5. APPENDICES

5.1 Appendix A. Simulations Illustrating Biases in Estimating General and Specific Effects Using Stepwise Procedures

The two simulated models presented here serve as examples of how stepwise estimation procedures can bias results and fail to detect existing age relations of specific factors. It is clear that such simulated examples can be set up in countless different ways, of which many might not exhibit the problems illustrated here. Rather than trying to give examples that resemble reality as close as possible, therefore, these simulations should serve to demonstrate a general problem of stepwise procedures: the assumptions incorporated in an earlier step influence the validity of results obtained in later steps if parameters are fixed from one step to the next. In the Salthouse (1998) procedure of first estimating and fixing a general effect, this leads to an over- rather than an underestimation of the general effect. In the Christensen et al. (2001) approach, different combinations of specific effects might lead to all sorts of biased patterns of results if the specific effect that is chosen to be fixed to zero is actually different from zero.

5.1.1 Salthouse (1998) and Allen et al. (2001) Procedure

The first model (Panel A in Figure A1) is a hierarchical model, just as Model 4 in Salthouse and Czaja (2000) and in Allen et al. (2001). The model was set up with nine indicator variables, V1 to V9 that load on three first-order factors, F1 to F3. These factors, in turn, load on one general factor (g), which was set up to explain half of each factor's age-independent variance. This implies that the other half is reliable specific variance, which is represented by the specific uniqueness (or disturbance) factors S1 to S3. This distribution of general and specific variance into equal proportions is more typical for young than for old samples—where usually the general factor will explain a larger portion of variance. Nonetheless, this model setup was chosen to show that even unusually strong specific age-associated effects could still be missed if the stepwise procedure was used. Standardized age-associated effects, therefore, were set to -.57 for the general and to -.57, +.20, and zero, for the specific factors, respectively. This means that the negative age correlations of the observed variables could be explained by a strong general decline,

which is still increased for the first three variables by an additional specific negative effect and counteracted to some degree by a positive age relation of the second specific factor for variables V4 to V6. For V1 to V3, the model-implied age correlation was calculated as $-.57*.61*.92 + (-.57*.61*.92) = -.64$, while for V4 to V6 it was $-.57*.81*.86 + (.20*.68*.86) = -.28$, and for V7 to V9, $-.57*.77*.87 = -.38$.
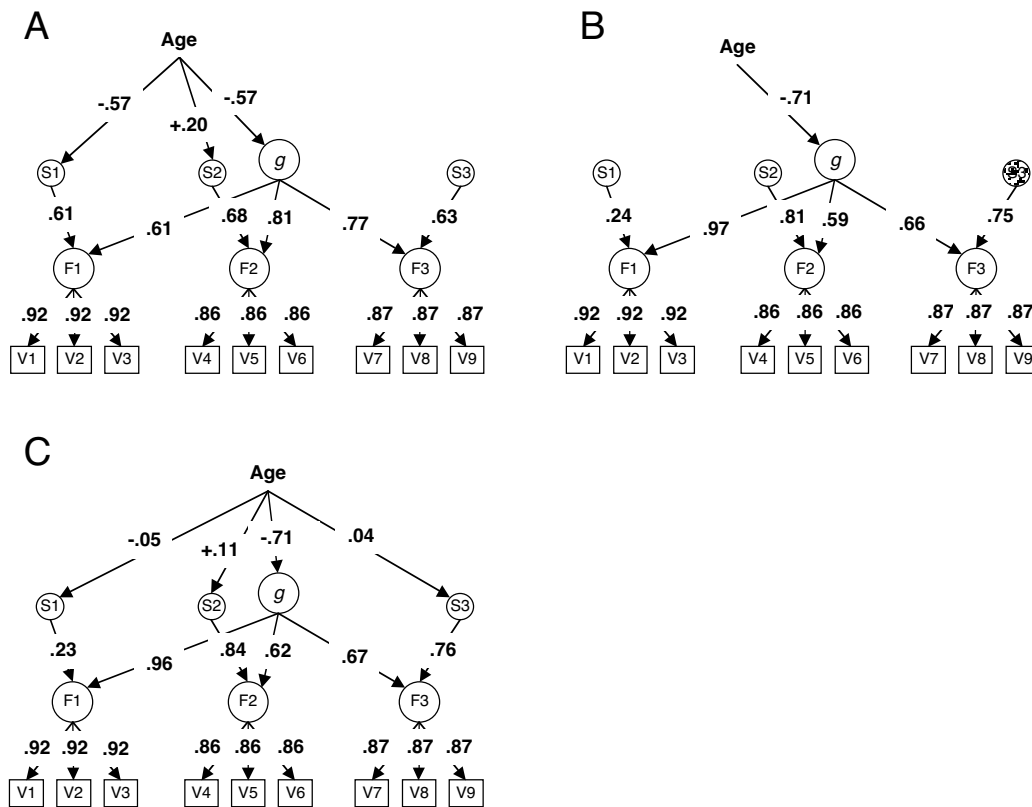

Figure A1. Simulated example illustrating the stepwise procedure by Salthouse (1998). A: Hierarchical model with one general and three first-order factors and true age-associated effects. B: Model with only an age-associated effect on the general factor specified. C: Estimates of specific effects with measurement model and general age-associated effect fixed to values estimated in the previous model. All parameter values are standardized estimates.

In this example, when the model with only a general effect was fitted to the data generated by the true model this resulted in a standardized parameter estimate of -.71 (Figure A1, Panel B), a non-trivial overestimation of the true effect, which was -.57. The estimates for the loadings of the first-order factors on the general factor were biased, with loadings being overestimated for the first (.97 instead of .61), and underestimated (.59 instead of .81) for the second and third first-order factors.

However, with a RMSEA value of .053 (χ^2 [32] = 44.89, assuming N = 500) this model was showing a fit that could be considered satisfactory[1]. Fixing the parameter estimates from this model and introducing paths from age to the specific factors, resulted in non-significant paths for the specific factors S1 and S3, and a significant path of +.11 to S2 (Figure A1, Panel C). The strong specific path from age to S1 in our true model, therefore, was completely absorbed by the general factor. The path to S2 of +.11 was smaller than the true effect of +.20.

5.1.2 Christensen et al. (2001) Procedure

The procedure used by Christensen et al. (2001) seeks for a non-significant specific effect that can be fixed to zero in a model where the general and all remaining specific effects are specified. The simulated example used here has a measurement model equivalent to our previous simulation example, but different values for the general and specific aging effects. The standardized general effect was set to -.49, and the effects on the three specific factors S1 to S3 to -.39, -.39, and -.44 (Figure A2, Panel A).

Testing a model with only the general and the specific effect on S3 resulted in an estimate of -.70 for the general and -.04 (n.s. for N = 500) for the specific effect (Figure A2, Panel B). In this incorrect model, the other two specific effects were obviously absorbed by the effect on the general factor, which was overestimated and therefore also reduced the specific effect that became non-significant. Continuing this procedure by fixing the third specific effect to zero and testing a model with the general effect and the other two specific effects led to an overestimation of the general effect with an estimate of -.73, and underestimation of the specific effects with non-significant estimates of .04 for the first two specific factors (Figure A2, Panel C).

---

[1] Because the model was fitted to a population covariance matrix instead of sample data, the RMSEA was calculated taking expected sample variability into account, i.e., RMSEA = Sqrt[χ^2(Min)/(df*N)]).
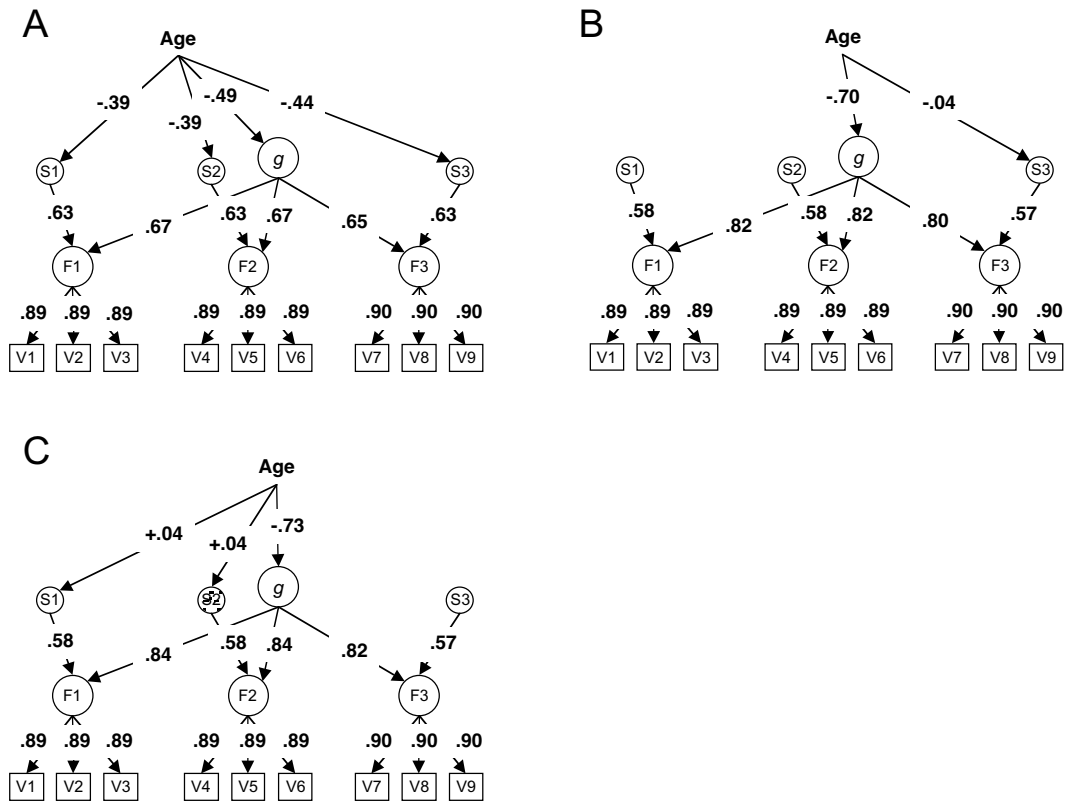
Figure A2.   Simulated example illustrating the stepwise procedure by Christensen et al. (2001). A: Hierarchical model with one general and three first-order factors and true age-associated effects. B: Model with only an age-associated effect on the general factor and the third specific factor specified. C: Estimates of general and specific effects with the age-associated effect on the third specific factor fixed to zero. All parameter values are standardized estimates.

