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The role of positivity and causality in interactions involving higher spin

Bert Schroer [1]

CBPF, Rua Dr. Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil

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Dedicated to Klaus Fredenhagen on the occasion of his 70th birthday

Abstract

It is shown that the recently introduced positivity and causality preserving string-local quantum field theory (SLFT) resolves most No-Go situations in higher spin problems. This includes in particular the Velo–Zwanziger causality problem which turns out to be related in an interesting way to the solution of zero mass Weinberg–Witten issue. In contrast to the indefinite metric and ghosts of gauge theory, SLFT uses only positivity-respecting physical degrees of freedom. The result is a fully Lorentz-covariant and causal string field theory in which light- or space-like linear strings transform covariant under Lorentz transformation.

The cooperation of causality and quantum positivity in the presence of interacting $s \geq 1$ particles leads to remarkable conceptual changes. It turns out that the presence of H-selfinteractions in the Higgs model is not the result of SSB on a postulated Mexican hat potential, but solely the consequence of the implementation of positivity and causality. These principles (and not the imposed gauge symmetry) account also for the Lie-algebra structure of the leading contributions of selfinteracting vector mesons.

Second order consistency of selfinteracting vector mesons in SLFT requires the presence of H-particles; this, and not SSB, is the raison d'être for H.

The basic conceptual and calculational tool of SLFT is the S-matrix. Its string-independence is a powerful restriction which determines the form of interaction densities in terms of the model-defining particle content and plays a fundamental role in the construction of pl observables and sl interpolating fields.

E-mail address: schroer@zedat.fu-berlin.de.

[1] Permanent address: Institut für Theoretische Physik, FU-Berlin, Arnimallee 14, 14195 Berlin, Germany.

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1. Introduction and history of the problem

The positivity property of quantum states guaranties the probabilistic interpretation of quantum theory. It enters the mathematical formalism through the identification of states with unit rays in a Hilbert space on which the quantum observables act as operators. In quantum field theory (QFT), or more generally for models with infinitely many degrees of freedom, it is often more appropriate to identify states with positive linear functionals on operator algebras. Thanks to the existence of a canonical construction[2] this formulation in terms of expectation values permits a return to the more common Hilbert space setting.

Its validity in quantum mechanics is guarantied by Heisenberg's canonical quantization of positions and momenta in conjunction with the von Neumann uniqueness theorem which insures that irreducible representations of the Heisenberg commutation relations are unitarily equivalent to the Schrödinger representation. Born's identification of the absolute square of the Schrödinger wave function with the probability density for finding a particle at a particular position connects positivity with spatial localization.

This situation undergoes significant changes in relativistic QFT where the positivity of field-quantization looses its "von Neumann protection" in the presence of higher spin $s \geq 1$. The first such clash with positivity was noticed by Gupta and Bleuler who observed that quantized *massless* vector potentials are incompatible with Hilbert space positivity. In the absence of interactions it is straightforward to restore positivity by passing from potentials to field strengths, but the use of local gauge invariance to preserve at least part of positivity in the presence of interactions leads to a loss of important physical operators and states.

This includes in particular all interacting fields which interpolate charge-carrying particles in the sense of large time scattering theory. Such *interpolating fields* play an indispensable role in connecting the causal localization- and quantum positivity-principles of QFT with observed scattering properties of particles. Their absence in quantum gauge theory (GT) is accompanied by a loss of mathematical tools of functional analysis. The proofs of structural properties as TCP and Spin&Statistics theorems use Hilbert space positivity in an essential way and have no substitute in indefinite metric Krein spaces. This reduces the use of GT to perturbative rules for dealing with indefinite metric- and ghost-degrees of freedom (the BRST formalism).

Positivity-obeying massive tensor potentials and their spinorial counterpart are provided by Wigner's unitary representation theory of positive energy particle representations of the (covering of the) Poincaré group, but they come with an increase of their short distance scale dimension[3] with spin $d_{sd} = s + 1$ which prevents their use in renormalized perturbation theory involving fields with higher spin $s \geq 1$. It turns out that this worsening of short distance behavior with increasing spin is accompanied by a m^{-s} divergence for small masses. Hence a formulation of QED in terms of positivity-maintaining point-local potentials is not possible.

[2] The "reconstruction theorem" in [1] is a special case of the more general Gelfand–Naimark–Segal ("GNS") reconstruction theorem [2].

[3] It is most conveniently obtained from property of the field's 2-ptfct $x \to \lambda x$ for $\lambda \to 0$.

In his well-known monograph Weinberg presents a systematic construction of the intertwiner functions which relate Wigner's spin s momentum space particle creation and annihilation operators $a^{\#}(p,s)$ associated with the unitary (m,s) representations with covariant pl free fields which act in the Wigner–Fock Hilbert space of the Wigner operators [3]. This interesting section in his book remained a torso since the (with increasing s) worsening short distance scale dimension of point-local fields prevents their use in renormalized perturbation theory as soon as $s \geq 1$.

In the main part of his book Weinberg uses the positivity-violating (but renormalizability-improving) gauge theoretic setting as obtained by Lagrangian quantization in which a perturbative inductive argument secures the positivity of gauge invariant operators. For this reason one does not find GT in presentations of nonperturbative QFT.

The independence of short distance dimensions of quantized gauge fields from spin/helicity is a consequence of the spin independence of the classical dimension $d_{cl} = 4$ of Lagrangians. For $s = 0, 1/2$ these fields agree with those obtained from the Wigner–Weinberg construction, but for $s \geq 1$ the equality of the short distance dimension with the classical dimension in terms of mass units ("engineering" dimension), namely $d_{sd} = d_{cl} = 1$ for integer and $3/2$ in case of half-integer spin, comes with an improvement of renormalizability at the price of the presence of unphysical degrees of freedom.

Whereas in older work [4] positivity problems for propagators of higher spin fields in GT have been at least partially addressed, more recent publications ([5], [6] and papers cited therein) are mainly concerned with classical geometric aspects of the Lagrangian gauge formalism for which these problems can be ignored.

The setting of string-local quantum field theory (SLFT) in the present article overcomes this conceptual gap between GT and constructions of fields based on Wigner's representation theory by providing a positivity maintaining causal perturbative QFT formalism which includes the important physical interpolating fields of particles whose large-time properties account for a unitary S-matrix and which are missing in GT. After almost 70 years of GT this amounts to a paradigmatic shift which does not only affect renormalized perturbation theory but also requires to extend the nonperturbative setting of "axiomatic QFT" as presented in [1].

A convenient starting point is to recall the construction of positivity obeying quantum fields Ψ_a in Weinberg's intertwiner formulation (for simplicity for massive tensor potentials):

$$\Psi_\alpha(x) = \int \Big(\sum_{s_3=-s}^{s} e^{ipx} v_{\alpha,s_3}(p) a^*(p,s_3) + \text{h.c.}\Big) d\mu_m(p) + h.c., \tag{1}$$
$$with\ \ d\mu_m(p) = \theta(p_0)\delta(p^2 - m^2) d^4p$$

The intertwiner functions $v(p)$ convert the Wigner creation/annihilation operators $a^{\#}(p,s_3)$ into covariant fields Ψ_α; their calculation uses only group theory [3]. They come with two indices, the s_3 which runs over the $2s+1$ values of the third component of the physical spin, and a tensor index $\alpha = (\mu_1, ..\mu_s)$ which refers to the $4s$ dimensional tensor representation of the Lorentz group of tensor degree s. The extension to fermions is straightforward but not needed for the problems addressed in the present work.

The momentum space Wigner creation/annihilation operators $a^{\#}(p,s_3)$, and hence also the covariant fields Ψ_α act in the Wigner–Fock Hilbert space obtained from the 1-particle Wigner representation by "second quantization".[4] Looking at the explicit form of the intertwiners and

[4] Note the difference to the standard use of "quantization" (in the words of Ed Nelson: "second quantization is a functor, whereas quantization is an art").

calculating the two-point function (2-ptfct) of Ψ one finds that the latter scales as $\lambda^{-2(s+1)}$ for $x \to \lambda x$ in the limit of small distances $\lambda \to 0$ which leads to assigning the short distance dimension $d_{sd} = s + 1$ to the point-local (pl) Ψ.

This construction via intertwiners permits much more flexibility than Lagrangian or functional integral quantization in converting the unique (m, s) Wigner operators into fields with different prescribed covariance- and causal localization-properties than quantization; includes in particular string-local (sl) covariant quantum fields with improved $(0 < d_{sd} < s + 1)$ short distance properties. Such fields are localized on causally separable (i.e. allowing relative causal positioning) semiinfinite space- or light-like strings (rays) $\mathcal{S} = x + \mathbb{R}_+ e$, $e^2 = -1$ or 0. Whereas in Weinberg's construction the covariance under Lorentz transformations is sufficient since causality, the construction of sl fields requires the *direct use of causal localization*. The resulting covariant fields extend the linear part of the pl relative causality class to sl (with pl considered as a special case of sl) and Wick-ordered products thereof constitute the nonlinear members.

This huge set of sl free quantum fields associated to an irreducible Wigner representation contains in particular a *sl tensor fields which is linearly associated with its pl counterpart*. This sl tensor field appears together with *s escort fields* with lower tensor degrees [7], [45]. Escorts are reminiscent of negative metric Stückelberg fields in gauge theory, except that they do not add unphysical degrees of freedom to the physical $a^{\#}(p, s_3)$ Wigner operators but only differ in their intertwiner functions.

Positivity and hence the unitarity of the S-matrix in the resulting string-local QFT (SLFT) is automatic[5] (no Nobel-prize worthy hard work as in gauge theory) and the chances to solve age-old infrared problems (large time scattering theory in QED, QCD confinement, ..) are significantly enhanced.[6] One prerequisite is the substitution of nonexistent positivity-maintaining pl potentials by sl counterparts and the according to the Weinberg–Witten No-Go theorem [9] missing $h \geq 1$ sl current and stress–energy tensors in [11] by suitably defined conserved sl substitutes. The smooth passing from massive sl two-point functions with $2s + 1$ degrees of freedom to their massless two-component helicity counterpart leads to a profound (indefinite metric- and ghost-free) understanding of the D-V-Z discontinuity problem [12,13].

An important step in the development of (SLFT) was the *construction of fields for the class of massless infinite spin Wigner representations* for which Yngvason's 1970 No-Go theorem excluded pl fields [14]. For this class Weinberg's group theoretic method is without avail; one rather had to resort to ideas from modular localization [34,35,15]. This paved the way for the construction of the simpler finite spin sl free fields, including the use of their short distance lowering and hence renormalization improving properties in interactions.

In the same work it was also realized that *finite* spin/helicity sl fields can also be obtained in a more direct way by integrating pl fields along semi-infinite lines. This direct construction is particularly useful for those sl potentials and their escorts which are linearly related to the pl spin $s \geq 1$ potentials. Different from the pl potentials which diverge in the massless limit, the corresponding sl potentials pass to corresponding finite helicity potentials which have no pl counterpart.

An important support for a string-extended QFT comes from work by Buchholz and Fredenhagen who used the setting of algebraic QFT [16] to show that models with particle states

[5] The causal separation properties of sl fields are more than enough for deriving linked cluster fall-off properties and insure the e-independence of the (on-shell) S-matrix.

[6] Recently Rehren showed that infinite spin fields can be obtained in terms of appropriately defined Pauli–Lubanski limits of finite spin escort fields [8].

separated by spectral gaps which fulfill certain consistency properties with respect to the local observables always contain interpolating operators localized in arbitrarily narrow spacelike cones (whose cores are strings). Perturbative SLFT is more specific by showing that in the presence of $s \geq 1$ particles positivity together with causal localizability leads to noncompact causal localization whose tightest localized covariant generating fields are string-local.

The combinatorial nature of perturbation theory per se does not require positivity and works also for gauge theory, but without positivity provided by a Wigner–Fock Hilbert space the quantum theory remains incomplete. SLFT reveals among other things that several limitations of gauge theory which are the cause of certain No-Go theorems of which the best known is the aforementioned Weinberg–Witten No-Go theorem (for a recent survey see [17]) are converted into Yes-Go statements in SLFT [11].

SLFT is the only formulation in which state-creating interpolating fields are separated from observables by spacetime localization properties. Whereas in the absence of interactions the localization of free fields associated to a Wigner representation ("kinematic localization") may be chosen at will, that of *interacting* fields in SLFT is determined by the particle content of the interacting theory: observables are pl and interpolating fields are sl.

The space- or light-like interpolating fields can be placed in spacelike separated positions which is a prerequisite for the application of the LSZ scattering theory. The particle states in which the expectation values of observables are measured are constructed in terms of suitably defined large time asymptotic limits to the vacuum. A theorem of large-time scattering theory insures that the dependence on the interpolating operator in the large time limit is contained in its vacuum-to-one particle matrixelement which is then removed by passing to the correctly normalized particle states. This implies in particular that the e-dependence of sl fields *does not affect particles and their scattering matrix*.

The so-called cluster decomposition property of correlation functions of fields plays an important role in the derivation of scattering properties. It is a consequence of a mass gap and the existence of an arbitrary large number of sl fields in relative spacelike position. This applies to spacelike strings and (with some stretch of geometric imagination) also holds for lightlike strings; it is however violated for timelike strings. In the present work the terminology "nonlocal" is avoided since its historical connotation may hinder to see that these fields can be brought into causally separated positions.

The reader is also reminded that the terminology "interaction density" instead of "interacting part of a Lagrangian" is not nitpicking; apart from interactions between $s < 1$ particles, interaction densities *constructed from sl Wigner fields* are never interacting parts of Lagrangian.

Another important point which requires attention is the fact that the kinematical sl localizations of free fields in terms of line integrals over $s \geq 1$ pl fields only serves to *construct interaction densities whose S-matrix is independent on string directions*. As mentioned the physical localization of the corresponding interacting fields is dynamical and generally different from that of their free counterpart used in the construction of the interaction density and the S-matrix.

A surprising property of SLFT is that in the presence of $s \geq 1$ particles its positivity and localization properties determine a unique model in terms of its particle content whenever such a theory exists. This is a result of the strong restriction which the string-independence of the S-matrix exerts on interaction densities.

The quest for an *intrinsic* formulation in which the umbilical cord to classical field theory provided by quantization has been cut is almost as old as QFT. In the first (still pre-renormalization) presentation of quantum electrodynamics at an international conference in 1929 [18] Pascual Jordan expressed this in the form of a plea for an intrinsic understanding of QFT which avoids the use of "(quasi)classical crutches"; a decade later his former collaborator Eugen Wigner took the first step in his famous classification of relativistic particles [19].

The second step was taken two decades later by Rudolf Haag [2] when he proposed an intrinsic formulation of QFT in terms of "causal nets of algebras" in which Wightman fields at best play the role of "coordinatizations" (in analogy to the use of coordinates in geometry).

With the arrival of the covariant formulation of quantized electrodynamics in the 50s, Jordan's dictum and its partial realization in Wigner's classification of noninteracting particles faded into the background; the new covariant computational rules of quantized electrodynamics took a firm hold and as a result the first covariant QFT was a positivity-violating gauge theory.

A somewhat unexpected aspect of these first successful calculations was the contrast between the precision of the experimentally verified perturbative results and the robustness of the calculated results against the use of quite different cutoff- and regularization-prescriptions, or even against different ways of implementing Lagrangian quantization (Gell-Mann–Low, Feynman path integrals, Bogoliubov's generating S-functional).

The presence of gauge theoretic indefinite metric degrees of freedom in interaction densities involving $s \geq 1$ particles led to conceptual problems. A formal proposal to overcome these shortcomings was made by Jordan [20]. It consisted in replacing the gauge dependent matter field[7] by the formally gauge invariant string-local composite field

$$\Psi(x) = \psi^K(x) \exp ig \int_0^\infty A_\mu^K(x_0, \ldots, x_3 + \lambda) d\lambda \tag{2}$$

where the K refers to the gauge dependent Lagrangian field which acts in an indefinite metric Krein space which in addition to physical degrees of freedom contains also indefinite metric quanta (scalar and longitudinal photons).

After the discovery of renormalized perturbation theory Mandelstam used such representation as a starting point in his attempt to construct a perturbation theory which avoids the use of potentials in favor of working directly with gauge invariant fields [22]. Subsequently Steinmann [23] studied the problem of recovering positivity by constructing such fields Ψ in higher order perturbation theory. Different proposals to recover positivity can be found in [24]. The constructions of such formally gauge invariant composite fields and their renormalization requires a lot of additional work and is of little interest unless it leads to new physical insights.

The SLFT perturbation theory in the present paper uses sl potentials with the s-independent short distance dimension $d_{sd} = 1$ which "live" in a physical Wigner–Fock particle space. The starting point is the observation that there exist sl vector potentials $A_\mu(x, e)$ localized on causally separable spacelike strings $\mathcal{S} = x + \mathbb{R}_+ e$ which together with their scalar sl "escorts" $\phi(x, e)$ are linearly related to their pl counterpart simple illustration is provided by the interaction density $L^P = A_\mu^P j^\mu$ of massive QED which is related to its pl counterpart as $A_\mu(x, e) = A_\mu^P(x) + \partial_\mu \phi(x, e)$.

Its use in an interaction density of e.g. massive QED $L^P = A_\mu^P j^\mu$ results in a relation $L^P = L - \partial^\mu \phi j_\mu$ in which the sl density $L(x, e)$ has an improved short distance dimension $d_{sd}(L) = 4$ (instead of $d_{sd}(L^P) = 5$) and accounts for the first order contribution to the (on-shell) S-matrix in the adiabatic limit $S = \int L$ to which the boundary term from $V_\mu = \phi j_\mu$ does not contribute.

This is in a nut-shell a perturbative implementation of the aforementioned abstract Buchholz–Fredenhagen theorem; it secures the existence of interpolating sl fields whose directional smear-

[7] Jordan used these fields for a pure algebraic derivation of Dirac's geometric magnetic monopole quantization [21].

ing provides the B-F operators localized in arbitrary narrow spacelike causally separable cones and insures that their large-time scattering limits results in e-independent Wigner particles and their S-matrix.

The extension of this first order L to higher orders involves time-ordered products in the interaction densities L respective L^P and leads to new powerful normalization conditions which ensure that the two different interaction densities lead to the same S-matrix. As a result of the with perturbative order growing number of counterterms, the L^P theory by itself is physically useless; but being "guided" by the sl L, V_μ pair it becomes a well-defined physically useful companion which shares not only its parameters but also its S-matrix and local observables with the $d_{sd}(L) = 4$ SLFT. Its only memory about its "unguided past" is the with perturbative order increasingly singular $d_{sd} \to \infty$ short distance dimension of its interpolating fields.

SLFT is an S-matrix theory in the sense that the particle content together with the string-independence of S determines (in all cases studied up to now uniquely) the form of the interaction density. In a second step the construction of the S-matrix is extended to that of pl and sl interacting fields.

Different from Lagrangian quantization the SLFT formalism does not prefer certain fields. All interacting fields which act in the same Wigner–Fock space and are members of the same causality class are on equal footing; which particle they interpolate depends only on the nontriviality on their vacuum-to-one-particle matrix elements.

Often new theoretical insights are the result of accidental observations. SLFT is not of this kind; what led to it is the rather deep connection of sl localization with *modular localization* theory. The terminology "string" used in quantizations of classical actions (Nambu–Goto actions, world-sheets, ..) bears no relation to the causal localization of string-local quantum fields in the present work.

A definition of causal localization which avoids such misunderstandings is that in terms of *modular localization.* In fact modular localization permits to identify *a pre-form of causal localization already within the Wigner positive energy representation space* [25] before "second quantization" converts it into the algebraic form of Einstein causality in QFT. *This idea paved the way for the construction of the QFT behind Wigner's infinite spin representation*.

Modular localization theory can be traced back to the Tomita–Takesaki modular theory of operator algebras of the 60s. It is one of a few mathematical theories to which physicists working on problems of statistical mechanics of open systems [26] made important contributions. It made its first appearance in the context with causal localization in the Bisognano–Wichmann theorem [27] which deals with modular properties of wedge-localized algebras. *Modular operator theory and modular localization requires positivity and hence cannot be applied to GT and Lagrangian quantization*.

As the result of accommodating thermal aspects and causal localization under one conceptual roof, it led to profound insights (thermal properties of "event horizons") into Hawking's black hole radiation [28]. A first survey about its history enriched by new results was presented by Borchers [29].

Modular localization also played an important role in the construction of QFTs from S-matrices of integrable models in $d = 1 + 1$ dimensions [30], [31]. Presently ideas from modular operator theory are being successfully applied to obtain a foundational understanding of entanglement entropy caused by causal localization (for a survey see [32], [33] and references therein).

Modular localization theory permits to extend Weinberg's intertwiner construction to Wigner's infinite spin representations and to obtain explicit expressions for the associated sl fields

[34–36], [37]. More recently these fields reappeared as "Pauli–Lubanski limits" of finite spin sl fields [8]. This made it possible to investigate physical properties of quantum matter through the study of its positivity-obeying causal localization structure and look for theoretical reasons why certain types of matter can not be seen in counters [38].

