

# Optimal non-Markovian composite search algorithms for spatially correlated targets

A. KLIMEK<sup>(a)</sup> and R. R. NETZ

*Freie Universität Berlin, Fachbereich Physik - 14195 Berlin, Germany*

received 1 November 2021; accepted in final form 24 January 2022  
published online 8 August 2022

**Abstract** – We study the efficiency of a wide class of stochastic non-Markovian search strategies for spatially correlated target distributions. For an uninformed searcher that performs a non-composite random search, a ballistically moving search is optimal for destructible targets, even when the targets are correlated. For an informed searcher that can measure the time elapsed since the last target encounter and performs a composite search consisting of alternating extensive ballistic trajectories and intensive non-Markovian search trajectories, the efficiency can be more than three times higher compared to a ballistic searcher. We optimize the memory function that describes the intensive non-Markovian search motion and find a single-exponential memory function to be optimal. In our extended search model the intensive search mode is activated when the distance between two consecutively found targets in the extensive search mode is smaller than a threshold length called the memory distance  $d_m$ . We find that a finite value of  $d_m$  quite generally leads to optimal search efficiency for correlated target distributions.



Copyright © 2022 The author(s)

Published by the EPLA under the terms of the [Creative Commons Attribution 4.0 International License](https://creativecommons.org/licenses/by/4.0/) (CC BY). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

**Introduction.** – Search problems are omnipresent in everyday life: immune cells search for viruses or bacteria [1], people search for natural resources [2] and animals forage for food [3–6], just to name a few examples. According to optimal foraging theory, the understanding of search processes reveals the driving forces behind the often complex search behavior of various organisms [7,8], but it can also inspire the optimization of search algorithms that might be useful in technology and computer science applications [9,10]. Thus, there have been many studies aiming to find the most efficient search strategy for different scenarios [11–16]. Without prior information on the spatial distribution of targets [17,18], the optimal search strategy typically includes random features [19]. To efficiently deal with target correlations, composite [20,21] and intermittent search algorithms were proposed [22]. A composite search, also known as an area-concentrated or area-restricted search, is comprised of alternating intervals of intensive and extensive search modes. In fact, many animals use composite search strategies [6,23,24]. In a popular class of models, the intensive search mode is typically activated after every target encounter and maintained for

the duration of the so-called giving-up time (GUT) [25]. Obviously, in order to perform a GUT-based composite search, the searcher needs the ability to detect target encounters and to measure time. A searcher that has these two abilities we call an informed searcher. There are also more complex search algorithms, where the searcher learns when to activate intensive search intervals [26], for example by spatial mapping of the found targets [27]. Clearly, such more sophisticated algorithms require more abilities of the searcher and are not considered by us.

In this paper we introduce a general non-Markovian composite search model that extends the GUT-composite model without requiring additional abilities of the searcher beyond measuring the time elapsed after the last target encounter. In our model, a single searcher performs a composite search and looks for immobile targets in continuous space and only detects direct encounters with targets within a given encounter radius, but has no ability of long-ranged target perception. The intensive search mode is described by the generalized Langevin equation that characterizes non-Markovian random motion by a memory function [28,29], while the extensive search mode consists of straight paths. As a generalization of the GUT model, the intensive search mode is turned on only if the spatial

<sup>(a)</sup>E-mail: [kanton@zedat.fu-berlin.de](mailto:kanton@zedat.fu-berlin.de) (corresponding author)

