We view the traders' market entry payoffs as functions, $V_a^*(k,\lambda)$ and $V_b^*(k,\lambda)$, of the exogenous parameters k and λ.

Consider a change in the type i traders' preferences such that as proposers in the ultimatum game they share the net surplus equally with the responder, instead of selfishly making the responder j indifferent between accepting and rejecting. One might suspect that this raises the expected utility V_j^* on side j of the market and lowers V_i^* on the other side. Yet, as we show in the next proposition, this conjecture is false. Similarly, suppose that all traders are selfish so that the proposer in the ultimatum game appropriates the entire gains from trade. In isolated bilateral bargaining then an increase in the probability λ of type a traders being selected as proposers would increase their expected utility to the detriment of type b traders. Yet, as Proposition 4 below shows, also this is not true when bargaining is embedded in the matching market environment. The reason is that the division of the net surplus in a match has repercussions on market entry.

Proposition 4. *In the matching market equilibrium $\mathscr{M}$, any change in the parameters (k,λ) affects the entry utilities on both sides of the market in the same way:*

$$\operatorname{sign}\left[V_a^*(k,\lambda) - V_a^*(k',\lambda')\right] = \operatorname{sign}\left[V_b^*(k,\lambda) - V_b^*(k',\lambda')\right]$$

for all (k,λ) and (k',λ').

This result is a straightforward implication of the balance condition (13) for equilibrium: In the steady state the mass of new traders entering the market has to be the same for both types. This obviously implies that in equilibrium the entry payoffs V_a^* and V_b^* must move in the same direction. Accordingly, the welfare implications of variations in (k,λ) can be evaluated by the Pareto efficiency criterion.

One of the most prominent results in search theory is Diamond's (1971) monopoly price paradox: In a market where buyers face search costs to find a seller and sellers make take–it–or leave–it price offers, the sellers will charge the monopoly price.[31] Further, when buyers are homogeneous and wish to buy a single unit of an indivisible good, the market will break down. This happens because the monopoly price leaves no rents for the buyers, who then will refrain from wasting search costs. This outcome is rather different from the competitive equilibrium and looks paradoxical because it holds even for arbitrarily small search costs, as long as these are positive. In our setting, it turns out that the monopoly price paradox does not occur if all traders are strongly fair, i.e., if $k_{a2} > 0.5$ and $k_{b2} > 0.5$. Otherwise, if $k_{i2} < 0.5$, by the reasoning of the monopoly price paradox trade collapses in the limit where all bargaining power rests on side i of the market[32]:

Proposition 5. *The matching market equilibrium $\mathscr{M}$ has the following properties:*

(i) $\partial V_a^*/\partial\lambda = \partial V_b^*/\partial\lambda = 0$ *if* $k_{a2} > 0.5$ *and* $k_{b2} > 0.5$,
(ii) $\lim_{\lambda\to 1} V_a^* = \lim_{\lambda\to 1} V_b^* = 0$ *if* $k_{a2} < 0.5$,
(iii) $\lim_{\lambda\to 0} V_a^* = \lim_{\lambda\to 0} V_b^* = 0$ *if* $k_{b2} < 0.5$.

Strongly fair traders split the bargaining surplus equally with the responder. If this happens in all matches, then the equilibrium is independent of which side is more likely to become the proposer. In contrast, if the proposer has only weak or no concerns for advantageous inequality, he offers the responder the minimal share that the latter is willing to accept. This reduces the responder's net benefit in a match to zero. In the limit $\lambda \to 1$, therefore, the type b agents have no bargaining power and their market entry payoff is zero if $k_{a2} < 0.5$. This implies that there are no active type b traders in the steady state. Consequently, agents of type a cannot find a trading partner and also get zero utility from entering the market. The matching market ends up in a no–trade equilibrium. If $k_{b2} < 0.5$, the same logic applies to the limit $\lambda \to 0$, where all bargaining power is on side b of the market.

Proposition 5 implies that the outcome with strongly inequity averse traders Pareto dominates the equilibrium with selfish traders if λ is either close to zero or close to one. For some intermediate range of the parameter λ, however, this ranking is typically reversed:

Proposition 6. *Consider k with $k_{a1} = k_{a2} = k_{b1} = k_{b2} = 0$ and k' with $0.5 < k'_{a2} \le k'_{a1}$ and $0.5 < k'_{b2} \le k'_{b1}$. There exists a $\mu > 0$ and an interval $(\underline{\lambda}, \overline{\lambda})$ with $0 < \underline{\lambda} < \overline{\lambda} < 1$ such that in the matching market equilibrium $\mathscr{M}$*

$$V_a^*(k,\lambda) > V_a^*(k',\lambda) \quad \text{and} \quad V_b^*(k,\lambda) > V_b^*(k',\lambda),$$

whenever $\bar{M}_a/\bar{M}_b \neq \mu$ and $\lambda \in (\underline{\lambda}, \overline{\lambda})$.[33]

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[31] See Bester (1988) for an analysis of a search market where sellers bargain with buyers rather than committing to posted price offers.

