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Experimental demonstration of measurement-device-independent measure of quantum steering

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Within the framework of quantum refereed steering games, quantum steerability can be certified without any assumption on the underlying state nor the measurements involved. Such a scheme is termed the measurement-device-independent (MDI) scenario. Here, we introduce a measure of steerability in an MDI scenario, i.e., the result merely depends on the observed statistics and the quantum inputs. We prove that such a measure satisfies the convex steering monotone. Moreover, it is robust against not only measurement biases but also losses. We also experimentally estimate the amount of the measure with an entangled photon source. As two by-products, our experimental results provide lower bounds on an entanglement measure of the underlying state and an incompatible measure of the involved measurement. Our research paves a way for exploring one-side device-independent quantum information processing within an MDI framework.

INTRODUCTION

Entanglement[1], steerability[2], and Bell nonlocality[3] are three types of quantum correlations which play essential roles in quantum cryptography, quantum teleportation, and quantum information processing[4–6]. The fact that quantum steering is treated as an intermediate quantum correlation between entanglement and nonlocality leads to a hierarchical relation among them. That is, all nonlocal states are steerable, and all steerable states are entangled, but not vice versa[7–9]. During the past decade, there have been many significant experimental works[10–16] and various theoretical results on quantum steering[17–22], including the correspondence with measurement incompatibility[23–27], one-way steering[28,29], temporal steering[30–34], continuous-variable steering[35–37], and measures of steering[38–43].

Bell nonlocality enables one to perform the so-called device-independent (DI) quantum information processing[5,44–47], i.e., one makes no assumption on the underlying state nor the measurements performed. From the hierarchical relation[7], it naturally leads to the fact that a Bell inequality can be treated as a DI entanglement witness. Nevertheless, not all entangled states can be detected by using a Bell inequality violation[48]. Recently, based on Buscemi's semi-quantum nonlocal games[49], Branciard et al.[50] proposed a collection of entanglement witnesses in the so-called measurement-device-independent (MDI) scenario. Compared with the standard DI scenario, there is one more assumption in an MDI scenario: the input of each detector has to be a set of tomographically complete quantum states instead of real numbers. Such a simple relaxation leads to that all entangled states can be certified by the proposed MDI entanglement witnesses[49,50]. This characterization gives rise to the recent works providing frameworks for MDI measures of entanglement[51–54],

non-classical teleportation[55], and non-entanglement-breaking channel verification[56–58].

Recently, Cavalcanti et al.[59] introduced another type of nonlocal game, dubbed as quantum refereed steering games (QRSGs). In each of such games, one player, denoted as Alice, is questioned and answers with real numbers, while the other player, saying Bob, is questioned with (isolated) quantum states but still answers with real numbers. They showed that there always exists a QRSG with a higher winning probability when the players are correlated by a steerable state[59]. Later, Kocsis et al.[60] experimentally proposed a QRSG and verified the steerability for the family of two-qubit Werner states in such a scenario, which is also referred to as an MDI scenario. Moreover, such a QRSG scenario can be used to generate the private random number by maximal violation of the higher dimensional steering inequality under the MDI framework[61,62].

Here we consider a variant of QRSGs, by which we propose the MDI steering measure (MDI-SM) of the underlying unknown steerable resource without accessing any knowledge of the involved measurements. We show that the MDI-SM is a standard measure of steerability, i.e., a convex steering monotone[41], by proving that it is equivalent to the previously proposed measures: the steering robustness[38] and the steering fraction[40]. Therefore, our proposed measure not only coincides with the degree of steerability of the underlying steerable resource, but also quantifies the degree of entanglement of the shared quantum state[38] and incompatibility of the measurements involved[23,26,63]. Furthermore, MDI-SM can be computed via a semidefinite program. We also show the MDI-SM is robust, in the sense that it can detect steerability in the presence of detection losses and biases[50–53].

Finally, we experimentally estimate the degree of steerability of the family of two-qubit Werner states in an MDI scenario. We consider that Alice performs three qubit-measurements in the mutually unbiased bases (MUBs) since they can be used to demonstrate the strongest steerability to Bob when Alice has three measurement settings[39]. On the other hand, Bob performs the Bell-state measurement (BSM) on his part of the state and the quantum inputs. Based on the observed correlations, the steerability of the family of two-qubit Werner states are quantified by solving a semidefinite program. As mentioned before, the experimental data naturally bounds the degree of entanglement of the underlying state, and the amount of measurement incompatibility of Alice's measurements. Compared with the previous experimental works[53,60,64–66] in the MDI scenarios, our method not only certifies the existence of entanglement and measurement incompatibility, but also bounds these quantities. Moreover, our experimental result roughly relates with the probabilities of successful subchannel discrimination in the MDI scenario[38,67].