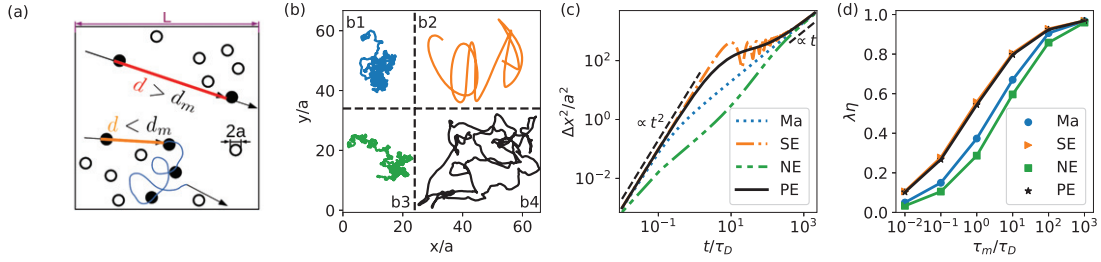


Fig. 1: (a) Schematics of the composite search model. Targets are denoted by circles with radius  $a$ , filled targets have been found already. The blue curved part of the lower trajectory represents the intensive search mode, which is activated since the distance between two consecutively found targets during extensive search is smaller than the memory distance  $d_m$ . In the upper trajectory the intensive search mode is not activated and the searcher keeps moving straight. (b) Example trajectories for the different memory kernels with fixed persistence time  $\tau_m = 0.1\tau_D$ . (b1) represents Markovian motion (eq. (6)), (b2) the single exponential kernel (eq. (7)) with  $\tau_\Gamma = 100\tau_D$ , (b3) the negative exponential (eq. (7)) with  $b = -0.1\tau_D^{-2}$  and  $\tau_\Gamma = 100\tau_D$  and (b4) the positive exponential kernel (eq. (7)) with  $b = 0.09\tau_D^{-2}$  and  $\tau_\Gamma = 100\tau_D$ . All trajectories have a simulation time of  $100\tau_D$  with a time step of  $0.001\tau_D$  and the initial velocity is drawn from a Boltzmann distribution. (c) Corresponding analytical mean squared displacement  $\Delta x^2$  of the different random walks displayed in (b) except for the NE kernel where more illustrative parameters of  $\tau_m = 0.1\tau_D$ ,  $b = -14\tau_D^{-2}$  and  $\tau_\Gamma = 7\tau_D$  are chosen. The dashed black lines represent the initial ballistic and the long time diffusive regimes, respectively. (d) Efficiency of non-composite random walks for uncorrelated targets depending on the persistence time  $\tau_m$ . Other random walk parameters are the same as in (b).

distance covered in extensive search mode between two found targets is smaller than a threshold value, the memory distance. Clearly, in the limit of very large memory distance our model reduces to the standard GUT model, for which the optimal GUT was assessed before [25,30]. For different correlated target distributions we optimize our model with respect to the parameters that characterize the memory function. We find that for correlated targets a memory function that is given by a single exponential shows the highest search efficiency.

### Model and methods. –

*System setup.* In our model a single searcher moves in a 2D box with periodic boundary conditions. There are  $N$  fixed targets inside the box and the searcher has a search radius  $a$ , as illustrated in fig. 1(a). The search radius can be interpreted as the range of vision, smelling, etc., or as the target size. The targets are immobile and their spatial correlation is determined by a preset radial distribution function (RDF). After a target is found it disappears but reappears after a recovery time of  $8000\tau_D$  in the case of correlated targets, which is the minimal time a composite searcher needs to traverse the periodically replicated simulation box size, where  $\tau_D$  is the diffusion time defined below. The finite recovery time prevents repeated finding of the same target, which would define a different search problem, but at the same time keeps the average target density constant [21]. In the case of uniformly randomly distributed targets, the found target recovers instantaneously at a new random position [31]. Therefore, for both correlated and uncorrelated scenarios, the targets are effectively destructible and cannot be repeatedly found. In order to compare different search strategies, the search efficiency  $\eta$  is defined as the number of found targets divided by the travelled distance, *i.e.*, the contour

length of the search trajectory. The target density  $\rho$  is determined by the number of targets  $N$  and the box edge length  $L$  as

$$\rho = \frac{N}{L^2}, \quad (1)$$

from which the mean free path length (MFPL) of the searcher follows as

$$\lambda = \frac{1}{\sigma\rho}. \quad (2)$$

Here  $\sigma$  is the search cross-section which depends on the spatial dimensionality and in 2D is given by  $\sigma = 2a$ . Throughout the paper we use  $N = 1000$  targets and a fixed target density  $a^2\rho = 1/200$ , giving rise to a MFPL of  $\lambda/a = 100$ . In our optimization we use the dimensionless efficiency  $\lambda\eta$ , which gives the number of found targets per MFPL: a searcher moving ballistically has an efficiency of one target per MFPL and thus  $\lambda\eta = 1$ , a value  $\lambda\eta > 1$  signals a search strategy that is more efficient than moving on a straight line.

*Random motion.* The random motion in the intensive search mode is described by the generalized Langevin equation (GLE), first derived by Mori and Zwanzig [32,33], which for a free particle with a memory kernel  $\Gamma(t)$  for each spatial direction reads

$$m\ddot{x}(t) = - \int_{-\infty}^t \Gamma(t-t')\dot{x}(t')dt' + F_R(t), \quad (3)$$

where  $m$  is the mass of the particle,  $\ddot{x}(t)$  is the acceleration and  $\dot{x}(t)$  the velocity at time  $t$ ,  $F_R(t)$  is a Gaussian random force and  $\gamma = \int_0^\infty \Gamma(t)dt$  is the friction coefficient. Note that in our linear model, the two spatial directions do not couple to each other. Rescaling time and space by  $\tau_D$  and  $a$  according to  $\tilde{t} = t/\tau_D$  and  $\tilde{x}(\tilde{t}) = x(t)/a$ , we obtain the

dimensionless version of the GLE in terms of the minimal number of rescaled variables

$$\tilde{x}(\tilde{t}) = - \int_{-\infty}^{\tilde{t}} \tilde{\Gamma}(\tilde{t} - \tilde{t}') \tilde{x}(\tilde{t}') d\tilde{t}' + \tilde{F}_R(\tilde{t}), \quad (4)$$

where  $\tau_m = m/\gamma$  is the persistence time and  $\tau_D = \gamma a^2/(k_B T)$  is the diffusion time and using  $\tilde{F}_R(\tilde{t}) = a\tau_D F_R(t)/(\tau_m k_B T)$ ,  $\tilde{\Gamma}(\tilde{t}) = \tau_D^2 \Gamma(t)/(\tau_m \gamma)$ . Without restricting the generality of the two-point correlation function that characterizes the random motion [34], we can assume the fluctuation dissipation theorem to hold and obtain

