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Collusion and Delegation under Information Control

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Abstract

This paper studies how information control affects incentives for collusion and optimal organizational structures in principal-supervisor-agent relationships. I consider a model in which the principal designs the supervisor's signal on the productive agent's private information and the supervisor and agent may collude. I show that the principal optimally delegates the interaction with the agent to the supervisor if either the supervisor's budget is large or the value of production is small. The principal prefers direct communication with the supervisor and agent if the supervisor's budget is sufficiently small and the value of production is high.

Keywords: Collusion, Information design, Delegation
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1 Introduction

In many organizations, a supervisor advises the principal on how to set the contractual terms for a productive agent with private information. Honest advice from the supervisor lowers the agent's informational advantage and allows the principal to reduce the agent's information rent. However, this creates scope for collusion as the agent is willing to pay the supervisor for biased advice that increases his rent. Public procurement is a prominent example of such a setting.[1] In many countries, procurement officers at central purchasing bodies advise public buyers and corruption is prevalent — often in the form of private suppliers paying bribes in exchange for highly priced public contracts.[2]

The threat of collusion influences the optimal organizational design of principal-supervisor-agent relationships. The extant literature studies whether hierarchical delegation is an optimal response to collusion, i.e., whether the principal can achieve the payoff from the optimal centralized mechanism by contracting only with the supervisor who in turn designs the agent's contract (Faure-Grimaud, Laffont and Martimort, 2003; Celik, 2009). The literature assumes that the supervisor's information about the agent is exogenous to the principal and draws a different conclusion of the optimality of delegation depending on the specific information structure that generates the supervisor's information.[3]

In this paper, I study whether delegation is an optimal response to collusion if the principal can influence what the supervisor learns about the agent. In many settings where the principal-supervisor-agent model can be applied, the development of information technology has made the allocation of information endogenous. In public procurement, most advanced economies have digitalized their procurement systems. With an e-procurement system, the allocation of information to the different

[1] OECD governments spend an average of 29% of total expenditure on public procurement (OECD, 2017).

[2] In a recent corruption scandal, an employee of the Italian central purchasing body, Consip, was allegedly bribed for the award of a public contract worth 2.7 billion Euro (ANSA, March 1, 2017). According to the OECD (2014), 57% of cases of foreign bribery payments were made to receive a public contract. See Di Tella and Schargrodsky (2003) for more detailed evidence of this type of procurement fraud in hospitals in Buenos Aires.

[3] Faure-Grimaud et al. (2003) prove that delegation is optimal in a model where the agent has one of two types and the supervisor may observe one of two signal realizations. Celik (2009) demonstrates that delegation may be strictly suboptimal when the agent has three types and the supervisor observes the element of a partition of the agent's type space in which the true type lies.

stakeholders is an important choice variable.[4] These technical developments bring up not only the question of whether information control and delegation are substitutes or complements, but also how information control could be used to fight collusion in supervisory institutions.

To study endogenous information, I consider a standard principal-supervisor-agent model as in Faure-Grimaud et al. (2003) and Celik (2009) and add information control of the principal over the supervisor. The agent can produce a good for the principal at a privately known cost. The supervisor observes a signal of the agent's costs. In the spirit of the literature on Bayesian persuasion (Kamenica and Gentzkow, 2011), the principal exerts information control by choosing the information structure that generates the supervisor's signal. Under centralization, the principal offers a grand contract to the supervisor and the agent. This centralized mechanism allows both the supervisor and the agent to send messages to the principal. The supervisor and agent collude by signing an enforceable side-contract that specifies side-payments and coordinates their behavior under the grand contract. Under delegation, the principal sets a grand contract under which only the supervisor can send messages. The supervisor and agent can still sign a side-contract which now serves as a sub-contract between the lower tiers of the hierarchy.

I show that the principal can implement the optimal centralized outcome by delegation if either the supervisor's budget is large or the principal's value of the good is small. In particular, the principal can extract the full surplus by delegation if the supervisor's budget is large enough. By contrast, centralization outperforms delegation if the supervisor's budget is sufficiently small and the principal's value of the good is large. Moreover, a partially informed supervisor is optimal for the principal whenever the supervisor's budget is strictly positive.

As pointed out by Faure-Grimaud et al. (2003), the key difference between centralization and delegation is the agent's outside option from rejecting the side-contract offered by the supervisor. Under centralization, the agent can reject the side-contract and still participate in the grand contract non-cooperatively. Under delegation, the agent has to accept the side-contract in order to participate in the grand contract. Thus, the principal can use the agent's rent from the non-cooperative equilibrium of the grand contract as an additional instrument under centralization. This instrument might be valuable as the agent's rent in the non-cooperative equilibrium of

[4]Twenty-nine OECD countries used a national e-procurement system in 2016 (OECD, 2017).

the grand contract determines his bargaining position within the colluding coalition. A higher rent improves the agent's bargaining position and makes it harder for the supervisor to find a profitable side-contract. If the instrument is valuable to the principal, centralization is better than delegation. Otherwise, delegation is as good as centralization.

With information control, the principal faces a trade-off between information elicitation and collusion prevention. If the supervisor receives additional information, the agent's informational advantage over the supervisor decreases. As long as the supervisor shares her information truthfully, this is beneficial to the principal. However, it also reduces information asymmetry in the colluding coalition and therefore enables the supervisor and the agent to collude more effectively. Faure-Grimaud et al. (2003) already note that the principal's payoff is maximal if the supervisor's signal has an intermediate precision. In this paper, I analyze how this trade-off can be optimally resolved in a model with an arbitrary type and signal space where precision cannot be captured by a single parameter.

In Section 4, I derive upper bounds on the principal's payoff for the cases of centralization and delegation. These upper bounds exceed the principal's payoff in the absence of a supervisor whenever the supervisor has a strictly positive budget. Thus, a supervisor may only be helpful to the principal if she can absorb some loss.

In Section 5, I present a combination of information structure and grand contract with which the principal attains the upper bound on the payoff with delegation. Under this combination, there is a cutoff cost level such that the good is produced if the cost is (weakly) below the cutoff and the good is not produced otherwise. The information structure generates a different signal realization for each type below the cutoff. The types above the cutoff randomly generate one of these signal realizations. This makes the signal noisy. Under the grand contract, the supervisor and agent receive for production a total payment equal to the cutoff. The agent is offered a price equal to the unique type below the cutoff that is possible after the signal realization, the supervisor keeps the difference between the cutoff and this type as a bonus if the agent accepts the offer. Without production, the supervisor has to make a transfer to the principal. Clearly, the agent receives no rent under this grand contract. The supervisor receives a positive rent with production and a negative rent without production. In order to minimize the supervisor's expected rent for any signal realization, the information structure pools more types above the cutoff into signal realizations that lead to a higher bonus under production. Thus, the bonus and the probability of production are negatively assorted. If the supervisor's budget is large enough, the principal can extract the full surplus in expectation by making the supervisor the residual claimant under production while setting a sufficiently negative payment without production.

In Section 6, I present the main results of this paper. First, delegation is optimal if either the supervisor's budget is large enough or the principal's value of the good is small. Second, centralization is superior to delegation if the supervisor's budget is sufficiently small and the principal's value of the good is high. As discussed above, delegation is inferior to centralization whenever the principal finds it optimal to use the agent's rent in the non-cooperative equilibrium of the grand contract as an instrument. If this is the case, the agent's rent is strictly positive and the principal's payoff is bounded away from the full surplus. As the principal can extract the full surplus under delegation if the supervisor's budget exceeds a threshold, a continuity argument implies that delegation remains optimal if the supervisor's budget lies in some region below the threshold.

Delegation is also optimal if the principal's value of the good is small. In that case, the cutoff separating producing and non-producing types is small and the probability of production is low. Even if the supervisor's budget is low, the principal can therefore effectively reduce the expected rent of the supervisor by setting negative payments for the supervisor in the relatively frequent case of no production. This is feasible under delegation and can be achieved as in the grand contract described above. By contrast, it is not feasible under delegation to impose losses on the supervisor with production as the supervisor can always avoid production (and the associated losses) by asking the agent for a prohibitively high bribe for the opportunity to produce. If the principal's value of the good is high, the mass of producing types is large and it is more effective to extract rents from the supervisor by imposing losses with production. I construct a combination of information structure and grand contract under centralization that allows the principal to do this. I show that this combination gives the principal a strictly higher payoff than the optimal payoff under delegation if the principal's value of the good is large and the supervisor's budget is sufficiently small. For this parameter range, the combination of information structure and grand contract is near-optimal as its payoff is a first-order approximation of the upper bound under centralization.

As noted above, this paper contributes to the literature on collusive supervision with adverse selection[5] (Faure-Grimaud et al., 2003; Celik, 2009; Mookherjee, Motta and Tsumagari, 2019) by introducing information control on the principal's side. The literature builds on the approach of Laffont and Martimort (1997, 2000) to mechanism design with collusion by modeling collusion as an enforceable side-contract between asymmetrically informed parties.[6,7] In contrast to Faure-Grimaud et al. (2003), Celik (2009), and this paper, Mookherjee et al. (2019) analyze a model of collusive supervision in which the colluding coalition can enter a side-contract before accepting the contract offered by the principal.[8] The participation decision of the agent and supervisor can therefore be part of the collusive agreement in the side-contract.[9] They show that delegation is strictly suboptimal in this setting.[10]

In this paper, I model the principal's control over the supervisor's signal in the spirit of the literature on Bayesian persuasion (Kamenica and Gentzkow, 2011; Rayo and Segal, 2010). Thus, the principal can choose an arbitrary signal design at no cost. In line with the literature on collusive supervision, I assume that the principal cannot himself observe the signal realization and that the signal is observed by the supervisor and the agent. Thus, the signal realization is public for the players of the mechanism set by the principal.[11]

Bergemann, Brooks and Morris (2015) and Roesler and Szentes (2017) study the implications of information design in models of bilateral trade. Bergemann et al. (2015) analyze the payoffs for the buyer and seller that can result from varying the information the seller possesses on the buyer's valuation. Roesler and Szentes (2017) study the optimal information acquisition of a buyer regarding her valuation. In the current paper, the trading relationship between principal and agent is intermediated

[5]Collusion and delegation with two productive agents is studied under moral hazard by Baliga and Sjöström (1998) and under adverse selection by Laffont and Martimort (1998).

[6]Che and Kim (2006) study the cost of collusion in a general mechanism design framework.

[7]Green and Laffont (1979) and Tirole (1986) study collusion with symmetric information, Crémer (1996), McAfee and McMillan (1992), and Caillaud and Jehiel (1998) consider specific mechanisms.

[8]Mookherjee, Motta and Tsumagari (2018) apply the model of Mookherjee et al. (2019) to develop a transaction cost-based theory of international vertical integration. Mookherjee and Tsumagari (2017) consider a variant of this model with a stronger form of collusion.

[9]Further papers that study this form of collusion are Mookherjee and Tsumagari (2004), Dequiedt (2007), Pavlov (2008), Che and Kim (2009), and Che, Condorelli and Kim (2018).

[10]Delegation is also often studied based on the model of Crawford and Sobel (1982). See Ivanov (2010) for an analysis of information control in this model.

[11]See also the literature on information design in games (Bergemann and Morris, 2013, 2016; Taneva, 2019; Mathevet, Perego and Taneva, forthcoming).

by a supervisor whose information can be varied.

This paper is also related to Ortner and Chassang (2018). They analyze a principal-monitor-agent model under moral hazard and show that corruption can be fought by introducing asymmetric information in the colluding coalition through the use of random transfers. In contrast to the present paper, it is therefore the terms of the contract and not the type of the agent which creates asymmetric information within the coalition. Inducing asymmetric information on transfers is costless in their setting as the principal only cares about expected transfers. Thus, the principal does not face a trade-off between information elicitation and collusion prevention. This trade-off is central to the analysis in this paper.[12]

The remainder of this paper is organized as follows. In Section 2 I illustrate the benefits of information control in a simple example. Section 3 introduces the general model. Section 4 sets up the principal's problem and provides benchmark and preliminary results. In Section 5 I characterize the optimal combination of information control and grand contract under delegation. Section 6 provides conditions for the optimality of either delegation or centralization under endogenous information. Section 7 discusses several extensions of the model. Section 8 concludes. All proofs can be found in the appendix.

2 An Illustrative Example

In this section, I present a simplified version of the general model to show how the principal can benefit from information control while delegating the interaction with the agent to the supervisor. The agent A can produce a good at cost θ which is uniformly distributed on $\{1, 2, 3\}$. The principal P values the good by $v \in (2, 3)$. The supervisor S observes a signal σ about θ and is endowed with a budget $\ell \geq 1$. In the absence of S, P optimally offers A the monopsony price $p^* = 1$ and makes an expected payoff of $\frac{1}{3}(v - 1)$.

If S is perfectly informed about A's cost and A is unable to pay bribes to S, P can extract the full surplus by delegating the interaction with A to S. In particular, S can be authorized to choose the price that P offers to A. As S is a disinterested party, she finds it optimal to pick $p = \theta$ if $\theta \leq 2$ and $p < 3$ if $\theta = 3$. Given this

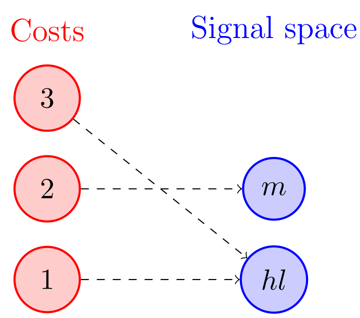
[12]Negenborn and Pollrich (2018) study the optimal use of asymmetric information about the contract in a general principal-supervisor-agent model.

behavior, P's expected payoff is the expected full surplus of $\frac{2}{3}v - 1$.

With collusion and perfect information of S about A's cost, this arrangement is prone to manipulation. In particular, S may promise A to always choose the maximal price P is willing to pay in exchange for a bribe. Under this form of collusion, P's payoff – given the maximal price $\overline{p}$ – is $\Pr(\theta \leq \overline{p})(v - \overline{p})$, weakly smaller than the monopsony payoff of $\frac{1}{3}(v-1)$. Thus, collusion destroys all benefits from supervision if S is perfectly informed about A.

Can P be better off if S knows less about A's costs? Suppose S perfectly learns A's cost whenever $\theta = 2$ but cannot distinguish $\theta = 1$ from $\theta = 3$. The information structure underlying S's signal is depicted in Figure 1. S either observes the signal realization $\sigma = m$ and knows that $\theta = 2$, or observes $\sigma = hl$ and updates her beliefs to the uniform distribution over $\{1, 3\}$.

Figure 1: Information structure of partially revealing signal



Signal realization hl is released if costs are 1 or 3. Signal realization m is released if costs are 2.

Furthermore, suppose P authorizes S to pick a price offer p to A. P pays S a transfer of $2-p$ if A accepts p, and S pays 1 to P if A rejects p. If the signal realization is m, S optimally offers a price of 2 and receives a payoff of zero as A always accepts. If the signal realization is hl, S optimally offers a price of 1. A accepts the offer if $\theta = 1$ and rejects if $\theta = 3$. For $\theta = 1$, S makes a profit of 1. For $\theta = 3$, S makes a loss of 1. This loss does not exceed her budget ℓ. Thus, S receives an expected payoff of $\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot (-1) = 0$ after the signal realization hl. It follows that S accepts the delegation contract after both signal realizations. The delegation contract is robust to collusion as the total payment of P to S and A depends only on the production decision. S therefore has no interest in increasing the price offer to A as this comes at her own expense.

Under this combination of a partially revealing information structure and a delegation contract, P extracts the full surplus even though S and A can collude. This follows from the observation that the production decision is efficient and neither S nor A receives a positive rent in expectation. Thus, a partially informative signal and delegation are optimal for P. This result extends to the general model as long as S's budget ℓ is large enough.

P's preferred information structure is non-monotone as it pools non-adjacent types into the same signal realization. Such non-monotone information structures naturally arise in settings where production costs are determined by two dimensions – a project-specific and an agent-specific dimension – and where the supervisor observes only one dimension.

Consider a procurement project that consists of two tasks. Each task is either of type a or of type b. If both tasks have type a (b), the procurement project is an a-project (b-project). If the tasks have different types, the procurement project is an m-project. A is either specialized in tasks of type a or of type b. A's costs are given by 1 plus the number of tasks in which it is not specialized. A knows its specialization and observes the project type. P knows neither the project type nor A's specialization and believes that the project type is independent from the specialization and that each project type and each specialization is equally likely. Figure 2 summarizes this description.

Figure 2: The determinants of A's cost θ

	a-specialist: $^1/_2$	b-specialist: $^1/_2$
a-project: $^1/_3$	1	3
m-project: $^1/_3$	2	2
b-project: $^1/_3$	3	1

θ depends on project type and specialization. Each project type and specialization is equally likely.

If S observes only the project type but not A's specialization, her learning process can be represented by the information structure in Figure 1. If S observes that the project is type a or b, she can infer that costs are going to be high or low. If S

observes that the project is of type m, she knows that costs are intermediate. Thus, P benefits from disclosing data about the project to S while hiding data about past performances of A.

3 The Model

Principal and agent

The principal, P, seeks to procure a single indivisible good. The agent, A, can produce the good at cost θ. A is privately informed about θ which is the realization of the random variable $\tilde{\theta}$ with distribution $F(\theta) = \Pr(\tilde{\theta} \leq \theta)$ on $\Theta \subseteq \mathbb{R}$ with $Conv(\Theta) = [\underline{\theta}, \overline{\theta}]$. P's value of the good is $v \in \mathbb{R}$ with $v > \underline{\theta}$. Given a transfer t from P to A, P's payoff is $vX - t$ and A's payoff is $t - \theta X$ where $X \in \{0, 1\}$ denotes the production decision. Production is efficient for $v \geq \theta$. The resulting expected full surplus is

$$\overline{W} \equiv \int_{\underline{\theta}}^{v} (v - \theta) dF(\theta).$$

If P and A are the only players, P cannot do better than to offer A the price[13]

$$p^* = p^*(v) \equiv \max \arg \max_{p \in \Theta} (v - p) F(p).$$

I denote the resulting monopsony payoff for P by $\underline{W}$.

Supervisor and Information Control

The supervisor S learns about A's cost θ by observing the realization σ of the signal $\tilde{\sigma}$. The signal realization is also observed by A but not by P.[14] S's payoff equals the net transfer she receives and does not depend on the production decision. S is endowed with a budget $\ell \in [0, \infty)$. She can never incur a loss that exceeds her budget.

Any signal can be represented by an information structure $I = (\Sigma, \mu)$. $\Sigma \subseteq \mathbb{R}$ is a set of signal realizations with the generic element $\sigma \in \Sigma$ and $\mu \in \Delta(\Sigma \times \Theta)$ is a probability measure on the set of possible realizations of cost and signal. The measure μ induces a conditional distribution $G(\theta|\sigma) = \Pr(\tilde{\theta} \leq \theta | \tilde{\sigma} = \sigma)$ and a marginal

[13]See for instance Section 2.2. in Börgers (2015) for a proof of the optimality of a posted price.
[14]This assumption follows Faure-Grimaud et al. (2003), Celik (2009), and Mookherjee et al. (2019).

distribution $H(\sigma) = \Pr(\tilde{\sigma} \leq \sigma)$. Following the literature on Bayesian persuasion, I only impose the requirement of Bayes-consistency on the information structure, i.e.,

$$\int_{\Sigma} G(\theta|\sigma)dH(\sigma) = F(\theta) \quad \forall\theta \in \Theta.$$

I denote by $\mathcal{I}$ the set of all Bayes-consistent information structures. For a given I, the support of the random variables $(\tilde{\sigma}, \tilde{\theta})$ and $\tilde{\theta}|\sigma$ are denoted by $Supp(\tilde{\sigma}, \tilde{\theta}) \subset \Sigma \times \Theta$ and $Supp(\tilde{\theta}|\sigma) \subset \Theta$, respectively.

P exerts information control by choosing the information structure $I \in \mathcal{I}$ that generates S's signal. This contrasts with the extant literature on collusion in principal-supervisor-agent relationships where some information structure in $\mathcal{I}$ is exogenously given. Following the literature on Bayesian persuasion, P can choose any information structure in $\mathcal{I}$ at zero cost.