RESULTS

MDI measure of steerability

Through this work, we assume that all quantum states act on a finite dimensional Hilbert space $\mathcal{H}$. The sets of density matrices and operators acting on $\mathcal{H}$ are denoted by $D(\mathcal{H})$ and $L(\mathcal{H})$, respectively. We denote the index sets of a finite number of elements by $\mathcal{A}$, $\mathcal{B}$, $\mathcal{X}$, and $\mathcal{Y}$. The probability of a specific index, say $a \in \mathcal{A}$, is denoted by $p(a)$.

In the MDI steering scenario, we consider two spatially separated parties, Alice and Bob, sharing a quantum state $\rho_{AB} \in D(\mathcal{H}_A \otimes \mathcal{H}_B)$ (see Fig. 1). During each round of the experiment, Alice receives a classical input $x \in \mathcal{X}$ and performs the corresponding measurement on her system with an outcome $a \in \mathcal{A}$. On the other hand, Bob performs a joint measurement on his system and a trusted input quantum state $\tau_y \in D(\mathcal{H}_{B_0})$, with $y \in \mathcal{Y}$. We note that the trustiness represents the state is well prepared and there is no side channel to transmit the state information. Their joint probability distributions can be expressed as: $p(a, b|x, \tau_y) = \mathrm{Tr}\left[(E_{a|x} \otimes E_b)(\rho_{AB} \otimes \tau_y)\right] \quad \forall a, b, x, y$, where $\{E_{a|x}\}_a \in L(\mathcal{H}_A)$ and $\{E_b\}_b \in L(\mathcal{H}_B \otimes \mathcal{H}_{B_0})$ are the positive-operator valued measurements (POVM) (i.e., the general quantum measurements) describing Alice's and Bob's measurement with the corresponding outcomes $\{a\}$ and $\{b\}$, respectively.

Within the framework of the resource theory of quantum steering[41], we concern more about the underlying assemblage[18] Bob receives rather than the shared quantum state. That is, we describe the obtained correlation by Bob's joint measurement $\{E_b\}$ on the quantum inputs $\{\tau_y\}$ and the assemblage $\{\sigma_{a|x}\}$:

$$p(a, b|x, \tau_y) = \mathrm{Tr}\left[E_b(\sigma_{a|x} \otimes \tau_y)\right]. \tag{1}$$

An assemblage $\{\sigma_{a|x}\}$ is a set of subnormalized quantum states defined by $\sigma_{a|x} = \mathrm{Tr}_A(\rho_{AB}E_{a|x} \otimes \mathrm{id})$[18], which includes both the information of Alice's marginal statistics $p(a|x) = \mathrm{tr}(\sigma_{a|x})$ and the normalized states $\hat{\sigma}_{a|x} = \sigma_{a|x}/p(a|x) \in D(\mathcal{H}_B)$ Bob receives. Here, id is the identity operator. The free state of the quantum steering (denoted as unsteerable assemblage) is the assemblage admitting a local-hidden-state (LHS) model[7], described by a deterministic strategy $D(a|x, \lambda)$ and pre-existing (subnormalized) quantum states $\{\sigma_\lambda\}$, such that $\sigma_{a|x} = \sigma_{a|x}^{US} = \sum_\lambda D(a|x, \lambda)\sigma_\lambda \, \forall a, x$. In particular, the set of all unsteerable assemblages LHS forms a convex set; consequently, for a given steerable assemblage $\{\sigma_{a|x}^S\}$, there always exists a set of positive semidefinite operators $\{F_{a|x} \succcurlyeq 0\}$, called a steering witness, such that $\mathrm{Tr}\sum_{a,x}F_{a|x}\sigma_{a|x}^S > \alpha$, while $\mathrm{Tr}\sum_{a,x}F_{a|x}\sigma_{a|x}^{US} \leq \alpha \quad \forall\{\sigma_{a|x}^{US}\} \in LHS$[16,18,38,39,42], where $\alpha :=$
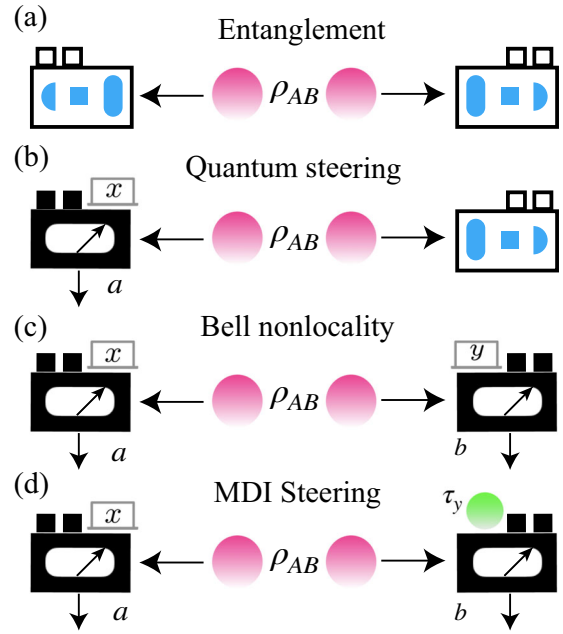