$$\langle F_R(t) F_R(0) \rangle = k_B T \Gamma(t). \quad (5)$$

For the special case

$$\Gamma(t) = 2\gamma\delta(t), \quad (6)$$

the GLE simplifies to the Markovian Langevin equation with instantaneous friction and describes a persistent random walk. When additionally the persistence time  $\tau_m$  goes to zero, overdamped Brownian diffusion is obtained. When the memory function  $\Gamma(t)$  differs from a delta peak, the GLE is via Markovian embedding mapped onto a multidimensional Markovian Langevin equation with additional degrees of freedom [35] and numerically solved by an Euler integrator to create trajectories [36]. We compare three different memory functions that are characterized by a finite memory time  $\tau_\Gamma$

$$\frac{\Gamma(t)}{m} = 2\alpha\delta(t) + b \exp(-t/\tau_\Gamma), \quad (7)$$

with  $\alpha = 1/\tau_m - b\tau_\Gamma$ . For  $b = 1/(\tau_\Gamma \tau_m)$  the single-exponential (SE) model [36] is obtained, characterized by  $\alpha = 0$ , whereas for  $\alpha \neq 0$  the positive-exponential (PE) model [37] for  $b > 0$  and the negative-exponential (NE) model [34] for  $b < 0$  are obtained, respectively, which additionally include a delta contribution at zero time. The expression for  $\alpha$  is a consequence of the properties of the velocity autocorrelation function [38] and assures that the integral over the kernel remains positive for  $\alpha \geq 0$  [34]. For  $b = 0$  one recovers the Markovian limit of eq. (6).

*Composite and non-composite search.* The composite random search consists of alternating segments of straight (ballistic) motion and intensive search motion described by the GLE, eq. (3). In the ballistic segment the searcher moves straight at constant velocity  $\sqrt{\pi}/1000 a/\tau_D$  (the actual value of the ballistic velocity is only relevant in connection with the recovery time and is not varied). When finding a target in the ballistic segment, the searcher remembers this event for a certain travel distance, which is defined as the memory distance  $d_m$ . If the searcher finds a second target before moving more than  $d_m$ , the motion switches to intensive search, which is described by the GLE, eq. (3), and stays active for the duration

of the GUT denoted by  $\tau_{GU}$ ; otherwise the searcher forgets the event of finding the target, as illustrated in fig. 1(a). After intensive search for a duration of  $\tau_{GU}$ , the searcher switches back to ballistic motion in a random direction. Our model becomes equivalent to the standard GUT searcher model in the limit of infinite memory distance  $d_m$ , in the limit of  $d_m = 0$  the searcher performs pure ballistic motion. For comparison we also consider non-composite non-Markovian search models, where the motion is always described by eq. (3), which corresponds to an infinite value of  $\tau_{GU}$ .

*Target distributions.* The target distribution strongly influences the most efficient search strategy and is created such that it obeys a preset RDF as explained below. An RDF is given by

$$g(r) = \frac{\rho(r)}{\rho_0}, \quad (8)$$

where  $\rho(r)$  is the target density at distance  $r$  from a central target,  $\rho_0$  is the mean density and an average over all central targets is taken. Uniformly randomly distributed targets are characterized by a constant RDF  $g(r) = 1$ . The RDF of a target distribution in a finite area  $a_{tot}$  fulfills the normalization condition

$$\int_0^{r_{max}} dr 2\pi r g(r) = a_{tot}, \quad (9)$$

where  $r_{max}$  is the maximal possible distance between two targets inside the regarded area. We focus on positively correlated targets that tend to form clusters. We choose an exponentially decaying RDF

$$g(r) = \mathcal{N}(Ae^{-r/l_c} + 1), \quad (10)$$

where  $l_c$  is the correlation length,  $A$  is the correlation strength and  $\mathcal{N}$  is a normalization factor assuring that eq. (9) holds. Since we use periodic boundary conditions, the RDF  $g(r)$  in eq. (8) is defined for all  $r$ , but for the normalization in eq. (9) we integrate the RDF only up to  $r_{max}$ . The algorithm used for creating target distributions starts from a random target distribution. Then one target is moved to a new random position; if the RDF is closer to the desired RDF in eq. (10) the new position is accepted, else other random positions are tried until one move is accepted. This process is then repeated for every target several times until the difference between the actual RDF and eq. (10) is below an error threshold, see fig. 2(e). The effect of increasing  $A$  can be seen in figs. 2(a) to (c).

For comparison, we also use circular-patch target distributions [18], see fig. 2(d). For these patch distributions, the number of targets per patch  $n$  and the patch radius  $R$  are chosen, then the patch centers are distributed on random positions and  $n$  targets are uniformly randomly distributed inside the patch radius  $R$  around each patch center. In the simulations each set of distribution parameters is used to create 500 different target distributions over which the search efficiency is averaged. Every simulation is  $10^5 \tau_D$  long and the searcher always starts in the middle of the simulation box (see figs. 2(a)–(d)).

