

# Automated Label Placement in Theory and Practice

Dissertation of Alexander Wolff  
Fachbereich Mathematik und Informatik  
Freie Universität Berlin

Supervisor: Dr. habil. Frank Wagner

External referees: Dr. Marc van Kreveld, Universiteit Utrecht,  
and Prof. Christopher Jones, University of Glamorgan.

Date of doctoral defense: May 28, 1999



# Contents

<b>Abstract</b>	<b>v</b>
<b>1 An Introduction to Label Placement</b>	<b>1</b>
1.1 Historic Development . . . . .	2
1.2 Theory . . . . .	3
1.3 . . . and Practice . . . . .	4
1.4 Quality . . . . .	4
1.5 Future Development . . . . .	5
1.6 Overview . . . . .	6
1.6.1 General Labeling, Compatible Representatives, and CSP .	6
1.6.2 Point Labeling: Label-Number Maximization . . . . .	7
1.6.3 Point Labeling: Label-Size Maximization . . . . .	8
1.6.4 Line Labeling . . . . .	8
1.6.5 Designing Geometric Algorithms . . . . .	9
<b>2 General Labeling: Label-Number Maximization</b>	<b>11</b>
2.1 Label Placement and CSP . . . . .	13
2.2 Maximum Variable-Subset CSP . . . . .	14
2.3 Irreducibility . . . . .	16
2.4 An Edge-Irreducibility Algorithm . . . . .	18
2.5 A General Label-Placement Algorithm . . . . .	28
<b>3 Point Labeling: Label-Number Maximization</b>	<b>31</b>
3.1 Comparing Various Models . . . . .	33
3.2 Fixed-Position Models . . . . .	42
3.2.1 Algorithm . . . . .	43
3.2.2 Experiments . . . . .	47
3.2.3 Results . . . . .	49
3.3 Slider Models . . . . .	61
3.3.1 NP-Hardness . . . . .	62

3.3.2	A Greedy Approximation Algorithm . . . . .	65
3.3.3	A Polynomial Time Approximation Scheme . . . . .	71
3.3.4	Implementation and Experimental Results . . . . .	75
<b>4</b>	<b>Point Labeling: Label-Size Maximization</b>	<b>81</b>
4.1	Rectangular Labels . . . . .	81
4.2	Circular Labels . . . . .	82
4.2.1	Previous Work . . . . .	84
4.2.2	Preliminaries . . . . .	84
4.2.3	Algorithm . . . . .	86
4.2.4	Analysis . . . . .	88
4.2.5	NP-Hardness . . . . .	91
<b>5</b>	<b>Line Labeling</b>	<b>97</b>
5.1	Previous Work . . . . .	99
5.2	A Buffer Around the Input Polyline . . . . .	100
5.3	A Candidate Strip . . . . .	102
5.4	Finding Good Label Positions . . . . .	105
5.5	Experimental Results . . . . .	106
5.6	Discussion and Extensions . . . . .	110
<b>6</b>	<b>Designing Geometric Algorithms</b>	<b>113</b>
6.1	Algorithm . . . . .	115
6.2	Step by Step Towards Good Design . . . . .	115
6.2.1	The Naive Approach . . . . .	116
6.2.2	Decoupling Algorithm and Data Organization . . . . .	116
6.2.3	Tightening Control . . . . .	118
6.2.4	Influencing Critical Decisions . . . . .	119
6.2.5	The Complete Interface . . . . .	121
6.3	Experiments . . . . .	124
6.3.1	Example Classes . . . . .	124
6.3.2	Results . . . . .	126
6.3.3	Evaluation . . . . .	127
	<b>Conclusion</b>	<b>131</b>
	<b>Zusammenfassung</b> (Summary in German)	<b>133</b>
	<b>Curriculum Vitae</b>	<b>135</b>
	<b>Bibliography</b>	<b>139</b>

# Curriculum Vitae

- 30.11.67 born in Stuttgart, Germany
- 2.6.87 A-levels at Mörke Gymnasium Ludwigsburg
- 1.7.87 – 28.2.89 alternative service
- 1.10.89 enrolment at Albert Ludwig University Freiburg;  
major in mathematics, minor in computer science
- 1.10.91 enrolment at Freie Universität Berlin;  
study focus: theoretical computer science
- 19.12.95 graduation from Freie Universität Berlin;  
title of Master's Thesis: "Map Labeling"
- 1.6.96 – 31.5.99 research assistant of Dr. Frank Wagner at Freie Universität  
Berlin; work on project "Efficient Algorithms for Map La-  
beling" funded by the German Science Foundation (DFG)
- 14.7. – 10.8.97 and
- 22.3. – 31.3.99 research guest of Dr. Marc van Kreveld, Utrecht University,  
The Netherlands
- 10.2.99 completion of PhD thesis
- 26.4. – 30.4.99 research guest of Prof. Peter Widmayer, ETH Zurich,  
Switzerland
- 28.5.99 doctoral defense, talk on "Parametrized Complexity—  
a New Approach for Hard Problems"