5.2   Appendix B. Examining the Parameter Dynamics of the Nested Factor Model

The simulations presented here were conducted to demonstrate that the nested factor model allows estimating age relations of the general and all specific group factors simultaneously. Furthermore, the influence of empirical heterogeneity in the relative contribution of general and specific variance to the indicator variables on parameter sensitivity and interdependence was investigated. By comparing across simulations, it is demonstrated that the proportionality constraint intrinsic to the hierarchical model relates directly to the problem of parameter dependency during estimation, which forced previous approaches to resort to the stepwise estimation procedure. The extent of the proportionality constraint imposed either by the model structure or by empirically observed covariance patterns is reflected in the parameter dynamics of the model.

The correlation matrix of parameter estimates contains information about these parameter dynamics (e.g., S.-C. Li, Lewandowsky, & DeBrunner, 1996). Depending on how sensitive a model is to changes in its parameter values (i.e., parameter sensitivity), different degrees of uncertainty are associated with the estimates and interpretations of the estimated parameter values thus should be made with caution (Green, 1977; S.-C. Li et al., 1996). The diagonal elements of the parameter covariance matrix, which can be obtained as a part of the output from SEM programs, contain information about parameter sensitivity. The square root of these elements gives an asymptotic estimate of the standard errors associated with the estimated parameter values and, therefore, about the precision of the estimates. High parameter sensitivity implies small standard errors and therefore high precision.

In addition, parameter estimates in a multi-parameter model in general are not independent of each other; rather they tend also to covary (i.e., parameter interdependence) to varying degrees during estimation. The off-diagonal elements of the parameter covariance matrix carry information about parameter inter-dependence. Furthermore, analyses that systematically vary the value of one parameter across a theoretically meaningful range and examine how the other parameters change accordingly can be used to examine these two aspects of parameter dynamics (i.e., parameter sensitivity and interdependence) of different models in detail.

5.2.1 Simulation Example of the Nested Factor Model

All simulations include nine variables, which all load on a general factor and on three orthogonal specific factors that each have three indicator variables. In all simulations, the standardized age-related effects were set to the same values as for the hierarchical model presented in Appendix A (i.e., -.57 for the general, and -.57, +.20, and .00 for the specific factors). The diagram presented in the panel A of Figure B1 shows the structure of a typical nested factor model with additional directed paths from the age variable specified.
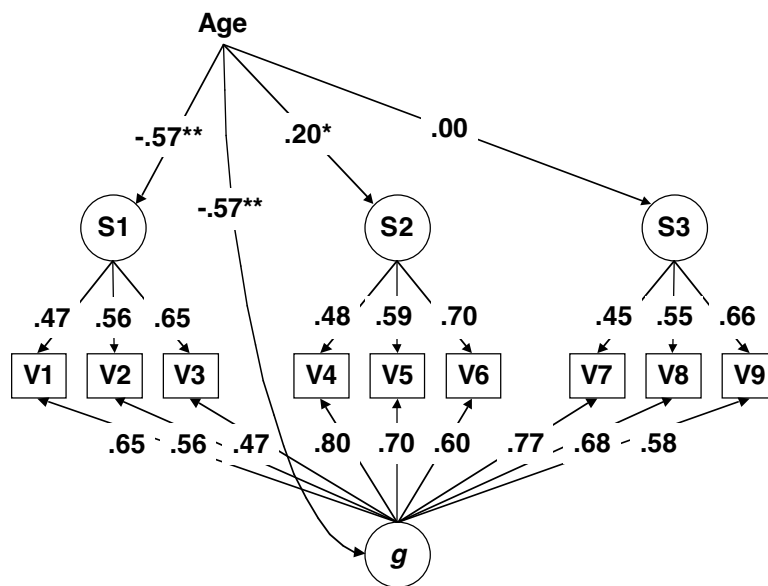
Figure B1.   Simulation example with the nested factor model and directed age-associated effects on the general and specific factors. All parameter values are standardized estimates.

For the first simulation, the unstandardized loadings of the variables on the specific factors were set to .50, .60, and .70 within each subset of indicator variables and their corresponding loadings on the general factor were set to .70, .60, and .50. These factor loadings were chosen as such to magnify the difference between the nested factor model and the hierarchical model, which constrains the ratio of general to specific variance to be equal across variables of a given first-order factor. To the contrary, these factor loadings express a pattern of disproportionality (or heterogeneity) across the indicator variables—higher loadings on the general factor were associated with lower loadings on the specific factors and vice versa. Fitting a nested factor model to the simulated covariance matrix with paths from age to all

four factors results in a recovery of the true parameter values. The problem of parameter linear dependency that is present in the hierarchical model is no longer a problem here.

5.2.2 Simulation Example With Empirical Proportionality of General and Specific Factor Loadings

As a comparison to the first simulation with unconstrained proportionality, a second example was set up with the unstandardized loadings set to .70, .60, and .50 for the general, and .35, .30, .25 for the specific factors within each triplet of variables—to simulate a strict proportionality constraint as in the hierarchical model. Trying to estimate the parameters of age-related effects in this model showed that the solution becomes inadmissible, due to parameter linear dependencies among the estimates of the general and specific age relations. Empirical proportionality in an unconstrained model, obviously, leads to the same problem of model identification as proportionality implied by the model structure.

5.2.3 Simulation Examples With Varying Degrees of Heterogeneity of the General to Specific Variance Ratio

In the first simulation of unconstrained proportionality—where the unstandardized loadings of the specific factors were set to .50, .60, and .70 within each subset of the indicator variables and their corresponding loadings on the general factor were set to .70, .60, and .50—the estimate of the general effect correlated between -.53 and -.57 with the estimates of the specific effects, which had correlations ranging from .43 to .49 among themselves. The standard errors associated with the estimates were relatively small (SE = .08 for the unstandardized general effect of -.70 and SE = .09 for the unstandardized specific effects of -.70 and +.20), indicating high parameter sensitivity.

In the third simulation set up to simulate a more strict—but not exact—proportionality constraint, the unstandardized loadings were .62, .60, and .58 for the general, and .58, .60, and .62 for specific factor within each triplet of variables. Fitting the nested factor model to the simulated covariance matrix with all possible age-associated effects again resulted in a perfect recovery of the true parameter values. However, the correlations of the parameter estimates were now -.97 between general and specific effects, and about .96 between different specific effects, indicating a

greater extent of parameter interdependence. Furthermore, the parameter sensitivity was markedly lower than in the first simulation, as reflected in the much larger standard errors associated with the estimates—the standard error was now $SE = .35$ for all three unstandardized estimates of general and specific effects, rendering the positive specific effect nonsignificant. This comparison showed that increasing the extent of the proportionality of the ratio of general to specific variance across the indicator variables—while keeping all other aspects of the simulation constant—resulted in less optimal parameter dynamics and a considerable increase in the uncertainty about the true effect sizes.

The negative covariation between the estimates of the general and specific effects found in the simulations in Appendix B implies that the uncertainty inherent in the estimate of the general effect is reflected in the uncertainty about the specific effects. If the true general effect in the population would be somewhat lower than its estimate, the specific effects would be expected to be accordingly higher, and vice versa. This inverse relationship stems from the fact that the nested factor model implies that the variances of the observed variables potentially can be explained jointly by two pathways—one through the general and one through the specific factors. The higher the estimate for one, the lower the other one has to be in order to account for the observed variance–covariance pattern. This means that while it is not impossible to disentangle general and specific effects in multivariate cross-sectional data, the separation of the two is associated with some uncertainty.