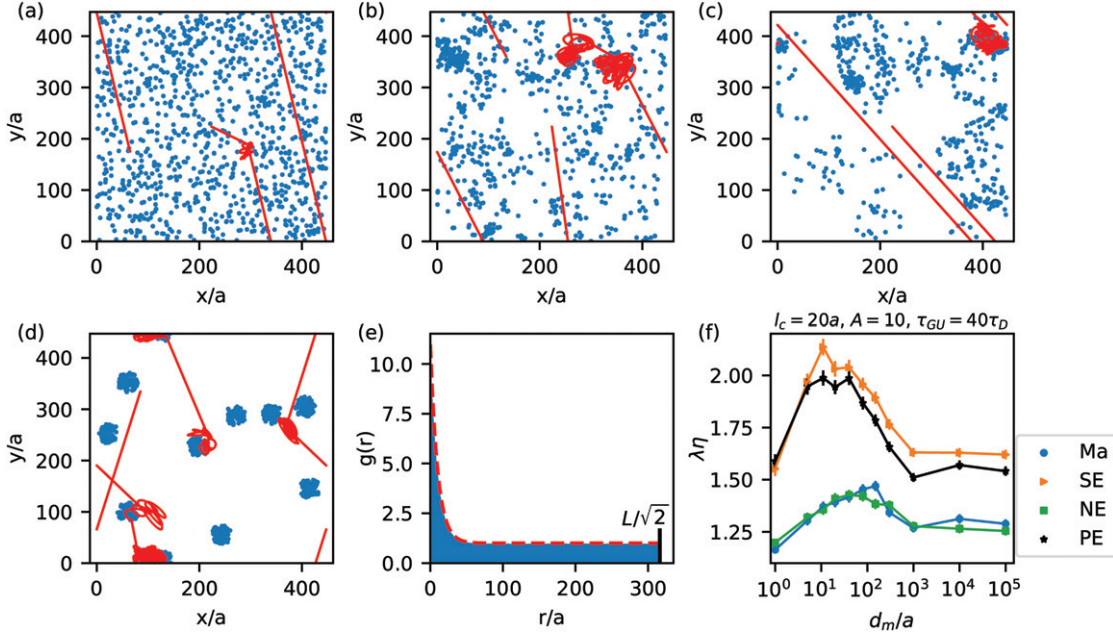


Fig. 2: Trajectories of a composite search where the intensive search mode is described by a single-exponential (SE) walk (eq. (7),  $\tau_m = 0.1\tau_D$ ,  $\tau_\Gamma = 100\tau_D$ ) with memory distance  $d_m = 40a$  and giving-up time  $\tau_{GU} = 100\tau_D$  for (a) uniformly random target distribution with  $A = 0$ , (b) correlated targets described by RDF parameters (eq. (10))  $A = 10$ ,  $l_c = 10a$ , (c) RDF parameters  $A = 50$ ,  $l_c = 10a$ , (d) targets that are distributed in ten circular patches with patch radius  $R = 20a$  and  $n = 100$  targets each. (e) The measured RDF from the target distribution in (b) (blue histogram) is compared with the preset RDF (red line) for  $A = 10$ ,  $l_c = 10a$ . (f) Comparison of the search efficiency as a function of the memory distance  $d_m$  for composite search strategies with different intensive search modes for  $\tau_{GU} = 40\tau_D$  and target distributions with correlation length  $l_c = 20a$  and strength  $A = 10$  (see eq. (10)). All other parameters are chosen to yield the highest efficiency which for the SE kernel leads to  $\tau_m = 0.1\tau_D$  and  $\tau_\Gamma = 100\tau_D$  and for the Markovian case (eq. (6)) to  $\tau_m = 1\tau_D$ . For the NE kernel the efficiency is highest in the Markovian limit ( $b \rightarrow 0$ ) and for the PE kernel in the SE kernel limit ( $\alpha \rightarrow 0$ , see eq. (7)). Thus, parameters close to the limits are chosen: for the NE kernel  $\tau_m = 1\tau_D$ ,  $b = -0.1\tau_D^{-2}$ ,  $\tau_\Gamma = 0.01\tau_D$ , for the PE kernel  $\tau_m = 0.1\tau_D$ ,  $\tau_\Gamma = 100\tau_D$ ,  $b = 0.09\tau_D^{-2}$  (see eq. (7)).