Allocations and payoffs

An allocation describes whether the good is produced and what transfers are exchanged between the parties. Formally, an allocation is given by

$$(X, t_S, t_A, \tau) \in \{0, 1\} \times \mathbb{R}^3,$$

where t_i is the transfer from P to $i \in \{A, S\}$ and τ is a side-transfer from A to S. The allocation (X, t_S, t_A, τ) leads to payoffs of $vX - t_A - t_S$ for P, $t_A - \tau - \theta X$ for A, and $t_S + \tau$ for S.[15]

A and S each have an outside option. If S chooses her outside option, it follows that $t_S = \tau = 0$ resulting in a payoff of zero for S. If A chooses his outside option, $X = \tau = t_A = 0$ and A's payoff is zero.

Centralization and delegation under collusion

Following Faure-Grimaud et al. (2003) and Celik (2009), I consider two forms of organizational design: centralization and delegation.

Under centralization, P directly communicates with A and S. P offers A and S

[15] As A knows σ and θ, all monotone transformations $u(\cdot)$ of A's payoff do not change the results. Thus, one may allow A to be risk-averse or to have a limited budget.

a (deterministic) grand contract

$$\beta = \Big(X(m_S, m_A), t_S(m_S, m_A), t_A(m_S, m_A)\Big)$$

which determines the production decision and transfers from P to S and A as functions of the messages m_S and m_A chosen by S and A from the sets M_S and M_A. If a party rejects the grand contract, A and S receive their outside options. Closely following the literature on collusion in mechanism design, I model collusion as an enforceable side-contract between S and A that coordinates the communication with P and specifies side-transfers. As in Faure-Grimaud et al. (2003) and Celik (2009), I assume that S proposes the side-contract to A in a take-it-or-leave-it offer.[16] Formally, S offers A a (deterministic) side-contract

$$\gamma = (\rho(m^{sc}; \sigma), \tau(m^{sc}; \sigma))$$

which determines the communication with P and the side-transfer through the reporting strategy $\rho : M^{sc} \times \Sigma \to M_S \times M_A$ and the payment rule $\tau : M^{sc} \times \Sigma \to \mathbb{R}$. Both ρ and τ are functions of the message m^{sc} chosen by A from the set M^{sc} and the signal realization σ which is common knowledge of the colluding parties. If A rejects the side-contract, A and S play the grand-contract non-cooperatively.

Under delegation, P communicates directly only with S via the grand contract while S communicates with A via the side-contract. Formally, P offers S a grand contract β with $M_A = \{m_A\}$ and S offers A a side-contract γ. Under delegation, the side-contract γ may be interpreted as a subcontract between P's main contractor S and the subcontractor A. If S rejects the grand contract, A and S receive their outside options. If S accepts the grand contract and A rejects the side-contract, A receives his outside option and S is forced to send a message that induces no production.

With this definition of delegation, P can make direct transfers to A. This contrasts with Faure-Grimaud et al. (2003) and Celik (2009) where the grand contract has to satisfy $t_A = 0$ under delegation. However, this difference is not substantial as t_A, t_S, and τ can be interpreted as the net transfers in a setting where P pays $t_A + t_S$ to S and S pays $t_A - \tau$ to A.[17] The advantage of this paper's definition of delegation is that the cases of delegation and centralization can be treated in the same framework

[16] I show in Section 7 that this assumption is not crucial for the results.

[17] I further comment on this equivalence after defining P's problem under delegation in Section 4.

by using a collusion-proofness principle in both instances. Moreover, this formulation fits the leading example of public procurement where private firms are paid by public buyers and not by the central purchasing body.

Timing and equilibrium concept

The timing of the game for the cases of centralization and delegation is the following:

t=0: P chooses an information structure $I \in \mathcal{I}$.

t=1: S and A observe I and σ. A furthermore observes θ.

t=2: P offers a grand contract β.

t=3: Under centralization, S and A each accept or reject P's offer β. Under delegation, only S accepts or rejects β.

t=4: S offers a side-contract γ to A.

t=5: A accepts or rejects γ.

I focus on *perfect Bayesian equilibria* (PBE) with *passive beliefs* in which S offers direct and truthful side-contracts whenever they are optimal.[18] In these equilibria, S does not update her belief about θ if A rejects the side-contract off the equilibrium path. Moreover, if the set of S's best responses to a grand contract β contains a direct and truthful side-contract, then S offers such a side-contract to A. This approach follows the concept of *weak collusion-proofness* in Laffont and Martimort (2000).

Remarks on the model

P exercises information control through public information design as both S and A observe the signal realization. An alternative modeling approach allows P to design signals that are privately observed by A and S.[19] There are two reasons for my modeling decision. First, the model remains close to the literature with exogenous information which assumes that the realization of the exogenously determined signal is observed by both A and S (Faure-Grimaud et al., 2003; Celik, 2009; Mookherjee et al., 2019). While the case of private signals is interesting, it is harder to compare

[18]See Fudenberg and Tirole (1991) for a definition of PBE.
[19]These signals might even be made dependent on a report of A.

to the literature which does not cover the case of private signals. Second, the model fits my leading application of public procurement where private firms typically have the right to know what data is collected about them by public authorities.

As in Faure-Grimaud et al. (2003) and Celik (2009), I restrict grand contracts and side-contracts to be deterministic. In the context of public procurement, deterministic mechanisms seem to comply better with the aim of rewarding public contracts in a transparent way. Moreover, the restriction to deterministic grand contracts may reflect the practical difficulty to commit to a stochastic mechanism.

As the subsequent analysis reveals, both assumptions turn out to be without loss of optimality if the supervisor has a sufficiently large budget. In this case, the principal can extract the full surplus with public information and deterministic mechanisms.

4 Preliminary Analysis

In this section, I use a collusion-proofness principle to formulate P's contracting problems under centralization and delegation. I then derive upper bounds on P's payoff for centralization and delegation. These upper bounds play a crucial role for the analysis of optimal combinations of information structure and grand contract in the subsequent sections.

Collusion-proofness principle

I first invoke a collusion-proofness principle. This approach follows Laffont and Martimort (1997) and allows me to restrict attention to direct and truthful grand contracts under which S offers A the direct and truthful *null side-contract* $\gamma_0 \equiv (\rho_0, \tau_0)$ with $\rho_0(\theta;\sigma) \equiv (\sigma, \sigma, \theta)$ and $\tau_0(\theta;\sigma) \equiv 0$ for all $(\sigma, \theta) \in Supp(\tilde{\sigma}, \tilde{\theta})$.

Lemma 1. *For any grand contract β, any information structure I, and any signal realization $\sigma \in \Sigma$, S has an optimal side-contract γ which is direct and truthful.*

For any equilibrium (with centralization or delegation) in which P offers the grand contract β and S offers the side-contract γ, there exists a payoff-equivalent equilibrium in which P offers $\beta_0 = \beta \circ \gamma$ with $M_S = \Sigma$ and $M_A = \Sigma \times \Theta$ under centralization and $M_S = \Sigma^2 \times \Theta$ under delegation, and S offers γ_0.[20]

[20]The usual revelation principle does not apply as the side-contract is deterministic. The proof uses a revelation principle in terms of payoffs due to Strausz (2003) and incorporates collusion-proofness.

For centralization, the collusion-proofness principle implies that it is optimal for P to offer a direct and truthful grand contract under which S does not benefit from non-trivial collusion with A. For delegation, the collusion-proofness principle implies that it is optimal for P to offer a direct grand contract under which S finds it optimal to offer a side-contract which truthfully conveys S's information σ and A's information (σ, θ) without using side-transfers.[21]

Centralization

Using the collusion-proofness principle, I now formulate P's optimization problem under centralization. The grand contract is acceptable to both S and A if

$$\mathbb{E}\left[t_S(\sigma, \sigma, \tilde{\theta})|\sigma\right] \geq 0 \quad (PC_S), \quad t_A(\sigma, \sigma, \theta) - \theta X(\sigma, \sigma, \theta) \geq 0 \quad (PC_A),$$

for all $(\sigma, \theta) \in Supp(\tilde{\sigma}, \tilde{\theta})$. Furthermore, S and A find it optimal to individually report their private information truthfully if

$$\begin{aligned}
&\mathbb{E}\left[t_S(\sigma, \sigma, \tilde{\theta})|\sigma\right] \geq \mathbb{E}\left[t_S(\hat{\sigma}_S, \sigma, \tilde{\theta})|\sigma\right] && (IC_S),\\
&t_A(\sigma, \sigma, \theta) - \theta X(\sigma, \sigma, \theta) \geq t_A(\sigma, \hat{\sigma}_A, \hat{\theta}) - \theta X(\sigma, \hat{\sigma}_A, \hat{\theta}) && (IC_A),
\end{aligned}$$

for all $\sigma, \hat{\sigma}_S \in \Sigma$, $(\sigma, \theta), (\hat{\sigma}_A, \hat{\theta}) \in Supp(\tilde{\sigma}, \tilde{\theta})$. S's limited budget (LB) implies that the transfer from P to S needs to satisfy $t_S(\sigma, \sigma, \theta) \geq -\ell$ for all $(\sigma, \theta) \in Supp(\tilde{\sigma}, \tilde{\theta})$. Finally, a grand contract is collusion-proof if S does not benefit from a non-trivial side-contract that A prefers over the null side-contract, that incentivizes A to report θ truthfully, and that respects S's limited budget. Formally, the condition (CP^c) requires

$$\begin{aligned}
&\gamma_0 \in \arg\max_{\gamma} \mathbb{E}\left[t_S(\rho(\tilde{\theta}; \tilde{\sigma})) + \tau(\tilde{\theta}; \tilde{\sigma})\right] \quad \text{s.t.}\\
&\quad t_A(\rho(\theta; \sigma)) - \tau(\theta; \sigma) - \theta X(\rho(\theta; \sigma)) \geq t_A(\sigma, \sigma, \theta) - \theta X(\sigma, \sigma, \theta), && (PC_A^{\gamma,c})\\
&\quad t_A(\rho(\theta; \sigma)) - \tau(\theta; \sigma) - \theta X(\rho(\theta; \sigma)) \geq t_A(\rho(\breve{\theta}; \sigma)) - \tau(\breve{\theta}; \sigma) - \theta X(\rho(\breve{\theta}; \sigma)), && (IC_A^{\gamma})\\
&\quad t_S(\rho(\theta; \sigma)) + \tau(\theta; \sigma) \geq -\ell, && (LB^{\gamma})
\end{aligned}$$

[21] The terms of the side-contract are not observable by P. Thus, P might ask A and S to report the terms of the side-contract to the grand contract. The collusion-proofness principle implies that P does not benefit from requesting these reports from A and S.

for all $\sigma \in \Sigma$, $\theta, \breve{\theta} \in Supp(\tilde{\theta}|\sigma)$.[22] A side-contract γ is feasible under centralization if it satisfies $(PC_A^{\gamma,c})$, (IC_A^{γ}), and (LB^{γ}). P's problem under centralization is

$$\mathbf{P}^c: \quad \max_{I,\beta} \mathbb{E}\left[vX(\tilde{\sigma},\tilde{\sigma},\tilde{\theta}) - t_S(\tilde{\sigma},\tilde{\sigma},\tilde{\theta}) - t_A(\tilde{\sigma},\tilde{\sigma},\tilde{\theta})\right]$$
$$\text{s.t.} \quad (PC_S),(PC_A),(IC_S),(IC_A),(LB),(CP^c).$$

A grand contract β is *feasible under centralization* if it satisfies all constraints in $\mathbf{P}^c$.

Delegation

Under delegation, P offers the grand contract to S only. Thus, the grand contract should respect S's limited budget constraint (LB) and participation constraint (PC_S). A grand contract is collusion-proof under delegation if S picks the null side-contract among all side-contracts that A prefers over the outside option, incentivize A to report θ truthfully, and respect S's limited budget. The constraint (CP^d) captures this formally:

$$\gamma_0 \in \arg\max_{\gamma} \mathbb{E}\left[t_S(\rho(\tilde{\theta};\tilde{\sigma})) + \tau(\tilde{\theta};\tilde{\sigma})\right] \quad \text{s.t.} \quad (IC_A^{\gamma}),(LB^{\gamma}),$$
$$t_A(\rho(\theta;\sigma)) - \tau(\theta;\sigma) - \theta X(\rho(\theta;\sigma)) \geq 0, \qquad (PC_A^{\gamma,d})$$

for all $\sigma \in \Sigma$, $\theta, \breve{\theta} \in Supp(\tilde{\theta}|\sigma)$.[23] A side-contract γ is feasible under delegation if it satisfies (IC_A^{γ}), (LB^{γ}), and $(PC_A^{\gamma,d})$. P's problem under delegation is given by

$$\mathbf{P}^d: \max_{I,\beta} \mathbb{E}\left[vX(\tilde{\sigma},\tilde{\sigma},\tilde{\theta}) - t_S(\tilde{\sigma},\tilde{\sigma},\tilde{\theta}) - t_A(\tilde{\sigma},\tilde{\sigma},\tilde{\theta})\right] \text{ s.t. } (PC_S),(LB),(CP^d).$$

A grand contract β is *feasible under delegation* if it satisfies the constraints (PC_S), (IC_S), (LB), and (CP^d). The constraints (PC_A) and (IC_A) are not part of problem $\mathbf{P}^d$. Under delegation, A cannot directly agree to participate in the grand contract. Instead, A participates in the grand contract if he accepts the side-contract. i.e., if the constraint $(PC_A^{\gamma,d})$ is satisfied. Thus, (PC_A) is not relevant under delegation. Similarly, A does not report to the grand mechanism under delegation. Therefore,

[22] By the principle of optimality, (CP^c) ensures that for all signal realizations $\sigma \in \Sigma$, $\rho_0(\theta;\sigma) = (\sigma,\sigma,\theta)$ and $\tau_0(\theta;\sigma) = 0$ are optimal for S given the posterior belief induced by σ.

[23] By the principle of optimality, (CP^d) ensures that for all signal realizations $\sigma \in \Sigma$, $\rho_0(\theta;\sigma) = (\sigma,\sigma,\theta)$ and $\tau_0(\theta;\sigma) = 0$ are optimal for S given the posterior belief induced by σ.

the constraint (IC_A) is not needed.

Finally, I come back to my earlier observation that it is immaterial whether we allow P to make direct transfers to A under delegation or not.[24] Formally, this observation follows from a change of variable in program $\mathbf{P}^d$ by using $t_{PS} \equiv t_A + t_S$ and $t_{SA} \equiv t_A - \tau$ instead of the variables t_A and t_S. The transfers t_{PS} and t_{SA} can then be interpreted as transfers from P to S and S to A in a setting where P cannot make direct transfers to A.

Difference between centralization and delegation

The key difference between centralization and delegation lies in A's participation constraints ($PC_A^{\gamma,c}$) and ($PC_A^{\gamma,d}$) for the side-contract. If A rejects the side-contract under centralization, he still receives the payoff from the non-cooperative equilibrium of the grand contract. Under delegation, a rejection of the side-contract implies that A receives his outside option.

By contrast, the additional constraints (PC_A) and (IC_A) do not harm P under centralization. To see this, note that A's participation constraint for the null side-contract under delegation ($PC_A^{\gamma_0,c}$) is equivalent to (PC_A). Moreover, the incentive constraint for A under the null side-contract ($IC_A^{\gamma_0}$) is equivalent to (IC_A).

Under centralization, P has one more instrument than under delegation. In particular, P can directly control A's payoff in the non-cooperative equilibrium of the grand contract. If P increases this payoff, the constraint ($PC_A^{\gamma,c}$) becomes tighter and the collusion-proofness constraint (CP^c) is relaxed. Thus, centralization is always at least as good as delegation. Moreover, delegation is optimal whenever P does not benefit from using A's rent as an instrument.

Benchmarks: Extreme information structures

Before I proceed with the analysis, I consider two benchmarks: the cases of uninformative and fully informative signal design. I show that P's optimal payoff is in both instances the monopsony payoff $\underline{W}$.

Independently of the information structure, P can always ignore S under centralization and guarantee a payoff of $\underline{W}$ by offering A the monopsony price p^*. Formally,

[24]Faure-Grimaud et al. (2003) and Celik (2009) do not allow such transfers under delegation.

this is equivalent to the grand contract[25]

$$\beta_{p^*} \equiv \Big(X(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}), t_S(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}), t_A(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta})\Big) = (\mathbf{1}_{\hat{\theta} \leq p^*}(\hat{\theta}), 0, p^* \cdot \mathbf{1}_{\hat{\theta} \leq p^*}(\hat{\theta})).$$

Under the non-cooperative equilibrium of this grand contract, A accepts the price offer if and only if $\theta \leq p^*$ and receives the payoff $\max\{p^* - \theta, 0\}$. Note that under any side-contract $\gamma = (\rho, \tau)$, the total payoff of A and S is given by $X(\rho(\theta; \sigma))(p^* - \theta)$ which is weakly lower than A's payoff under the non-cooperative equilibrium. As any side-contract needs to give A at least the non-cooperative payoff, S's payoff has to be weakly negative under collusion. It follows that collusion is not profitable for S. P can therefore achieve at least the monopsony payoff under centralization.

With an uninformative signal, the monopsony payoff is also an upper bound for P's payoff. To see this, note that any uninformative signal is equivalent to the case $\Sigma = \{\sigma\}$. Thus, σ can be omitted as an argument in the grand contract and the side-contract. The constraints (PC_A) and (IC_A) are therefore equivalent to

$$t_A(\theta) - \theta X(\theta) \geq \max\{0, t_A(\hat{\theta}) - \theta X(\hat{\theta})\}.$$

Maximizing P's expected payoff subject to these constraints is equivalent to the monopsony problem and results in a payoff of $\underline{W}$.

The monopsony payoff is also an upper bound for P if the signal is fully informative. In this case, the relationship between signals and types is bijective. Thus, we can again omit σ as an argument from the grand contract and the side-contract. As we can ignore the constraint (IC_A^{γ}) under complete information, the maximal side-transfer can be derived from $(PC_A^{\gamma,c})$ as

$$\tau(\theta) = t_A(\rho(\theta)) - \theta X(\rho(\theta)) - (t_A(\theta) - \theta X(\theta)).$$

Using this and the fact that (CP^c) requires $t_S(\theta) \geq t_S(\rho(\theta)) + \tau(\theta)$, we have

$$t_S(\theta) + t_A(\theta) - \theta X(\theta) \geq t_S(\rho(\theta)) + t_A(\rho(\theta)) - \theta X(\rho(\theta)).$$

Let $t(\theta) \equiv t_S(\theta) + t_A(\theta)$ and $\rho(\theta) = \hat{\theta}$. Note that (PC_S) and (PC_A) imply $t(\theta) -$

[25] The indicator function $\mathbf{1}_A(x)$ satisfies $\mathbf{1}_A(x) = 1$ if x satisfies A and $\mathbf{1}_A(x) = 0$ otherwise.

$\theta X(\theta) \geq 0$. Maximizing P's expected payoff subject to the resulting constraint

$$t(\theta) - \theta X(\theta) \geq \max\{0, t(\hat{\theta}) - \theta X(\hat{\theta})\}$$

is again equivalent to the monopsony problem and gives the payoff $\underline{W}$ to P.

Upper bounds on *P*'s payoff for any information structure

The constraints in P's optimization problems $\mathbf{P}^c$ and $\mathbf{P}^d$ simplify the structure of transfers in any feasible grand contract considerably. The simple structure of transfers allows me to derive upper bounds on P's payoff under centralization and delegation that hold for any combination of an information structure and a grand contract.

