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Abstract

This paper develops a two-step inference procedure to test for a local one-for-one relation of contemporaneous jumps in high-frequency financial data corrupted by market microstructure noise. The first step develops a new bivariate Lee-Mykland jump test for pre-averaged, intra-day returns. If a jump is detected in at least one of the two assets, then the second step tests for equal jump sizes. We apply the test procedure to pairs of nominal and inflation-indexed government bond yields at monetary policy announcements in the U.S., U.K., and Euro Area. The analysis provides new high-frequency evidence about the anchoring of inflation expectations and central banks' ability to push a measure of inflation expectations towards their inflation target.

Keywords: High-frequency statistics; pre-averaging; jump test; break-even inflation; anchoring of inflation expectations.

JEL: C58, C12, C32, E58

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1 Introduction

Inflation expectations are a major indicator to assess the credibility of central banks in achieving their main mandate of price stability. Since the global financial crisis, long-term inflation expectations in most advanced economies are persistently below the inflation targets of their respective central banks. Against this backdrop, in a recent speech, Borio (2020) questions central banks' ability to push inflation and inflation expectations back to their target.

This paper proposes high-frequency statistical methods to investigate the hypothesis that central bank announcements significantly move market interest rates but have no effect on inflation expectations. We zoom in on single policy announcements and study the event returns of nominal and inflation-indexed government bonds, whose spread defines a measure of inflation expectations, known as the break-even inflation.[1] We test for a local one-for-one relation of jumps between the nominal and inflation-indexed government bond yields at pre-scheduled monetary policy announcements. The one-for-one relation implies contemporaneous jumps of equal magnitude, and hence a non-moving spread. According to Gürkaynak et al. (2010), insignificant changes of long-horizon break-even inflation in response to news announcements indicate anchored inflation expectations. Therefore, our approach is named as a test for *cojump anchoring*.

Cojump anchoring requires testing for jumps in high-frequency financial data. Ait-Sahalia and Jacod (2009) and Lee and Mykland (2008) have established tests for the presence of jumps in discretely observed, univariate semimartingale models. An overview and empirical comparison of existing tests is provided by Dumitru and Urga (2012). Hansen and Lunde (2006) highlight that trading frictions, such as price discreteness and bid-ask bounces, play a non-trivial role at high observation frequencies. Commonly referred to as the market microstructure noise, these frictions keep observed prices away from discretely observed semimartingales and distort simple estimators of volatility and jumps. Therefore, Podolski and Ziggel (2010), Ait-Sahalia et al. (2012), Lee and Mykland (2012) and Bibinger et al. (2019) provide noise-robust, univariate jump tests in latent observation models. Extensions of the univariate methods to contemporaneous jumps, or cojumps, in bivariate semimartingale models have been proposed by Jacod and Todorov (2009) and Bibinger and Winkelmann (2015).

The cojump anchoring test procedure builds on a bivariate semimartingale model, observed in discrete time under market microstructure noise. We use the pre-averaging approach of

[1] Inflation-indexed bonds are bonds that compensate investors for inflation. They are considered as real investments. According to the Fisher equation, the spread between nominal and real yields is a measure of inflation expectations.

Podolskij and Vetter (2009) and Jacod et al. (2009) to account for the noise. We implement a two-step procedure. The first step tests, at a fixed point in time, the null hypothesis of no jump against the alternative of a jump in at least one of the two semimartingale processes. Neither a single univariate jump test nor a cojump test accomplishes the task. Therefore, we propose a new *bivariate jump test* for this purpose. The test statistic of the new bivariate jump test is the product of two univariate test statistics of Lee and Mykland (2012). We derive its limiting variance-gamma distribution under the null of no jumps. The alternative hypothesis reflects the hybrid nature of the bivariate test. In the case of a cojump, we obtain the same $n^{1/2}$ (divergence) rate as the cojump test of Bibinger and Winkelmann (2015), while in the case of a jump in only one asset, we obtain the $n^{1/4}$ rate of the univariate jump test of Lee and Mykland (2012).

Applying the bivariate jump test to the returns of nominal and inflation-indexed bonds locally around the time of a pre-scheduled policy announcement, the first-step procedure determines whether the announcement provides significant news that leads to a jump in either the nominal or the inflation-indexed bond, or both. Once a jump is detected, the second step then tests for equal event returns. We show that conditional on the rejection of the bivariate jump test, the univariate test of Lee and Mykland (2012), applied to the spread of the two processes, tests the null hypothesis of cojumps of equal magnitude and has power against a jump in the spread. If the equality of event returns is not rejected, we conclude that the bivariate jump test in the first step rejects the null of no jump because of a cojump of equal magnitude, in other words, cojump anchoring. A rejection of equal event returns indicates a significant jump of the spread of the two semimartingales, which points towards a significant jump in the break-even inflation in our application. In those cases, we further investigate if the direction of the jump pushes the inflation expectation measure closer to the central banks' inflation target.

Empirical evidence on how break-even inflation responds to monetary policy is mixed. Beechey and Wright (2009) regress 30-minute intra-day returns of U.S. nominal and inflation-indexed bonds on a monetary policy surprise variable and report a significant increase in break-even inflation in response to lower than expected interest rates. Gürkaynak et al. (2010) use the same approach to study the anchoring of inflation expectations with daily data from the U.S., Euro Area, U.K., and Sweden and find only little responses to monetary policy surprises. Gertler and Karadi (2015), Hanson and Stein (2015) and Nakamura and Steinsson (2018) implement vector autoregressive models with high-frequency identification techniques. They find an approximate one-for-one relation of U.S. nominal and inflation-indexed bond yields and no significant changes in the break-even inflation. These regression based approaches in fact estimate the average yield curve responses across all policy announcements over multiple years. Since most central banks nowadays have been very transparent and predictable in their policy communications and actions, the surprise component in the announcements has been reduced to a minimum. As a result, averaging across all policy announcements may neglect the impact of announcements that significantly move the break-even inflation. This highlights the potential benefit of high-frequency statistical methods which allow us to isolate the impact of individual events.

We use 30-second data from January 2015 to March 2020 on the U.S., U.K., and German government bonds with maturities closest to five and ten years. We also construct the five-year five-year forward rates using the duration approach of Shiller et al. (1983). The two-step testing procedure is implemented at the release time of interest rate decisions by the Federal Reserve (Fed), Bank of England (BoE), and European Central Bank (ECB). While there are substantial differences in the jump behavior of the nominal and inflation-indexed bonds across countries, some commonalities emerge. First, we find that the proportions of policy announcements that cause jumps in five-year five-year forward rates of nominal and inflation-indexed bonds range from 18% for the ECB to 44% for the Fed and 51% for the BoE. Cojump anchoring is prevailing among these announcements. Among all market-moving policy announcements, we find a similar percentage—around 60%—where we cannot reject a one-for-one relation of cojumps in the five-year five-year forward rates across the U.S., U.K., and Euro Area. Although the U.K. break-even inflation experiences the most frequent jumps, the German break-even inflation displays the largest average jump size in response to ECB's policy announcements. Our overall empirical results show that the monetary policy announcements by the ECB are the most successful in pushing the forward break-even inflation towards the inflation target. In contrast, the U.S. and U.K. forward break-even inflation display signs of de-anchoring. The majority of monetary policy announcements of the Fed and BoE pull the expectations measure away from the target.

The remainder of the paper is structured as follows. Section 2 introduces the econometric model and the two-step testing procedure for cojump anchoring. The finite sample properties of the new bivariate jump test are studied via simulations in Section 3. We use the two-step procedure to investigate cojump anchoring in the U.S., U.K., and Euro Area in Section 4. Section 5 concludes. Technical assumptions and proofs can be found in Appendices A and B, respectively. Additional empirical results for robustness checks are in Appendix C.

2 Econometric method

This section sets up the theoretical framework to test for equal jump sizes of two assets at some fixed point in time using intra-day data which includes noise from market microstructure. We first present the underlying model. Section 2.2 introduces the general testing procedure and hypotheses. The asymptotic distribution of pre-average event-returns is given in Section 2.3. From the asymptotic distribution, we derive the bivariate jump test (Section 2.4) and the test for equal event returns (Section 2.5). Our model and notation follows that of Jacod and Protter (2012) and Ait-Sahalia and Jacod (2014).

2.1 Underlying stochastic process

Let P_t denote the vector of efficient (log) prices of two assets on some normalized interval with some fixed start and end time, surrounding a fixed and known announcement time τ.[2] The efficient prices can be described by a continuous-time, bivariate Itô semimartingale on some filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$:

$$P_t = P_0 + \int_0^t b_s ds + \int_0^t \sigma_s dW_s + J_t, \quad t \in (0,1). \tag{1}$$

The continuous part consists of starting values P_0, a two-dimensional drift b_t, the 2×2-dimensional spot-covolatility $\Sigma_t = \sigma_t \sigma_t'$ and a two-dimensional standard Brownian motion W_t. J_t is a purely discontinuous process that can be completely characterized by its jumps. Assumptions about the different components of P_t are further formalized in Appendix A. Model (1) is fairly general and includes most of the popular models for asset prices in financial econometrics, particularly those introduced by Duffie and Kan (1996) for bond price processes. The occurrence of jumps is not restricted to the event time τ but can be distributed anywhere on $(0,1)$, having finite or infinite activity and variation. The statistical methods below remain valid, provided that covolatility estimation is robust to such jumps at $t \neq \tau$.

If we were able to observe the efficient price P_t, we could have observed all jumps directly. The price increment $\Delta P_t = P_t - P_{t-}$, $P_{t-} = \lim_{s<t, s\to t} P_s$ is zero in the case of no jump at time t, and $\Delta P_t = \Delta J_t$ otherwise. However, in any practical applications we have only finitely many observations n of the two assets. Observation times t_i, $i = 1, ..., n$ are discrete, synchronous and equally spaced across assets. We denote the sampling interval as $\Delta_n = t_i - t_{i-1} = 1/n$. Besides the finite n, we follow most of the microstructure literature and posit an additive, latent observation model at discrete time:

$$\tilde{P}_i = P_{t_i^n} + \epsilon_i. \tag{2}$$

We observe a noisy version of the efficient price process $P_{t_i^n}$, where $\epsilon_i \overset{\text{i.i.d}}{\sim} (0, \eta)$, $i = 1, ..., n$, is the market microstructure noise with a 2×2-dimensional covariance matrix η. Ait-Sahalia

[2]Note that for a zero coupon bond the yield is given by the negative of its log-price divided by its maturity. We therefore use the words log price and yields interchangeably.

and Yu (2009) find a negative relationship between the level of the noise terms and different liquidity measures. Less liquid assets usually have a larger noise variance. Distorting effects of volatility estimation through potential differences in the liquidity of nominal and inflation-indexed bonds are captured by the noisy observation model (2). Methods to test for significant noise are proposed by Ait-Sahalia and Xiu (2019), for example. Similar to the jump component, increments of the noise term do not vanish asymptotically. Hence, the conventional covolatility estimators such as realized volatility or bi-power variation are asymptotically dominated by the noise. Jump detection is more complicated in cases where jump returns are weakened by the noise while no-jump returns are amplified. Lee and Mykland (2012) highlight the importance of noise-robust jump tests compared with methods that do not account for market microstructure noise. In our setup, market microstructure is an i.i.d. process independent of P_t. We extend the model to more general setups in the simulation with endogenous and heteroskedastic noise. Nonsynchronicity of the multivariate data is of less importance because we focus on a fixed time point τ.