One should also mention a series of more recent publications [39], [40] in which covariant wave functions were constructed, but the much stronger result in the aforementioned work was overlooked. Relativistic wave equations for infinite spin appeared already in Wigner's 1948 paper ([41] 12.1–12.4). These different wave functions describe different covariant bases in Wigner's irreducible representation space. But for studying physical manifestations of matter one needs to know its causal localizability which in case of infinite spin does not follow from covariance and needs the use of modular localization theory as used in the cited 2006 papers. In this way the 1970 No-Go theorem [14] which excluded point-like localization was replaced by a sl Yes-Go theorem.

In fact the exclusion of linear pl fields is part of a more general No-Go theorem which rules out the possibility of constructing pl composites from the linear sl infinite spin fields. In its most general form the theorem excludes the existence of operator algebras localized in finite spacetime regions [42].

These theorems against pl localization of infinite spin matter may be seen as an extreme counterpart of the Weinberg–Witten No-Go theorem against the existence of higher helicity conserved currents and energy-momentum tensors. The difference is that there still exist W-W local charges, whereas in case of infinite spin there are no nontrivial operators localized in finite regions.

Recall that the raison d'être of a relativistic quantum *field* theory (for the difference between QFT and relativistic QM, see section 3 in [43]) is the realization of the "Nahewirkungsprinzip" (action in the neighborhood principle) of Faraday and Maxwell which culminated in Einstein's concept of relativistic covariance and causal localization. The positivity requirement of quantum probability turns the construction of models of QFT into a challenging problem which gauge theory did not solve.

It is the aim of this work to show how the recent SLFT formulation solves problems which have remained outside the range of GT (for a review of such problems see [17]). In [10], [11] this was already achieved for the problem behind the Weinberg–Witten (W-W) No-Go theorem [9] and the $s = 2$ van Dam–Veltman–Zakharov (D-V-Z) discontinuity [12,13]. Here we add the causality problems raised by Velo and Zwanziger (V-Z) [44].

SLFT's central point is however the presentation of a sl-based perturbation theory which in contrast to gauge theory preserves the Hilbert space positivity (no indefinite metric- and ghost-degrees of freedom) without destroying the causal separability of fields. It leads in particular to interesting different physical interpretations of interactions between vector mesons and Hermitian fields (Higgs models, but no SSB Higgs mechanism) [45].

The content of this paper is organized as follows.

The next section recalls and extends recent (partially already published) results concerning the construction of causally separable string-local free fields. It consists of 4 subsections which includes the construction of sl massless vector potentials and their canonically related massive counterpart.

The third section addresses the problem of interactions with external potentials. It is shown that the origin of the Velo–Zwanziger causality problem is the incorrect expectation that by modifying free field equations by adding linear couplings to external potential one preserves causality in the sense of causal propagation of Cauchy data. The solution of the V-Z problem has a close formal proximity to the solution of the Weinberg–Witten problem in [11].

Section 4 provides some background about modular localization. Its aim is to show that causal localization is incompatible with any form of quantization but important for understanding properties of causally localized quantum matter.

In section 5 the SLFT renormalization theory is applied to calculation of the S-matrix in various models involving vector mesons including some speculative remarks on $s \geq 2$ interactions.

Section 6 addresses problems of interacting sl fields in particular the model-dependent distinction between pl observables and sl interpolating fields.

The concluding remarks in section 7 summarize the new insights and present an outlook.

2. String-local tensor potentials and conservation laws

This section provides the kinematical prerequisites of SLFT i.e. the construction of those sl free fields which are used in later sections for the calculation of the S-matrix and interacting fields. The kinematic localization of free fields is not the same as the dynamic localization of their interacting counterparts (section 6).

2.1. Massless string-local potentials

The fact that even in the absence of interactions massless gauge potentials have no positivity-maintaining pl counterpart led to a more foundational re-thinking regarding the relation between positivity and causal localizability for which the solution of the massless infinite spin problem in terms of sl fields served as a role-model [34,35]. The finite helicity problem is simpler since in this case there exists only one covariant family of sl potentials $\hat{A}_\mu$ in the Wigner–Fock helicity Hilbert space whose field strength is the pl field strengths

$$\partial_\mu \hat{A}_\nu(x) - \partial_\nu \hat{A}_\mu(x) = F_{\mu\nu}(x) \tag{3}$$

They have the form of a semi-infinite line integrals (strings, rays)

$$\hat{A}_\mu(x) = A_\mu(x,e) := \int F_{\mu\nu}(x+\lambda e)e^\nu =: (I_e F_{\mu\nu})(x)e^\nu \tag{4}$$
$$U(a,\Lambda)A_\mu(x,e)U(a,\Lambda)^* = (\Lambda^{-1})_\mu^\nu A_\nu(\Lambda x, \Lambda e)$$

with e representing a space- time- or lightlike vector *which participate in the transformation under the homogeneous Lorentz group*. Causal localizability requires the possibility of placing an arbitrary large number of such sl fields in relative spacelike separated positions (denoted as $\times$). This excludes the timelike case but permits space- and light-like strings[8]

$$\left[A_\mu(x,e), A_\mu(x',e')\right] = 0, \ x + \mathbb{R}_0^+ e \times x' + \mathbb{R}_0^+ e' \tag{5}$$

Spacelike unit vectors e with $e^2 = -1$ are points on the $d = 1+2$ unit de Sitter space, whereas lightlike vectors e with $e^2 = 0$ may be identified with points on the two-dimensional celestial sphere. A closer examination shows that line integrals of *massless* field strengths along lightlike lines are ill-defined (see below) but well-defined (as distributions in e) for spacelike e; time-like lines would violate causal separability.

[8] This is less obvious in the lightlike case.

The derivation of nonperturbative theorems (PCT, Spin&Statistics, cluster properties, LSZ scattering theory, ..) does not need pl fields; what is important is the preservation of causal separability i.e. the fact that one can place an *arbitrary number* of sl fields into relative spacelike position.

The mathematical status of sl fields requires a more careful look at their singularity structure. For this purpose it is convenient to compute their 2-point function (2-pfct). Starting from that of the field strengths

$$\left\langle F_{\mu\nu}(x)F_{\kappa\lambda}(x')\right\rangle = \int e^{-ip(x-x')}M^{F_{\mu\nu},F_{\kappa\lambda}}(p)d\mu_0(p), \quad d\mu_m(p) = \frac{d^3p}{2\sqrt{\vec{p}^2+m^2}} \tag{6}$$

$$M^{F_{\mu\nu},F_{\kappa\lambda}}(p) = -p_\mu p_\kappa g_{\nu\lambda} + p_\mu p_\lambda g_{\nu\kappa} - p_\nu p_\kappa g_{\mu\lambda} + p_\nu p_\lambda g_{\mu\kappa}$$

and, using the fact that the λ-integration amounts to the Fourier transform of the Heavyside function and hence leads to distribution $(pe)^{-1}_{i\varepsilon} = \lim_{\varepsilon\to 0}(pe+i\varepsilon)^{-1}$ as boundary values of analytic functions, one obtains [11]

$$\left\langle A_\mu(x,-e)A_\nu(x',e')\right\rangle = \int e^{-ip(x-x')}M^{A_\mu,A_\nu}(p,e,e')d\mu_0(p) \tag{7}$$

$$M^{A_\mu,A_\nu}(p,e,e') = E_{\mu\nu}(-e,e') = -\eta_{\mu\nu} + \frac{p_\mu e_\nu}{(pe)_{i\varepsilon}} + \frac{e'_\mu p_\nu}{(pe')_{i\varepsilon}} - \frac{(ee')p_\mu p_\nu}{(pe)_{i\varepsilon}(pe')_{i\varepsilon}}$$

where the tensor $E_{\mu\nu}$ turns out to be an important building block of higher helicity 2-pfcts. The scaling degree d_{sd} is defined as the leading short distance contribution $\lambda^{-2d_{sd}}$ of the 2-ptfct under the scaling $\xi\to\lambda\xi$, $\xi = x-x'$ for $\lambda\to 0$ and can be directly read off from the large momentum behavior. Whereas $d_{sd}(F)=2$, the line integration lowers the degree to $d_{sd}(A)=1$.

A more detailed study shows that sl potentials and their 2-pfcts are well-defined as *distributions* in e, $e^2=-1$ (the unit de Sitter space) and x. All operators and correlation function are of homogeneous degree zero and hence the de Sitter differential can be written in the covariant form $d_e = de_\mu\frac{\partial}{\partial e_\mu}$. For lightlike $e's$ and $m>0$ the last term in (7) vanishes for $e'=-e$ and the distributional dependence of $A_\mu(x,e)$ on e changes to that of a function so that a directional testfunction smearing in e is not necessary. The identification of $e's$ in products of fields leads to a significant notational simplifications in perturbative calculations. The existence of momenta for which p is parallel to e excludes however massless limits lightlike strings.

For the timelike directions the denominators never vanish and no smearing is needed, but the causality requirement, namely the existence of an arbitrary number of causally separated sl fields, cannot be satisfied. Hence the choice $e=e_0=(1,0,0,0)$ leads to the nonlocal Coulomb- (or radiation-) potential with $A_0^C=0$ and spatial components

$$M^{A_i^CA_j^C} = \delta_{i,j} - \frac{p_ip_j}{\mathbf{p}^2} \tag{8}$$

"Freezing" this timelike string direction destroys the covariant transformation and one obtains a noncovariant inhomogeneous transformation law in which only the rotations and translations maintain their covariant appearance (see (12) below). Full covariance can be restored by letting the timelike direction participate in the Lorentz transformation, but the loss of causal localization remains. The Coulomb potential is used in quantum mechanics where relativistic covariance and causality play no role.

It is interesting to note that the Coulomb potential results also from *averaging a spacelike string over spatial directions in the $t=0$ plane orthogonal to the timelike e_0 vector*. There is no direct way to undo this directional averaging; one rather has to return from A^C to its covariant field strength F and obtain the associated sl potential as in (4). This directional averaging reveals a *close formal connection between the axial- and Coulomb-"gauge"*. Both potentials exist in the same Wigner–Fock helicity space, but only the covariant sl potential (4) is manifestly causal.

The use of sl potentials turns the so-called noncovariant axial- and lightcone-gauges into better manageable covariant Einstein-causal fields which act in a positivity maintaining Hilbert space.

It should be mentioned that in the literature the terminology "gauge" is used with two different meanings. In the covariant setting of QED perturbation theory it refers to a formal symmetry whose generator is a "gauge charge" which depends on unphysical indefinite metric degrees of freedom. On the other hand the Coulomb- or axial-gauge contains only the two helicity $h = \pm 1$ degrees of freedom and there is no symmetry-implementing gauge charge, although the additive contribution to the Lorentz transformation looks like a non-covariant gauge transformation (12) re-expressing the Lorentz-transformed $e = \Lambda e_0$ in terms of original e_0.

It is not the aim of this work to change historically grown terminology. Here the terminology "gauge" is exclusively used the situation in which unphysical degrees of freedom provide a covariant "gauge symmetry". Quantum gauge symmetry is not a physical symmetry (and consequently there is no physical sense in which it can be broken) but rather a formal tool to extract a physical theory as a subtheory from an unphysical formalism.

The large momentum behavior of the 2-pfct determines the short distance behavior of the field whereas the distributional behavior in e depends on the dimensionality of spacelike e-directions on which pe vanishes. The case of *lightlike* $e's$ is a bit more tricky. For massive p the pe denominator does not vanish since p and e only touch at lightlike infinity and as a result the sl fields are functions in e.

This changes in a radical way for massless p; in that case for each e there are lightlike $p's$ on which pe vanishes and as a result massless fields localized on lightlike strings do not even exist in the sense of distributions.[9] Lightlike sl fields have an interesting connection with light-cone quantization. In the massless case they reveal in a much clearer way the problematic nature of "lightcone quantization" [46].

The main purpose of this work is to offer a positivity- and causality-preserving alternative to gauge theory which avoids the use of the quantization parallelism to classical field theories by starting from Wigner's manifestly positivity-preserving particle representation theory. The important point is that spacetime localization properties already exist in the pre-form of *modular localization* within Wigner's particle theory. They can be used to construct pl or sl intertwiner functions which convert Wigner's creation and annihilation operators into covariant pl or sl free fields.

The perturbative construction of the S-matrix and of interacting sl fields does not need modular localization theory. For problems as *localization entropy [33], [32]* and nonperturbative constructions [31] its use is however indispensable. In the context of the present paper its importance is based on pinning causal localization to quantum positivity; whereas Lagrangian quantization of fields allows the presence of unphysical degrees of freedom, modular localization excludes them. More remarks on modular localization will be deferred to section 4.

The construction of sl potentials in terms of pl field strengths (4) permits an iteration to a scalar potential Φ

[9] I am indebted to Henning Rehren for drawing attention to the nonexistence of massless lightlike string localized fields.

$$A_\mu(x,e) - A_\mu(x,e') = \partial_\mu \Phi(x,e,e'), \;\; \Phi = (I_{e'} I_e F_{\mu\nu})(x) e^\mu e'^\nu \tag{9}$$

The Φ represents a field which is localized on the 2-dimensional *conic region* $\lambda e + \lambda' e', \; \lambda, \lambda' \geq 0$. In the massless limit this flux Φ is logarithmically divergent. The logarithmic divergence is expected to lead to an e, e' dependent continuous set of superselection rules which extend the Wigner–Fock helicity space.

This is reminiscent of the behavior of the exponential of a massive scalar free field in $d = 1+1$ in the massless limit[10] which played an important role in the work on "bosonization" of massless fermions and anyons [48]. In that case the massless limit of the properly mass-normalized exponentials leads to the superselection property

$$\left\langle e^{ia_1\varphi(x_1)} \ldots e^{ia_n\varphi(x_n)} \right\rangle = 0 \quad \text{if} \;\; \sum_1^n a_i \neq 0, \tag{10}$$

corresponding to a_i-"charge" conservation.

The "photon cloud" in the e-direction associated with $\exp ig\varphi$ is expected to cause a directional superselection rule which appears in the form of $e^{ig\varphi}\psi$ in the large time behavior of electric charge carrying fields and causes the modification of LSZ scattering theory. In this way one may hope to obtain a genuine spacetime understanding of the infrared momentum space recipes in [49].

The interest in this problem is also motivated by the existence of rigorous results derived from an appropriate formulation of the quantum Gauss law [50]. This theorem states that interacting electric charge-carrying operators ψ are accompanied by spacelike extended "photon clouds" whose different asymptotic conic directions correspond to a continuum of superselection sectors within the same charge-carrying sector. This is the cause a spontaneous breaking of Lorentz symmetry [51].

The existence of a continuum of superselection sectors for free photons would suggest the existence of large time asymptotic charge-carrying matter fields of the form $\psi_0 e^{ig\Phi}$ with ψ_0 a free matter field. Their large time asymptotic behavior is expected to play an important role in a future spacetime understanding of infrared properties which is outside the physical range of gauge theories.

For many applications it is useful to encode change in e (9) into changes of the Lorentz transformation law. A differential relation which is the basis for such conversion has the form ([11] Corollary 3.3)[11]

$$d_e A_\mu(x,e) = \partial_\mu u(x,e), \;\; d_e = \sum_i d_{e_i} \partial^{e_i} \tag{11}$$

where u is an exact de Sitter one-form $u = d_e \phi$. This conversion of directional de Sitter differentials into x-derivatives plays an important role in passing from interactions in the presence of a mass gap to their massless limit.

In the present context the formula for the change of $e's$ can be used to compute the additive change which is necessary in order to *maintain* the timelike e_0 direction of the Coulomb potential $A_i^C, \; A_0^C = 0$. The resulting affine transformation formula

[10] This infrared behavior was first observed in the coupling of a $d = 1+1$ current to the derivative of a massless scalar field ("infraparticle" [47]).

[11] As mentioned therein this remains well defined since sl fields and their correlation functions are homogeneous functions of degree zero in e and p.

$$U(a,\Lambda)A_i^C(x)U(a,\Lambda)^* = (\Lambda^{-1})_i{}^l A_l^C(\Lambda x + a) + (\Lambda^{-1})_i{}^\mu \partial_\mu \chi(x) \tag{12}$$

is equivalent to that obtained by starting from the Wigner helicity representation and using transverse polarization vectors [3].

A similar situation arises if one fixes an "axial" direction as e.g. $e = (0, 1, 0, 0)$. In this case the causal localizability is preserved in both descriptions. Ignoring the spacetime localization aspect and treating the axial direction as a noncovariant gauge misses the necessity of directional smearing (smearing around a point in de Sitter space) and probably contributed to the abandonment of the "axial gauge fixing". But what became a curse in the axial gauge fixing turns out to be a blessing in the covariant SLFT setting.

Covariant gauges as used in covariant perturbation theory always require the presence of ghost-extended indefinite metric BRST degrees of freedom setting which reduces the physical range. SLFT cuts the umbilical cord between perturbative Lagrangian quantization and classical gauge theory and restores positivity.

2.2. *A brief interlude, relation with concepts of algebraic QFT*

The simplest illustration of the interplay between positivity and causality is provided by the Aharonov–Bohm effect. To see this recall that Einstein causality is the statement that the algebra of operators localized in the causal complement $\mathcal{O}'$ of a spacetime region $\mathcal{O}$ belong to the commutant $\mathcal{A}(\mathcal{O})'$ algebra (the von Neumann algebra which consists of all operators which commute with $\mathcal{A}(\mathcal{O})$)

$$\begin{aligned} &\mathcal{A}(\mathcal{O}') \subseteq \mathcal{A}(\mathcal{O})' \quad \text{or} \quad \mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}(\mathcal{O}')', \quad \text{Einstein causality} \\ &\mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})', \quad \text{Haag duality} \end{aligned} \tag{13}$$

The second line defines the somewhat stronger Haag duality which states that an operator which commutes with all operators localized in the causal complement of $\mathcal{O}$ *must* belongs to $\mathcal{A}(\mathcal{O})$.

Einstein causality is a defining property of relativistic QFT, but Haag duality may be violated. In the absence of interactions such a violation can be excluded for massive QFT's but it does occur in the massless case when the $2s + 1$ spin degrees of freedom are converted into the $\pm h$ helicities. As observed in [52] (unpublished) Haag duality, which holds for simply connected spacetime regions, is violated for multiply connected regions as (genus one) tori.

In their proof the authors carefully avoid the use of gauge potentials associated to $m > 0$ massive is a property ($g \geq 1$ tori). This violation is an intrinsic property of the operator algebra generated by the field strength $F_{\mu\nu}$ of the $h = 1$ Wigner representation. But if one wants to understand this in terms of vector potentials one must use the positivity-maintaining sl potentials which preserve a somewhat hidden topological properties of Wilson loops which cause the breakdown of Haag duality while it upholds Einstein causality [45]. The indefinite metric potentials cannot distinguish between the two; only the localization in the presence of positivity is physical.

The cause of "eeriness" about the Aharonov–Bohm effect [53] (but also of its popularity) is that we erroneously interpret the intuitively accessible geometric Haag duality with the more abstract Einstein causality, thus forgetting that the latter also admits operators which have no unambiguous causal localization region (e.g. the magnetic flux through a surface with a fixed boundary). The ideal solenoid in the A-B setup closes at spacelike infinity, which in the conformal Wigner–Fock helicity world is a circle. In case of a finite tube one must place the electric circuit into a region of little magnetic backflow from north- to south-pole.

This is a strong reminder that it is not possible to separate causality from positivity and a warning not to confuse the "fake localization" of gauge dependent objects with genuine causal localization of quantum matter. It points to a potential source of misunderstanding involved in transferring the perfectly reasonable classical notion of *local* gauge symmetries to QFT by attributing a physical meaning to the formal observation that quantum gauge charges are "more local" than those corresponding to internal symmetries. It is also a reminder to rethink the physical meaning behind the terminology "gauging a model".

From (11) it follows that a Wilson loop[12] formed with $A_\mu(x,e)$ *is independent of* the choice of the direction e [45]. However it *retains a topological memory* of the string directions of the integrand which prevents a naive materialistic identification with a localization in a torus. One can choose $e's$ in such a way that this extension is spacelike with respect to any simply connected convex compact region.[13] Yet it is not possible to completely forget that the vector potential has a directional e-dependence. An elegant formulation of this $h \geq 1$ topological phenomenon directly based on field strengths and their duals in terms of "linking numbers" can be found in [54].

2.3. Massive string-local potentials

Before passing to the construction of massive sl fields it is helpful to recall the construction of their pl intertwiner functions $v(p)$ which convert the $m > 0$ Wigner creation and annihilation operators $a^\#(p,s)$ into covariant [3]. For the $s = 1$ Proca field they are the three polarization vectors $v_\mu(p,s_3)$ obtained by applying a rotation-free Lorentz boost to the spatial coordinate unit vectors. By definition they are Minkowski-orthogonal to p_μ and hence correspond to the 3 polarization vectors v obeying the completeness relation

$$\sum_{s_3=--1}^{1} v_\mu(p,s_3)v_\nu(p,s_3) = -\eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}$$
$$M^{A_\mu^P,A_\nu^P}(p) = -\pi_{\mu\nu}(p), \quad \pi_{\mu\nu}(p) = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}$$

where the $\pi_{\mu\nu}$ of the momentum space 2-pfct which also turns out to be the basic building block of all higher spin massive tensor potentials.