5.2.4 Simulation Example With Age as a Correlate

To demonstrate the use of age as a correlate of the nested factors, such a model was also fitted to the simulated data (see Figure B2). A comparison of its factor loadings to those of the age-as-a-predictor model (Figure B1) shows that for the general factor, the loadings of the indicator variables of the first factor were higher, and for those of the second factor lower. However, the loadings on the specific factors were almost exactly the same in both models. The age correlations of the nested factors were -.57 for the general and -.42, .10 and .00 for the specific factors. This shows that the patterns of age relations with the general and specific factors were similar across the two models. Thus, the age-as-a-covariate model shares with the age-as-a-predictor model the property of being able to detect specific age-associated effects. Furthermore, in the age-as-a-covariate model it is also possible to calculate how

much variance in chronological age could be explained by the nested general and specific factors.
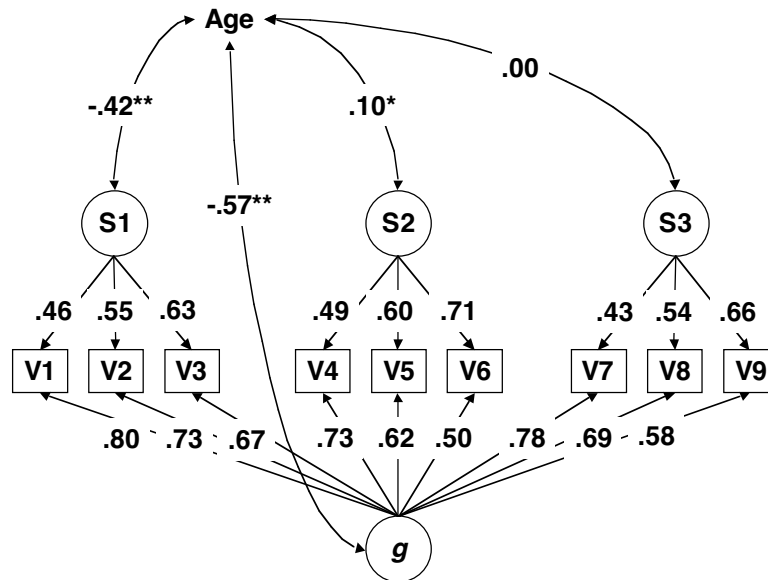
Figure B2. Simulation example with the nested factor model and the general and specific factors correlating with age. All parameter values are standardized estimates.

If age is treated as a dependent variable in the simulated model in Figure B2, 51% of the age variance can be predicted. This summary measure of the interrelatedness of the multivariate individual differences in the nine tasks and age can be easily calculated by summing up the squared age correlations of the orthogonal general and specific factors ($-.57^2 + -.42^2 + .10^2 = .51$).

5.3 Appendix C. Equivalence between Nested Factor Loading Matrix based on Age-Partialled Covariance Matrix and Nested Factor Loading Matrix of a Model Specifying Age-Associated Effects on All Factors

The covariance matrix of the indicator variables (Y) and age is given by

$$\Sigma = \begin{bmatrix} \Sigma_{YY} & \Sigma_{YAge} \\ \Sigma_{AgeY} & \sigma_{Age}^2 \end{bmatrix} = \begin{bmatrix} \Lambda_Y \left(\Gamma \sigma_{Age}^2 \Gamma' + \Psi \right) \Lambda_Y' + \Theta_\varepsilon & \Lambda_Y \Gamma \sigma_{Age}^2 \\ \sigma_{Age}^2 \Gamma' \Lambda_Y' & \sigma_{Age}^2 \end{bmatrix} \quad (1)$$

with

Σ_{YY}: the covariance matrix of the indicator variables,

Σ_{AgeY}: the covariance vector of the indicator variables with age,

σ_{Age}^2: the variance of age,

Λ_Y: the nested factor model factor-loading matrix,

Γ: a vector of age-associated effects on the nested factors,

Ψ: a diagonal matrix of residual factor variances,

Θ_ε: the covariance matrix of the error terms.

The age-partialled covariance matrix of the observed variables is given by

$$\Sigma_{YY \cdot Age} = \Sigma_{YY} - \Sigma_{YAge} \left(\sigma_{Age}^2 \right)^{-1} \Sigma_{AgeY} \quad (2)$$

Following Levin (1987), one obtains (3) by substituting (2) with elements from (1):

$$\Sigma_{YY} - \Sigma_{YAge} \left(\sigma_{Age}^2 \right)^{-1} \Sigma_{AgeY} =$$
$$\Lambda_Y \left(\Gamma \sigma_{Age}^2 \Gamma' + \Psi \right) \Lambda_Y' + \Theta_\varepsilon - \Lambda_Y \Gamma \sigma_{Age}^2 \Gamma' \Lambda_Y' \quad (3)$$
$$= \Lambda_Y \Psi \Lambda_Y' + \Theta_\varepsilon$$

Because Ψ is a diagonal matrix with its entries being the residual factor variances of the age-as-a-predictor model, the factor analytic solution for the age-partialled

covariance matrix differs only with respect to the factor variances—while the factor loading matrix is invariant.

5.4 Appendix D. Activities Coded in the Yesterday Interview and Their Correlations With Cognitive Ability Tasks

Table D1

Activities Coded in the Yesterday Interview and Correlations with Cognitive Ability Tasks

Activity Category	AN	LS	PR	SW	BE	PK	IP	DL	DS	WB	CA	PA	AR	MT
Rising	.00	.02	.05	.05	.03	.04	.06	-.01	.01	.05	.05	.14	.10	.12
Self-care	-.05	-.09	-.08	-.07	-.07	-.10	**-.13**	**-.16**	**-.17**	**-.14**	**-.14**	**-.12**	**-.14**	-.10
Eating	-.11	-.09	-.10	-.06	-.09	**-.12**	-.11	**-.13**	-.10	-.09	**-.15**	-.10	**-.14**	-.05
Going to bed	.11	**.16**	.07	.06	.05	.07	.05	.09	.06	.03	**.12**	.00	.06	.06
Other basic care	.10	**.13**	.11	.05	.08	**.12**	**.20**	.13	**.17**	.12	**.18**	.17	**.21**	**.17**
Shopping	.05	.07	.02	.06	-.01	.09	**.14**	**.16**	**.14**	**.16**	.13	.13	.13	.04
Light house chores	.02	.01	.00	.03	.00	.00	.05	.00	.00	.04	.01	.00	.00	.03
Major house chores	.02	-.04	-.01	-.02	-.01	-.02	.02	-.02	.03	-.02	.00	.02	.04	.03
Craft/needlework	.10	**.16**	**.15**	.11	**.12**	**.14**	**.13**	**.14**	**.19**	.12	**.13**	**.15**	**.14**	.11
Other chores	.05	.05	.00	.07	.08	.10	.02	.00	-.02	.01	.07	.07	.10	.08
Dealing with bank	.06	.03	.01	-.03	.06	.05	-.03	-.05	.00	.00	.03	.04	-.02	.05
Dealing with authorities	.05	.00	.10	.06	.09	.05	.04	.02	.04	.03	.08	.05	.06	.06
Dealing with post office	-.02	.04	.01	.05	.08	.08	.03	.03	.02	.01	.04	.05	.07	-.01
Dealing with other forms	-.03	.02	.03	.00	.01	.01	.03	.00	.02	.03	.03	.01	.06	.08
Medical treatment	-.04	-.07	-.07	-.03	-.07	-.06	-.04	-.04	-.04	-.01	-.07	-.03	-.03	-.01
Self-treatment	**.16**	**.17**	**.20**	.11	.11	**.17**	**.19**	**.21**	**.22**	**.14**	**.19**	**.18**	**.17**	**.20**
Cultural activity	.04	-.01	.07	.04	.03	.07	.05	.04	.09	.02	.02	.01	.02	.04
Educational activity	.18	.08	.09	.05	.09	.08	.07	.06	.07	.03	.11	**.12**	.05	.06
Sports	.04	.00	.02	.02	.04	.02	**.12**	.04	.09	.06	.09	.02	.08	.08
Creative activity	.05	.02	.03	.04	.04	.09	**.13**	.08	.08	.01	.10	.03	.05	.04
Garden/flower work	-.05	-.05	.00	-.01	.01	-.05	.01	.01	.04	.09	.03	.04	-.01	-.02
Walking	**.13**	.04	.06	.08	.05	.06	.07	.05	.05	.06	.05	.06	.05	.07
Excursion	.07	.06	.02	**.14**	**.16**	.11	.09	.08	.09	.10	.09	.07	.02	-.04
Reading	.10	.05	.05	**.14**	.09	.00	.06	.07	.07	.05	.03	.08	-.02	.05
Writing	**.13**	**.17**	.08	**.13**	.09	.09	**.15**	**.24**	**.21**	**.19**	**.14**	.09	**.13**	.08
Playing	-.06	-.06	-.01	-.01	.04	.02	-.08	-.04	-.02	-.09	-.05	-.04	-.01	-.01
Watching TV	.08	-.01	.05	.07	.03	.10	**.13**	.09	.03	.00	.09	-.04	.05	-.07
Listening radio/tape	-.03	-.06	.01	-.02	-.02	.04	-.01	-.04	.00	-.07	.00	-.01	.02	.01
Religious activity	.10	**.12**	.02	.06	.03	.02	.06	.03	.09	.08	.04	.09	.03	.02
Political activity	-.02	.03	.02	-.07	-.02	-.01	.02	.05	.05	.03	.07	-.06	-.06	-.02
Other leisure activity	.03	.04	-.01	.03	.04	.00	-.01	-.02	-.04	.00	.00	.00	-.02	-.01
Talking to people	.01	.01	.05	.01	.05	.11	.07	.06	.07	.06	.08	-.01	.07	.05
Visiting	.09	.10	.01	-.04	.03	.04	.06	.07	.09	.02	.02	.09	.08	-.01
Phone calls	.04	.05	.03	.06	.06	.01	.08	.05	.10	.05	.09	**.12**	**.12**	**.14**