## Results. –

*Non-composite search.* We first consider non-composite search strategies where the motion is without interruption described by the GLE in eq. (3) and not modified by interactions with the targets. We show trajectories of the four different random walk types we consider in fig. 1(b), namely (b1) the Markovian persistent random walk (Ma), (b2) the single-exponential memory walk (SE), (b3) the negative-exponential memory walk (NE) and (b4) the positive-exponential memory walk (PE). The observed differences between these random walk trajectories are clearly reflected by their mean squared displacements in fig. 1(c): the persistent random walk exhibits a single crossover between ballistic motion at short times and diffusive motion at long times, the SE walk is characterized by pronounced oscillations in the MSD that in the trajectory show up as circular motion pattern, the NE walk shows a second transient ballistic regime at long times that gives the trajectory a rather elongated shape (similar to active-particle motion and run-and-tumble walks) and the PE exhibits an intermediate plateau regime in the MSD which in the trajectory is reflected by sections where the walk seems to halt [39]. Thus, the different random

walk models we consider exhibit rather different motion patterns. In fig. 1(d) we present the rescaled efficiency  $\lambda\eta$  for the four different random walk types as described by the GLE, eq. (3), as a function of the rescaled persistence time  $\tau_m/\tau_D$ . It is seen that  $\lambda\eta$  increases monotonically with increasing  $\tau_m/\tau_D$ : the straighter the search trajectory is, the closer is the efficiency to the maximal possible value of  $\lambda\eta = 1$  for non-composite search. For a given value of  $\tau_m/\tau_D$ , the SE and PE walks have the highest efficiency, which is not surprising since their MSD's ballistic regimes are the longest. The maximal efficiency  $\lambda\eta = 1$  is reached in the limit  $\tau_m/\tau_D \rightarrow \infty$ , *i.e.*, for straight trajectories or for ballistic motion. This is generally true independent of the target distribution as long as the searcher is completely uninformed and has no additional information about the targets.

*Composite search.* For an informed searcher that keeps track of the elapsed time after it has last found a target, as implemented in our composite search model, the efficiency can become higher than one target per MFPL, *i.e.*,  $\lambda\eta > 1$ , for correlated targets. In fig. 2(f) the efficiency of all four different random walk types in the composite search scenario is shown as a function of the memory

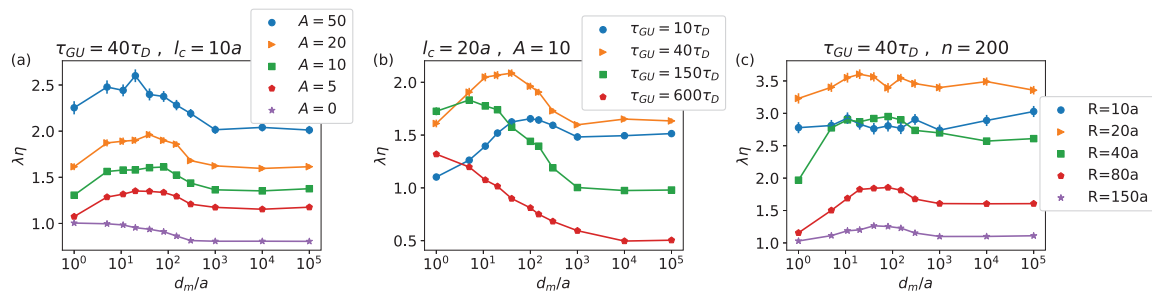


Fig. 3: Rescaled efficiency  $\lambda\eta$  as a function of the memory distance  $d_m$  for composite search using the single-exponential (SE) memory kernel, defined in eq. (7), for the non-Markovian motion in the intensive search modus with fixed memory time  $\tau_\Gamma = 100\tau_D$  and persistence time  $\tau_m = 0.1\tau_D$ . (a)  $\lambda\eta$  for fixed giving-up time  $\tau_{GU} = 40\tau_D$  and target distributions described by the RDF (eq. (10)) with fixed correlation length  $l_c = 10a$  and different values of the correlation strength  $A$ . (b)  $\lambda\eta$  for targets distributed according to the RDF (eq. (10)) with fixed  $l_c = 20a$  and  $A = 10$  for different values of the giving-up time  $\tau_{GU}$ . (c)  $\lambda\eta$  for fixed giving-up time  $\tau_{GU} = 40\tau_D$  and targets distributed in circular patches. The different symbols represent different patch radii  $R$  for fixed target number per patch  $n = 200$ .

distance  $d_m$  for a correlated target distribution characterized by a correlation length  $l_c = 20a$  and correlation strength  $A = 10$  for fixed giving-up time  $\tau_{GU} = 40\tau_D$ . All other parameters  $\tau_m$ ,  $b$  and  $\tau_\Gamma$  are optimized for each random walk type to yield the highest value of  $\lambda\eta$  for the optimal value of  $d_m$ , leading to parameter values given in the caption. Clearly, the non-Markovian SE walk leads to the highest search efficiency and significantly outperforms the Markovian persistent random walk. The PE walk has an efficiency that is much higher than the Markovian case and slightly lower than the SE walk for a given  $\tau_m$ , whereas NE walks have an efficiency that is slightly lower than Markovian walks. Interestingly, the efficiencies show a maximum at a finite memory distance  $d_m$  where the efficiency is significantly higher than in the limit  $d_m \rightarrow \infty$ , which represents the standard GUT searcher, and in the limit  $d_m \rightarrow 0$ , which corresponds to ballistic search and where the universal limit  $\lambda\eta = 1$  is found.

