RAND Journal of Economics
Vol. 52, No. 4, Winter 2021
pp. 859–883

Signaling versus Auditing

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We analyze a competitive labor market in which workers signal their productivities through education, and firms have the option of auditing to learn workers' productivities. Audits are costly and non-contractible. We characterize the trade-offs between signaling by workers and costly auditing by firms. Auditing is always associated with (partial) pooling of worker types, and education is used as a signal only if relatively few workers have low productivity. Our results feature new auditing patterns and explain empirical observations in labor economics like wage differentials and comparative statics of education choices. Our analysis applies also to other signaling problems, for example, the financial structure of firms, warranties, and initial public offerings.

1. Introduction

■ We investigate signaling in a market where the uninformed side of the market relies not only on informative signaling by the other side, but may itself acquire information by performing costly audits. Agents on the informed side of the market privately know their types and can choose publicly observable actions to signal their types. The uninformed agents make offers based on observed actions, but these offers can be withdrawn if an audit discovers unfavorable information. Our model thus extends the canonical model of Spence (1973), in which signaling is the only source of information, with auditing as a second source of information. We analyze the trade-offs between these two important sources of information: signaling by informed agents and information acquisition by uninformed agents. These trade-offs occur because in our model auditing is endogenously determined as part of the equilibrium. We fully characterize the set

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We thank an editor and two referees as well as Massimo Anelli, Matthew Backus, Yves Breitmoser, Daniel Gottlieb, Amir Habibi, Dominik Sachs, Sinan Sarpça, Klaus Schmidt, and Uta Schönberg for helpful comments. Bester and Lang gratefully acknowledge support by the Berlin Centre for Consumer Policies (BCCP) and by the Deutsche Forschungsgemeinschaft through CRC TRR 190 (project number 280092119). Li gratefully acknowledges support by WU Jiapei Award for Information Economics in 2018 (project number E18100686).

of equilibria as well as when signaling arises and when auditing arises. Moreover, we point out novel features of signaling in markets where the uninformed side has the option of auditing.

For our analysis, we use the framing of the labor market setting by Spence (1973), who first proposed education as a signaling device in labor markets: workers have private information about their own productivity, education is more costly for low than for high productivity workers and therefore can be used to signal productivities. Hence, firms can only infer workers' productivities from their education choices. In reality, besides looking at the workers' education, firms use sophisticated testing, assessment centers, and other instruments of auditing to learn workers' productivities.[1]

We combine these two features: workers have the option of signaling through education and firms have the option of conducting costly information acquisition by auditing workers. In particular, workers choose their education level and then firms announce job offers by posting wages for different education levels as in Spence (1973). After a worker applies for a job, firms choose whether or not to audit the applicant and then decide on hiring. We assume that an audit reveals a worker's productivity to the firm; but auditing is non-contractible and firms cannot pre-commit to auditing. We are interested in the strategic interaction between workers' signaling incentives and firms' auditing incentives. Although we frame our model and results in terms of labor markets, our analysis also applies to other environments that involve both signaling and auditing, as we point out below.

When auditing is unavailable, the most prominent equilibrium is the least-cost separating equilibrium in which workers with low and high productivity choose different education levels, and information is perfectly revealed through these signals. However, when the firms have the option to audit applicants, the competitive market has a unique equilibrium that exhibits very different features from the standard signaling model:[2] When auditing costs are large, the least-cost separating equilibrium is still the unique outcome, workers with different productivities choose different education levels and auditing does not occur in equilibrium; when auditing costs are small, the unique equilibrium outcome is partial pooling in which workers with different productivities choose the same education level and firms audit with positive probability. Hence, information is revealed partly through signaling and partly through auditing. When the fraction of low productivity workers is high, signaling by education is not used in equilibrium, and information is revealed solely through auditing.

The equilibrium results are intuitive and the driving force are the relative economic costs of information provision through the two channels. In a competitive market, firms' expected profits are driven down to zero in equilibrium. Workers with high productivity benefit from information revelation by being recognized as having high productivity, and receive a wage equal to their productivity subtracting expected costs of information acquisition. Information about workers' productivities is revealed either through workers' costly education or through the firms' costly auditing. With either channel, high productivity workers effectively bear the expected costs. For large auditing cost, information revelation is relatively cheap through signaling; hence, high productivity workers have an incentive to signal their productivity through education; when auditing costs are small, expected auditing costs passed through to high productivity workers are lower than signaling costs, and it is beneficial for them to refrain from signaling and to rely on firms' audits instead.

The equilibrium has interesting features which help us understand several puzzling stylized facts in the literature. In comparison to the standard signaling model, there is less or even no

[1] According to Dessler (2017, p. 210), more than 67% of employers tested applicants for various skills. For common testing and auditing procedures for applicants, see, for example, Noe et al. (2018, Ch. 6), Armstrong and Taylor (2017, p. 254-263), Dessler (2017, Ch. 6), or Martin (2012, pp. 207–208, 216–219).

[2] Naturally the firms' auditing strategy depends upon their beliefs about workers' productivities and firms have an incentive to audit only if the beliefs are sufficiently diffuse. The unique equilibrium is obtained by applying an extension by Bester and Ritzberger (2001) of the intuitive criterion of Cho and Kreps (1987) to refine beliefs and rule out counterintuitive equilibria. See Section 4 for more details and a discussion.

signaling by education in the equilibrium with auditing. When there is no signaling by education, equilibrium wages differ for individuals who are observably equivalent. These results correspond to robust empirical findings on horizontal wage differentials, described by Abowd et al. (1999, p. 254) as: "observably equivalent individuals earn[ing] markedly different compensation … is one of the enduring features of empirical analyses of labor markets."[3] Our explanation for this puzzling phenomenon is that the strategic choice of auditing requires some low productivity workers applying for better-paid jobs and getting accepted.[4]

With many high productivity workers, in our partial pooling equilibrium some high productivity workers signal through education whereas the remaining high productivity workers pool with low productivity workers on zero education, and firms audit some applicants with zero education. High productivity workers' education decreases as auditing costs decrease. Their education also decreases as their education costs increase. This again is an interesting feature that coincides with empirical findings (e.g., Castlemen and Long, 2016, or Dynarski, 2003) but contrasts with the standard signaling models where the amount of education does not depend on one's own education costs but on the education costs of other (lower) productivity workers.

With few high productivity workers, workers pool at zero education and no worker uses education as a signal in equilibrium. Furthermore, both types' payoffs are (sometimes strictly) higher in the pooling equilibrium with auditing than their respective payoffs in the separating equilibrium without auditing. Therefore, when firms can audit rather cheaply, workers indeed prefer not to signal. With vanishing auditing costs, the pooling equilibrium becomes more and more informative and converges to the complete information outcome: workers' expected payoffs converge to their productivities.

In comparison to the standard auditing models (see, e.g., Pollrich, 2017), our signaling environment changes the auditing pattern and "whom to audit." In the standard model, firms audit the applicants for the most appealing contract offer. The key conflict there is efficiency versus truth telling by the more efficient type. In our setting when both signaling by education and auditing are used in equilibrium, it is the applicants for low wages who are audited, whereas the high wage applicants are not audited. Auditing high wage applicants is unnecessary because they signal through their choice of education. The key trade-off in our setting are the relative economic costs of information provision through signaling versus auditing.

Our contribution can be helpful for analyzing other environments that involve strategic interactions between signaling and costly information acquisition. It applies to markets where the informed participants can choose some activity to signal their private information, and the uninformed participants compete with each other by their offers to the informed market side and have the option to acquire costly information.

Consider, for example, the model of Leland and Pyle (1977) in which entrepreneurs seek to sell their projects to investors. Each entrepreneur has private information about the future revenues of their project. As Leland and Pyle (1977) show, the equity participation of the entrepreneur can then be a signal of the project's quality. This is so because high-quality entrepreneurs have a higher incentive to retain a share of the revenues than low-quality entrepreneurs. Suppose now that investors can obtain information not only from observing the entrepreneur's equity share but also by auditing project quality. Our findings then indicate that in an equilibrium where investors audit, signaling plays a role only if the market share of high-quality projects is sufficiently high.

As another application, consider warranties. Sellers are privately informed about product quality, but they can offer warranties to signal the quality of their product. Offering warranties is costly, as products break down (Spence, 1977) or as they induce moral hazard by buyers

[3] See, for example, Thaler (1989), Song et al. (2019), Card et al. (2013), or Goux and Maurin (1999), for documentation of horizontal wage differentials.

[4] The strategic choice of auditing is essential here. In signaling models with exogenous information, all wage dispersion is explained by firms' heterogeneity or workers' observable characteristics, for example, education or grades.

(Lutz, 1989). In each case, offering a warranty is more expensive for sellers of low-quality products. Buyers observe the warranties and compete by placing bids for the product. Suppose now that buyers have the option to inspect the product's quality in addition to inferring it from the warranties. Our results then suggest that in any equilibrium with inspection sellers use less warranties as a signaling device than classical signaling models without inspections would imply. This is in line with the "mixed conclusions" of the empirical literature about warranties as signals of product quality (Riley, 2001, p. 455).

Finally, consider initial public offerings (IPOs). In the IPO process, the choice of investment bankers and board members may be a signal of firm value to potential investors (see, e.g., Titman and Trueman, 1986, and Certo et al., 2001 for theoretical analysis and empirical evidence). Prestigious investment bankers and board members, who are more accurate at evaluating information about the firm, are more costly. However, owners of high value firms are willing to pay a premium for hiring them to avoid underpricing. Our analysis suggests that for those IPOs where potential investors can learn the true firm value at relatively low cost, the firms may reduce their usage of high cost agents to signal firm value.

Related literature. Signaling has been one of the most influential theories in games of asymmetric information.[5] On the other hand, auditing has been recognized as effective in mitigating incentive problems, and has been studied extensively but mainly in isolation of signaling so far. By incorporating the option of costly auditing into the otherwise standard signaling setup of Spence (1973), we are able to analyze the trade-off between these two important sources of information. Our analysis provides clear predictions on how signaling interacts with auditing. In addition, our analysis provides new insights on the usage of signaling in comparison to the classical models without the strategic option of auditing. As is well known from the literature on inspection games, auditing is possible only in a mixed-strategy equilibrium.[6] We characterize in detail the resulting structure of (partial) pooling equilibria. Whereas in the literature pooling typically involves randomization only by low productivity workers, we show that auditing may in some situations induce high productivity workers to use mixed strategies. Remarkably, also full pooling can survive the intuitive criterion in contrast to other classical models of signaling. As one would expect, the availability of auditing makes high productivity workers better off, because their signaling effort is reduced. Perhaps surprisingly, low productivity workers also may gain from auditing and this gain strictly increases as the auditing cost decreases. The reason is that auditing makes it more attractive for high productivity workers to pool with the low productivity workers.

Our article relates to Stahl and Strausz (2017) who compare the efficiency of buyer certification versus seller certification.[7] They find that seller certification is more efficient. For this comparison, they consider signaling unobservable quality through prices following, for example, Wolinsky (1983) and Bagwell and Riordan (1991). In case of buyer certification, a monopolistic seller uses pricing to signal its product quality and uninformed buyers may acquire costly certification of product quality. In Stahl and Strausz (2017), the informed seller's price serves as a quality signal *and* at the same time specifies payments. In contrast, we add auditing to the canonical signaling model of Spence (1973) in which uninformed firms compete by offering wages and workers signal their productivity through education as a pure signaling device. In our richer model, we find a unique equilibrium depending on auditing costs and the fraction of high productivity workers in the population with full pooling, partial pooling by low productivity workers, or partial pooling by high productivity workers occurring.

Signaling as a separate activity allows us to make empirically testable predictions with respect to the amount of signaling which are impossible to obtain in models with signaling via prices. The reason is that education and wages are observable in our model, whereas in models

[5] See Kreps and Sobel (1994), Riley (2001), and Sobel (2009) for a review on the role of signaling and its applications in different fields.

[6] See Avenhaus et al. (2002) for a survey.

