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- H. Bester and J. Demuth (2011), "Signalling Rivalry and Quality Uncertainty", Freie Universität Berlin, School of Business & Economics Discussion Paper 2011/20
- J. Demuth (2012), "Regulation and Quality Incentives in Search Markets: Certification, Licensing and Minimum Prices", working paper
- J. Demuth and M. Mathea (2012), "Information Acquisition and the Demand for Hard Evidence", working paper

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Kurzzusammenfassung

Die vorliegende Dissertation enthält drei Arbeiten auf den Gebieten der Informations- und Industrieökonomie. Eine grundlegende Erkenntnis der informationsökonomischen Theorie ist, dass bei Vorliegen unvollkommener Information und unvollständiger Märkte, die Gleichgewichtsallokation eines Wettbewerbsmarktes im Allgemeinen nicht effizient im Sinne des Pareto-Kriteriums ist. Daraus können sich für die ökonomischen Akteure selbst Anreize ergeben, durch geeignete Maßnahmen oder die Nutzung geeigneter Institutionen private Information öffentlich zu machen oder Informationen zu akquirieren. Alternativ können Verbesserungen der Allokation eventuell durch äußere Eingriffe, also Regulierungsmaßnahmen, erzielt werden.

In der ersten Arbeit werden Informationssignale betrachtet. Dabei geht es darum, dass Verkäufer in einem Markt mit asymmetrischer Information, in dem ein Teil der Käufer die Qualität der angebotenen Güter nicht direkt beobachten kann, für sie vorteilhafte Information öffentlich machen wollen. Eine Möglichkeit für die Verkäufer diese private Information an die Käufer zu übermitteln, ist das Senden von Qualitätssignalen. Diese können z.B. Werbeaufwendungen oder, wie in unserem Fall, Preise sein. Glaubhafte Signale müssen die Eigenschaft haben, dass 1) die Verkäufer, die die Information über ihren Typ vermitteln wollen, einen Anreiz haben das Signal zu senden, d.h. der Vorteil größer als die Kosten des Signals ist, und 2) andere Typen, von denen sich der Signalgeber separieren will, keinen Anreiz haben das Signal zu senden, auch dann nicht, wenn sie von den Käufern für den vorteilhaften Typ gehalten werden. Das setzt voraus, dass die Kosten für das Signal für den besseren Typen kleiner sind als für den schlechteren Typen. In der Arbeit wird das Preissetzungsverhalten zweier konkurrierender Anbieter beschrieben, von dem einer etabliert ist und dessen Qualität allgemein bekannt ist und einer Marktneuling ist, dessen Qualität von einigen Käufern nicht beobachtet werden kann. In dieser Situation stellen die Preise beider Anbieter Signale dar, aus denen die uninformierten Käufer Rückschlüsse auf die Qualität des vom Marktneuling angebotenen Gutes ziehen können. Hauptergebnis unserer Arbeit ist, dass Gleichgewichte nur existieren, wenn sich ausreichend viele informierte Konsumenten im Markt befinden. Existierende Gleichgewichte entsprechen denen bei vollständiger Information. Dies hängt damit zusammen, dass im Gleichgewicht beide Wettbewerber die Qualität des Marktneulings durch ihre Preise signalisieren und damit jeder einzelne einen Anreiz hat zu seiner besten Antwort bei vollständiger Information abzuweichen.

In der zweiten Arbeit geht es um Regulierungsmaßnahmen in einem Suchmarkt, die Investionsanreize auf der Verkäuferseite, z.B. zur Steigerung der Qualität oder der

Ausbildung, beeinflussen. Auf dem Markt herrscht unvollständige Information über Handelsmöglichkeiten, die Suche nach potentiellen Handelspartnern verursacht Kosten. Haben sich ein Käufer und ein Verkäufer gefunden und stehen in Verhandlung, bestimmt sich die jeweilige Verhandlungsmacht der beiden Parteien wesentlich durch ihre Möglichkeiten alternative Handelspartner zu finden: Je einfacher sich ein alternativer Partner finden lässt, umso größer ist die Verhandlungsmacht. Umso höher nun die Verhandlungsmacht der Käufer, desto geringer fallen die Investitionsanreize für die Verkäufer aus. In dieser Situation profitieren die Käufer von geringen Preisen, leiden aber unter einer ineffizient niedrigen Qualität der im Markt befindlichen Güter. Es wird gezeigt, dass eine Marktzutrittsbeschränkung, z.B. durch Lizensierung, unter bestimmten Voraussetzungen zu Effizienzsteigerungen, und teilweise sogar zu einer Erhöhung der Konsumentenrente, führen kann. Des Weiteren wird gezeigt, dass Mindestpreise zu einer Senkung der Investitionen und eindeutig verminderter Markteffizienz führen.

In der dritten Arbeit werden im Rahmen eines Prinzipal-Agenten-Modells optimale Lohnverträge für die Delegation von Informationsakquise charakterisiert. Eine Investorin hat die Wahl zwischen unterschiedlichen Projekten und will über die Erfolgswahrscheinlichkeit eines Projekts Gewissheit erlangen. Dazu delegiert sie die Informationsbeschaffung an einen Experten. Da die Anstrengung des Experten dessen private Information darstellt, muss der Lohn sicherstellen, dass der Experte einen genügenden Anreiz hat Anstrengung zu leisten und wahrheitsgemäß zu berichten. Unsere Analyse konzentriert sich auf die Kommunikation zwischen Experte und Investorin. Wir nehmen dabei an, dass im Vertrag festgelegt werden kann, ob und in welchem Fall die Information über den Typ des Projektes mit überprüfbarer Evidenz, die zusätzliche Kosten verursacht, belegt werden muss. Je nach a priori Wahrscheinlichkeit eines erfolgreichen Projektes kann es nun optimal sein Evidenz im Fall eines positiven oder negativen Berichtes nachzufragen, oder dem Bericht ohne Evidenz zu folgen. Das Ergebnis kommt durch eine Abwägung zwischen den Kosten für Evidenz und einer Informationsrente, die dem Experten ohne Evidenznachfrage gezahlt werden muss, zustande.

"La denière démarche de la raison est de reconnaître qu' il y a une infinité de choses qui la surpassent"

—Blaise Pascal, Les Pensées

Contents

Chapter 1

Introduction

This thesis contains three essays in the realm of information economics and industrial organization. Although very different subjects are approached, all of them are concerned with the issue of market or organizational inefficiencies caused by imperfect information and the ways economic agents deal with them. Informational imperfections about diverse matters, such as *hidden information* about the quality of goods or investment opportunities, *hidden action* of an employed agent or imperfect information about opportunities to engage in trade, leading to *search frictions* and *holdup*, are covered.

It was in the 1950's when Arrow, Débreu and others successfully showed existence and Pareto optimality of a competitive general equilibrium (see e.g. Débreu (1959) or Hahn and Arrow (1971)) and thereby formalized the concept in Smith (1776) of an invisible hand guiding egoistic actors to a socially efficient allocation. In their work the assumptions of complete markets and symmetric information are explicitly made. This suggested the question of what happens if these assumptions were abandoned. Some authors at this time already examined consequences of imperfect information about qualities or prices on market outcomes. E.g. Scitovszky (1945) notices a contradiction between competitive behavior and the tendency of consumers to use the price of a good to infer its quality. He argues that prices can only be informative about quality if there is a sufficient number of consumers with expertise to identify high qualities in the market. Stigler (1961) challenges the *law of one price*, which predicts the price for a homogeneous good to be the same all over the market assured by arbitrage. He shows that if buyers do not have perfect information about all prices in the market, but have to sample sellers at a small cost, prices can disperse over a wide range. And Arrow (1963) identifies informational imperfections to have important implications for the economics of medical care.

Then in the 1970's the field of information economics got momentum with the seminal paper by Akerlof (1970). He displays an adverse selection effect crowding out the best qualities in a market if buyers have incomplete information about the quality of goods. The result that due to imperfect information markets might, partially or even completely, fail, illustrates the dependency of the paradigm of the competitive equilibrium to be efficient on the assumption of perfect information. Soon, informational models were applied to explain phenomena like the non-existence of a variety of insurances, credit rationing, involuntary unemployment, and many others [1]. More generally the Greenwald-Stiglitz-Theorem states that in markets with incomplete information, intervention of a central planner might improve market outcomes (see e.g. Stiglitz (1991)).

The first paper of my thesis "Signalling Rivalry and Quality Uncertainty" is concerned with signalling as a device to overcome the problem of adverse selection in a market with asymmetric information, in our case about the quality of a good. The concept of signalling was introduced by Spence (1973) in the context of labor markets. His idea that able workers use education as a signalling device to separate themselves from less able workers was soon applied to a variety of economic problems, e.g. by Leland and Pyle (1977) interpreting the retained stock of a company's owner in IPOs as a signal for the firms profitability or by Kreps and Wilson (1982) explaining limit pricing as a way for an market's incumbent to signal its strength to potential entrants. Our model extends Bagwell and Riordan (1991), who show that a monopolist might set prices above the standard monopoly price in order to signal its quality to uninformed consumers. In our setup there are two firms, one established with publicly known quality, and one entrant, whose quality can be observed only by some (expert) consumers. We examine signalling rivalry accruing from the firms' incentives to use their price decision to convey information about the quality of the entrant to uninformed consumers. We find that only full information equilibria exist in which both firms set prices as in a standard Bertrand competition setting with full information. The reason is that in all alternative separating equilibria, each firm has an incentive to free-ride on the signal of the rival firm by deviating to its full-information best response.

The second paper "Regulation and Quality Incentives in Search Markets: Certification, Licensing and Minimum Prices" examines underinvestment caused by a holdup problem in a search market. The insight that if negotiation takes place after involved agents made relationship specific investments, ex-post holdup leads to ex-ante underinvestment goes back to Williamson (1975). He argues that real world contracts between

[1] For insurances see e.g Rothschild and Stiglitz (1976), Stiglitz (1977) or Wilson (1977), for credit rationing Stiglitz and Weiss (1981), Bester (1985) or Gale and Hellwig (1985) and for involuntary unemployment Shapiro and Stiglitz (1984) or Yellen (1984)

cooperating partners are necessarily incomplete, i.e. they can not be conditioned on every possible future state of the world, which creates an opportunism problem. On the basis of this argument, he restates the idea of Coase (1937) that the trade-off between transaction costs in markets, which include costs due to opportunism, and transaction costs inside organizations defines the boundaries of firms.

In search markets imperfect information about exchange opportunities makes trade relations specific in the sense that switching to an alternative trading partner is costly. This makes ex-ante unspecific investments partially specific ex-post, because agents become locked-in into a relationship. Naturally, if the trading partner is not known when the investment decision is made, no contract can be written to compensate the seller. In this situation bargaining power in a match, and consequently the division of the additional surplus created through the seller's investment, depends on search conditions, especially on the number of agents on both sides of the market. In the second paper of this thesis it is shown that limiting the number of sellers through a licensing policy is potentially welfare increasing, because sellers can secure a greater share of the created surplus. Furthermore, it is shown that, because they increase the share of sellers' surpluses independently on their investment decisions, effective minimum prices decrease sellers' investments.

The third paper "Information Acquisition and the Demand for Hard Evidence" is concerned with informational asymmetries in two ways: It asks how an agent assigned to collect information is optimally incentivized to exert effort and to report his finding truthfully when his effort can not be observed by a principal and his report is not verifiable. We identify cases in which the principal should set incentives through wages, leaving an information rent to the agent, and others, in which she should demand hard evidence, which creates costs that are wasted from a welfare point of view. We use a simple principal-agent model, which is often used to describe economic situations in which problems of asymmetric information arise *after* a contract is signed.[2] In these models the principal has the bargaining power to offer a contract to the agent on her terms, but has to consider participation and incentive constraints: In order to induce the agent to participate, he has to be given at least his opportunity costs. When the agent acquires *hidden information* and/or undertakes *hidden action*, he might be given informational rents in order to incentivize truth-telling behavior and/or exertion of effort. This kind of models is applied in a variety of contexts, e.g. the relationship between a firm's owner and manager, firm and worker, insurer and insured or bank and borrower. We apply our setup to an investor-expert relationship. We find that the

[2] The modeling framework was first introduced by Hölmstrom (1979).

optimal contract will imply the agent to deliver hard evidence if it is not too costly and if the common prior of principal and agent is rather informative.

Chapter 2

Signalling Rivalry and Quality Uncertainty

2.1 Introduction

It is well established that in markets with asymmetric information firms may use prices, possibly in conjunction with additional marketing devices, to signal quality information to uninformed market participants. In particular, if only some fraction of consumers is informed about quality, then firms may signal their qualities to the uninformed by setting prices higher than under perfect information. The idea is high-quality firms suffer less from decreased sales to informed consumers due to price increases than low-quality firms. Therefore a high-quality firm can separate itself by setting a high price which is not profitable to imitate for the low-quality firm. Signalling thus leads to distorted pricing and an inefficient reduction in the supply of high-quality goods.

This paper studies an extension of the standard price signalling model to a durable goods duopoly. In this environment the equilibrium outcome is free of distortions and identical to the perfect information equilibrium. We obtain this conclusion for a horizontally and vertically differentiated duopoly market with price-setting competitors engaging in a game of signalling rivalry: An established incumbent, whose quality is known by all market participants, faces an entrant who is either supplying the same quality as the incumbent or a superior quality acquired through some product innovation. Both firms and some fraction of consumers know the entrant's quality. The uninformed consumers use prices set by *both* firms to infer quality information. An important feature of price competition is that the two firms have opposing interests in conveying information, because the incumbent gains and the entrant loses when observed prices make the uninformed consumers more pessimistic about the entrant's quality.

In our model consumers are confronted with two price signals concerning a single uncertain variable, the entrant's quality. For the analysis of equilibrium, we apply two standard refinements for the uninformed consumers' out-of-equilibrium beliefs. First, we use the 'intuitive criterion' of Cho and Kreps (1987) and show that this eliminates all equilibria in which both firms adopt a pooling strategy. This means that at least one firm must use a separating strategy that reveals the entrant's quality to the uninformed consumers. Interestingly, this conclusion can be derived by applying the intuitive criterion to the incumbent's rather than the entrant's pricing. The incumbent facing a low quality entrant can credibly deviate from pooling by setting a price that signals a low quality entrant, whereas under some parameter constellations the high quality entrant may not be able to avoid pooling by appealing to the intuitive criterion.

Second, in situations where one of the firms' pricing is informative we adopt the 'unprejudiced belief criterion' of Bagwell and Ramey (1991) to the pricing strategy of its competitor, because the intuitive criterion is no longer applicable. Under the unprejudiced belief criterion the consumers trust in the price signal of the non–deviating competitor whenever only one of the two firms selects an out-of-equilibrium price. This means that, given the other firm playing an equilibrium separating strategy, a deviating firm cannot influence beliefs by deviating to a non-equilibrium price and therefore always sets its best response price as under perfect information.

The unprejudiced belief criterion therefore excludes all separating equilibria with prices distorted from full-information prices. We show that these prices constitute the unique equilibrium outcome in our model as long as the fraction of informed consumers is not too small. If only rather few consumers are informed, there is no equilibrium satisfying our refinements. The reason is that either the low-type entrant could gain by deviating to the high-type equilibrium price or the incumbent playing against the high-type would deviate to the low-type equilibrium price. Thus the firms' price signals would become contradictory: The entrant would signal that his quality is high and the incumbent that the entrant's quality is low.

Related Literature

The standard prediction of the literature on price signalling is that quality uncertainty leads to distorted pricing for signalling purposes. The earliest contributions to this literature consider a market with a single seller. For example, Milgrom and Roberts (1986) show that a monopolist may use price and advertisement to convince consumers of the quality of a newly introduced product. In their model, which is based on repeat purchases of a non-durable good, prices can be distorted up– or downwards depending on expectations over future sales. Bagwell and Riordan (1991) consider a monopolist who produces a durable good whose quality may be high or low. The existence of informed consumers and cost differences between qualities allow the monopolist to signal high quality through an upward distorted price.[1] Basically, our model extends Bagwell and Riordan (1991) to a horizontally differentiated duopoly in which one of the two firms offers a quality that is known to the competitor but not to all consumers.

One strand of the literature extends the analysis of price signalling to oligopolistic markets under the assumption that firms have private information only about their own quality. They are not informed about the other firms and, therefore, have the same prior about their competitors' qualities as the uninformed consumers. Daughety and Reinganum (2007) and Daughety and Reinganum (2008) examine a horizontally and vertically differentiated duopoly and n–firm oligopoly, respectively. Price setting takes into account the ex-ante probabilities of rivals to be high– or low–quality types. Separating equilibria imply upward distorted prices, increasing in the ex-ante probability of firms being high–types. Similarly, Janssen and Roy (2010) show for a homogeneous oligopoly that fully revealing mixed strategy equilibria exist in which high–types distort prices upward and low–types randomize prices over an interval, thereby generating sufficient rents to avoid mimicry of the high–types.

Closer related to the information structure in our model is the other strand of the literature that assumes the oligopolists to be informed about their rivals' qualities. Hertzendorf and Overgaard (2001a) analyze price setting and advertising in a duopoly where qualities are perfectly negatively correlated and consumers only know that one firm offers high quality and the other low quality. They apply two refinements that lead to a unique separating and a unique pooling equilibrium. In the separating equilibrium, a high degree of vertical differentiation leads to upwards distorted prices and a low degree to downward distorted prices. Yehezkel (2008) introduces some informed consumers into a similar model and examines how pricing and advertising strategies depend on the fraction of informed consumers.

In Fluet and Garella (2002) the ex ante distribution of the firm's qualities is such that either both firms offer low quality or one firm offers low and the other high quality. The authors avoid the use of selection criteria and find multiple separating and pooling equilibria. For small quality differences separation can only be achieved with a combination of upward distorted prices and advertisement. This result is similar to a finding by

[1] Linnemer (2002) shows that in the same setup it would be in some cases more profitable for the high-type firm to combine price and advertising signals.

Hertzendorf and Overgaard (2001b), who show that fully revealing separating equilibria satisfying the unprejudiced belief condition do not exist.

These papers differ from our model in that they consider product differentiation only in the vertical dimension. This implies that the duopolists have a common interest in signalling *different* qualities since they earn zero profits if consumers believe that they both offer the same expected quality. In our model of signalling rivalry such a common interest does not exist because consumer preferences are differentiated horizontally between the firms, and in the vertical dimension all consumers have identical preferences. As a consequence, the incumbent always prefers the consumers to believe that the entrant's quality is identical to his own quality, whereas the entrant gains by convincing the consumers that he offers a superior quality. Another feature that distinguishes our model from the above literature is that the duopolist's are not in a symmetric position. Consumers are uninformed only about the entrant's and not about the incumbent's quality. They interpret the prices of both firms as signals only about the entrant's quality. In our analysis, we do not address expenditures on directly uninformative advertising as an additional signal. Since under our belief refinements only the full–information equilibrium without distortions survives, there is no role for dissipative advertising in equilibrium.

From a methodological perspective our analysis is closely related to Bagwell and Ramey (1991) and Schultz (1999). They study limit pricing by two incumbents to affect the entry decision of a third firm. The incumbents' prices signal their information about an industry–wide parameter. The third firm enters the market only if it concludes that the probability of a favorable state is sufficiently high. In the paper by Bagwell and Ramey (1991) the competitors have a common interest, both want to signal an unfavorable state in order to prevent entry. Introducing the unprejudiced belief refinement, the authors find that only non–distorted separating equilibria exist. Further, under additional assumptions the intuitive criterion of Cho and Kreps (1987) eliminates all equilibria with pooling. By applying the same belief refinements to our context, we arrive at similar conclusions for the qualitative features of equilibrium. Schultz (1999) considers a variation of Bagwell and Ramey (1991) where the incumbents have conflicting interests, i.e. one incumbent prefers the entrant to stay out of the market, whereas its competitor profits from entry. Again, separating equilibrium prices are not distorted. But due to signalling rivalry these equilibria only exist if the effect of entry on the incumbent's profits is relatively small. We obtain a related non–existence result in our model when the fraction of informed consumers is rather small.

This paper is organized as follows. In Section 2.2 we describe the model and, as a reference point, we derive the equilibrium under full information. Section 2.3 defines the Perfect Bayesian Equilibrium and explains the belief refinements of our analysis. In Section 2.4 we show that under our refinements only the full information equilibrium prices can survive in a signalling equilibrium and that such an equilibrium exists if the fraction of informed consumers is not too small. Section 2.5 provides concluding remarks.

2.2 The Model

We employ the demand structure of the standard Hotelling (1929) duopoly with the modification that the two firms may offer different qualities. One of the firms offers a quality that is publicly known by all market participants. For convenience, we call this firm the incumbent. The other firm, which we call the entrant, produces a quality that is known also by the competing incumbent. Yet, some fraction of potential consumers is not informed about the entrant's quality. In the terminology of Nelson (1970), the entrant's good is an *experience* good so that an uninformed consumer learns its true quality only after purchase. The uninformed consumers use the firms' prices to draw inferences about the entrant's quality. Accordingly, the price setting behavior of *both* firms takes into account that prices are quality signals.

There is a unit mass of consumers whose preference characteristic x is uniformly distributed on the interval $[0, 1]$. Each consumer purchases at most one unit of the good from either the incumbent I or the entrant E. Given the incumbent's quality q_I and the entrant's (expected) quality q_E, the valuation of a consumer with characteristic $x \in [0, 1]$ is

$$v_I(x) = q_I - tx, \quad v_E(x) = q_E - t(1-x) \tag{2.1}$$

for the incumbent's and the entrant's good. The parameter t reflects the degree of horizontal product differentiation. The two firms are also vertically differentiated if $q_I \neq q_E$. But the quality differential between the two firms affects the taste of all consumers in the same way, independently of their characteristic x. This aspect distinguishes our model from the price signalling models of Hertzendorf and Overgaard (2001a) and Fluet and Garella (2002) who similarly to Shaked and Sutton (1982) assume that consumers differ in their valuation of quality and that the goods are not horizontally differentiated. In what follows, we assume that the firms' qualities are sufficiently high so that each consumer buys one unit of the good.

All consumers observe the incumbent's price p_I and the entrant's price p_E. The critical consumer type $\tilde{x}$, who is indifferent between purchasing from firm I and firm E, is then determined by $v_I(\tilde{x}) - p_I = v_E(\tilde{x}) - p_E$, and by (2.1) we have

$$\tilde{x}(p_I, p_E, q_E - q_I) = \frac{p_E - p_I - (q_E - q_I) + t}{2t}. \tag{2.2}$$

All consumers with $x < \tilde{x}$ optimally buy the incumbent's good, whereas consumers with $x > \tilde{x}$ purchase from the entrant.