(table continues)

Table D1 (continued)

Activity Category	AN	LS	PR	SW	BE	PK	IP	DL	DS	WB	CA	PA	AR	MT
Other social activity	-.02	-.02	-.02	-.11	-.10	-.11	-.10	-.09	-.08	-.07	-.10	-.05	-.08	-.07
Regular paid work	**.12**	.10	.11	.06	**.12**	**.12**	.06	.10	.08	.10	.15	.09	.08	.06
Other work	.10	.06	**.12**	.07	.09	.10	.10	.02	.05	**.14**	.09	.11	.01	.11
Helping family member	.07	.11	**.13**	.02	.04	.08	.04	.10	.08	.08	.10	.04	.07	.01
Helping other person	.11	**.18**	**.12**	.04	.06	.08	.08	**.12**	**.13**	**.12**	.02	.11	**.12**	.08
Passive locomotion	**.19**	**.24**	**.23**	.10	**.14**	**.15**	**.23**	**.26**	**.25**	**.20**	**.24**	**.16**	**.17**	**.15**
Active locomotion	**.14**	.08	**.15**	-.01	.04	.09	**.18**	**.12**	**.15**	**.13**	**.16**	**.16**	**.18**	**.19**
Sleeping/day	-.09	-.05	-.05	-.01	-.01	-.07	-.10	-.09	-.11	-.10	**-.12**	-.06	**-.12**	-.06
Other passive behavior	**-.30**	**-.26**	**-.27**	**-.24**	**-.26**	**-.35**	**-.41**	**-.35**	**-.34**	**-.29**	**-.37**	**-.27**	**-.29**	**-.19**
Contemplate/passivity	**-.13**	**-.12**	-.11	**-.14**	-.11	**-.14**	**-.16**	**-.14**	**-.14**	**-.12**	-.10	**-.12**	-.10	-.06

Notes. Correlations of .09 or larger are significant with $p < .05$ (two-tailed), correlations of .12 or larger are significant with $p < .01$ (two-tailed) and in boldface. Activity categories showing at least three positive correlations with cognitive tasks at a significance level of $p < .01$ are set in boldface. AN = Analogies; LS = Letter Series; PR = Practical Problems; SW = Spot-a-Word; VC = Vocabulary; PK = Practical Knowledge; IP = Identical Pictures; DL = Digit Letter; DS = Digit Symbol; WB = Word Beginnings; CA = Categories; PA = Paired Associates; AR = Activity Recall; MT = Memory for Text.

5.5 Appendix E. Standardized Factor Loadings of the Nested Factor Model Estimated Separately for two Age Groups of the BASE Sample and for a Younger Adult Sample

Table E1

Standardized Nested Factor Model Loadings for Young-Old Group (Age < 85.15)

		g	S
Reasoning			
	AN	.63	.46
	LS	.64	.40
	PR	.73	.26
Knowledge			
	SW	.61	.29
	BE	.76	.61
	PK	.78	.14
Perceptual Speed			
	IP	.73	.30
	DL	.73	.59
	DS	.78	.45
Fluency			
	WB	.65	.45
	CA	.72	.38
Memory			
	PA	.58	.51
	AR	.66	.17
	MT	.53	.27

Notes. $N = 258$. g = General factor; S: Specific factors; AN = Analogies; LS = Letter Series; PR = Practical Problems; SW = Spot-a-Word; VC = Vocabulary; PK = Practical Knowledge; IP = Identical Pictures; DL = Digit Letter; DS = Digit Symbol; WB = Word Beginnings; CA = Categories; PA = Paired Associates; AR = Activity Recall; MT = Memory for Text.

Table E2

Standardized Nested Factor Model Loadings for Old-Old Group (Age > 85.15)

		g	S
Reasoning			
	AN	.56	.28
	LS	.64	.59
	PR	.62	.34
Knowledge			
	SW	.60	.51
	BE	.77	.43
	PK	.79	.22
Perceptual Speed			
	IP	.75	.42
	DL	.76	.53
	DS	.80	.44
Fluency			
	WB	.74	.24
	CA	.80	.19
Memory			
	PA	.63	.38
	AR	.73	.36
	MT	.56	.30

Notes. N = 258. g = General factor; S = Specific factors; AN = Analogies; LS = Letter Series; PR = Practical Problems; SW = Spot-a-Word; VC = Vocabulary; PK = Practical Knowledge; IP = Identical Pictures; DL = Digit Letter; DS = Digit Symbol; WB = Word Beginnings; CA = Categories; PA = Paired Associates; AR = Activity Recall; MT = Memory for Text.

Table E3

Standardized Nested Factor Model Loadings for Younger Adult Group (Age 25–69)

		g	S
Reasoning			
	AN	.65	.42
	LS	.56	.59
	PR	.38	.55
Knowledge			
	SW	.46	.35
	BE	.44	.90
	PK	.47	.32
Perceptual Speed			
	IP	.51	.38
	DL	.43	.66
	DS	.59	.66
Fluency			
	WB	.30	.57
	CA	.49	.40
Memory			
	PA	.70	.55
	AR	.46	.19
	MT	.41	.42

Notes. N = 171. *g* = General factor; S = Specific factors; AN = Analogies; LS = Letter Series; PR = Practical Problems; SW = Spot-a-Word; VC = Vocabulary; PK = Practical Knowledge; IP = Identical Pictures; DL = Digit Letter; DS = Digit Symbol; WB = Word Beginnings; CA = Categories; PA = Paired Associates; AR = Activity Recall; MT = Memory for Text.

5.6 Appendix F. Comparison of Unstandardized Factor Loadings in Age-as-a-Predictor Model, Age-as-a-Covariate Model, and in Age-Partialled Model

A comparison of the unstandardized factor loadings in the age-as-a-predictor model with those in a nested factor measurement model based on age-partialled covariances (model fit of the age-partialled model: $\chi^2[64] = 141.1$, RMSEA = .05, CFI = .99) of the 14 tasks showed that for the general factors in the two models, these loadings correlated .95; for the specific factor loadings the correlation was .99. The correlations of the factor loadings in the age-partialled model with those in the age-as-a-covariate model were .73 for general factors, and .99 for the specific factor loadings.

Because the fit of the age-as-a-predictor model was very good, the factor loading pattern for the general and specific factors were indeed very similar to the one based on age-partialled covariances. For the model with age as a correlate, however, the loadings on the general factor differed from the age-partialled solution. This was obviously due to the fact that age-associated variance was absorbed by the general factor, and not by the specific factors. The correlation of the specific factor loadings of the age as a correlate and the age-as-a-predictor model was $r = .998$. This shows that the interpretation based on the factor-loading pattern of the specific factors is invariant for the two kinds of models, while the general factor loadings differ considerably (the correlation between the general factor loadings was only $r = .77$).

Table F1

Unstandardized Factor Loadings in the Nested Factor Model With Age as a Covariate, Age as a Predictor, and Age-Partialled Measurement Model

		Age-as-a-Covariate Model		Age-as-a-Predictor Model		Age-partialled Model	
		g	s	*g*	s	*g*	s
Reasoning							
	AN	1.00	1.00	1.00	1.00	1.00	1.00
	LS	1.03	1.24	1.04	1.26	1.08	1.19
	PR	1.09	.84	1.07	.85	1.13	.80
Knowledge							
	SW	.91	1.00	1.08	1.00	1.10	1.00
	BE	1.13	1.22	1.34	1.24	1.39	1.14
	PK	1.21	.47	1.25	.52	1.29	.49
Perceptual Speed							
	IP	1.21	1.00	1.16	1.00	1.15	1.00
	DL	1.21	1.60	1.17	1.63	1.19	1.61
	DS	1.25	1.31	1.19	1.35	1.28	1.29
Fluency							
	WB	1.06	1.00	1.13	1.00	1.26	1.00
	CA	1.21	1.00	1.28	1.00	1.26	1.00
Memory							
	PA	.99	1.00	1.04	1.00	1.07	1.00
	AR	1.13	.54	1.12	.56	1.17	.54
	MT	.91	.61	.93	.66	.94	.68

Notes. N = 516. *g* = General factor; S = Specific factors; AN = Analogies; LS = Letter Series; PR = Practical Problems; SW = Spot-a-Word; VC = Vocabulary; PK = Practical Knowledge; IP = Identical Pictures; DL = Digit Letter; DS = Digit Symbol; WB = Word Beginnings; CA = Categories; PA = Paired Associates; AR = Activity Recall; MT = Memory for Text.