[7] We use the same refinement (Bester and Ritzberger, 2001) as Stahl and Strausz (2017).

of signaling via prices only prices are observable. First, we find less signaling in equilibrium than in classical models without the strategic option of auditing. Second, the education costs of high productivity workers determine their education level in line with the empirical findings discussed in the Introduction. Third, we find new auditing patterns with audits focusing on low wage contracts instead of high wage contracts in contrast to usual inspection games. Fourth, in our model auditing can yield a Pareto improvement for *all* workers. Finally, our model is applicable to other market environments discussed in the Introduction and in this section.

This article is also related to the literature on job-market signaling when firms can observe some additional *exogenous* information about workers' productivities besides the costly signal. Alos-Ferrer and Prat (2012) analyze a dynamic model in which the firm is able to extract information from noisy realizations of the worker's productivity *after* the worker is hired. In contrast, in our model firms can audit the worker's productivity *before* hiring. Feltovich et al. (2002) consider a model where, prior to making job offers, employers have access to grades and other information that is correlated with workers' productivity in addition to observing the education choices by workers. They point out countersignaling equilibria: only workers with intermediate productivity signal via education, whereas low and high productivity workers pool at zero education. Daley and Green (2014) fully characterize the set of equilibria in such a model.[8] Kurlat and Scheuer (2021) study a competitive equilibrium model with firms that are heterogeneous in their precision of evaluating additional information. Also in their model, some workers choose not to invest in signaling by education: some high productivity workers forgo signaling and are hired by better informed firms.

In these works, the additional information is exogenous, and the equilibrium outcome depends on the informativeness of this information and its correlation with the costly signal. In our model, information acquisition is strategic and firms have the option whether to conduct costly audits or not. Our equilibrium outcome depends on the interaction between the worker's signaling incentive and the firms' auditing incentive. Khalil and Lawarrée (1995, p. 442) stress the importance of this difference: "Contracts [are] very sensitive to strategic behavior" although they focus on screening instead of signaling. This strategic behavior allows us to explain wage differentials that do not depend on firms' heterogeneity or workers' observable characteristics, for example, education or grades.

Our article complements other applications of signaling games with information acquisition. Banks (1992) analyzes a setup where the monopolist knows *ex ante* its true marginal costs of production; the regulator observes the market price proposed by the monopolist and decides whether to verify the monopolist's marginal costs and to impose a regulatory price for the monopoly product. Mayzlin and Shin (2011) consider a firm strategically choosing the message content of costless advertisement to reveal the product's quality. Consumers can acquire additional information about the product's quality before purchasing. Relating to the countersignaling literature mentioned above, they prove that countersignaling also occurs in advertising with only mediocre quality firms engaging in informative advertising whereas high- and low-quality firms use uninformative advertising. The high-quality firms invite consumers to do their own information acquisition. Garfagnini (2017) analyzes a career-concerned worker signaling his type through overtime at work and the firm exercising oversight to identify low ability workers. The article focuses on the effects of oversight on effort provision under exogenous wages. Oversight increases or decreases effort depending on parameters. Ekmekci and Kos (2020) consider an uninformed sender who can covertly acquire information about his type before engaging in costly signaling. Thus, the receiver has to infer from the observed signal both whether the sender knows his type and what this type is. They show that, in equilibrium, the sender acquires information allowing for meaningful signaling. Hence, signaling also applies to settings with initially

[8] They show uniqueness of the equilibrium given an appropriate refinement and that their equilibrium has continuous limits in contrast to the classical signaling setup without exogenous information.

uninformed senders. These applications typically have very different features from the competitive job-market signaling environment we focus on.

This article is organized as follows. Section 2 contains the setup combining job-market signaling and costly auditing. In Section 3, we characterize the relation between firms' beliefs, the equilibrium wage, and firms' incentive to audit for different auditing costs. In Section 4, we present the extension of the intuitive criterion that we will use to refine the equilibria. Sections 5 and 6 fully characterize the set of equilibria and show uniqueness of the separating and the pooling equilibrium. Discussions and extensions can be found in Section 7. Section 8 contains concluding remarks. All formal proofs are relegated to an Appendix.

2. The model

■ We consider the following adaptation of the Spence (1973) signaling model, in which workers choose education as a signal and then firms compete for workers. There is a unit mass of workers, who differ in their innate productivity. We restrict attention to the case of two types, $i \in \{L, H\}$, of workers. A fraction $1 - \lambda \in (0, 1)$ of workers has type L and productivity $X_L > 0$; the remaining fraction λ has type H and productivity $X_H > X_L$. Each worker's productivity is private information.[9]

Before entering the job market, workers can choose an education level $y \geq 0$, which is publicly observable. The cost of education $c_i(y)$ is type dependent and differentiable. We follow the consensus in the literature and assume that

$$c_L(0) = c_H(0) = 0, \quad c'_L(y) > c'_H(y) > 0, \tag{1}$$

for all $y > 0$. Thus, for any $y > 0$, a low productivity worker has higher education costs than a high productivity worker. If a worker of type i with education choice y is employed at the wage w, his utility is $w - c_i(y)$.

There are at least two identical firms. Each firm faces no restriction on the amount of workers that it can hire and so the job market is competitive. Firms compete for workers by posting wage offers. We assume that all firms offer the "default" wage $w_L \equiv X_L$ as a part of their wage scheme. In addition to the default wage, each firm can make wage offers $w(y)$ for workers who have acquired education y. The assumption of the default wage helps us to simplify the exposition without affecting the equilibrium outcome as we discuss in Section 7.

Firms *a priori*, when making wage offers, only observe a worker's education choice y, but not his productivity. After receiving applications, however, they can choose to learn the productivity of a job applicant at the cost $k > 0$, and then make their hiring decision contingent on the observed productivity.[10] Firms cannot contractually commit to preemployment audits and they cannot make their wage offers contingent on their auditing choice and results in line with actual hiring practices. Also, workers cannot pay firms for being audited to receive a public certification of their productivity.[11] When a firm employs a worker with productivity X_i at the wage w, its profit is $X_i - w - k$ if it has performed an audit before hiring, and $X_i - w$ without an audit. If after auditing the firm decides not to hire the applicant, it incurs the loss $-k$.

Workers are free to opt for the default wage or to obtain some education y to apply for a wage offer $w(y)$ later. Obviously, firms will hire all workers that apply for the default wage w_L, independently of their education and without auditing. Note that w_L is the competitive wage for low productivity workers under perfect information and that, under imperfect information about

[9] Rather than considering a unit mass of workers, one may equivalently consider a single worker, whose productivity is randomly chosen by nature. If in our setting different fractions of type i workers take different decisions, this corresponds to a mixed strategy of type i in the setting with a single worker.

[10] For example, Guasch and Weiss (1981, p. 275) write that a "common practice is for firms to offer a wage for a given job classification, and to test applicants." They provide a model where testing job applications is contractible.

[11] A standard justification for auditing not to be contractible is that the firm's auditing activity is private information.

worker productivity, auditing applicants for w_L is clearly never optimal for a firm. Thus, each worker can always obtain a job at the wage w_L.

If a worker applies for a job offer with wage $w(y) > X_L$, the firm may choose to audit the applicant and then reject the application after auditing. In this sense, the wage offers $w(y) > X_L$ are non-binding. If a job applicant is rejected, he is perceived as a low productivity worker and gets the utility $X_L - s$, where $s \in (0, X_L)$ accounts for some delay or switching cost.[12] The cost s presents a penalty for a failed application: even if $w(y) - c_L(y) > X_L$, it may be attractive for a low productivity worker to apply for the default wage $w_L = X_L$ rather than $w(y)$, because if he is audited and rejected at $w(y)$ he only gets $X_L - s$.

We summarize the sequence of events of the signaling and auditing game as follows:

(i) Workers privately observe their productivity, X_L or X_H.
(ii) Workers either choose some education $y \geq 0$ or opt for the default wage $w_L = X_L$.
(iii) Firms compete by offering wages $w(y)$ for each observed education y.
(iv) Workers with education y choose for which offer $w(y)$ to apply.
(v) Firms take auditing decisions and then decide on hiring.

Up to stage (iv) this setting is essentially identical to Spence's (1973) classical signaling model. One minor difference is that workers can apply for the default wage w_L in stage (ii). However, it is easy to see that adding this option to Spence's (1973) original setting would not change anything in the outcome of the classical signaling model. In our setting, as we explain in more detail in Section 7, introducing the default wage merely simplifies the presentation of equilibrium. In stage (v) we add to the classical signaling model that firms have the option of auditing and not hiring a worker after auditing.

As in Spence (1973), firms are committed to their public wage announcements made before their hiring decision. This sequence of events looks quite natural in many labor markets. Indeed, in their handbook on personnel management Armstrong and Taylor (2017, p. 248) point out that the first step of the recruiting process is to define job requirements including "terms and conditions (pay, benefits, …)".[13] After attracting candidates, the next steps are sifting applications, interviewing, testing, assessing candidates, obtaining references, and checking applications. Apart from mirroring actual hiring practices, the assumption of commitment allows us to focus on the interaction of auditing and signaling in a competitive labor market. If, instead, a firm could renege on its initial wage announcement, it may appropriate some bargaining surplus in the wage negotiation with a job applicant. This would unravel wage competition between the firms, and also change their auditing behavior as well as the workers' incentives to invest in signaling.

In what follows, we analyze the perfect Bayesian equilibrium of this game. Obviously, firms' wage offers $w(y)$ will depend on the belief about the average productivity of workers that have selected education y. Also, as we show in the next section, this belief is important for the firms' auditing decisions. To simplify the exposition, we assume that all firms have identical out-of-equilibrium beliefs. Further, whenever a wage offer attracts workers, the average productivity of applicants for this wage is the same for all firms that make this offer. Hence, we can denote by $\mu(y) \in [0, 1]$ the firms' belief that a fraction $\mu(y)$ has high productivity among all workers who have chosen education y and apply for $w(y)$. We say that an equilibrium is *unique* if the workers' education choices and all wage offers that attract a positive mass of workers are uniquely determined.

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[12] As we show later, in equilibrium only low productivity workers will be rejected. A possible interpretation is that a rejected worker can only apply for the default wage and workers discount future wages by the factor $\delta \in (0, 1)$: workers have the utility loss $s = (1 - \delta)w_L$ if they are hired at a later date rather than immediately.

[13] Martin (2012, pp. 200–201), in a popular Human Resources textbook, confirms that the "job announcement should include …the salary and benefits attached to the position."

3. Wages and auditing

■ We first analyze how the firms' equilibrium wage offers and their auditing decisions depend on firms' belief $\mu(y)$ about workers with education y following the common logic of inspection games. By offering wages $w(y) > X_L$, firms aim to attract high productivity workers. Suppose a firm makes an offer $w \in (X_L, X_H]$. If the firm decides not to perform an audit, it optimally hires the applicant as long as his expected productivity is above the wage w. In contrast, with auditing a firm will hire only high productivity workers and reject low productivity workers. Given their belief, firms face the trade-off of incurring auditing costs and hiring high productivity workers only, or saving the auditing costs and hiring both types.

With auditing, the firm rejects low productivity applicants and its expected profit at the wage w is $\mu(y)(X_H - w) - k$, because only a fraction $\mu(y)$ of all workers is expected to have high productivity. Therefore, the competitive wage, which yields zero profits, equals

$$X_H - k/\mu(y)$$

when firms audit. Without auditing, the competitive wage equals the average productivity

$$X_L + \mu(y)(X_H - X_L),$$

because both types are hired. Comparing the competitive wage with and without auditing yields the cutoff $k = \mu(1-\mu)(X_H - X_L)$: for values of k below this cutoff, competition for high productivity workers forces firms to audit workers, because the resulting wage is higher than what they can offer without auditing; for values of k above the cutoff auditing will not occur under competition. The critical value $\mu(1-\mu)(X_H - X_L)$ attains a maximum of

$$\tilde{k} \equiv \frac{X_H - X_L}{4} \tag{2}$$

at the belief $\mu = 1/2$, that is, maximal uncertainty. Obviously, if the costs of auditing are too high, auditing cannot occur in equilibrium. As long as auditing cost $k > \tilde{k}$, irrespectively of the belief $\mu(y)$, the average productivity is higher than the benefits of hiring only high productivity workers with auditing. Therefore, competition precludes firms from auditing.