There are two possible qualities, q_L and q_H, with $0 < q_L < q_H$. The incumbent's quality is commonly known to be $q_I = q_L$. There is uncertainty, however, about the entrant's quality. Its quality is $q_E = q_H$ with ex ante probability λ and $q_E = q_L$ with probability $1 - \lambda$. One interpretation is that with probability λ the entrant has realized a product innovation which increases the 'standard' quality q_L by the amount $q_H - q_L$. We normalize the unit cost of producing quality q_L to zero and assume that the unit cost of producing quality q_H is $c > 0$.

By (2.2) only the quality differential between the two firms affects the consumers' demand decisions. Therefore, we can simplify notation by defining

$$\Delta \equiv q_H - q_L. \tag{2.3}$$

We assume that the high quality entrant has a competitive advantage because $c < \Delta$. Yet, to ensure an interior solution, we take the entrant's product innovation to be *non–drastic* so that

$$0 < \Delta - c < 3t. \tag{2.4}$$

This will guarantee that the incumbent's market share is positive even when he competes with the high quality entrant.

Both firms observe the realization of q_E before setting prices. In addition some fraction $\gamma \in (0, 1)$ of consumers becomes informed about the entrant's true quality before making demand decisions. Each consumer type x is equally likely to be informed. This means that in each subset of the consumers' characteristic set $[0, 1]$ the fraction of informed consumers is identically equal to γ.

The uninformed consumers use the observed prices p_I and p_E to draw inferences about the entrant's quality. We denote their posterior belief that the entrant's quality is $q_E = q_H$ by $\mu \in [0, 1]$. Thus the uninformed consumers anticipate that the entrant offers the *expected* quality $\mu q_H + (1 - \mu) q_L = q_L + \mu\Delta$. Since consumers are risk–neutral with respect to quality, for given prices p_I and p_E their demand decisions depend only on the *expected* quality difference between the two sellers.

In the uninformed consumers' expectation the quality difference $q_E - q_I$ is always equal to $\mu\Delta$, independently of the entrant's true quality. If the entrant's quality is $q_E = q_L$, the informed consumers know that $q_E - q_I = 0$. Therefore, by (2.2) the incumbent's and the entrant's market shares, D_{IL} and D_{EL}, are given by

$$\begin{aligned} D_{IL}(p_I, p_E, \mu) &= \gamma\tilde{x}(p_I, p_E, 0) + (1-\gamma)\tilde{x}(p_I, p_E, \mu\Delta), \\ D_{EL}(p_I, p_E, \mu) &= 1 - D_{IL}(p_I, p_E, \mu). \end{aligned} \tag{2.5}$$

If $q_E = q_H$, the informed consumers know that $q_E - q_I = \Delta$. In this case, the two sellers' market shares, D_{IH} and D_{EH}, are equal to

$$\begin{aligned} D_{IH}(p_I, p_E, \mu) &= \gamma\tilde{x}(p_I, p_E, \Delta) + (1-\gamma)\tilde{x}(p_I, p_E, \mu\Delta), \\ D_{EH}(p_I, p_E, \mu) &= 1 - D_{IH}(p_I, p_E, \mu). \end{aligned} \tag{2.6}$$

If the entrant's quality is $q_E = q_L$, it follows from (2.2) and (2.5) that the incumbent's profit $\Pi_{IL} = p_I D_{IL}$ and the entrant's profit $\Pi_{EL} = p_E D_{EL}$ are

$$\Pi_{IL}(p_I, p_E, \mu) = p_I \frac{t - (1-\gamma)\mu\Delta - p_I + p_E}{2t}, \tag{2.7}$$

$$\Pi_{EL}(p_I, p_E, \mu) = p_E \frac{t + (1-\gamma)\mu\Delta + p_I - p_E}{2t}. \tag{2.8}$$

If $q_E = q_H$, then by (2.2) and (2.6) the duopolists' profits, $\Pi_{IH} = p_I D_{IH}$ and $\Pi_{EH} = (p_E - c)D_{EH}$, are equal to

$$\Pi_{IH}(p_I, p_E, \mu) = p_I \frac{t - [\gamma + (1-\gamma)\mu]\Delta - p_I + p_E}{2t}, \tag{2.9}$$

$$\Pi_{EH}(p_I, p_E, \mu) = (p_E - c) \frac{t + [\gamma + (1-\gamma)\mu]\Delta + p_I - p_E}{2t}. \tag{2.10}$$

Note that for all $\gamma \in (0, 1)$ it is the case that

$$\frac{\partial \Pi_{EL}}{\partial \mu} > 0, \quad \frac{\partial \Pi_{EH}}{\partial \mu} > 0; \quad \frac{\partial \Pi_{IL}}{\partial \mu} < 0, \quad \frac{\partial \Pi_{IH}}{\partial \mu} < 0. \tag{2.11}$$

Irrespective of the true quality, the entrant always gains and the incumbent always loses when the uninformed consumers raise their belief that the entrant offers high quality. Since these consumers interpret the firms' prices as quality signals, price competition entails a signalling rivalry: The entrant has an incentive to choose a price that indicates high quality. This is in conflict with the incumbent's interest to convince consumers that the entrant offers low quality.

Before analyzing how the duopolists' signalling rivalry affects their price competition, we briefly describe the equilibrium under full information. The firms compete by simultaneously setting prices and their pricing strategies are contingent on the entrant's quality. If $q_E = q_L$, we denote the incumbent's and the entrant's price by p_{IL} and p_{EL}, respectively; if $q_E = q_H$ the firms' prices are denoted by p_{IH} and p_{EH}. When all consumers know the entrant's quality, the firms' profits can be calculated from (2.7)–(2.10) by setting $\mu \equiv 0$ for $q_E = q_L$ and $\mu \equiv 1$ for $q_E = q_H$.[2] The full information equilibrium prices $\hat{p} = ((\hat{p}_{IL}, \hat{p}_{EL}), (\hat{p}_{IH}, \hat{p}_{EH}))$ are then defined by the conditions for profit maximization so that

$$\begin{aligned} \Pi_{IL}(\hat{p}_{IL}, \hat{p}_{EL}, 0) \geq \Pi_{IL}(p, \hat{p}_{EL}, 0), \quad \Pi_{EL}(\hat{p}_{IL}, \hat{p}_{EL}, 0) \geq \Pi_{EL}(\hat{p}_{IL}, p, 0), \\ \Pi_{IH}(\hat{p}_{IH}, \hat{p}_{EH}, 1) \geq \Pi_{IH}(p, \hat{p}_{IH}, 1), \quad \Pi_{EH}(\hat{p}_{IH}, \hat{p}_{EH}, 1) \geq \Pi_{EH}(\hat{p}_{IH}, p, 1). \end{aligned} \tag{2.12}$$

for all $p \geq 0$. From the corresponding first–order conditions one can easily derive the solution

$$\hat{p}_{IL} = t, \quad \hat{p}_{EL} = t, \quad \hat{p}_{IH} = t - \frac{\Delta - c}{3}, \quad \hat{p}_{EH} = t + \frac{\Delta + 2c}{3}. \tag{2.13}$$

If $q_E = q_L$, both firms charge the same price and have the same market share $D_{IL} = D_{EL} = 1/2$. If $q_E = q_H$, the incumbent is disadvantaged against the entrant and, even though he sets a lower price, his market share $D_{IH} = (3t - \Delta + c)/(6t)$ is smaller than the entrant's share $D_{EH} = (3t + \Delta - c)/(6t)$.

2.3 Equilibrium and Belief Restrictions

We envisage the market to operate in three stages. First, both firms and a fraction γ of consumers observe the realization of the entrant's quality. Second, the firms compete simultaneously by setting prices. Finally, in the third stage the uninformed consumers use observed prices to update their beliefs about the entrant's quality, and all consumers decide whether to buy from the incumbent or the entrant.

In what follows we study pricing strategies of the firms and consumer beliefs that constitute a Perfect Bayesian Equilibrium of this game. The firms choose their prices contingent on their information about the entrant's quality, and the uninformed consumers' posterior probability of facing the high quality entrant is a function of the firms' prices. In equilibrium, each firm's price maximizes its profit and the uninformed consumer's posterior belief is consistent with Bayesian updating.[3]

[2] This is equivalent to setting $\gamma \equiv 1$.
[3] We restrict ourselves to pure strategy equilibria.

More formally, $(p^*, \mu^*) = ((p^*_{IL}, p^*_{EL}), (p^*_{IH}, p^*_{EH}), \mu^*)$ with $\mu^* \colon \mathbb{R}^2_+ \to [0, 1]$ is a *Perfect Bayesian Equilibrium* (PBE) if

(a) for $Q = L, H$

$$p^*_{IQ} = \text{argmax}_p \, \Pi_{IQ}(p, p^*_{EQ}, \mu^*(p, p^*_{EQ})), \tag{2.14}$$

$$p^*_{EQ} = \text{argmax}_p \, \Pi_{EQ}(p^*_{IQ}, p, \mu^*(p^*_{IQ}, p)), \tag{2.15}$$

and (b)

$$\mu^*(p^*_{IL}, p^*_{EL}) = 1 - \mu^*(p^*_{IH}, p^*_{EH}) = 0, \quad \text{if} \quad p^*_{IL} \neq p^*_{IH} \quad \text{or} \quad p^*_{EL} \neq p^*_{EH}, \tag{2.16}$$

$$\mu^*(p^*_{IL}, p^*_{EL}) = \mu^*(p^*_{IH}, p^*_{EH}) = \lambda, \quad \text{if} \quad p^*_{IL} = p^*_{IH} \quad \text{and} \quad p^*_{EL} = p^*_{EH}. \tag{2.17}$$

Equilibrium conditions (2.14) and (2.15) state that, for each quality $q_E \in \{q_L, q_H\}$, the incumbent and the entrant choose their prices to maximize profits, taking the competitor's price and the uninformed consumers' belief $\mu^*(\cdot)$ as given. Equilibrium conditions (2.16) and (2.17) require that on the equilibrium path the buyers' belief is consistent with Bayes' rule. The buyers become fully informed about the entrant's true quality not only in a *two–sided separating equilibrium*, where $p^*_{iL} \neq p^*_{iH}$ for both $i \in \{I, E\}$, but also in a *one–sided separating equilibrium*, where $p^*_{iL} \neq p^*_{iH}$ for some $i \in \{I, E\}$ and $p^*_{jL} = p^*_{jH}$ for $j \neq i$. Prices remain uninformative only if $p^*_{iL} = p^*_{iH}$ for *both* $i \in \{I, E\}$. In such a *pooling equilibrium* the posterior belief is equal to the a priori probability λ.

By (2.14) and (2.15), the uninformed consumers' quality expectations affect the duopolists' pricing decisions. But, conditions (2.16) and (2.17) impose restrictions on expectations only for prices that are actually chosen in equilibrium. Since out–of–equilibrium beliefs remain arbitrary, there are multiple equilibria, which are a typical feature of signalling games. This is so because the profit of a deviation from the equilibrium price depends on the uninformed consumers' interpretation of this deviation. For example, the incumbent may be deterred from changing its price simply because consumers would interpret this as a signal that the entrant's quality is high. Similarly, the entrant may be kept from changing its price if consumers view this as a signal of low quality. Without restrictions on consumer beliefs multiple equilibria with both upward and downward distorted prices can be found .

To avoid this problem, the literature usually applies refinements that impose restrictions on out–of–equilibrium beliefs. A prominent refinement is the 'intuitive criterion' of Cho and Kreps (1987), which has been used in a variety of price signalling games.[4] Unfortunately, this criterion is not generally applicable in the present context because it is defined for signalling games where each player has private information only about his own and not the other players' characteristics. In our model, however, the duopolists have *common* private information and not only the entrant's but also the incumbent's price may signal the entrant's quality. Therefore, the intuitive criterion cannot be used in our model if both firms' prices are informative. Nonetheless, it remains applicable if one of the firms' equilibrium prices is uninformative, i.e. if $p^*_{iL} = p^*_{iH}$ for some $i \in \{I, E\}$. In this case, the intuitive criterion can be used to refine beliefs for out–of–equilibrium prices of firm $j \neq i$.

Consider the incumbent in a situation where the entrant charges $p^*_{EL} = p^*_{EH}$ and the incumbent knows that the entrant's quality is low. Suppose the incumbent wishes to deviate to some price p_I if the uninformed consumers interpret p_I as a signal that indicates a low quality entrant. Then the idea of the intuitive criterion is that p_I should indeed convince the consumers that the entrant offers low quality if the following is true: If the incumbent knew that the entrant's quality is high, he would not gain from deviating to p_I even if the consumers would respond favorably for the incumbent by believing that p_I indicates a low quality entrant.

An analogous argument applies to the high quality entrant in a situation where the incumbent's pricing $p^*_{IL} = p^*_{IH}$ reveals no information. In this case, the intuitive criterion requires the uninformed consumers to believe that a price p_E signals high quality if for this belief deviating to p_E is profitable only for the high quality entrant and not for the low quality entrant.

More formally, the PBE (p^*, μ^*) satisfies the *intuitive criterion* if the following two conditions (a) and (b) are satisfied:

(a) If $p^*_{EL} = p^*_{EH} = p^*_E$, then $\mu^*(p_I, p^*_E) = 0$ for all p_I such that

$$\Pi_{IH}(p_I, p^*_E, 0) \leq \Pi_{IH}(p^*_{IH}, p^*_E, \mu^*(p^*_{IH}, p^*_E)) \tag{2.18}$$

and

$$\Pi_{IL}(p_I, p^*_E, 0) > \Pi_{IL}(p^*_{IL}, p^*_E, \mu^*(p^*_{IL}, p^*_E)). \tag{2.19}$$

(b) If $p^*_{IL} = p^*_{IH} = p^*_I$, then $\mu^*(p^*_I, p_E) = 1$ for all p_E such that

$$\Pi_{EL}(p^*_I, p_E, 1) \leq \Pi_{EL}(p^*_I, p^*_{EL}, \mu^*(p^*_I, p^*_{EL})) \tag{2.20}$$

[4] See, for example, Bagwell and Riordan (1991), Bagwell and Ramey (1991), Bester (1993), Bester and Ritzberger (2001).

and

$$\Pi_{EH}(p_I^*, p_E, 1) > \Pi_{EH}(p_I^*, p_{EH}^*, \mu^*(p_I^*, p_{EH}^*)). \tag{2.21}$$

As our analysis will show, the intuitive criterion eliminates all PBE in which both duopolists use a pooling strategy. Thus, only separating equilibria remain in which the entrant's quality is revealed to the uninformed buyers. As we have explained above, for this type of equilibrium the intuitive criterion is not generally applicable because, if one of the firms unilaterally deviates from its equilibrium pricing strategy, the buyers may still be able to infer the entrant's quality from the other firm's price.

As a refinement for situations where firm $i \in \{I, E\}$ defects from the equilibrium and firm $j \neq i$ uses a separating strategy $p_{jL}^* \neq p_{jH}^*$, we employ the 'unprejudiced belief criterion' introduced by Bagwell and Ramey (1991). The basic idea of this criterion is that upon observing an out–of–equilibrium price pair (p_I, p_E) the uninformed consumers rationalize their observation with the fewest number of deviations from the equilibrium strategies. Therefore, if a price pair occurs where one of the prices is out–of–equilibrium while the other price belongs to the separating pricing strategy of the competitor, the consumers believe that the entrant's quality is signaled by the competitor.

Actually, since there are only two types of the entrant, in our context it is sufficient to consider a simplified version of the unprejudiced belief criterion: If only the entrant chooses an out–of–equilibrium price p_E and the incumbent's equilibrium price p_{IH}^* indicates a high quality entrant, then the uninformed consumers should conclude that the entrant offers high quality; there are no belief restrictions if the incumbent's price p_{IL}^* signals low quality. Indeed, a high quality signal of the incumbent looks rather convincing since it is against his interest to admit that his competitor offers a superior good. An analogous reasoning applies when the uninformed consumers conjecture that the price p_I constitutes a unilateral deviation by the incumbent. In this situation, they should infer from the entrant's price p_{EL}^* that his quality is low; there are no belief restrictions if the entrant's price is p_{EH}^*. Again, this seems plausible because expecting high quality makes little sense if the entrant acknowledges that his quality is low.

More formally, the PBE (p^*, μ^*) satisfies the *unprejudiced belief criterion* if the following two conditions (a) and (b) are satisfied:

(a) If $p_{IL}^* \neq p_{IH}^*$, then $\mu^*(p_{IH}^*, p_E) = 1$ for all $p_E \neq p_{EL}^*$.

(b) If $p_{EL}^* \neq p_{EH}^*$, then $\mu^*(p_I, p_{EL}^*) = 0$ for all $p_I \neq p_{IH}^*$.

Notice that in a two–sided separating equilibrium the criterion does not impose belief restrictions on the out–of–equilibrium price constellations (p_{IH}^*, p_{EL}^*) and (p_{IL}^*, p_{EH}^*), under which the signals of the incumbent and the entrant appear contradictory. For these constellations it is not clear whether the incumbent or the entrant has deviated from his equilibrium strategy.

In what follows, we call a PBE (p^*, μ^*) that satisfies the intuitive and the unprejudiced belief criterion a *signalling equilibrium*. In the following section, we investigate the existence and properties of such an equilibrium.

2.4 Signalling Equilibria

Pooling Equilibria

We first consider pooling equilibria, in which the pricing strategies of both firms reveal no information about the entrant's quality. Let $p_I^* = p_{IL}^* = p_{IH}^*$ denote the incumbent's and $p_E^* = p_{EL}^* = p_{EH}^*$ the entrant's price in a pooling equilibrium. The uninformed consumers' belief then satisfies $\mu^*(p_I^*, p_E^*) = \lambda$.

We will show that the existence of pooling equilibria is not consistent with the intuitive criterion. This is so because after observing that the entrant offers low quality, the incumbent can gain by credibly signalling the entrant's true quality through some price $p > p_I^*$. Indeed, if $q_E = q_L$ the incumbent's gain from deviating to a price p that signals a low quality entrant is

$$\varphi_{IL}(p) \equiv \Pi_{IL}(p, p_E^*, 0) - \Pi_{IL}(p_I^*, p_E^*, \lambda). \tag{2.22}$$

If $q_E = q_H$, the incumbent's gain from deceiving the uninformed consumers by choosing p is

$$\varphi_{IH}(p) \equiv \Pi_{IH}(p, p_E^*, 0) - \Pi_{IH}(p_I^*, p_E^*, \lambda). \tag{2.23}$$

The following lemma shows that the incumbent's gain from signalling a low quality of the entrant by some price $p > p_I^*$ is higher when the entrant's true quality is low than when it is high. In fact, for some critical $p' > p_I^*$ the incumbent benefits from deviating to p' and inducing the belief $\mu(p', p_E^*) = 0$ only if he is not cheating.

Lemma 2.1 *(a) $\varphi_{IL}(p) - \varphi_{IH}(p)$ is strictly increasing in p, and $\varphi_{IL}(p_I^*) = \varphi_{IH}(p_I^*) > 0$. (b) There exists a unique $p' > p_I^*$ such that $\varphi_{IH}(p') = 0$.*

When the uninformed consumers' belief decreases from λ to zero, then at the price p_I^* the incumbent's demand increases by an amount which is independent of the entrant's true quality. This is so because the informed consumers' purchasing decisions are not affected and only some fraction of uninformed consumers switches to the incumbent. But if the incumbent raises its price above p_I^* he loses more informed consumers if $q_E = q_H$ than if $q_E = q_L$. Therefore, signalling a low quality entrant by a price p' that satisfies part (b) of Lemma 2.1 is attractive for the incumbent only if this signal is truthful. By the reasoning of the intuitive criterion, this makes it profitable for the incumbent to deviate from his pooling strategy.

Proposition 2.1 *There exists no signalling equilibrium* (p^*, μ^*) *such that* $p_{IL}^* = p_{IH}^*$ *and* $p_{EL}^* = p_{EH}^*$.

Interestingly, the conclusion that the intuitive criterion eliminates all pooling equilibria relies on the ability of the incumbent to credibly signal a low quality entrant rather than on the entrant's ability to provide a credible price signal of high quality. Indeed, one cannot use an analogous argument as in Lemma 2.1 to show that the high quality entrant always gains more than the low quality entrant from a price $p > p_E^*$ that the uninformed consumers interpret as a high quality signal. The reason is that the entrant's unit cost depends on his quality. If consumers become more optimistic and raise μ, then at a given price p_E^* the low and the high quality entrant's demand increases by the same amount. Yet, the low quality entrant's profit increases more than the high quality entrant's profit because the latter has a higher production cost and therefore a smaller profit margin. For some parameter constellations, this may prevent the high quality entrant to gain by deviating from a pooling strategy and appealing to the intuitive criterion.[5]

One–Sided Separating Equilibria

We now turn to the analysis of one–sided separating equilibria, in which one firm chooses a pooling and the other a separating pricing strategy. We will show that such equilibria typically do not exist, except for special parameter constellations. First, consider the case where the incumbent's price $p_I^* = p_{IL}^* = p_{IH}^*$ is independent of the entrant's quality, whereas the entrant chooses quality contingent prices p_{EL}^* and p_{EH}^* with $p_{EL}^* \neq p_{EH}^*$. Because in equilibrium the uninformed consumers infer the entrant's quality from his price, their beliefs satisfy $\mu^*(p_I^*, p_{EL}^*) = 0$ and $\mu^*(p_I^*, p_{EH}^*) = 1$.

The following lemma establishes necessary conditions for this type of equilibrium.

[5] This is related to the observation of Bagwell and Riordan (1991) that in a monopoly model pooling equilibria satisfying the intuitive criterion may exist for some range of parameter values.

Lemma 2.2 *Suppose that the prices p, with $p_I = p_{IL} = p_{IH}, p_{EL} \neq p_{EH}$, can be supported as a signalling equilibrium (p, μ) by some belief μ. Then p must satisfy*

$$p_{EL} = \text{argmax}_p \, \Pi_{EL}(p_I, p, 0), \tag{2.24}$$

$$p_I = \text{argmax}_p \, \Pi_{IH}(p, p_{EH}, 1) = \text{argmax}_p \, \Pi_{IL}(p, p_{EL}, 0), \tag{2.25}$$

$$p_{EH} \text{ maximizes } \quad \Pi_{EH}(p_I, p, 1) \quad \text{subject to} \quad \Pi_{EL}(p_I, p, 1) \leq \Pi_{EL}(p_I, p_{EL}, 0). \tag{2.26}$$

Condition (2.24) simply states that the low quality entrant's price reaction against p_I is not distorted by signalling considerations. Indeed, some price p not satisfying (2.24) can maximize the low quality seller's profit only if $\mu(p_I, p) > 0$. But this is inconsistent with an equilibrium where prices reveal the true quality. The same argument underlies the first condition in (2.25) for the incumbent's price when competing against the high quality entrant. The incumbent's price reaction against p_{EH} cannot be distorted because the consumers' belief that the entrant has high quality is already the worst possible belief from the incumbent's perspective.