2.2 The cojump anchoring test problem

We observe intra-day data of two assets a and b surrounding a fixed point in time τ. In this paper the two assets are a nominal and an inflation-indexed bond of the same maturity, and the time point τ is determined by a monetary policy announcement. Cojump anchoring is characterized by a one-for-one relationship in the nominal and inflation-indexed bond yields at the policy announcement time.

We consider a two-step procedure to disentangle the two key aspects of cojump anchoring. The first step determines whether the information released in the policy announcement triggers a significant jump in at least one of the bond yields. The path $P_t(\omega)$ of the two assets, observed through $\tilde{P}_i$, $i = 1, ..., n$, is assigned to either of the two sets:

$$\begin{cases} \Omega_\tau^{(C)} & = \{\omega : \text{ the processes } P_t^{(a)} \text{ and } P_t^{(b)} \text{ are continuous at } t = \tau\}, \\ \Omega_\tau^{(J)} & = \{\omega : \text{ at least one of the processes } P_t^{(a)} \text{ and } P_t^{(b)} \text{ jumps at } t = \tau\}. \end{cases} \tag{3}$$

The null hypothesis of no jump in assets a and b is distinguished from the alternative of a jump in either asset a or asset b, or both. We denote the null and alternative hypothesis by $\mathbb{H}_0^C(\tau) : P_t(\omega) \in \Omega_\tau^{(C)}$ and $\mathbb{H}_1^C(\tau) : P_t(\omega) \in \Omega_\tau^{(J)}$, respectively. If $\mathbb{H}_0^C(\tau)$ is rejected, we conclude that at least one of the two assets jumps at $t = \tau$, and proceed with the second step. Otherwise, we conclude that the policy announcement only contains information that are already expected by the market, which does not change either the nominal or the inflation-indexed government bond yields.

Conditioning on the rejection of the biviarate jump test in the first step, the second step then investigates whether the magnitude of event returns $\Delta P_\tau^{(a)}$ and $\Delta P_\tau^{(b)}$ are the same. We know from the first step that at least one of the two returns is dominated by its jump component. The following sets are considered:

$$\begin{cases} \Omega_\tau^{(E)} & = \{\omega: \text{ the processes have equal returns at } t = \tau: \ \Delta P_\tau^{(a)} = \Delta P_\tau^{(b)}\}, \\ \Omega_\tau^{(U)} & = \{\omega: \text{ the processes have unequal returns at } t = \tau: \ \Delta P_\tau^{(a)} \neq \Delta P_\tau^{(b)}\}. \end{cases} \tag{4}$$

Since the second step is conditioned on $\Omega_\tau^{(J)}$, the null hypothesis $\mathbb{H}_0(\tau) : P_t(\omega) \in \Omega_\tau^{(J)} \cap \Omega_\tau^{(E)}$ captures a cojump of equal magnitude in these two assets. In the context of the nominal and inflation-indexed bond yields, the null hypothesis represents cojump anchoring. The alternative $\mathbb{H}_1(\tau) : P_t(\omega) \in \Omega_\tau^{(J)} \cap \Omega_\tau^{(U)}$ implies a change in the spread of the two assets, and hence a jump in the break-even inflation.

In what follows, we estimate the event returns from noisy observations using a pre-averaging approach, and propose two different tests to distinguish the sets outlined in (3) and (4).

2.3 The bivariate distribution of pre-averaged returns

Given the noisy observation model (2), smoothing observed prices is the most intuitive approach to diminish the impact of market microstructure noise. Following the general pre-averaging approaches of Podolskij and Vetter (2009), Jacod et al. (2009) and Christensen et al. (2010), we use the mean of the observed price vector $\tilde{P}_i = (\tilde{P}_i^{(a)}, \tilde{P}_i^{(b)})'$,

$$\hat{P}_i = M_n^{-1} \sum_{j=i}^{(i+M_n-1)\wedge n} \tilde{P}_j, \quad i = 1, ..., n, \tag{5}$$

with window length $M_n = c\sqrt{n}$ and tuning parameter c. Since the microstructure noise is centered and serially uncorrelated, the vector of estimated returns at announcement time τ,

$$\Delta \hat{P}_{\lceil \tau n \rceil} = \hat{P}_{\lceil \tau n \rceil} - \hat{P}_{\lceil \tau n \rceil - M_n}, \tag{6}$$

is close to the latent increment of the efficient price ΔP_τ, and is no longer dominated by the noise. The window length balances the order of the noise and continuous component of the efficient prices. The index $\lceil \tau n \rceil$ denotes the smallest integer larger than τn and accounts for the case where the announcement time does not exactly match with the observation times of the processes.

The following proposition describes the limiting distribution of the estimated event returns $\Delta\hat{P}_{\lceil\tau n\rceil} = (\Delta\hat{P}^{(a)}_{\lceil\tau n\rceil}, \Delta\hat{P}^{(b)}_{\lceil\tau n\rceil})'$. We extend Lemma 1 of Lee and Mykland (2012) and Proposition 3.1 of Bibinger et al. (2019) from the univariate to the bivariate case.

Proposition 2.1 *Under the model presented in Section 2.1 and Assumptions 1 and 2 in Appendix A, the return vector (6) of pre-averaged prices (5) satisfies*

$$n^{1/4}\left(\Delta\hat{P}_{\lceil\tau n\rceil} - \Delta P_\tau\right) \xrightarrow{(st)} MN(0, \Gamma_\tau), \tag{7}$$

where the 2×2 *covariance matrix* Γ_τ *has elements*

$$\Gamma^{(i,j)}_\tau = 1/3\left(\Sigma^{(i,j)}_\tau + \Sigma^{(i,j)}_{\tau-}\right)c + 2c^{-1}\eta^{(i,j)}, \quad i,j = a,b. \tag{8}$$

This proposition shows that the simple pre-averaging approach consistently estimates the event return. The limiting spot variances $\Sigma^{(i,i)}_\tau$, $i = a,b$ and covariance $\Sigma^{(a,b)}_\tau$ can be random (in the case of stochastic volatility, for example), and therefore the limiting distribution in (7) is a mixed normal, MN. The return estimator is rate optimal, as indicated by $n^{1/4}$, which is a direct consequence of the choice of M_n and the order of continuous components of the efficient price P_t. The simple smoothing in (5) does not result in an efficient estimator though. An efficient estimator of a jump in a univariate setting is proposed by Bibinger et al. (2019) based on spectral statistics. As the variance-covariance matrix in (8) indicates, the diagonal and off-diagonal elements of Γ_τ have the same structure, provided that the microstructure noise displays a covariation $\eta^{(a,b)} \neq 0$ across the assets a and b. Γ_τ accounts for contemporaneous jumps in (co)volatility by referring to spot (co)volatility before ($\Sigma_{\tau-}$) and at (Σ_τ) the event; jumps in (co)volatility are studied by Bibinger and Winkelmann (2018) and Li and Todorov (2020), for example.

We apply the pre-averaging techniques proposed by Christensen et al. (2010) to estimate Γ_τ. Stable convergence (st) and a consistent estimator of the covariance Γ_τ provide a feasible, self-normalizing version of (7), which we will exploit below.

2.4 First step: The bivariate Lee-Mykland jump test

The Lee-Mykland test statistic is the estimated event return of a single asset, divided by the square root of its variance. The distributional result in Lee and Mykland (2012) to test for a jump in a single asset follows directly from Proposition 2.1:

$$n^{1/4}\Delta\hat{P}^{(i)}_{\lceil\tau n\rceil}(\Gamma^{(i,i)}_\tau)^{-1/2} \xrightarrow{(d)} N(0,1), \quad i = a,b. \tag{9}$$

For a jump-diffusion model observed under market microstructure noise, Lee and Mykland (2012) demonstrate the asymptotic normality of the univariate statistic under the null of no jump. Bibinger et al. (2019) extend the test to a more general semimartingale model.

In the bivariate model (2), we test for a jump in at least one of the two assets a and b using the product of two Lee and Mykland (2012) test statistics,

$$T^n(\tau; \Delta\tilde{P}_1^{(i)}, ..., \Delta\tilde{P}_n^{(i)}; \Delta\tilde{P}_1^{(j)}, ..., \Delta\tilde{P}_n^{(j)}) = \sqrt{n} \frac{\Delta\hat{P}_{\lceil \tau n \rceil}^{(i)}}{\sqrt{\Gamma_\tau^{(i,i)}}} \frac{\Delta\hat{P}_{\lceil \tau n \rceil}^{(j)}}{\sqrt{\Gamma_\tau^{(j,j)}}}, \quad i = a, b. \tag{10}$$

Under $\mathbb{H}_0^C(\tau)$ of no jump, the bivariate Lee-Mykland test statistics (10) is the product of two standard normal random variables with correlation

$$\rho = \Gamma_\tau^{(i,j)} / \sqrt{\Gamma_\tau^{(i,i)} \Gamma_\tau^{(j,j)}} \quad -1 \leq \rho \leq 1, \ i = a, b. \tag{11}$$

In the special case $i = j$, $T^n(\cdot)$ is simply the squared Lee-Mykland statistic. The limiting distribution of (10) is provided by the following proposition.

Proposition 2.2 *Given Proposition 2.1 and the test statistic $T^n(\cdot)$ in (10), the following asymptotic distribution applies under $\mathbb{H}_0^C(\tau)$:*

$$T^n(\tau; \Delta\tilde{P}_1^{(i)}, ..., \Delta\tilde{P}_n^{(i)}; \Delta\tilde{P}_1^{(j)}, ..., \Delta\tilde{P}_n^{(j)}) \xrightarrow{(d)} VG, \quad i, j = a, b, \tag{12}$$

where VG denotes the variance-gamma distribution with the respective shape, asymmetry, spread and location parameters $(1, \rho, \sqrt{1-\rho^2}, 0)$, and ρ is given by (11).