Fig. 1 Schematic illustration of the entanglement, quantum steering, Bell nonlocality, and MDI steering scenarios. A pair of entangled photons ρ_{AB} (pink balls) are shared between two spatially separated parties: Alice and Bob. They verify whether they share the entanglement, steering, and nonlocal resource by violating the entanglement witness, steering inequality, and Bell inequality, respectively. **a** In the entanglement certification task, Alice and Bob both perform characterized measurements (transparent box). **b** In the quantum steering scenario, one party performs uncharacterized measurements (black box) according to the classical input $\{x\}$, while the other party performs a set of characterized measurements. **c** In Bell nonlocality, Alice (Bob) receives the classical input $\{x\}$ ($\{y\}$) and returns the outcomes $\{a\}$ ($\{b\}$) with uncharacterized measurements. **d** In the MDI steering scenario, Bob's classical input $\{y\}$ of the steering scenario is replaced with quantum inputs $\{\tau_y\}$, removing the necessity of trustiness of the measurement device.

$\max_{\{\sigma_{a|x}^{US}\} \in LHS} \mathrm{Tr}\sum_{a,x}F_{a|x}\sigma_{a|x}^{US}$ is the local bound of the steering witness.

In what follows, we will construct the MDI-SM by using the aforementioned existence of a steering witness for any steerable assemblage. We start by considering a variant of QRSGs. Indeed, Eq. (1) can be treated as correlations obtained in a variant of QRSGs with steerable assemblages being a resource. We stress that, in the standard QRSGs, one instead treats a set of steerable states as a resource in such a game. These two resources are inequivalent because one can obtain the same assemblage from different states and measurements. With this, we define a payoff associated exclusively to a single Bob's outcome ($b = 1$) as

$$\mathcal{W}(\mathbf{P}, \beta) = \sum_{a,x,y} \beta_{a,1}^{x,y} p(a, 1|x, \tau_y), \tag{2}$$

where $\mathbf{P} := \{p(a, 1|x, \tau_y)\}$ is the experimentally observed statistics from an assemblage $\{\sigma_{a|x}\}$ based on Eq. (1) and $\beta := \{\beta_{a,1}^{x,y}\}$ is a set of real coefficients.

With the above definition, we prove that, given any steerable assemblage, there always exists a set of real coefficients β, such that the payoff $\mathcal{W}(\mathbf{P}, \beta)$ is strictly higher than those obtained from unsteerable assemblages. Details of the proof are given in Supplementary Note 1. In other words, the payoff $\mathcal{W}(\mathbf{P}, \beta)$ is effectively the same as the standard steering witness, in the sense that all steerable assemblages can be faithfully verified by a properly chosen $\mathcal{W}(\mathbf{P}, \beta)$. We note that the witness $\mathcal{W}(\mathbf{P}, \beta)$ can be seen as a generalization of a standard Bell inequality (see ref. [50]

for a similar formulation in the entanglement scenario), and is used to generalize the result of ref. [60], wherein the family of two-qubit Werner states is explicitly considered.