Lemma 2. *For any feasible grand contract under centralization or delegation, there exist the functions* t_A^0, t_A^1, t_S^0, $t_S^1 : \Sigma \to \mathbb{R}$ *such that for all* $(\sigma, \theta) \in Supp(\tilde{\sigma}, \tilde{\theta})$

i) $t_i(\sigma, \sigma, \theta) = X(\sigma, \sigma, \theta) t_i^1(\sigma) + (1 - X(\sigma, \sigma, \theta)) t_i^0(\sigma)$ *with* $i \in \{A, S\}$,

ii) $t_A^0(\sigma) \geq 0$, $p(\sigma) \equiv t_A^1(\sigma) - t_A^0(\sigma) \geq \sup\{\theta \in Supp(\tilde{\theta}|\sigma) : X(\sigma, \sigma, \theta) = 1\}$,

iii) $t_S^0(\sigma) \geq -\ell$, $t_S^1(\sigma) \geq -\ell$, $t_S^j(\sigma) < 0 \Rightarrow t_S^{j'}(\sigma) \geq 0$ *for* $j, j' \in \{0, 1\}$ *and* $j \neq j'$,

iv) $t_S^j(\sigma) + t_A^j(\sigma) = t^j$ *for* $j \in \{0, 1\}$.

Any feasible grand contract under delegation furthermore satisfies for all $\sigma \in \Sigma$

v) $t_S^1(\sigma) \geq 0$.

Result $i)$ states that the transfers to A and S in any feasible grand contract only depend on the signal realization and the production decision. The intuition behind this result is the following. A's transfer for a given signal realization has to be the same for all cost levels θ that lead to the same production decision. Otherwise, for any signal realization, A would only report the two cost levels $\hat{\theta}_1$ and $\hat{\theta}_0$ which maximize his transfer $t_A(\sigma, \sigma, \theta)$ among all costs levels that lead to production – for $\hat{\theta}_1$ – or no production – for $\hat{\theta}_0$. Similarly, if S's transfer was different between two cost levels that lead to the same production decision, S and A could sign a profitable side-contract in which they – for a given signal realization – would only report the cost levels that maximize the transfer $t_S(\sigma, \sigma, \theta)$ for the two cases of production and no production.

Result *ii*) makes the following point. Decompose A's transfer into a *fixum* $t_A^0(\sigma)$ and a *price* $p(\sigma)$ paid for production. For any feasible grand contract, the fixum has to be positive as A perfectly anticipates the production decision before deciding whether to participate in the grand contract. Moreover, the price $p(\sigma)$ needs to always cover A's production costs as A would otherwise overstate the costs to avoid production.

Result *iii*) states first that S's transfer to P can never exceed the bound ℓ – a direct implication of S's limited budget. Second, point *iii*) notes that S can never make both a loss with and without production as this would obviously violate S's participation constraint (PC_S).

Result *iv*) states that P can make the total transfer to S and A only contingent on whether or not the good is produced. Whenever the total transfer to S and A does not only depend on the production decision, S and A can sign a side-contract which leads to the same production decision as without collusion but coordinates their messages to P such that the highest total transfer to the colluding parties is generated. As the production decision remains unchanged, such a side-contract is always feasible. By fixing the total transfer, P induces a strong conflict of interest between S and A as any increase in the transfer of one of the parties has to decrease the transfer of the other party by the same amount.

Finally, result *v*) holds as S can avoid production under delegation by offering a side-contract which always sends a message that induces no production while no side-transfers are exchanged. This side-contract replicates A's outside option. Hence, A is willing to accept. Thus, P has to reward S for production under delegation.

The results of Lemma 2 can be used to derive upper bounds on P's payoff under delegation and centralization. Let x be the (ex-ante) probability of production induced by some grand contract, i.e.,

$$x \equiv \mathbb{E}[X(\tilde{\sigma}, \tilde{\sigma}, \tilde{\theta})],$$

and let $\overline{\theta}(x)$ be the x-quantile of the distribution F, formally defined as[26]

$$\overline{\theta}(x) \equiv \min\{\theta \in \Theta : F(\theta) \geq x\}.$$

[26]If F has a strictly positive density $f(\theta) = F'(\theta) > 0$, then F is invertible and $\overline{\theta}(x) = F^{-1}(x)$.

The maximal social surplus under a grand contract with production probability x is

$$B_1(x) \equiv \int_{\underline{\theta}}^{\overline{\theta}(x)} (v-\theta) dF(\theta)$$

as it is cost-minimizing to let only the types below the x-quantile produce. To state the following lemma concisely, I define[27]

$$B_2(x) \equiv x(v-\overline{\theta}(x)) + (1-x)\ell \quad \text{and} \quad B_3(x) \equiv x(v-\overline{\theta}(x)) + \int_{\overline{\theta}(x)-\ell}^{\overline{\theta}(x)} F(\theta)d\theta.$$

Lemma 3. *Under delegation, P's payoff from any information structure and any feasible grand contract with production probability x does not exceed*

$$B_d(x) \equiv \min\{B_1(x), B_2(x)\}.$$

Under centralization, P's payoff from any information structure and any feasible grand contract with production probability x does not exceed

$$B_c(x) \equiv \min\left\{B_1(x), \max\{B_2(x), B_3(x)\}\right\}.$$

As the participation of A and S in the grand contract is voluntary, P's expected payoff from any grand contract with probability of production x cannot exceed the maximal social surplus $B_1(x)$.

Next, I explain how the bound $B_2(x)$ arises. Due to Lemma 2, P pays expected total transfers of $\mathbb{E}[t_A(\tilde{\sigma},\tilde{\sigma},\tilde{\theta})+t_S(\tilde{\sigma},\tilde{\sigma},\tilde{\theta})] = xt^1 + (1-x)\,t^0$ under any feasible grand contract with production probability x. Moreover, Lemma 2 implies that S can never incur a loss with production under delegation. As this also has to hold for some signal realization $\sigma' \in \Sigma$ after which A produces for a cost weakly above $\overline{\theta}(x)$ and A can always guarantee himself a positive payoff, it follows that $t^1 = t_S^1(\sigma') + t_A^1(\sigma') \geq \overline{\theta}(x)$. Without production, S's loss is at most ℓ. Thus, $t^0 \geq -\ell$. Hence, P's payoff under delegation satisfies $x(v-t^1) - (1-x)t^0 \leq x(v-\overline{\theta}(x)) + (1-x)\ell = B_2(x)$.

While P cannot impose a loss on S with production under delegation, P may do so under centralization by setting $t_S^1(\sigma) < 0$ for some $\sigma \in \Sigma$. In the proof of the lemma, I show that P's payoff from imposing a loss on S with production under a

[27] I use the convention $\int_a^b z(x)dx = -\int_b^a z(x)dx$.

grand contract with production probability x cannot exceed the bound $B_3(x)$. Using integration by parts, this bound can be rewritten as

$$B_3(x) = \int_{\underline{\theta}}^{\overline{\theta}(x)} (v-\theta)dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}(x)-\ell} (\overline{\theta}(x) - \ell - \theta)dF(\theta).$$

Based on this reformulation, the bound $B_3(x)$ can be interpreted as the difference between the maximal social surplus generated under a grand contract with production probability x and rent payments of $\overline{\theta} - \ell - \theta$ for each type $\theta \leq \overline{\theta} - \ell$.

I explain in three steps why P cannot avoid these rent payments. First, for any grand contract with an ex-ante probability of production x, the highest producing type lies weakly above $\overline{\theta}(x)$. Thus, there exists a signal realization $\sigma \in \Sigma$ such that $p(\sigma) \geq \overline{\theta}(x)$. As S can at most incur a loss of ℓ, the total payment to A and S with production satisfies $t^1 \geq \overline{\theta}(x) - \ell$. Second, if S makes a loss in the case of production, P cannot also impose a loss on S without production as this would violate S's participation constraint. As A always receives weakly positive payments, this implies that the total payment to A and S without production satisfies $t^0 \geq 0$. Third, consider the cost level θ with $\theta \leq \overline{\theta}(x) - \ell$. If A produces for this cost level, A and S receive a joint rent of $t^1 - \theta \geq \overline{\theta}(x) - \ell - \theta$. This rent cannot be extracted as the joint payment to A and S without production t^0 is weakly positive.

5 Optimal Delegation

In this section, I present P's optimal combination of information structure and grand contract under delegation. In particular, I construct an information structure and a feasible grand contract under delegation which allow P to implement any ex-ante probability of production x with an expected payoff equal to the upper bound $B_d(x)$. The optimal combination of information structure and grand contract under delegation implements the production probability x_d at which the function $B_d(x)$ attains its maximum.[28]

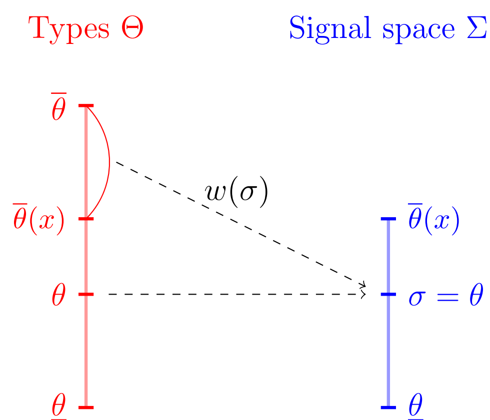
The key idea behind the optimal combination of information structure and grand

[28]Both functions $B_d(x)$ and $B_c(x)$ are upper semi-continuous and therefore attain a maximum on $[0, 1]$. This follows from $B_1(x)$ being continuous and $B_2(x)$ and $B_3(x)$ being upper semi-continuous as well as the fact that both the minimum as well as the maximum of two upper semi-continuous functions is also upper semi-continuous.

contract under delegation is the following. The signal provides enough information to allow S to extract all rents from A in the case of production. At the same time, the signal maintains S's uncertainty about the production decision and allows P to extract rents from S in the case of no production.

Next, I describe a combination of an information structure and a grand contract that implements a production probability x and leads to a payoff of $B_d(x)$ for P. Figure 3 depicts a *weighted information structure*. The signal space of the weighted information structure is the interval of types below the x-quantile, i.e., $\Sigma = [\underline{\theta}, \overline{\theta}(x)]$. If A's cost satisfies $\theta \leq \overline{\theta}(x)$, the signal realization $\sigma = \theta$ is drawn. If $\theta > \overline{\theta}(x)$, some $\sigma \in \Sigma$ is drawn according to the density function $w(\sigma)$. I refer to $w(\sigma)$ as a weighting function as it determines the relative weights with which the types above $\overline{\theta}(x)$ are pooled as noise into the signal realizations in Σ. The weighting function

Figure 3: Weighted information structure



A weighted information structure has the signal space $[\underline{\theta}, \overline{\theta}(x)]$. If $\theta \leq \overline{\theta}(x)$, then the signal realization $\sigma = \theta$ is generated. If $\theta > \overline{\theta}(x)$, some $\sigma \in [\underline{\theta}, \overline{\theta}(x)]$ is drawn from the density $w(\sigma)$.

$w(\cdot)$ characterizes the weighted information structure I_w and induces for any signal realization $\sigma \in \Sigma$ a conditional cdf[29]

$$G(\theta|\sigma) = \begin{cases} 0 & \text{if} \quad \theta < \sigma, \\ \frac{f(\sigma)}{f(\sigma)+(1-x)w(\sigma)} & \text{if} \quad \theta \in [\sigma, \overline{\theta}(x)], \\ \frac{f(\sigma)}{f(\sigma)+(1-x)w(\sigma)} + \frac{(1-x)w(\sigma)}{f(\sigma)+(1-x)w(\sigma)} \frac{F(\theta)-F(\overline{\theta}(x))}{1-F(\overline{\theta}(x))} & \text{if} \quad \theta > \overline{\theta}(x). \end{cases}$$

[29]The marginal cdf over signal realizations is given by $H(\sigma) = \int_{\underline{\theta}}^{\sigma} (f(\sigma') + (1-x)w(\sigma'))d\sigma'$.

I combine a weighted information structure with a grand contract under delegation that induces production if all reports coincide and lie below the threshold $\overline{\theta}(x)$, i.e.,

$$X(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = \mathbf{1}_{\hat{\sigma}_S = \hat{\sigma}_A = \hat{\theta} \leq \overline{\theta}(x)}(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}). \tag{1}$$

Under truthful reporting, the project is therefore realized whenever costs take the lowest possible value given the signal realization. Furthermore, the transfer to A covers the exact cost of production:

$$t_A(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = \hat{\theta} X(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}). \tag{2}$$

As the grand contract has to be feasible, Lemma 2 implies that the transfer to S is

$$t_S(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = X(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta})(t^1 - \hat{\theta}) + (1 - X(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}))t^0. \tag{3}$$

Proposition 1. *Under delegation, P can reach the payoff $B_d(x)$ for any probability of production $x \in [0,1]$ through the weighted information structure I_{w^x} with*

$$w^x(\sigma) = \frac{f(\sigma)\left[\min\{\overline{\theta}(x), \check{\theta}(x)\} - \sigma\right]_+}{\int_{\underline{\theta}}^{\min\{\overline{\theta}(x), \check{\theta}(x)\}} F(\sigma')d\sigma'}$$

and the grand contract β^x defined by the equations (1), (2), *and* (3) *with*

$$t^0 = -\min\left\{\frac{\int_{\underline{\theta}}^{\overline{\theta}(x)} F(\sigma)d\sigma}{1-x}, \ell\right\} \quad \textit{and} \quad t^1 = \overline{\theta}(x),$$

where $\check{\theta}(x)$ is defined as the unique solution to

$$\int_{\underline{\theta}}^{\theta} F(\sigma)d\sigma = (1-x)(\overline{\theta}(x) - \theta + \ell). \tag{4}$$

The combination of information structure I_{w^x} and grand contract β^x optimally resolves P's trade-off between information elicitation and collusion prevention under delegation. S's signal is very informative on A's costs in case the good is produced. This allows S to extract all rents from A by setting a price that equals A's cost under production. S earns the *bonus* $t^1 - \sigma$ for production after the signal realization $\sigma \in \Sigma$.

At the same time, the probability of a bonus payment $G(\sigma|\sigma)$ decreases in the bonus and enables P to extract S's rent by a payment in the case of no production. In contrast to the exact information on producing types, the optimal signal reveals no information about the relative likelihood of high cost types. As the type of A does not influence his payoff without production, P does not benefit from information about non-producing types.

I now explain Proposition 1 in more detail. I start by arguing that the null side-contract is feasible given the information structure and the grand contract in Proposition 1. From A's perspective, any grand contract that satisfies equations (1) and (2) is equivalent to a price offer equal to the signal realization. Thus, under the null side-contract, A's participation constraint ($PC_A^{\gamma_0,d}$) and incentive compatibility constraint ($IC_A^{\gamma_0}$) are satisfied and A never receives a positive rent.

Next, I show that S optimally responds to the information structure and the grand contract in Proposition 1 by offering the null side-contract. Given some weighted information structure I_w, a grand contract that satisfies equations (1)–(3), and the null side-contract, S receives an expected payoff after the signal realization σ of

$$\Pr(\tilde{\theta}=\sigma|\tilde{\sigma}=\sigma)(t^1-\sigma)+\Pr(\tilde{\theta}>\sigma|\tilde{\sigma}=\sigma)t^0=\frac{f(\sigma)(t^1-\sigma)+(1-x)w(\sigma)t^0}{f(\sigma)+(1-x)w(\sigma)}.$$

For $t^1=\overline{\theta}(x)$ and $t^0\leq 0$, S does not benefit from offering a side-contract under which A never produces, as S would then receive the negative payoff t^0. S would not benefit from a side-contract under which some types in the interval $(\overline{\theta}(x),\overline{\theta}]$ produce. This would require S to pay A a side-transfer τ with $\sigma+\tau>\overline{\theta}(x)$. S's payoff under production would then be negative as $t^1-\sigma-\tau=\overline{\theta}(x)-\sigma-\tau<0$. Together with $t^0\leq 0$, this implies that S's payoff under such a deviation cannot exceed zero.

P optimally designs the weighting function to minimize the transfer t^0 while satisfying S's participation constraint. Note that (PC_S) holds for some weighted information structure and grand contract satisfying equations (1)–(3) if

$$t^0=-\min_{\{\sigma\in\Sigma:w(\sigma)>0\}}\left\{\frac{f(\sigma)(\overline{\theta}(x)-\sigma)}{(1-x)w(\sigma)}\right\}.$$

As S's participation constraint is binding after the worst signal realization from S's perspective, P would like to make the worst signal as good as possible. In technical terms, P faces a max-min-problem. Ideally, P would like to choose $w(\cdot)$ and t^0 such that S's expected payoff is zero for all signal realizations. A weighting function which renders S's expected payoff constant across all signal realizations is

$$w(\sigma) = \frac{f(\sigma)(\overline{\theta}(x) - \sigma)}{(1-x)c},$$

where the constant $c \in \mathbb{R}_+$ – pinned down by $\int_\Sigma w(\sigma)d\sigma = 1$ – is given by

$$c = \frac{\int_{\underline{\theta}}^{\overline{\theta}(x)} F(\sigma)d\sigma}{1-x}.$$

If $c \leq \ell$, it is feasible to extract all rents from S by setting $t^0 = -c$. In this case, P's expected payoff equals the maximal social surplus under the ex-ante probability x, i.e., $B_1(x)$. If $c > \ell$, P cannot extract the full social surplus. However, as I show in the proof of Proposition 1, P can reach an expected payoff of $B_2(x)$ by setting $t^0 = -\ell$, $w(\sigma) = 0$ for signal realizations σ between $\check{\theta}(x)$ and the threshold $\overline{\theta}(x)$, and $w(\sigma)$ such that the expected payoff of S is constant for the remaining signal realizations.[30]

Proposition 1 implies three important results as immediate corollaries. First, the optimal combination of information structure and grand contract under delegation follows directly from the proposition.

Corollary 1. *An optimal combination of information structure and grand contract under delegation is given by* $(I_{w^{x_d}}, \beta^{x_d})$ *with* $x_d \in \arg\max_{x\in[0,1]} B_d(x)$.

Under the optimal grand contract with delegation, S receives a positive transfer from P with production and makes a payment to P without production. Note that this feature is shared with the optimal grand contract in Faure-Grimaud et al. (2003).

Second, Proposition 1 allows us to derive necessary and sufficient conditions for partial information revelation to be optimal under both delegation and centralization.

Corollary 2. *Under both centralization and delegation, an optimal information structure partially informs* S *about* A*'s cost* θ *if and only if* $x_d \in (F(\underline{\theta}), 1)$ *and* $\ell > 0$.

Recall from the analysis of the benchmarks that P can achieve at most a payoff of $\underline{W}$ under either an uninformative or a completely informative signal. Note that

[30]The uniqueness of $\check{\theta}(x)$ follows from intermediate value theorem. See proof for details.

$\underline{W} = \max_{x\in[0,1]} x(v - \overline{\theta}(x))$ and that $B_d(x)$ can be expressed as

$$B_d(x) = x(v - \overline{\theta}(x)) + \min\left\{\int_{\underline{\theta}}^{\overline{\theta}(x)} F(\theta)d\theta, (1-x)\ell\right\}.$$

Thus, $x_d \in (F(\underline{\theta}), 1)$ and $\ell > 0$ is equivalent to $\max_{x\in[0,1]} B_d(x) > \underline{W}$. Under mild conditions, P prefers a partially informative signal under delegation to either of the extreme information structures under centralization. As $B_c(x) = x(v - \overline{\theta}(x))$ for either $x \in \{F(\underline{\theta}), 1\}$ or $\ell = 0$, the same mild conditions are necessary and sufficient for the optimality of a partially informative signal under centralization. Without S, P can at most achieve the payoff $\underline{W}$. Thus, the corollary also provides necessary and sufficient conditions for P to benefit from hiring S under both centralization and delegation.

Third, P can extract the full surplus whenever S's budget is large enough.

Corollary 3. *If S's budget is large enough, P can extract the full surplus under delegation: If $v < \overline{\theta}$ and $\ell \geq \overline{\ell}(v) \equiv \overline{W}/(1 - F(v))$, then* $\max_{x\in[0,1]} B_d(x) = \overline{W}$. *If $v \geq \overline{\theta}$, then* $\lim_{\ell\to\infty} \max_{x\in[0,1]} B_d(x) = \overline{W}$.