[32] Recall that our assumption $k_{i2} \neq 0.5, i = a, b$ sidesteps the problem that the bargaining solution is not uniquely determined if $k_{a2} = k_{b2} = 0.5$ (cf. footnote 18). Without this assumption, (V_a^*, V_b^*) would be a set–valued function at $(k_{a2}, k_{b2}) = (0.5, 0.5)$ and upper hemicontinuous at this point: Any convex combination of $\lim_{(k_{a2},k_{b2})\uparrow(0.5,0.5)} (V_a^*, V_b^*)$ and $\lim_{(k_{a2},k_{b2})\downarrow(0.5,0.5)} (V_a^*, V_b^*)$ would satisfy the definition of the matching market equilibrium for $k_{a2} = k_{b2} = 0.5$.

[33] The parameter μ in this proposition is identical to the one used in Proposition 3 if the agents' preferences are given by k'. Thus, generically, i.e. as long as $\bar{M}_a/\bar{M}_b \neq \mu$, the matching equilibrium with strongly fair traders is not identical to the competitive equilibrium.

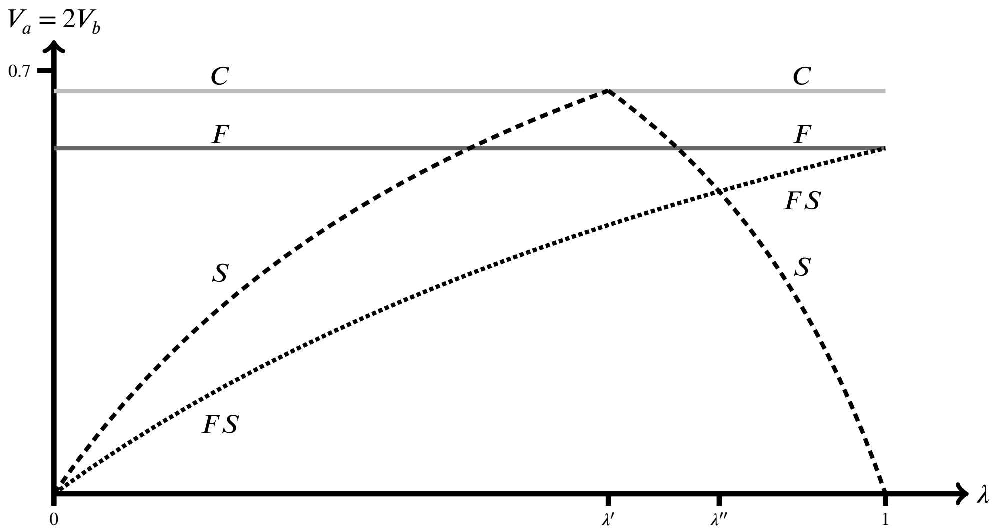

Fig. 2. A numerical example.

Unless by coincidence $\bar{M}_a/\bar{M}_b = \mu$, there exists a range of the parameter λ such that mutual selfishness guarantees all traders a higher utility than two–sided strong fairness.[34] To see why this is the case, consider purely selfish traders and set $\lambda' = \hat{V}_a$ and $1-\lambda' = \hat{V}_b$ so that each trader's probability to act as the proposer in a match is equal to his share of the unit surplus in the Walrasian equilibrium. In this situation, the matching market equilibrium with selfish traders replicates the Walrasian outcome: $V_a^* = \lambda' = \hat{V}_a$ and $V_b^* = 1-\lambda' = \hat{V}_b$ implies that, as in the competitive equilibrium, all active traders are matched so that $\alpha = \beta = 1$. Further, the proposer appropriates the full surplus in a match, because both sides of the market are selfish. Accordingly, in the absence of matching frictions, each agent's expected market entry payoff is simply the probability of acting as the proposer in the ultimatum game, which affirms that $V_a^* = \lambda'$ and $V_b^* = 1-\lambda'$. By the insight from Proposition 3 (i) on the Pareto dominance of the competitive equilibrium, this shows that the statement of Proposition 6 holds for $\lambda = \lambda'$. A simple continuity argument extends this to all values of λ in a neighborhood of λ'.

Proposition 6 refers to welfare when *all* traders are either selfish or fair. We next consider a situation where one type of traders is strongly fair, and the other type is selfish. If the fair traders are on the short side of the market, it turns out that everyone would gain if also the traders on the long side were strongly fair[35]:

Proposition 7. *Consider k with $k_{i2} > 0.5$ and $k_{j2} = 0$, $i, j \in \{a, b\}, j \neq i$, and k' with $k'_{a2} > 0.5, k'_{b2} > 0.5$. There exists a $\mu > 0$ such that if $\bar{M}_i/\bar{M}_j < \mu$, then in the matching market equilibrium $\mathscr{M}$*

$$V_a^*(k', \lambda) > V_a^*(k, \lambda) \quad \textit{and} \quad V_b^*(k', \lambda) > V_b^*(k, \lambda)$$

for all $\lambda \in (0, 1)$.

As $\bar{M}_i/\bar{M}_j < \mu$, by Proposition 3 the type j traders find themselves on the long side of the market and remain unmatched with positive probability. But, their likelihood of finding a trading partner would increase if as proposers in a match they left a larger share to the responder, because this would raise the attractiveness of market entry for type i agents. As a result, two–sided fairness would make both types better off than one–sided fairness on the short side of the market.