With a pl Proca potential A_μ^P one may associate two sl fields, the scalar sl field ϕ defined in terms of a line integral I_e of the e-projected Proca field $A_\mu^P e^\mu$ along e starting from the point x

$$\phi(x,e) = (I_e A_\mu^P)(x)e^\mu, \;\; a(x,e) = -m\phi(x,e) \tag{14}$$

and the sl vector potential A_μ in terms of the field strength of the Proca potential

$$A_\mu(x,e) = (I_e F_{\mu\nu})(x)e^\nu, \;\; F_{\mu\nu}(x) = \partial_\mu A_\nu^P - \partial_\nu A_\mu^P \tag{15}$$

whose massless limit coincides with (4).

The multiplication with m in (14) restores $d_{cl} = 1$ and removes the mass singularity, so that the $m = 0$ limit is an e-independent massless scalar $d_{sd}(a) = 1$ free field. In fact for all massive or massless sl tensor potentials $d_{sd} = 1$ and $3/2$ for halfinteger s whereas their pl counterparts increase linearly as $d_{sd} = s + 1$ or $d_{sd} = s$ and diverge like m^{-s} for $m \to 0$.

[12] By convoluting with a test function one can convert the Wilson loop integral into an operator localized on a solid torus.

[13] In case the solenoid has open ends the Wilson loop should avoid the region of the north–south magnetic backflow.

In particular the momentum space 2-pfct of the massive field strength and its massless associated sl vector potential are identical to their massless counterparts. This permits to lower the number of degrees of freedom by passing from $p \in H_m^\uparrow$ to $p \in V^\uparrow$ (and its "fattening" inversion, see below). In the pl setting this is not possible or can only be achieved in the presence of indefinite metric degree of freedom (the DVZ discontinuity, the WW problem).

It is instructive to look at this degree of freedom balance in more detail [10,11]. With the help of a p-dependent 4-matrix J (complex conjugation changes the sign of e, $tr =$ transposed)

$$J_\mu{}^\nu(p,e) = \eta_\mu{}^\nu - \frac{p_\mu e^\nu}{(pe)_{i\varepsilon}}, \quad \overline{J(p,-e)} = J(p,e) \tag{16}$$
$$M^{A_\mu(-e),A_\nu(e)} =: E_{\mu\nu}(e,e) = (J\pi J^{tr})_{\mu\nu}$$

the in e diagonal momentum space 2-pfct takes the form of the second line.[14] It shows that the positivity of the sl 2-pfct is inherited from the pl positivity. The rank of the E-matrix accounts for the degrees of freedom is 3 as a result of $J^{tr}e = 0$ and the additional relation $E_{\mu\nu}p^\nu = 0$ for $p \in V^+$ leads to a reduction from the three spin component to the two helicities $h = \pm 1$. This degree of freedom counting breaks down in the presence of indefinite metric.

This descend from $p \in H_m^\uparrow$ to V^+ permits an inversion, namely by continuous passing from momenta $p \in V^+$ to the mass shell H_m^+ ("fattening") one creates a new physical degree of freedom which together with former ± 1 accounts for the 3 degrees of freedom of spin $s = 1$. Such "magical" conversion of the particle content of two inequivalent Wigner representations can neither be achieved in terms of pl fields (no massless limit) or become contaminated by the presence of indefinite metric causing and ghost degrees of freedom of gauge theory. This is of particular interest in case of $s = 2$ [10,11] (see below). The use of sl fields is even more important for passing to the massless limit in the presence of interactions involving higher spins.

In the literature the terminology "fattening" had been used in connection with the Higgs model which describes the interaction between a massive vector meson with a massive real scalar field H as the result of spontaneous breaking of gauge symmetry (the Mexican hat potential). This idea contains two conceptual misunderstandings (which will be commented on in section 5 and the concluding remarks).

The real power of SLFT emerges in models of *selfinteracting* massive vector mesons where the preservation of 2nd order renormalizability requires the compensatory presence of a coupling to a Hermitian scalar H (Higgs) field[15] and imposes a Lie-algebra structure on the leading terms in the A_μ self-interactions. In section 5 we will provide the arguments.

An important property of the previously introduced pl and sl vector potential and its scalar escort $\phi(x,e)$ is their linear relation

$$A_\mu(x,e) = A_\mu^P(x) + \partial_\mu \phi, \quad \phi = -\frac{1}{m}a \tag{17}$$

This property justifies to call the $\phi's$ "escorts" of the sl potential, they share the same degrees of freedom. The appearance of the escort in form of a derivative is a consequence of Poincaré's lemma. The linear relation between fields corresponds to that between intertwiners (J as before):

$$J_\mu{}^\nu v_\nu(p) = v_\nu - p_\mu \frac{(ve)}{(pe)_{i\varepsilon}} \tag{18}$$

[14] Taking the same e would lead to the distributionally ill-defined denominator $pe_{-i\varepsilon} pe_{i\varepsilon}$.

[15] This (and not SSB) is the *raison d'être for the* H (section 5).

which follows directly from the definition (15). Each field contains the full information of the $(m, s = 1)$ Wigner representation; the encoding of $s = 1$ into a scalar is only possible within sl.

It is not accidental that the massive vector potentials which result from "fattening" their unique massless counterpart play a distinguished role in the new SLFT renormalization theory. Their smooth connection represents the higher spin analog of the smooth relation between $s < 1$ massless fields and their massive counterpart. The weakening of localization is necessary to preserve this smoothness in the presence of change of the number of degrees of freedom.

In the massless limit the $A_\mu^P(x)$ and $\phi(x, e)$ diverge as m^{-1} whereas the $A_\mu(x, e)$ and $a(x, e)$ stay infrared finite. The relation (u was introduced in (11))

$$\partial^\mu A_\mu = -ma, \quad d_e A_\mu = \partial_\mu u \tag{19}$$

$$u = -m^{-1} d_e a \tag{20}$$

leads to a divergence-free massless vector potential (Lorentz condition[16]) and a relation between two massless 1-forms in the de Sitter space of spacelike directions (that which remains of (17)). The purpose of the mass factors is to preserve the relation $d_{sd} = d_{cl}$ for all sl fields. The massless limit of u is logarithmically divergent.

Escorts (whose number increase with s) do not contain new degrees of freedom since, as the pl A^P, they are linear in the Wigner $s = 1$ creation/annihilation operators $a^\#(p, s_3)$ and only differ in their intertwiners. Rearrangements of degrees of freedom are quite common in quantum mechanical many-body problems.[17] Escorts are rearranged $s = 1$ degrees of freedom which carry the full content of $(m > 0, s)$ Wigner representations.

For $s \geq 2$ the sl fields lead to new properties. As a result of a possible relation with gravitation the case $s = 2$ is of special interest. The intertwiner of spin s Proca potentials (the P in $A^P_{...}$ refers to Proca or alternatively to pointlike) must be a divergence- and trace-free symmetric tensor; this is a consequence of the way the $2s + 1$ component subspace of spin is embedded in the $3s$-fold tensor product. Hence the intertwiners $v_{\mu_1 \dots \mu_s}(p, s_3)$ convert the symmetric trace-free s-fold tensor product of three-component spin 1 polarization vectors into covariant tensors of tensor-degree s.

For the momentum space $s = 2$ 2-pfct one obtains

$$M^{A^P_{\mu\nu}, A^P_{\kappa\lambda}}(p) = \frac{1}{2}\left[\pi_{\mu\kappa}\pi_{\nu\lambda} + \pi_{\mu\lambda}\pi_{\nu\kappa}\right] - \frac{1}{3}\pi_{\mu\nu}\pi_{\kappa\lambda} \tag{21}$$

where the numerical factors have their combinatorial origin in the symmetry and tracelessness and hence depend on the degrees of freedom. The sl 2-pfcts are of the same algebraic form and result by substituting $\pi_{\mu\nu} \to E_{\mu\nu}(-e, e)$ [10,11]. As for $s = 1$ this can be seen by passing from the Proca potential to the field strength (as stands for antisymmetrization)

$$F_{\mu_1\nu_1\mu_2\nu_2} = \underset{\mu\leftrightarrow\nu}{as}\ \partial_{\mu_1}\partial_{\mu_2} A^P_{\nu_1\nu_2} \tag{22}$$

and using the two-fold momentum space I operation to pass from the field strength to the potentials. Note that the symmetry of the Proca potential reduces the anti-symmetrization to a pairwise

[16] Note that this is an operator identity and not an imposed gauge condition.

[17] A well-known case is the appearance of Cooper pairs encounters in passing to the low temperature superconducting phase. Without this rearrangement classical vector potentials would not become short range inside a superconductor (the London effect).

operation $\mu_i \leftrightarrow \nu_i$. The resulting permutation properties of the resulting F are those of the linearized Riemann tensor.

The new phenomenon for $s > 1$ is that the massless limit of this field strength is not the same as that obtained directly from the massless $h = \pm 2$ Wigner representation. Correspondingly the sl potential associated with the massless limit of $F^{s=2}$ is different from that of $F^{|h|=2}$

$$A_{\nu_1\nu_2}(x,e) = (I_e^2 F^{s=2}_{\mu_1\mu_2\nu_1\nu_2})(x)e^{\mu_1}e^{\mu_2} \tag{23}$$

$$A^{(2)}_{\nu_1\nu_2}(x,e) = (I_e^2 F^{|h|=2}_{\mu_1\mu_2\nu_1\nu_2})(x)e^{\mu_1}e^{\mu_2} \tag{24}$$

This means in particular that the massive $s = 2$ sl potential obtained by fattening the $A^{(2)}$ is not the same as A although both account correctly for the $2s + 1$ spin degrees of freedom and share their Wigner–Fock Hilbert space. The massless limit of A splits into the direct sum of the two $|h| = 2$ degrees of freedom and the $h = 0$ contribution which is the remnant of the $s_3 = 0$ component. Conserved currents and stress–energy tensors preserve the number of degrees of freedom by converting the $\pm s_3$ components into $|h| = s_3$ helicities of a Wigner–Fock tensor product space.

In order to show how these results are related to the van Dam–Veltman–Zakharov discontinuity problem one must look at some details. Whereas fattening and taking the massless limit connect the 2-pfct of the 2-component massless helicity $|h| = 2$ potential $A^{(2)}$ with that of its 5-component $s = 2$ by deforming the momenta of the 2-pfct between $H_m^\uparrow$ and $V^\uparrow$, the massless limit of A is a cul de sac from which a return to the original massive pure $s = 2$ tensor potential is not possible.

The relation between the massless limit of A with that of $A^{(2)}$ are easily seen to have the following form

$$A^{(2)}_{\mu\nu}(x,e) = A_{\mu\nu}(x,e) + \frac{1}{2}E_{\mu\nu}(e,e)A^{(0)}(x,e) \tag{25}$$

$$E_{\mu\nu}(e,e) = \eta_{\mu\nu} + (e_\mu\partial_\nu + e_\nu\partial_\mu)I_e + e^2\partial_\mu\partial_\nu I_e^2$$

where the momentum space $E_{\mu\nu}$ has been rewritten as an integro-differential operator acting on a scalar sl field and the massless limit of $A^{(0)}$ is a (properly normalized) scalar escort. Combining this relation with that between the $s = 2$ pl field A^P, its sl counterpart A and the derivatives of escorts (the $s = 2$ analog of (17)) one obtains

$$A^P_{\mu\nu} = A_{\mu\nu} + \text{derivatives of escorts}$$

one concludes that in the adiabatic limit the interaction between "massive gravitons" and a trace-free energy-momentum tensor source $T_{\mu\nu}$ is [35]

$$\lim_{m\to 0}\int A^P_{\mu\nu}T^{\mu\nu} = \lim_{m\to 0}\int A_{\mu\nu}T^{\mu\nu} = \int (A^{(2)}_{\mu\nu} - \frac{\eta_{\mu\nu}}{\sqrt{6}}\varphi)T^{\mu\nu} \tag{26}$$

$$where\ \varphi(x) = \sqrt{\frac{3}{2}}\lim_{m\to 0} a^{(0)}(x,e) \tag{27}$$

The independence of the integrated massless $A^{(2)}$ contribution from the string direction follows from

$$\partial_{e_\kappa} A^{(2)}_{\mu\nu} = m^{-1}(\partial_\mu A^{(2)}_{\kappa\nu} + \partial_\nu A^{(2)}_{\mu\kappa}) \tag{28}$$

$$\partial_{e_\kappa} J_\mu{}^\nu = -\frac{p_\mu}{(pe)_{i\varepsilon}} J_\kappa{}^\nu$$

which in turn follows from the identity in the second line (for more details see [11]) and represents the $s = 2$ counterpart of the relation between de Sitter space 1-forms in (19).

The result confirms the van Dam–Veltman–Zakharov discontinuity: the massless limit of massive gravity differs from the result obtained directly with massless gravitons. But different from Zakharov's calculation which identifies this contribution as being the relic of a unphysical gauge theoretical degrees of freedom, the present calculation shows that it is really the massless footprint of the physical $s_3 = 0$ spin component. For the traceless stress–energy tensor of photons the last contribution vanishes whereas for couplings to matter (mercury perihelion) it remains.

This calculation permits a straightforward extension to any spin. The relation between the Proca potential, its sl counterpart and the associated sl escorts reads

$$A^P_{\mu_1...\mu_s} = A_{\mu_1...\mu_s} + sym.(\partial_{\mu_1}\phi_{\mu_2...\mu_s} + \partial_{\mu_1}\partial_{\mu_2}\phi_{\mu_3...} + \cdots + \partial_{\mu_1}\ldots\partial_{\mu_s}\phi) \tag{29}$$

where the $\phi_{\mu_1...\mu_i}$ is an $s-i$ fold iterated line integral along e of the spin s Proca potential and the symmetrization is over all indices and the ϕ are already symmetric by construction. For our purposes it is more convenient to use a different basis of escorts which are obtained by descending from the sl $A_{\mu_1...\mu_s}$ in terms of divergences

$$\begin{aligned} A^P_{\mu_1...\mu_s} &= A_{\mu_1...\mu_s} - sym.(\frac{\partial_{\mu_1}}{m}a^{(s-1)}_{\mu_2...\mu_s} + \frac{\partial_{\mu_1}\partial_{\mu_2}}{m^2}a^{(s-2)}_{\mu_3...} + \cdots + \frac{\partial_{\mu_1}\ldots\partial_{\mu_s}}{m^s}a^{(0)}) \\ ma^{(s-r)}_{\mu_r...\mu_s} &= -\partial^\mu a^{(s-r+1)}_{\mu\mu_r...\mu_s}, \quad a^{(s)}_{\mu_1...\mu_s} := A_{\mu_1...\mu_s} \end{aligned} \tag{30}$$

The second line shows that the a escorts start from the sl potential and descend by differentiation instead of descending from A^P by line-integration. The a has the same dimension $d_{sd} = 1 = d_{eng},\ d_{infr} = 0$, and are linear combinations of the ϕ escorts. As long as $m > 0$ each escort carries the full content of the Wigner spin s representation.

Although the $a's$ have a massless limit they still do not decouple. The van Dam–Veltman–Zakharov discontinuity shows that for $s = 2$ the $|h| = 2$ and $h = 0$ contributions stay together and have to be separated with the help of an integro-differential operation (25). The analogous situation in the general case is that the even and odd s_3 contributions remain coupled among themselves and can only be split in terms of their helicity content by the use of such integro-differential operations [11]. Naturally one can obtain a spin s vector potential from fattening a massless helicity h potential if $h = s$.

The tensor $v_{\mu_1..\mu_{|h|}}(q, e)$ which appears in the relation of the helicity h tensor field $A^{(|h|)}_{\mu_1..\mu_{|h|}}$ and the Wigner operator $a^\#(q, h)$ (which extends the construction of $A^{(2)}_{\mu\nu}$ in (25) to arbitrary helicity h). This e-dependent polarization tensor $v_{..}(q, e)$ replace the only up to re-gauging defined polarization tensor. If used in Weinberg's soft scattering limit of a massless particle with momentum q scattering on n massive particles with momenta $p_i, i = 1, ..n$ [17, 4.1], one obtains the same conclusions except that the gauge theoretic argument is replaced by the e-independence which follows in first order from the fact that the directional derivative with respect to e on these polarization tensors can be written as a spacetime derivative ∂_μ acting on such a tensor in analogy to (11). The use of sl polarization tensors instead of gauge symmetry is required by using positivity which guaranties the exclusive appearance of physical degrees of freedom.

The weakness of Lagrangian constructions of conserved currents and stress–energy tensors is that with the exception of low spins there is no guaranty that the so obtained classical expressions have the correct commutation relations with the quantum fields. It is much safer and easier to *start from the commutation relations between Wigner's generators* of the Poincaré group and the Wigner particle operators $a^\#(p, s_3)$ and to rewrite them with the help of the intertwiners into covariant commutation relations.

2.4. Infinite spin revisited

A simple illustration of such an "intrinsic quantum" construction of the stress–energy tensor has been recently presented in [8]. One starts from the expressions of the infinitesimal generators of translation $\mathbf{P}_\mu$ and Lorentz generators $\mathbf{M}_{\mu\nu}$ in terms of the Wigner operators $a^{\#}(p, s_3)$

$$\mathbf{P}_\mu = \int \sum_{s_3} a^*(p, s_3) p_\mu a(p, s_3) d\mu(p) \tag{31}$$

$$\mathbf{M}_{\mu\nu} = -i \int (\delta_{s_3 s_3'} p \wedge \partial_p + d(\omega)^t_{s_3 s_3'})_{\mu\nu} a^*(p.s_3) a(p, s_3') d\mu(p) \tag{32}$$

The first step is to rewrite the contribution of the spin component s_s to $\mathbf{P}_\mu$ as

$$\mathbf{P}_\mu = \iint d\mu(p) d\mu(p') \sum_{s_3, s_3'} (p_\mu a^*(p, s_3) \delta_{s_3 s_3'} (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}')(p_{10} + p_{20}) a(p', s_3') \tag{33}$$

$$(2\pi)^3 \delta(\mathbf{p} - \mathbf{p}') = \int e^{-i(\mathbf{p}-\mathbf{p}')x} d^3x = \int e^{-i(p-p')x} d^3x \tag{34}$$

where in the second line used the cancellation of the p_0 components.

What remains to do is to convert the Wigner operators via intertwiners into the covariant fields. For this one uses their completeness relation in order to write the unit operator in spin space as

$$g^{MN} v_{M s_3} \overline{v_{N s_s'}} = \delta_{s_3 s_3'}$$

where M and N represent the multi-tensor indices of the intertwiner. What remains is to use the Fourier transform (34) and pass from the Wigner operators to the fields. Using the fact that the $a^* a^*$ and aa contributions vanish as a result of the presence of $\overleftrightarrow{\partial}_0$ and that aa^* terms are absent in Wick-ordered products one verifies that

$$\mathbf{P}_\mu = \int \tilde{T}_{\mu 0}(x) d^3x, \quad \tilde{T}_{\mu\nu}(x) = -\frac{1}{4} \int : A^P_{\mu_1..\mu_s}(x) \overleftrightarrow{\partial}_\mu \overleftrightarrow{\partial}_0 A^{P,\mu_1..\mu_s}(x) : \tag{35}$$

where $\tilde{T}_{\mu\nu}$ is a contribution to the stress–energy tensor.

The full tensor density which generates all Poincaré transformations is of the form

$$T_{\mu\nu} = \tilde{T}_{\mu\nu} + \partial^\rho \Delta_{\mu\nu,\rho} \tag{36}$$

To compute the second contribution, which is also a bilinear expression in the A^P tensor fields, one starts from the bilinear expression for $M_{\mu\nu}$ in terms of the $a^{\#}$ Wigner operators which also contains a contribution the infinitesimal part of Wigner's little group. The representation of the Poincaré group generators in terms of pl stress–energy tensors may be rewritten in terms of their sl counterparts [11]. For recent results about constructing infinite spin fields and their E-M tensors as Pauli–Lubanski limits we refer to [8].

Rehren's construction of infinite spin quantum fields in terms of the Pauli–Lubanski limit is the most natural one; it corresponds to the use of the distinguished tensor potentials obtained by fattening its unique massless counterpart at fixed spin, except that it goes into the opposite direction at fixed P-L parameter.[18] The tensor field disappears in this limit and what remains (after appropriate adjustments) is the infinite family of escorts with arbitrary high tensor degree.

The nonexistence of the infinite spin tensor potential $A_{\mu_1\mu_2...\infty}$ accounts for the absence of a relation which converts the differential d_e into a spacetime divergence as well as the absence of a gauge theoretic formulation. This is the reason why infinite spin matter cannot interact with ordinary quantum matter [38].