5.7   Appendix G. Age-Partialled Correlations of General and Specific Factors in the Nested Factor Model With Indicators and a Latent Factor of Socioeconomic Status

Table G1

Age-Partialled Correlations of General and Specific Factors in the Nested Factor Model With Indicators and a Latent Factor of Socioeconomic Status

	g	R	K	S	F	M	χ^2 [5][a]
Income	.32***	-.03	.16	.03	-.07	.01	8.9
Social Class	.37***	-.02	.18*	.00	-.05	-.13	16.0**
Occupational Prestige	.42***	.01	.17*	-.05	.01	-.03	12.1
Education	.36***	.18*	.24***	.03	.06	-.07	27.5***
Latent Construct of SES	.56***	.06	.28**	-.02	-.01	-.10	25.0***

Notes. [a] χ^2-difference test of nested model comparison with a model that only specifies a correlation with the general factor. g = General factor; R = Reasoning; K = Knowledge; S = Perceptual Speed; F = Fluency; M = Memory.

Correlations in boldface were significant with * $p < .05$, ** $p < .01$, *** $p < .001$.

5.8 Appendix H. Age-Partialled Correlations of General and Specific Factors in the Age-as-a-Predictor Model With Items and Scale Scores of Openness for Experience From the NEO

Table H1

Age-Partialled Correlations of General and Specific Factors in the Nested Factor Model With Items and Scale Score of Openness for Experience From the NEO

Item content (Facet)	r_{Age}	g	R	K	S	F	M
„I don't like to waste my time daydreaming"[a] (Fantasy)	.12**	-.13*	-.01	-.08	.03	.04	.27**
„Once I find the right way to do something, I stick to it"[b] (Actions)	.05	-.20**	.13	-.03	-.06	-.01	.10
„Poetry has little or no effect on me"[a] (Aesthetics)	.03	-.04	.01	.20*	.17	.31**	.35**
„I often try new and foreign foods" (Actions)	-.22***	.16*	.00	.04	.12	.14	.09
„I have a very active imagination" (Fantasy)	-.15**	.31***	-.18*	.07	-.10	-.10	-.07
„I have a lot of intellectual curiosity" (Ideas)	-.14**	.34***	.00	.23**	-.05	-.01	-.04
Openness scale	-.10*	.17**	-.06	.12	.02	.09	.21*

Notes. [a] Item wording was changed to a positive statement in the German version. [b] Item was reflected in the total scale score. g = General factor; R = Reasoning; K = Knowledge; S = Perceptual Speed; F = Fluency; M = Memory

Correlations in boldface were significant with * $p < .05$, ** $p < .01$, *** $p < .001$.

5.9 Appendix J. Standardized Factor Loadings of the Nested Factor Model Estimated for Tasks Without Imputed Missing Values (*N* = 416) and With Missing Values Imputed by EM-Algorithm

Table J1

Standardized Factor Loadings of the Nested Factor Model Estimated for Tasks Without Imputed Missing Values

		g	S
Reasoning			
	AN	.65	.38
	LS	.67	.44
	PR	.72	.31
Knowledge			
	SW	.56	.45
	BE	.75	.49
	PK	.80	.20
Perceptual Speed			
	IP	.79	.29
	DL	.78	.55
	DS	.82	.40
Fluency			
	WB	.67	.35
	CA	.80	.35
Memory			
	PA	.64	.44
	AR	.74	.25
	MT	.61	.23

Notes. $N = 416$. *g* = General factor; S = Specific factors; AN = Analogies; LS = Letter Series; PR = Practical Problems; SW = Spot-a-Word; VC = Vocabulary; PK = Practical Knowledge; IP = Identical Pictures; DL = Digit Letter; DS = Digit Symbol; WB = Word Beginnings; CA = Categories; PA = Paired Associates; AR = Activity Recall; MT = Memory for Text.

Table J2

Standardized Factor Loadings of the Nested Factor Model With Missing Values Replaced by EM-Estimates

		g	S
Reasoning			
	AN	.68	.37
	LS	.70	.44
	PR	.75	.29
Knowledge			
	SW	.61	.43
	BE	.77	.49
	PK	.81	.19
Perceptual Speed			
	IP	.83	.29
	DL	.81	.52
	DS	.85	.37
Fluency			
	WB	.71	.31
	CA	.81	.30
Memory			
	PA	.66	.42
	AR	.76	.23
	MT	.61	.25

Notes. N = 516. *g* = General factor; S = Specific factors; AN = Analogies; LS = Letter Series; PR = Practical Problems; SW = Spot-a-Word; VC = Vocabulary; PK = Practical Knowledge; IP = Identical Pictures; DL = Digit Letter; DS = Digit Symbol; WB = Word Beginnings; CA = Categories; PA = Paired Associates; AR = Activity Recall; MT = Memory for Text.

5.10   Appendix K. Bootstrap Results for the Nested Factor Model With Age Correlations of the General and Specific Factors

Table K1

Bootstrap Standard Errors, Estimates of Parameter Bias, and 95% Confidence Intervals (CI; Percentile Method) of Parameter Estimates for Standardized Factor Loadings

Task	Factor	S.E.	S.E. of S.E.	Mean	Bias	S.E. of Bias	95%-CI Lower Bound	95%-CI Upper Bound	p
AN	g	.028	.001	.671	-.000	.001	.611	.724	.002
LS	g	.023	.001	.691	.001	.001	.645	.731	.002
PR	g	.022	.000	.730	-.000	.001	.684	.774	.002
SW	g	.028	.001	.613	-.001	.001	.560	.668	.002
BE	g	.020	.000	.760	-.002	.001	.719	.798	.002
PK	g	.018	.000	.811	-.002	.001	.772	.844	.002
IP	g	.017	.000	.814	-.000	.001	.780	.845	.002
DL	g	.019	.000	.816	.001	.001	.776	.852	.002
DS	g	.016	.000	.836	.001	.001	.802	.867	.002
WB	g	.025	.001	.716	.000	.001	.665	.763	.002
CA	g	.016	.000	.811	-.001	.001	.779	.842	.002
PA	g	.026	.001	.663	.000	.001	.607	.715	.002
AR	g	.021	.000	.761	.001	.001	.720	.800	.002
MT	g	.029	.001	.612	-.001	.001	.551	.665	.002
AN	R	.056	.001	.366	.000	.002	.259	.480	.002
LS	R	.063	.001	.448	-.004	.002	.332	.575	.002
PR	R	.052	.001	.314	.005	.002	.218	.424	.002
SW	K	.043	.001	.415	-.001	.001	.332	.497	.002
BE	K	.047	.001	.509	.003	.001	.424	.607	.002
PK	K	.039	.001	.201	.002	.001	.125	.277	.002
IP	S	.033	.001	-.303	.002	.001	-.367	-.236	.002
DL	S	.043	.001	-.479	.004	.001	-.559	-.395	.002
DS	S	.035	.001	-.400	.001	.001	-.468	-.337	.002
WB	F	.044	.001	-.347	.003	.001	-.427	-.253	.002
CA	F	.035	.001	-.268	.002	.001	-.330	-.196	.002
PA	M	.167	.004	.417	.014	.005	.232	.788	.002
AR	M	.070	.002	.232	.003	.002	.082	.365	.002
MT	M	.075	.002	.251	-.001	.002	.117	.400	.002

Notes.   g = General factor; M = Specific factor of Memory; F = Specific factor of Fluency; S = Specific factor of Perceptual speed; K = Specific factor of Knowledge; R = Specific factor of Reasoning; AN = Analogies; LS = Letter Series; PR = Practical Problems; SW = Spot-a-Word; VC = Vocabulary; PK = Practical Knowledge; IP = Identical Pictures; DL = Digit Letter; DS = Digit Symbol; WB = Word Beginnings; CA = Categories; PA = Paired Associates; AR = Activity Recall; MT = Memory for Text.

Table K2

Bootstrap Standard Errors, Estimates of Parameter Bias, and 95% Confidence Intervals (CI; Percentile Method) of Parameter Estimates for Age Correlations of the General and Specific Factors

Task	*S.E.*	*S.E.* of S.E.	Mean	Bias	*S.E.* of Bias	95%-CI Lower Bound	95%-CI Upper Bound	*p*
g	.053	.001	-.685	-.010	.002	-.811	-.594	.002
M	.145	.003	.196	.028	.005	-.003	.555	.054
F	.141	.003	-.313	-.029	.004	-.655	-.097	.002
S	.115	.003	-.045	-.022	.004	-.366	.136	.770
K	.098	.002	.351	.018	.003	.186	.562	.002
R	.100	.002	.071	.015	.003	-.083	.310	.500

Notes.   *g* = General factor; M = Specific factor of Memory; F = Specific factor of Fluency; S = Specific factor of Perceptual speed; K = Specific factor of Knowledge; R = Specific factor of Reasoning.