The full surplus is generated through the efficient production cutoff $\overline{\theta}(x) = v$. Under efficient production, S becomes the residual claimant of all surplus in the case of production. If $v < \overline{\theta}$, P can extract the full surplus through the payment t^0 whenever

$$\ell \geq \frac{\int_{\underline{\theta}}^{v}(v-\theta)dF(\theta)}{1-F(v)} = \frac{\overline{W}}{1-F(v)} = \overline{\ell}(v).$$

Thus, P can achieve the full surplus under delegation only if S can absorb at least a loss of $\overline{\ell}(v)$ which is a multiple of the full surplus with a factor of $\frac{1}{1-F(v)}$. If $v \geq \overline{\theta}$, P can set a cutoff just below $\overline{\theta}$. As ℓ grows large, the production probability x can be set arbitrarily close to 1, resulting in a payoff of almost $\overline{W}$.

Faure-Grimaud et al. (2003) note that if S is risk neutral, P can achieve the same profit as in the case where P can observe S's signal. Corollary 3 is reminiscent of this result as a sufficiently large budget for S may be viewed as an analogue to the risk neutrality of S.

Celik (2009) points out that the suboptimality of delegation in his model is linked to an informational *double marginalization* problem that arises under delegation.

With information control, P avoids the problem of double marginalization by giving S enough information about A to extract A's rent completely. P uses S's uncertainty over the production decision to extract rents from S and may even achieve the first-best surplus under delegation.

6 When is Delegation (sub)Optimal?

In this section, I provide conditions under which delegation is optimal. In particular, I show that delegation is optimal if either S's budget is large or the value of the good is small. Furthermore, I show that delegation is suboptimal if the value of the good is high and S's budget is sufficiently strict. I denote the median cost type by $m \equiv \overline{\theta}(\frac{1}{2})$.

Proposition 2. *Delegation is optimal if either*

i) S's budget is large , i.e., $\ell \geq \hat{\ell}(v)$ where $\hat{\ell} : [0, \infty) \to [0, \overline{\ell}(v))$ or

ii) the value of the good is small, i.e., $v < m$.

First, I explain why delegation is optimal if S's budget is large. I start with the following observation.

Lemma 4. *If delegation is suboptimal, P cannot extract the full surplus.*

Recall that the central difference between delegation and centralization is A's outside option when bargaining over the side-contract. Under centralization, A can reject the side-contract and participate in the grand contract non-cooperatively. Under delegation, A can only participate in the grand contract by accepting the side-contract. If centralization is superior to delegation, P finds it optimal to pay a positive rent to A in order to limit the possibility of S finding a profitable side-contract. As A's rent is strictly positive, P cannot extract the full surplus.

We know from Corollary 3 that P can (approximately) extract the full surplus under delegation if S's budget is sufficiently large. Moreover, P's optimal payoff under delegation is continuously increasing in ℓ as the bound $B_d(x)$ is continuously increasing in ℓ. Thus, there exists a threshold $\hat{\ell}$ below the threshold $\overline{\ell}$, such that delegation is optimal if $\ell \geq \hat{\ell}$.

Next, I explain why delegation is optimal if the value of the good lies below the median cost type. Corollary 2 shows that P only benefits from the presence of S if S

can incur a loss in some states of the world. Under centralization, P can impose a loss on S in the case of either no production or production. S's participation constraint prevents P from doing both. Under delegation, P can only impose a loss on S without production. As the maximal loss to S is limited by ℓ, P can reduce expected rent payments through imposing losses under a grand contract with production probability x by at most $x\ell$ if the loss is incurred under production and $(1-x)\ell$ if the loss is incurred without production. For $x < \frac{1}{2}$, P can reduce rents by more if S incurs a loss without production. As $x < \frac{1}{2}$ is optimal if $v < m$, delegation is optimal if $v < m$.

Reversing the argument above, centralization may be better than delegation if P wants to implement a high production probability and S's budget is small. Indeed, if $x > \frac{1}{2}$, P may reduce S's rent up to $x\ell$ if S incurs a loss with production and only up to $(1-x)\ell$ for a loss without production. However, this argument ignores that P has to pay a positive rent to A if S incurs a loss under production as A would otherwise accept any side-contract that avoids production. The next proposition shows that P can nevertheless benefit from centralization.

Proposition 3. *Suppose $f(\theta) = F'(\theta) > 0$ for all $\theta \in \Theta$. Delegation is suboptimal if*

i) the value of the good is high, i.e., $p^*(v) \in (m, \overline{\theta})$ and

ii) S's budget is sufficiently small, i.e., $\ell \in (0, \varepsilon)$ for some $\varepsilon > 0$.

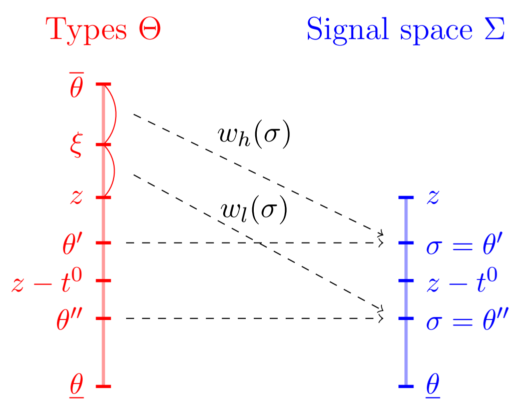
In particular, there exists a combination of an information structure and a feasible grand contract under centralization (I^c, β^c) for which P's expected payoff exceeds $\max_x B_d(x)$ and first-order approximates $\max_x B_c(x)$ if i) and ii) are satisfied.

I now describe the information structure I^c and the grand contract β^c which are formally defined in the proof of the Proposition.

Figure 4 depicts the information structure I^c. Given the cutoff cost $z \in \Theta$, the signal space Σ consists of the interval $[\underline{\theta}, z]$. A type $\theta \leq z$ generates the signal realization $\sigma = \theta$. The types $[z, \overline{\theta}]$ are separated into two intervals by the type $\xi > z$. A type in the interval $[\xi, \overline{\theta}]$ generates signal realizations in the interval $\Sigma_h = [z - t^0, z]$ according to the density $w_h(\sigma)$. A type in the interval $[z, \xi)$ generates signal realizations in $\Sigma_l = [\underline{\theta}, z - t^0]$ according to the density $w_l(\sigma)$ with $t^0 > 0$. Note that I^c is a weighted information structure for $\xi \in \{z, \overline{\theta}\}$.

A produces under the grand contract β^c for $\theta \leq z$. For a signal realization σ, the transfers are given by $t^1_A(\sigma) = \sigma + t^0$, $t^0_A(\sigma) = 0$, $t^1_S(\sigma) = z - \ell - \sigma$, and

Figure 4: Information structure under centralization



The information structure I^c.

$t^0_S(\sigma) = t^0$. Thus, A is effectively offered the price $\sigma + t^0$ and earns a margin of t^0 with production. Nevertheless, only types below z accept the price as I^c pools high producing types with high non-producing types and low producing types with low non-producing types. S incurs a loss with production for high signals in $[z - \ell, z]$ but receives a positive transfer without production. In the proof of the lemma, I specify the parameters z, t^0, ξ, $w_h(\sigma)$, and $w_l(\sigma)$ such that I^c is well-defined and β^c is feasible under centralization for a small ℓ. In particular, S does not benefit from a collusive agreement under which A is induced to not produce. For any σ, this would require S to pay t^0 to A for any cost level $\theta \in Supp(\tilde{\theta}|\sigma)$, not only for those types that would produce under the null side-contract. The functions $w_h(\sigma)$ and $w_l(\sigma)$ pool sufficiently many non-producing types into the signal realizations to make such a collusive agreement unattractive for S.

For a small ℓ, I^c and β^c allow P to reduce the total rent of S and A by approximately $x\ell$ with respect to the monopsony case. This implies that P achieves a higher payoff than under delegation if $x > \frac{1}{2}$. As $x > \frac{1}{2}$ is optimal if the monopsony price $p^*(v)$ exceeds the median, delegation is suboptimal in this case. Moreover, as $B_3(x) \simeq x(v - \overline{\theta}(x)) + x\ell$ for a small ℓ, (I^c, β^c) is near-optimal for a small ℓ and a high v.

There are two notable differences between (I^c, β^c) and the optimum under delegation (I^{x_d}, β^{x_d}). First, there is a difference between the grand contracts, which is in line with the previous discussion. Under β^c, S incurs a loss with production for the types

$\theta \in [z-\ell, z]$ and makes a gain without production. This is reminiscent of the optimal grand contract under centralization in Celik (2009) where S also only incurs a loss after a low-cost realization.In contrast, under the grand contract β^{x_d}, S only incurs a loss without production. Second, whereas I^{x_d} pools all non-producing types into the same signal realizations, there are two separate intervals of non-producing types under I^c. This is necessary as, under β^c, A receives a price which exceeds the cost θ by the fixed margin t^0. In order to avoid A producing for some θ above the cutoff z, I^c needs to ensure that for any signal realization, the distance between producing and non-producing types exceeds the margin t^0. Nevertheless, I^c is very similar to a weighted information structure. In particular, there is exactly one producing type in the support of each signal realization.

While I identify the near-optimal combination (I^c, β^c) for a small ℓ, I was unable to generally characterize optimal combinations of information structure and grand contract if delegation is suboptimal. If delegation is suboptimal, P prefers to pay a positive rent to A. The key challenge toward a complete characterization of optimal combinations of information structure and grand contract lies in finding the optimal allocation of rents across A and S. This problem is considerably more complicated than under delegation where all rents are extracted from A.

7 Extensions

7.1 Alternative bargaining protocols

Throughout the previous sections, the side-contract was set by S through a take-it-or-leave-it offer to A. In the context of public procurement, this assumption captures the situation in which a (corrupt) procurement officer has all the bargaining power vis-à-vis the private supplier.

In this section, I show that the results of the previous sections are robust with respect to the bargaining protocol. Following the approach of Laffont and Martimort (1997), I suppose a third party T offers the side-contract to S and A. To accommodate T, the timing of the game is adapted as follows:

t=0: P chooses an information structure $I \in \mathcal{I}$.

t=1: T, S, and A observe I and σ. A furthermore observes θ.

t=2: P chooses a grand contract β.

t=3: Under centralization, S and A each accept or reject P's offer β. If S or A rejects, both agents receive their outside option. Otherwise the game continues.

t=4: T offers a side-contract γ to S and A.

t=5: S and A each accept or reject T's offer. If both parties accept, the side-contract and the grand contract are played. If S or A rejects under centralization, the grand contract is played non-cooperatively. If S rejects under delegation, both parties receive the outside option. If A rejects under delegation, A receives the outside option and S is required to send a message that induces no production.

The framework with the third party T encompasses and generalizes the setting studied in the previous sections. If T has the same preferences as S, the situation is equivalent to the setting in which S offers the side-contract herself. If T has the same preferences as A, we are in the polar case in which A has all the bargaining power. I do not specify the preferences of the third party. Instead, I assume that T offers a side-contract that is Pareto efficient for the colluding coalition.[31] For a given grand β, there is typically a set of Pareto efficient side-contracts. I assume that P takes a cautious approach and evaluates the grand contract β by the smallest payoff that may arise for any Pareto efficient side-contract. Put differently, P assumes that T has adversarial preferences in the sense that the offered side-contract minimizes P's payoff conditional on being Pareto efficient.

Under centralization, I allow for equilibria in which the side-contract is rejected with positive probability. As Celik and Peters (2011) point out, equilibrium rejections of a mechanism can increase the set of implementable allocations in settings where non-participation in the mechanism triggers a default game played by agents. In such contexts, the rejection decision by an agent may convey information about his type and thereby influence the expected payoffs in the default game. Through this channel, rejections may expand the set of implementable allocations. As the grand contract constitutes the default game for S and A, there may exist a side-contract that can only achieve a Pareto improvement if it is rejected in equilibrium with positive probability.[32]

[31] This approach follows Che and Kim (2006).

[32] As S has no private information vis-à-vis A, a rejection by S does not lead to an update of beliefs. By contrast, rejections of A may alter S's belief about A's type.

Following a rejection of the side-contract under centralization, S and A update their beliefs and play the grand contract non-cooperatively. If there exist multiple continuation equilibria following a rejection of the side-contract, I assume that P can choose a continuation equilibrium. This assumption follows the usual approach in mechanism design which allows the principal to pick an equilibrium in case of equilibrium multiplicity. To account for rejections, I call a side-contract γ feasible under centralization if it satisfies (IC_A^γ) and (LB^γ). I do not require that γ satisfies participation constraints for A or S.

Under delegation, rejection of the side-contract does not lead to S and A playing a default game. Thus, it is not necessary to consider equilibrium rejections of the side-contract. Therefore, I call a side-contract feasible under delegation if it satisfies ($PC_A^{\gamma,d}$), (IC_A^γ), (LB^γ), and

$$\mathbb{E}[t_S(\rho(\tilde{\theta};\sigma)) + \tau(\tilde{\theta};\sigma)|\sigma] \geq 0, \quad \forall \sigma \in \Sigma. \qquad (PC_S^{\gamma,d})$$

A feasible side-contract γ is Pareto efficient under centralization (delegation) if there is no side-contract γ' which is feasible under centralization (delegation) and leads to a weakly higher expected payoff for S for all $\sigma \in \Sigma$ and to a weakly higher expected payoff for A for all $(\sigma, \theta) \in \Sigma \times \Theta$ and either to a strictly higher expected payoff for S for some $\sigma \in \Sigma$ or to a strictly higher payoff for A for some $(\sigma, \theta) \in \Sigma \times \Theta$.

Proposition 4. *Suppose the third party T may offer any feasible and Pareto efficient side-contract. Delegation is optimal if either $\ell \geq \hat{\ell}(v)$ or $v < m$. Delegation is suboptimal if $f(\cdot) = F'(\cdot) > 0$, $p^*(v) \in (m, \overline{\theta})$ and $\ell \in (0, \varepsilon)$ with some $\varepsilon > 0$.*

The combination of information structure I_{w^x} and grand contract β^x defined in Proposition 1 leads to the same production decision and gives P an expected payoff of $B_d(x)$ for any side-contract that is feasible under delegation. As S has a positive payoff under production and incurs a loss without production, S and A may potentially benefit from a side-contract that induces more production. However, any such side-contract requires A to produce for some type θ above the threshold $\overline{\theta}(x)$. Thus, A has to receive a total transfer above $\overline{\theta}(x)$ whenever the good is produced under the side-contract. As S and A only receive a total transfer of $t^1 = \overline{\theta}(x)$ under production, S would incur a loss and the side-contract cannot be feasible. By a similar argument, I show in the proof of the proposition that the combination of information structure I^c and a variant of the grand contract β^c give P the same expected payoff for any feasible side-contract under centralization. Thus, (I_{w^x}, β^x) and (I^c, β^c) perform well independently of the exact bargaining process between S and A.

7.2 Divisible output

In this section, I show that the combination of information control and delegated contracting also performs well with divisible output. In particular, P can extract the full surplus with divisible output under delegation if S has an unlimited budget and the smallest implemented quantity is sufficiently far away from zero.

I extend the main model by allowing for divisible output. Suppose A can produce any quantity $q \in Q \subset \mathbb{R}_+$ with $0 \in Q$ and $\max Q < \infty$ at a cost $c(\theta, q)$. The cost function $c(\theta, q)$ is increasing in both arguments, supermodular, and satisfies $c(\theta, 0) = 0$ for all $\theta \in \Theta$. P's gross utility is given by $v(q)$ with $v(0) = 0$. If A produces the quantity $q \in Q$ for the type $\theta \in \Theta$, the social surplus is given by $\phi(\theta, q) \equiv v(q) - c(\theta, q)$. An efficient production schedule is given by $\{q^*(\theta)\}_{\theta\in\Theta}$ with

$$q^*(\theta) \equiv \min\left\{\arg\max_{q\in Q} \phi(\theta, q)\right\}.$$

The efficient production schedule $q^*(\theta)$ is weakly decreasing due to supermodularity of the cost function. Suppose that $q^*(\theta) = 0$ for some $\theta \in \Theta$.[33] Define the cutoff $\theta_0 \equiv \inf\{\theta \in \Theta : q^*(\theta) = 0\}$ and denote the set of positive and efficient quantities by $Q^*_+ \equiv \{q \in Q \setminus \{0\} : \exists\theta \in \Theta, q^*(\theta) = q\}$ and its smallest element by $\underline{q}^* \equiv \inf Q^*_+$. The maximal social surplus for $\theta \in \Theta$ is $\phi^*(\theta) \equiv \phi(\theta, q^*(\theta))$ resulting in an expected full surplus of $\Phi^* \equiv \int_\Theta \phi^*(\theta) dF(\theta)$.

Proposition 5. *Suppose output is divisible and S has an unlimited budget. P can extract the full surplus under delegation if*

$$\underline{q}^* \geq \min\{q \in Q : \phi^*(\underline{\theta}) \leq c(\theta_0, q) - c(\underline{\theta}, q)\} \tag{5}$$

where $\min\{q \in Q : \phi^*(\underline{\theta}) \leq c(\theta_0, q) - c(\underline{\theta}, q)\} \in (0, q^*(\underline{\theta}))$.

I extend the idea behind the optimal combination of information structure and grand contract under delegation in Corollary 3 to the case of divisible output. In particular, P optimally sets a weighted information structure characterized by the

[33] The proof also describes how to deal with the case that $q^*(\theta) > 0$ for all $\theta \in \Theta$.

signal space $\Sigma = \Theta \setminus [\theta_0, \overline{\theta}]$ and the weighting function $w^*(\sigma) = \frac{f(\sigma)\phi^*(\sigma)}{\Phi^*}$ together with a grand contract under which P pays S a transfer of $v(q)$ for the delivery of a positive quantity $q \in Q^*_+$, S pays P a transfer of $\frac{\Phi^*}{\Pr(\theta \geq \theta_0)}$ in case of no production, and S is free to offer any sub-contract to A.

Under this delegation contract, S is the residual claimant in case of production. However, all of S's expected rent from production can be extracted in expectation through the payment without production if condition (5) is satisfied. This condition ensures that S cannot gain from inducing A to produce a positive quantity for $\theta \geq \theta_0$. If the type $\theta \geq \theta_0$ produces a quantity q for some signal realization σ, the type $\theta = \sigma$ receives a rent of at least $c(\theta_0, q) - c(\sigma, q)$. If this rent exceeds the social surplus $\phi^*(\sigma)$, S's payoff is negative with and without production. Thus, inducing a type $\theta \geq \theta_0$ to produce is unprofitable in this case. Condition (5) ensures that $\phi^*(\sigma) \leq c(\theta_0, q) - c(\sigma, q)$ for all $q \in Q^*_+$ and all $\sigma \in \Sigma$ as the inequality is the hardest to satisfy for the lowest signal $\sigma = \underline{\theta}$ and the quantity $\underline{q}^*$.

Condition (5) requires that the smallest positive and efficient quantity $\underline{q}^*$ is sufficiently far away from zero. If $\underline{q}^*$ is very close to zero, S can induce a type $\theta \geq \theta_0$ to produce $\underline{q}^*$ at very low costs in terms of rents to the types below θ. This is profitable as it allows S to save the transfer of $\frac{\Phi^*}{\Pr(\theta \geq \theta_0)}$ to P. Loosely speaking, condition (5) implies that the production decisions under which S is residual claimant are sufficiently different from the production decisions under which P extracts the rent of S.

Condition (5) is likely to be satisfied if A's cost function exhibits fixed costs or P's gross utility $v(q)$ is negative for small q. These instances might arise naturally in many applications. For a specific example, suppose the construction company A could construct an airport of size q for the municipality P. Then it is likely that A incurs some planning costs independently of the size of the airport and that the airport is only economically viable for P if it exceeds a certain minimal size.

7.3 Does information control make delegation optimal?

Consider a principal who gains information control over a supervisor. This change corresponds to moving from a situation with exogenous information – as in Faure-Grimaud et al. (2003) and Celik (2009) – to a setting with endogenous information – as studied in this paper. The results of the previous sections provide conditions under which delegation is optimal with information control. Under these conditions, gaining information control never induces the principal to change from delegation to centralization. However, these results do not tell us when a principal changes from centralization to delegation after obtaining information control.