Of physical relevance is the existence of conserved currents and energy-momentum tensor in the sense of bilinear forms [8]. Hence expectation values of E-M tensors and possible gravitational backreaction remain physically meaningful.

3. Causality and the Velo–Zwanziger conundrum

The Velo–Zwanziger conundrum is an alleged causality paradox which arose from the naive expectation that $s \geq 1$ quantum fields, whose free field equations are modified by linear pl couplings to external potentials, maintain their causal propagation. Formally it is closely related with the Weinberg–Witten No-Go theorem which excludes the existence of higher helicity conserved pl currents. This connection turns out to be useful for the solution of the V-Z conundrum.

3.1. Recalling the solution of the Weinberg–Witten problem and the associated local charges

In [11] it was shown that for massive $s \geq 1$ free field one can construct sl tensor potentials whose associated conserved sl currents have finite massless limits even when according to the Weinberg–Witten (W-W) theorem physical (gauge-invariant) pl currents do not exist.

In the massive case both the pl and sl currents are members of the same local equivalence class which consist of all Wick-ordered composites of pl fields and their related sl counterparts. Their relative causality reads

$$\left[j_\mu^P(x), j_\nu(x',e)\right] = 0 \; for \; x \times \mathcal{S}(x',e), \; \mathcal{S}(x',e) = x' + \mathbb{R}_+ e, \; e^2 = -1 \; or \; 0 \tag{37}$$

Their charge-densities differ by spatial divergences and hence they share the global $U(1)$ generators. In the massless limit the sl spin potentials pass continuously to their massless counterpart (not possible with pl potentials) which act in the conformally covariant helicity Wigner–Fock space. The sl currents are bilinear in the charge carrying sl potentials [11].

The two currents (37) share the same "engineering" dimension (classical dimension in terms of mass units) $d_{cl} = 3$, but possess different short distance scaling dimensions $d_{sd}(j_\mu^P) = 2(s+1)+1$ and $d_{sd}(j_\mu) = d_{cl} = 3$; this accounts for the fact that the sl j_μ allows a massless limit whereas j_μ^P diverges as $j_\mu^P \overset{m\to 0}{\sim} m^{-2s}$ (the W-W obstruction). As expected, the sl $j_\mu(x,e)$ admits a massless limit in which the $2s+1$ spin degrees of freedom decompose into a direct sum of s helicity and one scalar contribution so that the Wigner–Fock space turns into a tensor product of helicity spaces.

The presentation concerning the relation between pl and sl conservation laws in [11] was mainly focused on the stress–energy tensors (SET); in the following we present the corresponding problem for conserved currents. A convenient illustration is provided by the sl current with the lowest W-W helicity $h = 1$ as follows.

[18] In this way it selects a unique countable family of fields within the equivalence class of all relatively causal fields constructed in [35].

Using the linear relation $A^P_\mu(x) = A_\mu(x,e) - \partial_\mu\phi(x,e)$ between pl and its canonically associated sl field and the gradient of its escort derived in the previous section (17) one finds that the pl and sl currents are related as (omitting Wick-ordering)

$$j^P_\mu = iA^{P\nu}(x)^* \overleftrightarrow{\partial_\mu} A^P_\nu(x) = j_\mu(A(x,e)) + j_\mu(a(x,e)) + \partial^\kappa C_{\kappa\mu} \tag{38}$$

$$a(x,e) = m\phi(x,e),\ C_{\kappa\mu} = iA^*_\kappa \overleftrightarrow{\partial}_\mu\phi + i\phi^* \overleftrightarrow{\partial}_\mu\partial_\kappa\phi + h.c.$$

The first two contributions are conserved sl currents whose massless limit correspond to the current of the complex $s = 1$ sl field $A_\nu(x,e)$ (which replaces the nonexistent pl W-W current), and that of a complex scalar field $a(x) = \lim_{m\to 0} a(x,e)$. The m^{-2} W-W obstruction C does not contribute to the global charge.

The "obstructing" contribution $\partial^\kappa C_{\kappa\mu}$ carries both the leading short distance dimension $d_{sd} = 5$ and the m^{-2} divergence which is the culprit for the W-W problem. This kind of decomposition into s conserved $d_{sd}(j) = 3$ sl currents, which pass for $m \to 0$ to s sl helicity currents and a pl current of a scalar particle, exists for every spin $s \geq 1$.

Using the free field equation for A_μ and ϕ one verifies that C-contribution is of the form of a spatial divergence and hence does not contribute to the infinite volume limit of the charges [11]

$$Q(A^P) = Q(A) + Q(a) \tag{39}$$

i.e. the massless limit decomposes the three spin degrees into the ± 1 helicities of A_μ and $h = 0$ carried by a. Before this limit both sl fields A_μ and a account for the three $s = 1$ degrees freedom.

For pl currents there exists extensive literature on the problem of relation between conserved currents, local charges, and their global limits [55], [56], [57], [58]. The basic idea is to start from a conserved current and define

$$Q = \lim_{R\to\infty} Q(f_R, f_d),\quad Q(f_R, f_d) := j_0(f_R, f_d) \tag{40}$$

$$f_R(\mathbf{x}) = \begin{cases} 1 & |\mathbf{x}| < R \\ 0 & |\mathbf{x}| > R + r \end{cases} \tag{41}$$

$$f_d(x_0) \geq 0.\ suppf_d \subset |t| < d,\ \int f_d(x_0)dx_0 = 1 \tag{42}$$

One then uses the conservation law of the current to show that the commutator $[Q(f_R, f_d), A]$ for $A \in \mathcal{A}(\mathcal{O})$ is independent of the choice of the smearing function $g(x) = f_R(\mathbf{x}) f_d(x_0)$ as long as $\mathcal{O}$ remains inside their timelike extended shell structure (41).

The local charge $Q(g)$ which measures the charge of an operator A localized in $|\mathbf{x}| < R$ converges towards the generator Q of the global $U(1)$ symmetry. The concept of a local charge content becomes problematic in case of sl currents since the use of a rigid spacelike direction e does not allow the causal separation of $j_0(f_R, f_d)$ from the localization region of A. The heuristic idea for achieving such a separation would be to "comb" the strings emanating from the shell between R and $R + r$ into different directions so that they remain causally separated from $A \in \mathcal{A}(\mathcal{O})$. But then the strings emanating from points inside the shell would have to move to spacelike infinity outside the larger sphere and violate the localization *inside* the larger sphere.

In view of a recent proof of the so-called *split property* [59], which is known to secure the local implementation of global symmetries in massless $h \geq 1$ [60], there is no problem with the existence of local charges for QFT's with global symmetries; what is not clear is whether such charges can be described in terms of conserved currents.

Meanwhile K.-H. Rehren informed me that his student M. Heep constructed local charges from sl currents by appropriate use of conformal transformations. The idea is to construct a local charge operator localized in a half-space, that is then mapped to a sphere by a conformal transformation. In this way the strings become "curled" and end in the north pole.

Hence the W-W No-Go theorem excludes pl currents, but does not affect the causal localizability of charges in arbitrary small spacetime regions.

3.2. The V-Z conundrum arises from an incorrect implementation of causality

A simple class of models for a critical examination of the V-Z conundrum is provided by linear couplings of conserved currents to external vector potentials the relevant property of the sl current is its lowered short distance dimension. A suitable setting for such problems is obtained in terms of Bogoliubov's definition of the S-matrix and interacting local fields in terms of adiabatic limits of the Bogoliubov S-functional[19]

$$S := \lim_{g(x)\to g} S(g), \quad S(g) = T\exp i\int g(x)L_{int}(x)d^4x \tag{43}$$

$$A(x)|_{L_{int}} = \lim_{g(x)\to g} \frac{\delta}{i\delta f(x)} S^{-1}(gL)S(g(x)L_{int}(x) + fA)|_{f=0} \tag{44}$$

Here the interaction density L_{int} is a Wick-ordered product of not more than 4 free fields from the class of Wick-ordered composites of free fields and $A|_{L_{int}}$ the interacting counterpart of $A(x)$ which is either a free field or a Wick-ordered product of free fields (the terminology "free" is used for linear fields and Wick polynomials). The interacting field has the form of a power series in g with retarded products of n $L's$ which is retarded in $A(x)$. The linear Bogoliubov map $A \to A|_{L_{int}}$ does not preserve the algebraic structure but it maintains the property of causal separability. Hence fields constructed in this way maintain causality, and the solution of the V-Z problem consists in the proper computation of the interacting fields via (44).

The class of interactions with external potentials to be studied is of the form $L_{int} = L^P = U^\mu j^P_\mu$ with U^μ a external (classical) vector potential and j^P_μ a conserved current as before. For the current of a scalar complex free field φ there is no problem; its conserved current has $d_{sd}(j^P_\mu) = 3$ and hence (with $d_{sd}(L_{int}) = 3$) renormalizable. This is the model on which Velo and Zwanziger base their propagation picture: namely the scalar field obeys a linear field equation which is linear[20] in U^μ and they expect (erroneously, as will be seemed) that this holds independent of spin.

For $s = 1$ the $d_{sd}(L^P) = 5$ and hence the pl model is nonrenormalizable. To reduce the d_{sd} from 5 to 4 one uses the relation (38) which rewritten in terms of the interaction density reads

$$\begin{aligned} &L^P = j^P_\mu U^\mu = L - \partial^\kappa V_\kappa,\ C_{\kappa\mu} = iA^*_\kappa \overleftrightarrow{\partial}_\mu \phi + i\phi^* \overleftrightarrow{\partial}_\mu \partial_\kappa \phi + h.c \\ &L := j^s_\mu U^\mu - C_{\kappa\mu}\partial^\kappa U^\mu,\ \ V_\kappa = -C_{\kappa\mu}U^\mu,\ S^{(1)} \overset{a.l.}{=} \int L^P d^4x = \int L d^4x \end{aligned} \tag{45}$$

where the decomposition (38) of j^P_μ was used. Since $d_{sd}(C_{\kappa\mu}) = 4$ the power counting bound $d_d(L) = 4$ holds, the model is renormalizable and its first order S-matrix $S^{(1)}$ (the adiabatic limit

[19] Our use of the Bogoliubov's formalism is close to that in [61], [62], [63].

[20] Here and in the sequel linear stands for linear in U_μ and its derivatives.

of the interaction density) is the same for the two densities and hence string-independent (the suitably defined adiabatic limit of the ∂V vanishes).

The decomposition of j_μ^P (38), which previously served to solve the W-W problem (by converting the pl current into its for $m \to 0$ regular sl counterparts and a C-term, which carries the m^{-2} mass divergence but does not contribute to the global charge[21]), is now used to solve the V-Z causality problem. To achieve this one uses the fact that the C-term is a 4-divergence and disappears in the adiabatic limit which represents the S-matrix.

The two interaction densities L^P and L share the same S-matrix; whereas the pl $L^P(x)$ side insures that the S-matrix is that of a causal interaction, the sl $L(x,e)$ guaranties the renormalizability of $S^{(1)}$. The $L(x,e)$ together with $V_\mu(x,e)$ forms what will be referred to as a L, V_μ pair. The first order S-matrix ((45) second line), which is the adiabatic limit of the interaction density, is the same for L^P and L referred to as the linear relation (45). The L^P represents the heuristic content of the interaction, but as a result of its bad short distance behavior it is not suitable for perturbative calculations. The short distance improved L weakens the localization but retains enough of it to keep fields causally separated and to maintain scattering theory.

The remaining problem is the extension of this idea to higher order. For convenience of notation one uses a differential formulation of pl localization in the form of e-independence in the form $d_e(L - \partial V) = 0$. It is convenient to use lightlike $e's$ since in this case no smearing is needed. The problem in higher order is the time-ordering. For the e-independence of the S matrix one needs the ∂ to act outside the time-ordering e.g.

$$d_e(TL(1)L(2) - \partial_1^\mu TV_\mu(1)L(2) - \partial_2^\nu TL(1)V_\nu(2) + \partial_1^\mu\partial_2^\nu TV_\mu(1)V_\nu(2) \overset{?}{=} 0 \qquad (46)$$
$$d_eT(L(1) - \partial V(1))(L(2) - \partial V(2)) = 0$$

and higher order extensions involving one V_μ and $n-1$ $L's$.

This is generally not possible without creating "obstructions" of the form of delta contributions of the form $\delta(x_1 - x_2)d_eL_2(x_1, e)$ which are quadratic in U_μ and its derivatives. Higher order violations may lead to contributions of higher polynomial degree in U_μ and derivatives; it is a characteristic property of obstructions in models of external potential interactions that all obstructions remain bilinear in the quantum fields.

These obstructions are absorbed in the form of induced contributions into a modified Bogoliubov formalism by defining

$$L_{tot} = L + \frac{1}{2}L_2 + ..\frac{1}{n!}L_n + .. \qquad (47)$$

where L_n is of polynomial degree n in U_μ and its derivatives and remains quadratic in free fields. Note that induced contributions do not increase the number of parameters and hence must be distinguished from counterterms of pl renormalization theory.

In the $s = 1$ model (45) the L_2 contribution *can alternatively be encoded into a redefinition of time ordering*

$$T_0\partial_\mu\phi(x_1)\partial_\nu\phi(x_2) \equiv \partial_\mu\partial_\nu T_0\phi(x_1)\phi(x_2) \qquad (48)$$
$$T\partial_\mu\phi(x_1)\partial_\nu\phi(x_2) = T_0\partial_\mu\phi(x_1)\partial_\nu\phi(x_2) + icg_{\mu\nu}\delta(x_1 - x_2)$$
$$\partial^\mu T\partial_\mu\phi(x_1)\partial_\nu\phi(x_2) - T\partial^\mu\partial_\mu\phi(x_1)\partial_\nu\phi(x_2) = (1+c)\partial_\nu\delta(x_1 - x_2)$$

[21] Using conformal invariance of massless helicity representations one can also show the existence of local charges (see remarks in previous subsection).

and the validity of (46) for the T time-ordering requires to set $c = -1$. For $s > 1$ the kinematic T_0 time-ordering contains more derivatives and one has accordingly more $c's$ which must be numerically adjusted in such a way that the T time-ordering satisfies the higher order pair requirements beyond (46).

The following side-remark maybe helpful for the later extension of SLFT to a full QFT. The second order $A_\mu A^\mu \varphi^* \varphi$ term of scalar QED within the new SLFT can either be viewed as induced or encoded into a change of time-ordering for the derivatives of the complex scalar field. But not all obstructions can be absorbed in this way. The H-selfinteractions of the Higgs model is a *genuine* second order induced term which results exclusively from the implementation of the positivity and causality principle of QFT (rather than from an imposed Mexican hat interaction).

The verification of the higher order pair relations will be deferred to a more complete treatment of external potential problems. The expected result is:

Conjecture. *Couplings of conserved currents to external potentials fulfill the higher order time-ordered L, V_μ relation. For $s = 1$ the resulting field equations are quadratic in U_μ whereas for $s > 1$ they are of infinite order (expected since $d_{sd}(L) \geq 5$).*

The form of the linear causal field equations (in particular the higher order U contributions) is determined by the form of the induced contributions.

The external potential formalism and its formal connection with the solution of the W-W problem works in an analogous way for $s = 2$. The sl potential $A_{\mu\nu}(x, e)$ has two escorts, a vector a_μ and a scalar escort a which can be chosen in such a way that the operator dimension for all fields is identical to their classical dimension $d_{cl} = 1$ (or $3/2$ for half-integer spin)

$$
\begin{aligned}
& A^P_{\mu\nu} = A_{\mu\nu} + m^{-1}(\partial_\mu a_\nu + \partial_\nu a_\mu) + m^{-2}\partial_\mu \partial_\nu a \\
& j^P_\mu(x) = i A^{P*}_{\kappa\lambda} \overleftrightarrow{\partial_\mu} A^{P\kappa\lambda} = j^s_\mu(x, e) + \partial^\kappa C_{\kappa\mu} \\
& j^s_\mu(x, e) = i A^*_{\kappa\lambda} \overleftrightarrow{\partial_\mu} A^{\kappa\lambda} - 2i a^*_\kappa \overleftrightarrow{\partial_\mu} a^\kappa + i a^* \overleftrightarrow{\partial_\mu} a
\end{aligned} \tag{49}
$$

In this case $d_{sd}(L^P) = 7$ and $d_{sd}(j^\mu) = 3$ and hence L^P is by 3 units beyond the power-counting bound $d_{sd} = 4$. For $s = 1$ the $\partial^\kappa C_{\kappa\mu}$ carries the highest $d_{sd} = 7$ contribution. After an additional linear disentanglement between $A_{\mu\nu}$ and a one arrives at a decomposition of j^s_μ which in the massless limit represents the $h = 2, 1, 0$ helicity contributions [11].

The use of this decomposition in the rewriting of $L^P = j^P_\mu(x)U^\mu$ for $s = 2$ as a sl pair with $L^P = L - \partial V$ leads to a $d_{sd}(C_{\kappa\mu}) = 6$ contribution which contains bilinear in $\phi = m^{-2}a$ terms with more than 2 derivatives. In analogy to counterterms in every order in a nonrenormalizable full pl QFTs one expects to find induced terms in arbitrary high orders.

It is worthwhile to mention that there is also a gauge theoretic formulation in which the linear operator relation between the sl potential and its pl counterpart is replaced by the relation $A^K_\mu(x) = A^P_\mu(x) + \partial_\mu \phi^K$ where the K refers to the Krein space and the resort ϕ^K is the Stückelberg negative metric pl scalar which adds additional unphysical degrees of freedom to the indefinite Krein space. This is the formulation of the Uni Zürich group [64], [65] adapted to the presentation used in the present paper.

The gauge theoretic analog to the pair relation is $L^K = L^P + \partial^\mu V^K_\mu$. The model has a formal similarity with SLFT, but its pl interpolating fields are unphysical; positivity obeying interpolating fields are simply inconsistent with pl localization. The pair property works the same way, one only has to replace the d_e in (46) by the BRST $\mathfrak{s}$.

However negative metric degrees of freedom lead to an unphysical realization of appropriately nonlinear modified causal pl V-Z equation and should be rejected inasmuch as gauge dependent pl currents have been discarded by W-W in their No-Go theorem. Classical field theory is free of positivity requirements and gauge theoretic causal propagation is perfectly compatible with its principles. But the nonlinear dependence on external potential which was overlooked by V-Z is also needed for the classical propagation of Cauchy date.

Recently there have been attempts to solve the V-Z conundrum in terms of String Theory [68], [17]. These authors extract a system of pl equation in $d = 3 + 1$ via dimensional reduction from the Virasoro algebra in 10 dimensions and found nonlinear modifications in case of constant external fields.

But the fact that there is nothing stringy about their pl equations raises the old question: what do string-theorist really mean when they claim that their objects are stringy in spacetime. Does their use of the terminology "string" perhaps refer to a circular structure in a 10-dimensional target space whose Fourier components correspond to the irreducible Wigner components of the highly reducible superstring representation?

Their strings bear no relation with causal localization in spacetime but rather seem to refer to Born's quantum mechanical localization related to the spectral decomposition of the **x** operator arises. Their use of world-sheet and Nambu–Goto actions point into this direction and the way in which they think of their localized objects as vibrating in space strengthens this presumption. Causal localization in spacetime is very different (for more see next section).

The next section explores important aspects of causal localization which, although known to some experts, remained outside the conceptual radar screen of most particle physicists.

4. Particle wave functions and causal localization

There is no concept in particle physics which led to more misunderstandings than that of causal localization in spacetime. The strings of String Theory obtained e.g. from quantized world-sheet or Nambu–Goto actions bear no relation causal localization. A concept which reveals such misunderstandings and corrects them in the clearest possible way is "modular localization".

4.1. Newton–Wigner localization and its causality-providing modular counterpart

Wigner's theory of positive energy representations presents an interesting meeting ground of two very different localization concepts. On the one hand there is the quantum mechanical localization of dissipating wave packets whose center moves on relativistic particle trajectories. Its formulation in terms of quantum mechanical Born probabilities leads to the so-called Newton–Wigner localization [70]. For a scalar $m > 0$ particle

$$(\psi, \psi') = \int \bar{\psi} \overleftrightarrow{\partial_0} \psi' d^3x = \int \bar{\psi}_{NW} \psi'_{NW} d^3x \tag{50}$$

$$\text{hence} \quad \tilde{\psi}_{NW}(\mathbf{p}) = (2p_0)^{-1/2} \tilde{\psi}(\mathbf{p})$$

Hence an improper N-W eigenstate of the position operator $\mathbf{x}_{NW}$ has a mass-dependent extension of the order of a Compton wave length. In scattering theory, where only the large-time asymptotic behavior matters, such ambiguities in assigning relativistic quantum mechanical positions at finite times are irrelevant; the centers of wave packets of particles move on relativistic velocity lines and the probability to find a particle dissipates as t^{-3} along these lines for all inertial observers. In fact Wigner never thought of his Poincaré group representation theory as an entrance into causal QFT; for him it remained part of relativistic QM.[22]

The more recent discovery of *modular localization* shows that causality properties are dormant within Wigner's positive energy representation theory; they are reflected in properties of dense subspaces obtained by applying algebras of local observables $\mathcal{A}(\mathcal{O}$ to the vacuum state $\mathcal{H}(\mathcal{O}) = \mathcal{A}(\mathcal{O})\Omega$ and projecting the so obtained dense set of states of a QFT to the one-particle subspace $\mathcal{H}_{Wig}(\mathcal{O}) = E_1\mathcal{H}(\mathcal{O})$. That such spaces are dense in the Hilbert space (and consequently their projection in the one-particle subspace) is a special case of a surprising discovery made in the early in the early 60s (the Reeh–Schlieder theorem [1], [2]) which showed that the omnipresence of vacuum polarization confers to QFT *a very different notion of localization* from that of Born's quantum mechanical setting based on position operators.

