

# Summary

The main objective of this thesis is an investigation and description of the secular variation of the Earth's magnetic field between 1980 and 2000. In particular, in the first part of this study the phenomenon of geomagnetic jerks are investigated by means of a deterministic model, which basically gives a description of the secular variation as a parameterization of typical periodicities of the external field. I argue that the conclusion drawn from this approach is not valid for explaining jerks as caused by external events and not valid to describe the global secular variation.

In the second part a time-dependent model of the secular variation between 1980 and 2000 is developed. The endpoints of the time interval were chosen, because of the availability of high quality field models from satellite data for these epochs. The Gauss coefficients are expanded in time as function of cubic B-splines. This model is forced to fit field models from high quality vector measurements from MAGSAT in 1980 and ØRSTED in 2000. The methodology is new.

The model is a valuable extension of the hitherto existing time-dependent description of the secular variation, the *GUFM* which was valid until 1990. Unlike *GUFM* the model is based on observatory monthly means, and the knot spacing of the cubic B-Splines tighter than *GUFM*. Therefore it reveals a short term secular variation on subdecadal time scale, which was not as yet resolved. The model is also valuable to test the frozen flux hypothesis and to link some of the morphology of the radial field at the core-mantle boundary to the geodynamo.

The third part of this thesis deals with the inversion of the time-dependent field and secular variation model for different kinds of core surface flow. These flows allow prediction of the decadal change of the length of the day, an observable which is independent to geomagnetic data. The prediction of some of the flows have the right tendency, but differ in slope from the observed change of the length of the day.



# Zusammenfassung

In dieser Studie ist ein besonderes Augenmerk auf die Beschreibung des Erdmagnetfeldes und seiner Säkularvariation im Zeitraum von 1980 und 2000 gelegt worden. Dabei wurden die beiden Endpunkte des Zeitintervalls so gewählt, dass sie mit den Satellitenmissionen MAGSAT 1980 und CHAMP, ØRSTED in 2000 zusammenfallen. Diese Satellitenmissionen erlaubten es, räumlich hochaufgelöste Modelle des Erdmagnetfeldes zu erstellen.

Im ersten Teil dieser Studie befasse ich mich mit der Säkularvariation, beobachtet über 70 Jahre an drei verschiedenen Stationen: Eskdalemuir (Schottland), Hermanus (Südafrika) und Kakioka (Japan). Hier versuche ich die Säkularvariation mittels eines einfachen deterministischen Modells zu beschreiben, d. h. eine Beschreibung der Säkularvariation als (gewichtete) Überlagerung von Periodizitäten hervorgerufen durch Prozesse ausserhalb der Erdatmosphäre, z. B. 11-Jahres-Zyklus der Sonne, Sonnenrotation, etc. Dabei vermag dieser einfache Ansatz die Säkularvariation der einzelnen Stationen wiederzugeben, aber jedes Modell ist eben nur für eine bestimmte Station gültig, also ist somit die Aussagekraft dieser Modelle lokal beschränkt. Mehr noch; die Schlussfolgerung, dass die Säkularvariation und insbesondere, dass Geomagnetische Jerks durch externe Prozesse verursacht werden, ist nicht zutreffend.

Im zweiten Teil meiner Arbeit entwerfe ich ein Modell der Säkularvariation zwischen 1980 und 2000 basierend auf zeitlich variablen Gauss-Koeffizienten. Hier werden die bereits angesprochenen Hauptfeldmodelle der Satellitenmissionen verwendet, um das Modell der Säkularvariation in den Endpunkten festzulegen.

Anhand dieses Modells ist es mir möglich Eigenschaften, wie z. B. räumliche Ausbreitung, der drei bekannten Geomagnetischen Jerks dieses Zeitraumes (1983, 1991 und 1999) zu untersuchen. Weiterhin ermöglicht das Modell den Nachweis von magnetischer Diffusion an der Kern-Mantel-Grenze und stellt eine Weiterentwicklung des Modells für den Zeitraum 1590 – 1990 von Jackson et al. (2000) dar.

*Last but not least*, erlaubt das Modell Rückschlüsse auf die räumliche Konfiguration des Geodynamos; es weist auf die Existenz von Konvektionsrollen hin.

Im dritten Teil meiner Arbeit entwickle ich mögliche Modelle für die Fluidbewegung an der Kern-Mantel-Grenze zwischen 1980 – 2000 unter Verwendung meines Modells für das Hauptfeld und seiner Säkularvariation. Die allgemeine Vorstellung ist, dass die Säkular-

variation an der Kern–Mantel–Grenze durch Advektion und Diffusion erzeugt wird. Das wird genau durch die Induktionsgleichung beschrieben. Wichtig hierbei ist die Rolle der Diffusion, ich gehe darauf in den abschließenden Bemerkungen ein.

Aber die Fluidbewegung beschreibt nicht nur die Säkularvariation sondern auch eine davon unabhängige Observable, nämlich die Variation der Tageslänge.

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Table 1: List of Abbreviations and parameters used in this Thesis

EUV index	Extreme Ultra Violet index
$D_{ST}$ -index	Daily storm index
SSN	Sun spot numbers
SVE	Secular variation estimates
GUFM	There is no official transcription given by authors of Jackson et al. [2000], but the geomagnetic community tend to believe that this abbreviations stands for <i>Grand Unified Field Model</i> .
$a$	Earth's radius 6371.2 km
$c$	Earth's core radius 3485.0 km
$\Lambda$	Decadal change of the length of day
$\lambda_s$	Spatial regularization or damping parameter
$\lambda_t$	Temporal regularization or damping parameter
$\lambda_1, \lambda_2$	Regularization or damping parameter of the a priori information
$\lambda_v$	Regularization or damping parameter of the flow velocity
$\oint(\cdot)dS$	Integration over a sphere
$\int(\cdot)d\Sigma$	Integration over a cylinder