The second condition for p_I in (2.25) is implied by part (b) of the unprejudiced belief criterion. This criterion restricts the consumers' belief to $\mu(p, p_{EL}) = 0$ for all $p \neq p_I$. Further, Bayes' rule in (2.16) requires that $\mu(p_I, p_{EL}) = 0$. Thus, the incumbent's pricing has no impact on consumer beliefs when facing the low quality entrant, and so in this situation there are also no signalling distortions.

Finally, the constraint in condition (2.26) has to be satisfied because otherwise the low quality entrant would gain by imitating the high quality entrant's price. Further, the intuitive criterion implies that consumers infer high quality whenever the entrant gains by deviating to some price satisfying this constraint. Accordingly, the high quality entrant's price p_{EH} must solve the constrained maximization problem in (2.26).

Lemma 2.2 allows us to show that a one–sided separating equilibrium with $p^*_{EL} \neq p^*_{EH}$ exists at most for a single value of the parameter γ. Since there is no reason for why the fraction of informed consumers should be identical to this value, an equilibrium of this type effectively fails to exist.

Proposition 2.2 *For all $\gamma \neq t/(t + \Delta)$ there exists no signalling equilibrium (p^*, μ^*) such that $p^*_{IL} = p^*_{IH}$ and $p^*_{EL} \neq p^*_{EH}$.*

The nonexistence result stated in Proposition 2.2 is a straightforward implication of Lemma 2.2. The lemma shows that prices in a one–sided separating equilibrium have to satisfy four conditions. Yet, such an equilibrium determines only three prices. This means that not all conditions can hold simultaneously, unless the exogenous parameters accidentally make one of the conditions redundant. The following lemma shows that a similar observation applies to the other type of one–sided separating equilibria, in which the entrant adopts a pooling strategy $p^*_E = p^*_{EL} = p^*_{EH}$ and only the incumbent's prices p^*_{IL} and p^*_{EH} reveal the entrant's quality so that $\mu^*(p^*_E, p^*_{IL}) = 0$ and $\mu^*(p^*_E, p^*_{IH}) = 1$.

Lemma 2.3 *Suppose that the prices p, with $p_{IL} \neq p_{IH}, p_E = p_{EL} = p_{EH}$, can be supported as a signalling equilibrium (p, μ) by some belief μ. Then p must satisfy*

$$p_{IH} = \text{argmax}_p \, \Pi_{IH}(p, p_E, 1), \tag{2.27}$$

$$p_E = \text{argmax}_p \, \Pi_{EL}(p_{IL}, p, 0) = \text{argmax}_p \, \Pi_{EH}(p_{IH}, p, 1), \tag{2.28}$$

$$p_{IL} \textit{ maximizes } \quad \Pi_{IL}(p, p_E, 0) \quad \textit{subject to} \quad \Pi_{IH}(p, p_E, 0) \leq \Pi_{IH}(p_{IH}, p_E, 1). \tag{2.29}$$

We omit a proof of this lemma because it is analogous to the proof of Lemma 2.2. By our next proposition, also the implications the two lemmas are similar. In fact, Lemma 2.3 shows that a one–sided separating equilibrium with $p^*_{IL} \neq p^*_{IH}$ may exist merely under a single parameter constellation.

Proposition 2.3 *For all $\gamma \neq (3t\Delta - 2c\Delta - \Delta^2 - 3c^2)/(3t\Delta + 4c\Delta + 2\Delta^2)$ there exists no signalling equilibrium (p^*, μ^*) such that $p^*_{IL} \neq p^*_{IH}$ and $p^*_{EL} = p^*_{EH}$.*

Our results so far show that in a signalling equilibrium it cannot happen that one or both of the duopolists adopt a pooling strategy. In Proposition 2.1, the intuitive criterion rules out two–sided pooling. Propositions 2.2 and 2.3 eliminate one–sided pooling by combining the intuitive and the unprejudiced belief criterion. This leaves a two–sided separating equilibrium as the remaining candidate for a signalling equilibrium.

Two–Sided Separating Equilibria

In a two–sided separating equilibrium the uninformed consumers' equilibrium belief is $\mu^*(p^*_{IL}, p^*_{EL}) = 0$ and $\mu^*(p^*_{IH}, p^*_{EH}) = 1$ as $p^*_{IL} \neq p^*_{IH}$ and $p^*_{EL} \neq p^*_{EH}$. Since each firm's price is informative, the intuitive criterion is no longer applicable. Therefore, only the unprejudiced belief criterion plays a role in the following lemma which provides necessary and sufficient conditions for a two–sided separating equilibrium.

Lemma 2.4 *The prices p, with $p_{IL} \neq p_{IH}, p_{EL} \neq p_{EH}$, can be supported as a signalling equilibrium (p, μ) by some belief μ if and only if*

(a) p is identical to the perfect information equilibrium $\hat{p}$ in (2.13), and

(b) there exists some $\bar{\mu} \in [0, 1]$ such that

$$\begin{aligned} \Pi_{IH}(p_{IH}, p_{EH}, 1) &\geq \Pi_{IH}(p_{IL}, p_{EH}, \bar{\mu}), \\ \Pi_{EL}(p_{IL}, p_{EL}, 0) &\geq \Pi_{EL}(p_{IL}, p_{EH}, \bar{\mu}). \end{aligned} \tag{2.30}$$

By statement (a) of Lemma 2.4, in a two–sided separating equilibrium the firms' prices are identical to the outcome of price competition under full information of all market participants about the entrant's quality. Thus, even though prices act as signals, they are not distorted by incentive restrictions. This observation is a well–known implication of the unprejudiced beliefs refinement (see Bagwell and Ramey (1991)).[6] The idea is simply that the high quality entrant can ignore signalling effects when already the incumbent's price convinces the uninformed consumers of high quality. Similarly, the incumbent does not have to resort to distorted pricing to indicate a low quality entrant, because the entrant himself already reveals his quality through his price setting strategy. In a two–sided separating equilibrium, therefore, the firms' prices are determined as mutually undistorted best responses against the competitor and are thus identical to the full information equilibrium.

While prices are not distorted by signalling effects, statement (b) of Lemma 2.4 shows that they have to satisfy an incentive compatibility restriction, which is related to the signalling rivalry between the duopolists. The uninformed consumers will be perplexed when they observe the out–of-equilibrium price pair $(\hat{p}_{IL}, \hat{p}_{EH})$. These prices are contradictory because the incumbent's price signals a low quality entrant and the entrant's price a high quality. Also, it is not clear which firm has deviated from its equilibrium strategy. The prices $(\hat{p}_{IL}, \hat{p}_{EH})$ could originate from the equilibrium pair $(\hat{p}_{IH}, \hat{p}_{EH})$ because the incumbent has deviated to $\hat{p}_{IL}$; or they could originate from the equilibrium pair $(\hat{p}_{IL}, \hat{p}_{EL})$ because the entrant has deviated to $\hat{p}_{EH}$. Condition (2.30) states that there must be some belief $\bar{\mu} = \mu(\hat{p}_{IL}, \hat{p}_{EH})$ that deters both kinds of deviations. On the one hand, by the first inequality in (2.30), $\bar{\mu}$ must be high enough so as to make it unattractive for the incumbent to deviate from $\hat{p}_{IH}$ to $\hat{p}_{IL}$. On the other hand, the second inequality in (2.30) requires that $\bar{\mu}$ is small enough so that the entrant cannot gain by deviating from $\hat{p}_{EL}$ to $\hat{p}_{EH}$.

[6]Yehezkel (2006) proposes a generalization of the unprejudiced belief criterion that eliminates all possible separating equilibria but the full information outcome.

Whether condition (b) of Lemma 2.4 holds or not, depends on how large the fraction γ of informed consumers is. To state our next result, we define the critical parameter

$$\bar{\gamma} \equiv \frac{27\Delta t^2 + (\Delta - c)(3t\Delta + 15ct + 2c^2) - \Delta(\Delta^2 - c^2)}{27\Delta t^2 + 9\Delta^2 t + 18\Delta tc}. \tag{2.31}$$

Note that, since

$$\frac{\partial \bar{\gamma}}{\partial t} > 0, \qquad \lim_{t \to (\Delta - c)/3} \bar{\gamma} = \frac{\Delta^2 - c^2}{2\Delta^2 + c\Delta} > 0, \qquad \lim_{t \to \infty} \bar{\gamma} = 1, \tag{2.32}$$

our assumption (2.4) implies that $\bar{\gamma} \in (0, 1)$.

Proposition 2.4 *(a) Let $\gamma \geq \bar{\gamma}$. Then there exists a signalling equilibrium (p^*, μ^*) with $p^*_{IL} \neq p^*_{IH}$ and $p^*_{EL} \neq p^*_{EH}$. The prices p^* in this equilibrium are identical to the perfect information equilibrium $\hat{p}$. (b) If $\gamma < \bar{\gamma}$, there exists no signalling equilibrium (p^*, μ^*) such that $p^*_{IL} \neq p^*_{IH}$ and $p^*_{EL} \neq p^*_{EH}$.*

In a two–sided separating equilibrium prices are not distorted by signalling. The incumbent or the entrant can gain by a unilateral deviation only because this changes the uninformed consumers' beliefs. Therefore, a deviation is not profitable as long as not too many consumers are uninformed. This explains why $(\hat{p}, \mu^*)$ can constitute a signalling equilibrium for $\gamma \geq \bar{\gamma}$. If $\gamma < \bar{\gamma}$, then the firms' signalling rivalry is too intense to prevent profitable deviations: Either the incumbent will defect from the equilibrium if $q_E = q_H$, or the entrant will defect if $q_E = q_L$. As observed by Schultz (1999) in a different context, conflicting interests may thus rule out the existence of a two–sided separating equilibrium for some parameter constellations.

In Table 2.1 some numerical values illustrate how $\bar{\gamma}$ depends on c and t if $\Delta = 10$. Prices can be used as credible signals because of their effect on demand. Since the price sensitivity of demand is negatively related to the product differentiation parameter t, this implies that $\bar{\gamma}$ is increasing in t. An increase in the cost c of high quality raises the price differences $|\hat{p}_{IH} - \hat{p}_{IL}|$ and $|\hat{p}_{EL} - \hat{p}_{EH}|$. Therefore, a deviation of the incumbent from $\hat{p}_{IH}$ to $\hat{p}_{IL}$ or of the entrant from $\hat{p}_{EL}$ to $\hat{p}_{EH}$ is less profitable for high values of c. Consequently, if c is increased, a smaller fraction $\bar{\gamma}$ of informed consumers suffices for existence of a signalling equilibrium.

$\bar{\gamma}\|_{\Delta=10}$	$t = 2$	$t = 4$	$t = 6$
$c = 5$	0.35	0.51	0.6
$c = 7$	0.31	0.44	0.53
$c = 9$	0.23	0.34	0.43

Table 2.1: *Numerical values for* $\bar{\gamma}$.

2.5 Conclusion

Our analysis shows that a firm may not have to resort to distorted pricing to signal its quality to the uninformed consumers. If its quality is known to a competitor, then the prices of both firms become quality signals and signalling competition may lead to non–distorted pricing in equilibrium. Indeed, in our model only the full information equilibrium can survive under two belief refinements that have frequently been used in the literature.

This finding has obvious implications for other strategic choices. For example, consider the market entry decision of a firm whose quality is not publicly observable. In this situation our analysis indicates that entry decisions are not distorted when at least one of the incumbent firms learns the new firm's quality after it has entered the market. A similar conclusion obtains for *R&D* investments in product innovation when some consumers cannot observe whether the investment has been successful or not. As long as competing firms become informed about the outcome, our results suggest that the incentives for product innovation are not distorted by the presence of uninformed consumers.

Our analysis also reveals that the two refinements, which we adopt to restrict out–of–equilibrium beliefs, can become incompatible with existence of an equilibrium. When the fraction of informed consumers is too small in our model, there is no equilibrium satisfying both the intuitive criterion and the unprejudiced belief refinement. One way out of this problem would be to weaken these refinements. But it is not obvious how one should proceed along these lines because both refinements look rather appealing and convincing in the context of our model. Another approach would be modifying our model by assuming that the incumbent is not perfectly informed about the entrant's quality but that he receives noisy information. This would eliminate the problem of specifying beliefs for 'contradictory' price signals. With noisy information such signals would no longer be an out–of–equilibrium event in a two–sided separating equilibrium because it happens with positive probability that the incumbent receives information that the entrant's quality is low even though its quality is actually high. It may be interesting for future research to investigate whether with noisy information a signalling equilibrium always exists and whether it approaches the full information equilibrium as the noise becomes negligible.

2.6 Appendix

Proof of Lemma 1

(a) By (2.7) and (2.9) we have

$$\varphi_{IL}(p) - \varphi_{IH}(p) = \frac{(p - p_I^*)\gamma\Delta}{2t}, \tag{2.33}$$

and

$$\varphi_{IL}(p_I^*) = \varphi_{IH}(p_I^*) = \frac{(1-\gamma)\lambda\Delta p_I^*}{2t} > 0. \tag{2.34}$$

Since $\varphi'_{IL}(p) - \varphi'_{IH}(p) = \gamma\Delta/(2t) > 0$, this proves part (a).

(b) For all $p \geq p_E^* + t - \gamma\Delta$, $\varphi_{IH}(p) < 0$ because $D_{IH}(p, p_E^*, 0) = \Pi_{IH}(p, p_E^*, 0) = 0$. Since $\varphi_{IH}(p_I^*) > 0$, the intermediate value theorem therefore implies that there exist a $p' > p_I^*$ such that $\varphi_{IH}(p') = 0$. Moreover, p' is unique because $\varphi''_{IH}(p) = -1/t < 0$. Q.E.D.

Proof of Proposition 1

By Lemma 2.1 there exists a unique price $p' > p_I^*$ such that $\varphi_{IL}(p') > \varphi_{IH}(p') = 0$, i.e.

$$\Pi_{IH}(p', p_E^*, 0) = \Pi_{IH}(p_I^*, p_E^*, \lambda) \tag{2.35}$$

$$\Pi_{IL}(p', p_E^*, 0) > \Pi_{IL}(p_I^*, p_E^*, \lambda) \tag{2.36}$$

Thus p' satisfies conditions (2.18) and (2.19) of the intuitive criterion. This implies that $\mu^*(p', p_E^*) = 0$. Therefore, we have

$$\Pi_{IL}(p', p_E^*, \mu^*(p', p_E^*)) = \Pi_{IL}(p', p_E^*, 0) > \Pi_{IL}(p_I^*, p_E^*, \lambda) = \Pi_{IL}(p_I^*, p_E^*, \mu^*(p_I^*, p_E^*)). \tag{2.37}$$

Because the price strategies violate equilibrium condition (2.14) for $Q = L$, there cannot exist a signalling equilibrium with $p_I^* = p_{IL}^* = p_{IH}^*$ and $p_E^* = p_{EL}^* = p_{EH}^*$. Q.E.D.

Proof of Lemma 2

Since $p_{EL} \neq p_{EH}$ implies $\mu(p_I, p_{EL}) = 0$ and $\partial\Pi_{EL}/\partial\mu > 0$, it follows from equilibrium condition (2.15) that for all $p \geq 0$

$$\Pi_{EL}(p_I, p_{EL}, 0) \geq \Pi_{EL}(p_I, p, \mu(p_I, p)) \geq \Pi_{EL}(p_I, p, 0). \tag{2.38}$$

This proves that (2.24) must hold. Analogously, $\mu(p_I, p_{EH}) = 1$ and $\partial\Pi_{IH}/\partial\mu < 0$ imply by (2.14) that for all $p \geq 0$

$$\Pi_{IH}(p_I, p_{EH}, 1) \geq \Pi_{IH}(p, p_{EH}, \mu(p, p_{EH})) \geq \Pi_{IH}(p, p_{EH}, 1). \tag{2.39}$$

This proves that p_I must satisfy the first condition in (2.25).

Suppose that p_I does not satisfy the second condition in (2.25). Since part (b) of the unprejudiced belief criterion implies $\mu(p, p_{EL}) = 0$ for all $p \neq p_I$, then there exist some p such that

$$\Pi_{IL}(p_I, p_{EL}, \mu(p_I, p_{EL})) = \Pi_{IL}(p_I, p_{EL}, 0) < \Pi_{IL}(p, p_{EL}, 0) = \Pi_{IL}(p, p_{EL}, \mu(p, p_{EL})). \tag{2.40}$$

This is a contradiction to the condition that in equilibrium p_I has to satisfy (2.14) for $Q = L$.

Note that p_{EH} must satisfy the constraint in (2.26) because equilibrium condition (2.15) implies that

$$\Pi_{EL}(p_I, p_{EL}, 0) = \Pi_{EL}(p_I, p_{EL}, \mu(p_I, p_{EL})) \geq \Pi_{EL}(p_I, p_{EH}, \mu(p_I, p_{EH})) = \Pi_{EL}(p_I, p_{EH}, 1). \tag{2.41}$$

Suppose that p_{EH} does not solve the maximization problem in (2.26). Then there exists some p that satisfies the constraint in (2.26) and $\Pi_{EH}(p_I, p, 1) > \Pi_{EH}(p_I, p_{EH}, 1)$. Because part (b) of the intuitive criterion then implies $\mu(p_I, p) = 1$, this yields

$$\Pi_{EH}(p_I, p, \mu(p_I, p)) = \Pi_{EH}(p_I, p, 1) > \Pi_{EH}(p_I, p_{EH}, 1) = \Pi_{EH}(p_I, p_{EH}, \mu(p_I, p_{EH})), \tag{2.42}$$

a contradiction to equilibrium condition (2.15) for $Q = H$. Q.E.D.

Proof of Proposition 2

The first–order conditions for (2.24) and (2.25) in Lemma 2.2 are

$$\begin{aligned}
\frac{\partial\Pi_{EL}(p_I, p_{EL}, 0)}{\partial p_{EL}} &= \frac{t + p_I - 2p_{EL}}{2t} = 0, \\
\frac{\partial\Pi_{IH}(p_I, p_{EH}, 1)}{\partial p_I} &= \frac{t - \Delta + p_{EH} - 2p_I}{2t} = 0, \\
\frac{\partial\Pi_{IH}(p_I, p_{EL}, 0)}{\partial p_I} &= \frac{t + p_{EL} - 2p_I}{2t} = 0.
\end{aligned} \tag{2.43}$$

The solution of these equations is

$$p_I^* = t, p_{EL}^* = t, \; p_{EH}^* = t + \Delta. \tag{2.44}$$

If the constraint in (2.26) is not binding, we obtain from the first–order condition

$$\frac{\partial \Pi_{EH}(p_I^*, p_{EH}, 1)}{\partial p_{EH}} = \frac{2t + \Delta + c - 2p_{EH}}{2t} \tag{2.45}$$

that $p_{EH}^* = (2t+\Delta+c)/2$. This, however, is inconsistent with the last equation in (2.44) as $\Delta > c$. If the constraint in (2.26) is binding, then $\Pi_{EL}(p_I^*, p_{EH}^*, 1) = \Pi_{EL}(p_I^*, p_{EL}^*, 0)$. By (2.44) this equality is equivalent to

$$\frac{(\Delta + t)(t - \gamma\Delta)}{2t} = \frac{t}{2}. \tag{2.46}$$

From this equation it follows that the conditions of Lemma 2.2 are satisfied only if $\gamma = t/(t + \Delta)$. Q.E.D.

Proof of Proposition 3

From the first–order conditions for (2.27) and (2.28) in Lemma 2.3,

$$\begin{aligned}
\frac{\partial \Pi_{IH}(p_{IH}, p_E, 1)}{\partial p_{IH}} &= \frac{t - \Delta + p_E - 2p_{IH}}{2t} = 0, \\
\frac{\partial \Pi_{EL}(p_{IL}, p_E, 0)}{\partial p_E} &= \frac{t - 2p_E + p_{IL}}{2t} = 0, \\
\frac{\partial \Pi_{EH}(p_{IH}, p_E, 1)}{\partial p_E} &= \frac{t + \Delta + c - 2p_E + p_{IH}}{2t} = 0,
\end{aligned} \tag{2.47}$$

we obtain the solution

$$p_{IL}^* = \frac{3t + 2\Delta + 4c}{3}, \; p_{IH}^* = \frac{3t - \Delta + c}{3}, \; p_E^* = \frac{3t + \Delta + 2c}{3}. \tag{2.48}$$

If the constraint in (2.29) is not binding, we obtain from the first–order condition

$$\frac{\partial \Pi_{IL}(p_{IL}, p_E^*, 0)}{\partial p_{IL}} = \frac{\Delta + 2c + 6t - 6p_{IL}}{6t} \tag{2.49}$$

that $p_{IL}^* = (6t + \Delta + 2c)/6$. This, however, is inconsistent with the first equation in (2.48). If the constraint in (2.29) is binding, then $\Pi_{IH}(p_{IL}^*, p_E^*, 0) = \Pi_{IH}(p_{IH}^*, p_E^*, 1)$. By

(2.48) this is equivalent to

$$\frac{(2\Delta + 4c + 3t)(3t - 3\gamma\Delta - \Delta - 2c)}{18t} = \frac{(3t - \Delta + c)^2}{18t}. \tag{2.50}$$

Solving this equation for γ yields $\gamma = (3t\Delta - 2c\Delta - \Delta^2 - 3c^2)/(3t\Delta + 4c\Delta + 2\Delta^2)$. Thus, if γ does not satisfy this condition, also the conditions of Lemma 2.3 cannot hold.
Q.E.D.