Now consider auditing cost $k \leq \tilde{k}$. In this case, the equation $k = \mu(1-\mu)(X_H - X_L)$ has two solutions if costs $k < \tilde{k}$, and these solutions coincide for cost $k = \tilde{k}$:

$$\mu_1(k) \equiv \frac{1}{2} - \frac{1}{2}\left(\frac{X_H - X_L - 4k}{X_H - X_L}\right)^{1/2}, \quad \mu_2(k) \equiv \frac{1}{2} + \frac{1}{2}\left(\frac{X_H - X_L - 4k}{X_H - X_L}\right)^{1/2}. \tag{3}$$

The right part of Figure 1 shows how the cutoffs $\mu_1(k)$ and $\mu_2(k)$ for beliefs depend on the auditing costs k.

The following lemma describes how the firms' equilibrium behavior depends on their beliefs and auditing costs. The key message is that firms audit only when auditing costs are low and there is enough uncertainty so that they have diffuse beliefs about workers' productivities. Let $\rho(y) \in [0, 1]$ denote the probability that a firm will audit an applicant with education y.

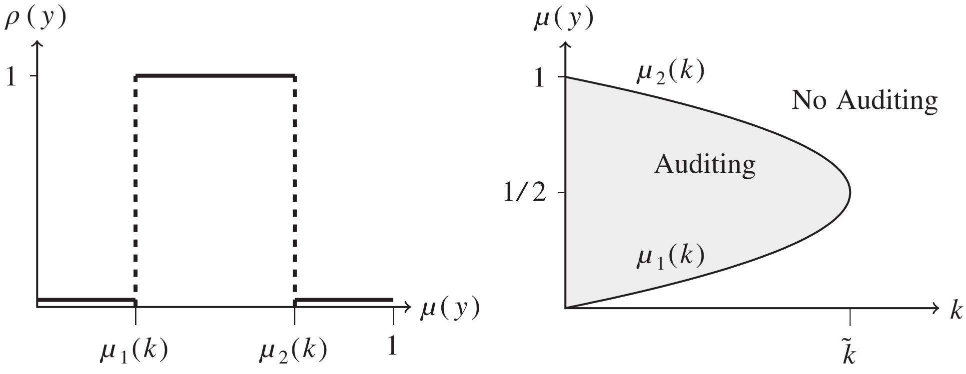
Lemma 1. After observing education y, in any equilibrium:

(i) if costs $k > \tilde{k}$ or beliefs $\mu(y) \notin [\mu_1(k), \mu_2(k)]$, then firms offer $w(y) = X_L + \mu(y)(X_H - X_L)$ and choose no auditing $\rho(y) = 0$.
(ii) if costs $k \leq \tilde{k}$ and beliefs $\mu(y) \in (\mu_1(k), \mu_2(k))$, then firms offer $w(y) = X_H - k/\mu(y)$ and choose full auditing $\rho(y) = 1$.
(iii) if costs $k \leq \tilde{k}$ and beliefs $\mu(y) \in \{\mu_1(k), \mu_2(k)\}$, then firms offer $w(y) = X_L + \mu(y)(X_H - X_L) = X_H - k/\mu(y)$ and any auditing $\rho(y) \in [0, 1]$ is optimal for them.

The equilibrium wage strictly increases in beliefs $\mu(y)$. At this wage, not auditing is uniquely optimal for the firms if $\mu(y) < \mu_1(k)$ or $\mu(y) > \mu_2(k)$ because their belief is

FIGURE 1

OPTIMAL AUDITING AND FIRMS' BELIEFS



relatively precise and workers are highly likely to have either high or low productivity. Only for more diffuse beliefs $\mu(y) \in (\mu_1(k), \mu_2(k))$ there is a high uncertainty regarding the workers' productivity, so that competition forces firms to audit their applicants. If $\mu(y) = \mu_1(k)$ or $\mu(y) = \mu_2(k)$, the firms are indifferent between auditing or not at the competitive wage. In this situation they optimally audit some arbitrary fraction $\rho(y) \in [0, 1]$ of applicants. The left part of Figure 1 shows how the optimal auditing probability $\rho(y)$ depends on beliefs $\mu(y)$ according to Lemma 1. As the right part of the figure illustrates, the range of beliefs where auditing is optimal shrinks as auditing costs k increase. For costs above $\tilde{k}$, firms never use auditing independently of beliefs as the interval $(\mu_1(k), \mu_2(k))$ collapses into an empty set.

The lemma implies that firms do not audit if workers' choice of education is fully revealing and firms are confident to infer workers' productivities from their education, which is the case if workers with different productivities have chosen different educations. Auditing can only occur in an equilibrium in which workers with different productivities have chosen the same education levels and thus workers' education choice is not fully revealing about their productivities.

Lemma 1 allows us to derive workers' expected utility from choosing education y given firms' belief. First consider high productivity workers. These workers will never be rejected and so always receive the wage $w(y)$ stated in the lemma. Therefore, their utility is

$$U_H(y|\mu(y)) \equiv \begin{cases} X_H - k/\mu(y) - c_H(y) & \text{if } k \leq \tilde{k} \text{ and } \mu(y) \in [\mu_1(k), \mu_2(k)], \\ X_L + \mu(y)(X_H - X_L) - c_H(y) & \text{otherwise.} \end{cases} \tag{4}$$

Note that the utility of high productivity workers is strictly increasing in the belief $\mu(y)$.

In contrast, a low productivity worker gets the wage $w(y)$ only if he is not audited. After an audit he is rejected and only gets $X_L - s$. Therefore, his expected utility depends on the audit probability $\rho(y)$. This probability, however, is arbitrary when Lemma 1 (iii) applies. For what follows, however, it is sufficient that for all other parameter combinations the utility of low productivity workers is well defined by

$$U_L(y|\mu(y)) \equiv \begin{cases} X_L - s - c_L(y) & \text{if } k \leq \tilde{k} \text{ and } \mu(y) \in (\mu_1(k), \mu_2(k)), \\ X_L + \mu(y)(X_H - X_L) - c_L(y) & \text{if } k \leq \tilde{k} \text{ and } \mu(y) \notin [\mu_1(k), \mu_2(k)], \\ X_L + \mu(y)(X_H - X_L) - c_L(y) & \text{if } k > \tilde{k}. \end{cases} \tag{5}$$

4. Belief refinements

■ As is well known, signaling games have a disconcerting multiplicity of equilibria. The reason is that the perfect Bayesian equilibrium in our context pins down the firms' beliefs only for equilibrium education choices. Therefore, multiple outcomes can be supported by out-of-equilibrium beliefs that deter any deviating education choice by interpreting it as a signal of low productivity. To rule out counterintuitive equilibria driven by such overly pessimistic beliefs, the literature has adopted belief refinements that impose plausible restrictions on out-of-equilibrium beliefs.

The standard refinement for Spence's (1973) model of education signaling is the *intuitive criterion* of Cho and Kreps (1987). If auditing is unavailable, it yields a unique prediction in the model with two worker types by ruling out pessimistic beliefs about their productivity for certain out-of-equilibrium education choices. Let U_L^* and U_H^* denote the equilibrium utility of low and high productivity workers, respectively. The idea of the intuitive criterion is that an out-of-equilibrium education choice y should be considered as a signal of a high productivity worker if—given this belief—only a high productivity worker has an incentive to deviate to education y:

Condition 1. For any out-of-equilibrium education y, if

$$U_H(y|1) > U_H^* \text{ and } U_L(y|1) < U_L^* \tag{6}$$

then beliefs $\mu(y) = 1$.

By the first inequality in (6), a high productivity worker gains by choosing education y if that education is interpreted as a signal of high productivity, whereas by the second inequality a low productivity worker loses by choosing education y even when he is considered to have high productivity. The intuitive criterion stipulates that education y is a convincing signal of high productivity in this situation. Thus, whenever (6) holds for some out-of-equilibrium education y, the equilibrium does not satisfy Condition 1: high productivity workers would gain by deviating to education y because the deviation increases their utilities $U_H(y|\mu(y)) = U_H(y|1) > U_H^*$.

The intuitive criterion is designed to refine out-of-equilibrium beliefs for signaling games without alternative sources of information. As Bester and Ritzberger (2001) argue, the intuitive criterion allows for auditing but will lead to multiplicity of equilibria. This is so because it specifies a deterministic beliefs restriction, $\mu(y) = 1$, when a deviating choice of education satisfies criterion (6). As we have seen in the previous section, however, firms will audit only if their belief is sufficiently diffuse. Therefore, even with arbitrarily small auditing costs, the intuitive criterion cannot induce auditing as a response to a deviating education choice.[14] To provide a more effective role for auditing, Bester and Ritzberger (2001) propose an extension of the intuitive criterion. We apply a slight modification because a low productivity worker's utility $U_L(\cdot)$ in (5) is not defined for beliefs $\mu \in \{\mu_1, \mu_2\}$:[15]

Condition 2. For any $\delta \in [0, 1]\backslash\{\mu_1, \mu_2\}$ and for any out-of-equilibrium education y, if

$$U_H(y|\delta) > U_H^* \text{ and } U_L(y|\delta) < U_L^* \tag{7}$$

then $\mu(y) \geq \delta$.

Condition 2 extends the idea of intuitive criterion to a situation where a deviation to education y is profitable only for high productivity workers when the firms believe that the deviation originates from high productivity workers with some probability δ. This belief is already rather pessimistic because low productivity workers have no incentive to deviate to education y. Condition 2 requires that the firms' belief should not be even more pessimistic than δ. Thus, whenever (7) holds, the equilibrium violates Condition 2 because utility $U_H(y|\mu(y)) \geq U_H(y|\delta) > U_H^*$

[14] Alternative refinements such as universal divinity in Banks (1992) and Condition $D1$ in Daley and Green (2014), also require off-equilibrium beliefs to be deterministic and therefore cannot induce auditing.

[15] Note that this makes Condition 2 less restrictive.

implies that high productivity workers would rather choose education y than the supposed equilibrium education. Notice that if (7) is satisfied for several values of δ, Condition 2 requires the out-of-equilibrium beliefs not to be more pessimistic than the highest of these values.

Condition 2 allows for belief $\mu(y) \in (0, 1)$ in the specification of out-of-equilibrium beliefs and, as a result, the firms may respond to a deviating education choice y by auditing if $\mu(y) \in [\mu_1(k), \mu_2(k)]$. Therefore, in contrast to the intuitive criterion, Condition 2 has the advantage of potentially inducing auditing after an out-of-equilibrium education choice. Clearly, Condition 2 is a stronger refinement than Condition 1, because it contains the latter as the special case $\delta = 1$. Therefore, if an equilibrium candidate survives Condition 2, it must also survive Condition 1. Conversely, if an equilibrium is ruled out by Condition 1, it will also be ruled out by Condition 2.

It turns out that in our context Conditions 1 and 2 yield the same unique outcome as long as auditing is too costly to occur in equilibrium. Yet, for small auditing costs only Condition 2 uniquely selects an equilibrium. More specifically, in the following section we first investigate the classical separating equilibrium selected by Condition 1. In this equilibrium firms do not audit job applicants because for the equilibrium education choices their beliefs are deterministic. Yet, we show that this equilibrium does not survive Condition 2 if auditing costs are small. In this case, Condition 2 selects a unique equilibrium with auditing as we show in Section 6. This implies some pooling because by Lemma 1 auditing can occur only if the firms' beliefs are diffuse.

5. Separating equilibrium

■ An equilibrium is called *separating* if education choices fully reveal a worker's productivity. Thus, the firms' beliefs for any education chosen in equilibrium are either zero or one, $\mu(y) \in \{0, 1\}$ for all education y contained in the support of workers' equilibrium education choices. As shown by Cho and Kreps (1987), the Spence (1973) model with two worker types has a unique equilibrium that satisfies the intuitive criterion of Condition 1. It is separating and has the following properties: all workers receive a wage equal to their productivity; low productivity workers choose zero education; high productivity workers choose education y_H^* defined by

$$c_L(y_H^*) = X_H - X_L. \tag{8}$$

This is the least-cost separating equilibrium in the sense that y_H^* is the lowest level of education such that low productivity workers cannot gain from imitating the high productivity workers' education choice to receive the wage X_H.

