Appendix F

Completion of Proof in Section 3.7.1: Evaluating $\vec{\Omega}$

Here, we show that assertion (3.118) is fulfilled, where Ω is defined by

$$\vec{\Omega} = \begin{pmatrix} \sum_{j\geq 2}\langle \mathcal{L}_y(\bar{C}_j u_j), \mathbf{1}\rangle_{\mu_x} \\ \sum_{j\geq 2}\langle \mathcal{L}_y(\bar{C}_j u_j), u_1(x,\cdot)\rangle_{\mu_x} \end{pmatrix}. \tag{F.1}$$

The coefficients $\bar{C}_j$ are given in (3.115)& (3.116). For $j \geq 2$ we obtain

$$\begin{aligned} \bar{C}_j &= \frac{1}{\lambda_j(x)}(H_{j0} + H_{j1})(t,\tau,x), \\ H_{j0} + H_{j1} &= \langle u_j, (-\partial_t + \mathcal{L}_y)(A_0 + A_1 u_1)\rangle_{\mu_x} = \langle u_j, \mathcal{L}_y(A_0 + A_1 u_1)\rangle_{\mu_x}. \end{aligned}$$

For $\vec{A} = (A_0, A_1)$ we use the representation $\vec{A} = (\mathbf{P}_0 + \mathbf{P}_1)\vec{A}(t=0) + \sum_{k\geq 2} e^{t\bar{\lambda}_k}\mathbf{P}_k\vec{A}(t=0)$ where $\mathbf{P}_0$ is the orthogonal projection onto $\text{span}\{(\mathbf{1},0)^T\}$ and $\mathbf{P}_1$ projects orthogonal onto $\text{span}\{\big(\mu_x(B_x^{(1)}) - \mu(B^{(1)}), \gamma_x\big)^T\}$. By exploiting $\chi_1 = \mu_x(B_x^{(1)}) + \gamma_x u_1$ we get

$$H_{j0} + H_{j1} = \sum_{k\geq 2} e^{t\bar{\lambda}_k} G_{jk}(\tau,x) + K_j \langle u_j, \mathcal{L}_y \chi_1\rangle_{\mu_x},$$

where G_{jk} are some functions independent of t and K_j is a constant. In Section 3.5.3 we have shown that $\langle u_j, \mathcal{L}_y\chi_1\rangle_{\mu_x}$ asymptotically has to vanish as $\epsilon \to 0$, see (3.71). Inserting the result into (F.1) we arrive at

$$\vec{\Omega} = \sum_{k\geq 2} e^{t\bar{\lambda}_k}\vec{J}_k(\tau,x) + \vec{K}(\tau,x), \qquad \vec{K} \ll 1.$$