In this section, I provide conditions on an exogenously given information structure such that delegation is suboptimal. Together with the conditions for the optimality of delegation with information control, these conditions ensure that delegation becomes optimal only after gaining information control. To state the result cleanly, I define – for a given information structure – the collection $\{p_a, x_a\}_{a\geq 0}$ of functions $p_a : \Sigma \to \mathbb{R}$ and $x_a : \Sigma \to [0, 1]$ with $p_a(\sigma) \in \arg\max_p (a - p)G(p|\sigma)$ and $x_a(\sigma) \equiv G(p_a(\sigma)|\sigma)$ for all $\sigma \in \Sigma$ and all $a \geq 0$.

Proposition 6. *Suppose the information structure $I \in \mathcal{I}$ is exogenously given. Delegation is suboptimal if for any collection of functions $\{p_a, x_a\}_{a\geq 0}$ there exists a signal realization $\overline{\sigma} \in \Sigma$ such that*

i) $p_a(\sigma) \leq p_a(\overline{\sigma})$ *and* $x_a(\sigma) \geq x_a(\overline{\sigma})$ *for all* $\sigma \in \Sigma, a \geq 0$,

ii) $\min\{Supp(\tilde{\theta}|\sigma)\} \neq p^*$, *and*

iii) there is no $a^* \geq 0$ *such that* $p_{a^*}(\overline{\sigma}) = p^*$ *and* $x_{a^*}(\sigma) = F(p^*)$ *for all* $\sigma \in \Sigma$.

Condition i) defines a worst signal realization for a given information structure. Suppose P pays S t^1 with production and t^0 without production while authorizing S to offer some price p to A. For some signal realization σ, S optimally offers A a price of $p_{t^1-t^0}(\sigma)$ which is accepted by A with probability $x_{t^1-t^0}(\sigma)$. If condition i) is satisfied, the signal realization for which S offers the highest price to A is also the signal after which A accepts the offer with the lowest probability.

Delegation is suboptimal for some exogenously given information structure if there exists a worst signal realization. After the worst signal realization $\overline{\sigma}$, S's payoff with production $t_1 - p(\overline{\sigma})$ and the probability of production $x(\overline{\sigma})$ are lower than for all other signal realizations. Thus, S's participation constraint is most restrictive after the worst signal realization and severely limits P's ability to extract rents from S for the other signal realizations. This effect is so strong that P prefers to contract directly with A and to ignore the additional information S might provide.

Condition ii) ensures that P cannot replicate the monopsony outcome under delegation. Condition iii) excludes the case in which the probability of production is identical for all signal realizations and the price after the worst signal equals the monopsony price. Note that this condition rules out the case where S is uninformed. Conditions i), ii), and iii) jointly imply that P's payoff under delegation is strictly below the monopsony payoff $\underline{W}$.

To illustrate the conditions in Proposition 6, I consider two common forms of information structures – information structures with additive noise and partitional information structures. I study when these information structures satisfy the conditions of Proposition 6. First, I consider an *information structure with additive noise*. Under such an information structure, A's cost is given by $\tilde{\theta} = \tilde{\sigma} + \tilde{\varepsilon}$ with $\tilde{\sigma}$ and $\tilde{\varepsilon}$ being independently distributed according to the continuous distribution functions $H(\sigma)$ and $Z(\varepsilon)$ with the supports $[\underline{\sigma}, \overline{\sigma}]$ and $[\underline{\varepsilon}, \overline{\varepsilon}]$. Second, I consider a *partitional information structure*. For such an information structure, the set Θ is partitioned and each signal $\sigma \in \Sigma$ is associated with exactly one element of the partition. I denote by $l(\sigma)$ and $h(\sigma)$ the lower and upper bound of the element of the partition associated with $\sigma \in \Sigma$. By convention, let $l(\sigma)$ and $h(\sigma)$ be increasing in σ.

Corollary 4. *Delegation is suboptimal for an information structure with additive noise if* $\log Z(\varepsilon)$ *is concave and* $\overline{\sigma} - \underline{\varepsilon} \neq p^*$. *Delegation is suboptimal for a partitional information structure if* $\log F(\theta)$ *is concave,* $F(h(\sigma)) - F(l(\sigma))$ *is increasing in* σ, *and* $l(\sigma) \neq p^*$ *and* $h(\sigma) \neq p^*$ *for all* $\sigma \in \Sigma$.

Corollary 4 shows that delegation is suboptimal under information structures with additive noise and partitional information structures under relatively mild assumptions. Thus, one can easily find cases in which delegation is suboptimal without and optimal with information control. This suggests that the introduction of information control might often be followed by a change from a centralized to a decentralized organizational structure.[34]

7.4 Ex-ante collusion

In this section, I consider the case of *ex-ante collusion* studied in Mookherjee et al. (2019). With ex-ante collusion, the side-contract between A and S does not only specify side-transfers and coordinated behavior in the grand mechanism, it also stipulates whether A and S participate in the grand mechanism in the first place. This contrasts with the assumption from the previous sections where I follow Faure-Grimaud

[34]In the model, centralization is always at least as good as delegation. However, if there is a cost of communication for the principal, delegation is strictly optimal under the appropriate conditions.

et al. (2003) and Celik (2009) in assuming that A and S cannot collude on their participation decisions regarding the grand contract. In the spirit of Mookherjee et al. (2019), I introduce ex-ante collusion by requiring that any grand contract includes the message e which results in no production and no transfer payments between P and A or S, i.e., $X = t_A = t_S = 0$.

A grand contract that is feasible under centralization or delegation is also *ex-ante collusion-proof* if the addition of the message e to the grand contract does not reduce P's payoff. If a grand contract is ex-ante collusion-proof, the option for A and S to coordinate their participation decision through the side-contract does not harm P. The following proposition provides a simple test of whether a feasible grand contract is ex-ante collusion-proof.

Proposition 7. *Consider a grand contract that is feasible under centralization or delegation. The grand contract is ex-ante collusion-proof if and only if* $t^0 \geq 0$.

The proposition is implied by the following argument. Without the exit message e, the colluding coalition can either coordinate on a message that induces production and a total payment of t^1 from P or send a message that induces no production and a total payment of t^0 from P. If the grand contract includes the exit message e, the coalition has the additional option of inducing no production and a total payment of zero from P through sending the message e. As the payoffs of A and S are common knowledge without production, the coalition benefits from the message e if $t^0 < 0$, i.e., if sending e allows the coalition to avoid making a payment to P. By contrast, if $t^0 \geq 0$, the coalition never gains from sending e instead of a message that induces no production and the total payment t^0.

Under an ex-ante collusion-proof grand contract, P cannot impose losses on S in case of no production. This result leads us to the following three observations.

Corollary 5. *P's payoff from any grand contract that is feasible under delegation and ex-ante collusion-proof lies weakly below the monopsony payoff.*

Under ex-ante collusion, the double marginalization problem with delegation cannot be resolved by information design. Under delegation, P cannot impose a loss on S in case of production as S can always avoid production by offering A a side-contract which sends a message that leads to no production for any report of A. With ex-ante collusion, P cannot impose a loss on S without production as S can always avoid the loss by using the exit message e in the side-contract. Lemma 3 shows that P only benefits from S if the latter bears some loss in some state of the world. Thus, P does not benefit from the presence of S if the grand contract is both ex-ante collusion proof and feasible under delegation.

Corollary 5 is closely connected to Proposition 2 in Mookherjee et al. (2019). They show that delegation is strictly inferior to not hiring S at all under the assumption that S is only partially informed about A. Moreover, Corollary 5 implies the following connection between the optimality of delegation and ex-ante collusion-proofness.

Corollary 6. *Consider a combination of an information structure and a grand contract that is optimal in the model without ex-ante collusion and suppose S's budget is strictly positive. The optimal grand contract is ex-ante collusion-proof if and only if delegation is suboptimal in the model without ex-ante collusion.*

The corollary is implied by the following argument. If the optimal grand contract is ex-ante collusion-proof, P cannot impose losses on S in case of no production. In order to benefit from S – which is possible given $\ell > 0$ – P has to impose a loss on S in case of production. However, this is only possible with centralization and, thus, delegation is suboptimal. Second, if delegation is suboptimal, P imposes a loss on S in case of production for at least one signal realization. It follows that P cannot impose a loss on S in case of no production to satisfy S's participation constraint. As S never makes a loss in case of no production, the exit message e is not helpful for S which implies that the grand contract is ex-ante collusion-proof.

Finally, I can make a statement about almost optimal combinations of information structures and grand contracts under ex-ante collusion if S's budget is small.

Corollary 7. *Suppose $F'(\theta) = f(\theta) > 0$ for all $\theta \in \Theta$. If S's budget is small, the combination of information structure I^c and grand contract β^c is near-optimal in the model with ex-ante collusion.*

The result is proved as follows. Lemma 3 implies that P's payoff is bounded by $B_3(x)$ for any grand contract with production probability x that satisfies $t_S^0(\sigma) \geq 0$ for some $\sigma \in \Sigma$. Any ex-ante collusion-proof grand contract satisfies $t_S^0(\sigma) \geq 0$ for all $\sigma \in \Sigma$. Thus, $B_3(x)$ is an upper bound on P's payoff under ex-ante collusion. The proof of Proposition 3 shows that P's payoff from (I^c, β^c) first-order approximates $\max_x B_3(x)$ for small ℓ. As β^c satisfies $t^0 > 0$, (I^c, β^c) is near-optimal with ex-ante collusion for small ℓ.

Mookherjee et al. (2018) study a model of collusive supervision with exogenous information, a binary production decision, and ex-ante collusion. As in the present paper, their optimal grand contract features a positive payment to S without production and a negative payment with production. As P is unable with ex-ante collusion to extract rents from A or S without production, P can only benefit from S by imposing a loss on S with production.

8 Conclusion

I consider a principal-supervisor-agent relationship in which the supervisor and agent can collude and the principal designs the supervisor's signal of the agent's private type. I study how the principal optimally uses information control to fight collusion and whether the principal should delegate the design of the agent's contract to the supervisor. The principal optimally chooses a partially informative signal and can extract the full surplus if the supervisor's budget is large enough. Delegation is an optimal response to collusion under information control if either the supervisor's budget is large or the principal's value from production is small. However, delegation is suboptimal if the agent's production is of high value to the principal and the supervisor's budget is sufficiently small.

I see two directions for further research. First, this paper studies the joint design of mechanisms and information under collusion with a focus on collusive supervision. It seems worthwhile to explore whether the insights extend to more general mechanism design environments. Second, this paper closely follows the literature on collusive supervision with exogenous information in assuming that the supervisor's signal is also observed by the agent. If the supervisor's budget is large, the principal does not benefit from inducing additional asymmetric information in the colluding coalition through the provision of private information to the supervisor. However, private information on the supervisor's side may be beneficial if the supervisor's budget is small. I leave these questions for future research.

Appendix

A Omitted proofs

Proof of Lemma 1: Collusion-proofness principle

I prove that S has an optimal, direct, and truthful side-contract for any β, I, and $\sigma \in \Sigma$. Given β, I, and $\sigma \in \Sigma$, S chooses a deterministic side-contract. The standard revelation principle does not apply to this setting as it may be impossible to replicate mixed reporting strategies to an indirect side-contract in a deterministic direct side-contract (Strausz, 2003). However, as the side-contract governs the interaction of one principal S and one agent A, a revelation principle in terms of payoffs due to Strausz (2003) applies: For any indirect side-contract, there exists a direct and truthful side-contract which gives both S and A at least the same payoffs than the indirect side-contract. Thus, for any β, there is a direct and truthful side-contract γ which is a best response for S. It follows that – given the definition of equilibrium in Section 3 – S offers a direct and truthful side-contract in any equilibrium, and that some direct and truthful side-contract is optimal for S if and only if there does not exist another truthful and direct side-contract which is strictly better for S.

I now prove the second part of the lemma. Consider a PBE with passive beliefs under either centralization or delegation in which P offers a grand contract β and S offers a side-contract γ. Let $\overline{u}_A(\sigma, \theta)$ denote A's payoff on the equilibrium path for each realization of $(\sigma, \theta) \in Supp(\tilde{\theta}, \tilde{\sigma})$. I want to argue that there exists another equilibrium in which P offers the grand contract $\beta_0 \equiv \beta \circ \gamma$ – with $M_S = \Sigma$ and $M_A = \Sigma \times \Theta$ under centralization, and $M_S = \Sigma^2 \times \Theta$ under delegation – and S offers the null side-contract γ_0. Given β_0, γ_0 is truthful as A's mapping from reports to payoffs remains the same as with β and γ. Furthermore, the null side-contract is optimal for S. Toward a contradiction, suppose that the null side-contract is suboptimal. Then there exists a side-contract γ^* such that for at least one $\sigma \in \Sigma$, γ^* gives A a payoff of at least $\overline{u}_A(\sigma, \theta)$ for any θ and gives S a strictly higher expected payoff than the null side-contract. However, this implies that the side-contract $\gamma^{**} \equiv \gamma \circ \gamma^*$ is a profitable deviation from γ if P offers the grand contract β. This leads to a contradiction. Finally, note that both equilibria are payoff-equivalent by construction. □

Proof of Lemma 2

I prove the result through a sequence of Lemmas.

Lemma A.1. *For any feasible grand contract, there exist the functions* $t_S^0, t_S^1, t_A^0, t_A^1 : \Sigma \to \mathbb{R}$ *such that* $t_i(\sigma,\sigma,\theta) = X(\sigma,\sigma,\theta)t_i^1(\sigma) + (1 - X(\sigma,\sigma,\theta))t_i^0(\sigma)$ *for* $i \in \{A, S\}$.

Proof. I start by proving the result for $i = A$. Fix some signal realization $\sigma \in \Sigma$. For any two types $\theta, \theta' \in Supp(\tilde{\theta}|\sigma)$ with $X(\sigma,\sigma,\theta) = X(\sigma,\sigma,\theta')$, $(IC_A^{\gamma_0})$ implies that $t_A(\sigma,\sigma,\theta) = t_A(\sigma,\sigma,\theta')$. Thus, there exist two functions $t_A^0 : \Sigma \to \mathbb{R}$ and $t_A^1 : \Sigma \to \mathbb{R}$ such that $t_A(\sigma,\sigma,\theta) = t_A^j(\sigma)$ whenever $\theta \in Supp(\tilde{\theta}|\sigma)$ satisfies $X(\sigma,\sigma,\theta) = j$ for $j \in \{0, 1\}$.

I prove the result for $i = S$ by contradiction. Suppose the statement does not hold. Then there exists a feasible grand contract, a signal $\sigma' \in \Sigma$, and two types $\theta', \theta'' \in Supp(\tilde{\theta}|\sigma')$ such that $X(\sigma',\sigma',\theta') = X(\sigma',\sigma',\theta'')$ and $t_S(\sigma',\sigma',\theta') > t_S(\sigma',\sigma',\theta'')$. Consider a side-contract γ' with $\rho(\theta;\sigma) = (\sigma,\sigma,\theta)$ for all $(\sigma,\theta) \neq (\sigma',\theta'')$, $\rho(\theta'';\sigma') = (\sigma',\sigma',\theta')$, and $\tau(\theta;\sigma) = 0$. Due to the argument for the case $i = A$, γ' leads to the same payoffs for A as γ_0. Thus, A reports truthfully to γ'. However, S has a strictly higher payoff under γ' than under γ_0. Thus, the grand contract is not feasible. □

Lemma A.2. *Any feasible grand contract satisfies without loss of optimality* $t_A^0(\sigma) \geq 0$ *and* $p(\sigma) \equiv t_A^1(\sigma) - t_A^0(\sigma) \geq \sup\{\theta \in Supp(\tilde{\theta}|\sigma) : X(\sigma,\sigma,\theta) = 1\}$.

Proof. I start by showing that a) $t_A^0(\sigma) \geq 0$, $\forall \sigma \in \Sigma$ with $\mathbb{E}[X(\sigma,\sigma,\tilde{\theta})|\sigma] < 1$, b) $p(\sigma) \geq \sup\{\theta \in Supp(\tilde{\theta}|\sigma) : X(\sigma,\sigma,\theta) = 1\}$, $\forall \sigma \in \Sigma$ with $\mathbb{E}[X(\sigma,\sigma,\tilde{\theta})|\sigma] \in (0, 1)$, and c) $t_A^1(\sigma) \geq \sup\{\theta \in Supp(\tilde{\theta}|\sigma) : X(\sigma,\sigma,\theta) = 1\}$, $\forall \sigma \in \Sigma$ with $\mathbb{E}[X(\sigma,\sigma,\tilde{\theta})|\sigma] = 1$.

$\mathbb{E}[X(\sigma,\sigma,\tilde{\theta})|\sigma] < 1$ implies that there exists some $\theta \in Supp(\tilde{\theta}|\sigma)$ with $X(\sigma,\sigma,\theta) = 0$. (PC_A) for centralization and $(PC_A^{\gamma_0})$ for delegation imply that $t_A(\sigma,\sigma,\theta) \geq 0$. Result a) then follows from Lemma A.1. $\mathbb{E}[X(\sigma,\sigma,\tilde{\theta})|\sigma] \in (0, 1)$ implies that there exist $\theta, \theta' \in Supp(\tilde{\theta}|\sigma)$ with $X(\sigma,\sigma,\theta) = 0$ and $X(\sigma,\sigma,\theta') = 1$. It follows from $(IC_A^{\gamma_0})$ that $p(\sigma) \geq \theta''$ for all θ'' such that $X(\sigma,\sigma,\theta'') = 1$, which proves b). For $\mathbb{E}[X(\sigma,\sigma,\tilde{\theta})|\sigma] = 1$, $X(\sigma,\sigma,\theta) = 1$ for all $\theta \in Supp(\tilde{\theta}|\sigma)$. (PC_A) for centralization and $(PC_A^{\gamma_0})$ for delegation imply c). The lemma then follows from setting $t_A^0(\sigma) = 0$ for $\mathbb{E}[X(\sigma,\sigma,\tilde{\theta})|\sigma] = 1$ and $p(\sigma) = 0$ for $\mathbb{E}[X(\sigma,\sigma,\tilde{\theta})|\sigma] = 0$. □

Lemma A.3. *Any feasible grand contract satisfies without loss of optimality* $t_S^j(\sigma) \geq -\ell$ *for* $j \in \{0, 1\}$ *and* $t_S^j(\sigma) < 0 \Rightarrow t_S^{j'}(\sigma) \geq 0$ *for* $j, j' \in \{0, 1\}, j \neq j'$.

Proof. $t_S^j(\sigma) \geq -\ell, \forall j \in \{0,1\}$ follows directly from (LB) for all $\sigma \in \Sigma$ such that $X(\sigma,\sigma,\theta)$ is not constant in θ. For any $\sigma \in \Sigma$ with $X(\sigma,\sigma,\theta)$ constant in θ, (LB) requires either $t_S^0(\sigma) \geq -\ell$ or $t_S^1(\sigma) \geq -\ell$. However, the other condition can be satisfied without affecting payoffs as they are never realized. Finally, note that (PC_S) and Lemma A.1 imply

$$\mathbb{E}\left[t_S(\sigma,\sigma,\tilde{\theta})|\sigma\right] = \mathbb{E}\left[X(\sigma,\sigma,\tilde{\theta})|\sigma\right] t_S^1(\sigma) + \left(1 - \mathbb{E}\left[X(\sigma,\sigma,\tilde{\theta})|\sigma\right]\right) t_S^0(\sigma) \geq 0.$$

Thus, either $t_S^0(\sigma)$ or $t_S^1(\sigma)$ has to be weakly positive. □

Lemma A.4. *Any feasible grand contract satisfies without loss of optimality* $t_S^j(\sigma) + t_A^j(\sigma) = t^j$ *for* $j \in \{0,1\}$.