An example Fig. 2 illustrates some of our findings by a numerical example: The masses of traders who arrive each period are $\bar{M}_a = 1$ and $\bar{M}_b = 2$; their opportunity costs of entering the market, r_a and r_b, are independently distributed uniformly on $[0, 1]$; the common discount factor is $\delta = 0.5$.[36]

For this specification, we obtain that $V_a = 2V_b$ both in the competitive equilibrium and in the matching market equilibrium as an immediate implication of the market clearing condition (5) and the balance condition (13), respectively. Therefore, along the C–C line, which represents the competitive equilibrium outcome, the type a agents get two–thirds and type b one-third of the unit surplus. The competitive equilibrium Pareto dominates the matching market outcome when all traders are strongly fair. This is illustrated by the F–F line, where by part (i) of Proposition 5 the traders' utilities do not depend on the bargaining power parameter λ. In contrast,

[34] Note that for $k'_{a2} > 0.5$ and $k'_{b2} > 0.5$, there are no inefficiency losses from inequity aversion because by Lemma 1 the net surplus is shared equally in a match.

[35] The parameter μ in the following proposition is identical to the one used in Proposition 3 if the agents' preferences are given by k'. Thus, by Proposition 3 (ii), $\bar{M}_i/\bar{M}_j < \mu$ implies that $M_i^* < M_j^*$ for the preference parameters k'.

[36] The discount factor is very low for illustrative purposes.

when all traders are purely selfish, the value of λ is important for the outcome as illustrated by the S–S curve: In the limits $\lambda \to 0$ and $\lambda \to 1$ the Diamond (1971) paradox emerges and by parts (ii) and (iii) of Proposition 5 the market collapses to no–trade. For values of λ close to λ', however, Proposition 6 applies and pure selfishness makes all traders better off than strong inequity aversion. Finally, the FS–FS line depicts the traders' utilities when type a on the short side of the market is strongly fair, whereas type b is selfish. For values of λ below λ'' type a's fairness is actually harmful: Both sides of the market would be better off with two–sided selfishness. Similarly, in line with Proposition 7, two–sided fairness Pareto dominates one–sided fairness for all values of $\lambda \in (0, 1)$.

7. Discussion

Fairness is an elusive concept because it refers to often subjective interpretations of justice and equality. But instead of advancing new concepts, our analysis focuses on the market implications of a well–established result from laboratory experiments: Contrary to the prediction of selfish behavior, the proposer's offer in the ultimatum game typically does not appropriate the entire bargaining surplus. Our theoretical model incorporates this observation by considering inequity averse traders to describe ultimatum bargaining embedded in a bilateral random matching market.

Agents are anonymous and cannot observe the past behavior of others; in a match they only look at the trade they are engaging in. Their bargaining behavior does not affect other agents and cannot influence inequality in the overall market. This seems consistent with how fairness preferences matter in laboratory experiments of the ultimatum game. Therefore, even in a market with many traders, we can view each match as a two–player ultimatum game and apply the concept of inequity aversion to the subgame perfect equilibrium of this game.[37]

Unlike in the isolated ultimatum game, in a market environment a player can find another trading partner in the event of disagreement. Therefore, in equilibrium traders on the two sides of the market may end up with different utilities, even if they fairly share the net gains from trade in each match. This is due to different match probabilities when the masses of active traders on the two sides of the market are not identical. One could argue that a matched trader feels jealous if his counterpart can more easily find an alternative match. It is also conceivable that he is annoyed that too many agents of his own type enter the market, thereby reducing his matching probability. However, it seems somewhat speculative to formalize the impact of such emotions on bargaining behavior, and our analysis abstracts from this issue.

In our model, inequity aversion affects the bargaining outcome in each match and, therefore, on an aggregate level the market equilibrium in the presence of matching frictions. This is rather different from the approach taken by Dufwenberg et al. (2011) who study a centralized competitive market where consumers have some general kind of other–regarding preferences over market outcomes. The focus of their analysis is to establish conditions under which social preferences do *not* affect the equilibrium allocation of consumption. It could be interesting to incorporate fairness preferences over market outcomes as an additional element into our model.[38] But it not obvious how to specify such preferences. If the market is symmetric, i.e. if $\bar{M}_a = \bar{M}_b$ and $F_a(\cdot) = F_b(\cdot)$, then it follows immediately from (5) and (13) that all active traders have the same utility, $V_a = V_b$, both in the competitive equilibrium and in the matching market equilibrium. Also welfare W_i in (2) is identical on both sides i of the market. Thus, inequity is not an issue for traders in a symmetric market. In asymmetric markets, however, the concepts of fairness and equal treatment are less clear and raise a number of questions[39]: Given that a surplus can only be realized in a bilateral exchange, how should fairness reflect the fact that the masses of traders on both sides of the market are not the same? To what extent is it consistent with fairness that traders on the short side enjoy higher utility than traders on the long side? Should fairness concepts take into account that traders have different entry costs? Is it unfair that welfare on one side of the market is higher if agents on the other side have higher entry costs? It seems difficult, perhaps impossible, to answer these questions with a widely acceptable formal concept of fairness preferences for market outcomes in asymmetric matching situations.

8. Conclusion

This paper relaxes the standard neoclassical assumption of egoistic preferences in a market context. It incorporates fairness motives and inequity aversion in a bilateral matching market, in which traders bargain over the terms of trade. The matching market exhibits frictions in the form of delay or waiting costs when a trader does not find a bargaining partner or when the parties in a match fail to reach an agreement. The level of market frictions is negatively related to the agents' discount factor.