The projection $\mathcal{H}_{Wig}(\mathcal{O})$ has the remarkable property that it can be constructed without the assistance of QFT *solely in terms of data from Wigner's representation theory* and that in the absence of interaction one can even revert the direction and obtain the net of causally localized subalgebras directly from that of modular localized Wigner subspaces [25].

In this way one does not only gain a more profound understanding of QFT but one also learns that Weinberg's pure group theoretic construction of *intertwiners* starting from Wigner's representation theory is part of a much more general setting which, if properly used, leads to an extension of perturbative renormalizability. This important concept of modular localization was not available during Wigner's lifetime (see remarks in the introduction).

The simplest way to see that the quantization of a relativistic classical particle associated with the action $\sqrt{-ds^2}$ does not lead to a covariant quantum theory is to remind oneself that there exists no operator $\mathbf{x}$ which is the spatial component of a covariant 4-vector [35]. The conceptual problem one is facing is better understood by first showing that causal localization bears no relation to Born's probabilistic quantum mechanical definition.

Starting from the quantum mechanical projectors $E(R)$ for $R \subset \mathbb{R}^3$ which appear in the spectral decomposition of $\mathbf{x}_{op}$

$$\mathbf{x}_{op} = \int \mathbf{x} dE(\mathbf{x}) \tag{51}$$

one has

$$E(R)E(R+\mathbf{a}) = 0 \quad \text{for} \quad R \cap (R+\mathbf{a}) = 0$$

Define $E(R+a) = U(a)E(R)U(a)^*$ for $a \in \mathbb{R}^4$. Assuming that this orthogonality relation has a causal extension in the sense that $E(R)E(R+a) = 0$ for spacelike separated $R \times R + a$ leads immediately to clash with positivity of the energy. This follows from the fact that the positivity of the energy leads to the analyticity of expectation values $(\psi, E(R)E(R+a)\psi)$ for $\mathrm{Im}\, a_0 > 0$ which in turn implies their identical vanishing (the Schwarz reflection principle) and with $||E(R)\psi||^2 = 0$ the triviality of such projectors $E = 0$.

A slight extension of the argument reveals that it can be dissociated from the position operator of quantum mechanics. It then states that in models with energy positivity *it is not possible*

[22] This perhaps explains why Wigner, inspite of his overpowering role in the development of 20th century quantum theory, never participated in the QED revolution and its QFT aftermath. For him his representation theory always remained part of relativistic quantum mechanics (the quantum mechanical Newton–Wigner localization in section 3). An interesting discussion can be found in Haag's memoirs [71] page 276.

to describe causal localization ("micro-causality") in terms of projectors and orthogonality of subspaces [72]. A profound intrinsic understanding of causal localization in QFT points into a very different direction from that obtained from the quantization of actions describing classical world lines or world sheets and Nambu–Goto action. The problems become insurmountable of one tries to construct actions in the presence of several of such objects and their distance.

Before presenting the relation to QFT it is worthwhile to mention a little known fact: it is perfectly possible to construct a relativistic description of interacting particles in relativistic QM build on macro-causality. For two particles this amounts to Poincaré group preserving modification in the center of mass system, but for more particles it is more complicated ([43] section 3). Apart from the fact that it leads to a Poincaré-invariant S-matrix, it does (unlike Schrödinger theory) not permit a description in terms of second quantization.

4.2. Mathematical properties of modular localization

To prepare the ground for causal localization it is helpful to start with some mathematical concepts concerning relations between real subspaces H (linear combination with reals) of a complex Hilbert space $H \subset \mathcal{H}$. The symplectic complement H' of a real space is defined as the closed real subspace ($\overline{H'} = H'$) defined in terms of the imaginary part of the scalar product in $\mathcal{H}$

$$H' = \{\xi \in \mathcal{H};\ \mathrm{Im}(\eta, \xi) = 0\ \forall \eta \in H\} \tag{52}$$

$$H_1 \subset H_2\ \Rightarrow H_1' \supset H_2' \tag{53}$$

which turns out to be the real orthogonal space on the real iH (only real linear combinations).

A closed real subspace H is called "standard" if it is both cyclic and separating

$$\begin{aligned} H \quad \text{cyclic:} \quad & \overline{H + iH} = \mathcal{H} \\ H \quad \text{separating:} \quad & H' \cap H = \{0\} \\ & (H + iH)' = H' \cap iH' \end{aligned} \tag{54}$$

Cyclicity and the separation property have a dual relation in terms of symplectic complements as written in the third line.

It is quite easy to obtain such standard spaces from covariant free fields. In the simplest case of a scalar field the Hilbert space $\mathcal{H}$ is the closure of the 1-particle Wigner space defined by the two-point function of the smeared fields

$$\begin{aligned} (f, g) = \langle A(f)^* A(g) \rangle &= \int \tilde{f}^*(p)\tilde{g}(p) d\mu(p) \\ \left[A(f)^*, A(g)\right] &= -i\, \mathrm{Im}\,(f, g) \end{aligned} \tag{55}$$

where $d\mu$ is the invariant measure on the positive mass hyperboloid. According to the Reeh–Schlieder theorem [2] the one-particle projection of the dense subspace of causally $\mathcal{O}$-localized states[23] is dense in the one-particle Wigner space. $\mathcal{O}$-localized real testfunctions define a dense real subspace $H(\mathcal{O})$ and causal disjointness corresponds to "symplectic orthogonality" and produces a *closed* real subspace.

$$H(\mathcal{O}') = H(\mathcal{O})' \tag{56}$$

[23] States localized in the spacetime region $\mathcal{O}$ are defined as the dense Reeh–Schlieder subspace obtained as $\mathcal{A}(\mathcal{O})\,|0\rangle$ where $\mathcal{A}(\mathcal{O})$ is the $\mathcal{O}$ localized subalgebra of $\mathcal{A}$.

As a side remark we mention that the same construction applied to a higher halfinteger spin field leads to a corresponding situation

$$ZH(\mathcal{O}') = H(\mathcal{O})', \quad Z = \frac{1 + iU(2\pi)}{1 + i} \tag{57}$$

where the unitary "twist" operator Z which is related to the factor -1 of the 2π rotation. The use of the twist operator allows to treat bosons and fermions under one common roof.

The important step for an intrinsic understanding of QFT (i.e. without the use of P. Jordan's "(quasi-)classical crutches" of quantization) is to *invert the previous construction: find a relation within Wigner's representation theory which permits to define $\mathcal{O}$-localized real subspaces which have the correct covariant transformation properties under Poincaré transformations [25].* For this purpose it is helpful to reformulate the above properties so that they take the form known from the mathematical Tomita–Takesaki theory of operator algebras which permits a direct connection of positive energy Wigner representations with a "local net of operator algebras". It provides a unified view in which Weinberg's pl intertwiner formalism and its sl extension are seen as two ways of generating the same free field theory. As shown in previous sections the improvement of short distance properties is the basis for a new perturbative renormalization setting.

To achieve this one needs an additional mathematical tool. The first step consists in a suitable extension of the modular concepts. A standard subspace H comes with a distinguished operator. With $D(Op)$ denoting the domain of definition of an operator one defines

Definition 1. A Tomita operator $\mathcal{S}$ is a closed antilinear densely defined involutive operator $D(\mathcal{S}) \subset \mathcal{H}$.

In physics one encounters such "transparent" operators only in QFT. It is easy to see that there exists a 1–1 correspondence between Tomita operators and standard subspaces H; $H \leftrightarrow \mathcal{S}$. This follows from the definition $\mathcal{S}(\xi + i\eta) = \xi - i\eta,\ \xi, \eta \in H$, whereas the opposite direction is a consequence of the definition $H = \ker\{\mathcal{S} - 1\}$.

As a result of involutiveness, the full content of Tomita operators is contained in their dense domains which coincides with their range ("transparency"). Hence modular theory may be alternatively formulated in terms of subspaces (54) $Dom\mathcal{S}$ of Tomita operators.

The polar decomposition of $\mathcal{S} = J\Delta^{1/2}$ of $\mathcal{S}$ into an anti-unitary J and a positive operator Δ with $D(S) = D(\Delta^{1/2})$ leads to the unitary modular group Δ^{it} acting in $\mathcal{H}$ and preserving the standard subspace $\Delta^{it}H = H$, whereas the modular conjugation maps into the symplectic complement $J\ H = H'$.

A Tomita operator appears in a natural way in Wigner's representation theory of positive energy representations of the Poincaré group $\mathcal{P}$. It is obtained by defining $\Delta^{it}_{W_0}$ in terms of the Lorentz boost operator which leaves the wedge $W_0 = \{x;\ z > 0, |t| < z\}$ invariant

$$\Delta^{it}_{W_0} = U(\Lambda_{W_0}(-2\pi t))$$
$$\mathcal{S}_{W_0} = J_{W_0}\Delta^{1/2}_{W_0}, \quad J_{W_0} = TCP \cdot R_\pi$$

together with an anti-unitary J obtained by multiplying the TCP reflection TCP with a π-rotation R in the x–y plane as written in the second line, taking W_0 into its causal complement.

The charge conjugation C maps an irreducible Wigner particle space into its charge conjugate and may need a doubling of the Wigner space whereas TP corresponds to the spacetime inversion $x \to -x$. The preservation of energy requires the time-reversal T to be anti-unitary. Massless representations need a helicity doubling $\pm h$.

Unbounded operators $\mathcal{S}$ whose dense domain is stable ("transparent" in the sense *domain* $=$ *range*) are somewhat unusual in quantum physics; they appeared first in quantum statistical mechanics [2] and later in searches for an intrinsic understanding of causal localization of QFT (without referring to Lagrangian quantization) [27]. In the present context the dense subspace of the Wigner space (possibly doubled by charge conjugation) corresponds to wave functions which are "modular localized" in the wedge W_0. Modular localization of Wigner wave functions is closely related to causal localization of fields and provides an extension of the Weinberg intertwiner formalism which includes Wigner's infinite spin class [35].

The construction proceeds as follows: start from Wigner's positive energy representation theory, define the Tomita operator S_{W_0} in the way described before, use the Poincaré transformations to construct a net of modular localized real subspaces $H(W)$ and use the second quantization functor (the Weyl or CAR functor) to pass to an interaction-free net of standard wave functions spaces $H(W)$ to *causally localized operator algebras $\mathcal{A}(W)$ acting in a Wigner–Fock Hilbert space.*

One also may directly construct real dense subspaces $H_{\mathcal{O}}$ and their complexified counterparts $D(\mathcal{S}_{\mathcal{O}}) = H_{\mathcal{O}} + iH_{\mathcal{O}} \subset \mathcal{H}$ corresponding to more general causally complete convex spacetime regions $\mathcal{O}_c$ as intersections:

$$H_{\mathcal{O}_c} = \bigcap_{W\supset\mathcal{O}_c} H_W \tag{58}$$
$$H_{\mathcal{O}} = \bigcup_{\mathcal{O}_c\subset\mathcal{O}} H_{\mathcal{O}_c}$$

whereas for more general regions the standard space is defined in terms of exhaustion from the inside (second line). For details we refer to [25].

The energy-positivity of the massive and the $\pm|h|$ massless Wigner representation classes plays an important role in establishing the isotony and causal localization of the "net of modular localized standard spaces"

$$\text{isotony:} \quad H_{\mathcal{O}_1} \subset H_{\mathcal{O}_2} \quad \text{if} \quad \mathcal{O}_1 \subset \mathcal{O}_2 \tag{59}$$
$$\text{causality:} \quad H_{\mathcal{O}_1} \subset H'_{\mathcal{O}_2} \quad \text{if} \quad \mathcal{O}_1 \times \mathcal{O}_2$$

where $\times$ denotes spacelike separation.

In the absence of interactions the passage from the spatial modular theory to its algebraic counterpart is almost trivial. One passes to the Wigner Fock space created by symmetrized tensor products and defines the $\mathcal{O}$-localized operator algebra as the von Neumann algebra generated by the Weyl operators

$$\mathcal{A}(\mathcal{O}) = \left\{e^{iA(h)}; h \in H_{\mathcal{O}}\right\}'', \quad [A(h_1), A(h_2)] = i\,\mathrm{Im}(h_1, h_2) \tag{60}$$

$$\text{with} \quad A(h) = \int \sum_{s_3=-s}^{s} (h(p,s_3)a^*(p,s_3) + h.c.)d\mu(p), \; h \in H_{\mathcal{O}} \tag{61}$$

in terms of the Wigner creation/annihilation operators[24] $a^{\#}(p,s_3)$ or their helicity counterparts. For half-integer spin or helicity the presence of the twist operator (57) leads to fermionic com-

[24] The "second quantization" counterparts of the Wigner wave functions.

mutation relation. In both cases the second quantization functor maps the spatial Tomita operator into its algebraic counterpart which acts in the ("second quantized") Wigner Fock space $\mathcal{H}$ associated with the Wigner one-particle space H

$$\mathcal{S}_{\mathcal{O}}^{alg} A\Omega = A^*\Omega, \ A \in \mathcal{A}(\mathcal{O})$$

In the sequel only the algebraic Tomita–Takesaki theory will be used and the superscript alg will be omitted for convenience.

Modular localized spaces $H(\mathcal{O})$ and their associated noninteracting field algebras $\mathcal{A}(\mathcal{O})$ are by construction "causally complete" $\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}'')$ and hence a fortiori Einstein causal (section 2 (13)). The causal completion $\mathcal{O}''$ is obtained by taking twice the causal complement $\mathcal{O} \to \mathcal{O}'$.

For wedge regions the modular group coincides with the unitary Wigner representation of the wedge-preserving Lorentz group; for all other regions the modular groups in $m > 0$ Wigner representations acts in a non-geometric ("fuzzy") way. It is believed that this nongeometric action becomes asymptotically (near the boundary of the causal completion) geometric.

Massless finite helicity theories have a larger set of regions related to Huygen's principle in which modular groups act in a geometric way; this includes all regions which are images of wedges under the action of conformal transformation as e.g. finite double cones. Sl potentials whose semiinfinite spacelike lines remain inside wedges may under suitable conformal transformation pass to potentials localized on finite elliptic curves which connect two points on different edges of double cones; they can be viewed as substitutes for the nonexistent pl coordinatization of double cones.

It may also happen that the standard spaces for compact spacetime regions $\mathcal{O}$ are trivial $H_{\mathcal{O}} = \{0\}$. This occurs precisely for the Wigner's *zero mass infinite spin representation* for which the tightest localized nontrivial spaces correspond to modular localization in arbitrary narrow spacelike cones. On the other hand zero mass *finite helicity* spaces are the most geometric representation since their modular groups continue to act geometrically even for double cones; in fact they correspond to conformal transformations which preserve double cones. Unlike finite helicity fields *infinite spin* representations are massless but *not conformal*.

Modular localization plays also an important role in the understanding of topological peculiarities of massless $h \geq 1$ free QFT in connection with toroidal regions (thickened Wilson loops). Last not least without their use it would not have been possible to discover the intrinsic noncompact localization of Wigner's infinite spin matter and the string-like nature of its generating causally localized fields.

This raises the question if modular theory preserves its constructive power in the presence of interactions. It turns out that in that case one is required to use the stronger operator algebraic modular theory. Interacting theories share the same modular groups $\Delta_{\mathcal{W}}^{it}$ with their noninteracting counterpart which is solely determined by the particle content.

The interaction enters through the dependence of the $J_{\mathcal{W}}$ on the interaction which, using the fact that the incoming TCP is related with its outgoing counterpart through the S-matrix S [69], [30], amounts to

$$J_{\mathcal{W}} = J_{\mathcal{W}}^{(in)} S \tag{62}$$

Again the starting point is a von Neumann operator algebra $\mathcal{A}$ acting in a Hilbert space $\mathcal{H}$ which contains a vector Ω which is cyclic and separating under the action of $\mathcal{A}$ (in QFT *the Reeh–Schlieder property* for $\mathcal{A}(\mathcal{O})$)

$$\text{cyclic:} \quad \mathcal{A}(\mathcal{O})\Omega \quad \text{is dense in} \quad \mathcal{H} \tag{63}$$
$$\text{separating:} \quad A\Omega = 0 \quad \Rightarrow \quad A = 0, \;\; A \in \mathcal{A}(\mathcal{O})$$

The definition $H = \overline{\mathcal{A}_{sa}(\mathcal{O})\Omega}$ (sa = selfadjoint part) or $\mathcal{H} = \overline{\mathcal{A}(\mathcal{O})\Omega}$ connects the algebraic modular theory with its previously presented spatial counterpart. But there remains an important difference: the map between standard subspaces and algebras is generally not injective whereas the algebraically generated subspaces (63) always are.

The interaction-free situation remains exceptional in that there exists a functorial relation between modular localized Wigner subspaces and interaction-free causally localized subalgebras defined in terms of the Weyl map (60). This functorial map is lost in the presence of interactions in which case the relation between modular theory and particles becomes more involved.

4.3. A critical perspective based on modular localization

Most of what is presently known about modular theory comes from the Bisognano–Wichmann theorem [27] which clarifies the modular properties of wedge-localized subalgebras $\mathcal{A}(W)$. In models with a complete particle interpretation one can use the modular theory of free fields to derive the B-W theorem in the presence of interactions [81]. Of special interest is the relation of the modular conjugation J with the TCP operator which is known to be connected to the S-Matrix [69]. This is particularly useful in $d = 1 + 1$ integrable models whose S-matrix is known (62).

For integrable models without bound states this led to an interpretation of the Zamolodchikov–Faddeev algebra in terms of modular localization in which the concept of "vacuum-polarization-free generators" (PFG)[25] plays an important role [30], [73]. This in turn led to existence proofs for certain $d = 1 + 1$ integrable models in terms of operator algebraic constructions based on modular theory [76]. For a recent account with many references to previous publications see [31]. These results complement those obtained in terms of the "bootstrap-formfactor" program in [75].

The important role of the S-matrix for modular localization in wedges has triggered attempts to reconstruct a full causal QFT from its "on-shell footprint" in form of its S-matrix [77,78]. These ideas are presently too weak for constructions in higher dimensions. Among the unexpected results of modular theory is the proof that models in $d = 1 + 2$ with anyonic statistics (or its nonabelian "plektonic" counterpart) have always nontrivial S-matrices [79].

Whereas the idea that a nonperturbative QFT is uniquely determined by its S-matrix remains still part of folklore, the S-matrix based perturbative SLFT construction in the previous section is the basis of the new perturbation theory. Different from the standard approach based on Lagrangian quantizations in which the S-matrix is obtained from the mass-shell restriction (the LSZ reduction formula) of time-ordered correlation functions of fields, SLFT inverts this situation by encoding the model-defining particle content into an interaction density of a perturbative defined S-matrix. This part of the construction uses exclusively pl or sl free fields which are directly related to the particles. Interacting (off-shell) quantum fields are constructed in a second step in which the higher order contributions to the interaction density (which were induced in the construction of S) serve as an input (44).

Each pl or sl field from the equivalence of free fields and their Wick-ordered composites has an interacting counterpart. Interacting fields which have nonvanishing matrixelements between

[25] The weakest assumptions under which PFGs exist were determined in [74].

the vacuum and a one-particle state have after appropriate normalization the same large-time in- and out-limits.

Finally we come to an important point whose clarification was promised at the beginning of this section namely the possible connection of sl quantum fields with String Theory. String theorists attribute a string-like localization to their objects without providing arguments in favor of causal localizability. Ideas based on worldlines, worldsheets, Nambu–Goto actions or strings start from classical relativistic mechanics and hope that quantization preserve these properties. This is evident from the way in which the covariant world-line action $\sqrt{-ds^2}$ in [80] is used to prepare the ground for the subsequent presentation of world-sheets and Nambu–Goto actions. The futile attempt to place two of such vibrating quantum mechanical strings into a relative spacelike position reveals the problems which ST has with causal localization in spacetime.

Fact is that, apart from Lagrangian quantization of $s < 1$ fields, only the Wigner representation theory (which cannot be accessed by quantization) contains the seed for causal localization. Modular localization as a pre-form of causal localization needs positivity and is inconsistent with gauge theory. One cannot declare an object as "stringy" because its classical action suggests this. This type of misunderstanding is clearly visible in Polchinski's use of the action of a relativistic particle as a preparatory step for relativistic worldsheet and Nambu–Goto actions.