The bivariate jump test has critical regions

$$\left\{ q_{\alpha/2}(VG) < T^n(\tau; \Delta\tilde{P}_1^{(i)}, ..., \Delta\tilde{P}_n^{(i)}; \Delta\tilde{P}_1^{(j)}, ..., \Delta\tilde{P}_n^{(j)}) < q_{1-\alpha/2}(VG) \right\}, \tag{13}$$

where $q_{\alpha/2}(VG)$ and $q_{1-\alpha/2}(VG)$ denote a lower and upper quantile of the variance-gamma distribution at significance level α. Hence the bivariate test has asymptotic level α under $\mathbb{H}_0^C(\tau)$, and rejects with probability 1 under $\mathbb{H}_1^C(\tau)$. The divergence of (10) has rate $n^{1/2}$ in the case of a cojump, and $n^{1/4}$ in the case of a single jump in either asset a or asset b.

Proposition 2.2 shows that the bivariate Lee-Mykland test statistic (10) with correlation ρ is variance-gamma distributed. The variance-gamma distribution with the specific parameters in Proposition 2.2 is the law of a normal variance-mean mixture with a $\chi^2(1)$ mixing density. It is equivalent to the distribution of an off-diagonal element of a Wishart matrix with one degree of freedom. The link between normal variance-mean mixtures and the variance-gamma distribution, which is a special case of generalized hyperbolic distributions, is reviewed by Barndorff-Nielsen et al. (1982).

Remark 1 A univariate jump test is a special case of Proposition 2.2. *The general asset $i, j = a, b$ notation in Proposition 2.2 provides a direct link between the bivariate jump test and the univariate Lee-Mykland jump test. If $i = j$, we have $\tilde{P}_t^{(i)} = \tilde{P}_t^{(j)}$, $P_t^{(i)} = P_t^{(j)}$ and $\rho = 1$. Hence, (10) is the square of a standard normally distributed test statistic for the univariate test of Lee and Mykland (2012). The squared statistic is $\chi^2(1)$ distributed, which is a special case of the variance-gamma distribution in Proposition 2.2.*

We compare the new bivariate jump test with the univariate jump test of Lee and Mykland (2012) in two specific examples below. This is not an easy task because these tests have different hypotheses. However, their test statistics are closely related, see Remark 1. For ease of exposition, we assume that the jump behavior of asset b at τ is known in Example 1 and 2.

Example 1 Bivariate vs. univariate jump test: only one asset jumps. *Suppose asset b does not jump at time τ and we want to test if asset a has a positive jump. Can we gain in power by using the proposed bivariate instead of the univariate Lee-Mykland test? Proposition 2.2 states that the test statistic (10) has the same $n^{1/4}$ divergence rate as (9) if only one asset jumps. However, critical values of the bivariate test depend on ρ. The smaller the correlation, the smaller the critical values. Critical values of the variance-gamma distribution are rather large compared with the standard normal distribution. For example, its 95% quantile is $q_{0.95}(VG) = 2.67$ for $\rho = 0.5$, and $q_{0.95}(VG) = 3.36$ for $\rho = 0.8$. These values are larger than the 95% quantile of the standard normal distribution which is 1.64. Thus, consider the two tests at 5% significance level, we only gain in power, if the normalized event return for asset b, $n^{1/4}\Delta\hat{P}^{(b)}_{\lceil \tau n \rceil}(\Gamma_\tau^{(b,b)})^{-1/2}$, is larger than $2.67/1.64 = 1.62$ for $\rho = 0.5$ and $3.36/1.64 = 2.05$ for $\rho = 0.8$. The probabilities of these events are 5.26% and 2.02%, respectively, as under the assumption that asset b does not jump, its normalized return $n^{1/4}\Delta\hat{P}^{(b)}_{\lceil \tau n \rceil}(\Gamma_\tau^{(b,b)})^{-1/2}$ is standard normally distributed.*

Example 2 Bivariate vs. univariate jump test: cojump. *Suppose asset b jumps at time τ with a jump size that is two times its standard deviation, $\Delta\hat{P}^{(b)}_{\lceil \tau n \rceil}(\Gamma_\tau^{(b,b)})^{-1/2} = 2$. We want to test if asset a also jumps at τ. The univariate test ignores the information about asset b. Given normality under the null of no jump in asset a, the univariate test rejects if $n^{1/4}\Delta\hat{P}^{(a)}_{\lceil \tau n \rceil}(\Gamma_\tau^{(a,a)})^{-1/2} > 1.64$, at the 5% level of significance. In contrast, the bivariate test benefits from the jump in asset b. With the 95% quantiles of the variance-gamma distribution of 2.67 and 3.36 for $\rho = 0.5$ and $\rho = 0.8$, respectively, we detect jumps with the bivariate jump test if $n^{1/2}\Delta\hat{P}^{(a)}_{\lceil \tau n \rceil}(\Gamma_\tau^{(a,a)})^{-1/2} > 1.33$ for $\rho = 0.5$, and $n^{1/2}\Delta\hat{P}^{(a)}_{\lceil \tau n \rceil}(\Gamma_\tau^{(a,a)})^{-1/2} > 1.68$ for $\rho = 0.8$. These values are obtained by dividing the critical values of the variance-gamma distribution by the standardized jump size of asset b. Additionally, the bivariate test scales with the larger factor $n^{1/2}$. This enables the detection of smaller jumps in asset a using the bivariate approach.*

Example 1 reveals that the bivariate test in Proposition 2.2 usually requires larger jump sizes than the univariate test if only one of the two assets jumps. Example 2 shows that the power gain of the bivariate test can increase as soon as assets a and b cojump.

Remark 2 Comparison with the local test for cojumps by Bibinger and Winkelmann (2015). *The test statistic proposed in Proposition 4.1 in Bibinger and Winkelmann (2015) is based on local quadratic variation estimates on a time interval surrounding τ. It is the estimation error of spot co-volatility under the null of no cojump. Therefore, they obtain a limiting normal distribution, which is not compatible with our result in Proposition 2.2. Under the alternative of a cojump, their statistic diverges with the same $n^{1/2}$ rate as the bivariate Lee-Mykland test. The test statistic (10) has an advantage of utilizing the information of the known jump time τ, whereas the cojump test of Bibinger and Winkelmann (2015) weights all observed returns over an interval around τ equally. On the other hand, the spectral methods are efficient while those of local means are not.*

2.5 Second step: A test for the equal jump sizes

Conditional on the rejection of the bivariate jump test, we test for equal event returns of assets a and b. The following corollary provides a high-frequency A/B test-type result.

Corollary 2.3 *Using the pre-averaging estimators (5) and Proposition 2.1, we consider the test statistic*

$$\bar{T}^n(\tau; \Delta\tilde{P}_1^{(i)}, ..., \Delta\tilde{P}_n^{(i)}; \Delta\tilde{P}_1^{(j)}, ..., \Delta\tilde{P}_n^{(j)}) = n^{1/4} \frac{\Delta\hat{P}^{(a)}_{\lceil \tau n \rceil} - \Delta\hat{P}^{(b)}_{\lceil \tau n \rceil}}{\sqrt{\Gamma_\tau^{(a,a)} + \Gamma_\tau^{(b,b)} - 2\Gamma_\tau^{(a,b)}}}. \tag{14}$$

In case of equal event returns, that is $P_t(\omega) \in \Omega_\tau^{(E)}$ in (2.1),

$$\bar{T}^n(\tau; \Delta\tilde{P}_1^{(i)}, ..., \Delta\tilde{P}_n^{(i)}; \Delta\tilde{P}_1^{(j)}, ..., \Delta\tilde{P}_n^{(j)}) \xrightarrow{(d)} N(0, 1), \tag{15}$$

the test for equal event returns of assets a and b has critical regions

$$\left\{ q_{\alpha/2}(\Phi) < \bar{T}^n(\tau; \Delta\tilde{P}_1^{(i)}, ..., \Delta\tilde{P}_n^{(i)}; \Delta\tilde{P}_1^{(j)}, ..., \Delta\tilde{P}_n^{(j)}) < q_{1-\alpha/2}(\Phi) \right\}, \tag{16}$$

where $q_{\alpha/2}(\Phi)$ and $q_{1-\alpha/2}(\Phi)$ denote a lower and upper quantile of the standard normal distribution. The test has asymptotic level α and rejects in the case of $P_t(\omega) \in \Omega_\tau^{(U)}$ with probability 1.

The test statistic (14) takes the difference in the event returns of the two assets, and divides it by the square root of the spot variance of the difference. Thus (15) with (14) is equivalent to the univariate Lee-Mykland test (9) applied to the spread of the two asset returns.

Corollary 2.3 provides a test for cojump anchoring if it is applied to nominal and inflation-indexed bond yields with the same maturity, conditional on the rejection of the bivariate jump test. In that case the set of equal event returns, $\Omega_\tau^{(E)}$ in (2.1), is reduced to cases where nominal and inflation-indexed bond yields cojump at time τ. Since τ refers to monetary policy announcement times, the alternative detects significant changes of the break-even inflation, and indicates central banks' ability to move the inflation expectations measure.

3 Simulation study

This section examines the finite-sample properties of the bivariate Lee-Mykland jump test proposed in Proposition 2.2. The test for equal high-frequency event returns in Corollary 2.3 relates to the univariate Lee-Mykland jump test and is investigated in simulation studies by Lee and Mykland (2012) and Bibinger et al. (2019). We consider observations on a three-hour time interval, with the time point τ in the middle of the three-hour interval. The simulation setup emulates that of Lee and Mykland (2012). The efficient price of asset a is simulated following the process

$$P_t^{(a)} = 1 + \int_0^t \sigma_s \, dW_s \, , t \in (0,1], \tag{17}$$

with a Heston-type stochastic volatility

$$d\sigma_s^2 = 0.0162 \left(0.8465 - \sigma_s^2\right) ds + 0.117 \, \sigma_s \, dB_s, \tag{18}$$

where B_t and W_t are two independent standard Brownian motions. We adopt the parameter values of Lee and Mykland (2012) and assume 252 trading days per year with 9 trading hours a day. The efficient price of asset b is simulated using

$$P_t^{(b)} = \rho \int_0^t dP_s^{(a)} + \sqrt{1-\rho^2} \int_0^t d\tilde{W}_s, \tag{19}$$

where ρ is the correlation between the returns of assets a and b, and $\tilde{W}_t$ a standard Brownian motion that is independent of B_t and W_t. The market microstructure noise is generated by

$$\epsilon_i^j = 0.0861 \Delta P_{t_i}^j + 0.06 \big(\Delta P_{t_i}^j + \Delta P_{t_{i-1}}^j\big) U_i, \quad i = 0, \ldots, n, \; j = a, b, \tag{20}$$

where $(U_i)_{0 \le i \le n}$ is a sequence of normally distributed random variables with expectation 0 and variance q^2. We consider two parameterizations of q, which governs the noise level (market quality parameter). The cross-correlation between P_t and the noise violates one of our theoretical assumptions. However, the simulation results will show that this correlation does not affect the