Fairness preferences are relevant for equilibrium welfare as agents discount future payoffs. But, the welfare effects are quite different from what one might expect from isolated bilateral bargaining outside of a market context. The reason is that the expected bargaining payoffs have feedback effects on market entry decisions and the traders' matching probabilities. Inequity aversion on either side of the market affects welfare on both sides always in the same way. Thus, any change in fairness preferences can be evaluated by the Pareto criterion. In some situations fairness is beneficial for welfare, whereas in others it can be harmful.

In the limit where the agents' discount factor tends to unity, they do not care about the timing of trade and so market frictions become irrelevant. The matching market outcome is then identical to the competitive equilibrium. Thus the frictionless limit of

[37] The same idea has been used, e.g., in Rubinstein and Wolinsky (1985) who study bargaining in an anonymous bilateral matching market and analyze the bargaining process in a match as the subgame perfect equilibrium with complete information.

[38] One conceptual problem for this is the fact that in our model the matching market outcome depends already on the agents' fairness attitudes in bargaining.

[39] A well–known principle of justice, dating back to Aristotle (384–322 B.C.), is that 'equals should be treated equally and unequals unequally'.

decentralized trade not only provides a justification for the Walrasian equilibrium of a centralized competitive market, but it also shows that fairness does not matter in a decentralized market with negligible market frictions.

The results of this paper are based on a very stylized specification of fairness in a matching market. All traders on the same side of the market are assumed to have identical preferences and in a match they are perfectly informed. This allows for a straightforward comparative statics analysis of changes in the degree of inequity aversion on either side of the market. Also, it simplifies the derivation of the steady state equilibrium, because the bargaining outcome is the same in all matches. Our analysis could be extended to the case of heterogeneous agents as long as agents are fully informed about each other's characteristics, as in Gale (1987). Perfect information ensures that an agreement is reached immediately, if there is a positive gain from trading. This avoids the problem of strategic bargaining under imperfect information, including the possibility of disagreement.

This paper is motivated by the empirical evidence from ultimatum bargaining and applies the popular theoretical model of inequity aversion of Fehr and Schmidt (1999). The ultimatum game is a useful theoretical starting point for analyzing the market implications of fairness because with selfish agents it would predict an extremely unfair division of the bargaining surplus, which is inconsistent with experimental observations. Real–world bargaining, however, may involve a sequence of offers and counteroffers rather than a single take–it–or–leave–it offer. In such cases, other forms of social preferences, such as reciprocity as a response to fair offers, may become relevant. Extensions of our model may address reciprocity and other kinds of social preferences in a market environment.

Declaration of competing interest

None.

Data availability

No data was used for the research described in the article.

Appendix A

This appendix contains the proofs of Propositions 1–7. For the proof of Lemma 1 we refer to Fehr and Schmidt (1999), p. 828. Some of the subsequent proofs employ the following lemma:

Lemma 2. *The variables $U^*_{aa}, U^*_{ab}, U^*_{ba}$, and U^*_{bb}, defined by (8), have the following properties: (i) $U^*_{aa} + U^*_{ba} \le 1$ and $U^*_{ab} + U^*_{bb} \le 1$ for all k, (ii) $U^*_{aa} = U^*_{bb} = 1$ and $U^*_{ab} = U^*_{ba} = 0$ if $k = 0$, (iii) $U^*_{aa} = U^*_{ba} = 0.5$ if $k_{a2} > 0.5$, and $U^*_{ab} = U^*_{bb} = 0.5$ if $k_{b2} > 0.5$, (iv) $U^*_{ab} = 0$ if $k_{b2} < 0.5$, and $U^*_{ba} = 0$ if $k_{a2} < 0.5$, (v) $U^*_{aa} \in (1/2, 1)$ if $k_{a2} < 0.5$ and $k_{b2} > 0$, and $U^*_{bb} \in (1/2, 1)$ if $k_{b2} < 0.5$ and $k_{a2} > 0$.*

Proof. (i) By (7), $U^*_{aa} = U_a(s^*_{aa}, s^*_{ba}) \le s^*_{aa}$ and $U^*_{ba} = U_b(s^*_{aa}, s^*_{ba}) \le s^*_{ba}$. As $s^*_{aa} + s^*_{ba} = 1$, this proves the first part of (i). The argument for the second part is analogous.

(ii) By (7) and Lemma 1, $U^*_{aa} = U_a(s^*_{aa}, s^*_{ba}) = s^*_{aa} = 1$ if $k = 0$. Therefore, $U^*_{ba} = U_b(s^*_{aa}, s^*_{ba}) = 1 - s^*_{aa} = 0$. An analogous argument proves that $U^*_{bb} = 1$ and $U^*_{ba} = 0$.

(iii) By Lemma 1, $s^*_{aa} = s^*_{ba} = 0.5$ if $k_{a2} > 0.5$, and $s^*_{ab} = s^*_{bb} = 0.5$ if $k_{b2} > 0.5$. Therefore, the statement follows immediately from (7).

(iv) By Lemma 1, $k_{b2} < 0.5$ implies $s^*_{ab} = \bar{s}_a = k_{a1}/(1 + 2k_{a1})$. Therefore, it follows from (7) that $U^*_{aa} = U_a(\bar{s}_a, 1 - \bar{s}_a) = 0$. An analogous argument applies to the second statement in (iv).