To investigate whether this outcome remains an equilibrium in our extension of Spence's (1973) setting, we define a critical auditing cost $\bar{k}$ by

$$\bar{k} = \tilde{k} \quad \text{if} \quad c_H(y_H^*) \geq 2\tilde{k}, \qquad \frac{\bar{k}}{\mu_2(\bar{k})} = c_H(y_H^*) \quad \text{if} \quad c_H(y_H^*) < 2\tilde{k}, \tag{9}$$

with $\tilde{k}$ given by (2).[16]

To understand the role of the critical auditing cost $\bar{k}$ for the equilibrium outcome, recall from the discussion of Lemma 1 that $k/\mu(y)$ is the auditing cost passed through to high productivity workers if the auditing cost is k and the firms' belief is $\mu(y)$. As $\mu_2(k)$ is the largest possible belief that induces an audit, $k/\mu_2(k)$ is the minimum auditing cost borne by high productivity workers if an audit takes place. In the second part of (9), $\bar{k}$ is the critical value of k at which this minimum auditing cost equals the education cost a high productivity worker incurs for the separating education y_H^*. For any auditing costs $k < \bar{k}$, high productivity workers are better off by bearing the expected audit cost than signaling by education y_H^* because $k/\mu_2(k) < c_H(y_H^*)$. As the following proposition shows, the parameter $\bar{k}$ is indeed critical for the existence of a separating equilibrium under Condition 2.

[16] Note that $\bar{k}$ is uniquely defined as $k/\mu_2(k)$ strictly increases in $k \in (0, \tilde{k}]$, equals zero for $k \to 0$, and $\tilde{k}/\mu_2(\tilde{k}) = 2\tilde{k}$. Further, $\bar{k} < \tilde{k}$ if $c_H(y_H^*) < 2\tilde{k}$.

Proposition 1.

(i) For all auditing costs k, there exists a unique separating equilibrium satisfying Condition 1: low productivity workers receive a wage of X_L and do not invest in education; high productivity workers receive a wage of X_H and choose education y_H^*. Auditing does not occur in equilibrium.
(ii) The separating equilibrium satisfies Condition 2 if and only if auditing costs are large, $k \geq \bar{k}$.

As indicated above, the statement in part (i) of the proposition is shown already in Cho and Kreps (1987) for a setting where the option of auditing is unavailable. To see that this equilibrium also persists if auditing is feasible, let firms simply believe that all low productivity workers choose an education below y_H^* and all high productivity workers choose an education of at least y_H^*. These beliefs are consistent with the equilibrium outcome and it is easily verified that they satisfy Condition 1. Also, according to Lemma 1 these beliefs imply that auditing occurs neither in equilibrium nor after out-of-equilibrium education choices. Thus, the equilibrium selected by Condition 1 is not affected by the feasibility of auditing. Even when the auditing cost is arbitrarily small, high productivity workers must invest education costs $c_H(y_H^*)$ to distinguish themselves from low productivity workers.

To complete the proof of Proposition 1, we show part (ii) by the following lemma:

Lemma 2. Suppose high productivity workers choose education y_H^* and receive a wage of X_H in equilibrium.

(i) If auditing costs are small, $k < \bar{k}$, then Condition 2 is not satisfied.
(ii) If auditing costs are large, $k \geq \bar{k}$, then Condition 2 is satisfied whenever low productivity workers receive equilibrium utility $U_L^* = X_L$.

Clearly, the equilibrium described in Proposition 1 (i) has all the properties required in Lemma 2, and so this proves Proposition 1 (ii).

By Lemma 2 (i), the separating equilibrium does not survive Condition 2 if auditing costs are small. The idea is that for some small out-of-equilibrium choice of education, there always exists some belief δ that induces auditing, and given this belief, high productivity workers benefit from such a deviation whereas low productivity workers lose from it due to auditing. By Condition 2, such a deviation should be interpreted as originating from high productivity workers with probability no smaller than δ. Given this belief, high productivity workers prefer such an audit-inducing deviation over education y_H^* because the auditing cost passed through to them is relatively low in comparison to the signaling cost associated with education y_H^*, thus destroying the separating equilibrium.

Lemma 2 (ii) implies that if auditing costs are large, the separating equilibrium satisfies Condition 2. The reason is that any deviation that does not induce auditing and is profitable for high productivity workers is also profitable for low productivity workers, and so there is no violation of Condition 2. If, however, a deviation induces auditing, it is never profitable for high productivity workers because the audit cost passed through to them is too high in comparison to the signaling cost from education y_H^*. Thus, there exists no deviation and belief such that the two inequalities in Condition 2 hold simultaneously, and therefore the separating equilibrium cannot be eliminated by Condition 2 when auditing costs are large.

In summary, applying Condition 2, Proposition 1 and Lemma 2 show that with the option of auditing, the least-cost separating equilibrium is supported as a plausible equilibrium if and only if auditing costs are large. Signaling by education is the effective channel of information provision, as in classic signaling models, and firms do not audit in equilibrium. Competition drives firms' profit down to zero, and the economic cost of information provision, through signaling or auditing, is directly or indirectly borne by high productivity workers, and therefore the cheaper way of information provision is supported in equilibrium. If auditing costs are small, however, the economic cost of auditing is lower than that of signaling, and the least-cost separating equilibrium fails to be an equilibrium satisfying Condition 2. Instead, there is a partial pooling equilibrium, in which workers use less signaling via education and auditing occurs with positive probabilities, as we show in the next section.

Note that Lemma 2 not only applies to the separating equilibrium in Proposition 1, it also applies to any equilibrium where some high productivity workers choose education y_H^* and receive the wage X_H. We will use this insight in the next section in Proposition 2 for the analysis of equilibria with auditing.

6. Pooling and auditing

■ Auditing requires some pooling. In a *pooling* equilibrium, some low productivity workers and some high productivity workers choose the same education y_P^*. Thus, an education of y_P^* does not fully reveal a worker's productivity. Denote the fraction of type $i \in \{L, H\}$ workers with education y_P^* by σ_i.[17] We say that *partial pooling* occurs at y_P^* if the fraction $\sigma_i < 1$ for some $i \in \{L, H\}$; otherwise, if all workers choose the pooling education, $\sigma_L = \sigma_H = 1$, we have *full pooling*.

It is well known from Cho and Kreps (1987) that pooling does not survive the intuitive criterion of Condition 1 without auditing. With auditing, however, pooling survives the intuitive criterion of Condition 1. Indeed, using their argument, we show that auditing must take place at any pooling education with positive probability. In addition, this auditing probability must be below one so that low productivity workers are willing to apply for the pooling wage with some risk of being detected as low productivity workers. As this result on auditing probabilities is implied already by the weaker Condition 1, it is clearly also true under the stronger Condition 2. Actually, Condition 2 uniquely determines the firms' beliefs at any pooling education.

Lemma 3. (i) Pooling can occur at an education y_P^* in an equilibrium under Condition 1 only if auditing costs satisfy $k \leq \tilde{k}$ and the equilibrium auditing probability satisfies $\rho(y_P^*) \in (0, 1)$.

(ii) Suppose pooling occurs at education y_P^* in an equilibrium satisfying Condition 2. Then firms' belief is $\mu(y_P^*) = \mu_2(k)$ and so the firms offer the wage

$$w(y_P^*) = X_L + \mu_2(k)(X_H - X_L) = X_H - k/\mu_2(k). \qquad (10)$$

Furthermore, pooling cannot occur at more than one education choice.

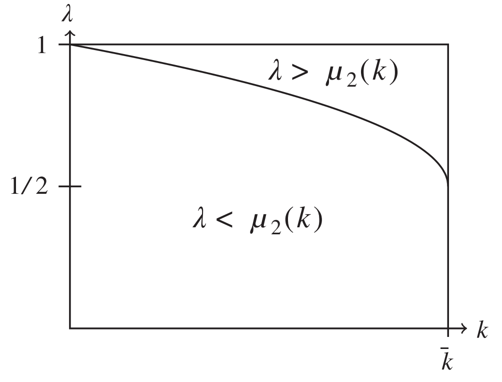
The intuition for why Condition 1 does not rule out pooling is that high productivity workers receives the pooling wage with certainty, whereas low productivity workers are rejected with positive probability equal to the auditing probability $\rho(y_P^*)$. Therefore, low productivity workers expect a wage from education y_P^* of less than $w(y_P^*)$. Hence, the increase in the wage from deviating to an education y with $\mu(y) = 1$ and $w(y) = X_H$ is higher for low productivity workers than for high productivity workers. Auditing in a pooling equilibrium, therefore, makes it more costly for high productivity workers to distinguish themselves by an education that low productivity workers are not willing to imitate. In fact, this makes it unattractive for high productivity workers to deviate from the pooling education to engage in signaling with a higher education that satisfies the requirements of Condition 1.

The intuition for Lemma 3 (ii) is as follows. The auditing probabilities imply that firms play mixed strategies in auditing. Hence, firms must be indifferent between auditing and not auditing an applicant with education y_P^*. Recall from Lemma 1 that firms are indifferent between auditing or not only under the beliefs $\mu_1(k)$ or $\mu_2(k)$. Condition 2 implies that firms must hold the more optimistic belief $\mu_2(k) \geq 1/2$ in a pooling equilibrium, and hence pins down the unique

[17] Lemma 3 below shows that pooling cannot occur at more than one education choice. Hence, our notation is without loss of generality.

FIGURE 2

CATEGORIES OF POOLING EQUILIBRIUM



equilibrium belief for any pooling education y_P^*, $\mu(y_P^*) = \mu_2(k)$.[18] Finally, pooling cannot occur at more than one education choice because the pooling wage specified in (10) is the same for any pooling education. As high productivity workers are always hired, it cannot be optimal for them to choose different educations with different costs to receive the same wage.

The following proposition shows that the unique equilibrium is pooling for small auditing costs. It further shows that the relation between the auditing cost k and the fraction λ of high productivity workers in the population classifies the pooling equilibrium into three categories: full pooling, partial pooling by high productivity workers, and partial pooling by low productivity workers.

Proposition 2. If auditing costs are small, $k < \bar{k}$, then there exists a unique equilibrium satisfying Condition 2:

(i) if there are many high productivity workers, $\lambda > \mu_2(k)$, then some high productivity workers choose the pooling education $y_P^* = 0$ and the remaining ones choose an education $\hat{y}_H \in (0, y_H^*)$. All low productivity workers choose the pooling education y_P^*.
(ii) if the fraction of high productivity workers is $\lambda = \mu_2(k)$, all workers of both types choose the pooling education $y_P^* = 0$.
(iii) if there are few high productivity workers, $\lambda < \mu_2(k)$, then some low productivity workers choose the pooling education $y_P^* = 0$ and the remaining ones apply for the default wage. All high productivity workers choose the pooling education $y_P^* = 0$.

For large auditing costs $k > \bar{k}$, a pooling equilibrium satisfying Condition 2 does not exist.

In the pooling equilibrium, firms audit workers with the pooling education y_P^* with some probability.[19] Firms have to hold the belief $\mu(y_P^*) = \mu_2(k)$ to be willing to audit applicants randomly. The condition $\mu(y_P^*) = \mu_2(k)$ on equilibrium beliefs determines the fraction of high and low productivity workers who apply for the pooling wage. Figure 2 illustrates the parameter sets for the three categories of pooling equilibrium. It depicts the line $\lambda = \mu_2(k)$ where full pooling occurs. Above the line, there is partial pooling by high productivity workers. Below the line, there is partial pooling by low productivity workers.

[18] Indeed, for the more pessimistic belief $\mu_1(k) < 1/2$, high productivity workers can benefit by deviating to some education $y' > y_P^*$ which induces more optimistic but also more diffuse beliefs. Under these beliefs firms audit with probability one so that only high productivity workers gain from deviating to education y'.

