# Appendix E

# Second Eigenvector of $\mathcal{L}^\epsilon$: Illustrative Example

Here, we refer to the last paragraph of Section 3.5.4 and compute the second eigenvector of the full dynamics' generator $\mathcal{L}^\epsilon$ in order to numerically compare the asymptotic result (3.75). To this end, we have chosen the potential $V = V(x, y)$ as

$$V(x,y) \;=\; 5(y^2-1)^2 \;+\; 1.25(y-x/2)^2. \tag{E.1}$$

The potential is shown in Figure E.1. The metastable decomposition for fixed $x$ is approximately given by $B_x^{(1)} = \{(x,y) \,:\, y \in \mathbf{R}^-\}$ and $B_x^{(2)} = \{(x,y) \,:\, y \in \mathbf{R}^+\}$. For the computation of the eigenvector, we choose $\epsilon = 0.005$ fixed and increase the inverse temperature $\beta$. Concerning the structure of the second eigenvector $u^\epsilon$, this can be considered equivalent to increasing the potential barrier in the direction of the fast variable $y$. For small values of $\beta$, the eigenvector $u^\epsilon(x,\cdot)$ is independent of the fast variable $y$, but clearly depends on $x$. The pictures seem to confirm the validity of the asymptotic strategy to derive $u^\epsilon$ that is outlined in the last paragraph of Section 3.5.4. For small values of $\beta$ we obtain

$$u^\epsilon(x,y) \;\approx\; a_1(x)\,\mathbf{1}_{B_x^{(1)}}(y) \;+\; a_2(x)\,\mathbf{1}_{B_x^{(2)}}(y),$$

whereas for $\beta = 8.5$ we approximately have

$$u^\epsilon(x,y) \;\approx\; a_1\,\mathbf{1}_{B^{(1)}}(x,y) \;+\; a_2\,\mathbf{1}_{B^{(2)}}(x,y),$$

where $a_1$ and $a_2$ now are independent of $x$. The pictures also reveal that the simple averaging procedure is inappropriate for $\beta \geq 2.5$.

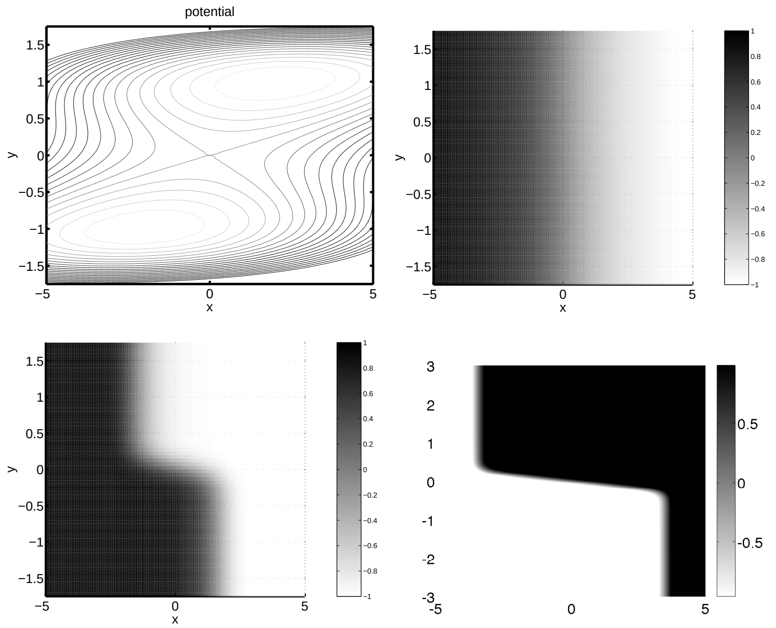


Figure E.1: Second eigenvector of $\mathcal{L}^\epsilon$ corresponding to the potential $V$ in (E.1) for $\epsilon = 0.005$ and $\beta = 1.0$ (right, top), $\beta = 2.5$ (bottom, left) and $\beta = 8.5$ (bottom, right).