(v) By Lemma 1, $k_{a2} < 0.5$ implies $s^*_{aa} = 1 - \bar{s}_b = 1 - k_{b1}/(1 + 2k_{b1})$. Therefore, $U^*_{aa} = U_a(1 - \bar{s}_b, \bar{s}_b) = (1 + k_{b1} - k_{a2})/(1 + 2k_{b1})$. As $k_{b1} \ge k_{b2} > 0$ and $k_{a2} \ge 0$, we have $U^*_{aa} < 1$. Further, $k_{a2} < 0.5$ implies $U^*_{aa} > 0.5$. An analogous argument applies to the statement about U^*_{bb}. □

Proof of Proposition 1. The solution of (10) and (11) yields

$$V^*_a = \frac{\alpha\left(\lambda U^*_{aa} + (1-\lambda)U^*_{ab}\right)}{\alpha\delta\left(\lambda U^*_{aa} + (1-\lambda)U^*_{ab}\right) + \beta\delta\left(\lambda U^*_{ba} + (1-\lambda)U^*_{bb}\right) + 1 - \delta}, \tag{15}$$

$$V^*_b = \frac{\beta\left(\lambda U^*_{ba} + (1-\lambda)U^*_{bb}\right)}{\alpha\delta\left(\lambda U^*_{aa} + (1-\lambda)U^*_{ab}\right) + \beta\delta\left(\lambda U^*_{ba} + (1-\lambda)U^*_{bb}\right) + 1 - \delta}. \tag{16}$$

Note that

$$\frac{\partial V^*_a}{\partial \alpha} > 0,\ \frac{\partial V^*_a}{\partial \beta} < 0,\ \lim_{\alpha\to 0} V^*_a = 0,\ \frac{\partial V^*_b}{\partial \alpha} < 0,\ \frac{\partial V^*_b}{\partial \beta} > 0,\ \lim_{\beta\to 0} V^*_b = 0. \tag{17}$$

Define

$$H(\alpha, \beta) \equiv F_a(V^*_a(\alpha, \beta))\bar{M}_a - F_b(V^*_b(\alpha, \beta))\bar{M}_b. \tag{18}$$

By (3) we have $\alpha M_a = \beta M_b$. Therefore, (12) implies that in equilibrium $H(\alpha,\beta)=0$. Further, by (3), $\max(\alpha,\beta)=1$. We next show that $H(\alpha,\beta)=0$ has a unique solution $(\alpha^*,\beta^*)\in(0,1]\times(0,1]$ such that $\max(\alpha^*,\beta^*)=1$. As $F_a'(V_a)>0$ and $F_b'(V_b)>0$, (17) implies that

$$\frac{\partial H(\alpha,\beta)}{\partial\alpha}>0,\quad \frac{\partial H(\alpha,\beta)}{\partial\beta}<0,\quad \lim_{\alpha\to 0} H(\alpha,\beta)<0,\quad \lim_{\beta\to 0} H(\alpha,\beta)>0. \tag{19}$$

First, consider the case $H(1,1)>0$. Then, by continuity of $H(\cdot)$, (19) implies that there exists a unique $\alpha^*\in(0,1)$ such hat $H(\alpha^*,1)=0$. Thus, if $H(1,1)>0$, $H(\alpha,\beta)=0$ has a unique solution $(\alpha^*,\beta^*)=(\alpha^*,1)$ satisfying $\max(\alpha^*,\beta^*)=1$. An analogous argument shows that if $H(1,1)\le 0$, there exist a unique (α^*,β^*) with $\alpha^*=1$ and $\beta^*\in(0,1]$ such that $H(\alpha^*,\beta^*)=0$ and $\max(\alpha^*,\beta^*)=1$. This proves that the matching probabilities (α^*,β^*) are uniquely determined in a matching market equilibrium.

Given (α^*,β^*), also the market entry utilities (V_a^*,V_b^*) are uniquely defined by (15) and (16). This in turn implies that (12) uniquely determines the numbers of traders (M_a^*,M_b^*) in the market. Finally, it is easily verified that (α^*,β^*) is consistent with (3) by (12), because $H(\alpha^*,\beta^*)=0$ implies $F_a(V_a^*)\bar{M}_a=F_b(V_b^*)\bar{M}_b$. □

Proof of Proposition 2. By (3) $\alpha M_a=\beta M_b$. Therefore, (12) implies that for any $\delta\in(0,1)$ the matching market equilibrium satisfies

$$F_a(V_a^*)\bar{M}_a=F_b(V_b^*)\bar{M}_b. \tag{20}$$

From (15) and (16) we obtain

$$V_a^*+V_b^*=\frac{\alpha\left(\lambda U_{aa}^*+(1-\lambda)U_{ab}^*\right)+\beta\left(\lambda U_{ba}^*+(1-\lambda)U_{bb}^*\right)}{\alpha\delta\left(\lambda U_{aa}^*+(1-\lambda)U_{ab}^*\right)+\beta\delta\left(\lambda U_{ba}^*+(1-\lambda)U_{bb}^*\right)+1-\delta}. \tag{21}$$

As $\max(\alpha,\beta)=1$, therefore

$$\lim\nolimits_{\delta\to 1}(V_a^*+V_b^*)=1. \tag{22}$$

Recall that in the competitive equilibrium $\mathscr{C}$ the values $\hat{V}_a$ and $\hat{V}_b$ are determined by (5). The first condition in (5) is identical to condition (20) for the matching market equilibrium $\mathscr{M}$. In the limit $\delta\to 1$, by (22) also the second condition in (5) is satisfied in the matching market equilibrium. As $F_a(\cdot)$ and $F_b(\cdot)$ are strictly increasing continuous functions, this proves the statement in Proposition 2. □