In the following we exclusively use the non-Markovian SE walk as it is in fig. 2(f) seen to be most efficient. In fig. 3(a) the composite search strategy using the SE walk is demonstrated to become more efficient as the target correlation strength  $A$  increases. Conversely, in the limit of uniformly randomly distributed uncorrelated targets, corresponding to  $A = 0$ , the composite search becomes less efficient than ballistic motion, which is reflected by the fact that  $\lambda\eta < 1$  for increasing memory distance  $d_m$ . The optimal memory distance  $d_m$  depends also quite sensitively on the giving-up time  $\tau_{GU}$ , as shown in fig. 3(b). The higher  $\tau_{GU}$  is, the shorter the optimal memory distance  $d_m$  becomes.

When searching for targets distributed in circular patches, the composite search efficiency can become even higher than for targets distributed according to the exponentially decaying RDF defined in eq. (10), as demonstrated in fig. 3(c). The efficiency is maximal for a patch radius around  $R = 20a$ , but there is no pronounced maximum in the efficiency as a function of the memory distance

$d_m$  unless the patch radius is so large that different patches start to overlap.

**Discussion.** – For destructible targets and without searcher-target interaction, *i.e.*, for non-composite search, there is no better search strategy than moving ballistically, which is true in the long time limit and for large search spaces [31,40]. This we demonstrate to hold also for non-Markovian random walks in fig. 1(d). This finding can be rationalized by noting that for destructible targets there is only a very small chance to find targets at places that have been visited before, thus, every crossing of the search trajectory with its own path makes the search less efficient. This holds even for the case of correlated targets: an uninformed searcher that samples a certain area very intensely without knowing if the target density is locally increased, will in the long time limit not increase its search efficiency.

In contrast, for correlated targets, a composite search model can perform better than moving just straight. Using non-Markovian motion described by the SE kernel for the intensive search mode yields the highest efficiency, see fig. 2(f), since the trajectory shape, shown in fig. 1(b), presumably samples target clusters most efficiently. This means that non-Markovian motion can be advantageous as the intensive search mode in a composite search. Besides, non-Markovian motion of the SE and PE type exhibits for given persistence time high search efficiencies also in the absence of searcher-target interaction, as shown in fig. 1(d). This might explain why some organisms exhibit non-Markovian motion, such as cancer cells [38] or slime mould [41,42], as an adaption to the need to search efficiently for resources. Clearly, evolutionary adaptation has been achieved in multiple ways, for instance by chemotaxis or other sensory abilities, which certainly can increase the search efficiency [25]. Nevertheless, in areas of low target gradients, the searcher needs to use search strategies that do not rely on chemotactic clues, in which case non-Markovian motion can be advantageous. While we focus

on exponentially decaying memory kernels, power-law kernels constitute an interesting model as well but are rather difficult to simulate by means of Markovian embedding and therefore are not considered by us.

The efficiency of a composite random search with searcher-target interaction strongly depends on the target distribution and increases with increasing target correlation strength  $A$ , as shown in fig. 3(a). For targets distributed in circular patches, the maximal search efficiency is particularly high, as seen by comparison of fig. 3(c) and (a). The introduction of the memory distance  $d_m$  leads to a higher certainty for the searcher to be in an area with high target density when switching to intensive search mode and therefore, the efficiency can be higher than the standard GUT search algorithm, which is recovered in the limit  $d_m \rightarrow \infty$ , as shown for the case when targets are distributed according to the RDF (eq. (10)) in fig. 2(f). In some sense, the concept of a memory distance constitutes a simple estimator of the local target density and allows the searcher to efficiently switch to intensive search. If targets are distributed in circular patches, the extra certainty to be inside a cluster obtained by the comparison with the memory distance  $d_m$  brings however no advantage, because finding a target is strictly equivalent to finding a patch. Therefore, for patchy target distributions the best strategy is to change to intensive search every time a target is found, *i.e.*,  $d_m \rightarrow \infty$ , as can be seen in fig. 3(c). Nonetheless, when the target patch radius becomes large, such that patches overlap, it once again becomes advantageous to have a finite memory distance, as seen in fig. 3(c). The overlap of patches leads to areas that are more dense in targets and thus a finite memory distance can increase the probability to search in such overlap regions. We have not varied the mean target density and have not considered search in three dimensions, but would assume that a maximum in the memory distance appears in higher dimensions and for different densities as well, so that the concept of a memory distance is likely also useful for efficient searching in these more general situations.

It is important to note that the concept of a memory distance does not require more abilities than the standard GUT searcher has already, but leads to higher or equal efficiencies. Hence, it is plausible that biological searchers that presumably perform GUT search, actually use composite search with a finite memory distance, especially when targets are correlated but not exclusively found in patches. As a matter of fact, this might apply to the behavior of ladybugs, which tend to increase  $\tau_{GU}$  after being initially fed with more or higher quality prey [43,44]. This indicates that the certainty of being inside a target cluster might be measured by successive prey consumption, which is captured in our composite search model that includes a finite memory distance. Our model is not designed for a specific organism or specific spatial scale and should be applicable to the search process of a single cell equally well as to the search process of insects

or mammals. Nevertheless, our model is rather simple and higher evolved organisms might actually use more sophisticated mechanisms. More complex models will certainly lead to even higher search efficiencies by using for instance target quality evaluation [26], mental mapping of all target positions [27], or additional sensory cues [25].