Now we stand in the position to introduce the MDI-SM for an unknown assemblage $\{\sigma_{a|x}\}$, denoted by

$$\mathcal{S}_1 := \max\{\mathcal{W}_1 - 1, 0\}, \tag{3}$$

with

$$\mathcal{W}_1 := \sup_{\beta, \mathbf{P}} \frac{\mathcal{W}(\mathbf{P}, \beta)}{\mathcal{W}_{\mathrm{LHS}}(\beta)}, \tag{4}$$

where $\mathcal{W}_{\mathrm{LHS}}(\beta) = \sup_{\mathbf{P} \in LHS} \mathcal{W}(\mathbf{P}, \beta)$ is the local bound for a given β. The physical meaning of the proposed measure is simple and the idea is very similar to that of the nonlocality fraction[68]: if the given correlation is unsteerable (i.e., it admits an LHS model), then $\mathcal{W}(\mathbf{P}, \beta) \leq \mathcal{W}_{\mathrm{LHS}}(\beta)$, and therefore $\mathcal{S}_1 = 0$. On the other hand, if the correlation is steerable, $\mathcal{W}(\mathbf{P}, \beta) > \mathcal{W}_{\mathrm{LHS}}(\beta)$, then $\mathcal{S}_1 > 0$. In Supplementary Notes 2 and 3, we further prove that:

- $\mathcal{S}_1$ is a steering monotone since it is equivalent to the steering fraction and the steering robustness.
- The optimal $\mathbf{P} := \{p(a, 1|x, \tau_y) = \mathrm{Tr}[E_1(\sigma_{a|x} \otimes \tau_y)]\}$ in Eq. (4) is obtained when Bob's measurement is the projection onto the maximally entangled state. That is, $E_1 = \left|\Phi_+^{BB_0}\right\rangle\left\langle\Phi_+^{BB_0}\right|$, with $\left|\Phi_+^{BB_0}\right\rangle = 1/\sqrt{d_B} \sum_{i=1}^{d_B} |i\rangle \otimes |i\rangle$.

After introducing our measure of the steerability in an MDI scenario, we proceed by considering the following two practical circumstances. First, one would like to estimate the degree of steerability of a given data table without any a priori knowledge about the experimental setup. Second, as the experimental apparatuses are inevitably erroneous in practical situations, how can one estimate the degree of steerability in the absence of the optimization of Bob's measurement? These two circumstances give rise to the attempt to estimate the degree of steerability of an experimentally observed correlation $\mathbf{P}$ when lacking the knowledge about the underlying assemblage.

In the case of an inaccessible assemblage, the optimization over $\mathbf{P}$ in Eq. (4) becomes not feasible. Consequently, the alternative quantity $\mathcal{W}_1^{\mathrm{LB}}(\mathbf{P}) := \sup_\beta \frac{\mathcal{W}(\mathbf{P}, \beta)}{\mathcal{W}_{\mathrm{LHS}}(\beta)}$ is a lower bound on $\mathcal{W}_1$, and

$$\mathcal{S}_1^{\mathrm{LB}}(\mathbf{P}) := \max\{\mathcal{W}_1^{\mathrm{LB}}(\mathbf{P}) - 1, 0\} \tag{5}$$

provides a lower bound on $\mathcal{S}_1$. Trivially, the bound becomes tight when Bob's measurement is the projection onto the maximally entangled state $E_1 = \left|\Phi_+^{BB_0}\right\rangle\left\langle\Phi_+^{BB_0}\right|$. Note that even if Bob's inputs do not form a complete set, Eq. (5) still provides a valid lower bound[51]. This can be understood from the fact that the set of tomographically complete inputs is a resource for Bob to demonstrate steerability in an MDI scenario. The lack of a completeness of quantum inputs can only decrease the degree of steerability.

Furthermore, to underpin the practical viability of our measure, we stress that the maximal value of Eq. (5) is computable via a semidefinite program (see Supplementary Note 4 for details):

given $\{p(a, 1|x, \tau_y)\}$ and $\{\tau_y\}$

$$\max_{\tilde{\beta}} \sum_{a,x,y} \tilde{\beta}_{a,1}^{x,y} p(a, 1|x, \tau_y) - 1$$

$$\mathrm{s.t.} \; d \times \mathrm{id} - \sum_{a,x,y} D(a|x, \lambda)\tilde{\beta}_{a,1}^{x,y} \tau_y \succcurlyeq 0 \quad \forall \lambda \tag{6}$$

$$\sum_y \tilde{\beta}_{a,1}^{x,y} \tau_y \succcurlyeq 0 \quad \forall a, x.$$

In the above equation, $d = d_B$ is the dimension of Bob's system. This program can be performed for a given experimentally observed correlation $\mathbf{P}$. Therefore, it works well particularly when Bob's measurement is the optimal one, i.e., the projection onto the

maximally entangled state. In this case, the solution of Eq. (6) gives the exact value of the MDI-SM defined in Eq. (3).