Proof. I prove this result by contradiction. Suppose the statement does not hold. Then there exists a feasible grand contract, two signal realizations $\sigma', \sigma'' \in \Sigma$ with $t_S^j(\sigma') + t_A^j(\sigma') < t_S^j(\sigma'') + t_A^j(\sigma'')$, and two types $\theta', \theta'' \in \Theta$ with $X(\sigma',\sigma',\theta') = X(\sigma'',\sigma'',\theta'') = j$. Define a side-contract γ such that

$$\rho(\theta;\sigma) = \begin{cases} (\sigma'',\sigma'',\theta'') & \text{if} \quad \sigma = \sigma', \theta \in \{\theta \in Supp(\tilde{\theta}|\sigma') : X(\sigma',\sigma',\theta) = j\}, \\ (\sigma,\sigma,\theta) & \text{otherwise;} \end{cases}$$

$$\tau(\theta;\sigma) = \begin{cases} t_A^j(\sigma'') - t_A^j(\sigma') & \text{if} \quad \sigma = \sigma', \theta \in \{\theta \in Supp(\tilde{\theta}|\sigma') : X(\sigma',\sigma',\theta) = j\}, \\ 0 & \text{otherwise.} \end{cases}$$

For A, γ is equivalent to γ_0. Thus, γ satisfies (PC_A^γ) and (IC_A^γ). For $\sigma = \sigma'$ and $\theta \in \{\theta \in Supp(\tilde{\theta}|\sigma') : X(\sigma',\sigma',\theta) = 1\}$, S's payoff is strictly larger under γ than under γ_0: $t_S^j(\sigma'') + t_A^j(\sigma'') - t_A^j(\sigma') > t_S^j(\sigma')$. For all other combinations of signal realization and type (σ,θ), S's total payoff is the same under γ and γ_0. Thus, γ is feasible as (LB^γ) is satisfied and the original grand contract is not feasible as (CP^z) is violated for $z \in \{c,d\}$. □

Lemma A.5. *Any grand contract that is feasible under delegation satisfies without loss of optimality* $t_S^1(\sigma) \geq 0$ *for all* $\sigma \in \Sigma$.

Proof. I prove the result by contradiction. Suppose there exists a feasible grand contract β under delegation and a signal realization $\sigma \in \Sigma$ such that $t_S^1(\sigma) < 0$. As

(PC_S) is equivalent to

$$\mathbb{E}\left[t_S(\sigma,\sigma,\tilde{\theta})|\sigma\right] = \mathbb{E}\left[X(\sigma,\sigma,\tilde{\theta})|\sigma\right] t_S^1(\sigma) + \left(1-\mathbb{E}\left[X(\sigma,\sigma,\tilde{\theta})|\sigma\right]\right) t_S^0(\sigma) \geq 0,$$

$t_S^1(\sigma) < 0$ implies $\mathbb{E}\left[X(\sigma,\sigma,\tilde{\theta})|\sigma\right] < 1$ and $t_S^0(\sigma) > 0$. Thus, there exists a type $\theta' \in Supp(\tilde{\theta}|\sigma)$ with $X(\sigma,\sigma,\theta') = 0$. S could therefore offer a feasible side-contract with $\rho(\theta;\sigma) = (\sigma,\sigma,\theta')$ and $\tau(\theta;\sigma) = 0$ for all $\theta \in Supp(\tilde{\theta}|\sigma)$ giving S a payoff of $t_S^0(\sigma) > \mathbb{E}\left[t_S(\sigma,\sigma,\tilde{\theta})|\sigma\right]$. Thus, (CP^d) is not satisfied and β is not feasible under delegation. □

Proof of Lemma 3

Fix a grand contract with production probability x. The participation constraints (PC_A) and (PC_S) imply that P's payoff cannot exceed $B_1(x)$. Points *ii)* and *v)* of Lemma 2 imply that under delegation $t^1 = t_S^1(\sigma)+p(\sigma)+t_A^0(\sigma) \geq \overline{\theta}(x)$. Points *ii)* and *iii)* of Lemma 2 imply $t^0 = t_A^0(\sigma) + t_S^0(\sigma) \geq -\ell$. Thus, P's payoff under delegation satisfies $x(v-t^1) - (1-x)t^0 \leq x(v-\overline{\theta}(x)) + (1-x)\ell$. It follows that P's payoff is bounded by $B_d(x) = \min\{B_1(x), B_2(x)\}$ under delegation.

Under centralization, P may set a grand contract with $t_S^1(\sigma) \geq 0$ for all $\sigma \in \Sigma$. Using the arguments from the last paragraph, P's payoff cannot exceed $B_d(x)$. Instead, consider a grand contract β with production probability x and $t_S^1(\sigma') < 0$ for some signal $\sigma' \in \Sigma$. I show that $t^0 \geq 0$ and $t^1 \geq \overline{\theta}(x) - \ell$. Point iii) of Lemma 2 implies $t_S^0(\sigma') \geq 0$. Point *ii)* of Lemma 2 implies $t_A^0(\sigma') \geq 0$. Thus, $t^0 = t_S^0(\sigma') + t_A^0(\sigma') \geq 0$. From point *ii)* of Lemma 2 it follows that there exists a signal realization $\sigma'' \in \Sigma$ such that $t_A^1(\sigma'') \geq p(\sigma'') \geq \overline{\theta}(x)$. Point *iii)* of Lemma 2 implies $t_S^1(\sigma'') \geq -\ell$. Thus, $t_S^1(\sigma'') + t_A^1(\sigma'') = t^1 \geq \overline{\theta}(x) - \ell$. (PC_S) and point *ii)* of Lemma 2 imply

$$\begin{aligned} &\mathbb{E}[t_S(\sigma,\sigma,\tilde{\theta}) + t_A(\sigma,\sigma,\tilde{\theta})|\sigma] \geq \mathbb{E}[t_A(\sigma,\sigma,\tilde{\theta})|\sigma] \\ &= \mathbb{E}[X(\sigma,\sigma,\tilde{\theta})t_A^1(\sigma) + (1-X(\sigma,\sigma,\tilde{\theta}))t_A^0(\sigma)|\sigma] \geq \mathbb{E}[X(\sigma,\sigma,\tilde{\theta})p(\sigma)|\sigma], \end{aligned}$$

and $t^0 \geq 0$ implies

$$t_A(\sigma,\sigma,\theta) + t_S(\sigma,\sigma,\theta) = X(\sigma,\sigma,\theta)t^1 + (1-X(\sigma,\sigma,\theta))t^0 \geq X(\sigma,\sigma,\theta)t^1.$$

These inequalities imply

$$\mathbb{E}[t_S(\sigma,\sigma,\tilde{\theta})+t_A(\sigma,\sigma,\tilde{\theta})|\sigma]\geq\mathbb{E}[X(\sigma,\sigma,\tilde{\theta})\max\{t^1,p(\sigma)\}].$$

Using the law of iterated expectations, I find

$$\mathbb{E}[t_S(\tilde{\sigma},\tilde{\sigma},\tilde{\theta})+t_A(\tilde{\sigma},\tilde{\sigma},\tilde{\theta})]\geq\mathbb{E}[X(\tilde{\sigma},\tilde{\sigma},\tilde{\theta})\max\{t^1,p(\tilde{\sigma})\}].$$

Due to $t^1\geq\overline{\theta}(x)-\ell$ and $X(\sigma,\sigma,\theta)p(\sigma)\geq X(\sigma,\sigma,\theta)\theta$, P's payoff is bounded by

$$\begin{aligned}
&\mathbb{E}[X(\tilde{\sigma},\tilde{\sigma},\tilde{\theta})(v-\max\{\overline{\theta}(x)-\ell,\tilde{\theta}\})]\\
\leq&\int_{\Sigma}\int_{\underline{\theta}}^{\overline{\theta}(x)-\ell}(v-\overline{\theta}(x)+\ell)d\mu(\sigma,\theta)+\int_{\Sigma}\int_{\overline{\theta}(x)-\ell}^{\overline{\theta}(x)}(v-\theta)d\mu(\sigma,\theta)\\
=&\int_{\underline{\theta}}^{\overline{\theta}(x)-\ell}(v-\overline{\theta}(x)+\ell)dF(\theta)+\int_{\overline{\theta}(x)-\ell}^{\overline{\theta}(x)}(v-\theta)dF(\theta)\\
=&x(v-\overline{\theta}(x))+\int_{\overline{\theta}(x)-\ell}^{\overline{\theta}(x)}F(\theta)d\theta.
\end{aligned}$$

The second line follows from the fact that the first line is maximal under the constraint $\int_{\Sigma}\int_{\Theta}X(\sigma,\sigma,\theta)d\mu(\sigma,\theta)=x$ for $X(\sigma,\sigma,\theta)=\mathbf{1}_{\theta\leq\overline{\theta}(x)}$. The third line follows from a change in the order of integration and $\int_{\Sigma}d\mu(\sigma,\theta)=dF(\theta)$. The last line follows from integration by parts. □

Proof of Proposition 1

To accommodate the possibility of mass points, let

$$\Delta_F(\theta)\equiv F(\theta)-\lim_{\theta'\nearrow\theta}F(\theta')\quad\text{and}\quad f(\theta)=\begin{cases}F'(\theta) & \text{if}\quad F'(\theta)\text{ exists;}\\ \Delta_F(\theta) & \text{if}\quad \Delta_F(\theta)>0;\\ 0 & \text{otherwise.}\end{cases}$$

Proof

I need to prove that *i*) I_{w^x} is a well-defined information structure, *ii*) β^x is feasible under delegation, and *iii*) P achieves an expected payoff of $B_d(x)$ using I_{w^x} and β^x.

Before doing so, I argue that $\check{\theta}(x)$ is uniquely defined. For $x=1$, the right-hand side of the equation in brackets in (4) is zero and the left-hand side is zero for $\theta = \underline{\theta}$ and strictly increasing for $\theta \in \Theta$. For $x < 1$, the left-hand side is weakly increasing for $\theta \geq \underline{\theta}$ and the right-hand side is strictly decreasing in θ. For $\theta = \underline{\theta}$, the left-hand side is zero whereas the right-hand side is weakly positive as $\underline{\theta} \leq \overline{\theta}(x)$ and $\ell \geq 0$. For $\theta > \underline{\theta}$, the left-hand side is strictly positive and the right-hand side becomes strictly negative as θ grows large. Thus, $\check{\theta}(x)$ is unique for each $x \in [0,1]$ according to the intermediate value theorem.

Next, I prove *i)* to *iii)*. Let $\underline{\theta}(x) \equiv \min\{\overline{\theta}(x), \check{\theta}(x)\}$. If $w^x(\cdot)$ is a well-defined weighting function, then i) is satisfied. Note that

$$\int_{\Sigma} w^x(\sigma)d\sigma = \frac{\int_{\underline{\theta}(x)}^{\overline{\theta}(x)} 0 d\sigma}{\int_{\underline{\theta}}^{\underline{\theta}(x)} F(\sigma)d\sigma} + \frac{\int_{\underline{\theta}}^{\underline{\theta}(x)} f(\sigma)(\underline{\theta}(x) - \sigma)d\sigma}{\int_{\underline{\theta}}^{\underline{\theta}(x)} F(\sigma)d\sigma} = 1.$$

Together with $w^x(\cdot) \geq 0$, this implies that I_{w^x} is well-defined.

Condition *ii)* holds if β^x satisfies the constraints (LB), (PC_S), (IC_S), and (CP^d). (LB) is satisfied as $t^1 - \sigma = \overline{\theta}(x) - \sigma \geq 0 \geq -\ell$ and $t^0 \geq -\ell$. S's expected payoff for $\sigma > \underline{\theta}(x)$ is $\overline{\theta}(x) - \sigma > 0$. For $\sigma \leq \underline{\theta}(x)$, it is

$$\begin{aligned}
&\frac{f(\sigma)(\overline{\theta}(x) - \sigma) + (1-x)w^x(\sigma)t^0}{f(\sigma) + (1-x)w^x(\sigma)} \\
&= \frac{f(\sigma)(\overline{\theta}(x) - \underline{\theta}(x)) + f(\sigma)(\underline{\theta}(x) - \sigma) + (1-x)w^x(\sigma)t^0}{f(\sigma) + (1-x)w^x(\sigma)} \\
&= \frac{f(\sigma)(\overline{\theta}(x) - \underline{\theta}(x)) + w^x(\sigma)\int_{\underline{\theta}}^{\underline{\theta}(x)} F(\sigma)d\sigma + (1-x)w^x(\sigma)t^0}{f(\sigma) + (1-x)w^x(\sigma)} = \overline{\theta}(x) - \underline{\theta}(x) \geq 0,
\end{aligned}$$

where the second equality follows from the definition of $w^x(\sigma)$ and the last equality follows from

$$\begin{aligned}
-t^0 &= \min\left\{\frac{\int_{\underline{\theta}}^{\overline{\theta}(x)} F(\sigma)d\sigma}{1-x}, \ell\right\} = \min\left\{\frac{\int_{\underline{\theta}}^{\overline{\theta}(x)} F(\sigma)d\sigma}{1-x}, \frac{\int_{\underline{\theta}}^{\check{\theta}(x)} F(\sigma)d\sigma}{1-x} - \overline{\theta}(x) + \check{\theta}(x)\right\} \\
&= \frac{\int_{\underline{\theta}}^{\underline{\theta}(x)} F(\sigma)d\sigma}{1-x} - \overline{\theta}(x) + \underline{\theta}(x).
\end{aligned}$$

Thus, (PC_S) is satisfied. (IC_S) is satisfied as any report with $\hat{\sigma}_S \neq \sigma$ leads with certainty to the worst possible payoff for S. It remains to check whether (CP^d)

is satisfied. Note that under β and γ_0, A receives a take-it-or-leave-it price offer of σ. Thus, $(PC_A^{\gamma_0})$ and $(IC_A^{\gamma_0})$ are satisfied. Furthermore, (LB) implies (LB^{γ_0}). Given that the total transfers to S and A are fixed to t^0 and t^1 conditional on the production decision, any side-contract γ that is strictly better than γ_0 for S needs to change the probability of production. For a given signal $\sigma \in \Sigma$, S may either lower the probability of production to zero or increase it to $q > G(\sigma|\sigma)$. In the first case, S's payoff is $t^0 \leq 0$. In the latter case, S needs to pay A a side-transfer of τ such that $\sigma + \tau > \overline{\theta}(x)$ whenever production takes place. Thus, S's expected payoff is $q(t^1 - \sigma - \tau) + (1-q)t^0 \leq 0$. Thus, (CP^d) is satisfied.

Finally, $iii)$ holds as P's expected payoff from β^x and I_{w^x} is

$$x(v - t^1) - (1-x)t^0 = x(v - \overline{\theta}(x)) + \min\left\{(1-x)\ell, \int_{\underline{\theta}}^{\overline{\theta}(x)} F(\sigma)d\sigma\right\} = B_d(x).$$

□

Proof of Proposition 2 and Lemma 4

First, I show that delegation is optimal if $\ell \geq \hat{\ell}(v)$ for some $\hat{\ell} : [0, \infty) \to [0, \overline{\ell})$. Let V_c and V_d denote P's optimal payoffs under centralization and delegation. Denote by x_c and x_d the optimal production probabilities under centralization and delegation. The following three lemmas prove Lemma 4.

Lemma A.6. *If $V_c > V_d$, then*

$$\Pr\Big((\tilde{\sigma}, \tilde{\theta}) \in \big\{\Sigma \times \Theta : t_S^1(\sigma) < 0, X(\sigma, \sigma, \theta) = 1\big\}\Big) > 0. \tag{6}$$

Proof. I prove the result by contradiction. Suppose $V_c > V_d$ and $\Pr\big((\tilde{\sigma}, \tilde{\theta}) \in \big\{\Sigma \times \Theta : t_S^1(\sigma) < 0, X(\sigma, \sigma, \theta) = 1\big\}\big) = 0$. Recall that $V_c \leq B_1(x_c)$. Due to Lemma 2, it holds that

$$t^1 = t_S^1(\sigma) + p(\sigma) + t_A^0(\sigma) \geq \sup_{\sigma \in \Sigma : t_S^1(\sigma) \geq 0} \Big\{\sup\{\theta \in Supp(\tilde{\theta}|\sigma) : X(\sigma, \sigma, \theta) = 1\}\Big\}$$

For $\Pr\Big((\tilde\sigma,\tilde\theta)\in\big\{\Sigma\times\Theta: t_S^1(\sigma)<0, X(\sigma,\sigma,\theta)=1\big\}\Big)=0$, it holds that

$$\sup_{\sigma\in\Sigma:t_S^1(\sigma)\geq 0}\Big\{\sup\{\theta\in Supp(\tilde\theta|\sigma): X(\sigma,\sigma,\theta)=1\}\Big\}\geq\overline{\theta}(x_c).$$

Together with $t^0\geq-\ell$, this implies $x_c(v-t^1)-(1-x_c)t^0\leq B_2(x_c)$. As $V_c\leq B_1(x_c)$,

$$x_c(v-t^1)-(1-x_c)t^0\leq\min\{B_1(x_c),B_2(x_c)\}=B_d(x_c)\leq\max_{x\in[0,1]}B_d(x)=V_d,$$

which gives a contradiction. □

Lemma A.7. *If* $\overline{W}-V_c<\varepsilon$ *for all* $\varepsilon>0$*, then for any* $\delta>0$

$$\Pr\Big((\tilde\sigma,\tilde\theta)\in\big\{\Sigma\times\Theta: X(\sigma,\sigma,\theta)=1, p(\sigma)-\theta>\delta\big\}\Big)=0. \tag{7}$$

Proof. I prove the result by contradiction. Suppose $\overline{W}-V_c<\varepsilon$ for all $\varepsilon>0$ and $\Pr\Big((\tilde\sigma,\tilde\theta)\in\big\{\Sigma\times\Theta: X(\sigma,\sigma,\theta)=1, p(\sigma)-\theta>\delta\big\}\Big)=\alpha>0$ for some $\delta>0$. Let $X_c(\sigma,\sigma,\theta)$ be the optimal production rule. Define $\Delta\equiv\big\{Supp(\tilde\sigma,\tilde\theta): X_c(\sigma,\sigma,\theta)=1, p(\sigma)-\theta>\delta\big\}$ and $\Delta^c\equiv Supp(\tilde\sigma,\tilde\theta)\setminus\Delta$. Note that

$$\begin{aligned}
V_c&=\iint\Big(X_c(\sigma,\sigma,\theta)(v-t_A^1(\sigma)-t_S^1(\sigma))-(1-X_c(\sigma,\sigma,\theta))(t_A^0(\sigma)+t_S^0(\sigma))\Big)d\mu(\sigma,\theta)\\
&\leq\iint X_c(\sigma,\sigma,\theta)(v-p(\sigma))d\mu(\sigma,\theta)\\
&\leq\iint_{\Delta^c}X_c(\sigma,\sigma,\theta)(v-\theta)d\mu(\sigma,\theta)+\iint_{\Delta}(v-\theta-\delta)d\mu(\sigma,\theta)\\
&\leq\int_{\underline{\theta}}^{v}(v-\theta)dF(\theta)-\iint_{\Delta}\delta d\mu(\sigma,\theta)=\overline{W}-\alpha\delta<\overline{W}
\end{aligned}$$

where the first inequality in the second line follows from (PC_S) and ii) of Lemma 2 and the second inequality follows from point ii) of Lemma 2 and $p(\sigma)>\theta+\delta$ for all $(\sigma,\theta)\in\Delta$. Thus, $\overline{W}-V_c>\alpha\delta$ which leads to a contradiction. □

Lemma A.8. *Suppose the information structure* I *and the grand contract* β *satisfy conditions* (6) *and* (7)*. Then* β *is not feasible under centralization.*

Proof. I prove the lemma by contradiction. Suppose (I,β) satisfy (6) and (7) and β is feasible under centralization. From (6) it follows that there exists a set of signals $\underline{\Sigma}$ with positive mass such that $\sigma\in\underline{\Sigma}$ implies $t_S^1(\sigma)<0$ and $\Pr(X(\sigma,\sigma,\tilde\theta)=1|\tilde\sigma=$

$\sigma) > 0$. If $X(\sigma,\sigma,\theta) = 1$ for all $\theta \in Supp(\tilde{\theta}|\sigma)$, then S's expected payoff after σ would be negative. Thus, (PC_S) implies that for all $\sigma \in \underline{\Sigma}$, there exists $\theta_\sigma^0 \in Supp(\tilde{\theta}|\sigma)$ such that $X(\sigma,\sigma,\theta_\sigma^0) = 0$ and $t_S^0(\sigma) > 0$. Furthermore, (7) implies that for all $\theta \in Supp(\tilde{\theta}|\sigma)$ with $X(\sigma,\sigma,\theta) = 1$ and all $\delta > 0$, it holds that $p(\sigma) - \theta < \delta$.