[19] See the proof of Proposition 2 for the derivation of these probabilities.

First, in Proposition 2 (i), if there are many high productivity workers, it is impossible that all high productivity workers choose the pooling education y_P^* to apply for the pooling wage because beliefs would be too optimistic, $\mu(y_P^*) > \mu_2(k)$, and firms would not audit candidates. Therefore, some high productivity workers have to engage in signaling by education $\hat{y}_H$ to sustain an equilibrium. Hence, there is partial pooling by high productivity workers. Some high productivity workers engage in signaling by education whereas the remaining ones get no education and choose the pooling contract. Thus, high productivity workers must be indifferent between receiving the pooling wage $w(y_P^*)$ and acquiring education $\hat{y}_H$ to receive wage X_H. Low productivity workers prefer getting no education $y_P^* = 0$ and applying for the pooling wage $w(y_P^*)$ to education $\hat{y}_H$ that would yield the wage X_H. In fact, by Condition 1, it is easy to see that they have to be indifferent between these two options: if low productivity workers strictly preferred applying for the pooling wage $w(y_P^*)$ over choosing education $\hat{y}_H$, then high productivity workers could appeal to the intuitive criterion of Condition 1 that an education choice slightly below $\hat{y}_H$ should also be considered as a convincing signal of high productivity.

Second, in Proposition 2 (ii), if the fraction of high productivity workers is $\lambda = \mu_2(k)$, there is full pooling. Full pooling means that all workers get the same education of zero. This case occurs only under non-generic parameter combinations because full pooling requires that the fraction λ of high productivity workers in the population equals the belief cutoff, $\mu_2(k) = \lambda$. Thus, for a given audit cost k, full pooling is possible at most for a single value of the fraction λ of high productivity workers. Full pooling can be viewed as a limit case $\sigma_H \to 1$ of the first category. Alternatively, it can be viewed as the limit case $\sigma_L \to 1$ of the third equilibrium category.[20]

Third, in Proposition 2 (iii), if there are few high productivity workers, it is impossible that all low productivity workers choose the pooling education y_P^* to apply for the pooling wage because beliefs would be too pessimistic, $\mu(y_P^*) < \mu_2(k)$. Therefore, some low productivity workers have to opt for the default wage to sustain an equilibrium. Hence, there is partial pooling by low productivity workers. Some low productivity workers choose the pooling education $y_P^* = 0$ whereas the remaining ones opt for the default wage. Thus, low productivity workers must be indifferent between receiving the default wage and choosing the pooling education to obtain the pooling wage, where they are rejected with some probability. We confirm our results for an alternative specification without a default wage in Section 7. As Figure 2 shows, Proposition 2 (i) and (ii) only apply for a sufficiently high fraction of high productivity workers, that is, λ sufficiently large. Thus, if costs k are sufficiently small or if the fraction of high productivity workers is $\lambda \leq 1/2$, the equilibrium is always as described in part (iii).

The intuition why Condition 2 selects pooling as the unique equilibrium for small auditing costs is as follows: for small auditing costs, the economic costs of auditing are lower than those of signaling and for high productivity workers the pooling wage $w(y_P^*) = X_H - k/\mu_2(k)$ is more attractive than their payoff $X_H - c_H(y_H^*)$ in the separating equilibrium. Therefore, competition among the firms induces pooling of workers at the pooling wage, and less signaling is used in comparison to the least-cost separating equilibrium in Proposition 1. In particular, in part (i), some high productivity workers signal through education and the remaining ones pool with the low productivity workers. Both signaling and auditing are used for information provision in equilibrium. On the other hand, in part (iii), all high productivity workers and some low productivity workers pool, education as a signal is not used in equilibrium, and audits are the only source of information. Low costs of auditing eliminate signaling by education. As Condition 2 implies Condition 1, this equilibrium also survives Condition 1.[21]

Notice the wage differentials in equilibrium. In Proposition 2 (iii), some low productivity workers receive the pooling wage $w(y_P^*) > X_L$, but the remaining ones receive a wage equal to

[20] As the two approaches have different solutions for the auditing probabilities, the auditing probability is not uniquely determined under full pooling. Any auditing probability between the solutions of the first or the third category can support full pooling.

[21] If auditing costs are small, both the separating equilibrium in Proposition 1 and the pooling equilibrium in Proposition 2 are supported under Condition 1, whereas only the pooling equilibrium is supported under Condition 2.

their low productivity. The firms' strategic choice of auditing endogenously creates wage differentials for low productivity workers, even though there are no search frictions in our model. This provides a nice explanation for the anomalies of large wage differentials among observably equivalent individuals that are well documented in the empirical labor literature, as discussed in the Introduction. So far, a consistent theory explaining these wage differentials without the assumption of implausible large search frictions or firm heterogeneity has been missing.

Finally, in Proposition 2 (i), firms audit the workers who apply for the relatively lower pooling wage $w(y_P^*)$ and the workers that apply for the high wage X_H are not audited. This stands in contrast to the common auditing pattern in the literature that usually applicants for higher wages are audited. The reason is that only high productivity workers apply for the high wage and they perfectly reveal their productivities by their choice of education $\hat{y}_H$. Thus, auditing them to learn their productivity is unnecessary.

Comparative statics of the pooling equilibrium. We now turn to the comparative statics of the pooling equilibrium with respect to the auditing costs k.

Proposition 3. If auditing costs are small, $k < \bar{k}$, then the pooling equilibrium has the following properties:

(i) if there are many high productivity workers, $\lambda > \mu_2(k)$, then the fraction of high productivity workers choosing pooling is decreasing and the auditing probability $\rho(y_P^*)$ is increasing in costs k. Further, the education $\hat{y}_H$ increases in auditing costs k and decreases in high productivity workers' education costs.[22]

(ii) if there are few high productivity workers, $\lambda < \mu_2(k)$, then the fraction of low productivity workers choosing pooling is increasing and the auditing probability $\rho(y_P^*)$ is decreasing in costs k. Further, $\lim_{k \to 0} \sigma_L = 0$ and $\lim_{k \to 0} \rho(y_P^*) = (X_H - X_L)/(X_H - X_L + s) \in (0, 1)$.

In the pooling equilibrium, firms' beliefs have to satisfy the condition $\mu(y_P^*) = \mu_2(k)$ with the cutoff $\mu_2(k)$ decreasing in auditing costs. If there are many high productivity workers, less high productivity workers need to pool when costs increase. Analogously, if there are few high productivity workers, more low productivity workers are required to pool.

The comparative statics of the auditing rate with respect to auditing costs depend on the equilibrium category.[23] The reason is that the indifference condition for low productivity workers differs between the categories. If there are many high productivity workers, low productivity workers must be indifferent between applying for the pooling wage and imitating a high productivity worker's education $\hat{y}_H$. As auditing costs increase, the deviation utilities $X_H - c_L(\hat{y}_P)$ of low productivity workers decrease faster than the pooling wage, and so auditing probabilities are increasing in auditing costs. In contrast, if there are few high productivity workers, the firms' auditing rate has to make low productivity workers indifferent between applying for the pooling wage and opting for the default wage. If costs increase, then the pooling wage decreases, and low productivity workers need to be audited less frequently to stay indifferent.

Interestingly, the equilibrium education $\hat{y}_H$ in Proposition 2 (i) depends on the education costs of high productivity workers. This contrasts with the standard result without auditing in (8) that high productivity workers' education costs do not matter for their equilibrium education. With auditing, however, more expensive education lowers the education level necessary to make high productivity workers indifferent between the lower pooling wage without education and the higher separation wage with education. Low productivity workers remain in the pooling contract as their utilities increase due to a lower auditing probability.

[22] We say that high productivity workers' education costs increase if the new education costs are larger than the initial education costs for all education levels: $c_H^{\text{New}}(y) > c_H(y)$ for all $y > 0$.

[23] However, the equilibrium auditing rate decreases in the delay cost s independently of the equilibrium category: s reduces the low productivity worker's expected payoff from applying for the pooling wage at which firms audit applicants with positive probability in all categories.

If instead the auditing costs decrease, less high productivity workers obtain an education and the amount of education obtained decreases. Hence, the lower the auditing costs, the less signaling by education occurs. The reason is that lower auditing costs make auditing more attractive for firms. Therefore, more high productivity workers choose pooling and forgo any education without reducing firms' incentives to audit. At the same time, lower auditing costs imply higher wages whenever audits are involved. Thus, less education is required to make high productivity workers indifferent between the higher separation wage with that education and the lower pooling wage without education.

7. Discussion

■ In this section we discuss some implications and extensions of our analysis.

□ **The limit $k \to 0$.** Under perfect information about workers' productivities, each worker receives a wage equal to their productivity without signaling by education or costly auditing. It seems a reasonable conjecture that the equilibrium under asymmetric information resembles the perfect-information outcome for sufficiently small auditing costs. Nonetheless, as Proposition 1 (i) shows, this is not true if merely Condition 1 is assumed. We now show, however, that the conjecture holds under the stronger Condition 2. Recall that for small auditing costs, a separating equilibrium satisfying Condition 2 does not exist, whereas the pooling equilibrium as described in Proposition 2 survives. The following result shows that workers' equilibrium payoffs approach the outcome under perfect information when auditing costs become negligible.

Proposition 4. If auditing costs are small, $k < \bar{k}$, then the pooling equilibrium in Proposition 2 has the following property: in the limit $k \to 0$, the belief $\mu(y_P^*)$ tends to unity and the payoffs of high and low productivity workers become identical to their payoffs under perfect information. That is $\lim_{k \to 0} U_H^* = X_H$ and $\lim_{k \to 0} U_L^* = X_L$.

In the separating equilibrium under Condition 1, high productivity workers invest $c_H(y_H^*)$ in education independently of k. Condition 2 yields the more plausible implication that the perfect-information equilibrium is obtained when auditing costs vanish. In this limit, auditing occurs with positive probability by Proposition 3 (ii); but this does not reduce welfare because auditing costs are zero in the limit.

□ **Welfare.** Next, we address the welfare implications of auditing. If auditing is impossible, or equivalently, if auditing costs are large, then Proposition 1 describes the outcome under Condition 2: in the separating equilibrium without audits, high productivity workers get the utility $X_H - c_H(y_H^*)$ and low productivity workers get X_L. If, however, auditing is feasible and auditing costs are small, the equilibrium selected by Condition 2 is the pooling equilibrium in Proposition 2. As the following proposition shows, for small auditing costs this equilibrium yields a Pareto improvement relative to the separating equilibrium.

Proposition 5. If auditing costs are small, $k < \bar{k}$, then the workers' utilities, U_H^* and U_L^*, have the following properties in the pooling equilibrium in Proposition 2:

(i) if there are many high productivity workers, $\lambda > \mu_2(k)$, then $U_H^* > X_H - c_H(y_H^*)$ and $U_L^* > X_L$. Further, U_H^* and U_L^* are decreasing in k.
(ii) if there are few high productivity workers, $\lambda < \mu_2(k)$, then $U_H^* > X_H - c_H(y_H^*)$ and $U_L^* = X_L$. Further, U_H^* is decreasing in k.

Interestingly, as part (i) of the proposition shows, not only high but also low productivity workers may gain from the feasibility of auditing: if there are many high productivity workers, they subsidize low productivity workers. This happens in the pooling equilibrium because for these parameter values some high productivity workers do not pool with the low productivity workers but choose some education $\hat{y}_H < y_H^*$. A low productivity worker's payoff from choosing education $\hat{y}_H$ and getting the wage X_H is higher than their low productivity X_L. Therefore, to keep them from deviating to $\hat{y}_H$, their expected payoff from pooling must be higher than their low productivity X_L. Moreover, both high and low productivity workers' gains from auditing increase as auditing costs decrease because lower auditing costs reduce education $\hat{y}_H$ by Proposition 3 (i).

□ **Binding wage offers.** In our model, firms make non-binding wage announcements at stage (*iii*). In particular, they can reject any application at stage (*v*). Now suppose that firms can make also binding wage offer at stage (*iii*) in addition to non-binding wage announcements. Thus, they have to hire any applicant who applies for such a binding wage offer. Notice that a firm will never audit applicants for a binding wage offer because it cannot make use of the acquired information. Does this modification of the model affect the equilibria determined in Propositions 1 and 2?