Finally, we would like to show that the MDI-SM is robust against detection losses. To see this, we consider the average loss rate of Bob's measurement $\eta \in [0, 1]$. The observed correlation in this case is $p_\eta(a, 1|x, \tau_y) = \eta \cdot p(a, 1|x, \tau_y)$, shrinking the MDI-SM by η, i.e., $\eta \cdot \mathcal{S}_1$. As can be seen above, the shrinking quantity $\eta \cdot \mathcal{S}_1$ is still able to detect steerability in an MDI scenario with arbitrary detection losses and provide a lower bound on the steerability of the underlying assemblage (see refs. [50,53] for similar discussions in the MDI entanglement scenario).

Experimental results

In the following, we will experimentally demonstrate how to estimate, in an MDI manner, the degree of steerability of the underlying steerable resource given by Alice's three measurement settings with the two dimensional MUBs acting on the two-qubit Werner states, namely $\rho_{AB} = v|\psi^-\rangle\langle\psi^-| + \left(\frac{1-v}{4}\right) \mathrm{id}$, with visibility $0 \leq v \leq 1$, singlet state $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle)$, and id being the identity operator.

The experimental setup is schematically shown in Fig. 2. Further details are given in "Methods". Specifically, after sending the two-qubit Werner state ρ_{AB} to Alice and Bob, we obtain the set of probability distributions $\{p(a, b|x, \tau_y)\}$ [described in Eq. (1)] by which Alice performs measurements in the Pauli bases X, Y, and Z, on her part of the system, while Bob performs the joint measurement on his part of the system and his quantum inputs τ_y. Bob's tomographically complete set of quantum inputs is composed of eigenstates of the three Pauli matrices. The joint measurement performed by Bob is the BSM, i.e., the optimal measurement, so that the value of the measure $\mathcal{S}_1$ can be achieved.

Due to our experimental setup, we further show that for the underlying assemblage $\{\sigma_{a|x}\}$ being a qubit, all of the four measurement operators $\{E_b\}_{b=1,2,3,4}$ of the BSM are optimal for Bob, i.e., the produced correlation for each b leads to the maximum value of Eq. (4). Further discussions on the two-qubit case are given in Supplementary Note 5. Therefore, Eq. (5) can be modified into the following form:

$$\mathcal{S}^{\mathrm{LB}}(\mathbf{P}) := \max\left\{\frac{1}{4}\sum_{b=1}^4 \mathcal{W}_b^{\mathrm{LB}}(\mathbf{P}) - 1, 0\right\}, \tag{7}$$

where $\mathcal{W}_b^{\mathrm{LB}}(\mathbf{P}) := \sup_{\beta'} \frac{\mathcal{W}(\mathbf{P}, \beta')}{\mathcal{W}_{\mathrm{LHS}}(\beta')}$ with $\beta' := \{\beta_{a,b}^{x,y}\}_{a,x,y}$ for each b. When there is a detection bias between the four detectors of the BSM, Eq. (7) also provide a valid lower bound on the proposed measure. More specifically, consider that we have four detectors with the biased detection rates of ξ_1, ξ_2, ξ_3, and ξ_4, respectively, with $\sum_b \xi_b = 4$ and $\xi_b \geq 0$ $\forall b$. For the ideal case, $\xi_b = 1$ for all b. When there exists some bias, the observed correlation will be $\xi_b \cdot p(a, b|x, \tau_y)$. Obviously, this correlation also reveals the steerability of the underlying resource, i.e.,

$$\mathcal{S}_\xi^{\mathrm{LB}}(\mathbf{P}, \{\xi_b\}) := \max\left\{\frac{1}{4}\sum_{b=1}^4 \mathcal{W}_{b,\xi}^{\mathrm{LB}}(\mathbf{P}, \{\xi_b\}) - 1, 0\right\}$$
$$:= \max\left\{\frac{1}{4}\sum_{b=1}^4 \xi_b \cdot \sum_{axy} \beta_{a,b}^{*,x,y} p(a, b|x, \tau_y) - \xi_b, 0\right\}$$
$$\leq \max\left\{\frac{1}{4}\sum_{b=1}^4 \xi_b \mathcal{W}_b^{\mathrm{LB}}(\mathbf{P}) - 1, 0\right\}$$
$$= \mathcal{S}^{\mathrm{LB}}(\mathbf{P}), \tag{8}$$