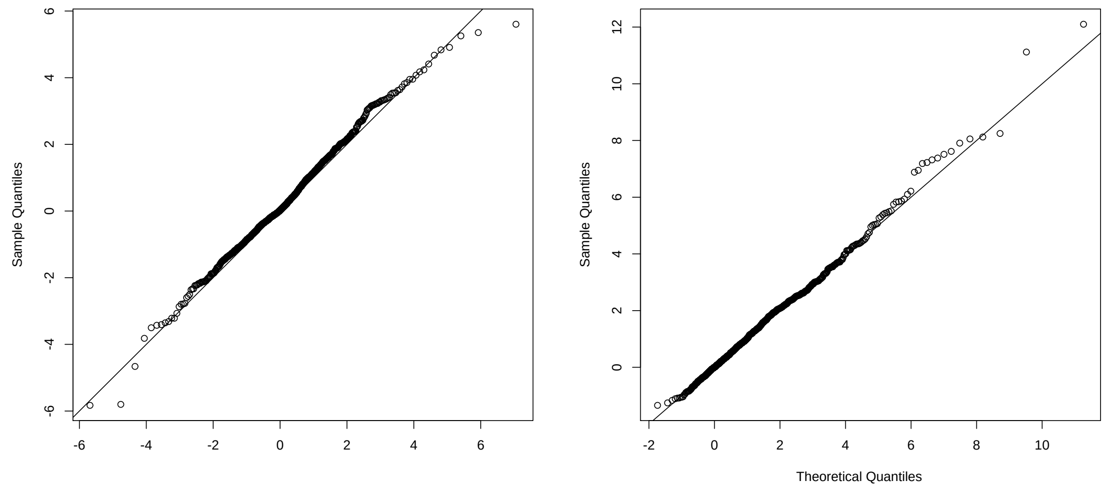

Figure 1: QQ-Plot of bivariate jump test with the variance-gamma distribution.
Simulation under the hypothesis of no jump with $n = 10800$ observations. The figure indicates the difference of the distribution under the null for a correlation of 0.1 (left) and 0.7 (right) between assets a and b. The straight line is the diagonal.

finite sample performance of the test. Therefore, we work with this more general and realistic setting. We estimate q using the noise estimator suggested in Proposition 1 of Lee and Mykland (2012). The (co)volatility is estimated using the pre-averaging approach of Christensen et al. (2010). The pre-averaging for the bivariate Lee-Mykland test statistics refers to a block size $M_n = c\sqrt{n}$. The constant c is chosen according to Table 5 of Lee and Mykland (2012).

We first examine the finite sample distribution of the test statistic (10) under the null hypothesis of no jump. The QQ-plots in Figure 1 compare the the simulated test statistic (10) with the quantiles of the limiting variance-gamma distribution for two different values of ρ, 0.1 (left) and 0.7 (right). The sampling frequency is 1-second, which corresponds to $n = 10800$ observations in the three-hour interval. The QQ-plots suggest that the empirical distributions of the bivariate Lee-Mykland test statistic closely follow its limiting theoretical distribution. The x-axes of the two plots in Figure 1 show that for lower correlation between the continuous components of assets a and b, the quantiles of the limiting variance-gamma distribution shift to smaller values. Holding other parameters fixed, smaller values of ρ will leads to lower critical values on both tails of the variance-gamma distribution, and hence make it easier to detect jumps when two assets have event returns of the same sign, and harder when the two event returns have the opposite signs.

We then investigate the size and power properties of the proposed bivariate jump test under different scenarios. Jumps of varying magnitude are added to the observed prices at the time point τ in the middle of the three-hour interval. We experiment with different jump sizes,

Table 1: Size and power of the bivariate Lee-Mykland test.

Jump size	Size	Power (single jump)			Power (cojump)		
$\Delta X^{(a)}_{\tau}$	0σ	0.2σ	0.6σ	1.2σ	0.2σ	0.6σ	1.2σ
$\Delta X^{(b)}_{\tau}$	0σ	0σ	0σ	0σ	0.2σ	0.6σ	1.2σ
Noise level: $q = 0.0001$							
30-sec (n=360)	0.059	0.539	0.777	0.892	0.722	1.000	1.000
5-sec (n=2160)	0.041	0.673	0.876	0.935	0.996	1.000	1.000
1-sec (n=10800)	0.048	0.767	0.914	0.959	1.000	1.000	1.000
Noise level: $q = 0.001$							
30-sec (n=360)	0.050	0.514	0.760	0.877	0.713	0.999	1.000
5-sec (n=2160)	0.046	0.661	0.878	0.933	0.993	1.000	1.000
1-sec (n=10800)	0.040	0.751	0.904	0.959	1.000	1.000	1.000

Tests are conducted at nominal level of 5% on a three-hour block with $n = 360, 2160, 10800$ observations using 2000 iterations. The number of observations are indicated in parentheses in the first column. σ is the volatility ($\sqrt{0.8465/(252 \times 3)}$ of asset a and asset b on the three-hour block. The correlation between assets a and b is set to $\rho = 0.7$. Results for different ρ are very similar to those reported in this table. $c = 1/8$ is used to determine M_n.

sampling frequencies, and noise levels, and present the results in Table 1. The correlation ρ is fixed to be $\rho = 0.7$. In our setup, different values of ρ have relatively small effects on the simulation results. The tests are conducted at a 5% nominal level of significance. Each column of the table examines a different combination of jump sizes, while each row examines a different sampling frequency. The top and bottom panels show two different noise levels. We start with zero jump sizes, which represent the size of the bivariate jump test. Under the null hypothesis of no jumps in assets a and b, the rejection rate is close to the nominal level of $\alpha = 0.05$ in all cases. Sampling frequency and noise level do not seem to affect the size of the bivariate jump test.

The power of the bivariate jump test depends on a few factors. Firstly, in the case of a single-asset jump, larger jump sizes lead to higher power, which is expected. Secondly, keeping the jump size fixed, having a cojump substantially increase the power of the test compared to a single-asset jump. Consistent with the discussions in Examples 1 and 2, this demonstrates the fast divergence of the test statistic when both assets have jumps, compared with the slower divergence rate when only one asset has jump. Thirdly, higher noise level, shown in the bottom panel of Table 1, only marginally reduce the test power, especially when the sampling frequency is high. Finally, the test power converges to 1 quickly as the sampling frequency increases. Overall the simulation results in Table 1 demonstrate very good size and power property of our bivariate jump test.

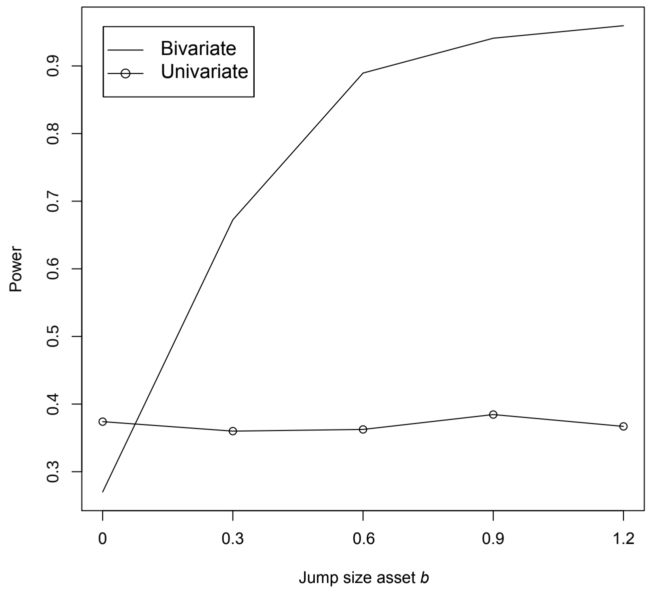

Figure 2: Power comparison of the univariate and bivariate Lee-Mykland jump test.
Tests are conducted at level $\alpha = 0.05$ using 2000 iterations. We set $\sigma = \sqrt{0.8465/(252 \times 3)}$ for the volatility of assets a and b on the three hour block with $n = 360$ observations. The x-axis displays the jump size of asset b as a fraction of σ. The jump size of asset a is fixed at 0.1σ, while the jump size of asset b increases from zero to 1.2σ. The univariate test tests asset a for a jump at time point τ. The correlation between assets a and b is $\rho = 0.7$.

Figure 2 provides a comparision between the bivariate and univariate Lee-Mykland test. In analog to Example 1 and 2, we fix the jump size of asset a, and the correlation $\rho = 0.7$. We chose a jump size of 0.1σ, and sampling frequency of 30-second, which corresponds to $n = 360$. Clearly such a small jump size is relatively difficult to distinguish from all other returns on the three-hour time interval. Figure 2 depicts the percentage of rejecting the null hypothesis of no jump using either the univariate or bivariate tests as the jump size of b varies. The univariate test ignores any information on asset b. Therefore, the power curve of the univariate Lee-Mykland test in Figure 2 is flat, and only depends on the fixed jump of asset a. The rejection rate of the univariate test is close to 40%. In contrast, the bivariate test takes both assets into account. As Example 1 suggests, if asset b does not jump (x-axis is zero), using the bivariate test leads to some loss in power in detecting a jump in asset a. However, the power of the bivariate test surpasses that of the univariate test quickly when a jump is added to asset b. For instance, when asset b has a jump of size 0.3σ, the bivariate test has almost 70% power. This is consistent with Example 2 that cojumps in both assets leads to substantial power gain using the bivaraite test. Figure 2 shows that the power of the bivariate test increases to 1 when the size of the contemporaneous jump of asset b gets larger.

4 Empirical evidence

In this section we study the pre-scheduled monetary policy announcements of the Federal Reserve (Fed), European Central Bank (ECB) and Bank of England (BoE). We first introduce the high frequency data on the nominal and inflation-indexed government bonds. The two-step testing procedure introduced in Section 2 is then used to test for cojump anchoring. This exercise provides some insights on central banks' ability in altering market's inflation expectation through its policy communication.

4.1 Government bond data

The Fed, ECB and BoE communicate their interest rate decision and other key policy measures at pre-scheduled announcement times. The Fed and ECB usually have their monetary policy meetings every six weeks, while BoE changed from monthly meetings to a 6-week interval in 2017. We obtain the list of announcement days and times of these three central banks from January 2015 to March 2020. In total, we study 43 announcements of the Fed, 44 announcements of ECB and 51 of BoE.

Intra-day high-frequency data on government bonds for the U.S., Germany, and U.K. are extracted from Thomson Reuters tick history database (via Refinitiv DataScope Select). Since the Euro Area countries do not issue joint bonds, we use German government bonds to examine the impact of ECB announcements. The German inflation-indexed bonds are linked to the Euro Area harmonized consumer price index. For each pair of country and central bank, we download quotes on a three-hour time interval around the publication time of the press release. At each announcement time, the nominal and inflation-indexed bonds with maturities close to five and ten years are chosen. We compute five-year five-year forward rates from the five-year and ten-year bond yields using the approach of Shiller et al. (1983). All bond yields are synchronized to 30-second observations, which corresponds to $n = 360$ in the simulation.