This does not exclude the use of the quantum mechanical Newton–Wigner localization to describe the dissipation of wave-packets. One may even construct a macro-causal Poincaré-invariant multi-particle theory which satisfies cluster decomposition properties and leads to a Lorentz-invariant scattering matrix ([43] section 3). Such a construction disproves the conjecture that relativistic particles and cluster properties alone will lead to QFT.

Proposals which avoid quantization of classical actions and try to find causal quantum matter in the target space of certain models of $d = 1 + 1$ conformal QFTs are less easy to dismiss. One such proposal is based on the Virasoro-algebra of a supersymmetric 10-component chiral conformal current model (the "superstring"). Its target space contains an algebraic structure which leads to a highly reducible unitary Wigner representation of the Poincaré group [83].

This is primarily a group theoretical observation which (apart from the ten spacetime dimensions) fits well into Majorana's 1932 project of finding algebraic structure which, in analogy to the $O(4, 2)$ hydrogen spectrum, describes wave functions of families of higher spin/helicity particles. But why should one believe that the corresponding fields are sl in the absence of any supporting argument? Is the terminology perhaps related to picturing the *tower* of representations containing different masses and spins as Fourier components of a kind of *internal* circle? In that case the use of "string" for something bears no relation to spacetime and would be misleading.

Looking at the ST literature one gets frustrated about the disproportionate relation between its conceptual poverty as compared to its mathematical richness which its vague physical pictures lead to in the hands of mathematicians. The word "string" should be more than an "epitheton ornans" for a physically insufficiently understood mathematical formalism.

With additional conceptual care one can also avoid a widespread misunderstanding in the physical interpretation of the AdS_{n+1}-CFT_n isomorphism. It was certainly consequential to complement the observation of equality of the symmetry groups of the two spacetimes by the verification of a stronger Einstein causality-preserving isomorphism between the two QFTs. But unlike classical field theory the timelike completion property[26] in QFT $\mathcal{A}(\mathcal{O}'') = \mathcal{A}(\mathcal{O})$, (which roughly speaking describes causal propagation) *is not a consequence of the spacelike Einstein*

[26] The causal completion $\mathcal{O}''$ is the result of taking twice the causal complement.

causality. Formally it is equivalent to Haag duality $\mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})'$ from which it results by taking the commutant on both sides and rewriting $\mathcal{A}(\mathcal{O}')'$ by applying Haag duality to the (generally noncompact) region $\mathcal{O}'$.

In the old days [86], [87] it was shown that Einstein causality and the causal dependency property (formally equivalent to Haag duality) are *independent* requirements. Causality without Haag duality occurs if there are "too many" degrees of freedom as in the case of the generalized free field; a phenomenon which has no classical analog (since the notion of quantum *degrees of freedom* has no classical counterpart).

This manifests itself in a sort of "poltergeist effect" in that there may be *more quantum degrees of freedom streaming into the dependency region as time moves on than there were in the original appropriately defined initial ("Cauchy") data*; in operator-algebraic notation $\mathcal{A}(\mathcal{O}) \subsetneq \mathcal{A}(\mathcal{O}'')$ while Einstein causality $\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{O}')'$ is preserved.

In fact it is quite easy to construct Einstein-causal models for which this completion property is violated. Generalized free fields with a suitably large κ behavior of their Kallen–Lehmann spectral functions (containing a much larger cardinality of degrees of freedom than free fields) were used at the beginning of the 60s[27] to show that the *causal shadow property* represents a separate requirement (initially called "the time-slice property").

The heuristic picture is that "squeezing" a QFT with the natural cardinality of degrees of freedom corresponding to a *n+1*-dimensional QFT into an *n* dimensional spacetime ("holographic image") causes an "overpopulation"; this is precisely what happens in the AdS_{n+1}-CFT_n case [85]. The simplest illustration is obtained by projecting a free AdS field and noting that the resulting conformal field is a generalized free field of the kind used in [86], [87]. In the *opposite* direction i.e. starting from a "normal" CFT one expects an "anemic" degree of freedom situation on the AdS side. As shown in [84] this is precisely what happens; in fact there are no degrees of freedom at all in compact AdS regions (double cones); to find any one has to pass to infinitely extended wedge-like regions in AdS.

The overpopulation of degrees of freedom distinguishes "holographic projection" from "normal" lower dimensional QFT and this explains the quotation marks in "pathology"; a holographic projection maintains the degrees of freedom of the original QFT and in this way prevents the conformal side to be a normal QFT. This is a point which had been ignored in most post Maldacena work.

A helpful viewpoint concerning such "overpopulated" models which one obtains by dimension reducing projections is to not consider them as autonomous QFTs but to view them rather as stereographic projections of the original QFT.

The degree of freedom issue is not limited to the AdS-CFT isomorphism but *affects all attempts to extend (quasi)classical Kaluza–Klein dimensional reductions of lowering extra dimensions to full-fledged causal QFT*. To the extent that such pictures are compatible with the principles of QFT[28] they correspond to perform a stereographic projection on the original theory and not to pass to a lower dimensional QFT. This poses the question whether the fashion around extra dimensions would have occurred with a higher conceptual awareness about the subtle nature of positivity and causal localization of relativistic quantum matter which forbid the use of naive quasiclassical arguments.

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[27] I am sometimes asked about the origin of this terminology. The answer is simple, it accounts for the fact that Haag had this idea already before I entered the collaboration; my contributions consisted in providing calculations involving generalized free fields.

[28] *Massaging* Lagrangians is not the same as passing from one QFT to another.

A breakdown of Haag duality for entirely different reasons occurs in local nets generated by massless helicity $h \geq 1$ free field strengths. This "topological duality violation" was mentioned in section 2 where its relation to the Aharonov–Bohm effect and linking numbers was explained. It is lost in the positivity-violating gauge theoretical setting which cannot distinguish between the Haag duality and the somewhat coarser Einstein causality.

In his book on Local Quantum Physics Haag proposes an interesting extension from the established one-fold duality for observable algebras to causally separated two double cone localized algebras. In fact he explores the possibility of the existence of a homomorphism from the orthocomplemented lattice of causally complete regions in Minkowski space into that of von Neumann algebras of observables ([2], Tentative Postulate 4.2.1). For a region which consists of two spacelike separated double cones K_1, K_2 this requires $\mathcal{A}(K_1 \vee K_2) = \mathcal{A}(K_1) \vee \mathcal{A}(K_2)$.

Haag notices with a certain amount of disappointment that duality is violated for doubly localized observable algebras associated to the conserved currents of free fields. This follows from the existence of a pair $\psi(f)\psi(g)^*$ with $supp\ f \subset K_1,\ supp\ g \subset K_2$ which commutes with $\mathcal{A}(K_1 \vee K_2)$ but is not in $\mathcal{A}(K_1) \vee \mathcal{A}(K_2)$. He viewed this as a shortcoming of free fields which he expected to disappear in the presence of interactions. He uses the idea of a "gauge bridge" between $\psi(f)$ and $\psi(g)^*$ as a hint that a future positivity-maintaining replacement of gauge theory may satisfy this strengthened form of causality ("Haag duality") may.

The string-bridges of SLFT do precisely this i.e. they prevent that Haag duality of two causally separated double cones is violated by a charge–anticharge pair. In contrast to gauge bridges which have no material content (since they can be changed by gauge transformation implemented by indefinite metric gauge charges) string-bridges consist of quantum matter. Hence the existence of string bridges provides a local method to distinguish an interacting net from a free one.

Another somewhat more metaphoric way of saying the same is that SLFT results from gauge theory by applying Occam's razor to indefinite metric- and ghost-degrees of freedom. Gauge theory is its best placeholder within the setting of Lagrangian quantization.

Interestingly the same string bridges which save Haag duality of observables also allow to view interpolating fields as resulting from space- or light-like limits in which the anticharge component is disposed of at infinity but leaves a trail of quantum matter behind.

Whereas all sl fields in the absence of interactions can be obtained as semiinfinite line integrals from pl fields,[29] this breaks down in the presence of interactions. In that case the necessarily sl localized interpolating $s < 1$ fields receive their sl localization metaphorically speaking through being "infected" by their contact with higher order $s \geq 1$ sl potentials with which they share the interaction density. This will be exemplified in a number of models in the next section.

For anybody who has grown up with Haag's way of looking at QFT it is deeply satisfying that the model-dependent division between gauge invariant observables and gauge dependent interpolating fields corresponds to the SLFT localization dichotomy between pl observables and sl interpolating fields.

The fault line, which unfortunately still separates the ST community from those who are working on the successful but still largely unfinished project of QFT, runs precisely alongside the issue of causality and its refinements. The recent progress on entanglement entropy [32], [33] requires a profound understanding of causality in the context of operator algebras. The fact that these ideas are presently also leading to a revision of perturbative QFT raises hopes that this schism will be overcome.

[29] Note that the interaction density and the S-matrix only uses such free fields.

5. Renormalization in the presence of massive sl vector mesons

5.1. Remarks on scalar QED; induced interactions and counterterms

As mentioned in the introduction SLFT differs both in its concepts as well as in its calculational techniques from Lagrangian- (or Euclidean action)-based quantization theories. In section 3 these differences played a role in the solution of the Velo–Zwanziger causality conundrum and in the present section they will be exemplified in a full QFT.

The simplest nontrivial model for illustrating these differences (for reasons which will become clear in the sequel) is scalar QED.

Being an S-matrix-based QFT, the starting point of SLFT is an S-matrix which, following Bogoliubov, is formally written as a time ordered product of an interaction density $L(x,e)$ as in (43). The construction of interacting sl fields uses Bogoliubov's map (44) which converts free fields into their interacting counterparts whose large-time asymptotic behavior reproduces the scattering amplitudes associated to S. In this section we will be exclusively interested in the S-matrix; localization properties of interacting fields will be mentioned in section 6.

The simplest nontrivial model for which the preservation of positivity requires the use of sl localized fields is *scalar QED*. Here "nontrivial" refers to the appearance of a 2nd order induced term $A\cdot A\varphi^*\varphi$ and a 4th order counterterm $(\varphi^*\varphi)^2$ which has no counterpart in spinorial QED.

The independence of the large-time LSZ limits of causally separable fields on their original localization is the basis of the SLFT perturbation theory. The following is in part a recollection of arguments presented in section 3.

In lowest order we may start with the pl interaction density $L^P = A^P_\mu j^\mu,\ \ j^\mu = i\varphi^*\overleftrightarrow{\partial^\mu}\varphi$ and use the linear relation with its short distance improved sl counterpart and its escort $A^P_\mu = A_\mu - \partial_\mu\phi$ to write

$$L^P = L - \partial^\mu V_\mu,\ with\ V_\mu = j_\mu\phi \tag{64}$$

$$S^{(1)} = \int L^P(x)d^4x = \int L(x,e)d^4x \tag{65}$$

This solves two problems in one stroke, the highest short distance contribution to L^P has been encoded into a divergence which drops out in the adiabatic limit (second line) so that S is string-independent (the left hand side) as well as renormalizable (the right hand side).

Actually one may forget the L^P and formulate the SLFT construction solely in terms of a L, V_μ pair fulfilling the $L-\partial V=0$ pair condition. It turns out that the L, V_μ pair corresponding to vector mesons interacting with lower spin particles and possibly among themselves is uniquely determined: the interaction is completely determined in terms of its particle content! In other words the L^P can be defined in terms a SLFT pair which is in turn defined of the particle content.

For the formulation of the higher order pair property it is convenient to use the differential form of the pair property (as in section 3) and write $d_e(L-\partial^\mu V_\mu)=0$. Further simplification is obtained by using the "Q-formalism" with

$$\begin{aligned} &Q_\mu = d_e V_\mu = j_\mu u,\ u = d_e\phi \\ &\quad d_e L - \partial^\mu Q_\mu = 0 \end{aligned} \tag{66}$$

The Q_μ turns out to have a better $m\to 0$ behavior; it is only logarithmically divergent and $\partial^\mu Q_\mu$ remains finite. The logarithmic infrared divergence is a perturbative spacetime indication that the massless limit of vacuum expectation values cannot be described in terms of Wigner–Fock space which is simply a tensor product of a helicity space of photons with that of charge-carrying Wigner particles [50]. In other words the S-matrix based perturbative SLFT formalism indicates that its massless limits needs a (presently unknown) extended formulation of scattering theory.

A notational simplification for the higher order pair conditions is obtained by using lightlike strings (not possible for $m = 0$). In this case the massive sl fields are functions in e rather than distributions and hence all e may be set equal. The second order pair condition reads

$$d_e TLL' - \partial^\mu TQ_\mu L' - \partial'^\mu TLQ'_\mu = 0 \tag{67}$$

and the extension to higher than second order is straightforward. They have no counterpart in the standard pl setting and account for the strength of SLFT as compared to the standard pl perturbation theory.

Violations of these relations are referred to as *obstructions*. The Bogoliubov S-matrix formalism is preserved by encoding these obstructions into a redefinition of the interaction density $L \to L_{tot} = L + L_2 + ..$ just as it was done for external potential interactions in section 3.

The time-ordering T which fulfills (67) is not necessarily the "kinematic" time ordering T_0 which is defined by attaching a $-i2\pi(p^2 - m^2 - i\varepsilon)$ denominator to the momentum space 2-ptfct. The scaling rule of renormalization requires that T and T_0 share the same scaling degree which in the presence of two derivatives leaves a normalization freedom

$$\left\langle T\partial_\mu\varphi^*\partial'_\nu\varphi'\right\rangle = \left\langle T_0\partial_\mu\varphi^*\partial'_\nu\varphi'\right\rangle + icg_{\mu\nu}\delta(x-x') \tag{68}$$

which leads to

$$\partial^\mu\left\langle T\partial_\mu\varphi^*\partial'_\nu\varphi'\right\rangle - \left\langle T\partial^\mu\partial_\mu\varphi^*\partial'_\nu\varphi'\right\rangle = i(1+c)\partial'_\nu\delta(x-x') \tag{69}$$

with an initially undetermined c.

The fulfillment of the second order pair requirement (67) in the tree approximation fixes $c = -1$. The action of S on one-particles states as the identity operator $S|p\rangle = |p\rangle$ takes care of the contribution from 2 contractions. The change of T_0 to T in $Tj_\mu j'_\nu = T_0 j_\mu j'_\nu - g_{\mu\nu}\delta$ in all $Tj_\mu j_\nu$ accounts for the occurrence of the second order induced $A_\mu A^\mu|\varphi|^2$ term

$$TLL' = T_0LL' - i\delta(x-x')L_2 \tag{70}$$

$$L_2 = gA_\mu A^\mu|\varphi|^2 \tag{71}$$

which is usually attributed to the implementation of gauge symmetry, but here it follows from the causality and positivity principle of interpolating fields which guaranties the e-independence of S.

The reason for using the kinematical time-ordering T_0 instead of T is the comparison with GT. In SLFT it is more natural to use T in which case the second order L_2 remains encoded in TLL'.

As already pointed out in section 3, GT has a *formally* similar structure. This is most clearly visible in a setting of gauge theory which avoids the standard Lagrangian quantization of gauge theory (for spins $s \geq 1$ see [4]) in favor of a perturbative S-matrix formulation as in [64], [65]. The physical $A_\mu(x,e)$ and its escort $\phi(x,e)$ correspond to the gauge potential A^K_μ and the Stückelberg field ϕ^K acting in a ghost extended Krein space; the authors show that $A^K - \phi^K$ has properties expected from A^P_μ.

The two S-matrix-based constructions share the same improved short distance behavior, but they achieve this in a very different way. Whereas in gauge theory this is the result of enforced compensations between positive and negative probabilities in intermediate states, the ultraviolet improvement in SLFT accomplishes this by lessening the tightness of causal localization (but not abandoning it!) and in this way reducing the strength of vacuum polarization which is the only physical way to describe particles in terms of physical (i.e. not gauge) interpolating fields.

Though both SLFT and gauge theory have the same short distance dimensions and probably even share their Callen–Symanzik equations (and the related asymptotic freedom property encoded in the beta-function), gauge theory *cannot account for the physics at finite distances let alone at long distances*; infrared properties and the problem of confinement remain outside its physical range.

Last not least the functional-analytic and operator-algebraic methods used in deriving nonperturbative theorems from basic principles are not available in Krein spaces. For this reason gauge theory is shunned in books addressing the conceptual structure of QFT [1], [2]. The perturbative gauge theoretic construction of a unitary S-matrix reveals this tension between conceptual clarity and the efficiency of calculations which account for experimental observations; it is a blessing for the impressive achievements of the Standard Model but a curse for a on the principles of positivity and causal localizability formulated QFT of the books.

Considering these conceptual deficiencies the perturbative calculations of a gauge-invariant S-matrix of the Standard Model is a truly impressive achievement. The idea that it represents a successful placeholder of an unknown QFT is quite old and there have been many failed attempts to find the real thing. The close formal analogy between gauge theory and SLFT suggest that both may even exist side by side in a Krein extended Wigner–Fock space containing additional indefinite metric degrees of freedom.[30]

Presently there exist no higher than second order SLFT calculation. Higher order loop calculations in SLFT are much more laborious than calculations in gauge theory. The gauge theoretic 4th order calculation establishes the existence of a $c(\varphi^*\varphi)^2$ counterterm. In contrast to the second order $A \cdot A\varphi^*\varphi$ contribution its strength c is a new parameter which is not determined by electromagnetism of the e.g. π^+ meson.[31]

Could this counterterm in GT be an induced contribution in SLFT? This question is not as crazy as it sounds. The above L^P theory is by itself nonrenormalizable; its short distance dimensions and the number of counterterms increase with perturbative order. Yet if "guided" in the above sense by a L, V_μ pair it shares the finite number of possible free varying parameters with the SLFT L description.

The still missing answers to such questions are not only owed to the fact that the number of theoreticians who are presently working on SLFT problems can be counted on one hand but they also find their explanation in that the necessary calculations are more involved than those based on pl fields. The sl setting of QFT is the only known way to uphold the principles of QFT for *all* fields.

The SLFT approach also touches on an old mathematical problem which arose from QFT in the late 60s. The question was whether fields with $d_{sd} = \infty$ (polynomial unbounded) fields as e.g. Wick-ordered exponential functions of pl fields as $\exp g\varphi$ have a well-defined mathematical status. This led Jaffe to extend the notion of Schwartz distributions to a general class of distributions which still allows smearing with a dense set of compact localized Schwartz test functions.

The SLFT guided construction of the L^P pl setting requires to identify pl observables in both settings and suggests to identify the interpolating state creating fields of the L^P theory with Jaffe

[30] J. Mund seems to have discovered such a "hybrid" formulation (private communication).

[31] Using such a model to describe electromagnetic interactions of charge-carrying pions one usually sets $c = 0$.

fields. They correspond to the well-behaved sl interpolating fields: the two theories share not only the S-matrix but also their local observables whereas the states in the L^P theory remain singular in the sense of Jaffe. Such singular fields are not required to have the usual domain properties which one needs to generate operator algebras from fields so that the algebraic localization of compact spacetime regions is fully accounted for by observables.

After having exemplified the main difference between gauge theory and SLFT in the model of scalar QED, the following subsections will present low order calculations in other models in which vector mesons couple to lower spin matter fields and among themselves. This includes in particular the Higgs models for which, different from the standard treatment, the form of the Mexican hat potential and its spontaneous symmetry breaking is not imposed but rather *induced* as a consequence of e-independence of S. Even more surprising is that the division into observables and sl interpolating is very different from what one naively expects: neither the field strength $F_{\mu\nu}$ nor the Higgs field is a pl observable.

5.2. The perturbative S-matrix in the SLFT setting

The appropriate formalism for the direct perturbative calculation of the on-shell S-matrix is based on the adiabatic limit of Bogoliubov's operator-valued time-ordered $S(g)$ functional. Its adjustment to SLFT has been mentioned in (43) in section 3 and further explored in the previous subsection.

Time-ordering of quantum fields mathematically represented by operator-valued distributions is characterized in terms of properties among which the causal factorization is the physically most important one. The Epstein–Glaser formalism [82] provides a perturbative computational scheme in which the time-ordering of $n+1$ pl interaction densities is inductively determined in terms of the nth ordered time-ordered product. The formulation in the presence of sl fields is more involved and has not been carried out beyond second order. Preliminary results reveal that a systematic nth order construction requires the use of new concepts [95].

The E-G perturbation theory for the S-matrix can be extended to sl fields (44). The result is a formula which maps a field in the local equivalence class of Wick-ordered composites of free fields into the equivalence class of "normal ordered" relative local interacting fields which act in the same Wigner–Fock Hilbert space but are nonlocal with respect to their free counterparts. Nowhere does this formalism refer to Lagrangian quantization. For gauge theory this was first carried out in [61] where it was shown the time-ordering of the S-matrix passes to that of retarded products in terms of fields.