Consider now the side-contract γ given by

$$\rho(\theta;\sigma) = \begin{cases} (\sigma,\sigma,\theta_\sigma^0) & \text{if} \quad \sigma \in \underline{\Sigma}, \\ (\sigma,\sigma,\theta) & \text{otherwise;} \end{cases} \quad \text{and} \quad \tau(\theta;\sigma) = \begin{cases} -\varepsilon & \text{if} \quad \sigma \in \underline{\Sigma}, \\ 0 & \text{otherwise.} \end{cases}$$

where $\varepsilon > 0$. This side-contract is identical to the null side-contract for $\sigma \notin \underline{\Sigma}$. Thus, the side-contract is feasible for $\sigma \notin \underline{\Sigma}$. For $\sigma \in \underline{\Sigma}$, the side-contract leads to a constant production decision $X = 0$ and a constant payment to A of $t_A^0(\sigma)+\varepsilon$. Thus, it satisfies (IC_A^γ). Any type $\theta \in Supp(\tilde{\theta}|\sigma)$ with $X(\sigma,\sigma,\theta) = 0$ wants to accept γ as it increases A's payoff from $t_A^0(\sigma)$ to $t_A^0(\sigma)+\varepsilon$. Any type $\theta \in Supp(\tilde{\theta}|\sigma)$ with $X(\sigma,\sigma,\theta) = 1$ wants to accept this side-contract as it leads to a payoff of $t_A^0(\sigma) + \varepsilon \geq t_A^0(\sigma) + p(\sigma) - \theta$, as $p(\sigma) - \theta < \delta$ for any $\delta > 0$. Finally, note that γ leads to the same expected payoff for S as the null side-contract for $\sigma \notin \underline{\Sigma}$ and leads to an expected payoff of $t_S^0(\sigma) - \varepsilon$ for $\sigma \in \underline{\Sigma}$ with

$$t_S^0(\sigma) - \varepsilon > \mathbb{E}[X(\sigma,\sigma,\tilde{\theta})|\sigma]t_S^1(\sigma) + (1 - \mathbb{E}[X(\sigma,\sigma,\tilde{\theta})|\sigma])t_S^0(\sigma)$$

for ε sufficiently close to zero. Thus, γ is feasible and strictly profitable for S. It follows that (CP^c) is not satisfied and therefore β is not feasible. This gives a contradiction. □

Point i) of Proposition 2 results from Lemma 4 and the following two observations. First, $\max_{x\in[0,1]} B_d(x)$ is continuous and increasing in ℓ as $B_2(x)$ is continuous and increasing in ℓ. Second, due to Corollary 3, $\max_{x\in[0,1]} B_d(x) = \overline{W}$ for $v < \overline{\theta}$ and $\ell \geq \overline{\ell}$ and $\lim_{\ell\to\infty} \max_{x\in[0,1]} B_d(x) = \overline{W}$ for $v \geq \overline{\theta}$. Thus, that there exists a function $\hat{\ell} : [0,\infty) \to [0,\overline{\ell}(v))$ which gives for each v a bound $\hat{\ell}(v)$ such that delegation is optimal if $\ell \geq \hat{\ell}(v)$.

Second, I prove that delegation is optimal if $v < m$. Let $x_3 \in \arg\max_{x\in[0,1]} B_3(x)$. Note that $x_3 \leq F(v)$ as $B_3(x) = B_1(x) - \int_{\underline{\theta}}^{\overline{\theta}(x)-\ell} F(\theta)d\theta$ is strictly decreasing for $x > F(v)$ due to $B_1(x)$ being strictly decreasing for $x > F(v)$ and $\int_{\underline{\theta}}^{\overline{\theta}(x)-\ell} F(\theta)d\theta$

being weakly increasing. Due to $v < m$, we have $F(\overline{\theta}(x_3)) \leq \frac{1}{2}$ and

$$B_3(x_3) = x_3(v - \overline{\theta}(x_3)) + \int_{\overline{\theta}(x_3)-\ell}^{\overline{\theta}(x_3)} F(\theta)d\theta \leq x_3(v - \overline{\theta}(x_3)) + \int_{\overline{\theta}(x_3)-\ell}^{\overline{\theta}(x_3)} F(\overline{\theta}(x_3))d\theta$$
$$\leq x_3(v - \overline{\theta}(x_3)) + \ell(1 - F(\overline{\theta}(x_3))) = B_2(x_3).$$

Now, I can prove $\max_{x\in[0,1]} B_c(x) = \max_{x\in[0,1]} B_d(x)$ for $v < m$ by contradiction. Suppose $\max_{x\in[0,1]} B_c(x) > \max_{x\in[0,1]} B_d(x)$ and let $x_z \in \arg\max_{x\in[0,1]} B_z(x)$ with $z \in \{c, d\}$. $B_d(x_d) < B_c(x_c)$ implies $B_2(x_c) < B_3(x_c)$ as $B_2(x_c) \geq B_3(x_c)$ would imply $B_c(x_c) = \min\{B_1(x_c), B_2(x_c)\} = B_d(x_c) \leq B_d(x_d)$ which contradicts $B_d(x_d) < B_c(x_c)$. $B_2(x_c) < B_3(x_c)$ implies $B_c(x_c) = B_3(x_c)$ as $B_3(x) \leq B_1(x)$ for all $x \in [0, 1]$. It follows that $B_d(x_d) < B_c(x_c) = B_3(x_c) \leq B_3(x_3) \leq B_2(x_3) \leq B_d(x_d)$ which is a contradiction. Thus, $V_c = V_d$ for $v < m$. □

Proof of Proposition 3

I construct the combination of information structure and grand contract (I^c, β^c) described in the main text. Define β^c by

$$X(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = \mathbf{1}_{\hat{\sigma}_S=\hat{\sigma}_A=\hat{\theta}\leq z}(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}), \quad t_A(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = p(\hat{\theta})X(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}), \tag{8}$$
$$t_S(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = \begin{cases} X(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta})(t^1 - p(\hat{\theta})) + (1 - X(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}))t^0 & \text{if} \quad \hat{\sigma}_S = \hat{\sigma}_A \\ -\ell & \text{if} \quad \hat{\sigma}_S \neq \hat{\sigma}_A \end{cases}$$
$$\text{with} \quad t^0 = \frac{\int_{z-\ell}^{z}(F(z) - F(\theta))d\theta}{1 - F(z)}, \quad t^1 = t^0 + z - \ell, \quad p(\sigma) = \sigma + t^0, \quad z \in \Theta.$$

In order to specify the information structure, I define the weighting function

$$w(\sigma) = \frac{f(\sigma)[\sigma - z + \ell]_+}{\int_{z-\ell}^{z}(F(z) - F(\theta))d\theta}$$

which satisfies $w(\cdot) \geq 0$ and

$$\int_{\underline{\theta}}^{z} w(\sigma)d\sigma = \int_{z-\ell}^{z} \frac{f(\sigma)(\sigma - z + \ell)}{\int_{z-\ell}^{z}(F(z) - F(\theta))d\theta}d\sigma = 1,$$

and the type $\xi \in \Theta$ which satisfies

$$\int_{z-t^0}^{z} w(\sigma)(1-F(z))d\sigma = 1 - F(\xi).$$

Note that $\xi = z$ if $t^0 \geq \ell$ or $z - t^0 \leq \underline{\theta}$. Otherwise, $\xi \in (z, \overline{\theta})$. The information structure I^c is now constructed as follows. For any $\theta \leq z$, the signal realization $\sigma = \theta$ is generated. Any $\theta \in [\xi, \overline{\theta}]$ generates some $\sigma \in \Sigma_h = [z - t^0, z]$ according to the density $w_h(\sigma) = \frac{1-F(z)}{1-F(\xi)} w(\sigma)$. Any $\theta \in [z, \xi)$ generates some $\sigma \in \Sigma_l = [\underline{\theta}, z - t^0]$ according to the density $w_l(\sigma) = \frac{1-F(z)}{F(\xi)-F(z)} w(\sigma)$.[35] Thus, I^c is well-defined for a small ℓ as $w_l(\sigma), w_h(\sigma) \geq 0$,

$$\int_{z-t^0}^{z} w_h(\sigma)d\sigma = \int_{z-t^0}^{z} \frac{1-F(z)}{1-F(\xi)} w(\sigma)d\sigma = 1, \text{ and}$$

$$\begin{aligned}\int_{\underline{\theta}}^{z-t^0} w_l(\sigma)d\sigma &= \int_{\underline{\theta}}^{z-t^0} \frac{1-F(z)}{F(\xi)-F(z)} w(\sigma)d\sigma = \frac{1-F(z)}{F(\xi)-F(z)}\left(1 - \int_{z-t^0}^{z} w(\sigma)d\sigma\right) \\ &= \frac{1-F(z)}{F(\xi)-F(z)}\left(1 - \frac{1-F(\xi)}{1-F(z)}\right) = 1.\end{aligned}$$

Next, I prove that β^c is feasible under centralization given I^c. (PC_A) and (IC_A) are satisfied due to β^c being equivalent to a price offer $p(\sigma)$ to A whenever $Supp(\tilde{\theta}|\sigma) \cap (\sigma, \sigma + t^0) = \emptyset$ under I^c. $Supp(\tilde{\theta}|\sigma) \cap (\sigma, \sigma + t^0) = \emptyset$ holds if $\xi \geq z + t^0$. For $\ell = 0$, $t^0 = 0 < \xi = \overline{\theta}$. As ξ is continuously decreasing in t^0, it follows that $\xi \geq z + t^0$ for ℓ small.

Next, I show that (PC_S) is satisfied. As cost levels below $z - \ell$ are perfectly revealed, the expected payoff of S for $\sigma < z - \ell$ is $z - \ell - \sigma \geq 0$. For $\sigma \geq z - \ell$, the expected payoff is

$$\begin{aligned}&\Pr(\tilde{\theta} = \sigma|\tilde{\sigma} = \sigma)(t^1 - p(\sigma)) + (1 - \Pr(\tilde{\theta} = \sigma|\tilde{\sigma} = \sigma))t^0 \\ &= \frac{f(\sigma)(z-\ell-\sigma)}{f(\sigma) + w(\sigma)(1-F(z))} + \frac{w(\sigma)(1-F(z))}{f(\sigma) + w(\sigma)(1-F(z))} \frac{\int_{z-\ell}^{z}(F(z) - F(\theta))d\theta}{1-F(z)} \\ &= \frac{f(\sigma)(z-\ell-\sigma) + f(\sigma)(\sigma - z + \ell)}{f(\sigma) + w(\sigma)(1-F(z))} = 0.\end{aligned}$$

[35] If $\xi = z$, costs in $[\underline{\theta}, z - t^0]$ are perfectly revealed and $w_l(\sigma)$ can be ignored.

Furthermore, (IC_S) is satisfied, as S never has an incentive to unilaterally misreport the signal realization. This would lead to $\hat{\sigma}_A \neq \hat{\sigma}_S$ and a certain payoff of $-\ell$ to S.

In order to prove that β^c is feasible, it remains to show that (CP^c) is satisfied. Note first that any side-contract γ with $\hat{\sigma}_A \neq \hat{\sigma}_S$ can never be better than the null side-contract γ_0 as it leads to the minimal total payoff of $-\ell$ to S and A. Some γ with $\hat{\sigma}_A = \hat{\sigma}_S$ can only be better for S than γ_0 if it induces a different production decision than γ_0. This follows from the fact that the sum of transfers to S and A is fixed conditional on the production decision (t^1 if good is produced, t^0 otherwise). Given γ_0 and some signal realization σ, only the type $\theta = \sigma$ produces and receives a transfer of $p(\sigma) = \sigma + t^0$. A side-contract γ that induces the type $\theta = \sigma$ to not produce requires S to make a side-transfer of at least $-\tau = t^0$ to A. S's expected payoff from γ is therefore given by $t^0 + \tau = 0$ which is not better than the payoff from γ_0. For any signal realization $\sigma \geq z - \ell$, a side-contract may also change the production decision by inducing some type $\theta' \in Supp(\tilde{\theta}|\sigma)$ with $\theta' > \sigma$ to produce the good. This requires that S pays A a side-transfer $-\tau \geq \theta' - p(\hat{\sigma})$. Thus, S's payoff under production satisfies $t^1 - p(\hat{\sigma}) + \tau \leq t^1 - \theta' = t^0 + z - \ell - \theta' < t^0 + z - \ell - \sigma \leq t^0$ which is worse than under γ_0. It follows that β^c satisfies (CP^c).

Finally, I show that P's maximal expected payoff from (I^c, β^c) exceeds the optimal payoff under delegation and approximates the upper bound under centralization if ℓ is close to zero and $p^*(v) \in (m, \overline{\theta})$. P's maximal expected payoff from (I^c, β^c) is

$$\max_z F(z)(v - t^1) - (1 - F(z))t^0 = \max_z F(z)(v - z + \ell) - \frac{\int_{z-\ell}^{z}(F(z) - F(\theta))d\theta}{1 - F(z)}.$$

I denote the solution to this problem by $z_c(\ell)$ and the value by $V_c(\ell)$. Equivalently, I denote P's maximal payoff under delegation by $V_d(\ell) = \max_z \tilde{B}_d(z)$ where

$$\tilde{B}_d(z) \equiv B_d(F(z)) = F(z)(v - z) + \min\left\{\int_{\underline{\theta}}^{z} F(\theta)d\theta, (1 - F(z))\ell\right\}$$

and I denote by $z_d(\ell)$ the optimal cutoff under delegation. First, I prove $V_c(\ell) > V_d(\ell)$ for $p^* \in (m, \overline{\theta})$ and small ℓ. Note that $\tilde{B}_d(z) = F(z)(v - z) + (1 - F(z))\ell$ for ℓ sufficiently small. Note furthermore that $V_c(\ell)$ and $V_d(\ell)$ are continuous in ℓ, $z_c(0) = z_d(0) = p^*$, and $V_c(0) = V_d(0) = \underline{W}$. From the envelope theorem, it follows that $V_c'(0) = F(z_c(0)) = F(p^*)$ and $V_d'(0) = 1 - F(z_d(0)) = 1 - F(p^*)$. From $p^* > m$, it follows that $V_c(\ell) > V_d(\ell)$ for $\ell \in (0, \varepsilon)$ if ε is sufficiently small.

Second, I prove that (I^c, β^c) is approximately optimal for $p^* > m$ and $\ell \in (0, \varepsilon)$ for ε sufficiently small. As $V_c(\ell) > V_d(\ell)$, it is enough to prove that $V_c(\ell)$ approximates the upper bound $V_3(\ell) \equiv \max_z B_3(F(z))$. As $V_3(0) = \underline{W}$ and $V_3'(0) = F(p^*)$, $V_c(\ell)$ is a first-order approximation of $V_3(\ell)$ around $\ell = 0$. Thus, (I^c, β^c) is near-optimal for $p^* > m$ and ℓ close to zero. $\square$

Proof of Proposition 4

I show that Propositions 1 to 3 extend to the setting with T. First, I argue that P's expected payoff in the model with T remains bounded by $\max_x B_d(x)$ under delegation and by $\max_x B_c(x)$ under centralization. The following argument applies to centralization and delegation. Fix some feasible grand contract β. The collusion-proofness constraint (CP^z) with $z \in \{c, d\}$ implies that γ_0 is the optimal feasible side-contract for S. Thus, γ_0 is feasible and Pareto efficient. As P assumes that T chooses the worst Pareto efficient side-contract from P's perspective, P's payoff cannot exceed the payoff from β and γ_0. Hence, T's payoff is bounded by $\max_x B_d(x)$ under delegation and by $\max_x B_c(x)$ under centralization.

Proposition A.1. *For any $x \in [0,1]$, the combination of information structure I_{w^x} and grand contract β^x gives P an expected payoff of $B_d(x)$ for any side-contract that is feasible under delegation.*

Proof. I start with three preliminary observations: First, I_{w^x} is well-defined as shown in the proof of Proposition 1. Second, as β^x satisfies (CP^d), γ_0 is feasible under delegation and Pareto efficient in the model with T. Third, for a given γ, define $q(\theta;\sigma) \equiv X(\rho(\theta;\sigma))$ and $\kappa(\theta;\sigma) \equiv t_A(\rho(\theta;\sigma)) + \tau(\theta;\sigma)$. A's payoff for the true type (σ, θ) and the report $\hat{\theta}$ to the side-contract is given by $\kappa(\hat{\theta};\sigma) - \theta q(\hat{\theta};\sigma)$. By standard arguments, (IC_A^γ) implies that $q(\theta;\sigma)$ has to be of the form $q(\theta;\sigma) = \mathbf{1}_{\theta \le y(\sigma)}$ for some $y(\sigma) \in Supp(\tilde{\theta}|\sigma) \cup \{\underline{\theta} - \varepsilon\}$ with $\varepsilon > 0$.

I now prove that all side-contracts that are feasible under delegation lead to identical production decisions under β^x, i.e., $q(\theta;\sigma) = X(\sigma,\sigma,\theta) \Leftrightarrow y(\sigma) = \sigma$ for all $\sigma \in \Sigma$. Toward a contradiction, suppose there exists a side-contract that is feasible under delegation such that either i) $y(\sigma) > \sigma$ or ii) $y(\sigma) < \sigma$.

In case i), any type $\theta \le y(\sigma)$ needs to receive a rent of at least $y(\sigma) - \theta$ for reporting truthfully. For a type $\theta \le y(\sigma)$, S's payoff is therefore bounded by $t^1 - \theta - (y(\sigma) - \theta) = t^1 - y(\sigma) = \overline{\theta}(x) - y(\sigma) < 0$. For all $\theta > y(\sigma)$, S's payoff is bounded by $t^0 < 0$. Thus,

S's expected payoff over all $\theta \in Supp(\tilde{\theta}|\sigma)$ is also strictly negative which implies that $(PC_S^{\gamma,d})$ is not satisfied. This is a contradiction.

In case ii), S's expected payoff is at most t^0 for any $\theta \in Supp(\tilde{\theta}|\sigma)$. As $t^0 < 0$, $(PC_S^{\gamma,d})$ is not satisfied leading to a contradiction.

All feasible side-contracts under delegation implement the production decision $X(\sigma, \sigma, \theta)$ which leads to an expected payoff for P of $x(v - t^1) - (1 - x)t^0 = B_d(x)$. □

Proposition A.2. *Delegation is optimal in the model with T if either $\ell \geq \hat{\ell}(v)$ or $v < m$.*

Proof. As the model with T allows for more general forms of collusion, P's payoff is weakly lower than in the main model. Proposition A.1 implies that P can achieve the payoff $\max_x B_d(x)$ also in the model with T. Together with Proposition 2, this implies the result. □

Proposition A.3. *Suppose $f(\cdot) = F'(\cdot) > 0$ for all $\theta \in \Theta$. Delegation is suboptimal in the model with T if $p^*(v) \in (m, \overline{\theta})$ and $\ell \in (0, \varepsilon)$ for some $\varepsilon > 0$.*

Proof. Given the feasible information structure I^c, I construct a variation of the grand contract β^c such that P's payoff in the model with T is arbitrarily close to the payoff from (I^c, β^c) in the main model. Together with Proposition 3, this proves the result. In particular, consider the sequence of grand contracts $\{\beta^c_\eta\}$ indexed by $\eta > 0$ and defined as β^c in (8) with the sole difference that $t^1 = t^1_\eta = t^0 + z - \ell + \eta$. Thus, $\lim_{\eta \to 0} \beta^c_\eta = \beta^c$.