where $\beta_{a,b}^{*,x,y}$ is the optimal set of coefficients for the biased correlation $\xi_b \cdot p(a, b|x, \tau_y)$.
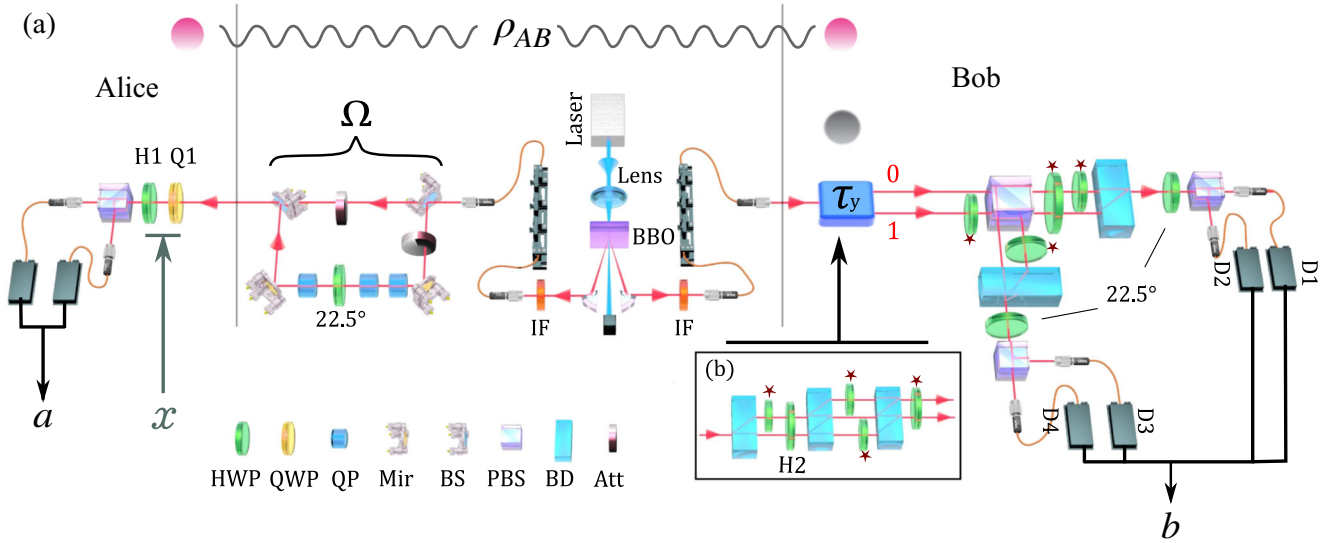
Fig. 2 Schematic drawing of the experimental setup. a The singlet state of a pair of photons $\frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle)$ is generated by a spontaneous parametric down-conversion process, where H (V) represents the horizontally (vertically) polarized direction. The Werner state is prepared by adding white noise (denoted by Ω) to the system. Then one of the photons is sent to Alice, who uses Q1, H1, and PBS to perform the measurement x. The other photon is sent to Bob with an additional qubit system τ_y encoded on the photon's path degree of freedom '0' and '1'. We emphasize the preparation of the trusted quantum system in panel **b**. Now Bob performs a complete Bell-state measurement on the equivalent two-qubit systems, i.e., measuring the polarization directions and the spatial paths of the single particle, and returns an outcome b. At the end, a set of probability distributions $\{p(a, b|x, \tau_y)\}$ is obtained to quantify the degree of steerability of the steerable resource. BBO barium borate crystal, HWP(H) half-wave plate, IF interference filter, Att attenuator, Mir mirror, QP quartz plate, QWP(Q) quarter-wave plate, PBS polarizing beam splitter, BS beam splitter, BD beam displacer. The star represents that the HWP's axis is oriented at 45˚.
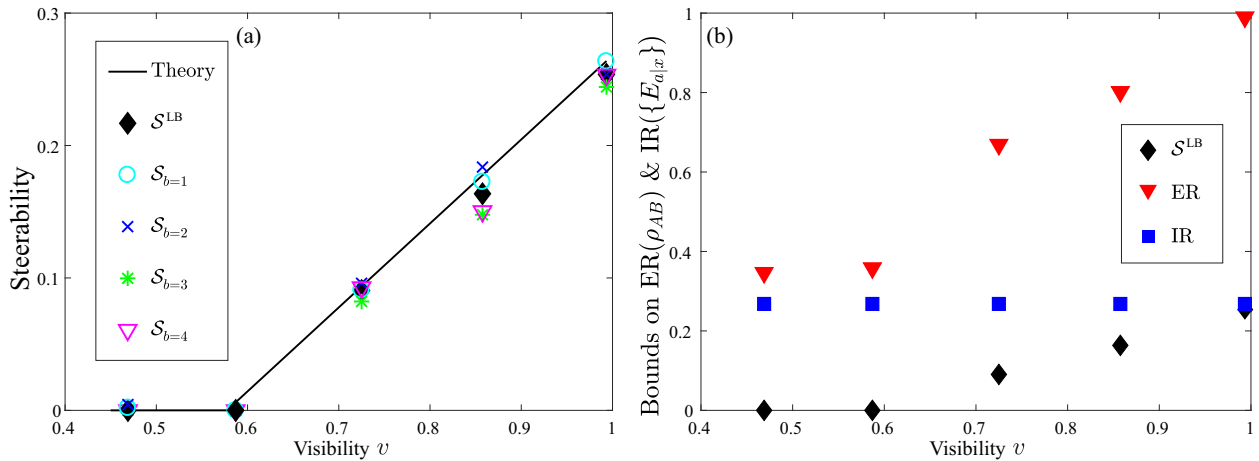