Proposition 6. Suppose firms can make binding wage offers. Then Propositions 1 and 2 are still valid.

Binding wage offers do not change the equilibria we discussed above. Further, binding wage offers cannot create new equilibria. This follows directly from Lemma 3, which shows that the intuitive criterion does not allow for any equilibrium contracts with pooling and no audits.

□ **Default wage.** Firms compete by making wage offers and so in equilibrium each firm will earn zero expected profit. Therefore, it is always optimal for a firm to offer the "default" wage $w_L = X_L$ and to hire all applicants for w_L without an audit. Obviously, if a worker decides to apply for the default wage, it is not optimal to acquire education.

It may look a bit unusual from the perspective of the standard signaling by education model that firms offer the default wage $w_L = X_L$ even before workers take education decisions. This, however, only serves to simplify the specification of the firms' beliefs. Without this simplification we would have to make the firms' beliefs contingent not only on the observed education choice but also on the wage for which a worker applies.

Note that the default wage affects the equilibrium only in Proposition 2 (iii). Recall that in Proposition 2 (iii) some low productivity workers pool with the high productivity workers at education $y_P^* = 0$ and apply for the pooling wage $w(y_P^*) > w_L = X_L$, whereas the remaining low productivity workers apply for the default wage $w_L = X_L$. In this equilibrium, workers distinguish themselves not by education but only by the wage they apply for. This outcome can be supported as an equilibrium when there is no default wage and *all* wage offers are made after the workers' education choices by specifying beliefs in the following way: irrespective of the chosen education, firms believe that a worker has low productivity if he applies for a wage offer $w \leq X_L$; if a worker with education y applies for a wage offer $w(y) > X_L$ beliefs are specified as in Section 6. Given these beliefs, competition results in the two wage offers $w_L = X_L$ and $w(y_P^*) > X_L$. As $y_P^* = 0$, education is not used as a signal in equilibrium. However, auditing supports partial pooling as in Proposition 2 (iii): all applicants for w_L are hired without audits, whereas applicants for the pooling wage $w(y_P^*)$ are audited with probability $\rho(y_P^*)$. This auditing probability makes low productivity workers indifferent between applying for the higher pooling wage $w(y_P^*)$ and the lower wage w_L. As high productivity workers are not rejected after an audit, all high productivity workers apply for the higher pooling wage $w(y_P^*)$.

8. Conclusions

■ In signaling models à la Spence (1973), firms can only infer workers' productivities from their education choices. In reality, firms additionally use costly auditing to learn workers' productivities. We characterize the trade-offs between workers' signaling incentives and firms' auditing incentives. Under an extension of the intuitive criterion, there exists a unique equilibrium. When auditing costs are large, the least-cost separating equilibrium is the unique outcome, and information is revealed solely through signaling. When auditing costs are small, the equilibrium outcome is a partial pooling one in which workers with different productivities choose the same education level and firms audit with positive probability. Hence, information is revealed partly through signaling and partly through random audits. When there are many low productivity workers in the population, signaling by education is not used in equilibrium, and information is revealed solely through random audits. Furthermore, as auditing costs vanish, the equilibrium with auditing becomes more and more informative and approaches the perfect-information outcome.

Without implausible large search frictions, it is difficult to explain large wage differentials between observably equivalent individuals. These wage differentials, however, are well documented in empirical analyses of labor markets. We show that combining signaling with auditing in equilibrium yields the wage differentials for individuals who are observably equivalent. Even controlling for unobservable productivities, these wage differentials between individuals persist. The strategic choice of auditing requires some low productivity workers to get better-paid jobs. Hence, some low productivity workers apply for more attractive job offers. The anticipated auditing for these offers ensures that not all low productivity workers have an incentive to apply for these offers. Consequently, even if workers could apply repeatedly, they have no incentive to leave less-paid jobs even in the absence of search frictions.

Furthermore, our partial pooling equilibrium differs in two essential aspects from the standard signaling models. First, in contrast with the standard signaling model, education choices are governed by individual education costs if any education is acquired in equilibrium. Second, firms do not always audit the applicants for the highest wages. The first result mirrors empirical estimates of elasticities of education with respect to schooling cost, as in, for example, Castlemen and Long (2016) and Dynarski (2003). The second result provides an exciting new twist on how to think about auditing in both theoretical and applied work.

Our analysis considers the role of auditing in a signaling environment, in which the informed agents move first by choosing a signal. An alternative setting is the competitive screening model developed by Rothschild and Stiglitz (1976). In this setting the uninformed players first announce offers, and then each informed player chooses among offers. In the labor market context of our article this would mean that firms compete by making education contingent wage offers. Then each worker chooses the offer which is best for him and acquires the required education. Our analysis of auditing could be applied to such an environment by assuming that firms make offers but maintain the right to audit applicants before taking hiring decisions. It seems reasonable that auditing will occur also in a screening version of our model if audit costs are sufficiently small. However, it is an open question how this affects competitive screening by education in equilibrium. An analysis in this direction will provide interesting insights in comparison to the existing contributions on auditing, for example, Khalil and Lawarrée (1995), Khalil (1997) as well as Pollrich (2017), who study screening with auditing by a monopolistic mechanism designer. Ball and Kattwinkel (2019) generalize this setting by considering general and type-specific audits.

Appendix A

This Appendix contains the proofs of Lemmas 1–3 and of Propositions 2–6. The proof of Proposition 1 is substantiated in the main text.

Proof of Lemma 1. Obviously, no firm offers a wage $w > X_H$ to employ workers at this wage. Consider any wage $w \in (X_L, X_H]$. If the firm audits an applicant, its profits are $\mu(y)(X_H - w) - k$ because after the audit it will optimally reject type L and hire type H. Hiring an applicant without an audit yields profits $\mu(y)X_H + (1 - \mu(y))X_L - w$. Therefore the firm's profits are

$$\Pi \equiv \max\,[\mu(y)(X_H - w) - k,\ \mu(y)X_H + (1 - \mu(y))X_L - w,\ 0]. \tag{A1}$$

Suppose that there is auditing, that is, $\rho(y) > 0$. It follows that $\Pi = \mu(y)(X_H - w) - k$. Then $\mu(y)(X_H - w) - k \geq 0$, which contradicts $\mu(y) = 0$. Thus, $\mu(y) > 0$ must hold, which in turn implies

$$w \leq X_H - k/\mu(y). \tag{A2}$$

In addition, $\mu(y)(X_H - w) - k \geq \mu(y)X_H + (1 - \mu(y))X_L - w$, which implies

$$k \leq (1 - \mu(y))(w - X_L) \leq (1 - \mu(y))(X_H - k/\mu(y) - X_L) \tag{A3}$$

where the second inequality holds because of (A2). Inequality (A3) is equivalent to

$$k \leq \mu(y)(1 - \mu(y))(X_H - X_L) \leq \tilde{k}, \tag{A4}$$

where the second inequality holds because $\mu(y)(1 - \mu(y))$ attains its maximum at $\mu(y) = 1/2$. Note that (A4) yields a contradiction to $k > \tilde{k}$. Thus, for $k > \tilde{k}$, there is no auditing and $\rho(y) = 0$ for any wage $w \in (X_L, X_H]$ and any beliefs $\mu(y) \in [0, 1]$. Therefore, Bertrand competition implies $w(y) = \mu(y)X_H + (1 - \mu(y))X_L$.

At any wage $w \leq X_L$ firms optimally accept all applicants without an audit. Therefore, Bertrand competition implies $w(y) = \mu(y)X_H + (1 - \mu(y))X_L \geq X_L$. This is only consistent with $w \leq X_L$ for $\mu(y) = 0$ and $w(y) = X_L$.

The argument above shows that $\rho(y) = 0$ if the first inequality in (A4) is violated. As $\mu(y)(1 - \mu(y))$ is strictly concave in $\mu(y)$, $k > \mu(y)(1 - \mu(y))(X_H - X_L)$ is equivalent to $\mu(y) \notin [\mu_1(k), \mu_2(k)]$. This proves part (i).

Part (ii) follows by an analogous argument because auditing is optimal if $k < \mu(y)(1 - \mu(y))(X_H - X_L)$, which is equivalent to $\mu(y) \in (\mu_1(k), \mu_2(k))$. Finally, part (iii) holds because firms are indifferent between auditing and not auditing if $k = \mu(y)(1 - \mu(y))(X_H - X_L)$, which is equivalent to $\mu(y) \in \{\mu_1(k), \mu_2(k)\}$.

Bertrand competition implies that firms make zero profits and $\Pi = 0$. Therefore,

$$w(y) = \max\,[X_H - k/\mu(y), \mu(y)X_H + (1 - \mu(y))X_L] \tag{A5}$$

completing the proof. *Q.E.D.* □

Proof of Lemma 2. (i) Suppose that for $k < \bar{k}$ there is an equilibrium with the properties stated in the lemma that satisfies Condition 2. Then, by definition of $\bar{k}$ in (9), $c_H(y_H^*) - k/\mu_2(k) > 0$ because $k/\mu_2(k)$ is increasing in k and hence $k/\mu_2(k) < \bar{k}/\mu_2(\bar{k})$. Therefore, there must exist an out-of-equilibrium education $y' > 0$ close enough to zero such that

$$c_H(y') < c_H(y_H^*) - k/\mu_2(k). \tag{A6}$$

Suppose a worker chooses education y' and firms' beliefs are $\delta = \mu_2(k) - \epsilon'$ with $\epsilon' > 0$. Then, for ϵ' sufficiently small, $\delta \in (\mu_1(k), \mu_2(k))$. Therefore, by (4) and (5),

$$U_H(y'|\delta) = X_H - \frac{k}{\mu_2(k) - \epsilon'} - c_H(y'), \quad U_L(y'|\delta) = X_L - s - c_L(y'). \tag{A7}$$

Because in equilibrium type H chooses y_H^* and receives the wage X_H and type L can always apply for $w_L = X_L$, we have

$$U_H^* = X_H - c_H(y_H^*), \quad U_L^* \geq X_L. \tag{A8}$$

For ϵ' sufficiently small, therefore (A6)–(A8) imply that

$$U_H(y'|\delta) > U_H^*, \quad U_L(y'|\delta) < U_L^*. \tag{A9}$$

By Condition 2, therefore, $\mu(y') \geq \delta$. As $U_H(y'|\mu)$ is increasing in μ, we thus obtain by (A9) that $U_H(y'|\mu(y')) \geq U_H(y'|\delta) > U_H^*$, a contradiction to type H workers optimally choosing y_H^* in equilibrium.

(ii) We show that (7) cannot hold if $k \geq \bar{k}$, which means that Condition 2 does not apply. As this condition imposes no restrictions for beliefs $\delta \in \{\mu_1(k), \mu_2(k)\}$, we consider only beliefs $\delta \notin \{\mu_1(k), \mu_2(k)\}$. This implies that $\rho(y') \in \{0, 1\}$ for an out-of-equilibrium choice y'. First consider the case $\rho(y') = 1$. By Lemma 1, this case requires that $k \in [\bar{k}, \tilde{k}]$ and $\delta \in (\mu_1(k), \mu_2(k))$. Note that, for any education y' and any $k \in [\bar{k}, \tilde{k}]$

$$c_H(y') \geq 0 \geq c_H(y_H^*) - \frac{k}{\mu_2(k)}. \tag{A10}$$

As $\delta < \mu_2(k)$, we obtain by (4) that

$$U_H(y'|\delta) = X_H - \frac{k}{\delta} - c_H(y') < X_H - \frac{k}{\mu_2(k)} - c_H(y') \leq X_H - c_H(y_H^*) = U_H^*, \tag{A11}$$

where the last inequality follows from (A10). This shows that the first inequality in (7) cannot hold.