It is instructive to compare our results with Lévy walk search strategies which have been amply studied. An uninformed Lévy walk search strategy, characterized by a power-law step length distribution of the form  $p(l) \propto l^{-\mu}$ , does not outperform ballistic motion in an unconfined search space for destructible immobile targets [31]. In contrast, when the search space is confined or targets are non-destructible, there are even uninformed search strategies that can be more efficient than moving on a straight line, for instance Lévy walks [31] or composite random walks [45]. Moreover, if a searcher-target interaction is introduced in a Lévy search model, namely if the searcher stops at every found target [31,46], Lévy search strategies lead to higher efficiencies in the case when targets are clustered or non-destructible. In fact, the most efficient GUT Lévy search strategy for different target distributions that involve circular patches was shown to consist of segments of straight motion (corresponding to  $\mu \rightarrow 1$ ) interrupted by pure diffusion (corresponding to  $\mu \geq 3$ ) as the intensive search mode [25]. Since diffusion can be represented by Markovian motion with low persistence time  $\tau_m$  and the Markovian intensive search strategy is in fig. 2(f) shown to be less efficient than motion described by the SE kernel (eq. (7)), we conclude that the most efficient composite search using an SE kernel is more efficient than the most efficient GUT Lévy search strategy for targets distributed in patches as well as targets distributed according to eq. (10).

**Conclusions.** – We analyze the search efficiencies of rather general non-Markovian composite random search strategies and different target distributions. We reconfirm that uninformed searchers which do not interact with the targets exhibit an upper bound for the efficiency of one target per MFPL when targets are destructible, which is reached for a ballistic searcher moving straight in one direction. Hence, the efficiency of non-composite random walks that are described by the GLE (eq. (3)) is the highest for very large persistence times but for finite persistence time depends on the type of the utilized memory kernel: for instance the SE kernel (eq. (7)) and the PE kernel (eq. (7)) lead to higher efficiencies, especially for long memory time  $\tau_\Gamma$ , compared to the Markovian case (eq. (6)). Our results suggest that composite search with a non-Markovian intensive search mode can be more efficient than a Lévy walk when searching for correlated targets. Furthermore, the non-Markovian composite search using the SE kernel (eq. (7)) for the intensive search mode leads to higher search efficiencies than the Markovian motion (eq. (6)). For rather smooth spatial target distributions described by the RDF (eq. (10)), a finite memory distance

$d_m$  leads to higher search efficiencies than the standard GUT searcher which is recovered in the limit  $d_m \rightarrow \infty$ . This is an interesting finding, since usage of a finite memory distance does not require the searcher to have more abilities beyond being able to measure the time elapsed since the last target encounter. Thus, we suggest that simple organisms, searching for correlated targets without the ability to probe the environment, are likely to use a composite search with a finite memory distance in order to increase their search efficiency.

\*\*\*

We acknowledge the funding by the Deutsche Forschungsgemeinschaft (DFG) through grant CRC 1114 “Scaling Cascades in Complex Systems”, Grant No. 235221301, Project B03, and by the ERC Advanced Grant No. 835117.

*Data availability statement:* The data that support the findings of this study are available upon reasonable request from the authors.