Proof of Proposition 3. (i) As shown in the proof of Proposition 1, the equilibrium values (α^*,β^*) are uniquely determined by $\max(\alpha^*,\beta^*)=1$ and $H(\alpha^*,\beta^*)=0$ in (18). By definition of $H(\cdot)$ this implies that $(\alpha^*,\beta^*)=(1,1)$ if and only if

$$\frac{\bar{M}_a}{\bar{M}_b}=\mu\equiv\frac{F_b(V_b^*(1,1))}{F_a(V_a^*(1,1))}. \tag{23}$$

Thus, whenever $\bar{M}_a/\bar{M}_b\neq\mu$ either $\alpha^*<1$ or $\beta^*<1$. By (21), $V_a^*+V_b^*$ is strictly increasing in α and β. Therefore, $\bar{M}_a/\bar{M}_b\neq\mu$ implies

$$\begin{aligned}&V_a^*(\alpha^*,\beta^*)+V_b^*(\alpha^*,\beta^*)<V_a^*(1,1)+V_b^*(1,1)=\\&\frac{\lambda\left(U_{aa}^*+U_{ba}^*\right)+(1-\lambda)\left(U_{ab}^*+U_{bb}^*\right)}{\delta\left[\lambda\left(U_{aa}^*+U_{ba}^*\right)+(1-\lambda)\left(U_{ab}^*+U_{bb}^*\right)\right]+1-\delta}\le 1,\end{aligned} \tag{24}$$

where the last inequality follows from Lemma 2 (i). As $\hat{V}_a+\hat{V}_b=1$ by (5), (24) implies $V_i^*<\hat{V}_i$ for at least some $i\in\{a,b\}$. Suppose $V_j^*\ge\hat{V}_j$ for $j\neq i$. Then by (5)

$$F_j(V_j^*)\bar{M}_j\ge F_j(\hat{V}_j)\bar{M}_j=F_i(\hat{V}_i)\bar{M}_i>F_i(V_i^*)\bar{M}_i, \tag{25}$$

because $F_j'(\cdot)>0$ and $F_i'(\cdot)>0$. But this yields a contradiction to (12) because (3) implies $\alpha M_a=\beta M_b$ and so the matching equilibrium has to satisfy $F_a(V_a^*)\bar{M}_a=F_b(V_b^*)\bar{M}_b$. This proves part (i) of the Proposition.

(ii) By (18) and (23), $\bar{M}_a/\bar{M}_b>\mu$ implies $H(1,1)>0$. By the proof of Proposition 1, then in equilibrium $\alpha^*<1$. Therefore, by (3) we obtain $M_a^*>M_b^*$. An analogous argument proves the second part of statement (ii). □

Proof of Proposition 4. By the balance condition (13),

$$\begin{aligned}&F_a(V_a^*(k,\lambda))\bar{M}_a=F_b(V_b^*(k,\lambda))\bar{M}_b,\\&F_a(V_a^*(k',\lambda'))\bar{M}_a=F_b(V_b^*(k',\lambda'))\bar{M}_b\end{aligned} \tag{26}$$

for all (k,λ) and (k',λ'). Suppose, for example, that $V_a^*(k,\lambda)>V_a^*(k',\lambda')$ and $V_b^*(k,\lambda)\le V_b^*(k',\lambda')$. Because $F_a'(\cdot)>0$ and $F_b'(\cdot)>0$, then

$$F_a(V_a^*(k,\lambda))\bar{M}_a > F_a(V_a^*(k',\lambda'))\bar{M}_a = F_b(V_b^*(k',\lambda')\bar{M}_b \geq F_b(V_b^*(k,\lambda))\bar{M}_b, \tag{27}$$

by the second equality in (26). Thus, (27) yields a contradiction to the first equality in (26). Analogous arguments prove that $V_a^*(k,\lambda) < V_a^*(k',\lambda')$ implies $V_b^*(k,\lambda) < V_b^*(k',\lambda')$, and that $V_a^*(k,\lambda) = V_a^*(k',\lambda')$ implies $V_b^*(k,\lambda) = V_b^*(k',\lambda')$. □

Proof of Proposition 5. (i) If $k_{a2} > 0.5$ and $k_{b2} > 0.5$, then $U_{aa}^* = U_{ba}^* = U_{ab}^* = U_{bb}^* = 0.5$ by Lemma 2 (iii). Therefore, (15) and (16) imply

$$V_a^* = \frac{\alpha}{\alpha\delta + \beta\delta + 2(1-\delta)}, \quad V_b^* = \frac{\beta}{\alpha\delta + \beta\delta + 2(1-\delta)}. \tag{28}$$

As shown in the proof of Proposition 1, the equilibrium values (α^*, β^*) are uniquely determined by $\max(\alpha^*, \beta^*) = 1$ and $H(\alpha^*, \beta^*) = 0$ in (18). By (28), therefore, α^* and β^* do not depend on λ. This proves part (i) of the Proposition.