Figures 3 and 4 depict two examples of our 30-second bond yields data over a 1.5-hour window on two announcement dates. Figure 3 shows German bond yields around an ECB announcement time at 13:45 on December 8th, 2016, and Figure 4 displays U.K. bond yields around a BoE announcement time at 12:00 on June 20th, 2019. The left panel plots the five-year nominal (blue) and inflation-indexed (orange) bond yields, and the right panel plots the ten-year bond yields. In each plot the range of the y-axis for the nominal and inflation-indexed bond yields is set to be the same. It is evident from both figures that at their respective announcement times, there is a sizable change in all four bond yields. Difference in the changes between the nominal and inflation-indexed bond yields determines the direction of jump in the break-

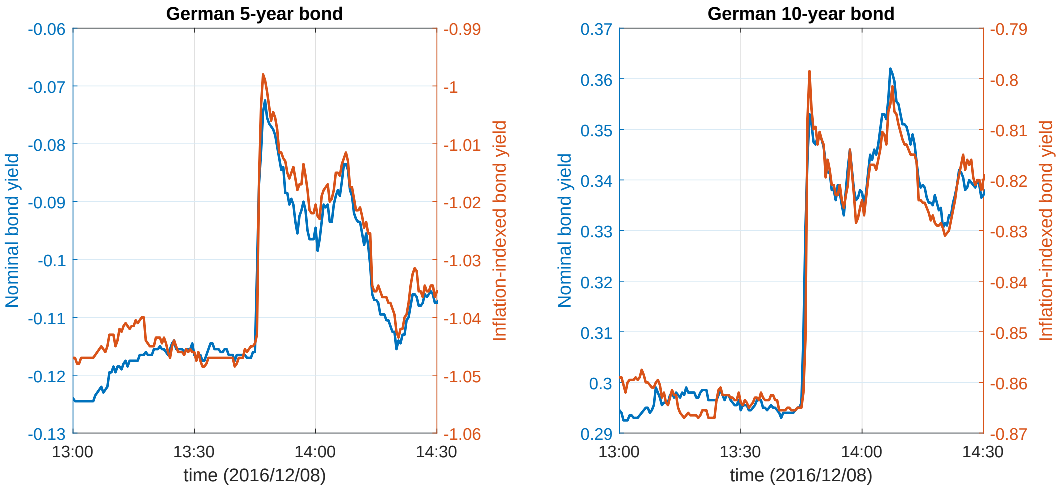

Figure 3: German bond yields on 2016/12/08 around an ECB announcement at 13:45. These bond yields are 30-second observations over the 1.5-hour interval around the monetary policy announcement time. The left panel plots the 5-year bond yields, and the right panel plots the 10-year bond yields. The nominal bond (blue) uses the left axis on each plot, and the inflation-indexed bond (orange) uses the right axis. The ISIN for the 4 bonds used are: DE0001102317 for five-year nominal, coupon 1.5%, maturity 2023/05/15; DE0001030542 for five-year inflation-indexed, coupon 0.1%, maturity 2023/04/15; DE0001102390 for ten-year nominal, coupon 0.5%, maturity 2026/02/15; DE0001030567 for ten-year inflation-indexed, coupon 0.1%, maturity 2026/04/15.

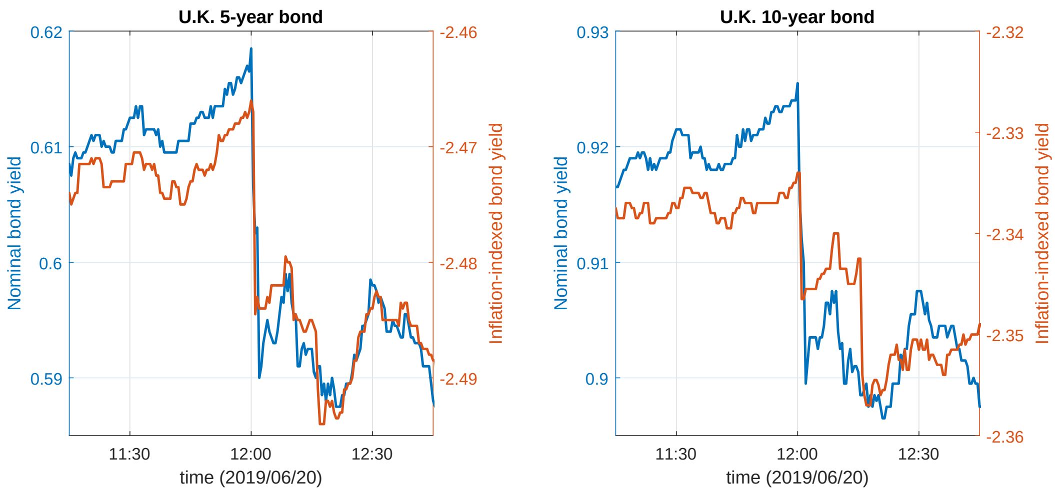

Figure 4: U.K. bond yields on 2019/06/20 around a Bank of England announcement at 12:00. These bond yields are 30-second observations over the 1.5-hour interval around the monetary policy announcement time. The left panel plots the 5-year bond yields, and the right panel plots the 10-year bond yields. The nominal bond (blue) uses the left axis on each plot, and the inflation-indexed bond (orange) uses the right axis. The ISIN for the 4 bonds used are: GB00BHBFH458 for five-year nominal, 2.75% Treasury Gilt 2024; GB0008983024 for five-year inflation-indexed, 2.5% Index-linked Treasury Stock 2024; GB00B24FF097 for ten-year nominal, 4.75% Treasury Gilt 2030; GB0008932666 for ten-year inflation indexed, 4.125% Index-linked Treasury Stock 2030.

even inflation. More importantly, Figures 3 and 4 show that bonds with different maturity and interest rate structure can behave very differently. The German five-year bond yields bounce back in much larger magnitude than the ten-year bond yields. In the case of the U.K. government bonds, the nominal bond yield has a much larger jump size than the inflation-indexed bond at both five-year and ten-year maturities. These differences in the size of jumps lead to changes in the break-even inflation. Lastly, we should note that the window length to allow for gradual jumps plays an important role in the two-step testing procedure, because the bond yields may continue to adjust within the few minutes after the announcement time. We use the 5-minute window to estimate the size of the jumps, but also experiment with alternative choice of 2-minute window as a robustness check. These results are in Appendix C.

Table 2 presents some descriptive statistics of the nominal and inflation-indexed bonds from the three countries. The top panel of Table 2 reports the average size of 30-second yield changes from the noisy observations over the three-hour time interval. We take the absolute value of yield changes and present their medians in basis points in Panel A. The U.S. bonds exhibit the largest changes on average, while German bonds have the smallest movements. The second panel in Table 2 displays the median of estimated event returns in basis points across all policy announcements. The event returns are estimated using the pre-averaging approach in (5) with $c = 1/18$, which corresponds to pre-averages over one-minute time windows. To account for delayed responses and gradual adjustments, we use a five-minute cut-out window. That is, the event return is the difference of the average yields before the event time and five minutes after the event time. Event returns are substantially higher than the average 30-second returns shown in Panel A, indicating higher likelihood of jumps at announcement times.

Panel C of Table 2 uses the estimated event returns across all monetary policy announcements for each country, and reports the correlation coefficients between the nominal and inflation-indexed bonds with the same maturity. In general there is a high degree of comovements between the nominal and inflation-indexed bonds. The correlation coefficient is almost one for event returns in the five-year bond yields for all three countries, but decreases by varying degrees for the ten-year maturity bonds. The correlation for five-year five-year forward rates is the lowest, with only around 0.5 for German and British bonds. The last two panels of Table 2 report the estimated local volatility and noise level using the estimators of Christensen et al. (2010) with window length of six minutes. The estimated local volatility is of comparable magnitudes across different countries and maturities. The noise level is higher for the five-year five-year forward rate, but in general quite similar with a level around 0.001. The block size of the pre-averaging approach is chosen according to the low noise level, as the average over one-minute window essentially takes two 30-second observations.

Table 2: Descriptive statistics of government bond yields on monetary policy announcement days from 2015 to 2020.

Maturity	U.S. Nominal	U.S. Indexed	Germany Nominal	Germany Indexed	U.K. Nominal	U.K. Indexed
Panel A: Median absolute 30-sec returns						
5Y	0.29	0.28	0.11	0.08	0.20	0.20
10Y	0.20	0.21	0.10	0.04	0.20	0.20
5Y5Y	0.30	0.32	0.16	0.09	0.27	0.20
Panel B: Pre-averaged absolute event returns						
5Y	1.38	1.67	0.34	0.38	0.61	0.57
10Y	1.17	1.18	0.37	0.36	0.72	0.60
5Y5Y	0.91	0.95	0.43	0.46	0.85	0.58
Panel C: Correlation of pre-averaged event returns						
5Y	0.99		0.98		0.98	
10Y	0.97		0.78		0.87	
5Y5Y	0.76		0.48		0.55	
Panel D: Local volatility σ						
5Y	0.073	0.084	0.058	0.070	0.063	0.056
10Y	0.067	0.076	0.074	0.080	0.067	0.063
5Y5Y	0.078	0.087	0.088	0.110	0.071	0.072
Panel E: Noise level $\sqrt{\eta^{(j,j)}}$						
5Y	0.0011	0.0012	0.0008	0.0010	0.0008	0.0006
10Y	0.0011	0.0012	0.0009	0.0009	0.0008	0.0007
5Y5Y	0.0020	0.0023	0.0016	0.0018	0.0011	0.0010

The table shows median statistics based on 30-second yield changes across five-year (5Y) and ten-year (10Y) nominal and inflation indexed bonds and five-year five-year forwards (5Y5Y) on monetary policy announcement days from 2015 to 2020. Returns are in basis points. Observed absolute returns refer to all observations across the three hour time intervals. The event return is the Lee and Mykland (2012) estimate at the press release time. Volatility is estimated with the pre-averaging approach of Christensen et al. (2010). The standard deviation of the noise (noise level) is estimated with the estimator of Christensen et al. (2010).

Table 3: Rejections of the bivariate jump test in the nominal and inflation-indexed bonds (LM2) and rejections of equal jump size (1for1) at monetary policy announcements in 2015–2020.