5.11   Appendix L. Bootstrap Results for the Nested Factor Model Predicting Writing Activity

Table L1

Bootstrap Standard Errors, Estimates of Parameter Bias, and 95% Confidence Intervals (CI; Percentile Method) of Parameter Estimates for Standardized Regression Weights of Writing and Basic Competency on the General and Specific Factors

DV	Predictor	*S.E.*	*S.E.* of *S.E.*	Mean	Bias	*S.E.* of Bias	95%-CI Lower Bound	95%-CI Upper Bound	*p*
BaCo	*g*	.031	.001	.587	.002	.001	.529	.648	.002
Writing	BaCo	.051	.001	.055	.002	.002	-.047	.158	.313
Writing	M	.109	.002	.090	-.002	.003	-.126	.294	.419
Writing	F	.103	.002	.308	.001	.003	.106	.517	.005
Writing	S	.073	.002	.295	-.003	.002	.146	.443	.002
Writing	K	.067	.002	.064	.001	.002	-.073	.197	.369
Writing	R	.086	.002	.168	-.002	.003	-.008	.339	.053
Writing	*g*	.069	.002	.072	.000	.002	-.069	.200	.313

Notes.   DV = Dependent Variable; *g* = General factor; M = Specific factor of Memory; F = Specific factor of Fluency; S = Specific factor of Perceptual speed; K = Specific factor of Knowledge; R = Specific factor of Reasoning; BaCo = Basic Competency.

5.12 Appendix M. SAS Program Code of the Monte Carlo Simulation Study

```
*florian schmiedek          ;
*october 2001               ;


*                               CODE STRUCTURE

*part one  : data generation                                       ;
*part two  : model estimation                                      ;
*part three: analysis of results                                   ;


*                               VARIABLE LIST

*      n_subj: sample size                                         ;
*      n_ind : number of indicators per group factor (3/4)         ;
*      g   :          variance explained by general factor (.4, .5, .6)        ;
*                     variance explained by specific factor = .8-g  ;
*      disp :         disproportionality of general vs. specific factor loadings  ;
*                     (.02 = +/-.02, .06 = +/-.06, .10 = +/-.10)   ;
*      spec :         model specification: 1=correct, 0=specific effect omitted ;
*      x_(i)_(j):     predictor set indicator number j of group factor i        ;
*      elg_(i)_(j):   estimated loading of predictor set indicator number j
*                      of group factor i on g                                 ;
*      elf_(i)_(j):   estimated loading of predictor set indicator number j
*                      of group factor i on group factor                 ;
*      e_(i)_(j):     uniqueness of predictor set indicator number j of group factor ;

*options mprint symbolgen mlogic;   *(can be used for debugging macros);

%global i j n_subj n_f n_ind n_i_y spec number n_result;
%global i_size i_ind i_g i_disp i_replic effect_s;


libname samples 'f:\age_fx_sim\second_sim\samples';
run;

libname result 'f:\age_fx_sim\second_sim\results';
run;


proc format;                      *trick to solve problems with negative fixed factor
loadings in CALIS;
        value signfmt -1='-' 1='+';
run;


****************************************************************************;
*                           PART ONE: Data generation           ;
****************************************************************************;

%macro generate;
%let number=%eval(&number+1);

proc iml;
        seed=10000+&number;

        %* this part of syntax works only if number of group factors(n_f)=3 !!!!  ;
                %do j=0 %to &n_ind-1;
                        l_g=(sqrt(&i_g*.1-&i_disp*.01+&j*(&i_disp*.02/(&n_ind-1))));
                        l_s=(sqrt(.8-l_g*l_g));
                        lambda_x=lambda_x//(l_g||l_s||{0 0});
                %end;
                %do j=0 %to &n_ind-1;
                        l_g=(sqrt(&i_g*.1-&i_disp*.01+&j*(&i_disp*.02/(&n_ind-1))));
                        l_s=(sqrt(.8-l_g*l_g));
                        lambda_x=lambda_x//(l_g||{0}||l_s||{0});
                %end;
                %do j=0 %to &n_ind-1;
                        l_g=(sqrt(&i_g*.1-&i_disp*.01+&j*(&i_disp*.02/(&n_ind-1))));
                        l_s=(sqrt(.8-l_g*l_g));
```

170

```
                                    lambda_x=lambda_x//(l_g||{0 0}||l_s);
                        %end;
                        ;

                error=diag(j(3*&n_ind,1,.2));

                %* calculation of model implied covariance matrix;
                cov_x=lambda_x*t(lambda_x) + error;

                age_fx={-.7071 &effect_s 0 0};
                age_cov=lambda_x*t(age_fx);

                cov_true=cov_x||age_cov;
                cov_true=cov_true//(t(age_cov)||{1});

        m=j(&n_subj,(3*&n_ind+1),seed);
        y=normal(m);

        c=root(cov_true);  %* cholesky decomposition;
        x=y*c;

        %*creating variable names:              ;
        varnames={
        %do i_fac=1 %to 3;
                %do i_x=1 %to &n_ind;
                        "x_&i_fac._&i_x"
                %end;
        %end;
        "age"};

        create samples.sim&number from x [colname=varnames];
        append from x;
        create cov_true from cov_true;
        append from cov_true;
         create lam_true from lambda_x;
        append from lambda_x;

quit;
run;

%mend generate;


*****************************************************************************************;
*                          PART TWO: Model estimation                        ;
*****************************************************************************************;

%macro est;
proc calis data=samples.sim&number
                outram=out_ram outstat=out_stat outest=out_est
        method=&method maxiter=200 cov noprint;
                lineqs
                %* measurement model of predictor factors;
                %do i=1 %to 3;
                        %do j=1 %to &n_ind;
                                x_&i._&j = l_g_&i._&j f_g + l_f_&i._&j f_&i + e_&i._&j
                                %if (&i=3 and &j=&n_ind) %then %do;
                                ;
                                %end;
                                        %else %do;
                                                        ,
                                        %end;
                        %end;
                %end;

        std
                %do i=1 %to 3;
                        %do j=1 %to &n_ind;
                                e_&i._&j = er_&i._&j,
                        %end;
                        %end;

                %do i=1 %to 3;
                        f_&i
                %end;
                        = &n_f * 1.00,
                f_g = 1.00,
```

```
                                    age = var_age;

                        cov
                                f_g age = cov_g
                                %if &spec=1 %then %do;
                                        ,
                                        f_1 age = cov_1,
                                        f_2 age = cov_2,
                                        f_3 age = cov_3;
                                %end;
                                %if &spec=0 %then %do;
                                        ;
                                %end;
                        bounds
                                %do i=1 %to 3;
                                %do j=1 %to &n_ind;
                                        0 <= l_g_&i._&j,
                                                0 <= l_f_&i._&j
                                                %if (&i=3 and &j=&n_ind) %then %do;
                                                        ;
                                                %end;
                                                %else %do;
                                                        ,
                                                %end;
                                %end;
                        %end;

        %mend est;


        ****************************************************************************************;
        %macro append;
          %* creating new line of output file;
          %let n_result=%eval(&n_result+1);
          %let n_var=%eval(3*&n_ind+1);

                proc iml;
                use out_ram;
                read all into ram;
                use out_stat;
                read all into stat;
                        use out_est;
                read all into est;
                use cov_true;
                read all into cov_true;

                        use lam_true;
                read all into lam_true;
                        t_g=lam_true[,1];   %*vector of true g-loadings;
                        t_f=lam_true[,2]+lam_true[,3]+lam_true[,4];
                        %*vector of true specific factor loadings;

                        num =&number;
                        n_var=&n_var;
                        rmr     =ram[(4+&n_f+2+2*3*&n_ind)+6,4];
                        df      =ram[(4+&n_f+2+2*3*&n_ind)+9,4];
                        chi2    =ram[(4+&n_f+2+2*3*&n_ind)+11,4];
                        chi2_p  =ram[(4+&n_f+2+2*3*&n_ind)+12,4];
                        chi2null=ram[(4+&n_f+2+2*3*&n_ind)+13,4];
                        rmsea   =ram[(4+&n_f+2+2*3*&n_ind)+14,4];
                        closefit=ram[(4+&n_f+2+2*3*&n_ind)+17,4];
                        ecvi    =ram[(4+&n_f+2+2*3*&n_ind)+18,4];
                        cfi     =ram[(4+&n_f+2+2*3*&n_ind)+21,4];
                        mcdo    =ram[(4+&n_f+2+2*3*&n_ind)+28,4];
                        bbnnfi  =ram[(4+&n_f+2+2*3*&n_ind)+29,4];
                        bbnfi   =ram[(4+&n_f+2+2*3*&n_ind)+30,4];
                        pars_ind=ram[(4+&n_f+2+2*3*&n_ind)+31,4];


                fit=rmr||df||chi2||chi2_p||chi2null||rmsea||
                                closefit||ecvi||cfi||mcdo||bbnnfi||
                                bbnfi||pars_ind;


                        %* reading age variance and age covariances;

                        var_age=ram[45+5*&n_f*&n_ind,4];
```

```
%if &spec=1 %then %do;
        cov_g =ram[45+5*&n_f*&n_ind+1,4];
        cov_s1=ram[45+5*&n_f*&n_ind+3,4];
                cov_s2=ram[45+5*&n_f*&n_ind+5,4];
                cov_s3=ram[45+5*&n_f*&n_ind+7,4];
%end;
%if &spec=0 %then %do;
        cov_g =ram[45+5*&n_f*&n_ind+1,4];
                cov_s1=.;
                cov_s2=.;
                cov_s3=.;
%end;

%* calculating age correlations;
                corr_g =cov_g/sqrt(var_age);
                corr_s1=cov_s1/sqrt(var_age);
                corr_s2=cov_s2/sqrt(var_age);
                corr_s3=cov_s3/sqrt(var_age);

        %*detecting improper solutions;
                improper=0;
                if (abs(corr_g)>1)  then improper=1;
                if (abs(corr_s1)>1) then improper=1;
                if (abs(corr_s2)>1) then improper=1;
                if (abs(corr_s3)>1) then improper=1;


        %* reading measurement models;

start1=45+2*&n_f*&n_ind;
%*first row of exogenous factor loading estimates;
end1  =45+5*&n_f*&n_ind-3;
%*end of factor loading estimates      ;
start2=51+3*&spec+5*&n_f*&n_ind;
%*first row of exogenous error variance estimates;
end2  =51+3*&spec+6*&n_f*&n_ind-1;

do i=start1 to end1 by 3;
        l_g=l_g//ram[i,4];
        l_f=l_f//ram[i+1,4];
end;

                e_x=ram[start2:end2,4];

%* detecting heywood cases;
        heywood=0;
        if sum(e_x-abs(e_x))<0 then heywood=1;


%* calculating distortion of exogenous measurement model
        (absolute values of loadings have to be used because
            factors may have reversed(negative) loadings);
%* mean squared parameter bias;
        s_g=t(abs(l_g)-t_g)*(abs(l_g)-t_g);
        s_f=t(abs(l_f)-t_f)*(abs(l_f)-t_f);

        spb=(1/(2*&n_f*&n_ind))#(s_g+s_f);

%* relative mean squared parameter bias;
        rs_g=(abs(l_g)-t_g)#(abs(l_g)-t_g)/t_g;
        rs_f=(abs(l_f)-t_f)#(abs(l_f)-t_f)/t_f;
                rspb=(1/(2*&n_f*&n_ind))#(j(1,&n_f*&n_ind,1)*rs_g+j(1,&n_f*&n_ind,1)*rs_
                f);

        %* reading covariance of age covariance parameters;
                if &spec=1 then do;
                        %let row1=%eval(2*&n_f*&n_ind+18);
                        %let col1=%eval(2*&n_f*&n_ind+3);

                        var_g =est[&row1 +1,&col1 +1];
                        var_s1=est[&row1 +2,&col1 +2];
                        var_s2=est[&row1 +3,&col1 +3];
                        var_s3=est[&row1 +4,&col1 +4];

                        if var_g<0  then var_g=0;
                        if var_s1<0 then var_s1=0;
```