(ii) By Lemma 2 (iv) we have $U_{ba}^* = 0$ if $k_{a2} < 0.5$. Therefore, (16) implies $\lim_{\lambda\to 1} V_b^* = 0$. Thus $\lim_{\lambda\to 1} F_b(V_b^*)\bar{M}_b = 0$. By (20) therefore also $\lim_{\lambda\to 1} F_a(V_a^*)\bar{M}_a = 0$, which implies $\lim_{\lambda\to 1} V_a^* = 0$. An analogous argument proves part (iii) of the Proposition. □

Proof of Proposition 6. Consider k' with $k'_{a2} > 0.5$ and $k'_{b2} > 0.5$. As shown in the proof of Proposition 5, the equilibrium values α^* and β^* do not depend on λ, and by (28)

$$V_a^*(k') = \frac{\alpha^*}{\alpha^*\delta + \beta^*\delta + 2(1-\delta)}, \quad V_b^*(k') = \frac{\beta^*}{\alpha^*\delta + \beta^*\delta + 2(1-\delta)}. \tag{29}$$

If $\alpha^* = \beta^* = 1$, then $V_a^*(k') = V_b^*(k') = 1/2$. Therefore, the value of μ defined in (23) becomes

$$\mu \equiv \frac{F_b(1/2)}{F_a(1/2)} \tag{30}$$

for k'. Whenever $\bar{M}_a/\bar{M}_b \neq \mu$, it cannot be the case that $\alpha^* = \beta^* = 1$, because in equilibrium $F_a(V_a^*(k')\bar{M}_a = F_b(V_b^*(k')\bar{M}_b$. Thus, $\bar{M}_a/\bar{M}_b \neq \mu$ implies either $\alpha^* < 1$ or $\beta^* < 1$.

Now consider k with $k = 0$ and let $\hat{\alpha}$ and $\hat{\beta}$ denote the associated equilibrium matching probabilities. For $k = 0$, we obtain from Lemma 2 (ii) that $U_{aa}^* = U_{bb}^* = 1$ and $U_{ab}^* = U_{ba}^* = 0$. Therefore, (15) and (16) imply

$$V_a^*(k,\lambda) = \frac{\hat{\alpha}\lambda}{\hat{\alpha}\delta\lambda + \hat{\beta}\delta(1-\lambda) + (1-\delta)}, \qquad V_b^*(k,\lambda) = \frac{\hat{\beta}(1-\lambda)}{\hat{\alpha}\delta\lambda + \hat{\beta}\delta(1-\lambda) + (1-\delta)}. \tag{31}$$

If $\lambda = 1/2$, then

$$V_a^*(k,1/2) = V_a^*(k'), \quad V_b^*(k,1/2) = V_b^*(k') \quad \text{for} \quad \hat{\alpha} = \alpha^* \quad \text{and} \quad \hat{\beta} = \beta^*. \tag{32}$$

As the matching market equilibrium is unique by Proposition 1, this implies immediately that the equilibrium for (k,λ) coincides with the equilibrium for k' if $\lambda = 1/2$.

To prove the Proposition, we first consider the case $\bar{M}_a/\bar{M}_b < \mu$ so that $M_a^* < M_b^*$ by Proposition 3. Thus $\alpha^* = 1$ and $\beta^* < 1$ in the equilibrium for k'. By the above argument then also $\hat{\alpha} = 1$ and $\hat{\beta} < 1$ in the equilibrium for k if $\lambda = 1/2$. We first show that $\hat{\beta}$ is strictly increasing in λ as long as $\hat{\beta} < 1$. Indeed, it is easily verified that

$$\frac{\partial V_a^*(k,\lambda)}{\partial\lambda} > 0, \quad \frac{\partial V_a^*(k,\lambda)}{\partial\hat{\beta}} < 0, \quad \frac{\partial V_b^*(k,\lambda)}{\partial\lambda} < 0, \quad \frac{\partial V_b^*(k,\lambda)}{\partial\hat{\beta}} > 0. \tag{33}$$

Thus, if $\hat{\beta}$ were not strictly increasing in λ, $V_a^*(k,\lambda)$ would be increasing and $V_b^*(k,\lambda)$ would be decreasing in λ, a contradiction to Proposition 4.

By (31), $\hat{\alpha} = 1$ and $\hat{\beta} < 1$ implies that

$$\frac{\partial[V_a^*(k,\lambda) + V_b^*(k,\lambda)]}{\partial\lambda} = \frac{(1-\delta)(\hat{\alpha} - \hat{\beta})}{[\hat{\alpha}\delta\lambda + \hat{\beta}\delta(1-\lambda) + (1-\delta)]^2} > 0. \tag{34}$$

Further

$$\frac{\partial[V_a^*(k,\lambda) + V_b^*(k,\lambda)]}{\partial\hat{\beta}} = \frac{(1-\delta)(1-\lambda)}{[\hat{\alpha}\delta\lambda + \hat{\beta}\delta(1-\lambda) + (1-\delta)]^2} > 0. \tag{35}$$

As $\hat{\beta}$ is strictly increasing in λ this implies that $V_a^*(k,\lambda) + V_b^*(k,\lambda)$ is strictly increasing in λ as long as $\hat{\beta} < 1$. Therefore, by Proposition 4, both $V_a^*(k,\lambda)$ and $V_b^*(k,\lambda)$ are strictly increasing in λ as long as $\hat{\beta} < 1$. Thus, there exists an interval $(\underline{\lambda}, \bar{\lambda})$ with $\underline{\lambda} = 1/2 < \bar{\lambda} < 1$ such that

$$V_a^*(k,\lambda) > V_a^*(k,1/2) = V_a^*(k'), \quad V_b^*(k,\lambda) > V_b^*(k,1/2) = V_b^*(k') \tag{36}$$

for all $\lambda \in (\underline{\lambda}, \overline{\lambda})$. This proves Proposition 6 for the case $\bar{M}_a/\bar{M}_b < \mu$. An analogous argument for the case $\bar{M}_a/\bar{M}_b > \mu$, with $\alpha^* < 1$ and $\beta^* = 1$ completes the proof. □