Maturity	Fed		ECB		BoE	
	LM2	1for1	LM2	1for1	LM2	1for1
5Y	23 (0.53)	4 (0.17)	8 (0.18)	1 (0.13)	24 (0.47)	12 (0.50)
10Y	23 (0.53)	6 (0.26)	11 (0.25)	3 (0.27)	25 (0.49)	13 (0.52)
5Y5Y	19 (0.44)	7 (0.37)	8 (0.18)	3 (0.38)	26 (0.51)	11 (0.42)

The bivariate Lee-Mykland (LM2) jump test is applied to the nominal and inflation-indexed bonds. The univariate Lee-Mykland test (1for1) is applied to the break-even inflation, conditional on the rejection of LM2. We use a 1% level of significance and a 5-minute jump window at the announcement time to allow for delayed or gradual adjustments. The tests are implemented with the same choice of tuning parameters as in the simulations.

4.2 Cojump anchoring

We test policy announcements of the three central banks using the government bond data introduced above. In the first step of the test procedure, we apply the bivariate Lee-Mykland jump test from Proposition 2.2 to the nominal and inflation-indexed bonds with the same maturity. Conditional on a rejection of the hypothesis of no jump in the two bond yields, we test for equal jump sizes using Corollary 2.3. Parameter values for pre-average return and volatility estimation are chosen as described in Sections 3 and 4.1. One advantage of the statistical methods based on high frequency data is that we can zoom in on each individual announcement separately, and analyze their potentially distinct effects on government bond yields and the break-even inflation. We examine locally around each policy announcement whether the cojump anchoring holds.

Table 3 summarizes the results from both steps of the testing procedure at 1% significance level. We provide the number of rejections of the first and second step tests, as well as the percentage of rejections in parenthesis. Columns denoted by 'LM2' refer to the bivariate Lee-Mykland jump test, while the columns entitled '1for1' indicate the test for equal event returns conditional on the rejection of the bivariate jump test. We find that 50% of the monetary policy announcements by the Fed and BoE are accompanied by a respective jump in the U.S. and U.K. bond yields. In contrast, only 18% to 25% of the policy announcements by the ECB trigger significant jumps in German bonds, which is only about half as often as in the U.S. and U.K. cases.

In the case of the U.S., the cojump anchoring hypothesis is rejected on 4 and 6 policy announcements for the five-year and ten-year U.S. treasury bond yields, respectively, accounting for only 17% and 26% of all market-moving policy announcements. The ECB announcements show similar proportions of rejecting cojump anchoring, with 13% and 27% of all market-moving announcements for the five-year and ten-year German bonds, respectively. The highest fraction where cojump anchoring is rejected is displayed by the BoE announcements. Roughly 50% of all market-moving announcements do not lead to equal jump sizes in the nominal and inflation-indexed bonds. Besides evidence based on the spot rates, the five-year five-year forwards allow for a more detailed analysis of long-horizon inflation expectations since they remove the short to medium term fluctuations. Interestingly, results become more homogeneous at the five-year five-year forward horizon. Despite different number of jumps detected in the first-step using the bivaraite jump test, the percentage of cojump anchoring is quite similar across the three central banks. Around 40% of the market-moving policy announcements lead to different jump sizes in the nominal and inflation-indexed five-year five-year forward rates. In other words, 60% of the time we find cojump anchoring in the five-year five-year forward rate across the three countries. This percentage is higher than those of the five-year and ten-year bond yields for the Fed and ECB announcements, but lower for the BoE announcements.

In summary, we find that although there are varying percentages of announcements that lead to jumps in the government bond yields among different countries, a large proportion of these market-moving announcements are accompanied with same jump sizes in the nominal and inflation-indexed bonds. Hence the empirical results provide strong evidence for cojump anchoring and a small number of jumps in the break-even inflation. The U.K. government bonds display the most frequent rejections of a one-for-one relation of jumps in nominal and inflation-indexed bonds, and thus the most frequent changes in the break-even inflation. In what follows we zoom in onto the set of policy announcements where we reject the cojump anchoring hypothesis and investigate how the break-even inflation moves.

4.3 Jumps in break-even inflation

The Fed, ECB and BoE have implemented the same inflation target of 2% annual rate in the U.S., Euro Area and U.K., respectively. Since the break-even inflation at five-year and ten-year horizons are consistently below 2% over our sample period from 2015 to 2020, we are particularly interested in the situations of rejecting cojump anchoring with an upward jump in the break-even inflation.

Table 4 shows the percentage of positive and negative jumps in the break-even inflation, as well as the average jump sizes in basis points. We distinguish between upward jumps 'BEI(+)', which indicate jumps that push the break-even inflation up towards the inflation target, and downward jumps 'BEI(-)', which move the break-even inflation away from the inflation target.

Table 4: Average jump size in the break-even inflation in basis points at monetary policy announcements in 2015–2020.

Maturity	Fed		ECB		BoE	
	BEI(+)	BEI(-)	BEI(+)	BEI(-)	BEI(+)	BEI(-)
5Y	1.13	$--$	$--$	-1.20	0.53	-1.34
	(1.00)	(0.00)	(0.00)	(1.00)	(0.25)	(0.75)
10Y	1.09	-0.67	2.24	$--$	1.32	-0.92
	(0.50)	(0.50)	(1.00)	(0.00)	(0.38)	(0.62)
5Y5Y	1.96	-1.51	4.53	$--$	2.91	-1.81
	(0.43)	(0.57)	(1.00)	(0.00)	(0.45)	(0.55)

The table shows the average jump return in basis points of break even inflation at monetary policy announcements. In parenthesis below is the share of positive vs. negative jumps in break-even inflation. Jump sizes are estimated using 5-minute window at the announcement times.

The following results stand out. First, across maturities and countries, the average jump size for upward jumps is usually larger than that of downward jumps in the break-even inflation, with only one exception of the U.K. five-year horizon. Second, although German bonds have the lowest number of jumps in the break-even inflation among the three countries, their average jump sizes are always the highest across different maturities. Based on the average jump size and direction of jumps, the ECB has the strongest control over its long-horizon break-even inflation. All three jumps in the Euro Area ten-year and five-year five-year forward break-even inflation are upward jumps with average magnitudes of 2.24 and 4.53 basis points, respectively. Last but not least, the share of positive vs. negative jumps and their average magnitudes differ across countries, suggesting that these three central banks have different abilities in affecting their domestic inflation expectations. The U.S. has higher proportion of positive jumps in shorter horizon break-even inflation, but U.K. has higher percentages of positive jumps in the longer horizon. The U.S. and U.K. five-year five-year forward break-even inflation have roughly the same proportions of 45% upwards and 55% downward jumps. Among these 45% of positive jumps, the BoE announcements push U.K. break-even inflation by 2.91 basis points upwards on average, while announcements of the Fed move U.S. break-even inflation up by 1.96 basis points.

Figure 5 depicts the jump sizes in the nominal and inflation-indexed bond yields using the five-year five-year forward rates for each country.[3] The 45-degree line indicates the null hypothesis of cojump anchoring, i.e. same jump size in the nominal and inflation indexed bond yields. Each dot represents a policy announcement where the null of no jump in the first step of the

[3]We have also plotted the jump sizes for the five-year and ten-year nominal and inflation-indexed bond yields. These results are available upon request.

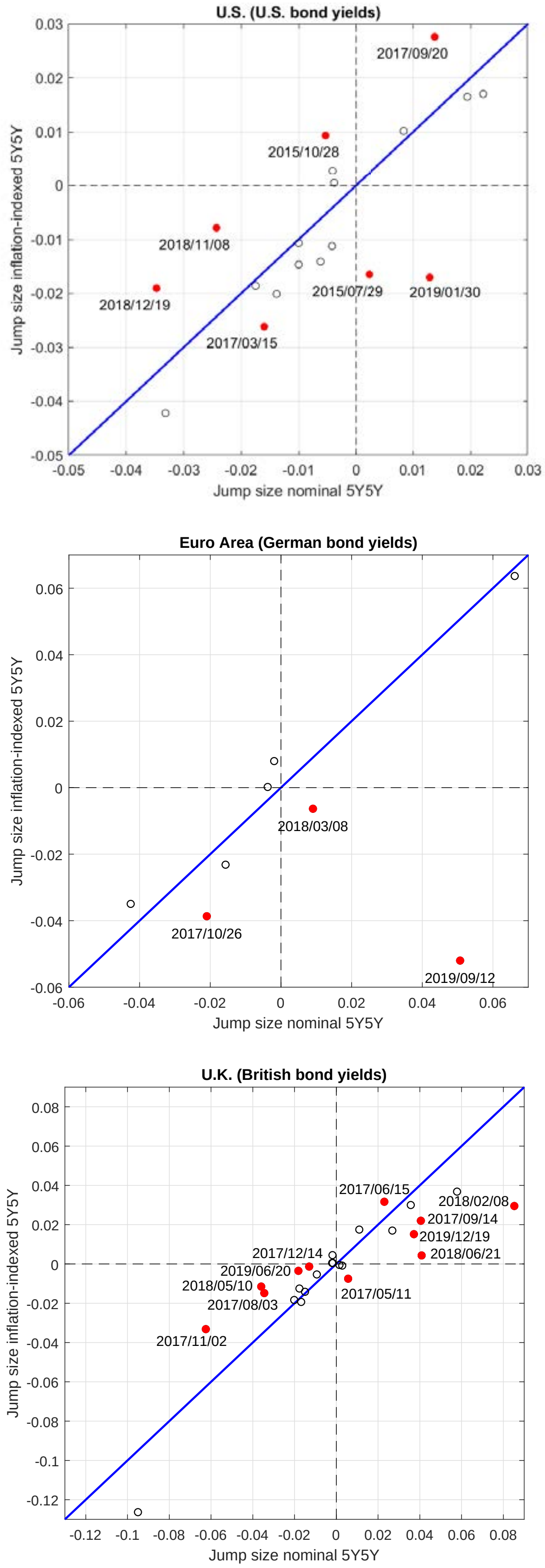

Figure 5: Jump sizes of the nominal and inflation-indexed five-year five-year forwards. Dots represent monetary policy event returns that reject the null of the bivariate jump test. Rejection of the one-for-one relation of nominal and inflation-indexed yields (cojump anchoring) is highlighted in red and denoted by the date of the policy announcement. The event returns are estimated using 5-minute jump window. The one-for-one relation is indicated by the 45-degree line.

bivariate jump test is rejected. The announcements where the second step of testing for equal jump size is rejected are denoted by red solid dots with the announcement dates labeled. The majority of jumps detected are quite close to the 45-degree line, indicating similar jump sizes in the nominal and inflation-indexed bond yields. The more far away the jump sizes are from the 45-degree line, the more likely the cojump anchoring is rejected. We observe the largest movements in the break-even inflation when the nominal and inflation-indexed bonds jump in opposite directions. For instance, the announcement of the Fed on January 30th, 2019 and the ECB announcement on September, 12th 2019 shift the nominal bond yields upwards and the inflation-indexed bond yields downwards, leading to an upward jump in the five-year five-year break-even inflation.