Now consider the case where $\rho(y') = 0$ given the belief δ after observing y'. Then, by Lemma 1, the first inequality in (7) is equivalent to

$$U_H(y'|\delta) = X_L + \delta(X_H - X_L) - c_H(y') > U_H^* = X_H - c_H(y_H^*). \tag{A12}$$

Further, by $U_L^* = X_L$ and the definition of y_H^* in (8), the second inequality in (7) requires that

$$U_L(y'|\delta) = X_L + \delta(X_H - X_L) - c_L(y') < U_L^* = X_L = X_H - c_L(y_H^*). \tag{A13}$$

Therefore,

$$c_H(y_H^*) - c_H(y') > (1-\delta)(X_H - X_L) > c_L(y_H^*) - c_L(y'). \tag{A14}$$

Thus, $y_H^* > y'$ by the first inequality because $(1-\delta)(X_H - X_L) \geq 0$. Therefore, (A14) implies that

$$c_H(y_H^*) - c_H(y') = \int_{y'}^{y_H^*} c_H'(y)\,\mathrm{d}y > c_L(y_H^*) - c_L(y') = \int_{y'}^{y_H^*} c_L'(y)\,\mathrm{d}y. \tag{A15}$$

As this contradicts the assumption in (1) that $c_L'(y) > c_H'(y) > 0$, this shows that (7) in Condition 2 cannot hold. *Q.E.D.* □

Proof of Lemma 3. We first show that $\rho(y_P^*) \in (0,1)$. Suppose that $\rho(y_P^*) = 0$. Then the equilibrium utilities are

$$U_L^* = w(y_P^*) - c_L(y_P^*), \quad U_H^* = w(y_P^*) - c_H(y_P^*). \tag{A16}$$

As $c_L'(y) > c_H'(y)$ and $w(y_P^*) < X_H$, there exists a $y' > y_P^*$ such that

$$U_H(y'|1) = X_H - c_H(y') > U_H^* \text{ and } U_L(y'|1) = X_H - c_L(y') < U_L^*. \tag{A17}$$

Therefore, Condition 1 implies that $\mu(y') = 1$. Then the first inequality in (A17) implies that all H types would choose y' rather than y_P^*, a contradiction.

Suppose that $\rho(y_P^*) = 1$. Then $U_L^* = X_L - s - c_L(y_P^*)$ because type L is always detected. However, by applying for the default wage $w_L = X_L$, type L can ensure themselves the utility $X_L > U_L^*$, a contradiction.

As $\rho(y_P^*) > 0$, it follows from Lemma 1 that $k \leq \tilde{k}$ because firms will never audit if $k > \tilde{k}$. This completes the proof of (i).

Above arguments and Lemma 1 imply $\mu(y_P^*) \in \{\mu_1(k), \mu_2(k)\}$. If $k = \tilde{k}$, then $\mu_1(k) = \mu_2(k)$ and the claims in (ii) follow directly. Now consider the case $k < \tilde{k}$, where $\mu_1(k) < \mu_2(k)$, and suppose $\mu(y_P^*) = \mu_1(k)$. Then equilibrium utilities satisfy

$$U_H^* = X_H - k/\mu_1(k) - c_H(y_P^*), \quad U_L^* \geq X_L. \tag{A18}$$

Consider out-of-equilibrium education $y' = y_P^* + \epsilon$ and the belief $\delta = (\mu_1(k) + \mu_2(k))/2$. As $\delta \in (\mu_1(k), \mu_2(k))$, with this belief the firms will choose $\rho = 1$ and so

$$U_H(y'|\delta) = X_H - \frac{k}{\delta} - c_H(y'), \quad U_L(y'|\delta) = X_L - s - c_L(y'). \tag{A19}$$

As $\delta > \mu_1(k)$ we obtain for ϵ sufficiently small that (7) applies and so Condition 2 implies that $\mu(y') \geq \delta$. Thus, $U_H(y'|\mu(y')) \geq U_H(y'|\delta) > U_H^*$, a contradiction to the equilibrium requirement that choosing y_P^* is optimal for type H. This implies that $\mu(y_P^*) = \mu_2(k)$. The wage in (10) follows from Lemma 1 (iii). As different values of y_P^* would result in the same wage $X_H - k/\mu_2(k)$ with different education costs for high productivity workers, only a single y_P^* can be supported in a pooling equilibrium.*Q.E.D.* □

To prove Proposition 2, we first prove the following claim:

Claim A1. If pooling occurs at y_P^* in an equilibrium satisfying Condition 2, then $y_P^* = 0$ and $\sigma_i = 1$ for at least one type $i \in \{L, H\}$ and for all $k < \bar{k}$.

Proof of Claim A1. Suppose there is a $y > 0$ chosen only by type L workers. Then $w(y) = X_L$ and so their utility is $X_L - c_L(y) < X_L$. As type L can opt for $w_L = X_L$, this yields a contradiction. Suppose there are two education levels $0 < y' < y''$ chosen only by type H. Then $w(y') = w(y'') = X_H$ and so $c_H(y') < c_H(y'')$ yields a contradiction to y'' being optimal for type H. Consequently, if pooling occurs at y_P^* in an equilibrium satisfying Condition 2, then there is no education $y > 0$ chosen only by type L workers and at most one education $\hat{y}_H > 0$ chosen only by type H workers.

Suppose $y_P^* > 0$. Then the equilibrium utilities satisfy

$$U_H^* = X_H - k/\mu_2(k) - c_H(y_P^*), \quad U_L^* \geq X_L. \tag{A20}$$

Consider the out-of-equilibrium education $y' > 0$ and some belief $\delta \in (\mu_1(k), \mu_2(k))$ so that with this belief the firms will choose $\rho(y') = 1$. Then for δ sufficiently close to $\mu_2(k)$ and y' sufficiently close to zero we have

$$U_H(y'|\delta) = X_H - \frac{k}{\delta} - c_H(y') > U_H^*, \quad U_L(y'|\delta) = X_L - s - c_L(y') < X_L \leq U_L^*. \tag{A21}$$

Therefore Condition 2 implies that $\mu(y') \geq \delta$. Thus, $U_H(y'|\mu(y')) \geq U_H(y'|\delta) > U_H^*$, a contradiction to the equilibrium requirement that choosing y_P^* is optimal for type H. This implies that $y_P^* = 0$.

Suppose $\sigma_L \in (0,1)$ and $\sigma_H \in (0,1)$. Then some fraction $1-\sigma_L$ of type L workers apply for $w_L = X_L$ and some fraction $1-\sigma_H$ of type H workers choose some $\hat{y}_H$ to receive the wage $w(\hat{y}_H) = X_H$. This requires that type L cannot gain from choosing $\hat{y}_H$:

$$X_L \geq X_H - c_L(\hat{y}_H). \tag{A22}$$

Thus, $\hat{y}_H \geq y_H^*$ by (8). If $\hat{y}_H > y_H^*$, the inequality in (A22) is strict. It is easily verified that then $\hat{y}_H$ does not satisfy Condition 1, which is implied by Condition 2. Therefore, the equality $\hat{y}_H = y_H^*$ is valid. For $k < \bar{k}$, this yields a contradiction to Lemma 2 (i). Thus, for all $k < \bar{k}$, $\sigma_L \in (0,1)$ and $\sigma_H \in (0,1)$ cannot hold simultaneously in a pooling equilibrium satisfying condition 2.*Q.E.D.* □

Proof of Proposition 2. According to Claim A1 above this proof, we can restrict attention to three equilibrium categories. This allows the following simplified notation. In a pooling equilibrium, the firms' beliefs at y_P^* are determined by Bayes' rule as

$$\mu(y_P^*) = \frac{\sigma_H \lambda}{\sigma_L(1-\lambda) + \sigma_H \lambda} \in (0,1), \tag{A23}$$

because a fraction λ of all workers has type H.

(i) If $\lambda > \mu_2(k) = \mu(y_P^*)$, Bayes' rule in (A23) implies that $\sigma_H < \sigma_L$. Therefore, by Claim A1, $\sigma_H \in (0,1)$ and $\sigma_L = 1$. This means that type L at least weakly prefers choosing y_P^* to applying for $w(y_P^*)$, and being audited with probability $\rho(y_P^*)$, over opting for $w_L = X_L$. Also, type H must be indifferent between receiving $w(y_P^*)$ and choosing education $\hat{y}_H$ to receive wage X_H:

$$X_L \leq \left(1 - \rho(y_P^*)\right) w(y_P^*) + \rho(y_P^*)(X_L - s), \quad w(y_P^*) = X_H - c_H(\hat{y}_H). \tag{A24}$$

Note that the second condition uniquely determines the education level $\hat{y}_H$. As an additional equilibrium requirement, type L should have no incentive to choose $\hat{y}_H$. In fact, by Condition 1, it is easy to see that he has to be indifferent between applying for $w(y_P^*)$ and choosing $\hat{y}_H$ to obtain the wage X_H: if type L strictly preferred applying for $w(y_P^*)$ over choosing $\hat{y}_H$, then type H could appeal to the intuitive criterion of Condition 1 that also an education choice slightly below $\hat{y}_H$ should be considered as a convincing signal of high productivity. To rule out that this destroys the equilibrium, it must be the case that

$$\left(1 - \rho(y_P^*)\right) w(y_P^*) + \rho(y_P^*)(X_L - s) = X_H - c_L(\hat{y}_H). \tag{A25}$$

As $w(y_P^*) = X_H - k/\mu_2(k)$ by Lemma 3, the second condition in (A24) is equivalent to $c_H(\hat{y}_H) = k/\mu_2(k)$. As $k/\mu_2(k)$ is increasing in k and $k < \bar{k}$, we obtain from (9) that $c_H(\hat{y}_H) < c_H(y_H^*)$. Therefore, $\hat{y}_H < y_H^*$. This, together with (8), implies that

$$X_H - c_L(\hat{y}_H) > X_H - c_L(y_H^*) = X_L. \tag{A26}$$

Therefore the first condition in (A24) is implied by (A25). Finally, as $w(y_P^*) = X_H - c_H(\hat{y}_H) > X_H - c_L(\hat{y}_H)$ and $X_L - s < X_L < X_H - c_L(\hat{y}_H)$, (A25) has a unique solution $\rho(y_P^*) \in (0,1)$. This proves that the equilibrium conditions (A24) and (A25) are satisfied for unique values of $w(y_P^*)$, $\hat{y}_H$, and $\rho(y_P^*)$.

The equilibrium can be supported by beliefs $\mu(y) = \mu_2(k)$ for $y < \hat{y}_H$, and $\mu(y) = 1$ for $y \geq \hat{y}_H$. To see that these beliefs satisfy Condition 2, note that type H certainly cannot gain by choosing some $y' > \hat{y}_H$ because their equilibrium payoff is $X_H - c_H(\hat{y}_H)$. Suppose he could gain by deviating to some $y' \in (0, \hat{y}_H)$. Then the wage for workers with education y' must satisfy $w(y') > w(y_P^*)$. This requires the belief $\mu(y') > \mu_2(k)$, which implies that $\rho(y') = 0$. This means that also type L will be hired at $w(y')$ after choosing y'. By (A24) type H gains from choosing y' if $w(y') - c_H(y') > X_H - c_H(\hat{y})$. However, because by (1) $c_H(\hat{y}) - c_H(y') < c_L(\hat{y}) - c_L(y')$, this implies $w(y') - c_L(y') > X_H - c_L(\hat{y})$. Therefore, by (A25) also type L would gain by choosing y'. This proves that the equilibrium survives Condition 2.

(ii) If $\lambda = \mu_2(k) = \mu(y_P^*)$, Bayes' rule in (A23) implies that $\sigma_H = \sigma_L$. Therefore, Claim A1 implies $\sigma_L = \sigma_H = 1$. This is simply the limiting case of (i) and the same arguments as above can be applied.