Proof of Proposition 7. Without loss of generality, let $i = a$ and $j = b$. Consider k' with $k'_{a2} > 0.5, k'_{b2} > 0.5$ and let α^* and β^* denote the associated equilibrium matching probabilities. As shown in the proof of Proposition 5, the equilibrium values α^* and β^* do not depend on λ, and by (28)

$$V_a^*(k') = \frac{\alpha^*}{\alpha^*\delta + \beta^*\delta + 2(1-\delta)}, \quad V_b^*(k') = \frac{\beta^*}{\alpha^*\delta + \beta^*\delta + 2(1-\delta)}. \tag{37}$$

If $\alpha^* = \beta^* = 1$, then $V_a^*(k') = V_b^*(k') = 1/2$. Therefore, the value of μ defined in (23) becomes

$$\mu \equiv \frac{F_b(1/2)}{F_a(1/2)} \tag{38}$$

for k'. By the proof of Proposition 3, $\bar{M}_a/\bar{M}_b < \mu$ implies $M_a^* < M_b^*$ and so, by (3), $\alpha^* = 1$ and $\beta^* < 1$.

Now consider k with $k_{a2} > 0.5$ and $k_{b2} = 0$ and let $\tilde{\alpha}$ and $\tilde{\beta}$ denote the associated equilibrium matching probabilities. We obtain from Lemma 2 (iii)–(v) that $U_{aa}^* = U_{ba}^* = 0.5$, $U_{ab}^* = 0$ and $U_{bb}^* \in (1/2, 1)$. Therefore, (15) and (16) imply

$$V_a^*(k,\lambda) = \frac{\tilde{\alpha}\lambda 0.5}{\tilde{\alpha}\delta\lambda 0.5 + \tilde{\beta}\delta(\lambda 0.5 + (1-\lambda)U_{bb}^*) + (1-\delta)}, \tag{39}$$

$$V_b^*(k,\lambda) = \frac{\tilde{\beta}(\lambda 0.5 + (1-\lambda)U_{bb}^*)}{\tilde{\alpha}\delta\lambda 0.5 + \tilde{\beta}\delta(\lambda 0.5 + (1-\lambda)U_{bb}^*) + (1-\delta)}. \tag{40}$$

As $\partial V_a^*(k,\lambda)/\partial\lambda > 0$,

$$V_a^*(k,\lambda) < V_a^*(k,1) = \frac{\tilde{\alpha}}{\tilde{\alpha}\delta + \tilde{\beta}\delta + 2(1-\delta)}. \tag{41}$$

Suppose now, in contradiction to Proposition 7, that $V_a^*(k,\lambda) \geq V_a^*(k')$ for some $\lambda \in (0,1)$. Then by (29) and (41)

$$\frac{\tilde{\alpha}}{\tilde{\alpha}\delta + \tilde{\beta}\delta + 2(1-\delta)} > \frac{\alpha^*}{\alpha^*\delta + \beta^*\delta + 2(1-\delta)}. \tag{42}$$

Recall that $\alpha^* = 1$ and $\beta^* < 1$. As the left-hand side of (42) is increasing in $\tilde{\alpha}$ and decreasing in $\tilde{\beta}$, it cannot be the case that the equilibrium matching probabilities for k satisfy $\tilde{\alpha} < 1$ and $\tilde{\beta} = 1$. Thus $\tilde{\alpha} = 1$ and $\tilde{\beta} \leq 1$. Indeed, for $\tilde{\alpha} = \alpha^* = 1$ it follows from (42) that $\tilde{\beta} < \beta^*$.

By (39) and (40), we have

$$\frac{\partial(V_a^* + V_b^*)}{\partial\lambda} = \frac{2(1-\delta)[\tilde{\alpha} + \tilde{\beta}(1 - 2U_{bb}^*)]}{[\tilde{\alpha}\delta\lambda + \tilde{\beta}\delta(\lambda + 2(1-\lambda)U_{bb}^*) + 2(1-\delta)]^2} > 0, \tag{43}$$

because $\tilde{\alpha} = 1, \tilde{\beta} < 1$, and $U_{bb}^* < 1$. This implies

$$\begin{aligned} V_a^*(k,\lambda) + V_b^*(k,\lambda) &< V_a^*(k,1) + V_b^*(k,1) \\ &= \frac{\tilde{\alpha} + \tilde{\beta}}{\tilde{\alpha}\delta + \tilde{\beta}\delta + 2(1-\delta)} < \frac{\alpha^* + \beta^*}{\alpha^*\delta + \beta^*\delta + 2(1-\delta)}, \end{aligned} \tag{44}$$

where the last inequality holds because $\tilde{\alpha} = \alpha^* = 1$ and $\tilde{\beta} < \beta^*$. By (29) we thus obtain

$$V_a^*(k,\lambda) + V_b^*(k,\lambda) < V_a^*(k') + V_b^*(k'). \tag{45}$$

By Proposition 4 therefore $V_a^*(k,\lambda) < V_a^*(k')$ and $V_b^*(k,\lambda) < V_b^*(k')$, a contradiction to $V_a^*(k,\lambda) \geq V_a^*(k')$. □

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