5 Conclusion

This paper is motivated by the empirical question of how a measure of inflation expectations responds to monetary policy announcements. We propose a high-frequency statistical inference procedure to test for *cojump anchoring*. Cojump anchoring is represented by a one-for-one change in the nominal and inflation-indexed government bond yields of the same maturity locally around the time of a pre-scheduled monetary policy announcement.

The two key aspects of cojump anchoring form the basis of the proposed two-step testing procedure. Firstly, cojump anchoring is conditioned on the existence of a sufficiently large monetary policy surprise. We require that the policy announcement under study contains new information that substantially moves at least one of the two government bond yields. The set of announcements that only contains expected information but no surprise is removed. Secondly, within the set of market-moving announcements, we examine if the changes in the nominal and inflation-indexed bond yields have the same size. If the nominal and inflation-indexed bonds have the same movement around a policy announcement, the spread between them, also known as the break-even inflation, has zero response to the announcement. In sum, cojump anchoring is characterized by movements of the same size in the nominal and inflation-indexed government bond yields and hence a non-moving spread between them.

A two-step inference procedure based on noisy high-frequency data is proposed to test for cojump anchoring. The first step develops a new bivariate jump test based on the univariate Lee-Mykland test statistics for the hypothesis that at least one of the two asset price series has a jump. This test fills in the gap between a univariate jump test and a bivariate cojump test that requires both assets to jump. We show that under the null hypothesis of no jump, the test statistic has a limiting variance-gamma distribution. The alternative hypothesis reflects the hybrid nature of the bivariate test. The test statistic has the same divergence rate as the univariate Lee and Mykland (2012) test in the case of a single-asset jump, and a faster $n^{1/2}$ rate in the case of a cojump, which is the same as the cojump test of Bibinger and Winkelmann (2015). Conditional on the rejection of the bivariate jump test, the second step applies the univariate Lee-Mykland test to the spread of the two processes to examine equal event returns. The null hypothesis of equal event returns signals a cojump of the same size in the nominal and inflation-indexed bond yields but no jump in the break-even inflation, and hence cojump anchoring.

We apply the two-step procedure to five-year and ten-year government bond data of the U.S., Germany, and U.K. to test for cojump anchoring at their respective central banks' monetary policy announcement times. All announcements from January 2015 to March 2020 by the Federal Reserve, European Central Bank, and Bank of England have been analyzed. We find that among all announcements that contain unexpected news arrival to the market, cojump anchoring is prevailing in the U.S. and Euro Area break-even inflation. The U.K. break-even inflation exhibits the most frequent jumps. The five-year five-year forward rates reveal some commonalities across the three countries. Around 60% of market-moving announcements are accompanied by cojump anchoring, while the remaining 40% lead to jumps in the break-even inflation in the five-year five-year forward rates. We also examine the percentages of positive and negative jumps in the break-even inflation, and their average jump sizes. As all three central banks aim to push the inflation expectation up towards the 2% target rate, ECB appears to be the most successful in achieving this goal in that they have the largest jump size and highest percentage of positive jumps at ten-year and five-year five-year forward rates.

The methodology proposed in this paper is motivated by the empirical problem of investigating changes in the nominal and inflation-indexed government bond yields, but can also be applied in other contexts. Although our testing procedure specifies the second step conditional on the rejection of no jump in the first step bivariate jump test, the two tests in the two steps are in fact independent and can be used separately. The notion of *cojump anchoring* also exists in other financial applications. For example, stocks cross-listed in different stock exchanges are expected to have the same movements under the different listings. We can use the cojump anchoring testing procedure to examine the synergies between them, especially when at the time when company announcements are released. The cojump anchoring test provides an alternative view of price discovery across markets. We leave this for future research.

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A Assumptions

First, we are more precise about the underlying semimartingale model. We consider (1) on some filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$. The jumps J_t in (1) are split into compensated (small) jumps and finitely many large jumps:

$$J_t = \int_0^t \int_{\mathbb{R}} \delta(s,z) \mathbb{1}_{\{|\delta(s,z)|\leq 1\}} (\mu - \nu)(ds, dz) + \int_0^t \int_{\mathbb{R}} \delta(s,z) \mathbb{1}_{\{|\delta(s,z)|>1\}} \mu(ds, dz), \tag{21}$$

with the jump size function δ, defined on $\Omega \times \mathbb{R}_+^2 \times \mathbb{R}^2$, and the Poisson random measure μ, which is compensated by $\nu(ds, dz) = \lambda(dz) \otimes ds$ with a σ-finite measure λ. For the smoothness of the elements of the drift $b_t^{(i)}$ and $\sigma_t^{(i,j)}$, $i, j = a, b$ of spot squared volatility $\Sigma_t = \sigma_t \sigma_t'$, we require

Assumption 1 *In (1), for assets $i, j = a, b$, the drift $(b_t^{(i)})_{t\geq 0}$ is a locally bounded process. The volatilities never vanish, $\inf_{t\in[0,1]} \sigma_t^{(i,i)} > 0$ almost surely. For all $0 \leq t + s \leq 1$, $t \geq 0$, some constants $C_n, \tilde{C}_n > 0$, some $\beta > 1/2$ and for a sequence of stopping times T_n increasing to ∞, we have that*

$$\left| \mathbb{E}\left[\sigma^{(i,j)}_{(t+s)\wedge T_n} - \sigma^{(i,j)}_{t\wedge T_n} \,|\mathcal{F}_t\right] \right| \leq C_n \, s^{\beta} \,, \tag{22}$$

$$\mathbb{E}\Big[\sup_{t\in[0,s]} |\sigma^{(i,j)}_{(t+t)\wedge T_n} - \sigma^{(i,j)}_{t\wedge T_n}|^2 \Big] \leq \tilde{C}_n \, s \,. \tag{23}$$

We impose the following regularity conditions on the jumps

Assumption 2 *Assume for the predictable function δ in (21) that $\sup_{\omega,x} |\delta(t,x)|/\gamma(x)$ is locally bounded with a non-negative deterministic function γ that satisfies*

$$\int_{\mathbb{R}^2} (\gamma^r(x) \wedge 1) \lambda(dx) < \infty \,, \tag{24}$$

with jump activity index r, $0 \leq r < 4/3$.

B Proofs

Proof of Proposition 2.1. This proof fills the missing gaps of the proof of Proposition 3.1 of Bibinger et al. (2019) for our purposes. We state here only the crucial extensions of the covariance. Consistency and mixed normality of the pre-average estimator in the general semimartingale setting for $d = 1$ is shown in Bibinger et al. (2019), and carry over to the bivariate setting. Hence the missing part which proves Proposition 2.2 is the covariance between the pre-average jumps of two assets $i, j = a, b$, $i \neq j$ at a time point τ We rewrite the vector of pre-averaged returns of the observed price processes in terms of increments,

$$M_n^{-1} \left(\sum_{k=0}^{M_n-1} \tilde{P}_{\lceil \tau n \rceil + k} - \sum_{k=-M_n}^{-1} \tilde{P}_{\lceil \tau n \rceil + k} \right)$$
$$= \sum_{k=1}^{M_n-1} \Delta \tilde{P}_{\lceil \tau n \rceil + k} \frac{M_n - k}{M_n} + \sum_{k=0}^{M_n-1} \Delta \tilde{P}_{\lceil \tau n \rceil - k} \frac{M_n - k}{M_n} \,, \tag{25}$$

with $\Delta\tilde{P}_{\lceil\tau n\rceil}=\tilde{P}_{\lceil\tau n\rceil}-\tilde{P}_{\lceil\tau n\rceil-1}$.

$$\begin{aligned}\mathrm{Cov}\Bigg[&\sum_{k=1}^{M_n-1}\Delta\tilde{P}^{(a)}_{\lceil\tau n\rceil+k}\frac{M_n-k}{M_n}+\sum_{k=0}^{M_n-1}\Delta\tilde{P}^{(a)}_{\lceil\tau n\rceil-k}\frac{M_n-k}{M_n}\\ &,\sum_{k=1}^{M_n-1}\Delta\tilde{P}^{(b)}_{\lceil\tau n\rceil+k}\frac{M_n-k}{M_n}+\sum_{k=0}^{M_n-1}\Delta\tilde{P}^{(b)}_{\lceil\tau n\rceil-k}\frac{M_n-k}{M_n}\Bigg]\\ &=\sum_{k=1}^{M_n-1}\mathrm{E}\left[\Delta\tilde{P}^{(a)}_{\lceil\tau n\rceil+k}\Delta\tilde{P}^{(b)}_{\lceil\tau n\rceil+k}\right]\left(1-\frac{k}{M_n}\right)^2\\ &+\sum_{k=0}^{M_n-1}\mathrm{E}\left[\Delta\tilde{P}^{(a)}_{\lceil\tau n\rceil-k}\Delta\tilde{P}^{(b)}_{\lceil\tau n\rceil-k}\right]\left(1-\frac{k}{M_n}\right)^2,\end{aligned}$$

with uncorrelated increments on disjoint intervals. We separately study the asymptotic behavior of the components of $\Delta\tilde{P}_{\lceil\tau n\rceil}$. With the notation $\Delta^n_{\lceil\tau n\rceil-k}X=X_{(\lceil\tau n\rceil-k)/n}-X_{(\lceil\tau n\rceil-(k-1))/n}$, $\Delta\tilde{P}_{\lceil\tau n\rceil}=\Delta^n_{\lceil\tau n\rceil}b+\Delta^n_{\lceil\tau n\rceil}C+\Delta^n_{\lceil\tau n\rceil}J+\Delta\epsilon_{\lceil\tau n\rceil}$, with drift, b_t, Brownian component, C_t, jumps, J_t and noise, ϵ_t. The higher order n of the drift part allows us to neglect the drifts. For the Brownian component, Itô isometry, $\mathrm{E}[\int_0^t\sigma_s^{(a,a)}dW_s^{(a)}\int_0^t\sigma_s^{(b,b)}dW_s^{(b)}]=\int_0^t\mathrm{E}[\sigma_s^{(a,a)}\sigma_s^{(b,b)}]\rho_s^{(a,b)}ds$, and the smoothness of the volatility and correlation imply that

$$\begin{aligned}\mathrm{E}\left[\Delta^n_{\lceil\tau n\rceil+k}C^{(a)}\Delta^n_{\lceil\tau n\rceil+k}C^{(b)}|\mathcal{F}_\tau\right]&=\mathrm{E}\left[\int_{((\lceil\tau n\rceil+(k-1))/n)}^{((\lceil\tau n\rceil+k)/n)}\sigma_s^{(a,b)}ds|\mathcal{F}_{t_\tau}\right]+\mathcal{O}_P(n^{-2})\\ &=\frac{\rho_\tau^{(a,b)}\sigma_\tau^{(a)}\sigma_\tau^{(b)}}{n}+\mathcal{O}_P\left(\sqrt{\frac{M_n}{n}}n^{-1}\right)\end{aligned}$$

for $k=1,...,M_n-1$. Similarly, we obtain to the left of τ

$$\begin{aligned}\mathrm{E}\left[\Delta^n_{\lceil\tau n\rceil-k}C^{(a)}\Delta^n_{\lceil\tau n\rceil-k}C^{(b)}|\mathcal{F}_\tau\right]&=\mathrm{E}\left[\int_{((\lceil\tau n\rceil-(k-1))/n)}^{((\lceil\tau n\rceil-k)/n)}\sigma_s^{(a,b)}ds|\mathcal{F}_{t_\tau}\right]+\mathcal{O}_P(n^{-2})\\ &=\frac{\rho_{\tau-}^{(a,b)}\sigma_{\tau-}^{(a)}\sigma_{\tau-}^{(b)}}{n}+\mathcal{O}_P\left(\sqrt{\frac{M_n}{n}}n^{-1}\right).\end{aligned}$$