Proof of Lemma 4

We first show that (a) and (b) must hold in a signalling equilibrium (p, μ). By (2.14)

$$\Pi_{IH}(p_{IH}, p_{EH}, 1) \geq \Pi_{IH}(p, p_{EH}, \mu^*(p, p_{EH})) \geq \Pi_{IH}(p, p_{EH}, 1) \tag{2.51}$$

for all $p \geq 0$, where the second inequality follows from $\partial\Pi_{IH}/\partial\mu < 0$. Similarly, (2.14) and part (b) of the unprejudiced belief criterion imply

$$\Pi_{IL}(p_{IL}, p_{EL}, 0) \geq \Pi_{IL}(p, p_{EL}, \mu^*(p, p_{EL})) = \Pi_{IL}(p, p_{EL}, 0) \tag{2.52}$$

for all $p \neq p_{IH}$. By continuity of $\Pi_{IL}(\cdot, p_{EL}, 0)$, therefore also

$$\Pi_{IL}(p_{IL}, p_{EL}, 0) \geq \Pi_{IL}(p_{IH}, p_{EL}, 0). \tag{2.53}$$

By an analogous argument it follows from (2.14), $\partial\Pi_{EL}/\partial\mu > 0$, and part (a) of the unprejudiced belief criterion that

$$\Pi_{EL}(p_{IL}, p_{EL}, 0) \geq \Pi_{EL}(p_{IL}, p, 0), \quad \Pi_{EH}(p_{IH}, p_{EH}, 1) \geq \Pi_{EH}(p_{IH}, p, 1) \tag{2.54}$$

for all $p \geq 0$. By (2.51)–(2.54), p satisfies the conditions that define $\hat{p}$ in (2.12). This proves that (p, μ) must satisfy claim (a) that $p = \hat{p}$. Note that by (2.14) and (2.15)

$$\begin{aligned} \Pi_{IH}(p_{IH}, p_{EH}, 1) &\geq \Pi_{IH}(p_{IL}, p_{EH}, \mu(p_{IL}, p_{EH})), \\ \Pi_{EL}(p_{IL}, p_{EL}, 0) &\geq \Pi_{EL}(p_{IL}, p_{EH}, \mu(p_{IL}, p_{EH})). \end{aligned} \tag{2.55}$$

This proves that statement (b) holds for $\bar{\mu} \equiv \mu(p_{IL}, p_{EH})$.

Next we show that $(\hat{p}, \mu)$ is a signalling equilibrium for some μ only if (b) holds. Note that the intuitive criterion does not apply to $\hat{p}$ because $\hat{p}_{IL} \neq \hat{p}_{IH}$ and $\hat{p}_{EL} \neq \hat{p}_{EH}$.

In line with the unprejudiced belief criterion, define

$$\mu(\hat{p}_{IH}, p) \equiv 1 \text{ for all } p \neq \hat{p}_{EL}, \quad \mu(p, \hat{p}_{EL}) \equiv 0 \text{ for all } p \neq \hat{p}_{IH}, \quad \mu(\hat{p}_{IH}, \hat{p}_{EL}) \equiv \lambda. \tag{2.56}$$

Further, if (2.30) in part (b) of the lemma holds for $p = \hat{p}$ we can set

$$\mu(\hat{p}_{IL}, p) \equiv 0 \text{ for all } p \neq \hat{p}_{EH}, \quad \mu(p, \hat{p}_{EH}) \equiv 1 \text{ for all } p \neq \hat{p}_{IL}, \quad \mu(\hat{p}_{IL}, \hat{p}_{EH}) \equiv \bar{\mu}. \tag{2.57}$$

The beliefs for all other price pairs (p_I, p_E) play no role in the definition of a PBE and so they are arbitrary. Since $\mu(\hat{p}_{IL}, \hat{p}_{EL}) = 0$ and $\mu(\hat{p}_{IH}, \hat{p}_{EH}) = 1$ by (2.56) and (2.57), these beliefs satisfy Bayes rule (2.16) in part (b) of the definition of a PBE. Further since $\hat{p}$ satisfies (2.12) and (2.55) holds for $p = \hat{p}$, it is easily verified that $(\hat{p}, \mu)$ satisfies also the conditions (2.14) and (2.15) for profit maximization in part (a) of the definition of a PBE. This proves that $\hat{p}$ and the beliefs μ in (2.56) and (2.57) constitute a signalling equilibrium if (2.30) in part (b) of the lemma holds for $p = \hat{p}$. If the latter condition does not hold, then there is no belief $\mu(p_{IL}, p_{EH})$ that satisfies both conditions in (2.55) for $p = \hat{p}$. In this case, there exists no PBE (p, μ) with $p = \hat{p}$ because at least one of the conditions (2.14) and (2.15) for profit maximization is violated. Q.E.D.

Proof of Proposition 4

By Lemma 2.4 it is sufficient to show that for $p = \hat{p}$ (2.30) has a solution $\bar{\mu} \in [0, 1]$ if and only if $\gamma \geq \bar{\gamma}$. Using $\hat{p}$ in (2.13), the first inequality in (2.30) is equivalent to

$$\frac{(3t - \Delta + c)^2}{18t} \geq \frac{3t + 2c + \Delta(1 - 3\gamma) - 3\bar{\mu}\Delta(1 - \gamma)}{6}. \tag{2.58}$$

Solving this inequality for $\bar{\mu}$ yields

$$\bar{\mu} \geq \bar{\mu}_I(\gamma) \equiv \frac{9t\Delta(1 - \gamma) - (\Delta - c)^2}{9t\Delta(1 - \gamma)}. \tag{2.59}$$

By (2.13) the second inequality in (2.30) is equivalent to

$$\frac{t}{2} \geq \frac{(3t + 2c + \Delta)(3t - 2c - \Delta + 3\bar{\mu}\Delta(1 - \gamma))}{18t}. \tag{2.60}$$

Solving this inequality for $\bar{\mu}$ yields

$$\bar{\mu} \leq \bar{\mu}_E(\gamma) \equiv \frac{(\Delta + 2c)^2}{3\Delta(1 - \gamma)(3t + 2c + \Delta)}. \tag{2.61}$$

The inequalities (2.59) and (2.61) admit a solution $\bar{\mu}$ if and only if $\bar{\mu}_I(\gamma) \leq \bar{\mu}_E(\gamma)$. It is easily verified that $\bar{\gamma}$, as defined in (2.31), satisfies $\bar{\mu}_I(\bar{\gamma}) = \bar{\mu}_E(\bar{\gamma})$. Note that $\bar{\mu}_I(0) < 1, \bar{\mu}_E(0) > 0$, $\bar{\mu}'_I(\gamma) < 0$ and $\bar{\mu}'_E(\gamma) > 0$. Since $\bar{\gamma} \in (0,1)$, this implies that there exists a $\bar{\mu} \in [\bar{\mu}_I(\gamma), \bar{\mu}_E(\gamma)] \cap [0,1]$ if and only if $\gamma \geq \bar{\gamma}$. Q.E.D.

Chapter 3

Regulation and Quality Incentives in Search Markets: Certification, Licensing and Minimum Prices

3.1 Introduction

In many markets search and bargaining have a non-negligible influence on market outcomes, e.g. in markets for labor, services, real estate, antiques, etc. In case of labor markets search models are extensively used in order to explain unemployment, wage distributions, job market turnover and the like (see e.g. Rogerson, Shimer, and Wright (2005)). In this paper, which extends the random matching market model of Rubinstein and Wolinsky (1985) by a pre-entry quality decision, we examine the effects of specific market regulations on supplier's investments.

Certification, licensing and minimum prices are common regulation policies in all sorts of markets. In an environment of asymmetric information, certification allows buyers of a product to observe sellers' qualities [1]. The certifier is a credible third party, taking a fee for its certification service and reporting quality information truthfully. Sellers acquire the certificate voluntarily and pay the fee. All sellers, independent of their certification decision, are allowed to sell their products in the market. By licensing, we understand a policy by which sellers are only allowed to enter the market if they acquire a license. The license can be given by the state or a trade organization either randomly or conditioned on quality. Different to certification, sellers without a license are not allowed to enter the market. Consequently, by limiting the number of licenses

[1] We think of an *experience* good in the terminology of Nelson (1970). Alternatives to certification and licensing, like brands or warranties will not be considered (see e.g. Dranove and Jin (2010)).

the supply gets rationed. Minimum prices set a minimum floor on prices. All sellers can offer their goods, but are not allowed to ask a price smaller than that price floor.

We begin with a characterization of a steady state matching market equilibrium without certification, i.e. without opportunity for sellers to differentiate themselves from the buyers' perspective. In this situation sellers have no incentive to invest in quality. If certification is available and not too costly, cost efficient sellers invest and earn a markup compared to low-quality sellers. Based on this market environment, we discuss the effects of licensing and minimum-prices on sellers' investment decisions.

The paper adds two main insights to the existing literature. First, we show that welfare in search markets that are characterized by substantial quality investments can be increased by limiting the number of sellers through a licensing policy. The search and bargaining structure leads to a holdup of sellers' investments in quality which can be mitigated by enforcing the sellers' bargaining power through an amelioration of their search conditions.

Second, it is shown that the introduction of an effective minimum price lowers investment incentives for sellers and thereby lowers welfare. In order to get this result we show how the minimum price affects the price distribution through a shift in bargaining power from buyers to sellers. This provides a search-theoretical foundation for the so called *knock-on* or *ripple* effect of price regulations on the price distribution.

Related Literature

The adverse selection problem causing the failure of high-quality supply in markets with asymmetric information was first described by Akerlof (1970). How it can be mitigated through quality certification by a credible third party was examined by Viscusi (1978).

Many economists emphasize the role of licensing as a device to avoid competition, which is mainly demanded by insiders of professional groups (see e.g. Moore (1961), Friedman and Friedman (1962), Rottenberg (1962), Stigler (1971), Posner (1974) or Maurizi (1974)). More recent research looks on licensing as a way to mitigate inefficiencies caused by asymmetric information, e.g. Leland (1979) or Shapiro (1986) (see Kleiner (2000) for an overview). In Leland (1979) licensing serves to establish a minimum quality standard and thereby mitigates the adverse selection problem directly. Shapiro (1986) focuses on licensing of specific input factors, which sets a minimum standard on supplier's investments that consequently lowers the marginal cost of quality provision. But the models of Leland (1979) and Shapiro (1986), which are set up in a spot market environment, do not explain why licensing might be more advantageous in some case than certification. Both policies allow high-quality sellers to be identified by consumers. But through licensing, low-quality sellers are prevented from market entry, which, in general, is welfare decreasing.

In this paper, we argue that the function of licensing to limit market entry, in some circumstances, has positive welfare effects because it decreases inefficiencies caused by the holdup of quality investments in a search market environment. The argument was made before, e.g. in Kleiner (2000), but we don't know about an attempt to model the effect explictly so far. Specifically, we address the trade-off between higher quality and higher prices for consumer welfare. We don't aim to identify in which situations licensing is better than certification. Instead, we abstract from the potential of licensing to mitigate inefficiencies caused by adverse selection and concentrate on its potential to mitigate inefficiencies caused by the holdup of quality investments. The question which policy, if any, is suited better to decrease market inefficiencies depends very much on the specifics of the market, in our model on the matching technology, and will be left open.

Underprovision of high quality in our model stems from a holdup of sellers' quality investments in the ex-post search and bargaining process. The surplus generated by sellers' investments is shared between sellers and buyers depending on the relative numbers of agents and on the time value of search. The holdup of investments in the context of asset specificity is presented for e.g. by Williamson (1985) or Grout (1984). Different from Grossman and Hart (1986) or Hart and Moore (1990), mitigating the hold-up by a reallocation of property rights is not feasible in our context because investments take place before a matching partner is determined.

Acemoglu and Shimer (1999) look at the holdup problem in a matching market from the buyer's perspective, where it is causing underinvestment of firms in productivity. They show that a competitive setting of wages in combination with observable productivity levels leads to efficient investment decisions. Acemoglu (1996) argues that social increasing returns in human capital accumulation can be caused by search frictions: A higher level of the human capital stock increases firms' incentives for investments in productivity which increases workers' incentives to invest in human capital. Bester (2009) examines investment incentives on both sides of the market with free entry and finds equilibrium investments to be socially inefficient, especially for the long side of the market, but converging to the first-best as search frictions become negligible. Ramey and Watson (2001) study investment decisions after a match formed and find that search frictions cause positive effort incentives to keep the relationship productive as a breakup of the relation would be costly.

We find that minimum prices have a *knock-on* or *ripple* effect on the price for the high-quality good which is proportionally smaller than the direct implication on the price for the low-quality good. In the case of labor markets Gramlich (1976) and Grossman (1983) predict the same effect of minimum wage regulations on the wage distribution. Gramlich (1976) explains it in a neoclassical framework by the substitution of labor demand from low-skilled workers to higher-skilled ones. Consequently, increased demand for high-skilled labor leads to increased wages for this group of workers. Grossman (1983) extends this analysis by an efficiency wage argument: A smaller wage gap between high- and low-skilled labor disincentives high-skilled workers, which causes employers to increase their wages too.

Different to these papers, we explain the effect of a minimum price on the price distribution as the outcome of bargaining, taking into account search frictions. The magnitude of the knock-on effect is characterized by the agents' search conditions, the quality gap between sellers and the relative numbers of qualities in the market. This might deliver an useful framework for empirical work on wage distributions. Neumark, Schweitzer, and Wascher (2004) show that the knock-on effect can be empirically observed, especially for wages nearly above the minimum wage.

The effect of the minimum price on the supplier's investment decision also connects to a strand of literature examining the relationship between minimum wages and investments in human capital. Our model captures only general investments. Related to labor markets these might be school education and general training which is not firm specific. Results of existing research on the effects of minimum wages on schooling are ambiguous. Agell and Lommerud (1997) argue that minimum wages have a positive incentive effect on intermediate talented workers, who just reach the primary market at the minimum wage, and a negative effect on the lowest talented workers, who will not enter the primary market at the minimum wage. Acemoglu and Pischke (1998) explain the sponsoring of general training by employers as a reaction to wages that are distorted in favor of unskilled workers in economies with wage floors.

In section 2 we introduce the matching market model. In section 3 we discuss the steady-state market equilibria with and without certification. The potential welfare raising effect of a licencing policy is shown in section 4. The effect of a minimum wage on the market equilibrium and quality incentives is presented in section 5. Section 6 concludes the paper.

3.2 The Model

We model a decentralized market which encompasses search and bargaining as a dynamic matching market in the spirit of Rubinstein and Wolinsky (1985). Market outcomes are described by steady state equilibria, in which inflows of sellers and buyers into the market are equalized with outflows from the market. The matching market approach is extended by a quality decision made by cost differentiated sellers. The equilibrium prices determined by the conditions in the search market influence incentives for sellers to invest in quality before entering.

The masses of sellers and buyers active in the matching market are S and B, respectively. Sellers offer one unit of either a high-quality good valued q_H or a low-quality good valued q_L by buyers, with $q_H > q_L > 0$. We denote the difference in qualities by $\Delta q \equiv q_H - q_L$. The sellers' valuations for both goods are zero. Before entering the market, sellers decide which quality to supply. Sellers have costs for the production of a low-quality good that are normalized to 0. Costs c to produce the high quality are distributed uniformly on $[0, 1]$. In the categorization of Che and Hausch (1999) investments in quality are *cooperative* because the seller's investment improves the value for the buyer only.

It is assumed that initially there is asymmetric information in the market, so that buyers can not observe the quality when they are matched with a seller. To be recognized as high quality, the seller has to acquire a certificate confirming quality at cost Z. High quality will be produced by a seller if the difference in payoffs $\Delta v \equiv v_{SH} - v_{SL}$ for high and low qualities in the market is greater than individual costs c for the production of the high quality good plus certification costs Z. Because c in uniformly distributed on $[0, 1]$, a fraction $\lambda = \max\{0, \min\{\Delta v - Z, 1\}\}$ of sellers with costs smaller or equal $\Delta v - Z$ enters the market with high quality and the fraction $1 - \lambda$ enters with low quality.

Matching Process

Time is discrete and infinite. In each period each agent meets at most one agent of the opposite group. Matches are random, i.e. buyers can not direct their search to high-quality sellers, because they don't learn the sellers' qualities before they get matched. The number of matches in one period is determined by a matching function $M(S, B)$, where S and B are the masses of active sellers and buyers, respectively. $M(S, B)$ is assumed to have the following properties:

Assumption 1

$$(i) \qquad \frac{\partial M(S,B)}{\partial S} \geq 0, \ \frac{\partial M(S,B)}{\partial B} \geq 0 \tag{3.1}$$

$$(ii) \qquad M(0,B) = M(S,0) = 0 \tag{3.2}$$

$$(iii) \qquad M(S,B) \leq min\{S,B\} \tag{3.3}$$

The number of matches is non-decreasing in the number of agents in the market. If there is only one type of agents active in the market, matches cannot happen. As each agent can be matched at most to one agent of the other type, the number of matches is bounded to the minimum number either of sellers or buyers.

The expected matching probabilities for individual sellers and buyers in each period are given by $\alpha = M(S,B)/S$ and $\beta = M(S,B)/B$, respectively. A special case used for the examination of the licensing policy is *efficient* matching with $M(S,B) = \min\{S,B\}$. Matched agents bargain over the price for the seller's good. If bargaining is successful, i.e. seller and buyer agree on a price, trade takes place. After trade, agents leave the market and are replaced by the same types in the next period. If there is no agreement, both agents continue their search in the following period.

Bargaining

Depending on search conditions in the matching market, and consequently on relative bargaining strengths, sellers and buyers realize payoffs v_{SQ} and v_B, respectively. The index $Q \in \{H,L\}$ indicates if the seller offers high or low quality. Future payoffs are discounted by a factor $\delta \in (0,1)$ per period. When a seller and buyer are matched, each party makes a take-it-or-leave-it price offer with probability 1/2. As alternative matches can happen only in later periods, switching the partner involves costs, which means that agents in a match are partially locked-in in the relation. This gives the party who makes the price offer some market power. The offers are made such that the other party is just indifferent between agreement and continued search, whereby we assume that this offer is accepted. Consequently, the seller proposes to the buyer a price p_{SQ} such that $q_Q - p_{SQ} = \delta v_B$ and the buyer proposes to a seller a price p_{BQ} such that $p_{BQ} = \delta v_{SQ}$.

Expected Payoffs

The seller has a probability α to be matched. If he is matched, he makes a price offer himself or receives a price offer, each with probability one half. If he is not matched, search goes on in the following period with the payoff from search discounted. The expected payoff v_{SQ} of a seller with quality $Q \in \{H, L\}$ in the matching market is implicitly given by

$$v_{SQ} = \alpha \left(\frac{1}{2} p_{SQ} + \frac{1}{2} p_{BQ} \right) + (1-\alpha)\delta v_{SQ} \quad (3.4)$$

$$= \frac{\alpha}{2} (q_Q + \delta v_{SQ} - \delta v_B) + (1-\alpha)\delta v_{SQ}. \quad (3.5)$$

A buyer is matched with a seller with probability β. If matched the buyer bargains with a high-quality seller with probability λ and with a low-quality seller with probability $1-\lambda$ and makes a price-offer or receives a price-offer, each with probability one half. If not matched, search goes on and the buyer gets the discounted expected value of continued search. The buyers' expected payoff is given by

$$v_B = \beta \left[\lambda \left(q_H - \frac{1}{2} p_{SH} - \frac{1}{2} p_{BH} \right) + (1-\lambda) \left(q_L - \frac{1}{2} p_{SL} - \frac{1}{2} p_{BL} \right) \right] \quad (3.6)$$
$$+ (1-\beta)\delta v_B$$

$$= \frac{\beta}{2} \left[\lambda (q_H - \delta v_{SH} + \delta v_B) + (1-\lambda)(q_L - \delta v_{SL} + \delta v_B) \right] + (1-\beta)\delta v_B. \quad (3.7)$$

Market Equilibria

We solve for steady-state equilibria in which the inflow of agents equals the outflow for each type of agent. In the Perfect Bayesian Equilibria (PBE) considered, all bargains lead to trade, so that no type of agent will accumulate in the market. At the stage of quality decision, sellers take the equilibrium number of high- and low-quality sellers in the market as given. Furthermore, in equilibrium buyers' quality expectations are correct.

Social Welfare

To evaluate different regulations with the unregulated market outcome, we need a measure of social welfare. The perspective taken throughout the analysis is the following: In each period, buyers and sellers leave the market after a successful match. Then, the same numbers of buyers and sellers enter with expected payoffs of $v_{SH} - c - Z$, v_{SL} and

v_B. The social welfare created per period is

$$W = M(S,B)[v_B + \lambda(v_{SH} - \lambda/2 - Z) + (1-\lambda)v_{SL}], \tag{3.8}$$

the number of matches times the expected payoffs of buyers and high- and low-quality sellers when they enter the market minus the average cost to produce a high-quality good of $\lambda/2$ and certification costs Z.

3.3 Market Equilibrium with and without Certification

3.3.1 Equilibrium without Certification

If there is no opportunity to acquire certification or it is too costly to get a certificate, there is no equilibrium with $\lambda > 0$: Without certification, buyers' quality expectation would not be influenced by the sellers' individual quality decisions. Therefore no seller would have an incentive to invest. The only equilibrium is one with sellers supplying low quality. As buyers anticipate $\lambda = 0$, there is no incentive for sellers to provide high quality at some cost as long they can't be distinguished from low-quality sellers. Solving equations 3.5 and 3.7 for $\lambda = 0$ leads to the following equilibrium without certification.

Proposition 3.1 *If certification is not available or too costly, i.e.* $Z > \bar{Z}$ *with*

$$\bar{Z} \equiv \frac{\alpha \Delta q}{2(1-\delta) + \alpha\delta}, \tag{3.9}$$

there is an unique PBE in which all sellers provide low quality ($\lambda^* = 0$)*and all sellers enter the matching market.*

In equilibrium agents payoffs are given by

$$v_{SL} = \frac{\alpha q_L}{2(1-\delta) + (\alpha+\beta)\delta} \tag{3.10}$$

and

$$v_B = \frac{\beta q_L}{2(1-\delta) + (\alpha+\beta)\delta}. \tag{3.11}$$

Their payoffs increase in agents' own search probabilities and in δ and decrease in the search probabilities of the other side.

3.3.2 Equilibrium with Certification

If certification is available and its costs smaller than $\bar{Z}$, the sellers with lowest costs c to produce high quality will invest. In order to profit from the provision of high quality these sellers will apply for certification at cost Z. As the certification allows buyers to differentiate between high and low quality sellers, price proposals and payoffs for both types will be different. Low quality sellers will only stay in the market if they can expect nonnegative payoffs. This is only the case if the quality difference is small relative to the absolute value q_L of the low-quality good, or if the market is not too competitive, i.e. for a sufficient small value of δ.