The power-counting restriction of renormalizability $d_{sd}(L^P) \leq 4$ is violated if one of the spin/helicity of the particles is ≥ 1. For interactions involving particles with highest spin $s = 1$ the $d_{sd}(L^P) = 5$ there are two ways to recover renormalizability. Either by converting L^P into a "gauge pair" L^K, V_μ^K which requires the extension of the Wigner–Fock space by indefinite metric degrees of freedom and possibly BRST ghosts, or one maintains the physical degrees of freedom (and with it the positivity of the Wigner–Fock Hilbert space) by converting L^P into sl L, V_μ pair.

For the rest of the paper we will stay with models which are sl renormalizable $d_{sd}(L) \leq 4$. This includes all couplings whose particle content consists of $s = 1$ coupled to $s < 1$ and among themselves. In the case of massless sl vector potentials the escort ϕ diverges as m^{-1} and the large-time LSZ derivation of the S-matrix breaks down (the on-shell restrictions of correlations develop logarithmic $m \to 0$ singularities) and with it the S-matrix based SLFT construction.

However some remnants of the SLFT construction can be saved; the exact one-form $d_e\phi$ and hence also the $Q_\mu = d_e V_\mu$ is only logarithmically divergent and $\partial^\mu Q_\mu$ remains convergent. Hence even in case of breakdown of the S-matrix as a result of infrared problems the L, Q_μ pair condition

$$d_e L - \partial_\mu Q^\mu = 0, \;\; Q_\mu = d_e V_\mu \tag{72}$$

remains a nontrivial condition. In fact it is this weaker formulation of e-independence which corresponds to the BRST invariance of gauge theory.

In the previous subsection it was shown that, although the second order pair condition in its original form is violated, it is possible to encode the obstructing contribution L_2 into a redefinition of the interaction density. It is helpful to formulate this idea in a model-independent way.

The definition of second order obstruction against the naive form of the L, Q_μ pair property reads (using lightlike $e's$ which can be identified)

$$\begin{aligned} O^{(2)} &:= d_e TLL' = T\partial^\mu Q_\mu L' - \partial^\mu TQ_\mu L' + TL\partial'^\mu Q'_\mu - \partial'^\mu TLQ'_\mu \\ O^{(2)} &= \delta(x-x')d_e L_2(x,e) \end{aligned} \tag{73}$$

Encoding them into interaction density one obtains

$$L_{tot} := L + gL_2, \quad S(g) = T\exp\int ig(x)L_{tot}(x,e)d^4x \tag{74}$$

This change of bookkeeping which converts higher order obstruction into induced contributions L_n amounts a change of $L \to L_{tot}$ in the Bogoliubov $S(g)$ is important. It affects the higher orders; the third order obstruction is now

$$O^{(3)}(g,g,g) = d_e\left[TL(g)L_2(g^2) + \frac{i}{3}TL(g)L(g)L(g)\right] \tag{75}$$

In models of interacting $s = 1$ vector mesons as the Higgs model or scalar massive QED the third order obstruction vanishes in the adiabatic limit and the induced contributions account for the Mexican hat potential. As a consequence the terms in this potential are induced and not postulated for the purpose of implementing SSB. This will be explicitly verified in the following subsections.

The L, Q_μ pair condition and its higher order extension within the sl Bogoliubov–Epstein–Glaser setting is also meaningful for $d_{sd}(L) > 4$. The before mentioned "minimal" models contain only induced contributions but their number increases with the perturbative order. By definition of minimal there are no higher order counterterm parameters so the model depends only on those parameters which are already present in the interaction density L.

The conceptual and mathematical superior aspects of SLFT poses the question whether it is possible to pass directly from SLFT to pl ultra fields, thus avoiding the pl counterterm formalism. This problem will come up again in connection with cubic $h = 2$ selfinteractions in the next section (84).

5.3. External source models

Consider a vector potential coupled to an conserved classical current j_μ [11]. The S-matrix and the interacting vector potential are

$$L^P = A_\mu^P j^\mu = A_\mu j^\mu - \partial_\mu(\phi j^\mu),\ hence\ L = A_\mu j^\mu,\ V_\mu = \phi j_\mu \tag{76}$$
$$S_e(g) = T\exp i\int g(x)L(x,e) \overset{g(x)\to g}{\to} S = \exp ig\iint j^\mu i\Delta_F j'_\mu : \exp ig\int A_\mu^P j^\mu :$$
$$A_\mu^{ret}(x,e,e') = S_e{}^{-1}(g)\frac{-i\delta}{\delta f_\mu(x,e')}S_e(g, j\to j+f)|_{f=0} = A_\mu(x,e) + \int G_{\mu\mu'}^{ret} j^{\mu'}$$
$$G_{\mu\mu'}^{ret}(x,e;x',e') = (-\eta_{\mu\mu'} - \partial_\mu e_{\mu'} I_{e'} + \partial_{\mu'} e'_\mu I_{-e} + (ee')\partial_\mu\partial_{\mu'} I_e I_{-e'})G^{ret}(x-x')$$

The direct use of L^P with $d_{sd}(L^P) = d_{sd}(A_\mu^P) = 2$ leads to a delta function ambiguity $g_{\mu\nu}c\delta(x - x')$ in the time-ordered A_μ^P propagator which accounts for a replacement $i\Delta_F \to i\Delta_F + \frac{c}{m^2}\delta$ in the second line. This in the pl formulation undetermined counterterm renormalization parameter in the S-matrix and in $A_\mu^{P,ret}$ is absent in the less singular sl formulation.

In that case S is independent of c and (by use of current conservation) the interacting field does not depend on e'. As expected the field strength remains pl. Hence the avoidance of the direct use of A_μ^P in the calculation maintains the predictive power of the model. If needed one can convert the sl setting with the help of $\phi(x,e)$ to a A_μ^P. In contrast to the directly calculated A_μ^P this via sl determined pl potential is "better".

Passing from external source to *external potential* problems the differences between the direct pl results and those obtained via the sl detour are much stronger (section 3).

5.4. Hermitian H coupled to a massive vector potential

The coupling of a vector potential to a Hermitian scalar matter field H comes with a new phenomenon. In addition to a change of the time-ordered product of the H-field there is now a genuine *induction* of H-selfinteractions.

The "germ" of an interaction density (the "ignition") for an A_μ, H field content is the $mA\cdot AH$ coupling, where the vector meson mass factor m accounts for the classical dimension $d_{eng} = 4$ and also indicates that the model has no nontrivial Maxwell limit (the reason why it was discovered a long time after QED). Its sl operator dimension is $d_{sd} = 3$, hence the germ is a superrenormalizable sl interaction density. The first order L, Q_μ pair property ($Q_\mu = d_e V_\mu$) requires the presence of the escort ϕ also in L and leads to (L, Q_μ relation easy to check)

$$L = m\left\{A\cdot(AH + \phi\overleftrightarrow{\partial}H) - \frac{m_H^2}{2}\phi^2 H\right\} + U(H),\ U(H) = mc_1H^3 + c_2H^4$$
$$V_\mu = A_\mu\phi H + \frac{1}{2}\phi^2\overleftrightarrow{\partial_\mu}H,\quad d_e(L - \partial^\mu V_\mu) = 0,\ L^P = L - \partial^\mu V_\mu. \tag{77}$$

A systematic determination of this first order pair L, V_μ pair starting from the simplest coupling (the "germ") $gmA\cdot AH$ of the A-H particle content and a general ansatz for L and V_μ containing all kinematically possible $d_{sd} \le 4$ terms (19 terms in L) which can be formed from H, A_μ and its escort ϕ shows that (77) is (up to ∂_μ divergence terms and exact d_e differentials) is the unique solution [88]. However a verification that the L, V_μ pair satisfies the pair condition requires only the use of free field equations and the relations between A_μ and its escort and will be left to the reader.

The first order pair condition does not determine the strength of the H-selfinteractions since e-independent contributions to L simply pass through the pair condition. The necessity of their presence which includes the determination of the c_i in (77) is seen in second and third order.

This “induction” of additional contributions with well-defined numerical coefficients is a new phenomenon of SLFT; there is a formal similarity with the imposition of the second order BRST gauge invariance on the S-matrix [64] but the essential difference is that the e-independence of S is a consequence of the positivity and causal localization principle of QFT.

For the S-matrix one only needs the second order tree component to the obstruction $O^{(2)}$ in (73) In addition to a second order change of the time ordering of the propagator involving derivatives of H which parallels that in (68) one now encounters a genuine second order induction (74)

$$L_2 = g[(m_H^2 + 3c_1 m^2)H^2\phi^2 - \frac{m_H^2}{4}\phi^4 + c_2 H^4] \tag{78}$$

Finally the vanishing of the third order tree contribution fixes the values of c_1, c_2 in terms of the three physical parameters of its field content which were already present in the germ namely g, m, m_H. To allow for a comparison with the Higgs mechanism we write the result in the form

$$L_{tot}^{(2)} = mA\cdot(AH + \phi\overleftrightarrow{\partial}H) - V(H,\phi),\ V = g\frac{m_H^2}{8m^2}(H^2 + m^2\phi^2 + \frac{2m}{g}H)^2 - \frac{m_H^2}{2g}H^2 \tag{79}$$

where $L_{tot}^{(2)} = L + \frac{g}{2}L_2$. The appearance of a quadratic mass term is the result of writing the interaction density as if it would be part of a classical Lagrangian of gauge potentials. The reader may fill in the details of the straightforward calculations by himself or look up the more detailed presentation in [88].

Apart from a mass contribution the V looks like a field-shifted Mexican hat potential. *But different from the Higgs mechanism it has not been obtained by postulating a Mexican hat potential and subjecting it to a shift in field space*. It is rather *induced* by a renormalizable A, H *field content* and it is the *unique renormalizable QFT with this field content*. There is simply no room for imposing a Mexican hat potentials since the induction of the H and ϕ selfinteractions is a consequence of e-independence of the S-matrix which in turn is a consequence of scattering theory involving $d_{sd} = 1$ causally separable space- or light-like strings.

The SSB picture of the Higgs model also reveals another common misunderstanding, this time about SSB. The Mexican hat potential together with the shift in field space is *not the definition* of SSB but *rather a way to implement* such a situation *when it is possible*. The definition of SSB is rather the *existence of a locally conserved current whose global charge diverges*. This is only possible in the presence of massless Goldstone bosons and all verbal attempts to make SSB consistent with a mass gap (a photon becoming fattened to a vector meson by eating a Goldstone) only obscure the interesting correct understanding.

QFT is not a theory which can create the masses of its model-defining field content. In particular SSB is not about creating finite masses from an initially massless situation; to the contrary it is about how to place a massless particle (the Goldstone boson) into an interaction density so that the current conservation remains that of a symmetric theory but some local charges are prevented to converge in the infinite volume limit to a finite global charge (the definition of SSB). The only known The prescription of a field shift on a Mexican hat potential as the “Higgs mechanism” has to be seen in a historical context; it helped to overcome the formal problems which one faces when one tries to extend Lagrangian quantization from Maxwell’s theory of charge-carrying fields to a situation in which a vector potential couples to a Hermitian matter fields. There are numerous historical illustrations of situations for which important discoveries were made through formal manipulations which were later replaced by a derivation which is consistent with the principles of QFT. Incorrect placeholder are useful but only up to the discovery of the real reasons.

A model of QFT is uniquely fixed in terms of its field content. The SLFT setting (which seems to be the only one consistent with all principles of QFT) for a A_μ, H field content starts with a $A_\mu A^\mu H$ as the simplest coupling and the rest is done by induction using the $L.Q_\mu$ pair property which converts the heuristic physical content of the ill-defined pl interaction density into the physically superior SLFT setting where the "induction" resulting from the implementation of the pair property to all orders unfolds the full content of SLFT.

5.5. Selfinteracting vector mesons

It is straightforward to check that there is no renormalizable L, Q_μ pair for a self-coupled singlet $(A \cdot A)^2$. The principles of QFT as embodied into the pair condition admit however selfinteractions between multiplets ("colored") of vector potentials while imposing strong restrictions on the "multi-colored" coupling parameters. In this case the germ is a FAA selfinteraction and the general ansatz for the construction of a L, V_μ pair which includes the "colored" escorts is of the form

$$L = \sum (f_{abc} F_c^{\mu\nu} A_{a,\mu} A_{b,\nu} + h_{abcd} A_{a,\mu} A_{b,\nu} A_c^\mu A_d^\nu) + \text{terms in } A_{a,\mu} \text{ and } \phi_a' s \tag{80}$$

where the couplings and the masses of the vector mesons are initially freely variable parameters but, as expected, the first and second order pair condition places strong restrictions on them [88], among other things the f and h are interrelated in the same way (Jacobi identities of reductive Lie-algebras) as in gauge theory [64]; in particular the A–ϕ and ϕ–ϕ couplings depend also on the masses of the vector mesons. The main distinction to gauge theory is that these properties are direct consequences of the principles of QFT and do not arise in the course of the gauge theoretic extraction of physics from a unphysical (positivity-violating) description through the imposition of gauge invariance.

The most interesting aspect of the SLFT formulation is that there remains a renormalizability destroying second order induced selfinteraction which, if left uncompensated, destroys renormalizability even though the interaction density fulfills the power counting restriction of renormalizability. The way to overcome this is to compensate this $d_{sd} = 5$ term with a second order contribution from a A–H interaction with a scalar Higgs field.[32] This is a totally different situation from the abelian A–H interaction for which such all second order terms stay within the power counting bound. Neither case bares any physical resemblance to spontaneous symmetry breaking since in both cases the field shifted Mexican hat potential is second order induced.

The idea of short distance compensations between contributions from different spins arose in connection with supersymmetry. Although not invented for this purpose, SUSY does improve the short distance behavior somewhat but not enough to guaranty the renormalizability and preservation of supersymmetry in higher orders. The situation of selfinteracting vector mesons is different, in that the preservation of renormalizability is the raison d'être for the H. Nature does not have to decide between a symmetry and its SSB, rather the existence of the H is directly connected to the preservation of its positivity and causality principles or in other words a massive A_α^μ field content by itself is not consistent.

[32] A $s \geq 1$ field would worsen the second order short distance behavior.

Gauge symmetry is not a physical symmetry so there is nothing to break; all these physically incorrect pictures evaporate if one maintains causality *and* positivity which is perturbation theory is only possible by starting with an sl L, V_μ or Q_μ pair property. The fiber bundle like Lie structure of the f_{abc} couplings is not the result of an imposed symmetry it rather arises from the string-independence of the S-matrix which in turn is a result of LSZ scattering theory of interacting causally separable positivity obeying quantum fields; hence the situation is very different from the superselection structure of unitary representation classes of observable algebras which leads to the notion of inner symmetries. This shows that quantum causality is much more fundamental than its classical Faraday–Maxwell–Einstein counterpart.

Having thus strengthened the conceptual understanding of interactions between vector mesons in the Standard Model one may ask whether SLFT contains also messages about their coupling to matter. In recent work [89] it was shown that SLFT does not only restrict the selfcouplings between vector mesons and requires the presence of a Higgs particle in the presence of selfinteracting massive mesons but it also restricts their coupling to the Fermion currents and their chirality properties. This is of particular interests for massive $W_\pm.Z$ vector mesons and the photon, a case for which the authors explain the restrictions from SLFT in detail.

5.6. The pair condition for higher spins

The extension of SLFT S-matrix construction to that of interacting higher spins $s \geq 2$ is an important issue about which one presently knows little. There have been quite extensive investigations in a gauge theoretic equivalent of the pair condition by Scharf [64]. In view of formal similarities with SLFT it is interesting to take a closer look at some of his results.

Scharf looked at the simplest $s = 2$ selfinteraction which is of a cubic form trh^3 where $h_{\mu\nu}$ is the $s = 2$ massless tensor field. The physical interest in this model is connected with the use of $h_{\mu\nu}$ as a linear approximation of the gravitational $g_{\mu\nu}$ field. As in SLFT, the short distance dimension of integer spin gauge fields is equal to their classical dimension in terms of mass units namely $d_{sd} = 1$. In [64] it was shown that there exists no gauge theoretic trilinear selfinteraction L^K with $d_{sd}(L^K) = 3$ without involving derivatives of $h_{\mu\nu}$, its trace h^μ_μ as well as ghost fields and their anti-ghost. He found a cubic interaction density of $d_{sd}(L^K) = 5$ which is above the power-counting bound of renormalization, but still presents a huge reduction from the $d_{sd}(L^P) = 11$.

Taking into account that gravitational coupling carries a dimension and expanding the Einstein–Hilbert Lagrangian in a suitable way using $\kappa = \sqrt{32\pi G}$ as an expansion parameter, he arrived at a formal connection of the classical expansion with the quantum-induced correction up to second order; this was later extended to all tree orders [64,90]. The agreement of tree approximations with classical perturbation theory is not unexpected in itself, but in the present context it relates two competing ideas, one being of classical geometric origin (the Einstein–Hilbert action) and the other the gauge theory of selfinteracting $h = 2$ particles.

Christian Gaß showed recently (private communication) that SLFT provides a simpler version of such a cubic selfinteractions in the form

$$L = \kappa(2\partial_\rho h^{\kappa\lambda}\partial_\sigma h_{\kappa\lambda} + 4\partial_\beta h^\alpha_\rho \partial_\alpha h^\beta_\sigma)h^{\rho\sigma}, \quad h_{\mu\nu} := A^{(2)}_{\mu\nu} \tag{81}$$

$$d_e A^{(2)}_{\mu\nu} = \partial_\mu a_\nu + \partial_\nu a_\mu, \tag{82}$$

where $h_{\mu\nu} = A^{(2)}_{\mu\nu}$ is the sl helicity 2 potential from (25) section 2 (which already played a role in solving the D-V-Z discontinuity problem [11]). Using the relation between d_e and ∂_μ of the second line one easily verifies that $d_e L$ is of the form $\partial^\mu Q_\mu$ i.e. the above L belongs to a L, Q_μ pair. Since massless $h \geq 1$ fields are intrinsically sl, the corresponding minimal models are expected to be "ultra-distributions" which are localizable in spacelike cones.

For $h = 1$ there exists no colorless selfinteraction whereas for $h = 2$ the situation seems to be reverse since the existence of colored cubic selfinteractions can be excluded [91]. A proof based on Scharf's S-matrix gauge formalism can be found in [92].

The fact that there are no *renormalizable* $s = 2$ selfcouplings does not exclude the possibility to find sl L, Q_μ pairs of interactions between sl $h_{\mu\nu}$ with lower spin fields as H or/and A_μ. An ansatz for L which generalizes the A_μ, H particle content of the abelian Higgs model would be of the form ($h_{\mu\nu}$ massive)

$$L = mgh_{\mu\nu}h^{\mu\nu}H + U(H, h, \phi) \tag{83}$$

where the first term represents the "ignition" i.e. the simplest renormalizable ($d_{sd} = 3$) interaction associated with a h, H particle content and U contains all the remaining possible at most quadrilinear couplings between $h_{\mu\nu}$, its 5 escorts ϕ_μ, ϕ and H. Their coupling strengths are determined from the first or second order ("induction") pair condition.

The L, Q_μ pair may be uniquely determined, but it is rather improbable that $d_{sd}(L) \leq 4$. It would be premature to dismiss L, V_μ pairs with $d_{sd}(L) > 4$. The example of pl models with $d_{sd}(L^P) = 5$, which in the standard pl renormalization theory leads to a with perturbative order increasing number of renormalization parameters but under the guidance of an S-matrix-equivalent sl pair turns into an improved formalism. This upgraded L^P description contains $d_{sd} \to \infty$ pl fields but shares its parameters, the S-matrix and its pl local observables with the SLFT renormalization theory.

Presently our understanding of the consequences of the higher order SLFT pair requirements is too scarce to say anything credible about L, V_μ pairs with $d_{sd}(L) > 4$. A clarification of this important issue will be left to future research.

6. Dynamical string-localization of interacting fields

Free massive pl fields can not only be converted into their sl counterparts by integration along strings but the directional $e^\mu \partial_\mu$ differentiation on sl free fields permits also the return to its pl form. Together with their Wick-ordered composites they form the local equivalence (sl extended Borchers-) class $\mathcal{B}$ of free fields (pl fields are viewed as special cases of sl).

Recall that for the construction of the S-matrix corresponding to a prescribed particle content one uses pl fields for $s < 1$ and those special $s \geq 1$ massive sl potentials which were constructed in section 2.3 by "fattening" their uniquely defined sl massless counterpart. Together with the uniquely defined pl Proca potential and a scalar sl field referred to as the escort they constitute a triple of relatively causally localized fields which act in the massive $s = 1$ Wigner–Fock space. They fulfill a linear relation which is the basis for the construction of renormalizable sl interaction densities with string-independent S-matrices.

This "kinematic" sl localization of $s \geq 1$ free fields is important for the construction of the S-matrix ala Bogoliubov. But it does not account for the *physical localization of the interacting fields* which is not in the hands of the calculating physicists but is determined by the particle content of the model. To distinguish between the two the localization of the interacting fields will be referred to as "dynamic localization".

To understand this important point it is helpful to recall the form of the *Bogoliubov map* which relates the pl or sl Wick-ordered free fields from the local equivalence class of free fields $\mathcal{B}$ to that of normal ordered interacting fields $\mathcal{B}|_L$ (44). For pl gauge theoretic interactions densities L^K this problem has been studied in [61].