## REFERENCES

- [1] BELTMAN J. B., MARÉE A. F. M. and DE BOER R. J., *Nat. Rev. Immunol.*, **9** (2009) 789.
- [2] KOLESAR P., *J. Oper. Res. Soc.*, **33** (1982) 153.
- [3] VISWANATHAN G. M., DA LUZ M. G. E., RAPOSO E. P. and STANLEY E. H., *The Physics of Foraging: An Introduction to Random Searches and Biological Encounters* (Cambridge University Press) 2011.
- [4] PERRY G. and PIANKA E. R., *Trends Ecol. Evol.*, **12** (1997) 360.
- [5] KAREIVA P. and ODELL G., *Am. Nat.*, **130** (1987) 233.
- [6] WEIMERSKIRCH H., PINAUD D., PAWLOWSKI F. and BOST C.-A., *Am. Nat.*, **170** (2007) 734.
- [7] PYKE G. H., PULLIAM H. R. and CHARNOV E. L., *Q. Rev. Biol.*, **52** (1977) 137.
- [8] PYKE G. H., *Annu. Rev. Ecol. Syst.*, **15** (1984) 523.
- [9] PARPINELLI R. S., LOPES H. S. and FREITAS A. A., *IEEE Trans. Evol. Comput.*, **6** (2002) 321.
- [10] MASRI S. F., BEKEY G. A. and SAFFORD F. B., *Appl. Math. Comput.*, **7** (1980) 353.
- [11] KHADEM S. M. J., KLAPP S. H. L. and KLAGES R., *Phys. Rev. Res.*, **3** (2021) 023169.
- [12] LEVERNIER N., TEXTOR J., BÉNICHOU O. and VOITURIEZ R., *Phys. Rev. Lett.*, **124** (2020) 080601.
- [13] RO S. and KIM Y.-W., *Phys. Rev. E*, **96** (2017) 012143.
- [14] HOLLINGER G., SINGH S., DJUGASH J. and KEHAGIAS A., *Int. J. Rob. Res.*, **28** (2009) 201.
- [15] TEJEDOR V., VOITURIEZ R. and BÉNICHOU O., *Phys. Rev. Lett.*, **108** (2012) 088103.
- [16] PLYULIN V. V., BLACKBURN G., LOMHOLT M. A., WATKINS N. W., METZLER R., KLAGES R. and CHECHKIN A. V., *New J. Phys.*, **21** (2019) 103028.
- [17] SCHARF I., KOTLER B. and OVADIA O., *Evol. Ecol.*, **23** (2009) 245.
- [18] HUISMAN T. J., *Search Performance and the Spatial Resource Distribution* (Wageningen University) 2014.
- [19] VISWANATHAN G. M., RAPOSO E. P. and DA LUZ M. G. E., *Phys. Life Rev.*, **5** (2008) 133.
- [20] BENHAMOU S., *Ecology*, **88** (2007) 1962.
- [21] BENHAMOU S., *J. Theor. Biol.*, **159** (1992) 67.
- [22] BÉNICHOU O., LOVERDO C., MOREAU M. and VOITURIEZ R., *Rev. Mod. Phys.*, **83** (2011) 81.
- [23] CARTER M. C. and DIXON F. G., *J. Anim. Ecol.*, **51** (1982) 865.
- [24] BELL W. J., *Annu. Rev. Entomol.*, **35** (1990) 447.
- [25] NOLTING B. C., HINKELMAN T. M., BRASSIL C. E. and TENHUMBERG B., *Ecol. Complex.*, **22** (2015) 126.
- [26] KRAKAUER D. C. and RODRIGUEZ-GIRONES M. A., *J. Theor. Biol.*, **177** (1995) 417.
- [27] FRONHOFFER E. A., HOVESTADT T. and POETHKE H.-J., *Oikos*, **122** (2013) 857.
- [28] CAROF A., VUILLEUMIER R. and ROTENBERG B., *J. Chem. Phys.*, **140** (2014) 124103.
- [29] KLIPPENSTEIN V., TRIPATHY M., JUNG G., SCHMID F. and VAN DER VEGT N. F. A., *J. Phys. Chem. B*, **125** (2021) 4931.
- [30] CHUPEAU M., BÉNICHOU O. and REDNER S., *Phys. Rev. E*, **95** (2017) 012157.
- [31] BARTUMEUS F., DA LUZ M. G. E., VISWANATHAN G. M. and CATALAN J., *Ecology*, **86** (2005) 3078.
- [32] MORI H., *Prog. Theor. Phys.*, **33** (1965) 423.
- [33] ZWANZIG R., *Phys. Rev.*, **124** (1961) 983.
- [34] NETZ R. R., *Phys. Rev. E*, **101** (2020) 022120.
- [35] ZWANZIG R., *Nonequilibrium Statistical Mechanics* (Oxford University Press) 2001, pp. 143–160.
- [36] KAPPLER, J., DALDROP J. O., BRÜNIG F. N., BOEHLE M. D. and NETZ R. R., *J. Chem. Phys.*, **148** (2018) 014903.
- [37] NETZ R. R., *J. Chem. Phys.*, **148** (2018) 185101.
- [38] MITTERWALLNER B. G., SCHREIBER C., DALDROP J. O., RÄDLER J. O. and NETZ R. R., *Phys. Rev. E*, **101** (2020) 032408.
- [39] MITTERWALLNER B. G., LAVACCHI L. and NETZ R. R., *Eur. Phys. J. E*, **43** (2020) 1.
- [40] JAMES A., PLANCK M. J. and BROWN R., *Phys. Rev. E*, **78** (2008) 051128.
- [41] TAKAGI H., SATO M. J., YANAGIDA T. and UEDA M., *PLoS ONE*, **3** (2008) e2648.
- [42] LI L., COX E. C. and FLYVBJERG H., *Phys. Biol.*, **8** (2011) 046006.
- [43] NAKAMUTA K., *J. Insect Physiol.*, **31** (1985) 849.
- [44] NAKAMUTA K., *Appl. Entomol. Zool.*, **17** (1982) 501.
- [45] PINA-GARCIA C. A., GU D. and HU H., *International Conference on Intelligent Robotics and Applications* (Springer, Berlin) 2011, pp. 620–629.
- [46] VISWANATHAN G. M., AFANASYEV V., BULDYREV S. V., HAVLIN S., DA LUZ M. G. E., RAPOSO E. P. and STANLEY H. E., *Physica A*, **282** (2000) 1.