173

```
                        if var_s2<0 then var_s2=0;
                        if var_s3<0 then var_s3=0;

                        cov_g1=est[&row1 +2,&col1 +1];
                        cov_g2=est[&row1 +3,&col1 +1];
                        cov_g3=est[&row1 +4,&col1 +1];
                        cov_12=est[&row1 +3,&col1 +2];
                        cov_13=est[&row1 +4,&col1 +2];
                        cov_23=est[&row1 +4,&col1 +3];

                        se_g  =sqrt(var_g);
                        se_s1 =sqrt(var_s1);
                        se_s2 =sqrt(var_s2);
                        se_s3 =sqrt(var_s3);

                        if var_g=.  then se_g  = 0;
                        if var_s1=. then se_s1 = 0;
                        if var_s2=. then se_s2 = 0;
                        if var_s3=. then se_s3 = 0;

                        corr_g1 =cov_g1/(sqrt(var_g)#sqrt(var_s1));
                        corr_g2 =cov_g2/(sqrt(var_g)#sqrt(var_s2));
                        corr_g3 =cov_g3/(sqrt(var_g)#sqrt(var_s3));
                        corr_12 =cov_12/(sqrt(var_s1)#sqrt(var_s2));
                        corr_13 =cov_13/(sqrt(var_s1)#sqrt(var_s3));
                        corr_23 =cov_23/(sqrt(var_s2)#sqrt(var_s3));

                        if var_g=0 then do;
                                corr_g1 =0;
                                corr_g2 =0;
                                corr_g3 =0;
                        end;
                        if var_s1=0 then do;
                                corr_g1 =0;
                                corr_12 =0;
                                corr_13 =0;
                        end;
                        if var_s2=0 then do;
                                corr_g2 =0;
                                corr_12 =0;
                                corr_23 =0;
                        end;
                        if var_s3=0 then do;
                                corr_g3 =0;
                                corr_13 =0;
                                corr_23 =0;
                        end;
                end;
                else do;
                        se_g  =0;
                        se_s1 =0;
                        se_s2 =0;
                        se_s3 =0;
                        corr_g1=0;
                        corr_g2=0;
                        corr_g3=0;
                        corr_12=0;
                        corr_13=0;
                        corr_23=0;
                end;

        nr_subj=&n_subj;
        nr_ind =&n_ind;
        replic =&i_replic;

        cond_g =&i_g;
        disp   =&i_disp;
                effect_s=&effect_s;
        spec   =&spec;
        %if &method=lsml %then %do;
                meth=1;
        %end;

          %* the vectors with (different numbers of) factor loadings and error terms
          have to be
          re-formatted to fit into the results file:    ;
```

```
              %do i_1=1 %to 3;          %* max number of group factors;
                    %do i_2=1 %to 4;   %* max number of indicators;
                            %if (&i_1<=&n_f and &i_2<=&n_ind) %then %do;
                                    l_g2=l_g2||l_g[(&i_1-1)*&n_ind+&i_2,1];
                            %end;
                            %else %do;
                                    l_g2=l_g2||{.};
                            %end;
                    %end;
              %end;
              %do i_1=1 %to 3;          %* max number of group factors;
                    %do i_2=1 %to 4;   %* max number of indicators;
                            %if (&i_1<=&n_f and &i_2<=&n_ind) %then %do;
                                    l_f2=l_f2||l_f[(&i_1-1)*&n_ind+&i_2,1];
                            %end;
                            %else %do;
                                    l_f2=l_f2||{.};
                            %end;
                    %end;
              %end;
              %do i_1=1 %to 3;                  %* max number of group factors;
                    %do i_2=1 %to 4;          %* max number of indicators;
                            %if (&i_1<=&n_f and &i_2<=&n_ind) %then %do;
                                    e_x2=e_x2||e_x[(&i_1-1)*&n_ind+&i_2,1];
                            %end;
                            %else %do;
                                    e_x2=e_x2||{.};
                            %end;
                    %end;
              %end;


        new=num||replic||nr_subj||nr_ind||
                    cond_g||disp||effect_s||spec||
                    meth||
                    fit||heywood||improper||
              l_g2||l_f2||e_x2||
                    var_age||cov_g||cov_s1||cov_s2||cov_s3||
                    corr_g||corr_s1||corr_s2||corr_s3||
                    corr_g1||corr_g2||corr_g3||corr_12||corr_13||corr_23||
                    se_g||se_s1||se_s2||se_s3||
              spb||rspb;

        create new_line from new ;
        append from new;
quit;

   %* appending new line to output file;
   %if &n_result=1 %then %do;
   data result.results;
        set new_line ;
   %end;
   %else %do;
   proc append base=result.results new=new_line;
   %end;


%mend append;

*********************************************************************************;

%macro loops;
%let number  =0;
%let n_result=0;
%let n_f      =3;

%do i_replic=1 %to 100;

%do i_size=1 %to 4;
        %if &i_size=1 %then %let n_subj=125;
        %if &i_size=2 %then %let n_subj=250;
        %if &i_size=3 %then %let n_subj=500;
        %if &i_size=4 %then %let n_subj=1000;

        %do n_ind=3 %to 4;

                    %do i_g=4 %to 6 %by 1;
```