One important result is that this perturbatively defined linear Bogoliubov map preserves the relative causality of fields but not the algebraic structure. This is in agreement with algebraic QFT which is based on the idea that the full physical content of QFT in the presence of interactions is contained in the net of spacetime localized algebras [2]. What is shared between $\mathcal{B}$ and $\mathcal{B}|_L$ in case of massive vector potentials is the Wigner–Fock Hilbert space in which these fields act.

This transfer of pl causality undergoes significant changes in the presence of sl fields. As in the calculations in the previous section one uses a lightlike e, in this case no directional testfunction smearing is necessary.

For the understanding of changes in localization caused by the Bogoliubov map it is not necessary to enter the details of perturbative renormalization. It suffices to understand the relations between free fields in $\mathcal{B}$ which result from the assumption that their interacting images of the Bogoliubov map into the target spaces $\mathcal{B}|_{L_{tot}}$ and $\mathcal{B}|_{L^P_{tot}}$ coalesce. Hence one may omit the prefactors S^{-1} in the Bogoliubov maps and write

$$S(g(x)L^P_{tot} + \lambda f\varphi_g)|_{\lambda=0} \overset{a.l.}{\simeq} S(g(x)L_{tot} + \lambda f\varphi)|_{\lambda=0} \tag{84}$$

$$\varphi_g|_{L^P_{tot}} = \varphi|_{L_{tot}}, \quad \varphi_g(x,e) = \varphi(x,e) + \sum_{k=1}^{N} \varphi_k(x,e) \tag{85}$$

where the $\varphi_g|_{L^P_{tot}}$ refers to the interacting image of φ_g under the L^P_{tot} Bogoliubov map.

The information about the localization of an interacting field $\varphi|_{L_{tot}}$ is contained in the left hand side[33] whereas its renormalizability status (finite or infinite d_{sd}) can be read off on the right hand side. Fields which are renormalizable and at the same time pl in the L^P setting represent observables whereas renormalizable fields which are sl on the L^P_{tot} side are sl interpolating fields.

The formal combined map of $\mathcal{B}$ into itself is highly non-linear and generally changes localization properties; this is the price for the preservation of renormalizability. The $\varphi_k(x,e)$ in (85) are determined by the induction

$$\varphi_{k+1}(x,e) = ig\int T(L^P_{tot}(x') - L_{tot}(x',e))\varphi(x,e) = ig\int [\partial'^{\mu} T]V_{tot,\mu}(x',e)\varphi_k(x,g)d^4x' \tag{86}$$

where $[\partial'^{\mu}, T]$ denotes the difference between the ∂ acting outside and inside the time-ordering which either vanishes or contributes a δ-term.

In massive QED this conversion (85) has no effect on pl observables; fields as A^P_μ and $F = \mathrm{curl}\, A^P$ simply pass through since with $V_\mu = \phi j_\mu$ and $\varphi_0 = A^P_\mu$ the right hand side (86) vanishes and hence

$$A^P_\mu(x)|_{L^P} = A^P_\mu(x)|_L, \quad F_{\mu\nu}|_{L^P} = F_{\mu\nu}|_L \tag{87}$$

The idea underlying such conversions was first used by Mund [93] in the context of massive spinor QED. He calculated higher orders for the charge-carrying ψ (spinor or complex scalar) and found consistency with

33 The important point is that the L^P Bogoliubov map preserves localization; hence one can use it to find out whether the image of φ under L is sl or pl.

$$\psi(x)|_{L^P} = e^{ig\phi(x,e)}\psi(x)|_L \tag{88}$$
$$\psi(x)|_L = e^{-ig\phi(x,e)}\psi(x)|_{L^P} \tag{89}$$

The formula is reminiscent of gauge transformation, however its physical content is quite different.

A particularly interesting application of the conversion formalism arises in the Higgs model. Different from massive QED, neither the $s = 1$ field $A^P_\mu|_{L^P}$, nor H_{L^P} are local observables. Using the form of V_μ in (77) one finds that H is transformed into H_1

$$H_1(x,e) = -\int [\partial'^\mu, T]V_\mu(x')H(x)d^4x' = \frac{1}{2} :\phi^2(x,e): \tag{90}$$

i.e. H is against naive expectations not an observable but rather represents a sl interpolating field. The same holds for A^P or its $F = \mathrm{curl}\, A^P$.

Allowing additive composite modifications $H \to H + polyn(H, A^P)$ which preserve the asymptotic scattering state of the H-particle does not change the situation. The same holds for the A^P_μ or $F = \mathrm{curl}\, A^P$. Hence both fields which are linearly related to the particle content of the model are interpolating fields and do not represent observables. The fact that $A^P_\mu|_{L^P}$ and F are observables[34] in massive QED but not in the Higgs model shows that the observable-interpolating field dichotomy is not a kinematic property.

Hence fields representing local observables in the Higgs model are necessarily composite. A composite field which exists in every model is the interaction density $L^P_{tot}|_{L^P_{tot}} = L_{tot}|_{L_{tot}}$. The right hand side was calculated in (79) and the computation of L^P will be contained in a forthcoming publication [88].

A better understanding about the singular structure of pl fields may shed new light on the localizability of $d_{sd} = \infty$ fields which arose in connection with summing up graphical structures in certain nonrenormalizable models [94]. This problem was taken up by Arthur Jaffe [66], [67] who discovered a new class of singular distributions which still permit smearing with a dense set of compact supported Schwartz testfunctions. These Jaffe distributions had no impact on QFT because the unguided pl nonrenormalizability with its infinite number of renormalization counterterm parameters does not present a well-defined arena for physical applications. Such $d_{sd} = \infty$ pl fields are probably too singular to generate operator algebras, but they may still create physical states in the SLFT-guided L^P formalism.

In section 3.2 we sketched the application of this formalism to interactions with external potentials. Such interactions do not lead to loop contributions. This simplicity of only induced contribution promises an interesting mathematically controllable playground for the study of the pl localization properties of observables and that of sl interpolating fields.

SLFT is presently the last step in a process of dissociating QFT from its historic ties with Lagrangian quantization. When shortly after the discovery of renormalized QED Arthur Wightman presented his "axiomatic" formulation of QFT in terms of pl fields, it appeared to be the most appropriate intrinsic formulation which can be extracted from Lagrangian quantization and Wigner's representation theory [1]. In his algebraic formulation of Local Quantum Physics (LQP) Rudolf Haag proposed a setting of QFT based on a net of localized operator algebras representing observables which removed the last vestiges of quantization [2].

[34] The line integral over an observable commutes with "switching on" the interaction and does not represent an interpolating fields.

The next step was taken in the 80s by Buchholz and Fredenhagen who showed that the existence of observable algebras and suitably defined particle states guaranties the presence of operators localized in arbitrary narrow spacelike wedges (whose cores are strings) which create these particle states from the vacuum [16]. These constructions were too far removed from the exigencies of renormalized perturbation theory in order to have a direct impact on calculations.

As a result it took more than 30 years to incorporate these observations into a new sl perturbation theory in whose discovery the understanding of the noncompact localization of Wigner's infinite spin matter was an important catalyzer [35]. Fortunately one does not have to go through the details of this history in order to do perturbative calculations. But what may be interesting to note is that, different from Wightman's extraction of his axiomatic setting from what one learned from the mathematically rather ill-defined rules of Lagrangian quantization, the construction of SLFT took the opposite path by converting ideas from LQP into perturbatively accessible computations.

Its most remarkable physical property is that observables are distinguished from interpolating fields in terms of localization, which is of course to be expected in a theory based on causal localization, but which GT could not accomplish.

There remains the question of how GT with its lack of quantum positivity for interpolating fields achieves to be such an amazingly successful description. This will be commented on in the concluding remarks.

7. Resumé, loose ends and an outlook

SLFT is a formulation of QFT in which renormalizable interacting fields maintain the tightest possible localization which is compatible with quantum positivity and causality. In contrast to gauge theory its physical range is not limited to local observables and the S-matrix but also includes string-local interpolating fields which mediate between the causality principles of QFT and the string-independent scattering properties of particles. All degrees of freedom are provided by Wigner's particle representation theory.

As described in the introduction the discovery of SLFT was triggered by the construction of sl free fields associated to Wigner's positive energy infinite spin representation [35]. Yngvason's 1970 No-Go theorem [14] precluded the existence of pl fields. It turned out that Wigner's massless infinite spin representation presents a much stronger barrier against pl localization than that observed by Weinberg and Witten in massless finite helicity representations. The Weinberg–Witten No-Go theorem excludes the existence of conserved higher helicity pl currents and energy-momentum tensors; in view of the absence of massless pl vector potentials and the fact that the existence of pl massless limits depends on the short distance dimension d_{sd} this is hardly surprising.

The infinite spin case excludes the existence of pl composites; more general: the causal localization of infinite spin matter is necessarily noncompact [42] in concordance with smearing sl fields with directionally compact localized test functions $f(x,e),\ e^2 = -1$. Closely related is that *infinite spin matter cannot interact with ordinary (finite spin) quantum matter*, but through its energy-momentum tensor its backreaction on classical gravity may lead to a noncompact form of gravity. Quantum inertness combined with gravitational reactivity are properties attributed to dark matter [38]. Since the sl infinite spin energy-momentum tensor is known as a bilinear form [8] such a calculation appears feasible.

The existence of sl infinite spin field with *finite* d_{sd} suggested that the renormalizability destroying $d_{sd} = s + 1$ increase of short distance dimension can be avoided by using sl fields. This was the start for the construction of sl potentials for finite s, h which provided the positivity preserving (the Gupta–Bleuler degrees of freedom avoiding) $d_{sd} = 1$ potentials. As mentioned in section 3 the absence of pl currents does not exclude the existence of local charges which are localized in arbitrary small spacetime regions.

The weakening of causal localization in SLFT should not be misunderstood as (what is commonly referred to as) "nonlocal".[35] The use of covariant semi-infinite space- or light-like half-lines does not get into conflict with the causality prerequisites of scattering theory (namely the possibility of placing an arbitrary number of fields in relative spacelike positions), nor is the derivation of important structural theorems (TCP, Spin&Statistics) impeded.

Among the continuously many sl potentials only one for each s plays a role in SLFT perturbation theory. The key observation for its construction is that the equation curl $A = F$ for a *sl massless field* $A_\mu(x,e)$ *acting in the Wigner–Fock helicity space* associated to the $(m = 0, h = \pm 1)$ Wigner representation has a unique solution which replaces the positivity violating pl potential of GT.

By a process referred to as "fattening" (section 2) this solution selects among the many possible massive sl potentials (which act in the $s = 1$ Wigner–Fock Hilbert space of the unique pl Proca potential) a distinguished sl vector potential. Together with a canonically constructed scalar sl potential $\phi(x,e)$ (the escort) one obtains a triple of linear related fields $A_\mu - \partial_\mu\phi = A_\mu^P$ which act in the $s = 1$ Wigner Fock space and belong to the linear part of the causal equivalence class of (Wick-ordered) free fields associated to the Wigner representation $(m > 0, s)$.

The string independence expressed as the pair relation $d_e(A - \partial\phi) = 0$ is the basis for constructing a renormalizable interaction density $L(x,e)$ which couples the $s = 1$ sl A and ϕ fields to lower spin free fields which remain pl. Together with a suitably defined vector density V_μ one arrives at the pair relation $d_e(L - \partial V) = 0$ which insures the string-independence of the S-matrix which is obtained by taking the adiabatic limit of time-ordered product of the interaction density. The lowest order pair relation may need an extension by induced terms which result from the implementation of higher order pair conditions. This is a new phenomenon which has no counterpart in the old pl perturbation theory.

The interacting quantum fields associated to this S-matrix are constructed in terms of the Bogoliubov map which converts pl or sl fields from the causal equivalence class of Wick-ordered free fields into their normal ordered interacting counterpart. The restriction to pl $s < 1$ and sl $s = 1$ free fields is only necessary in the construction of the S-matrix; the Bogoliubov map can be applied to *any* (pl or sl, elementary or composite) field in the free field class.

Its interacting target fields have in general a different localization from their source fields. The target localization has to be determined in the L^P setting (see previous section). Renormalizable ($d_{sd} < \infty$) fields in the L target space (independent of their pl or sl localization) represent observables if their L^P source fields are pl; otherwise they represent interpolating sl fields.

SLFT has been applied up to second order to all models in which vector mesons interact with themselves or with $s < 1$ particles. The by far conceptually most demanding and interesting QFT is the Higgs model which in its most simple (abelian) form is the QFT in which a vector meson interacts with a $s = 0$ Hermitian field. The first order pair turns out to be uniquely fixed and its second order implementation induces a H selfinteraction which looks as if it would come from spontaneous symmetry breaking on a postulated Mexican hat selfinteraction. The conceptual difference to SLFT is enormous.

[35] The authors of [10] had problems with referees who rejected the work with the argument that SLFT is nonlocal.

A similar but somewhat more elaborate second order calculation for selfinteracting massive vector mesons reveals that the coupling structure of the *leading* $d_{sd}=4$ *contributions* up to second order is that of a reductive Lie-algebra. The surprise is that, different from gauge theory, this apparent symmetry in the d_{sd} leading contribution has not been imposed. In fact it is not even a symmetry in the sense in which this terminology is used to describe unitary implemented inner symmetries.

Whereas symmetries and their spontaneous or complete breaking of selfinteracting scalar particles can be freely imposed, there are strong restrictions from first principles on the form of $s\geq 1$ SLFT selfinteractions which leave no such freedom; the form of selfinteractions in the presence of $s\geq 1$ is fully determined by the particle content of the model and not at the disposition of the calculating theorist. The use of the positivity violating gauge symmetry obscures this important insight. The chirality theorem [89] shows that these principles also affects the coupling of selfinteracting vector potentials to Dirac fermions.

Another somewhat unexpected property is that renormalizable interaction sl densities L may produce second order sl $d_{sd}=5$ contributions which, if left uncompensated, destroy the e-independence of S as well as renormalizabilty. The only way to save such a model is to enlarge its particle content by a A–H interaction which induces a compensating second order A selfinteraction. *This, and not SSB, is the raison d'être for the presence of an H-particle in models of selfinteracting massive vector mesons.*

The application of the SLFT perturbation theory to the Higgs model leads to other foundational questions whose answer may be trendsetting for the development of QFT. The basic interaction density $A\cdot AH$ for a A_μ, H particle content (the "ignition" from which the L, V_μ pair requirement uniquely induces all other contributions) is superrenormalizable since $d_{sd}(AAH)=d_{cl}=3$. One does not expect that interactions induced by superrenormalizable couplings lead to higher order counterterms with new coupling parameters. A 4th order confirmation of this expectation does presently not exist (neither in SLFT nor in the gauge theoretic SSB setting).

Even more important is to find out if $SLFT$ permits an extension to $s\geq 2$. The remarks in section 5.6 on $s=2$ selfinteractions show that $d_{sd}(L)=5$. To conclude that the theory is useless because it violates the power-counting bound is premature since (previous section) the main reason for dismissing interaction densities is that they lead to a with perturbative order increasing number of coupling parameters and not the fact that there are fields with an increasing short distance scaling degree. For the acceptance of a model it suffices that its S-matrix is well-defined and that its observables remain pl with bounded d_{sd}, independent of whether the d_{sd} of its sl interpolating fields increase with perturbative order.

Candidates with $s=2$ potentials $A_{\mu\nu}$ and superrenormalizable "ignition" of the form $A_{\mu\nu}A^{\mu\nu}H$ or $A^{\mu\nu}A_\mu A_\nu$ and induced L, V_μ pairs couplings with $d_{sd}(L)=5$ are expected to exist. As long as the number of counterterm coupling parameters does not increase with perturbative order and the physical predictability is maintained there is no obvious reason for their exclusion of such L, V_μ pairs. Only further research can resolve these challenging problems.

Can SLFT shed some light on the perplexing question *why GT inspite of its obvious conceptual shortcomings*[36] *remained such an amazingly successful theory?* This paradigmatic question may have a positive answer. The L^K, V_μ^K pair property is a consequence of the relation

[36] Positivity is an indispensable property which secures the probability interpretation of quantum theory.

$A_\mu^P = A_\mu^K + \partial_\mu \phi^{P,K}$ where K refers to the Krein space of GT and $\phi^{P,K}$ is a scalar pl "hybrid" escort which mediates between the P and K formalism [96].

In the BRST formulation used in [64] the fields act in a Stückelberg- and ghost-extended BRST space. The physical space, to which the action of A_μ^P can be restricted, is defined in terms of BRST cohomology and observables are defined as objects invariant under the BRST operation $\mathfrak{s}$ ($\mathfrak{s}O = 0$ for observables and $\mathfrak{s}S = 0$ for the S-matrix).

The advantage of the hybrid formulation proposed by Mund [96] is that, different from the formalism used in [64], Stückelberg- and ghost-degrees of freedom are not needed. Instead of spaces which are embedded in the sense of BRST cohomology one deals with factorization through Gupta–Bleuler subspaces.

An explicit expression for the mixed hybrid escort $\phi^{P,K}$ was recently calculated by Mund (private communication). The Proca potential lives in the transverse subspace to the mass shell $p^\mu \psi_\mu(p) = 0$ which is embedded in the Krein space whereas the living space of fields is the full 4-component Krein space. The triple relation can the be used to define a L^K, V_μ^K pair which is S-matrix-related to the physical L^P formulation.

A favorable situation for studying *infrared phenomena* arises from the hybrid triple $A_\mu^{sl}(x,e) = A_\mu^K(x) + \partial_\mu \phi^{sl,K}(x,e)$. The A_μ^S (without) lives on the physical subspace of the Gupta–Bleuler Krein space. All three contributions have a massless limit, but $\phi^{s,K}$ without the derivative has the typical logarithmic infrared divergence known from scattering theory of charge-carrying particles. Its exponential $\exp ig\phi^{S,K}(x,e)$ seems to provide the kind of directional superselection rule of photon "clouds" whose presence is required by a theorem [50]. This picture is a closer analog of (10) than the $\exp ig\Phi(x,e,e')$ constructed in (9).

This hybrid pair description does not only explain the close relation of a (from ghosts and negative metric Stückelberg fields liberated) Gupta–Bleuler GT with the positivity preserving SLFT, but it also shows that GT plays a useful constructive role for a better understanding of SLFT in QED. The hybrid relation reveals that the physical origin of quantum gauge theory is that one cannot squeeze causally spacetime localized pl vector potentials into the Wigner momentum space; this is only possible by permitting a noncompact but still causally separating localization.

In particular it contains information about the change of the Wigner particle space for the $\mathcal{B}|_L$ operators (previous section) in the massless limit. Whereas the fields in B live in a Wigner–Fock helicity space, their interacting images in $\mathcal{B}|_L$ act on a larger space for whose construction one needs to form line integrals on indefinite Gupta–Bleuler potentials $A_\mu^K(x)$ (still indefinite) and convert them into complex exponential fields (the photon clouds) whose associated Hilbert space is expected to show a similar infrared structure as the exponentials $\exp ig\varphi(x)$ of the indefinite logarithmic divergent massless $d = 1 + 1$ scalar fields (the φ-clouds).

The hope is to obtain a spacetime understanding of infrared phenomena including the large-time behavior which replaces that of the LSZ scattering theory. This includes the vanishing of scattering between charge-carrying particles with only a finite number of outgoing photons.

This cannot be described solely in terms of free matter fields, rather the exponential sl dependent photon cloud fields must play an important role. Similar to the φ-clouds in a two dimensional model (10) they are expected to "soften" the mass-shell singularity and account for the zero probability for the emission of a finite number of photons in collisions of charged particles whereas a perturbative expansion which ignores this change of the mass-shell leads to the logarithmic infrared singularities. As often, the devil is in the details.

SLFT also suggests that behind confinement there could be a more radical *off-shell* perturbative logarithmic infrared divergence of massless selfinteracting gluons. Such off-shell divergences are absent in covariant gauges of nonabelian GT, but off-shell long distance singular behavior of self-interacting gluons *in SLFT* may be stronger than in GT [45], [38].

Most problems of SLFT remain unsolved; on particular the present state of knowledge about higher order perturbative renormalization is insufficient. Its strengths are that the new ideas passed many tests and that the promise to transcend the conceptual limitations of GT is too tempting to resist.

This may be attributed in part to the fact that its underlying ideas are in embryo and the number of researchers who know about their existence and decided to study them is still very small. There is no lack of researchers working on foundational problems of QFT extending the pioneering work of Wightman, Haag and others. Most theoreticians use the existing gauge theoretic formalism to solve problems of high energy particle physics or cosmology. During the last 5 decades a lot of time has been invested in research on speculative ideas as String Theory, Multiverses, Supersymmetry and alike; the incentive was obviously to continue the success of the first three decades of QFT in which such speculative way of proceeding was very successful and which led to most of our by now household goods.

The lack of any tangible results of these attempts led meanwhile to feelings of somberness. The new insights into QFT provided by SLFT raise the question why loose time with speculative ideas if we still know so little about our most successful theory?

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