Fig. 3 Results of the MDI-SM and the estimation of entanglement and measurement incompatibility. a The MDI experimental demonstration of estimating steerability of the family of two-qubit Werner states when considering Alice has three measurement settings. The theoretical prediction of the MDI-SM is plotted in the black line. The tailored estimator $\mathcal{S}^{LB}(\mathbf{P})$ described in Eq. (7) for this experiment is marked as diamonds. The MDI-SM in Eq. (3) are marked using circles, crosses, stars, and open triangles. **b** MDI lower bounds on the degree of entanglement and incompatibility. The diamond symbols in **a** and **b** represent the same quantity. We use the tailored estimator $\mathcal{S}^{LB}(\mathbf{P})$ as lower bounds on the entanglement robustness (ER) of the underlying state and the incompatibility robustness (IR) of Alice's measurements. The actual values of these two quantities are represented by closed triangles and squares, respectively. By using the Monte Carlo algorithm, we obtain the standard deviations of $\mathcal{S}_b^{LB}(\mathbf{P})$ in the value around 0.007 and the standard deviations of $\mathcal{S}^{LB}(\mathbf{P})$ in the value around 0.004 for three measurement settings by error propagation.

Our experimental estimation on $\mathcal{S}_1$ is plotted in Fig. 3a. As can be seen, although $\mathcal{S}^{LB}(\mathbf{P})$ in Eq. (7) may not perform the best among the other fine-grained terms $\mathcal{S}_b^{LB}(\mathbf{P}) := \mathcal{W}_b^{LB}(\mathbf{P}) - 1$, it is the most suitable one in the sense that the variance from the theoretical prediction is the smallest. Besides, some fine-grained terms wrongly detect the existence of steerability due to the overestimation caused by the detection bias (i.e., the estimation of

steerability in Fig. 3a when the visibility is lower than $1/\sqrt{3}$). With Eq. (8), such overestimation will not occur when we use the quantity $\mathcal{S}^{LB}(\mathbf{P})$. Therefore, our estimation on the MDI-SM is robust against not only detection biases but also losses.

Except for estimating the degree of steerability of the underlying assemblage in an MDI scenario, here we show that our experimental results directly bound the degree of entanglement

ER(ρ_{AB}) of the underlying state and the degree of measurement incompatibility IR($\{E_{a|x}\}$) of Alice's measurements. We briefly recall these two quantities in Supplementary Note 6. The result is shown in Fig. 3b. The detail of the quantum state tomography to access these two quantities are also shown in Supplementary Note 7. Our results are based on the fact that the steering robustness of the assemblage SR($\{\sigma_{a|x}\}$) is a lower bound on the entanglement robustness ER(ρ_{AB})[38] and incompatibility robustness IR ($\{E_{a|x}\}$)[23,26,63]. Therefore, as $\mathcal{S}^{LB}(\mathbf{P})$ is a lower bound on the steering robustness, $\mathcal{S}^{LB}(\mathbf{P})$ is also used to provide a lower bound on ER(ρ_{AB}) and IR($\{E_{a|x}\}$).