The increments in *iid* noise contribute

$$\begin{aligned}\mathrm{E}\left[\Delta\epsilon^{(a)}_{\lceil\tau n\rceil-k}\Delta\epsilon^{(b)}_{\lceil\tau n\rceil-k}|\mathcal{F}_\tau\right]&=\mathrm{E}\left[(\epsilon^{(a)}_{\lceil\tau n\rceil-k}-\epsilon^{(a)}_{\lceil\tau n\rceil-(k-1)})(\epsilon^{(b)}_{\lceil\tau n\rceil-k}-\epsilon^{(b)}_{\lceil\tau n\rceil-(k-1)})\right]\\ &=2\eta^{(a,b)}.\end{aligned}$$

Finally, in conjunction with the identities

$$\begin{aligned}\sum_{k=1}^{M_n-1}\left(1-\frac{k}{M_n}\right)^2&=\frac{1}{3}M_n-\frac{1}{2}+\frac{1}{6}M_n^{-1},\\ \sum_{k=0}^{M_n-1}\left(1-\frac{k}{M_n}\right)^2&=\frac{1}{3}M_n+\frac{1}{2}+\frac{1}{6}M_n^{-1},\end{aligned}$$

we obtain the asymptotic covariance of event returns of asset a and b:

$$\sqrt{M_n}\mathrm{E}\left[\Delta\hat{P}^{(a)}_{\lceil\tau n\rceil}\Delta\hat{P}^{(b)}_{\lceil\tau n\rceil}\right] \to \left(\frac{\rho^{(a,b)}_{\tau}\sigma^{(a)}_{\tau}\sigma^{(b)}_{\tau}}{3} + \frac{\rho^{(a,b)}_{\tau-}\sigma^{(a)}_{\tau-}\sigma^{(b)}_{\tau-}}{3}\right) c + 2\eta^{(a,b)}. \tag{26}$$

■

Proof of Proposition 2.2. The distributional result in (12) is a direct consequence from Proposition 2.1, which implies for asset a and b that both $n^{1/4}\Delta\hat{P}^{(i)}_{\lceil\tau n\rceil}(\Gamma^{(i,i)}_{\tau})^{-1/2}$, $i = a, b$, are for $n \to \infty$ standard normally distributed under the null hypothesis of no jumps. Let $d\tilde{W}$ be the increment of a Brownian motion defined on a canonical orthogonal extension of the original probability space and rewrite

$$\Delta\hat{P}^{(i)}_{\lceil\tau n\rceil}(\Gamma^{(i,i)}_{\tau})^{-1/2} = \rho\Delta\hat{P}^{(j)}_{\lceil\tau n\rceil}(\Gamma^{(j,j)}_{\tau})^{-1/2} + n^{1/4}\sqrt{1-\rho^2}d\tilde{W}, \tag{27}$$

with $\rho = \Gamma^{(i,j)}_{\tau}(\Gamma^{(i,i)}_{\tau}\Gamma^{(j,j)}_{\tau})^{-1/2}$ the correlation of event returns of asset $i, j = a, b$. The $n^{1/4}$ scaling in (27) is needed to balances the order of the second and first summand. Plugging (27) into the test statistic (10), we obtain

$$\begin{aligned} &T^n(\tau; \Delta\tilde{P}^{(i)}_1, ..., \Delta\tilde{P}^{(i)}_n; \Delta\tilde{P}^{(j)}_1, ..., \Delta\tilde{P}^{(j)}_n) \\ &\quad = \sqrt{n}\,\rho\left(\frac{\Delta\hat{P}^{(i)}_{\lceil\tau n\rceil}}{\sqrt{\Gamma^{(i,i)}_{\tau}}}\right)^2 + n^{3/4}\sqrt{1-\rho^2}\frac{\Delta\hat{P}^{(i)}_{\lceil\tau n\rceil}}{\sqrt{\Gamma^{(i,i)}_{\tau}}}d\tilde{W}. \end{aligned} \tag{28}$$

The transformation highlights that $T^n(\cdot)$ is under the null hypothesis of no jumps a normal variance-mean mixture where the mixing density is a χ^2 with one degree of freedom. The corresponding probability density is therefore the variance-gamma distribution with parameters defined by (28); see Cui et al. (2016).

■

The variance gamma distribution $VG(1, \rho, \sqrt{1-\rho^2}, 0)$ in (12) has the explicit form

$$f(x) = \frac{1}{\pi\sqrt{1-\rho^2}}\exp\left(\frac{\rho x}{1-\rho^2}\right)K_0\left(\frac{|x|}{1-\rho^2}\right), \quad x \in \mathbb{R}, \tag{29}$$

with $K_0(\cdot)$ the modified Bessel function of the second kind of order zero.

C Empirical results with 2-minute jump window

Table A.1: Rejections of the bivariate jump test in the nominal and inflation-indexed bonds (LM2) and rejections of equal jump size (1for1) at monetary policy announcements in 2015–2020.

Maturity	Fed		ECB		BoE	
	LM2	1for1	LM2	1for1	LM2	1for1
5Y	24 (0.55)	6 (0.25)	9 (0.20)	1 (0.11)	14 (0.27)	7 (0.50)
10Y	18 (0.41)	5 (0.28)	7 (0.16)	1 (0.14)	16 (0.31)	9 (0.56)
5Y5Y	14 (0.32)	5 (0.36)	5 (0.11)	1 (0.20)	16 (0.31)	12 (0.75)

The bivariate Lee-Mykland (LM2) jump test is applied to the nominal and inflation-indexed bonds. The univariate Lee-Mykland test (1for1) is applied to the break-even inflation, conditional on the rejection of LM2. We use a 1% level of significance and a 2-minute jump window at the announcement time to allow for delayed or gradual adjustments. The tests are implemented with the same choice of tuning parameters as in the simulations.

Table A.2: Average jump size in the break-even inflation in basis points at monetary policy announcements in 2015–2020.

Maturity	Fed		ECB		BoE	
	BEI(+)	BEI(-)	BEI(+)	BEI(-)	BEI(+)	BEI(-)
5Y	0.95 (0.83)	−0.65 (0.17)	−− (0.00)	−1.43 (1.00)	0.43 (0.29)	−0.74 (0.71)
10Y	0.69 (0.60)	−0.65 (0.40)	3.04 (0.25)	−− (0.00)	3.00 (0.33)	−0.84 (0.67)
5Y5Y	1.10 (0.60)	−1.58 (0.40)	6.44 (0.60)	−− (0.00)	4.80 (0.33)	−1.44 (0.67)

The table shows the average jump return in basis points of break even inflation at monetary policy announcements. In parenthesis below is the share of positive vs. negative jumps in break-even inflation. Jump sizes are estimated using 2-minute window at the announcement times.

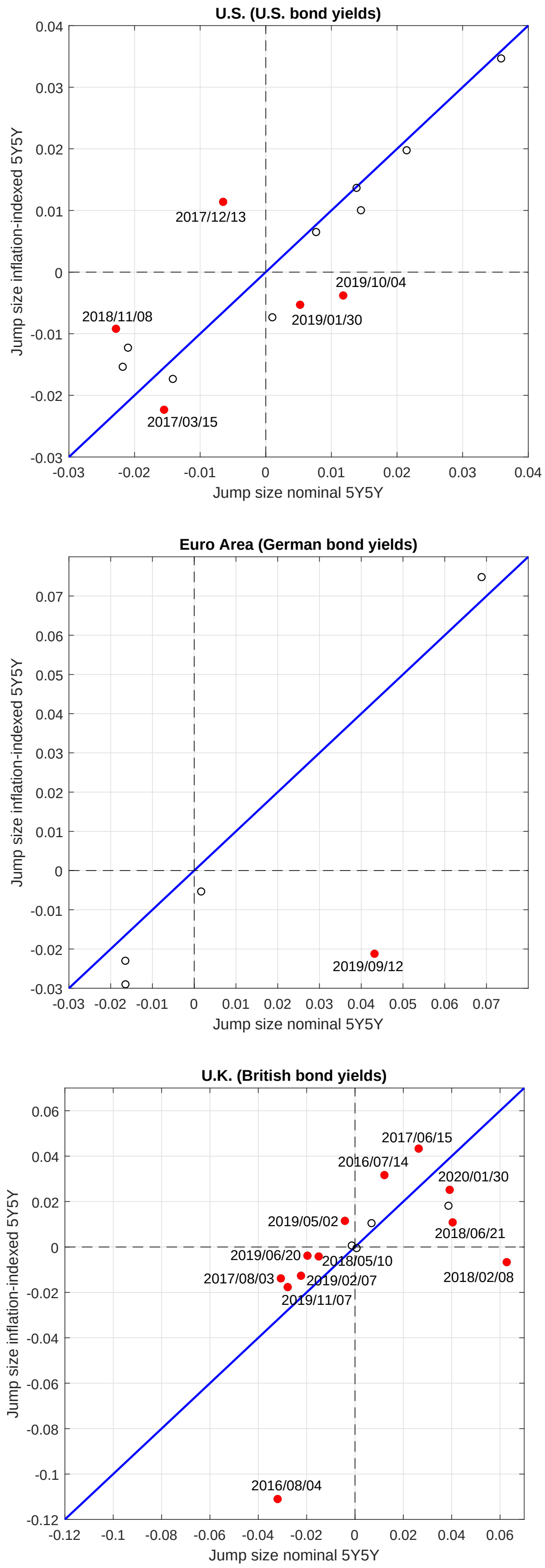

Figure A.1: Jump sizes of the nominal and inflation-indexed five-year five-year forwards. Dots indicate monetary policy event returns that reject the null of the bivariate jump test. Rejection of the one-for-one relation of nominal and inflation-indexed yields (cojump anchoring) is highlighted in red and denoted by the date of the policy announcement. The event returns are estimated using 25-minute jump window. The one-for-one relation is indicated by the 45-degree line.

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