Summary and Outlook

In three-dimensional space-time there may be particle types, whose nature is neither bosonic nor fermionic. Rather, they have a more general statistics, which can be described by a representation of the braid group. These particles have been termed plektons or, if the representation is one-dimensional, anyons [Wil82]. Proposals that such particles, or quasi-particles, may be relevant for the explanation of two-dimensional condensed matter phenomena such as the fractional quantum Hall effect [Lau83, Hal84, ASW84, ZHK89, FK91] and high-T_c superconductivity [Lau88, Wil90] have raised a strong interest of physicists in braid group statistics. Yet, the quantum theory of such particles is still poorly understood, and none of the so far proposed models is physically completely satisfying, in particular from the viewpoint of local quantum physics.

In this thesis we hope to have contributed to the understanding of the quantum physics of particles with braid group statistics in $2+1$ dimensions. The discussion is model-independent and based on the assumptions of local quantum physics. Let us briefly recapitulate the results. Under the assumption that the observable algebra satisfies the so-called Bisognano-Wichmann property [BW75], we have established a PCT theorem for plektons and a spin-statistics theorem for anyons. The structure of the space of scattering states has been further clarified. It is known [FM91, FGR96] to be fixed by the relevant representation of the braid group and by the rules which determine the charges contained in the scattering states (the 'fusion' rules). We have presented unitary Møller operators W^+ and W^- from the space of outgoing and incoming scattering states, respectively, into one and the same reference Hilbert space. Thus in particular, outgoing states can now naturally be compared with incoming states via the S-matrix $S = (W^+)^* W^-$. Both Møller operators translate the physical representation of the covering group of the Poincaré group on the spaces of outgoing and incoming scattering states, respectively, into the canonical representation [MS95] on the reference Hilbert space. Consequently, our S-matrix commutes with the physical representation of the covering group of the Poincaré group, as has to be required. As a new result, we have constructed the 'incoming PCT-operator', i.e. the product of the PCT-operator with the S-matrix.

In the case of anyons, the fusion rules mentioned above are trivial and the relevant representation of the braid group are classified by a finite set of complex numbers, namely the statistics phases of the particles under consideration. Thus in the anyonic case our results concerning the Hilbert space of scattering states yield explicit formulae for the representation of the universal covering group of the Poincaré group and for the 'incoming PCT-operator'. For anyons one can construct a field algebra from the observable algebra and a suitable set of sectors. We have established the new result that the Bisognano-Wichmann property of the observable algebra extends to this anyonic field algebra. In particular, the PCT-operator coincides with the modular conjugation of the field algebra associated to the standard wedge region (0.16) up to a 'twist operator', and the modular operator can be expressed in terms of the representation of the boosts leaving this wedge invariant. In effect, we have thereby constructed the 'incoming' Tomita-operator associated to any given wedge region. This operator is characteristic of the field algebra associated to the region, and being supplied with the family of incoming Tomita-operators for all wedge regions may be helpful in the construction of models.

We have finally addressed the issue of free relativistic fields for anyons, which have not been constructed yet. It is known that fields for anyons cannot be localizable in bounded regions of space-time, but only in regions which extend to space-like infinity. Here we have characterized 'free fields' by the requirement that they are localizable in such regions, and that they create only single particle vectors from the vacuum. (This requirement implements the idea that free fields should establish a 'second quantization functor': all relevant information of the model should be already encoded in the single particle space.) However, we have shown that under a mild regularity condition, such fields cannot exist in the case of anyons.

The following picture of a model with trivial S-matrix which comes close to second quantization is conceivable and not excluded by our no-go result. There may be free fields in the above sense associated to wedge regions, but localization within smaller regions is associated with particle production. An analogous phenomenon is exhibited by the $U(1)$ current algebra in $1+1$ dimensions. Further, the construction of such wedge-localized fields may be based on our present results on the Tomita-operators associated to such regions. Technically, the construction might proceed in two steps. First, one solves the linear problem of determining the Tomita-operators associated to intersections of wedges. The crucial step is then to find the 'correct' associated algebras. In general, these are far from being unique, but one may hope that the requirements from local quantum physics are restrictive enough to find reasonable candidates. On the way through 'flatland' towards such a construction one has the possibility to employ and further develop concepts of modern quantum field theory alternative to the common Lagrangian method, which might turn out to be fertile also in $3+1$ dimensions.