Proposition 3.2 *If $Z \leq \bar{Z}$, then there exists a $\delta^c(\alpha, \beta, q_L, q_H, Z) \in [0, 1]$ such that for*

$$\delta \leq \delta^c(\alpha, \beta, q_L, q_H, Z) \tag{3.12}$$

there is an unique PBE with a fraction

$$\lambda^* = \frac{\alpha \Delta q}{2(1-\delta) + \alpha\delta} - Z \tag{3.13}$$

of high quality sellers and a fraction $1 - \lambda^*$ of low quality sellers, all participating in the market.
A sufficient condition for the existence of the equilibrium is given by

$$\frac{\Delta q(\Delta q - Z)}{q_L} \leq \frac{\alpha}{\beta}. \tag{3.14}$$

The agents' payoffs in the matching market are given by

$$v_{SH}^* = \frac{\alpha\{[2(1-\delta) + \alpha\delta]q_H + \beta\delta(1-\lambda^*)\Delta q\}}{[2(1-\delta) + \alpha\delta][2(1-\delta) + (\alpha+\beta)\delta]}, \tag{3.15}$$

$$v_{SL}^* = \frac{\alpha\{[2(1-\delta) + \alpha\delta]q_L - \beta\delta\lambda^*\Delta q\}}{[2(1-\delta) + \alpha\delta][2(1-\delta) + (\alpha+\beta)\delta]}, \tag{3.16}$$

$$v_B^* = \frac{\beta[\lambda^* q_H + (1-\lambda^*)q_L]}{2(1-\delta) + (\alpha+\beta)\delta} \tag{3.17}$$

High quality sellers with cost parameter c make a net profit of $V_{SH}(c) = v_{SH}^* - c - Z$. Again, agents' payoffs are increasing in δ and own search probabilities, while decreasing in the search probabilities of the opposite side. Furthermore, the sellers' payoffs decrease in λ as buyers have a higher probability to be matched to a high-quality seller, which enforces buyers' bargaining power. Consequently, for buyers the payoff is increasing in λ.

We do not characterize the equilibrium for $\delta > \delta^c$, when buyers do not trade with low-quality sellers, but continue their search in the market until they meet a high-quality seller. This equilibrium would depend much on the matching technology. In general some low-quality sellers will then stay out of the market. If matching probabilities are not changing if less sellers enter, then only high-quality sellers will enter the market and a higher proportion of sellers decides to invest. If search conditions for sellers are ameliorating when some sellers stay out of the market, there will be a mixed equilibrium with more sellers investing than in the full-participation equilibrium and some low-quality sellers entering the market until their expected profit of market participation is zero. For the examination of licensing and minimum prices we will assume the market initially to be in the full-participation equilibrium.

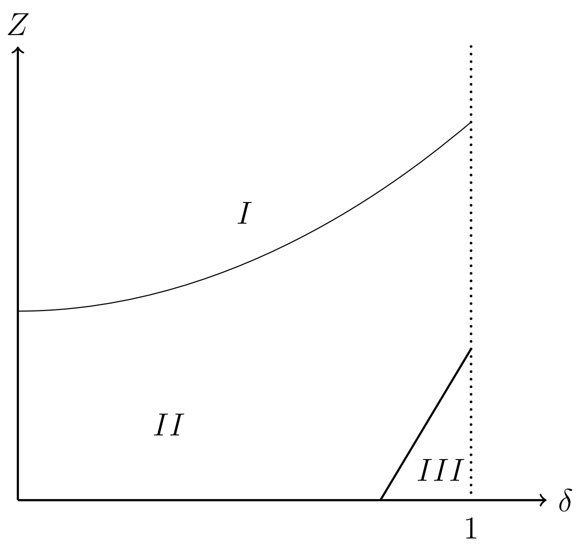

Figure 3.1: (I) Equilibrium without certification, (II) Certification equilibrium with full participation, (III) Certification Equilibrium with partial participation

The possibility of certification induces a fraction of sellers to invest in quality, and thereby increases welfare. But, market equilibria, either with all sellers entering the market or only with high- or low-quality sellers entering, suffer from underprovision of quality due to a holdup problem: the social product added by investing sellers is distributed between sellers and buyers in the bargaining process. Consequently, sellers invest only if their marginal cost for high quality does not exceed their marginal fraction of additionally created surplus. However, when the market becomes frictionless, i.e. δ

goes to one, quality decisions become socially efficient, i.e. all investments that increase the welfare measure W defined in equation 3.8 are made.

Proposition 3.3 *For $\delta \to 1$, the quality decisions of sellers become socially efficient.*

In this situation, sellers can appropriate the whole surplus generated by their quality investments.

3.4 Licensing

Licensing describes a policy by which sellers are allowed to enter the search market only if they receive a license. The license can be given either randomly to sellers or conditioned on quality. Depending on relative numbers of buyers and sellers and on the efficiency of the matching technology $M(S, B)$, excluding sellers from market activity might improve market efficiency, even if all sellers have an incentive to participate in the market. There is a trade-off between higher incentives for sellers to invest in quality when α increases due to a smaller number of sellers on one hand and a decreasing total number of matches, which lowers welfare, on the other.

In the following we limit ourselves to random licensing. It is known from Leland (1979) that excluding low quality types from the market might increase welfare in an asymmetric information environment if more consumers end up with high qualities. We want to stress the potential positive effect of limiting the number of sellers as a way to mitigate the holdup problem. It can be shown to be relevant even for random licensing in a full information environment, i.e. for $Z = 0$. If there is asymmetric information and it is possible to exclude primarily low-qualities, this would increase the positive welfare effect further. In the following propositions we use the efficient matching technology.

The first proposition states that in case there are more sellers than buyers in the market, limiting the number of sellers increases welfare.

Proposition 3.4 *For random licensing and efficient matching, if sellers are on the long side of the market, i.e. $S > B$, it is welfare maximizing to limit the number of sellers to $S = B$.*

Decreasing the number of sellers enforces the bargaining position of the remaining sellers and thereby increases their incentives to invest. For the efficient matching technology investments are optimal if the number of sellers becomes less or equal the number of buyers. But, decreasing the number of sellers S below the number of buyers B would lead to a smaller number of matches, thus $S = B$ would be the best policy.

Moreover, if the incentive effect for sellers to switch to high quality supply is great compared to the disadvantageous shift in the rent distribution for buyers and δ is sufficiently small, $S = B$ also maximizes consumer rent $M(S, B)v_B$.

Proposition 3.5 *For random licensing and efficient matching, if $S > B$, and if*

$$\frac{\Delta q^2}{q_L} \cdot \frac{4 - (6 - \delta)\delta}{(2 - \delta)^2 \delta} \geq 1, \tag{3.18}$$

limiting the number of sellers to $S = B$ maximizes consumer rent.

To be on the short side of the market means for buyers to be in a favorable bargaining position, which is weakened when the number of sellers decreases. So, for consumer rent there is a trade-off between increased investment incentives for sellers, leading to a higher average quality in the market and weakened bargaining power for buyers, leading to higher prices.
For less efficient matching technologies, a number $S > B$ will be socially optimal. The optimal number is determined by equalizing the positive marginal effect of an increased number of matches and the negative effect of decreased incentives for sellers to invest.

3.5 Minimum Prices

In this section we will examine the effect of the introduction of a minimum price p^m, which is directly effective only for the low-quality good, on the bargained price for the high-quality good. In order to highlight this *knock-on* effect, we will first take the quality decision by sellers as given and denote the fraction of high-quality sellers as $\hat{\lambda}$. In a second step it will be shown that the minimum price disincentives the marginal investing sellers to invest in quality, so that in equilibrium the fraction of high-quality goods in the market decreases.

The presence of the minimum price worsens the outside option of a buyer bargaining with a high-quality seller, which leads to an increased price also for the high quality good. This knock-on effect on the price for the high-quality good is smaller than the initial increase in the price for the low-quality good. For this reason, the difference of market payoffs between high- and low-quality sellers decreases, so that incentives to invest in quality decrease. In order to study the effect, we will consider only parameter constellations in which all sellers participate in the bargaining equilibrium of section 3 and neglect certification costs.

The price proposed by a buyer to a low-quality seller in the full-participation equilibrium of section 3 is $p^*_{BL} = \delta v^*_{SL}$. We now look at an equilibrium in which the price for the low-quality good is at least p^m, which is set exogenously, while the price for the high-quality good is bargained as before. The outside option of the buyer is then decreased by the possibility to meet a low-type seller in the future, who will get at least the minimum price, instead of p^*_{BL}.
We call the minimum price effective if it is above p^*_{BL}. If the minimum price is set between p^*_{BL} and p^m_0, all agents stay in the market. For prices above $p^m_0 = q_L - \delta v_B$, buyers will not accept to trade with low-quality sellers. In the following proposition we describe the effect of an increase of the minimum price on the price $\hat{p}_H$ for the high-quality good.

Proposition 3.6 *An increase Δp^m of the minimum price in the interval $p^m \in (p^*_{BL}, p^m_0)$ leads to an increase of the average bargained price $\hat{p}_H = (p_{SH} + p_{BH})/2$ given by*

$$\frac{\partial \hat{p}_H}{\partial p^m}\Delta p^m = \frac{\beta(1-\hat{\lambda})\delta(1-\delta+\alpha\delta)}{[2(1-\delta)+\alpha\delta][1-\delta+\beta\delta]-\alpha\beta\delta^2\hat{\lambda}}\Delta p^m. \tag{3.19}$$

The knock-on factor $\partial \hat{p}_H/\partial p^m$ is smaller than 1 and increasing in α, β and δ and decreasing in $\hat{\lambda}$.

The increase of the price for the high-quality good depends positively on the search probabilities and on δ. Furthermore, the knock-on effect is stronger if the fraction of high-quality sellers λ is small, i.e. more sellers are directly affected by the minimum price. The fact that the price for the high-quality increases less than the minimum price means that the difference in payoffs for high- and low-quality sellers in the market is decreasing. This leads to the last proposition.

Proposition 3.7 *If a fraction $\lambda^* \in (0,1)$ supplies high-quality goods at the minimum price $p^m \in (p^*_{BL}, p^m_0)$ effective for the low-quality good, an increase of the minimum price leads to a matching market equilibrium with fewer sellers investing in high quality, i.e. $d\lambda^*/dp^m < 0$.*

An increase of the minimum price shifts bargaining power and thus rents from buyers to sellers. Because low quality sellers profit more from the minimum price, investment incentives are decreased. Buyers are harmed by both, less high-quality goods in the market and a weaker bargaining position. The number of matches is the same with and without minimum price, while average quality is lower. The minimum price clearly lowers welfare.

3.6 Conclusion

Certification, licensing and price floors are among the most common policy instruments for market regulation. This paper highlights the impact of these regulations on suppliers' investment decisions in search markets. Our analysis focuses on small quality differences, so that sellers in the certification equilibrium where not excluded from market participation. We also assumed, besides the dynamic quality decision of suppliers, a static demand and supply. In the longer run these might change in reaction to the policies applied.

We showed that the need for certification with asymmetric information leads to a higher degree of underinvestment than it would be the case for symmetric information. Sellers have to bear the full cost of certification, but can not increase their share of surplus at the bargaining stage more than in a symmetric situation.

In order to reduce the adverse effect of investment holdup, limiting the number of sellers was shown to increase market welfare, and in some cases also consumer rent if the number of sellers is high compared to the number of buyers. The model considers only sellers' investments. If buyers could also invest, reducing the number of sellers would have a negative effect on their investments, the aggregated effect on both sides of the market could go in both directions. This means that our result is more meaningful for the quality of goods and services, e.g. craftsmanship, but also for labor markets in which firm's investments in productivity play a minor role. On one hand, the positive effect of licensing on quality might lead to a more favorable assessment of institutions like professional guilds that increase sellers' bargaining power through licensing policies. On the other hand, if new technologies decrease search frictions, formerly useful licensing policies might become less so.

We do not answer the question which policy, certification or licensing, is better suited to encounter informational asymmetries in markets. It will depend on the specifics of the market. In general, if search frictions are non-negligible and quality investments barely profitable, while buyers would highly profit from them, licensing might be more beneficial than certification despite the disadvantage of excluding sellers, whose low-quality goods might still create additional welfare.

Effective minimum prices have a negative effect on investment incentives in our model. But again, this conclusion is valid only for markets with negligible buyer investments. For buyers' investments the opposite is true, an effective minimum price would incentivize higher investments. We do not consider minimum prices to keep low-quality sellers out of the market, which would happen if the market becomes too competitive for low-quality sellers to engage in profitable trades. Due to the importance of unemployment in labor markets, the application of our results to the effects of minimum wages on human capital investments should be done with caution. The positive knock-on effect on the price distribution is in accordance with prior theoretical and empirical research.

3.7 Appendix

Proof of Proposition 1

Without certification sellers can not credibly offer a high quality good. Buyers will only accept to pay a price based on average quality, which does not depend on the single seller's quality decision. This means that no seller has an incentive to invest and buyers anticipate that there will be no high-quality good in the market, the fraction of high-quality will be $\lambda^* = 0$. The equilibrium is determined by the simultaneous solution to equations 3.5 and 3.7. Equilibrium payoffs are given in equations 3.10 and 3.11. By inequation 3.9 no seller has an incentive to deviate to investing and acquiring a certificate as the difference in payoffs is $v_{SH} - Z - v_{SL} = \frac{\alpha(q_H - q_L)}{2(1-\delta)+\alpha\delta} - Z < 0$ even for the seller with the smallest investment costs. Q.E.D.

Proof of Proposition 2

Solving equations 3.5 and 3.7 and taking into account the sellers' quality decisions determining the fraction of high-quality goods $\lambda = v_{SH} - Z - v_{SL}$, one gets agents' payoffs in equations 3.15 to 3.17 and expression 3.13 in the proposition. Low-quality sellers will only enter the market if they make a nonnegative payoff $v^*_{SL} \geq 0$. This is the case for $\delta = 1$ and consequently for all $\delta \in [0,1]$ if 3.14 is fulfilled. If 3.14 is not fulfilled, it is true for $\delta \leq \delta^c(\alpha, \beta, q_L, q_H, Z) \in [0,1]$ with

$$\delta^c = \frac{\beta\Delta q(\alpha\Delta q - 2Z) + 4q_L(2-\alpha) - \Delta q\sqrt{\beta^2(\alpha\Delta q - 2Z)^2 + 8q_L\alpha(2-\alpha)}}{2(2-\alpha)[(2-\alpha)q_L - \beta\Delta q Z]} \tag{3.20}$$

Q.E.D.

Proof of Proposition 3

For $\delta \to 1$ it follows that $\lambda^* \to \Delta q - Z$. For the welfare function W as defined in equation 3.8, the optimal λ is given by

$$\lambda = \frac{(\alpha+\beta)\Delta q}{2(1-\delta) + (\alpha+\beta)\delta} - Z. \tag{3.21}$$

In the limit, for $\delta \to 1$, the sellers' decisions $\lambda^* = \Delta q - Z$ are equal to the welfare optimizing λ. Q.E.D.

Proof of Proposition 4

For the case of random licensing and efficient matching, on one hand, if $S > B$, as α is non-increasing in S and $M(S,B) = B$ for all S, limiting the number of sellers to $S = B$ will increase market efficiency. On the other hand, if $S < B$, α is equal to one for all S and the number of matches is increasing in S as $M(S,B) = S$. This means that, if $S > B$, limiting the number of sellers to $S = B$ through licensing will maximize welfare. Q.E.D.

Proof of Proposition 5

For efficient matching consumer rent is given by $M(S,B)v_B^*(\lambda^*)$,

$$M(S,B)v_B^*(\lambda^*) = M(S,B)\frac{\beta\left\{q_L + \frac{\alpha\Delta q}{2(1-\delta)+\alpha\delta}\Delta q\right\}}{2(1-\delta)+(\alpha+\beta)\delta}. \tag{3.22}$$

For efficient matching and $S > B$, we have M(S,B)=B, $\alpha = B/S$ and $\beta = 1$. Differentiating with respect to S leads to the sufficient condition 3.18 guaranteeing consumer rent to increase as S decreases. Decreasing S below B would lead to less matches and a worse bargaining position for buyers and at the same time not changing quality incentives. So, $S = B$ maximizes consumer rent. Q.E.D.

Proof of Proposition 6

With an effective minimum price at work, buyers' proposals change from $p_{BL} = \delta v_{SL}$ to p^m. Sellers' proposals are made as before. From solving equations 3.5 and 3.7 using $p_{BL} = p^m$, the equilibrium expected price $\hat{p}_H$ for the high-quality good is determined as

$$\hat{p_H} = \frac{[1-\delta+\alpha\delta][2(1-\delta)q_H + \beta\delta(1-\hat{\lambda})(\Delta q + p^m)]}{[2(1-\delta)+\alpha\delta][1-\delta+\beta\delta] - \alpha\beta\delta^2\hat{\lambda}}. \tag{3.23}$$

Taking the first derivative with respect to p^m leads to the proposition. The equilibrium with all matches leading to trade only exists if buyers have no incentive to refuse trade due to a price which exceeds their outside option of continued search, i.e. it has to be true that $q_L - p^m \geq \delta v_B$. This condition is fulfilled with equality for

$$p_0^m = \frac{(2-\beta)[2(1-\delta)+\alpha\delta] + 2\alpha\beta\delta\hat{\lambda}q_L - 2\beta\hat{\lambda}\Delta q}{(2-\beta)[2(1-\delta)+\alpha\delta] + \beta\hat{\lambda}(2+\alpha\delta)}. \tag{3.24}$$

The marginal effects of α, β, δ and $\hat{\lambda}$ on the knock-on factor $m \equiv \partial\hat{p}_H/\partial p^m$ are given by the first partial derivatives with respect to these parameters:

$$
\begin{aligned}
\frac{\partial m}{\partial \alpha} &= \frac{\beta(1-\delta)\delta^2(1-\hat{\lambda})[1-(1-\beta(1+\hat{\lambda})\delta)]}{\{[2-(2-\alpha)\delta][1-(1-\beta)\delta]-\alpha\beta\hat{\lambda}\delta^2\}^2} > 0 \qquad (3.25)\\
\frac{\partial m}{\partial \beta} &= \frac{(1-\delta)\delta(1-\hat{\lambda})[1-(1-\alpha)\delta)][2-(2-\alpha)\delta]}{\{[2-(2-\alpha)\delta][1-(1-\beta)\delta]-\alpha\beta\hat{\lambda}\delta^2\}^2} > 0\\
\frac{\partial m}{\partial \delta} &= \frac{2(1-\delta)^2+4\alpha(1-\delta)+\alpha[\alpha+\beta(1+\hat{\lambda})]}{\{[2-(2-\alpha)\delta][1-(1-\beta)\delta]-\alpha\beta\hat{\lambda}\delta^2\}^2} > 0\\
\frac{\partial m}{\partial \alpha} &= -\frac{\beta(1-\delta)\delta[1-(1-\alpha)\delta][2-(2-\alpha-2\beta)\delta]}{\{[2-(2-\alpha)\delta][1-(1-\beta)\hat{\lambda}]-\alpha\beta\hat{\lambda}\delta^2\}^2} < 0.
\end{aligned}
$$

Q.E.D.

Proof of Proposition 7

The fraction λ of high-quality sellers is determined by adding the equilibrium condition $\lambda = \Delta v(\lambda, p^m)$ to the equation system solved in proposition 6. As

$$
\frac{\partial \Delta v(\lambda, p^m)}{\partial p^m} = -\frac{\alpha(1-\delta)[2(1-\delta)+(\alpha+\beta)\delta]}{[1-\delta+\alpha\delta]\{[2(1-\delta)+\alpha\delta][2(1-\delta)+\beta\delta]-\alpha\beta\lambda\}} < 0 \qquad (3.26)
$$

for all λ and p^m, from $\lambda^* = \Delta v(\lambda^*, p^m)$ it follows that $d\lambda^*/dp^m < 0$. Q.E.D.

Chapter 4

Information Acquisition and the Demand for Hard Evidence

4.1 Introduction

Rational decision making in a world of imperfect information requires an assessment of expected utilities associated with feasible choices. In the context of investment decisions, the challenge is to find projects promising superior returns. From an investor's perspective there are on the one hand assets that are relatively safe, like sovereign bonds, which do not require much expertise. On the other hand, there are riskier projects, for which an investor might find it profitable to acquire information before she decides to get involved. Either because of time constraints or because of inferior expertise, the investor might delegate the information gathering to an (expert-) agent.

In our analysis we study optimal wage contracts to induce an agent to acquire information about the profitability of a risky project and to report his findings truthfully. In the first part of the paper, we examine whether the investor, who is in the following referred to as the principal, can induce effort less costly by restricting the set of messages which can be reported by the agent. This is motivated by a result of Szalay (2005). We find that in our context, with an agent inherently indifferent about the principal's choice, it is never more profitable for the principal to use this strategy. Second, we show that if the agent can collect (socially wasteful) hard evidence to support his report, it should be demanded by the principal if it is not too costly to acquire and if the common prior of the principal and agent with which success or failure happens is relatively informative.

The demand for hard evidence could be interpreted as a form of costly state-verification. Without hard evidence, the principal has to incentivize the agent to exert effort and to tell the truth. In our model, these incentives are induced by a wage contract which is conditioned on the outcome of the project. But, because the outcome is only informative if the project is chosen by the principal, she has to leave an information rent to the agent. Alternatively, the principal can ask the agent to make his report verifiable by delivering hard evidence, which causes costs payed by the agent, but ultimately borne by the principal. Which way to induce effort and truth telling is optimal depends on the costs for effort and evidence and the frequency of necessary demands for evidence.

Much of the existing literature is concerned with a principal delegating formal authority to choose between alternative projects to an agent with superior information. We concentrate our analysis on the communication between agent and principal. Because in our model the choice of project by the principal is determined by the agent's report, delegation of the project's choice would not change the results.

Related Literature

Early literature examining the moral hazard problem in incentive contracts was provided amongst others by Harris and Raviv (1979), Hölmstrom (1979) and Grossman and Hart (1983). In these papers, the principal can neither observe the state of nature nor an agent's productive input, but some (output-) variable influenced by both. Assumed that the output distribution for increased effort first-order dominates the distribution for lower effort levels, second-best solutions can be reached through suitable wage contracts that balance incentives and risk-sharing.

As Lambert (1986) points out, in the case of an agent exerting effort to acquire new information about alternative projects that might be undertaken, this first-order dominance relation is not valid anymore. In his paper, he examines the delegation of a choice between a risky and a risk-free project to a risk-averse port-folio manager. If the manager can not communicate his private information to the principal, in some cases, in trading-off wage and risk, he will decide against the risky project in spite of its profitability for the investor. Only if communication is possible and verifiable, a contract can be designed to align the agent's decision with the principal's interest. Demski and Sappington (1987) and Malcomson (2009) study a similar delegation problem in a more general setting with an arbitrary number and a continuum of possible returns, respectively. Different from these papers, we assume communication between principal and agent to be possible, but only verifiable by demanding costly evidence.