(iii) If $\lambda < \mu_2(k) = \mu(y_P^*)$, Bayes' rule in (A23) implies that $\sigma_H > \sigma_L$. Therefore, Claim A1 implies $\sigma_L \in (0,1)$ and $\sigma_H = 1$. Type L workers must be indifferent between opting for the wage $w_L = X_L$ and choosing y_P^* to apply for $w(y_P^*)$, where they are rejected with probability $\rho(y_P^*)$:

$$X_L = \left(1 - \rho(y_P^*)\right) w(y_P^*) + \rho(y_P^*)(X_L - s). \tag{A27}$$

As $w(y_P^*)$ is given by Lemma 3 (ii), this equation uniquely determines the likelihood $\rho(y_P^*) \in (0,1)$ of being audited when applying for $w(y_P^*)$. The equilibrium can be supported by the belief $\mu(y) = \mu_2(k)$ for all y. To show that Condition 2 is not violated, suppose that type H could gain by deviating to some $y' > 0$. Then the wage for education y' must satisfy $w(y') > w(y_P^*)$, which by the argument above implies also type L will be hired at $w(y')$ after choosing y'. For type H to gain, it must be the case that

$$w(y') - c_H(y') > w(y_P^*) = X_H - \frac{k}{\mu_2(k)} \geq X_H - c_H(y_H^*), \tag{A28}$$

where the last inequality holds by (9) because $k < \bar{k}$ and $k/\mu_2(k)$ is increasing in k. Note that this implies $y' < y_H^*$ because $w(y') \le X_H$. For type L to lose from deviating to y', it must be the case that

$$w(y') - c_L(y') < X_L = X_H - c_L(y_H^*), \tag{A29}$$

where the equality holds by definition of y_H^* in (8). By (A28) and (A29), $c_H(y_H^*) - c_H(y') > c_L(y_H^*) - c_L(y')$, a contradiction to assumption (1). This proves that (7) in Condition 2 cannot hold.

The arguments in (i)–(iii) show that for all $k < \bar{k}$ there exists a unique pooling equilibrium satisfying Condition 2. Suppose that an equilibrium with pooling at some education y_P^* exists also for $k > \bar{k}$. By Lemma 3 (i) this implies $\rho(y_P^*) \in (0, 1)$ and therefore, by Lemma 1 it must be the case that $k \le \tilde{k}$. For $k > \bar{k}$, $k/\mu_2(k) > c_H(y_H^*)$ by (9). Therefore, for $\epsilon > 0$ sufficiently small, the type H's equilibrium utility satisfies

$$U_H^* = X_H - k/\mu_2(k) < X_H - c_H(y_H^* + \epsilon) \tag{A30}$$

by Lemma 3 (ii). As type L can always apply for $w_L = X_L$, by (8) we have

$$U_L^* \ge X_L = X_H - c_L(y_H^*) > X_H - c_L(y_H^* + \epsilon). \tag{A31}$$

Thus for the education $y_H^* + \epsilon$ the inequalities in (6) are satisfied, which by Condition 1 implies $\mu(y_H^* + \epsilon) = 1$. Then by (A30), type H would deviate to $y_H^* + \epsilon$. This proves that for $k > \bar{k}$ already Condition 1, which is weaker than Condition 2, precludes the existence of a pooling equilibrium. *Q.E.D.* □

Proof of Proposition 3. (i) If $\lambda > \mu_2(k) = \mu(y_P^*)$, $\sigma_L = 1$ by Proposition 2 (i) and so (A23) yields

$$\sigma_H = (1 - \lambda)\mu_2(k)/\lambda(1 - \mu_2(k)). \tag{A32}$$

This proves that $\partial\sigma_H/\partial k < 0$ because $\mu_2'(k) < 0$. The auditing rate $\rho(y_P^*)$ is determined by (A25). By Lemma 3 (ii), this is equivalent to

$$c_L(\hat{y}_H) - \frac{k}{\mu_2(k)} = \rho(y_P^*)\left(X_H - X_L - \frac{k}{\mu_2(k)} + s\right). \tag{A33}$$

The term in brackets on the right-hand side decreases in k. The derivative of the left-hand side is

$$\frac{\partial c_L(\hat{y}_H)}{\partial y}\frac{\partial \hat{y}_H}{\partial k} - \frac{\partial k/\mu_2(k)}{\partial k} > \frac{\partial c_H(\hat{y}_H)}{\partial y}\frac{\partial \hat{y}_H}{\partial k} - \frac{\partial k/\mu_2(k)}{\partial k} = 0, \tag{A34}$$

because $c_H(\hat{y}_H) = k/\mu_2(k)$ by the definition of $\hat{y}_H$ in (A24). Therefore the left-hand side increases in k. This implies that $\rho(y_P^*)$ increases in k because both sides of the equation are positive.

The education $\hat{y}_H$ obtained by some type H workers is determined by the indifference condition $X_H - k/\mu_2(k) = X_H - c_H(\hat{y}_H)$. Hence, $\hat{y}_H$ decreases as type H workers' education costs increase as defined in footnote 22. In addition, $\hat{y}_H$ increases in k because $k/\mu_2(k)$ increases in k. Hence, the amount of education $\hat{y}_H$ increases in auditing costs k.

(ii) If $\lambda < \mu_2(k) = \mu(y_P^*)$, $\sigma_H = 1$ by Proposition 2 (iii) and so (A23) yields

$$\sigma_L = \lambda(1 - \mu_2(k))/(1 - \lambda)\mu_2(k). \tag{A35}$$

This proves that $\partial\sigma_L/\partial k > 0$ because $\mu_2'(k) < 0$. As $\lim_{k\to 0}\mu_2(k) = 1$, we obtain $\lim_{k\to 0}\sigma_L = 0$. By (A27) and Lemma 3 (ii)

$$\rho(y_P^*) = \frac{X_H - X_L - k/\mu_2(k)}{X_H - X_L - k/\mu_2(k) + s}. \tag{A36}$$

As $k/\mu_2(k)$ increases in k, $\rho(y_P^*)$ decreases in k. Moreover, the limit of $\rho(y_P^*)$ for $k \to 0$ is as stated in the proposition because $\lim_{k\to 0} k/\mu_2(k) = 0$. *Q.E.D.* □

Proof of Proposition 4. First, notice that $\mu(y_P^*) = \mu_2(k)$ and that in the limit $\lim_{k\to 0}\mu_2(k) = 1$ and hence, $\lambda < \mu_2(k)$ for $k \to 0$. Proposition 2 (iii) shows $U_H^* = w(y_P^*) = X_H - k/\mu_2(k)$ and $U_L^* = X_L$ by equation (A27). Therefore $\lim_{k\to 0} U_H^* = \lim_{k\to 0} X_H - k/\mu_2(k) = X_H$ and $\lim_{k\to 0} U_L^* = X_L$.*Q.E.D.* □

Proof of Proposition 5. (i) Suppose $\lambda > \mu_2(k)$. Proposition 2 (i) proves that $U_H^* = X_H - c_H(\hat{y}_H)$ and $\hat{y}_H < y_H^*$. Therefore, $U_H^* = X_H - c_H(\hat{y}_H) > X_H - c_H(y_H^*)$. In addition, $\hat{y}_H$ is determined by $c_H(\hat{y}_H) = k/\mu_2(k)$ according to the proof of Proposition 2 (i). As $\mu_2(k)$ decreases and $k/\mu_2(k)$ increases in k, $\hat{y}_H$ increases and $U_H^* = X_H - c_H(\hat{y}_H)$ decreases in k.

Proposition 2 (i) also proves that U_L^* is determined by (A25) and, hence, $U_L^* = X_H - c_L(\hat{y}_H)$. Recall the definition of y_H^* with $X_L = X_H - c_L(y_H^*)$. Therefore, $\hat{y}_H < y_H^*$ implies $U_L^* = X_H - c_L(\hat{y}_H) > X_H - c_L(y_H^*) = X_L$ and U_L^* decreases in k because $\hat{y}_H$ increases in k.

(ii) Suppose $\lambda < \mu_2(k)$. Proposition 2 (iii) proves $U_H^* = w(y_P^*) = X_H - k/\mu_2(k)$. The definition of $\bar{k}$ implies $\bar{k}/\mu_2(\bar{k}) \le c_H(y_H^*)$. As $k/\mu_2(k)$ increases in k and $k < \bar{k}$, $U_H^* = X_H - k/\mu_2(k) > X_H - \bar{k}/\mu_2(\bar{k}) = X_H - c_H(y_H^*)$. Moreover, $U_H^* = X_H - k/\mu_2(k)$ decreases in k. Proposition 2 (iii) also proves that U_L^* is determined by (A27). Therefore, $U_L^* = X_L$.*Q.E.D.* □

Proof of Proposition 6. In the fully separating equilibrium of Proposition 1, there are no audits in equilibrium. Hence, for $k \geq \bar{k}$ it does not matter whether wage offers are binding or non-binding. The reasoning of Lemma 2 is also unaffected by additional binding wage offers.

Suppose for $k < \bar{k}$ there is an additional equilibrium besides the one in Proposition 2 with a binding offer $w(y')$ for some education y' inducing the belief $\mu(y')$. By Lemma 1 there are no audits even for non-binding wage offers if $\mu(y') \notin [\mu_1(k), \mu_2(k)]$. Thus, binding offers with $\mu(y') \notin [\mu_1(k), \mu_2(k)]$ are effectively already included in our analysis leading to Proposition 2. This implies that $\mu(y') \in [\mu_1(k), \mu_2(k)]$ and so $\mu(y') \in (0, 1)$. As Lemma 3 shows, however, pooling at some y' implies that auditing occurs with probability $\rho(y') > 0$. Because auditing is never optimal for a binding offer, this means that $w(y')$ cannot be an equilibrium offer. Consequently, for $k < \bar{k}$ there cannot be an additional equilibrium with a binding wage offer.

Next, we verify that the pooling equilibrium in Proposition 2 remains an equilibrium by arguing that the firm has no incentive to deviate to make a binding wage offer at stage (iii). Begin with the case $\lambda = \mu_2(k)$. All workers choose education $y_P = 0$ and beliefs are $\mu(y_P) = \mu_2(k)$. If the firm deviates by making a binding wage offer below $X_H - k/\mu_2(k)$, the deviation attracts only type L workers, if it attracts any workers at all, and the deviation is unprofitable. If the binding wage offer is above or equal to $X_H - k/\mu_2(k)$, the deviation attracts all types of workers, but the deviation is unprofitable because by the definition of $\mu_2(k)$

$$k = \mu_2(k)(1 - \mu_2(k))(X_H - X_L) \Leftrightarrow k/\mu_2(k) = (1 - \mu_2(k))(X_H - X_L) \Leftrightarrow$$

$$X_L + \mu_2(k)(X_H - X_L) = X_H - k/\mu_2(k).$$

Hence, the average productivity cannot exceed the deviation wage and the deviation is unprofitable for the firm.

Now turn to the case $\lambda < \mu_2(k)$. All workers choose the default contract or education $y_P = 0$ and beliefs are $\mu(y_P) = \mu_2(k)$. If the binding wage offer is below $X_H - k/\mu_2(k)$, the deviation attracts only type L workers, if it attracts any workers at all, and the deviation is unprofitable. If the binding wage offer is above or equal to $X_H - k/\mu_2(k)$, the deviation attracts all types of workers, but the deviation is unprofitable, due to the definition of $\mu_2(k)$ as above.

Finally, consider the case $\lambda > \mu_2(k)$. Some type H workers choose education $\hat{y}_H$, all remaining workers choose education $y_P = 0$ and beliefs are $\mu(y_P) = \mu_2(k)$. If a firm offers a binding wage for education $\hat{y}_H$, such a deviation is unprofitable because a binding wage that is attractive for workers with $\hat{y}_H$ must attract type L workers as well. If a firm offers a binding wage for education y_P and the binding wage offer is below $X_H - k/\mu_2(k)$, the deviation attracts only type L workers, if it attracts any workers at all, and the deviation is unprofitable. If the binding wage offer for education y_P is above or equal to $X_H - k/\mu_2(k)$, the deviation attracts all types of workers with education y_P, but the deviation is unprofitable, due to the definition of $\mu_2(k)$ as above. The deviation does not attract any workers with education $\hat{y}_H$, as long as the wage is below X_H because education costs are sunk. *Q.E.D.* □

ACKNOWLEDGEMENTS

■ Open Access funding enabled and organized by Projekt DEAL.

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