DISCUSSION

In this work, we consider a variant of QRSGs, by which we introduce a measure of steerability in a MDI scenario, i.e., without making assumptions on the involved measurements nor the underlying assemblage. The only characterized quantities are the observed statistics and a tomographically complete set of quantum states for Bob. Through this, all steerable assemblages can be witnessed, in contrast to the fact that only a subset of steerable assemblages can be detected in the standard DI scenario. We further show that it is a convex steering monotone by proving the equivalence to the steering fraction as well as the steering robustness. Therefore, the MDI-SM provides a lower bound on the degree of entanglement of the unknown quantum state and measurement incompatibility of the involved measurements. Besides, our approach is able to detect steerability in an MDI scenario with arbitrary detection losses and provide a lower bound on the steerability of the underlying assemblage.

Moreover, we tackle two optimization problems in Eq. (4). That is, the optimal measurement and MDI steering witness used for MDI-SM are obtained, or equivalently, we obtain the optimal strategies for the variant of QRSGs. At first glance, it seems to be a difficult problem to obtain the optimal measurement, since Bob has to optimize over all possible measurements. However, we show that the projection onto the maximally entangled state is always an optimal one for any steerable resource. The optimal MDI steering witness (the variant QRSGs), on the other hand, can be efficiently computed by semidefinite programming. Finally, we provide an experimental demonstration of estimating the degree of steerability. The result also bounds the degree of entanglement, and incompatibility in an MDI scenario. We have also proposed an improved MDI-SM which decreased the effect of some detection biases between Bob's detectors.

This work also reveals some open questions: It is interesting to investigate whether our method can be modified to all steerable assemblages in a standard DI scenario with the approach recently proposed in refs. [69,70]. More recently, the DI certification of all steerable states has experimentally been implemented by self-testing an ancilla entangled pair[71]. It is also interesting to propose practical applications with the MDI scenario (or even a fully DI scheme following the work of refs. [69–71]). Since the formulation of the standard steering scenario can be applied to certify the security of quantum keys[72], one can ask if this is also the case in the MDI scenario.

METHODS

Experimental estimation of MDI-SM

The system state is encoded on the polarization (H, V) where H(V) represents the horizontally (vertically) polarized direction of the photon. Through a spontaneous parametric down-conversion process, we generate pairs of maximally entangled photons' state $\frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle)$. The Werner state is prepared by dephasing the photons to a completely mixed state with probability $(1 - v)$[73–75]. On Bob's side, a trusted device shown in Fig. 2b prepares the auxiliary qubit τ_y on the path degree of freedom of his owned photon. Note that, although we encode Bob's shared state (that

with Allice) and his quantum input in the same photon, these two states are indeed in different degrees of freedom. More specifically, these two states are prepared by different preparation devices, one for creating the bipartite quantum state ρ_{AB} while the other for generating τ_y. That is to say, in our MDI scenario under consideration, the former preparation device is not trusted while the latter is trusted.

On Alice's side, she uses the quarter-wave plate Q1, the half-wave plate H1 combined with a polarization beam splitter to perform a measurement according to the value of x, and returns the outcome a to the referee. While Bob needs to implement the optimal joint measurement, i.e., BSM on two degrees of freedom of the same particle (the polarization and the path degree of freedom), similar to the former works[53,76,77]. This method avoids the entangled measurement on two particles, which is a tough task with 50% efficiency in linear optics[78,79]. All the experimental details can be found in Supplementary Note 7. Moreover, a joint-measurement apparatus does not receive any information of the input quantum state before performing the measurement. More specifically, there is no side channel which transmits any information of the state to the measurement apparatus. Such protocol is physically and realistically more reliable than a situation where a referee prepares a trust quantum input to Bob. See Supplementary Note 7 for more experimental details.

DATA AVAILABILITY

All data not included in the paper are available upon reasonable request from the corresponding authors.

CODE AVAILABILITY

All code not included in the paper are available upon reasonable request from the corresponding authors.

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H.Y.K. and S.L.C. contributed equally to the development of the theoretical analysis and conceived the project; G.Y.X. supervised the experiment; G.Y.X. and Y.Y.Z. designed the experiment; Y.Y.Z. conducted the experiment and collected data with the help from G.Y.X.; Y.Y.Z. and H.Y.K. analyzed the experimental data with the help from G.Y.X., C.F.L., and G.C.G.; H.Y.K., S.L.C., and H.B.C. proved the theoretical results; Y.N.C. and F.N. supervised the research. All authors contributed to the writing of the manuscript.

COMPETING INTERESTS

The authors declare no competing interests.

ADDITIONAL INFORMATION

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