Kihlstrom (1988) extends the analysis of Lambert (1986) by an adverse selection problem caused by a heterogeneity of expert's types. By assuming the expert's costs to acquire information to be private, Osband (1989) and Köhler (2004) go in a similar direction. In Osband (1989), besides the moral hazard problem, socially inefficient results are generated by adverse selection as the principal faces a trade-off between signal precision and the payment of an information rent to experts with lower marginal effort costs. Köhler (2004) finds that if effort costs are unknown to the principal, than different from Lambert (1986) and our model, it is generally not optimal to reward the agent only when his recommendation is confirmed.

Dybvig, Farnsworth, and Carpenter (2010) is an attempt to examine the expert problem adapted to port-folio theory. They find that optimal contracting in port-folio management encompasses incentive payments relative to benchmark port-folios and a restriction of investment choices.

Another strand of literature related to our paper is concerned with cheap-talk-models in the spirit of Crawford and Sobel (1982). If ex-ante there is neither conflict nor common interest, as in our model, informative cheap-talk equilibria do not exist (see Sobel (2010)). Aghion and Tirole (1997) and Dessein (2002) ask when the principal prefers the delegation of formal authority over communication if principal and agent differ in their preferences over available projects. Aghion and Tirole (1997) specifically take into account the moral hazard problem to incentivize agent's effort.

Some recent papers examine the moral-hazard topic of information acquisition in institutional design: Dewatripont and Tirole (1999) argue that in situations when the principal delegates impartial decision making, e.g. in the judicial system, efforts in information acquisition can be increased through delegation to advocates defending opposite cases. Li (2001) demonstrates how a commitment to conservatism, e.g. in the approval of new drugs, mitigates a free-rider problem of information acquisition in group decisions. Szalay (2005) finds that restricting the action space of an agent to extreme actions might improve his incentives for information acquisition if he suffers from wrong decisions himself.

Using a similar setup like ours, Gromb and Martimort (2007) show that if collusion can be avoided, information acquisition through the delegation to two agents implies lower agency costs than the delegation only to one agent. We examine in which circumstances costly verification of an agent's report should be demanded by the principal. This relates to literature concerned with contracting and state-verification, especially Townsend (1979) and Gale and Hellwig (1985). Townsend (1979) looks in a rather general setting on contracts specifying reports of an agent leading either to unverified approval or costly state verification by the principal. Gale and Hellwig (1985) show that under specific assumptions the standard debt contract with state-verification only in case of bankruptcy is the optimal contract for outside financing of risky projects with asymmetric information. Different from these models, in our setup the true state of the unknown project is observed at the last stage and can be contracted on if it is chosen by the principal. This allows us to write an incentive compatible contract without state-verification at the cost of an information rent left to the agent. Therefore the principal's decision to include state-verification in the contract becomes endogenous.

In section 2 we introduce the model. In section 3 we compare contracts requiring full message sets with contracts requiring only partial message sets. In section 4 we characterize and compare contracts with full and partial evidence obligations, respectively. Using the results from sections 3 and 4, in section 5 we finally endogenize the principal's decision when to include a hard evidence obligation in the offered contract. Section 6 concludes the paper.

4.2 The Model

We consider a principal (she) and agent (he) model. Principal and agent are both risk-neutral, but the agent is protected by limited liability. The principal can choose between two alternative investment projects, project I and some (outside) project I'. For project I exists a good state of nature G and a bad one B. The state of nature is not observable, but the a priori probability α of the good state is common knowledge.

Project I succeeds with probability 1 in state G and generates the net profit R_S and fails with probability 1 in state B and generates the net profit R_F. The success and failure likelihood of project I' is independent of the state of nature. It succeeds with probability $\gamma \in (0,1)$ and yields the profit R'_S and it fails with probability $1-\gamma$ and its profit is R'_F. The expected profit of I' is $R' = \gamma R'_S + (1-\gamma)R'_F$. We just consider the not trivial case such that $R_S > R' > R_F$. We define $\bar{\alpha} \equiv (R' - R_F)/(R_S - R_F)$. Without any further information acquisition the principal chooses project I if $\alpha R_S + (1-\alpha)R_F \geq R'$ or in short $\alpha \geq \bar{\alpha}$. Although the success or failure of the project is observable at the last stage and hence contractible, the principal's choice of project is not directly observed by the agent and can not be contracted on.

The principal decides to hire an agent if this promises her an additional expected profit greater than the expected wage she has to pay. If the agent exerts effort $e \in [0,1]$ at cost k he observes the true state with probability e. With probability $1-e$ he observes nothing. If he does not exert effort, he has no cost and does not observe the true state with probability 1. The agent's effort choice and his observation are his private information. After his effort decision, the agent reports his observation by choosing a message $m \in \{N, B, G\}$, indicating either that he is uninformed (N) or that he observed the bad (B) or good (G) state. Contingent on the report m, the principal pays the agent a wage w_{mS} in the event of success and w_{mF} in the event of failure. By limited liability wages have to be non–negative.

The sequence of events is the following: After a contract has been signed, the agent exerts effort e or 0. The agent observes either G, B or N and reports some $m \in \{N, B, G\}$. The principal uses the agent's report to update her belief about the state of nature and then chooses between I and I'. The project outcome is realized and the principal pays the agent.

In the second part of the paper, we extend the model by the possibility for the agent to collect hard evidence for his observation at cost c. The principal can include the obligation to collect evidence for the agent contingent on his message into the contract.

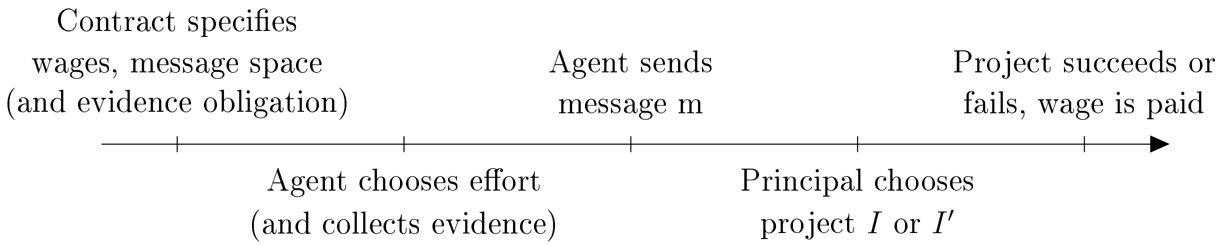

Figure 4.1: The Sequence of Events

4.3 Contracts without Evidence Acquisition

4.3.1 Full Message Set

We start with a characterization of optimal wage contracts that induce the agent to exert effort and to report his information truthfully by sending one of the three massages $m \in \{N, B, G\}$. On one hand, wages for informative reports, G or B, need to be set high enough to give the agent an incentive to exert the effort to acquire information. On the other hand, as the agent who exerted effort might not have found out the true state, the expected wage for an uninformative report N need to be set high enough to avoid that the uninformed agent choses to report G or B. Due to limited liability, it can not be reached by punishing wrong reports with negative wages.

Depending on the principal's decision when the state of nature stays uncertain after the agent's report, we have to consider the two cases in which she chooses either I or I'.

Case $\alpha R_S + (1-\alpha)R_F \geq R'$

Given that the agent is induced to exert effort and to report truthfully, it will be shown that the principal chooses project I, whenever the agent reports either G or N. If the agent reports B, she chooses always I'.

Given a contract $\{w_{GS}, w_{GF}, w_{NS}, w_{NF}, w_{BS}, w_{BF}\}$, the agent's expected payoff if he exerts effort, reports truthfully and expects the principal to choose I if he reports N is

$$\Pi_A^E = e[\alpha w_{GS} + (1-\alpha)(\gamma w_{BS} + (1-\gamma)w_{BF})] + (1-e)[\alpha w_{NS} + (1-\alpha)w_{NF}] - k. \quad (4.1)$$

The first part of his payoff function is realized if the agent becomes informed with probability e and reports either G or B, the second part is realized if the agent stays uninformed with probability $1-e$ and reports N. The parameter k is the effort cost to acquire information.

If the agent exerts no effort and reports N truthfully, his payoff is

$$\Pi_A^N = \alpha w_{NS} + (1-\alpha) w_{NF}. \tag{4.2}$$

Consequently, the agent exerts effort iff

$$\Pi_A^E - \Pi_A^N = e[\alpha(w_{GS} - w_{NS}) + (1-\alpha)(\gamma w_{BS} + (1-\gamma) w_{BF} - w_{NF})] - k \geq 0. \tag{4.3}$$

The principal's payoff if she induces the agent to exert effort and if she chooses I if the agent reports N is

$$\begin{aligned} \Pi_P^E &= e[\alpha(R_S - w_{GS}) + (1-\alpha)(R' - \gamma w_{BS} - (1-\gamma) w_{BF})] \\ &\quad +(1-e)[\alpha(R_S - w_{NS}) + (1-\alpha)(R_F - w_{NF})] \end{aligned} \tag{4.4}$$

The first part of the principal's payoff is realized if the agent is informed and reports either G or B, the second part is realized if the agent reports N and the principal chooses project I.

In order to induce the agent to report truthfully, besides condition (4.3), the following incentive constraints have to be fulfilled:

$$\begin{aligned} &(IC_G) \qquad w_{GS} \geq \max[w_{NS}, \gamma w_{BS} + (1-\gamma) w_{BF}, 0] \qquad (4.5) \\ &(IC_N) \qquad \alpha w_{NS} + (1-\alpha) w_{NF} \geq \max[\alpha w_{GS} + (1-\alpha) w_{GF}, \gamma w_{BS} + (1-\gamma) w_{BF}, 0] \\ &(IC_B) \qquad \gamma w_{BS} + (1-\gamma) w_{BF} \geq \max[w_{GF}, w_{NF}, 0]. \end{aligned}$$

The expected wage when the agent reports truthfully has to be at least the wage he would get if he reported one of the two alternatives, and has to be at least 0 due to limited liability.

Lemma 4.1 *For $\alpha \geq \bar{\alpha}$ an optimal wage contract $\{w_{GS}^*, w_{GF}^*, w_{NS}^*, w_{NF}^*, w_{BS}^*, w_{BF}^*\}$ which induces the agent to exert effort and to report truthfully is given by*

$$\begin{aligned} \gamma w_{BS}^* + (1-\gamma) w_{BF}^* &= \frac{k}{e(1-\alpha)} \\ w_{GS}^* &= \frac{k}{e\alpha(1-\alpha)} \\ w_{GF}^* &= 0 \\ w_{NS}^* = w_{NF}^* &= \frac{k}{e(1-\alpha)}. \end{aligned} \tag{4.6}$$

Under an optimal wage contract the principal always chooses project I when the agent reports N.

Proofs are given in the Appendix.
As both, the principal and the agent, are risk-neutral, the optimal contract is independent of the specific distribution between wages w_{BS} and w_{BF}. Only the optimal expected wage after a report B is specified.

The principal's expected payoff if she hires the agent and sets an optimal wage contract is

$$\Pi_P^{E*} = \alpha R_S + (1-\alpha) R_F + (1-\alpha) e (R' - R_F) - k - \frac{k}{e(1-\alpha)} \tag{4.7}$$

The first two terms reflect the payoff the principal would get if she would not hire the agent, but always invest in project I. The third term describes additional profits from choosing I' in case that the agent identifies a bad project I and the principal earns R' instead of R_F. The last two terms are the costs of information acquisition k and the information rent $k/[e(1-\alpha)]$ which is left to the agent.

If the principal abstains from hiring the agent, her payoff is

$$\Pi_P^N = \alpha R_S + (1-\alpha) R_F. \tag{4.8}$$

The principal hires the agent iff

$$\Delta \bar{\Pi}_P = \Pi_P^{E*} - \Pi_P^N \geq 0. \tag{4.9}$$

The maximum expected additional profit $\Delta \bar{\Pi}_P$ generated by the agent's information is reached for $\alpha = \bar{\alpha}$. At this point, the agent's employment has the maximum worth to the principal and the agent is hired for effort costs up to

$$k^{max} \equiv \frac{e^2 (R' - R_F)(R_S - R')^2}{(R_S - R_F)[R_S - R_F + e(R_S - R')]}. \tag{4.10}$$

For $\alpha > \bar{\alpha}$, the agent is hired iff $\alpha \leq \alpha^{max}$ with

$$\alpha^{max} \equiv 1 - \frac{\sqrt{k[k + 4(R' - R_F)]} + k}{2e(R' - R_F)}. \tag{4.11}$$

For values of α near to one, bad projects I are very unlikely, while the information rent becomes very high. Consequentially, the agent is only hired if α is not above the threshold defined by expression (4.11).

We now turn to the alternative case, for which project's I expected profit is less than for project I', assessed by the prior success probability α.

Case $\alpha R_S + (1 - \alpha)R_F < R'$

Given that the agent is induced to exert effort and to report truthfully, it will be shown that the principal chooses project I, whenever the agent reports G. If the agent reports B or N, she chooses I'.
The agent's payoff if he exerts effort, reports truthfully and expects the agent to choose I' if he reports N is

$$\Pi_A = e[\alpha w_{GS} + (1 - \alpha)(\gamma w_{BS} + (1 - \gamma)w_{BF})] + (1 - e)[\gamma w_{NS} + (1 - \gamma)w_{NF}] - k \tag{4.12}$$

and if he exerts no effort, but reports truthfully,

$$\Pi_A^N = \gamma w_{NS} + (1 - \gamma)w_{NF}. \tag{4.13}$$

The agent exerts effort iff

$$\Pi_A^E - \Pi_A^N \geq 0. \tag{4.14}$$

The principal's payoff if she induces the agent to exert effort is

$$\begin{aligned} \Pi_P^E &= e[\alpha(R_S - w_{GS}) + (1 - \alpha)(R' - \gamma w_{BS} - (1 - \gamma)w_{BF})] \\ &\quad + (1 - e)[R' - \gamma w_{NS} - (1 - \gamma)w_{NF}]. \end{aligned} \tag{4.15}$$

In order to induce the agent to report truthfully, besides condition (4.14), the following incentive constraints have to be fulfilled:

$$
\begin{aligned}
&(IC_G) \qquad w_{GS} \geq \max[\gamma w_{NS} + (1-\gamma)w_{NF}, \gamma w_{BS} + (1-\gamma)w_{BF}, 0] && (4.16)\\
&(IC_N) \qquad \gamma w_{NS} + (1-\gamma)w_{NF} \geq \max[\alpha w_{GS} + (1-\alpha)w_{GF}, \gamma w_{BS} + (1-\gamma)w_{BF}, 0] &&\\
&(IC_B) \qquad \gamma w_{BS} + (1-\gamma)w_{BF} \geq \max[w_{GF}, \gamma w_{NS} + (1-\gamma)w_{NF}, 0]. &&
\end{aligned}
$$

Lemma 4.2 *For $\alpha < \bar{\alpha}$ an optimal wage contract $\{w^*_{GS}, w^*_{GF}, w^*_{NS}, w^*_{NF}, w^*_{BS}, w^*_{BF}\}$ is given by*

$$
\begin{aligned}
w^*_{BS} &= w^*_{BF} = \frac{k}{e(1-\alpha)} && (4.17)\\
w^*_{GS} &= \frac{k}{e\alpha(1-\alpha)}\\
w^*_{GF} &= 0\\
w^*_{NS} = w^*_{NF} &= \frac{k}{e(1-\alpha)}
\end{aligned}
$$

Under an optimal wage contract the principal always chooses project I' when the agent reports N.

Proof: Analog to the proof of Lemma 4.1.

The principal's payoff if she hires the agent and sets an optimal wage contract is

$$\Pi_P^{E*} = R' + \alpha e(R_S - R') - k - \frac{k}{e(1-\alpha)} \tag{4.18}$$

The first term R' is the principal's profit if she would not hire the agent and always choose project I'. The second term describes her additional profit if she hires the agent who might find a good project I which earns her R_S instead of R'. The wage costs are the same as in the previous case.
If the principal abstains from hiring the agent, her payoff is

$$\Pi_P^N = R'. \tag{4.19}$$

The principal hires the agent iff

$$\Delta\bar{\Pi}_P = \Pi_P^{E*} - \Pi_P^N \geq 0 \tag{4.20}$$

The principal hires the agent never if $k > k^{max}$ as defined in equation (4.10) and for

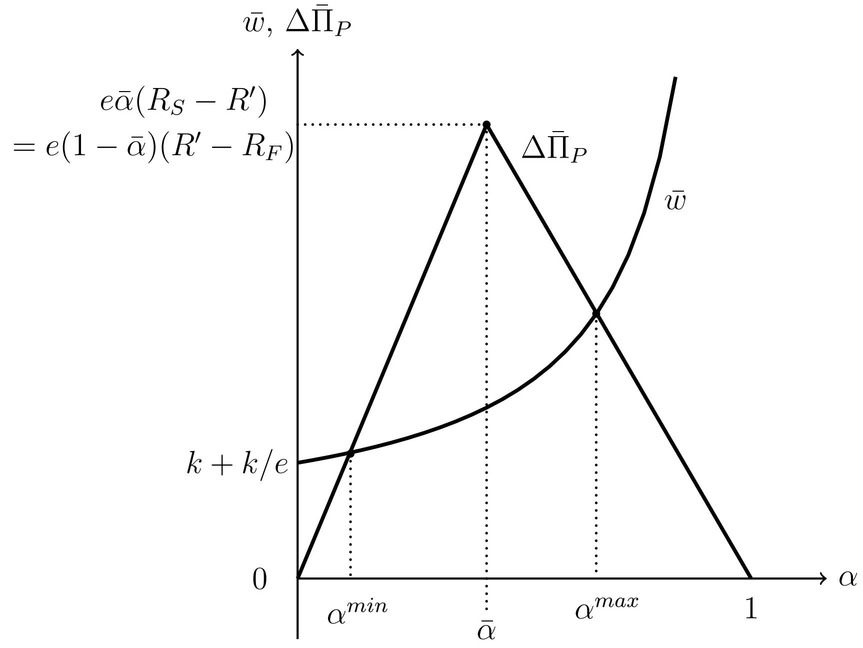

Figure 4.2: Expected additional profit and wage of the optimal contract with the full information set

$k \leq k^{max}$ she hires the agent iff $\alpha \geq \alpha^{min}$ with

$$\alpha^{min} \equiv \frac{1}{2} - \frac{\sqrt{k^2 - 2(2+e)k(R_S - R') + e^2(R_S - R')^2} - k}{2e(R_S - R')}. \tag{4.21}$$

Summarizing the two cases, the principal is made better off by hiring the agent and setting a wage contract specified in Lemma 4.1 or 4.2 compared to an uninformed decision iff $\alpha \in [\alpha^{min}, \alpha^{max}]$. In figure 4.2 we show an example of expected additional profits $\Delta\bar{\Pi}_P$ and wages $\bar{w}$ of an optimal contract. Figure 4.3 shows the range of α-values for which the agent is hired at a given effort cost k. The higher k, the smaller is the interval $[\alpha^{min}, \alpha^{max}]$ in which the principal finds it profitable to hire the agent.

4.3.2 Partial Information Sets

We will now consider partial information sets, for which the contract allows the agent only to send a message $m \in \{G, N\}$ or $m \in \{B, N\}$, respectively.[1]

[1] A $\{G, B\}$ - contract can easily be identified as inferior to the full information set contract: If the agent is uninformed about the project, the full information set contract induces the agent to report N and pays an expected wage set such that the agent earns the same as if he reported G or B. Under the $\{G, B\}$ - contract the agent is forced to guess between G and B, but still earns the same expected

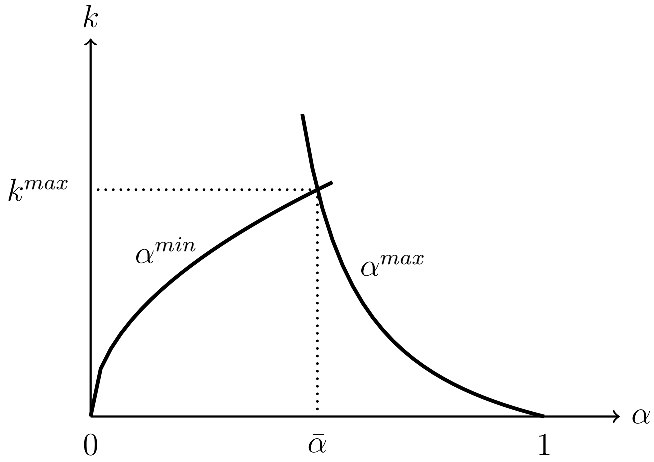

Figure 4.3: For given effort costs k, α^{min} and α^{max} are the minimum and maximum values of α, respectively, for which the principal hires the agent

$\{G, N\}$ - Contracts

A $\{G, N\}$-contract is set up such that the agent reports G when he observes G and reports N when he observes B or nothing. For the principal it is never profitable to offer a contract if she will invest independently whether the report is G or N. After the agent exerted effort and reported N, the principal updates her beliefs about the probability of I to be a success to $\alpha' = [\alpha(1-e)]/[\alpha(1-e)+(1-\alpha)] < \alpha$. It follows that the principal will never hire the agent, if $\alpha' R_S + (1-\alpha')R_F \geq R'$ or $\alpha \geq (R' - R_F)/[Rs - R_F - e(R_S - R')]$.

Case $\alpha' R_S + (1 - \alpha')R_F < R'$

The principal chooses project I if the agent reports G. If the agent reports N, she chooses I'. The agent's payoff if he exerts effort is

$$\Pi_A^E = e[\alpha w_{GS} + (1-\alpha)(\gamma w_{NS} + (1-\gamma)w_{NF})] + (1-e)[\gamma w_{NS} + (1-\gamma)w_{NF}] - k \quad (4.22)$$

and if he exerts no effort

$$\Pi_A^N = \gamma w_{NS} + (1-\gamma)w_{NF}. \quad (4.23)$$

The agent exerts effort iff

wage. At the same time, if the principal finds it optimal to follow the agent's report, she will make inefficient decisions when the agent guesses G and $\alpha < \bar{\alpha}$ or when the agent guesses B and $\alpha \geq \bar{\alpha}$.

$$\Pi_A^E - \Pi_A^N \geq 0. \tag{4.24}$$

The principal's expected net payoff if she induces the agent to exert effort is

$$\begin{aligned}\Pi_P^E &= e[\alpha(R_S - w_{GS}) + (1-\alpha)(R' - \gamma w_{NS} - (1-\gamma)w_{NF})] \\ &\quad +(1-e)(R' - \gamma w_{NS} - (1-\gamma)w_{NF}).\end{aligned} \tag{4.25}$$

In order to induce the agent to inform the principal truthfully about his information, besides condition (4.24), the following incentive constraints have to be fulfilled:

$$\begin{aligned}&(IC_G) \qquad w_{GS} \geq \max[\gamma w_{NS} + (1-\gamma)w_{NF}, 0] \\ &(IC_N) \qquad \gamma w_{NS} + (1-\gamma)w_{NF} \geq \max[\alpha w_{GS} + (1-\alpha)w_{GF}, 0] \\ &(IC_B) \qquad \gamma w_{NS} + (1-\gamma)w_{NF} \geq \max[w_{GF}, 0]\end{aligned} \tag{4.26}$$

Lemma 4.3 *An optimal wage contract* $\{w_{GS}^*, w_{GF}^*, w_{NS}^*, w_{NF}^*\}$ *is given by*

$$\begin{aligned}w_{GS}^* &= \frac{k}{e\alpha(1-\alpha)} \\ w_{GF}^* &= 0 \\ w_{NS}^* = w_{NF}^* &= \frac{k}{e(1-\alpha)}\end{aligned} \tag{4.27}$$

Under an optimal wage contract the principal always chooses project I' when the agent reports N.