175

```
                        %do i_disp=2 %to 10 %by 4;
                                %do i_effect=1 %to 9;
                                        %if &i_effect=1 %then %let effect_s=.3;
                                        %if &i_effect=2 %then %let effect_s=.2;
                                        %if &i_effect=3 %then %let effect_s=.1;
                                        %if &i_effect=4 %then %let effect_s=.0;
                                        %if &i_effect=5 %then %let effect_s=-.1;
                                        %if &i_effect=6 %then %let effect_s=-.2;
                                        %if &i_effect=7 %then %let effect_s=-.3;
                                        %if &i_effect=8 %then %let effect_s=-.4;
                                        %if &i_effect=9 %then %let effect_s=-.5;

                        %generate;

                        %* correctly specified model;
                                %let spec  =1;
                        %let method =lsml;
                        %est;
                        %append;

                        %* misspecified model;
                        %let spec  =0;
                        %let method =lsml;
                                %est;
                                %append;

                                %end;
                        %end;
                %end;
        %end;
%end;
%end;
%mend loops;

%loops;


******************************************************************************.
*                       PART THREE: Analysis of results       ;
*****************************************************************************;

data result.final;
        set result.results;
        rename
                col1  = number
                col2  = Replic
                col3  = nr_subj
                col4  = nr_ind
                col5  = cond_g
                col6  = disp
                col7  = effect_s
                col8  = spec
                col9  = method
                col10 = rmr
                col11 = df
                col12 = chi2
                col13 = chi2_p
                col14 = chi2null
                col15 = rmsea
                col16 = closefit
                col17 = ecvi
                col18 = cfi
                col19 = mcdo
                col20 = bbnnfi
                col21 = bbnfi
                col22 = pars_ind
                col23 = Heywood
                col24 = improper
                col25 = l_g_1_1
                col26 = l_g_1_2
                col27 = l_g_1_3
                col28 = l_g_1_4
                col29 = l_g_2_1
                col30 = l_g_2_2
                col31 = l_g_2_3
                col32 = l_g_2_4
```

```
                col33 = l_g_3_1
                col34 = l_g_3_2
                col35 = l_g_3_3
                col36 = l_g_3_4
                col37 = l_f_1_1
                col38 = l_f_1_2
                col39 = l_f_1_3
                col40 = l_f_1_4
                col41 = l_f_2_1
                col42 = l_f_2_2
                col43 = l_f_2_3
                col44 = l_f_2_4
                col45 = l_f_3_1
                col46 = l_f_3_2
                col47 = l_f_3_3
                col48 = l_f_3_4
                col49 = e_1_1
                col50 = e_1_2
                col51 = e_1_3
                col52 = e_1_4
                col53 = e_2_1
                col54 = e_2_2
                col55 = e_2_3
                col56 = e_2_4
                col57 = e_3_1
                col58 = e_3_2
                col59 = e_3_3
                col60 = e_3_4
                col61 = var_age
                col62 = cov_g
                col63 = cov_s1
                col64 = cov_s2
                col65 = cov_s3
                col66 = corr_g
                col67 = corr_s1
                col68 = corr_s2
                col69 = corr_s3
                col70 = corr_g1
                col71 = corr_g2
                col72 = corr_g3
                col73 = corr_12
                col74 = corr_13
                col75 = corr_23
                col76 = se_g
                col77 = se_s1
                col78 = se_s2
                col79 = se_s3
                col80 = spb
                col81 = rspb;
        run;

        proc print data=results;
        run;
```

5.13   Appendix N. Influence of Sample Size and Number of Indicator Variables on the Number of Heywood Cases and Improper Solutions

Table N1

Influence of Sample Size on the Number of Heywood Cases and Improper Solutions

	Sample Size				χ^2 [*df*]
	125	250	500	1000	
Heywood cases	160	3	0	0	467.8 [3]
Improper solutions	1103	800	544	304	560.8 [3]
Total Number of Runs	8100	8100	8100	8100	

Table N2

Influence of the Number of Indicator Variables on the Number of Heywood Cases and Improper Solutions

	Number of Indicators		χ^2 [*df*]
	3	4	
Heywood cases	118	45	32.9 [1]
Improper solutions	1356	1395	0.6 [1]
Total Number of Runs	16200	16200	

5.14 Appendix O. Main Effects and Interactions of Factors Examined in the Monte Carlo Study on Standard Errors of the Estimates of General and Specific Age-Associated Effects

Table O1

ANOVA for the Standard Error of the Estimate of the Age Correlation of the General Factor

Source	SS (Type III)	df	MSe	F	p	η^2
Corrected Model	40.42	173	.23	241.55	.00	.59
Intercept	231.98	1	231.98	239847.47	.00	.89
NS	14.74	3	4.91	5078.26	.00	.34
NI	.60	1	.60	618.97	.00	.02
D	19.38	2	9.70	10020.92	.00	.41
G	3.19	2	1.60	1650.46	.00	.10
ES	1.16	8	.15	150.18	.00	.04
NS * NI	.06	3	.02	20.49	.00	.00
HS * D	.45	6	.08	78.03	.00	.02
NI * D	.33	2	.17	171.84	.00	.01
NS * NI * D	.02	6	.00	2.69	.01	.00
G * D	.58	4	.15	150.58	.00	.02
ES * D	.46	16	.00	29.44	.00	.02
NS * G * D	.11	18	.01	6.35	.00	.00
NS * ES * D	.13	72	.00	1.85	.00	.01
NI * G * D	.04	6	.01	6.38	.00	.00
NI * ES * D	.03	24	.00	1.25	.18	.00
Error	28.35	29312	.001			
Total	284.01	29486				
Corrected Total	68.77	29485				

Notes. NS = Number of subjects; NI = Number of indicator variables; D = Disproportionality of indicator variance composition; G = Variance explained by *g* factor; ES = Effect size of specific effect R^2 = .59 (Adjusted R^2 = .59).

Table O2

ANOVA for the Standard Error of the Estimate of the Age Correlation of the Specific Factor

| Source	SS (Type III)	df	MSe	F	p	η^2
Corrected Model	65.28	173	.38	170.98	.00	.50
Intercept	332.95	1	332.95	150879.02	.00	.84
NS	19.52	3	6.51	2947.98	.00	.23
NI	1.37	1	1.37	619.84	.00	.02
D	38.34	2	19.17	8686.35	.00	.37
G	.11	2	.05	24.17	.00	.00
ES	3.49	8	.44	197.79	.00	.05
NS * NI	.17	3	.06	26.29	.00	.00
HS * D	.70	6	.12	53.06	.00	.01
NI * D	.56	2	.28	126.01	.00	.01
NS * NI * D	.03	6	.00	1.99	.06	.00
G * D	.52	4	.13	58.45	.00	.01
ES * D	.65	16	.04	18.30	.00	.01
NS * G * D	.15	18	.01	3.83	.00	.00
NS * ES * D	.37	72	.01	2.34	.00	.01
NI * G * D	.00	6	.00	.20	.98	.00
NI * ES * D	.05	24	.00	.90	.61	.00
Error	64.68	29312	.00			
Total	437.49	29486				
Corrected Total	129.96	29485				

Notes. NS = Number of subjects; NI = Number of indicator variables; D = Disproportionality of indicator variance composition; G = Variance explained by *g* factor; ES = Effect size of specific effect R^2 = .50 (Adjusted R^2 = .50).

5.15   Appendix P. Bias in Estimates of General and Specific Age-Associated Effects and Distribution of General Effect Estimates

Table P1

True and Average Estimated Age Correlations of the General and Specific Factors Excluding Runs With Improper Solutions

Factor	True Age Correlation	Age Correlation Estimate	*SD* of Parameter Estimates	*N*
Specific Factor 1	-.50	-.51	.12	3300
Specific Factor 1	-.40	-.41	.14	3298
Specific Factor 1	-.30	-.31	.15	3306
Specific Factor 1	-.20	-.23	.16	3261
Specific Factor 1	-.10	-.13	.17	3286
Specific Factor 1	.00	-.04	.18	3279
Specific Factor 1	.10	.07	.18	3255
Specific Factor 1	.20	.17	.18	3262
Specific Factor 1	.30	.26	.18	3239
Specific Factor 2	.00	-.02	.17	29486
Specific Factor 3	.00	-.02	.16	29486
General Factor	-.70	-.68	.12	29486

General Factor Age Correlation Estimate
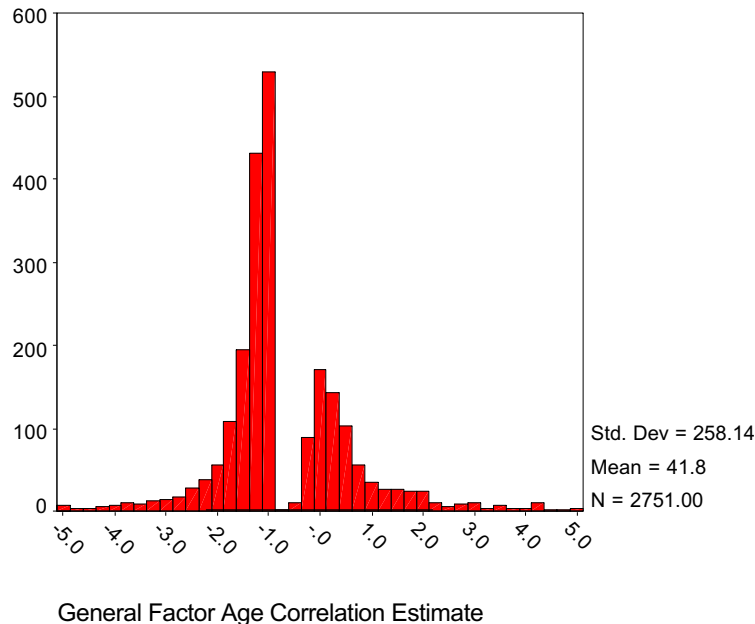
Figure P1.   Frequencies of different estimates for the age correlation of the general factor in runs with improper solutions. Only runs with estimates of the general factor age correlation between -5 and 5 are included.