Proof: Analog to the proof of Lemma 4.1.

The principal's expected net payoff if she hires the agent and sets an optimal wage contract is

$$\Pi_P^{E*} = R' + \alpha e(R_S - R') - k - \frac{k}{e(1-\alpha)} \tag{4.28}$$

If the principal abstains from hiring the agent, her net payoff is either

$$\Pi_P^N = \alpha R_S + (1-\alpha)R_F \tag{4.29}$$

if $\alpha \geq \bar{\alpha}$ or

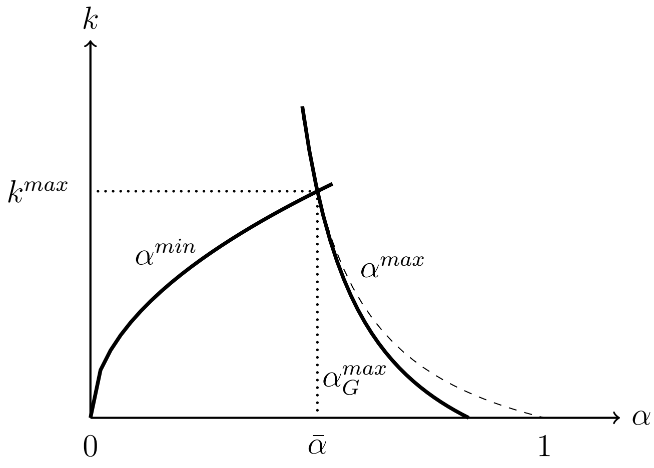

Figure 4.4: For given effort costs k, α^{min} and α_G^{max} are the minimum and maximum values of α, respectively, for which the principal hires the agent

$$\Pi_P^N = R' \tag{4.30}$$

if $\alpha < \bar{\alpha}$. The principal hires the agent iff

$$\Pi_P^{E*} - \Pi_P^N \geq 0 \tag{4.31}$$

The principal hires the agent, for $k \leq k^{max}$ iff $\alpha^{min} \leq \alpha \leq \alpha_G^{max}$ with α^{min} as defined in equation (4.21) and

$$\begin{aligned} \alpha_G^{max} \equiv & \frac{1}{2} - \frac{1}{2e[R_S - R_F - e(R_S - R')]} \{ek + R_F - R' + \\ & + \sqrt{e[e^3(R_S - R')^2 + e(R_S - R' - k)^2 - 4k(R_S - R_F) - 2e^2(R_S - R')(R_S - R' + k)]}\Big\}. \end{aligned} \tag{4.32}$$

Figure 4.4 shows that the interval $[\alpha^{min}, \alpha_G^{max}]$ for which the principal finds it profitable to hire the agent is smaller than for the full message set for all effort costs k.

$\{B, N\}$ - Contracts

A $\{B, N\}$ - contract is set up such that the agent reports B when he observes B and reports N when he observes G or nothing. For the principal it is never profitable to offer a contract if she will never invest independently whether the report is B or N. After the agent exerted effort and reported N, the principal updates her beliefs about the probability of I to be a success to $\alpha' = \alpha/[\alpha + (1-\alpha)(1-e)] > \alpha$.

It follows that the principal will never hire the agent if $\alpha' R_S + (1-\alpha')R_F < R'$ or $\alpha < [(1-e)(R'-R_F)][Rs - R_F - e(R'-R_F)]$.

Case $\alpha' R_S + (1-\alpha')R_F \geq R'$

The principal chooses project I if the agent reports N. If the agent reports B, she chooses I'.

The agent's payoff if he exerts effort is

$$\Pi_A^E = e[\alpha w_{NS} + (1-\alpha)(\gamma w_{BS} + (1-\gamma)w_{BF})] + (1-e)[\alpha w_{NS} + (1-\alpha)w_{NF}] - k \quad (4.33)$$

and if he exerts no effort

$$\Pi_A^N = \alpha w_{NS} + (1-\alpha)w_{NF}. \quad (4.34)$$

The agent exerts effort iff

$$\Pi_A^E - \Pi_A^N \geq 0. \quad (4.35)$$

The principal's payoff if she induces the agent to exert effort is

$$\begin{aligned} \Pi_P^E &= e[\alpha(R_S - w_{NS}) + (1-\alpha)(R' - \gamma w_{BS} - (1-\gamma)w_{BF})] \\ &\quad +(1-e)[\alpha(R_S - w_{NS}) + (1-\alpha)(R_F - w_{NF})] \quad (4.36) \end{aligned}$$

In order to induce the agent to inform the principal truthfully about his information, besides condition (4.35), the following incentive constraints have to be fulfilled:

$$\begin{aligned} &(IC_G) \qquad w_{NS} \geq \max[\gamma w_{BS} + (1-\gamma)w_{BF}, 0] \quad (4.37) \\ &(IC_N) \qquad \alpha w_{NS} + (1-\alpha)w_{NF} \geq \max[\gamma w_{BS} + (1-\gamma)w_{BF}, 0] \\ &(IC_B) \qquad \gamma w_{BS} + (1-\gamma)w_{BF} \geq \max[w_{NF}, 0] \end{aligned}$$

Lemma 4.4 *An optimal wage contracts* $\{w_{BS}^*, w_{BF}^*, w_{NS}^*, w_{NF}^*\}$ *is given by*

$$\begin{aligned} w^*_{NS} &= \frac{k}{e\alpha(1-\alpha)} \\ w^*_{NF} &= 0 \\ \gamma w^*_{BS} + (1-\gamma) w^*_{BF} &= \frac{k}{e(1-\alpha)} \end{aligned} \tag{4.38}$$

Under an optimal wage contract the principal will always chooses project I when the agent reports N.

Proof: Analog to the proof of Lemma 4.1.

The principal's payoff if she hires the agent and sets the optimal wage contract is

$$\Pi_P^{E*} = \alpha R_S + (1-\alpha)R_F + (1-\alpha)e(R' - R_F) - k - \frac{k}{e(1-\alpha)} \tag{4.39}$$

If the principal abstains from hiring the agent, his payoff is either

$$\Pi_P^N = \alpha R_S + (1-\alpha)R_F, \tag{4.40}$$

if $\alpha \geq \bar{\alpha}$ or

$$\Pi_P^N = R', \tag{4.41}$$

if $\alpha < \bar{\alpha}$. The principal hires the agent, if

$$\Pi_P^{E*} - \Pi_P^N \geq 0 \tag{4.42}$$

For $k \leq k_{max}$ the principal hires the agent iff $\alpha_B^{min} \leq \alpha \leq \alpha^{max}$ with α^{max} as defined in equation (4.11) and

$$\alpha_B^{min} \equiv 1 - \frac{\sqrt{e}(R_S - R' - k) + \sqrt{e[k^2 + (R_S - R')^2 - 2k(R_S - 3R' + 2R_F)] - 4k(R_S - R_F)}}{2\sqrt{e}[R_S - R_F - e(R' - R_F)]}. \tag{4.43}$$

Figure 4.5 shows that the interval $[\alpha_B^{min}, \alpha^{max}]$ for which the principal finds it profitable to hire the agent is smaller than for the full message set for all effort costs k.

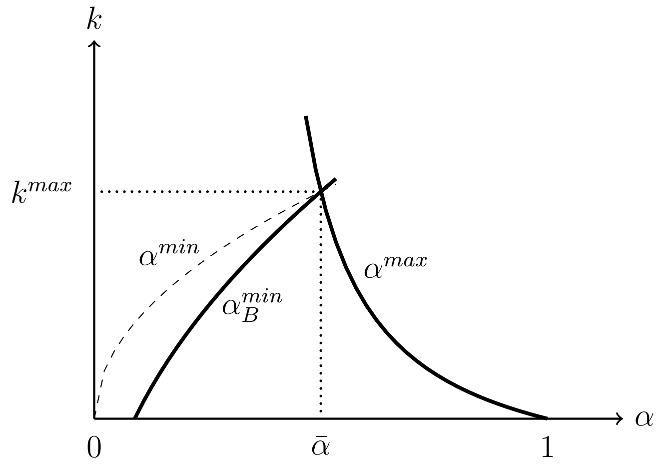

Figure 4.5: For given effort costs k, α_B^{min} and α^{max} are the minimum and maximum values of α, respectively, for which the principal hires the agent

4.3.3 Optimal Information Set

To decide which information set is optimal to use, we compare profits for different intervals of α-values. The cost to induce the agent to exert effort is the same for all information sets. The principal has to pay the agent the cost k to collect information and an information rent $k/[e(1-\alpha)]$.

Proposition 4.1 *Full information disclosure is not dominated by partial information disclosure. The agent is hired for $k \leq k^{max}$ iff $\alpha^{min} \leq \alpha \leq \alpha^{max}$.*

In the proposition, it is shown that with an agent who is inherently indifferent which project is chosen by the principal and who is only motivated by conditioned wage payments, it is not possible to reduce incentive payments by a restriction of the message set. For $\alpha < \bar{\alpha}$, the full message set contract and $\{G, N\}$-contract have the same expected profitability, for $\alpha \geq \bar{\alpha}$, the full message set contract and $\{B, N\}$-contract have the same expected profitability.

4.4 Contracts with the Possibility of Evidence Acquisition

4.4.1 Full Evidence Acquisition

We now consider the full information set contract and study optimal contracts with evidence acquisition. If the agent reports G or B, the contract obliges him, after having spent the cost k to acquire information, to collect evidence at an additional cost c, so that the correctness of information becomes verifiable before the investment decision is made.

Case $\alpha R_S + (1-\alpha)R_F \geq R'$

The principal chooses project I if the agent reports either G or N. If the agent reports B, she chooses I'.

The Agent's payoff if he exerts effort is

$$\Pi_A^E = e[\alpha w_{GS} + (1-\alpha)(\gamma w_{BS} + (1-\gamma)w_{BF}) - c] + (1-e)[\alpha w_{NS} + (1-\alpha)w_{NF}] - k, \quad (4.44)$$

where the evidence cost c has to be additionally borne by the agent if he reports either G or B. If the agent exerts no effort, his profit is

$$\Pi_A^N = \alpha w_{NS} + (1-\alpha)w_{NF}. \quad (4.45)$$

The agent exerts effort iff

$$\Pi_A^E - \Pi_A^N \geq 0. \quad (4.46)$$

The principal's payoff if she induces the agent to exert effort is

$$\begin{aligned} \Pi_P^E &= e[\alpha(R_S - w_{GS}) + (1-\alpha)(R' - \gamma w_{BS} - (1-\gamma)w_{BF})] \\ &\quad + (1-e)[\alpha(R_S - w_{NS}) + (1-\alpha)(R_F - w_{NF})] \end{aligned} \quad (4.47)$$

In order to induce the agent to inform the principal truthfully about his information, besides condition (4.46), the following incentive constraints have to be fulfilled:

$$\begin{aligned} &(IC_G) \qquad w_{GS} - c \geq \max[w_{NS}, 0] \\ &(IC_N) \qquad \alpha w_{NS} + (1-\alpha)w_{NF} \geq 0 \\ &(IC_B) \qquad \gamma w_{BS} + (1-\gamma)w_{BF} - c \geq \max[w_{NF}, 0]. \end{aligned} \quad (4.48)$$

Now the incentive constraints only have to guarantee that it is not more profitable for the informed agent to imitate an uninformed agent and that all wages are nonnegative.

Lemma 4.5 *An optimal contract* $\{w^*_{GS}, w^*_{GF}, w^*_{NS}, w^*_{NF}, w^*_{BS}, w^*_{BF}\}$ *is given by*

$$\begin{aligned} w^*_{GF} &= 0 \\ w^*_{GS} &= \frac{k}{e} + c \\ \gamma w^*_{BS} + (1-\gamma) w^*_{BF} &= \frac{k}{e} + c \\ w^*_{NS} &= w^*_{NF} = 0 \end{aligned} \tag{4.49}$$

The principal's payoff if she hires the agent and sets an optimal wage contract is

$$\Pi^{E*}_P = \alpha R_S + (1-\alpha)R_F + (1-\alpha)e(R' - R_F) - k - ec \tag{4.50}$$

If the principal abstains from hiring the agent, her payoff is

$$\Pi^N_P = \alpha R_S + (1-\alpha)R_F. \tag{4.51}$$

The principal hires the agent iff

$$\Pi^{E*}_P - \Pi^N_P \geq 0 \tag{4.52}$$

For $k \leq e(R' - R_F - c)$, the principal hires the agent iff

$$\alpha \leq \hat{\alpha}^{max}_F \equiv 1 - \frac{k + ec}{e(R' - R_F)}$$

Case $\alpha R_S + (1-\alpha)R_F < R'$

The principal chooses project I only if the agent reports G. If the agent reports N or B, she chooses I'.
The agent's payoff if he exerts effort is

$$\Pi_A = e[\alpha w_{GS} + (1-\alpha)(\gamma w_{BS} + (1-\gamma)w_{BF}) - c] + (1-e)[\gamma w_{NS} + (1-\gamma)w_{NF}] - k \tag{4.53}$$

and if he exerts no effort

$$\Pi^N_A = \gamma w_{NS} + (1-\gamma)w_{NF}. \tag{4.54}$$

The agent exerts effort if

$$\Pi^E_A - \Pi^N_A \geq 0. \tag{4.55}$$

The principal's payoff if he induces the agent to exert effort is

$$\begin{aligned} \Pi_P^E &= e[\alpha(R_S - w_{GS}) + (1-\alpha)(R' - \gamma w_{BS} - (1-\gamma)w_{BF})] \\ &\quad +(1-e)[R' - \gamma w_{NS} - (1-\gamma)w_{NF}]. \end{aligned} \tag{4.56}$$

In order to induce the agent to inform the principal truthfully about his information, besides condition (4.55), the following incentive constraints have to be fulfilled:

$$\begin{aligned} &(IC_G) \qquad w_{GS} - c \geq \max[\gamma w_{NS} + (1-\gamma)w_{NF}, 0] \\ &(IC_N) \qquad \gamma w_{NS} + (1-\gamma)w_{NF} \geq 0 \\ &(IC_B) \qquad \gamma w_{BS} + (1-\gamma)w_{BF} - c \geq \max[\gamma w_{NS} + (1-\gamma)w_{NF}, 0]. \end{aligned} \tag{4.57}$$

Lemma 4.6 *An optimal contract* $\{w_{GS}^*, w_{GF}^*, w_{NS}^*, w_{NF}^*, w_{BS}^*, w_{BF}^*\}$ *is given by*

$$\begin{aligned} w_{GF}^* &= 0 \qquad (4.58) \\ w_{GS}^* &= \frac{k}{e} + c \\ w_{BS}^* &= w_{BF}^* = \frac{k}{e} + c \\ w_{NS}^* &= w_{NF}^* = 0 \end{aligned} \tag{4.59}$$

Proof: Analog to Lemma 4.5.

The principal's payoff if she hires the agent and sets an optimal wage contract is

$$\Pi_P^{E*} = R' + \alpha e(R_S - R') - k - ec \tag{4.60}$$

If the principal abstains from hiring the agent, her payoff is

$$\Pi_P^N = R'. \tag{4.61}$$

The principal hires the agent iff

$$\Pi_P^{E*} - \Pi_P^N \geq 0. \tag{4.62}$$

If $k \leq e(R_S - R' - c)$, the principal hires the agent iff

$$\alpha \geq \hat{\alpha}_F^{min} \equiv \frac{k + ec}{e(R_S - R')}$$

4.4.2 Partial Evidence Acquisition

G-Evidence

If the agent reports G, he has to collect evidence at cost c, so that the correctness of information becomes verifiable before the investment decision is made.

Case $\alpha R_S + (1-\alpha)R_F \geq R'$

The principal chooses project I, if the agent reports either G or N. If the agent reports B, she chooses I'.

The agent's payoff if he exerts effort is

$$\Pi_A^E = e[(\alpha w_{GS} - c) + (1-\alpha)(\gamma w_{BS} + (1-\gamma)w_{BF})] + (1-e)[\alpha w_{NS} + (1-\alpha)w_{NF}] - k \quad (4.63)$$

and if he exerts no effort

$$\Pi_A^N = \alpha w_{NS} + (1-\alpha)w_{NF}. \quad (4.64)$$

The agent exerts effort iff

$$\Pi_A^E - \Pi_A^N \geq 0. \quad (4.65)$$

The principal's payoff if she induces the agent to exert effort is

$$\begin{aligned} \Pi_P^E &= e[\alpha(R_S - w_{GS}) + (1-\alpha)(R' - \gamma w_{BS} - (1-\gamma)w_{BF})] \\ &\quad + (1-e)[\alpha(R_S - w_{NS}) + (1-\alpha)(R_F - w_{NF})] \end{aligned} \quad (4.66)$$

In order to induce the agent to inform the principal truthfully about his information, besides condition (4.65), the following incentive constraints have to be fulfilled:

$$
\begin{aligned}
&(IC_G) \qquad w_{GS} - c \geq \max[w_{NS}, \gamma w_{BS} + (1-\gamma) w_{BF}, 0] && (4.67)\\
&(IC_N) \qquad \alpha w_{NS} + (1-\alpha) w_{NF} \geq \max[\gamma w_{BS} + (1-\gamma) w_{BF}, 0] \\
&(IC_B) \qquad \gamma w_{BS} + (1-\gamma) w_{BF} \geq \max[w_{NF}, 0].
\end{aligned}
$$

Lemma 4.7 *An optimal contract* $\{w^*_{GS}, w^*_{GF}, w^*_{NS}, w^*_{NF}, w^*_{BS}, w^*_{BF}\}$ *is given by*

$$
\begin{aligned}
w^*_{GF} &= 0 && (4.68)\\
w^*_{GS} &= \frac{k}{\alpha e} + c \\
w^*_{BS} &= w^*_{BF} = 0 \\
w^*_{NS} &= w^*_{NF} = 0.
\end{aligned}
$$

The principal's payoff if she hires the agent and sets the optimal wage contract is

$$\Pi^{E*}_P = \alpha R_S + (1-\alpha) R_F + (1-\alpha) e (R' - R_F) - k - \alpha e c \tag{4.69}$$

If the principal abstains from hiring the agent, her payoff is

$$\Pi^N_P = \alpha R_S + (1-\alpha) R_F. \tag{4.70}$$

The principal hires the agent iff

$$\Pi^{E*}_P - \Pi^N_P \geq 0 \tag{4.71}$$

If $k \leq e(R' - R_F)$, the principal hires the agent iff

$$\alpha \leq \hat{\alpha}^{max}_G \equiv 1 - \frac{k + ec}{e(R' - R_F + c)} \tag{4.72}$$

Case $\alpha R_S + (1-\alpha) R_F < R'$

The principal chooses project I only if the agent reports G. If the agent reports N or B, she chooses I'.

The agent's payoff if he exerts effort is

$$\Pi_A = e[\alpha(w_{GS} - c) + (1-\alpha)(\gamma w_{BS} + (1-\gamma) w_{BF})] + (1-e)[\gamma w_{NS} + (1-\gamma) w_{NF}] - k \tag{4.73}$$

and if he exerts no effort

$$\Pi_A^N = \gamma w_{NS} + (1-\gamma) w_{NF}. \tag{4.74}$$

The agent exerts effort iff

$$\Pi_A^E - \Pi_A^N \geq 0. \tag{4.75}$$

The principal's payoff, if she induces the agent to exert effort is

$$\begin{aligned} \Pi_P^E &= e[\alpha(R_S - w_{GS}) + (1-\alpha)(R' - \gamma w_{BS} - (1-\gamma) w_{BF})] \\ &\quad +(1-e)[R' - \gamma w_{NS} - (1-\gamma) w_{NF}] \end{aligned} \tag{4.76}$$

In order to induce the agent to inform the principal truthfully about his information, besides condition (4.75), the following incentive constraints have to be fulfilled:

$$\begin{aligned} (IC_G) &\qquad w_{GS} - c \geq \max[\gamma w_{NS} + (1-\gamma) w_{NF}, \gamma w_{BS} + (1-\gamma) w_{BF}, 0] \\ (IC_N) &\qquad \gamma w_{NS} + (1-\gamma) w_{NF} \geq \max[\gamma w_{BS} + (1-\gamma) w_{BF}, 0] \\ (IC_B) &\qquad \gamma w_{BS} + (1-\gamma) w_{BF} \geq \max[\gamma w_{NS} + (1-\gamma) w_{NF}, 0]. \end{aligned} \tag{4.77}$$

Lemma 4.8 *An optimal contract* $\{w^*_{GS}, w^*_{GF}, w^*_{NS}, w^*_{NF}, w^*_{BS}, w^*_{BF}\}$ *is given by*

$$\begin{aligned} w^*_{GF} &= 0 \\ w^*_{GS} &= \frac{k}{\alpha e} + c \\ w^*_{BS} &= w^*_{BF} = 0 \\ w^*_{NS} &= w^*_{NF} = 0. \end{aligned} \tag{4.78}$$

Proof: Analog to Lemma 4.7.

The principal's payoff if she hires the agent and sets an optimal wage contract is

$$\Pi_P^{E*} = R' + \alpha e(R_S - R') - k - \alpha ec \tag{4.79}$$

If the principal abstains from hiring the agent, her payoff is

$$\Pi_P^N = R'. \tag{4.80}$$

The principal hires the agent iff

$$\Pi_P^{E*} - \Pi_P^N \geq 0 \tag{4.81}$$

If $k \leq e(R_S - R' - c)$, the principal hires the agent iff

$$\alpha \geq \hat{\alpha}_G^{min} \equiv \frac{k}{e(R_S - R' - c)} \tag{4.82}$$

B-Evidence

If the agent reports B, he has to collect evidence at cost c, so that the correctness of information becomes verifiable before the investment decision is made.

Case $\alpha R_S + (1-\alpha)R_F \geq R'$

The principal chooses project I, if the agent reports either G or N. If the agent reports B, she chooses I'.
The agent's payoff if he exerts effort is

$$\Pi_A^E = e[(\alpha w_{GS}) + (1-\alpha)(\gamma w_{BS} + (1-\gamma)w_{BF}) - c] + (1-e)[\alpha w_{NS} + (1-\alpha)w_{NF}] - k \tag{4.83}$$

and if he exerts no effort

$$\Pi_A^N = \alpha w_{NS} + (1-\alpha)w_{NF}. \tag{4.84}$$

The agent exerts effort iff

$$\Pi_A^E - \Pi_A^N \geq 0. \tag{4.85}$$

The principal's payoff if she induces the agent to exert effort is

$$\begin{aligned}\Pi_P^E \quad = \quad & e[\alpha(R_S - w_{GS}) + (1-\alpha)(R' - \gamma w_{BS} - (1-\gamma)w_{BF})] \\ & +(1-e)[\alpha(R_S - w_{NS}) + (1-\alpha)(R_F - w_{NF})]\end{aligned} \tag{4.86}$$

In order to induce the agent to inform the principal truthfully about his information, besides condition (4.85), the following incentive constraints have to be fulfilled:

$$
\begin{aligned}
&(IC_G) && w_{GS} \geq \max[w_{NS}, 0] && (4.87)\\
&(IC_N) && \alpha w_{NS} + (1-\alpha)w_{NF} \geq \max[\alpha w_{GS} + (1-\alpha)w_{GF}, 0] &&\\
&(IC_B) && \gamma w_{BS} + (1-\gamma)w_{BF} - c \geq \max[w_{NF}, w_{GF}, 0]. &&
\end{aligned}
$$

Lemma 4.9 *An optimal contract* $\{w^*_{GS}, w^*_{GF}, w^*_{NS}, w^*_{NF}, w^*_{BS}, w^*_{BF}\}$ *is given by*

$$
\begin{aligned}
w^*_{GS} &= w^*_{GF} = 0 \qquad (4.88)\\
w^*_{NS} &= w^*_{NF} = 0\\
w^*_{BS} &= w^*_{BF} = \frac{k}{(1-\alpha)e} + c.
\end{aligned}
$$

Proof: Analog to Lemma 4.7.

The principal's payoff if she hires the agent and sets the optimal wage contract is

$$\Pi^{E*}_P = \alpha R_S + (1-\alpha)R_F + (1-\alpha)e(R' - R_F) - k - (1-\alpha)ec \qquad (4.89)$$

If the principal abstains from hiring the agent, her payoff is

$$\Pi^N_P = \alpha R_S + (1-\alpha)R_F. \qquad (4.90)$$

The principal hires the agent iff

$$\Pi^{E*}_P - \Pi^N_P \geq 0 \qquad (4.91)$$

If $k \leq e(R' - R_F - c)$, the principal hires the agent iff

$$\alpha \leq \hat{\alpha}^{max}_B \equiv 1 - \frac{k}{e(R' - R_F - c)}. \qquad (4.92)$$

Case $\alpha R_S + (1-\alpha)R_F < R'$

The principal chooses project I only if the agent reports G. If the agent reports N or B, she chooses I'.

The agent's payoff if he exerts effort is

$$\Pi_A = e[\alpha w_{GS} + (1-\alpha)(\gamma w_{BS} + (1-\gamma)w_{BF} - c)] + (1-e)[\gamma w_{NS} + (1-\gamma)w_{NF}] - k \qquad (4.93)$$

and if he exerts no effort

$$\Pi_A^N = \gamma w_{NS} + (1-\gamma) w_{NF}. \tag{4.94}$$

The agent exerts effor, iff

$$\Pi_A^E - \Pi_A^N \geq 0. \tag{4.95}$$

The principal's payoff if she induces the agent to exert effort is

$$\begin{aligned} \Pi_P^E &= e[\alpha(R_S - w_{GS}) + (1-\alpha)(R' - \gamma w_{BS} - (1-\gamma)w_{BF})] & (4.96) \\ &+(1-e)[R' - \gamma w_{NS} - (1-\gamma)w_{NF}] & (4.97) \end{aligned}$$

In order to induce the agent to inform the principal truthfully about his information, besides condition (4.95), the following incentive constraints have to be fulfilled:

$$\begin{aligned} (IC_G) &\qquad w_{GS} \geq \max[\gamma w_{NS} + (1-\gamma)w_{NF}, 0] & (4.98) \\ (IC_N) &\qquad \gamma w_{NS} + (1-\gamma)w_{NF} \geq \max[\alpha w_{GS} + (1-\alpha)w_{GF}] \\ (IC_B) &\qquad \gamma w_{BS} + (1-\gamma)w_{BF} - c \geq \max[\gamma w_{NS} + (1-\gamma)w_{NF}, w_{GF}, 0]. \end{aligned}$$

Lemma 4.10 *An optimal contract* $\{w_{GS}^*, w_{GF}^*, w_{NS}^*, w_{NF}^*, w_{BS}^*, w_{BF}^*\}$ *is given by*

$$\begin{aligned} w_{GS}^* &= w_{GF}^* = 0 & (4.99) \\ w_{NS}^* &= w_{NF}^* = 0 \\ w_{BS}^* &= w_{BF}^* = \frac{k}{(1-\alpha)e} + c. \end{aligned}$$

Proof: Analog to Lemma 4.7.

The principal's payoff if she hires the agent and sets the optimal wage contract is

$$\Pi_P^{E*} = R' + \alpha e(R_S - R') - k - (1-\alpha)ec \tag{4.100}$$

If the principal abstains from hiring the agent, her payoff is

$$\Pi_P^N = R'. \tag{4.101}$$

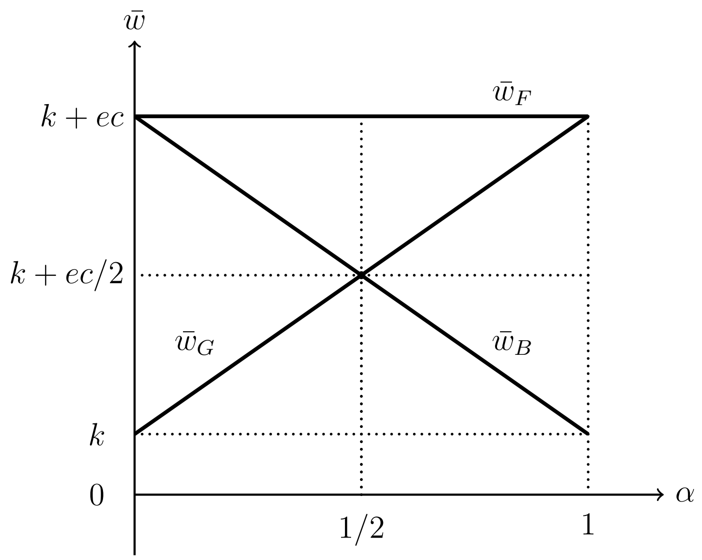

Figure 4.6: Expected wages $\bar{w}_F$ for full evidence, $\bar{w}_G$ for G-evidence and $\bar{w}_B$ for B-evidence

The principal hires the agent iff

$$\Pi_P^{E*} - \Pi_P^N \geq 0. \tag{4.102}$$

If $k \leq e(R_S - R' - c)$, the principal hires the agent iff

$$\alpha \geq \hat{\alpha}_B^{min} \equiv \frac{k}{e(R_S - R' - c)}. \tag{4.103}$$

Figure 4.6 shows the expected wages for all three evidence contracts depending on α.

We can now compare the different evidence contracts.

4.4.3 Optimal Evidence Acquisition

We will now compare the different evidence contracts. We find that the principal will never use the full evidence contract as truthful reporting without leaving an information rent to the agent can be guaranteed by G- or B-evidence only. These contracts imply lower expected wages, because the evidence cost c has to be borne by the agent less often than for the full evidence contract.

Proposition 4.2 *The full evidence contract is never more profitable than the G-evidence or B-evidence contract.*

The only advantage of the full evidence contract compared to the partial contracts is that the principal can induce incentives by both wages w_{GS} and w_B. [2].

4.5 Optimal Contracts

In the following we leave the dominated full evidence contract and the dominated partial information contracts out of consideration. The first contract described, with the full information set, is now referred to as No-evidence contract. In the preceding analysis we found the expected wage $\bar{w}^*$, the principal pays the agent in an optimal contract if she hires the agent, to be

$$\bar{w}^* = \begin{cases} k + \alpha ec & \text{for G-evidence} \\ k + \frac{k}{e(1-\alpha)} & \text{for No-evidence} \\ k + (1-\alpha)ec & \text{for B-evidence,} \end{cases}$$

and the expected additional return on the principal's investment $\Delta\bar{\Pi}_P$ to be

$$\Delta\bar{\Pi} = \begin{cases} e\alpha(R' - R_F) & \text{for } \alpha < \bar{\alpha} \\ e(1-\alpha)(R_S - R') & \text{for } \alpha \geq \bar{\alpha}. \end{cases}$$

By comparing the expected wages for the contracts, we get the following proposition.

Proposition 4.3 *The cost minimizing contract that induces the agent to acquire information and to report truthfully is*

for $c < \frac{4k}{e^2}$*:*

G - evidence for $\alpha < \frac{1}{2}$
B - evidence for $\alpha \geq \frac{1}{2}$

for $c \geq \frac{4k}{e^2}$*:*

G-evidence for $0 \leq \alpha < \alpha_{GN}$
No-evidence for $\alpha_{GN} \leq \alpha \leq \alpha_{BN}$

[2] This implies that with a risk-averse agent and sufficient low evidence costs c, the full evidence contract could be more profitable than the partial contracts, for some range of α

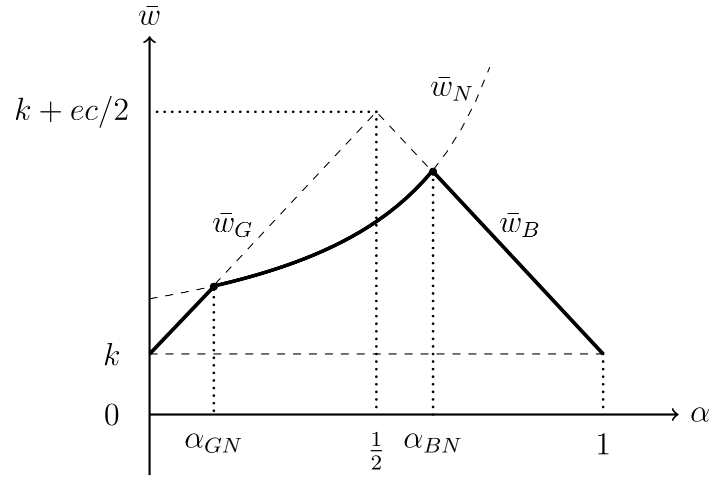

Figure 4.7: $c \geq \frac{4k}{e^2}$: Dashed lines: Expected wages for G-, B- and No-evidence, Solid line: Expected wage for the optimal contract

B-evidence *for* $\alpha_{BN} < \alpha \leq 1$*, where*

$$\alpha_{GN} \equiv \frac{1}{2}\left(1 - \sqrt{1 - \frac{4k}{e^2 c}}\right) \tag{4.104}$$

$$\alpha_{BN} \equiv 1 - \sqrt{\frac{k}{e^2 c}}. \tag{4.105}$$

For sufficient low evidence costs $c < \frac{4k}{e^2}$, it is always better to implement an evidence contract. Further the principal asks the report with the lower probability to be made with evidence, with G- evidence for $\alpha < \frac{1}{2}$ and B - evidence for $\alpha \geq \frac{1}{2}$. For higher evidence costs the No-evidence contract can become optimal. For some intermediate values of α, paying the information rent is less costly than compensating the agent for his evidence collection. With increasing evidence costs, the boundaries of the interval of α-values for which no-evidence is optimal $[\alpha_{GN}, \alpha_{BN}]$ converge to the boundary values 0 and 1. Figure 4.7 shows an example with evidence costs for which all three cases show up. The last proposition states that the agent is only hired when expected additional expected profits are at least the expected wage of the optimal contract.

Proposition 4.4 *The principal hires the agent iff* $\Delta\bar{\Pi} \geq \bar{w}^*$ *using the cost minimizing contract from proposition 4.3.*

In figure 4.8 we show an example in which we have five distinct intervals of α values. For α-values near to zero and one the agent is not hired by the principal. For interme-

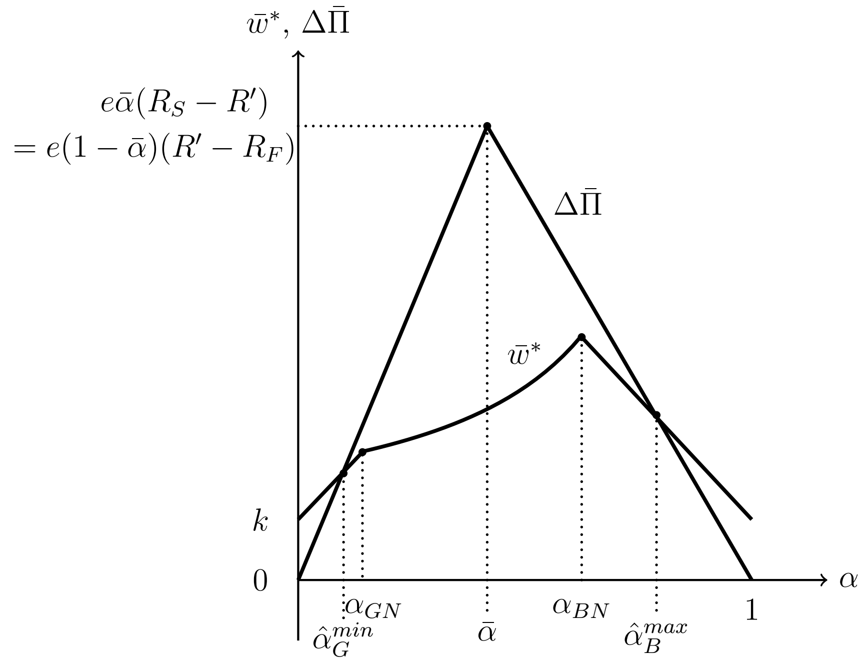

Figure 4.8: Optimal expected wage $\bar{w}^*$ and additional payoff $\Delta\bar{\Pi}_P$

diate values there is some interval with G-evidence, an interval for which no-evidence is optimal, and an interval with B-evidence.

4.6 Conclusion

We characterize wage contracts inducing an expert-agent to exert effort and to report his acquired information truthfully. We find that the principal would ask the agent to report either one of three signals or alternatively only one of two signals with different meaning for high and low prior probabilities of successful projects. The contract with three signals and the best contract with two signals create the same expected payoff for the principal. Restricting the message space can not be used to create effort incentives at a lower cost. Furthermore the contract might include the obligation for the agent to support his report with hard evidence for positive findings if the a priori probability for good projects is small, and for negative findings if it is great.

In our analysis it makes no difference if the agent or the principal pays for evidence. Our model differs from existing literature about state-verification by Townsend (1979) and Gale and Hellwig (1985) in the way that the true state becomes common knowledge if the unknown project is realized and can be contracted on. But, because the true state is not observed by the principal if the outside project is chosen, effort incentives can only be created by giving the agent an information rent. This makes the decision for costly state-verification, namely the demand for hard evidence, endogenous in our model. State-verification is chosen by the principal if it is not too costly and has to be executed only with a small probability, which is the case if the prior is rather informative.

4.7 Appendix

Proof of Lemma 1

By condition 4.3 incentives to exert effort are created by the wages w_{GS}, w_{BS} and w_{BF}, while w_{NS} and w_{NF} disincentive the agent. So the principal wants to increase the former wages sufficiently to induce the agent to exert effort and to decrease the latter as far as possible. This is bindingly constrained by (IC_N). Consequently, one gets:

$$\alpha w_{NS}^* + (1-\alpha)w_{NF}^* = \alpha w_{GS}^* = \gamma w_{BS}^* + (1-\gamma)w_{BF}^*. \tag{4.106}$$

Replacing $\alpha w_{NS} + (1-\alpha)w_{NF}$ and $\gamma w_{BS} + (1-\gamma)w_{BF}$ in condition 4.3 by αw_{GS}, for the agent's incentive to exert effort, it follows that $w_{GS}^* \geq k/[e\alpha(1-\alpha)]$. In order to maximize Π_P^E, w_{GS}^* is set to $w_{GS}^* = k/[e\alpha(1-\alpha)]$. The wage w_{GF}^* is never realized in equilibrium and is set to 0.

Furthermore, we have to show that it is optimal for the principal to set $w_{NS}^* = w_{NF}^*$. Assume $w_{NS}^* \neq w_{NF}^*$ and $\alpha \neq \gamma$, then the principal will choose project I after receiving a report N iff

$$\alpha \geq \widetilde{\alpha} \equiv \frac{R' - R_F - \gamma(w_{NS}^* - w_{NF}^*)}{R_S - R_F - (w_{NS}^* - w_{NF}^*)}. \tag{4.107}$$

Because the principal's decision can be anticipated by the agent, w_{NS} and w_{NF} have to be set such that $\Pi_A^E - \Pi_A^0 \geq e[\alpha(w_{GS} - w_{NS}) + (1-\alpha)(w_B - w_{NF})] - k \geq 0$ if the principal will choose I and such that $\Pi_A^E - \Pi_A^0 \geq e[\alpha w_{GS} + (1-\alpha)w_B - \gamma w_{NS} - (1-\gamma)w_{NF}] - k \geq 0$ if the principal will choose I'. Taking into account incentive constraint (IC_N), the expected wage in both cases is the same: $\bar{w} = k + k/[e(1-\alpha)]$. But, as the efficient decision is to choose project I whenever $\alpha \geq \bar{\alpha}$, an optimal contract implies $w_{NS}^* = w_{NF}^*$, because it commits the principal to make the efficient project choice. Alternatively, to make sure that the principal has an incentive to choose I, she could set any wages w_{NS}^* and w_{NF}^* such that $\alpha w_{NS}^* + (1-\alpha)w_{NF}^* = k/[e(1-\alpha)]$ and $\alpha \geq \widetilde{\alpha}$. Q.E.D.

Proof of Proposition 1

We have to proof that full disclosure is at least as profitable as the $\{G, N\}$- and $\{B, N\}$-disclosure rules for any α:

For $\alpha < \alpha^{min}$ and $\alpha > \alpha^{max}$, for all three disclosure rules it is optimal not to hire the agent. Because costs are the same for all disclosure rules, in the following we only consider revenues.

$\alpha^{min} \leq \alpha < \alpha_B^{min}$:

Only full disclosure and $\{G, N\}$ are profitable, both generate the same profit.

$\alpha_B^{min} \leq \alpha < \bar{\alpha}$:

All three disclosure rules are profitable. Full disclosure and $\{G,N\}$ generate $R' + e\alpha(R_S - R')$, $\{B,N\}$ generates $\alpha R_S + (1-\alpha)R_F + e(1-\alpha)(R' - R_F)$. The first profit is greater than the second iff $(1-e)[\alpha R_S + (1-\alpha)(R_F - R')] \leq 0$. This is the case for the considered interval.

$\bar{\alpha} \leq \alpha < \alpha_G^{max}$:

All three disclosure rules are profitable. Full disclosure and $\{B,N\}$ generate $\alpha R_S + (1-\alpha)R_F + e(1-\alpha)(R' - R_F)$, $\{G,N\}$ generates $R' + e\alpha(R_S - R')$, . The first profit is greater than the second iff $(1-e)[\alpha R_S + (1-\alpha)(R_F - R')] \geq 0$. This is the case for the considered interval.

$\alpha_G^{max} \leq \alpha \leq \alpha^{max}$:

Only full disclosure and $\{B,N\}$ are profitable, both generate the same profit. Q.E.D.

Proof of Lemma 5

For an uninformed agent it is not possible anymore to imitate the informed agent. Therefore the wage for a N report can be set to 0. The wages w_{GS}^*, w_{BS}^* and w_{BF}^* have to be set high enough to fulfill inequality 4.46. In order to maximize the principal's profit the condition should be fulfilled with equality. To induce the informed agent to tell the truth, w_{GS}^*, w_{BS}^* and w_{BF}^* have to be at least c. This can easily be guaranteed by setting w_{GS}^* equal to $\gamma w_{BS}^* + (1-\gamma)w_{BF}^*$. Q.E.D.

Proof of Lemma 7

Optimally the incentive to exert effort is completely provided by the wage w_{GS}. For a wage of zero for a B report as well as for a N report, the agent has no incentive to report untruthfully when he observes B or N, as switching between these two reports does not increase his payoff and for a G report he would need hard evidence. Q.E.D.

Proof of Proposition 2

The result immediately follows from comparing the expected profits and wages of G-evidence and B-evidence contracts with the full-evidence contract. All contracts create the same additional payoff, but the expected wage is only $k + \alpha c$ and $k + (1-\alpha)c$ for the G-evidence and B-evidence contract, respectively, while it is $k + c$ for the full evidence contract. Q.E.D.

Proof of Proposition 3

First, comparing expected wages for the G-evidence and B-evidence contract, it is apparent that G-evidence is preferred by the principal for $\alpha \in \left(0, \frac{1}{2}\right)$ and B-evidence for $\alpha \in \left[\frac{1}{2}, 1\right)$. Second, the expected wage without evidence $k + \frac{k}{e(1-\alpha)}$ is smaller or equal than the expected wage for the G-evidence contract $e\alpha c$, iff $\alpha \leq \alpha_{GN} \equiv \frac{1}{2}\left(1 - \sqrt{1 - \frac{4k}{e^2 c}}\right)$. It is smaller or equal than the expected wage for the B-evidence contract $e(1-\alpha)c$, iff

$\alpha \geq \alpha_{BN} \equiv 1 - \sqrt{\frac{k}{e^2 c}}$. For $c \geq \frac{4k}{e^2}$, we have $\alpha_{GN} \in (0, \frac{1}{2})$ and $\alpha_{BN} \in (\frac{1}{2}, 1)$. Thus, we have the three intervals described in the second part of the proposition. For $c < \frac{4k}{e^2}$, the cost of the contract without evidence is greater than the cost of G-evidence for all $\alpha \in (0, \frac{1}{2})$ and greater than the cost for B-evidence for all $\alpha \in \left[\frac{1}{2}, 1\right)$. Consequently, G-evidence is optimal for $\alpha \in (0, \frac{1}{2})$ and B- evidence is optimal for $\alpha \in (\frac{1}{2}, 1)$. Q.E.D.

Proof of Proposition 4

It is trivial that the principal will hire the agent if and only if the additional expected profit is greater than the expected wage she has to pay the agent. Q.E.D.

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