In a first step, I show that any grand contract β^c_η is feasible under centralization in the main model given I^c whenever β^c is feasible under centralization and η is close to zero. First, note that (PC_A) and (IC_A) are identical under β^c_η and β^c. Second, observe that (PC_S) and (IC_S) are less stringent under β^c_η than under β^c. Third, one can show that β^c_η satisfies (CP^c) by taking exactly the same steps that prove that β^c satisfies (CP^c) in the proof of Proposition 3. The only difference is that S's payoff from a deviation in which the good is produced for $\theta > z$ is given by $t^1_\eta - p(\hat{\sigma}) + \tau \leq t^1_\eta - \theta = t^0 + z - \ell - \theta + \eta < t^0$, where the last inequality holds for η close to zero. Finally, note that S's expected payoff given the signal realization σ is

$$\Pr(\tilde{\theta} = \sigma|\sigma)(t^1_\eta - p(\sigma)) + \Pr(\tilde{\theta} > \sigma|\sigma)t^0 = [z - \ell - \sigma]_+ + \Pr(\tilde{\theta} = \sigma|\sigma)\eta \qquad (9)$$

and that P's expected payoff from (I^c, β^c_η) given by $\Pr(\tilde{\theta} = \sigma|\sigma)(v - t^1_\eta) - \Pr(\tilde{\theta} > \sigma|\sigma)t^0$ converges to the profit from (I^c, β^c) as η goes to zero.

In a second step, I show that following a rejection of γ, truthful reporting strategies to any grand contract β^c_η remain mutual best responses for arbitrary beliefs. Suppose A reports truthfully. If S reports $\hat{\sigma}_S \neq \sigma = \hat{\sigma}_A$, then S's payoff is $-\ell$. As this is the lowest possible payoff, a truthful report by S is a best response to truthful reporting by A. Next, suppose S reports truthfully. In this case, (IC_A) implies that truthful reporting is a best response for A.

In a third step, I demonstrate that P's expected payoff from I^c and β^c_η for η close to zero is the same for any side-contract that is feasible under centralization. As in the proof of Proposition A.1, I show this by proving that all equilibria with feasible side-contracts lead to the same production decision $X(\sigma, \sigma, \theta)$ defined in (8). Consider an equilibrium in which P sets I^c and β^c_η, S and A accept β^c_η, T offers γ, and S and A accept γ with probabilities $\alpha_S(\sigma)$ and $\alpha_A(\theta;\sigma)$. Following a rejection of γ, the production decision is $X(\sigma, \sigma, \theta)$ as truthful reporting strategies are mutual best responses for S and A. Thus, I turn to the case in which the side-contract γ is accepted with positive probability, i.e., $\alpha_S(\sigma) > 0$ and $\alpha_A(\sigma, \theta) > 0$ for some $(\sigma, \theta) \in \Sigma \times \Theta$. Define $q(\theta;\sigma) \equiv X(\rho(\theta;\sigma))$ and $\kappa(\theta;\sigma) \equiv t_A(\rho(\theta;\sigma)) + \tau(\theta;\sigma)$. By standard arguments, (IC^γ_A) implies that $q(\theta;\sigma)$ has to be of the form $q(\theta;\sigma) = \mathbf{1}_{\theta \leq y(\sigma)}$ for some $y(\sigma) \in \; Supp(\tilde{\theta}|\sigma, acceptance) \cup \{\underline{\theta} - \varepsilon\}$ with $\varepsilon > 0$. If $q(\theta;\sigma) = X(\sigma, \sigma, \theta)$ $\Leftrightarrow y(\sigma) = \sigma$, S and A are both indifferent between accepting and rejecting the side-contract. Note that $y(\sigma) > \sigma$ with $\Pr(\tilde{\theta} \in (\sigma, y(\sigma)]|\sigma, acceptance) = 0$ and $y(\sigma) < \sigma$ with $\Pr(\tilde{\theta} = \sigma|\sigma, acceptance) = 0$ are both equivalent to $y(\sigma) = \sigma$. Thus, there remain the cases i) $y(\sigma) > \sigma$ with $\Pr(\tilde{\theta} \in (\sigma, y(\sigma)]|\sigma, acceptance) > 0$ and ii) $y(\sigma) < \sigma$ with $\Pr(\tilde{\theta} = \sigma|\sigma, acceptance) > 0$.

In case i), any type $\theta \leq y(\sigma)$ needs to receive a rent of at least $y(\sigma) - \theta$ for reporting truthfully. Note that $y(\sigma) - \sigma > t^0$ by construction of I^c and $y(\sigma) - \theta > 0$ for all $\theta < y(\sigma)$. Thus, all $\theta \in Supp(\tilde{\theta}|\sigma)$ with $\theta < y(\sigma)$ accept the side-contract in equilibrium. It follows that S's expected payoff is

$$\Pr(\tilde{\theta} \leq y(\sigma)|\sigma)(t^1_\eta - y(\sigma)) + \Pr(\tilde{\theta} > y(\sigma)|\sigma)t^0$$

where the second summand follows from the fact that S's payoff is t^0 for any $\theta > y(\sigma)$ independently of whether this type accepts or rejects the side-contract. This term is strictly smaller than S's expected payoff from rejecting the side-contract given in (9) as $p(\sigma) = \sigma + t^0 < y(\sigma)$ and $t^1_\eta - y(\sigma) = t^0 + z - \ell + \eta - y(\sigma) < t^0$ for η close to zero due to $y(\sigma) > z$. Thus, $\alpha_S(\sigma) > 0$ is suboptimal which gives a contradiction.

In case ii), the type σ receives a rent of at least t^0 as A would otherwise reject γ. As this rent is earned without production, all types $\theta \in Supp(\tilde{\theta}|\sigma)$ with $\theta > \sigma$ have to receive the same rent and strictly prefer to accept γ. Thus, S's expected payoff from accepting γ is at most

$$\begin{aligned} &\Pr(\tilde{\theta} = \sigma|\sigma)(\alpha_A(\sigma;\sigma)(t^0 - t^0) + (1 - \alpha_A(\sigma;\sigma))(t^1_\eta - p(\sigma)) + \Pr(\tilde{\theta} > \sigma|\sigma)(t^0 - t^0) \\ = &\Pr(\tilde{\theta} = \sigma|\sigma)(1 - \alpha_A(\sigma;\sigma))(z - \ell - \sigma + \eta). \end{aligned}$$

As $\alpha_A(\sigma, \sigma) > 0$, this is strictly smaller than S's expected payoff from rejecting the side-contract given in (9). Thus, $\alpha_S(\sigma) > 0$ is suboptimal which gives a contradiction.

It follows that the production decision is $X(\sigma, \sigma, \theta)$ in any equilibrium. As β^c_η is feasible under centralization given I^c in the main model, there exists an equilibrium in which T offers the null side-contract γ_0 and P's expected payoff converges to the payoff from (I^c, β^c) in the main model as η goes to zero. □

Proof of Proposition 5

I assume $\{\theta \in \Theta : q^*(\theta) = 0\} \neq \emptyset$. At the end of the proof, I sketch how the result can be extended to the case with $\{\theta \in \Theta : q^*(\theta) = 0\} = \emptyset$.

Suppose P sets a weighted information structure with the signal space $\Sigma = Q \setminus [\theta_0, \overline{\theta}]$ and the weighting function $w^*(\sigma) = \frac{\phi^*(\sigma) f(\sigma)}{\Phi^*}$. Furthermore, suppose P offers S a delegation contract that specifies a payment schedule of

$$t(q) = v(q) \cdot \mathbf{1}_{q \in Q^*_+} - \frac{\Phi^*}{\Pr(\theta \geq \theta_0)} \cdot \mathbf{1}_{q \in Q \setminus Q^*_+}$$

for the delivery of a quantity $q \in Q$. Upon acceptance of the delegation contract, S offers A a sub-contract that specifies a quantity $q(\hat{\theta};\sigma)$ and a transfer $\tau(\hat{\theta};\sigma)$ from S to A as functions of the commonly observed signal realization σ and a report of A about the type. Thus, A's payoff is $\tau(\hat{\theta};\sigma) - c(\theta, q(\hat{\theta};\sigma))$. Due to supermodularity of the cost function, incentive compatibility implies that $q(\theta;\sigma)$ has to be weakly decreasing in θ. At first, suppose S offers a sub-contract with $q(\theta;\sigma) = q \cdot \mathbf{1}_{\theta=\sigma}$. S optimally implements this production rule with $\tau(\theta;\sigma) = c(\sigma, q) \cdot \mathbf{1}_{\theta=\sigma}$ which gives an expected payoff of

$$\Pr(\tilde{\theta} = \sigma|\sigma)(v(q) - c(\sigma, q)) - \Pr(\tilde{\theta} > \sigma|\sigma)\frac{\Phi^*}{\Pr(\theta \geq \theta_0)}.$$

This expression is maximized for $q = q^*(\sigma)$ which results in an expected payoff of

$$\frac{f(\sigma)\phi^*(\sigma) - \Pr(\theta \geq \theta_0)w^*(\sigma)\frac{\Phi^*}{\Pr(\theta \geq \theta_0)}}{f(\sigma) + \Pr(\theta \geq \theta_0)w^*(\sigma)} = 0.$$

Next, suppose S chooses a production rule with $q(\theta;\sigma) > 0$ for some $\theta \geq \theta_0$. Let y denote the highest type that produces a positive quantity. Individual rationality implies that $\tau(y;\sigma) \geq c(y, q(y;\sigma))$. Thus, all types $\theta < y$ can at least earn a rent of $c(y, q(y;\sigma)) - c(\theta, q(y;\sigma))$ by reporting $\hat{\theta} = y$. Thus, S's expected payoff is bounded from above by

$$\begin{aligned}\int_{\underline{\theta}}^{y} (v(q(\theta;\sigma)) - c(\theta, q(\theta;\sigma)) - c(y, q(y;\sigma)) + c(\theta, q(y;\sigma)))dG(\theta|\sigma) \\ - \Pr(\tilde{\theta} > y|\sigma)\frac{\Phi^*}{\Pr(\theta \geq \theta_0)}.\end{aligned} \qquad (10)$$

Next, note that

$$\begin{aligned} & v(q(\theta;\sigma)) - c(\theta, q(\theta;\sigma)) - c(y, q(y;\sigma)) + c(\theta, q(y;\sigma)) \qquad (11) \\ \leq \; & v(q(\theta;\sigma)) - c(\theta, q(\theta;\sigma)) - c(y, \underline{q}^*) + c(\theta, \underline{q}^*) \\ \leq \; & v(q(\theta;\sigma)) - c(\underline{\theta}, q(\theta;\sigma)) - c(y, \underline{q}^*) + c(\underline{\theta}, \underline{q}^*) \\ \leq \; & \phi^*(\underline{\theta}) - c(\theta_0, \underline{q}^*) + c(\underline{\theta}, \underline{q}^*) \end{aligned}$$

where the first inequality follows from supermodularity and $q(y;\sigma) \geq \underline{q}^*$, the second inequality follows from supermodularity and $\theta \geq \underline{\theta}$, and the third inequality follows from the definition of $\phi^*(\theta)$, $c(\theta, q)$ being increasing in θ, and $y \geq \theta_0$. As $c(\theta_0, q) - c(\underline{\theta}, q)$ is decreasing in q due to supermodularity, condition (5) implies that the last line in equation (11) is weakly negative. Thus, the integrand in the first line of equation (10) is weakly negative which implies that the total expression is strictly negative. It follows that S optimally offers a sub-contract with the production rule

$q(\theta;\sigma) = q^*(\theta)$ and P's expected payoff is given by

$$\mathbb{E}\left[\mathbf{1}_{\theta<\theta_0}(v(q^*(\theta)) - v(q^*(\theta))) + \mathbf{1}_{\theta\geq\theta_0}\frac{\Phi^*}{\Pr(\theta \geq \theta_0)}\right] = \Phi^*.$$

Moreover, note that $\min\{q \geq 0 : \phi^*(\underline{\theta}) \leq c(\theta_0, q) - c(\underline{\theta}, q)\} \in (0, q^*(\underline{\theta}))$ due to

$$\begin{aligned} c(\theta_0, 0) - c(\underline{\theta}, 0) = 0 < \phi^*(\underline{\theta}) &= v(q^*(\underline{\theta})) - c(\theta_0, q^*(\underline{\theta})) + c(\theta_0, q^*(\underline{\theta})) - c(\underline{\theta}, q^*(\underline{\theta})) \\ &< \phi^*(\theta_0) + c(\theta_0, q^*(\underline{\theta})) - c(\underline{\theta}, q^*(\underline{\theta})) = c(\theta_0, q^*(\underline{\theta})) - c(\underline{\theta}, q^*(\underline{\theta})). \end{aligned}$$

Finally, consider the case $\{\theta \in \Theta : q^*(\theta) = 0\} = \emptyset$. I sketch how to prove that P can virtually extract the full surplus. If $\Pr(\tilde{\theta} = \overline{\theta}) > 0$, change the transfer in case of production to $\tilde{v}(q) \equiv v(q) - \phi^*(\overline{\theta})$ and pool a probability mass $\varepsilon < \Pr(\tilde{\theta} = \overline{\theta})$ of the type $\overline{\theta}$ into the signal space $\Sigma = \Theta$ according to the weighted information structure with cutoff $\overline{\theta}$ and weighting function $w^*(\sigma) = \frac{f(\sigma)(\phi^*(\sigma) - \phi^*(\overline{\theta}))}{\Phi^* - \phi^*(\overline{\theta})}$. All arguments from above can now be applied to find that P achieves an expected profit of $\Phi^* - \varepsilon\phi^*(\overline{\theta})$ which converges to Φ^* as $\varepsilon \to 0$.

If $F(\cdot)$ is continuous at $\overline{\theta}$, set a cutoff θ_0 close to $\overline{\theta}$, change the transfer in case of production to $\tilde{v}(q) \equiv v(q) - \phi^*(\theta_0)$ and use the weighted information structure with cutoff θ_0 and weighting function $w^*(\sigma) = \frac{f(\sigma)(\phi^*(\sigma) - \phi^*(\theta_0))}{\Phi^* - \phi^*(\theta_0)}$. All arguments from above can now be applied to find that P achieves an expected profit of $\Phi^* - (1 - F(\theta_0))\phi^*(\theta_0)$ which converges to Φ^* as $\theta_0 \to \overline{\theta}$. □

Proof of Proposition 6

Consider the game defined in Section 3 with the sole difference that some information structure $I \in \mathcal{I}$ is exogenously fixed at $t = 0$. Suppose I satisfies conditions i) and ii). I prove that P's expected payoff from any feasible grand contract under delegation lies strictly below the monopsony payoff $\underline{W}$. As P can achieve $\underline{W}$ under centralization, this proves that delegation is suboptimal.

Consider an equilibrium with delegation. First, I show that $p(\sigma) \in \arg\max_p (t^1 - t^0 - p)G(p|\sigma), \forall \sigma \in \Sigma$ in any such equilibrium. Suppose not. Then there exists some $\sigma \in \Sigma$ with $p(\sigma) \notin \arg\max_p (t^1 - t^0 - p)G(p|\sigma)$. For this signal realization, S's expected payoff satisfies

$$\begin{aligned} & x(\sigma)(t^1 - t_A^1(\sigma)) + (1 - x(\sigma))(t^0 - t_A^0(\sigma)) \\ = \;& G(p(\sigma)|\sigma)(t^1 - t^0 - p(\sigma)) + t^0 - t_A^0(\sigma) \\ < \;& G(p^o(\sigma)|\sigma)(t^1 - t^0 - p^o(\sigma)) + t^0 - t_A^0(\sigma) \end{aligned}$$

where $x(\sigma) = G(p(\sigma)|\sigma)$ and $p^o(\sigma) \in \arg\max_p (t^1 - t^0 - p)G(p|\sigma)$. The expected payoff in the third line is achievable for S by offering A a side-contract with $X(\rho(\theta;\sigma)) = \mathbf{1}_{\theta \leq p^o(\sigma)}$ and $\tau(\theta;\sigma) = (p(\sigma) - p^o(\sigma))\mathbf{1}_{\theta \leq p^o(\sigma)}$. Thus, $p(\sigma) \notin \arg\max_p (t^1 - t^0 - p)G(p|\sigma)$ cannot hold in equilibrium.

Second, note that $p(\sigma) \in \arg\max_p (t^1 - t^0 - p)G(p|\sigma)$ implies $t^1 - t^0 \geq p(\sigma)$ with $t^1 - t^0 = p(\sigma)$ only if $p(\sigma) = \min\{Supp(\tilde{\theta}|\sigma)\}$. Thus, $p(\overline{\sigma}) \neq \min\{Supp(\tilde{\theta}|\overline{\sigma})\}$ implies $t^1 - t^0 > p(\overline{\sigma})$.

In a third step, I show that P's expected payoff in any equilibrium with delegation is strictly smaller than the monopsony payoff $\underline{W}$. P's expected payoff satisfies

$$\begin{aligned} & \int_\Sigma \left(x(\sigma)(v - t^1 + t^0) - t^0\right) dH(\sigma) && (12) \\ \leq \;& \int_\Sigma \left(x(\sigma)(v - t^1 + t^0) + x(\overline{\sigma})(t^1 - t^0 - p(\overline{\sigma}))\right) dH(\sigma) \\ = \;& \int_\Sigma \left(x(\sigma)(v - p(\overline{\sigma})) - (x(\sigma) - x(\overline{\sigma}))(t^1 - t^0 - p(\overline{\sigma}))\right) dH(\sigma) \\ \leq \;& \int_\Sigma x(\sigma)(v - p(\overline{\sigma}))dH(\sigma) = \int_\Sigma G(p(\sigma)|\sigma)(v - p(\overline{\sigma}))dH(\sigma) \\ \leq \;& \int_\Sigma G(p(\overline{\sigma})|\sigma)(v - p(\overline{\sigma}))dH(\sigma) = F(p(\overline{\sigma}))(v - p(\overline{\sigma})) \\ \leq \;& \underline{W} \end{aligned}$$

where the first inequality follows from S's participation constraint for $\overline{\sigma}$ given by $t^0 \geq -x(\overline{\sigma})(t^1 - t^0 - p(\overline{\sigma}))$, the second inequality follows from $t^1 - t^0 \geq p(\overline{\sigma})$ and $x(\overline{\sigma}) \leq x(\sigma)$ for all $\sigma \in \Sigma$, and the third inequality follows from $p(\overline{\sigma}) \geq p(\sigma)$ for all $\sigma \in \Sigma$. Next, note that $\min\{Supp(\tilde{\theta}|\overline{\sigma})\} \neq p^*$ implies that we are in one of the following cases. Either $p^* \neq p(\overline{\sigma})$ or $p^* = p(\overline{\sigma}) \neq \min\{Supp(\tilde{\theta}|\overline{\sigma})\}$ and $t^1 - t^0 > p(\overline{\sigma})$. In the first case, the fourth inequality of (12) is strict. In the second case, the second inequality of (12) is strict if $x(\sigma) > x(\overline{\sigma})$ for a positive mass of signal realizations. It remains to show that there is a strict inequality in (12) for the case where $p(\overline{\sigma}) = p^*$

and $x(\sigma) = x(\overline{\sigma})$ for almost all signal realizations. If $x(\overline{\sigma}) < F(p^*)$, the fourth inequality is strict. Finally, the case $p(\overline{\sigma}) = p^*$ and $x(\sigma) = x(\overline{\sigma}) = F(p^*)$ is ruled out by condition iii). □

Proof of Corollary 4

Consider an information structure with additive noise. Note that

$$G(p|\sigma) = \Pr(\tilde{\theta} \leq p|\sigma) = \Pr(\tilde{\varepsilon} \leq p - \sigma|\sigma) = Z(p - \sigma).$$

The problem $\max_p (a - p)Z(p - \sigma)$ has the first-order condition

$$a - p - \frac{Z(p - \sigma)}{z(p - \sigma)} \geq 0.$$

By logconcavity of $Z(\varepsilon)$, the left-hand side is decreasing in p and increasing in σ. Thus, $p_a(\sigma)$ is increasing in σ. Next, consider the equivalent problem $\max_x (a - \sigma - Z^{-1}(x))x$ with the first-order condition

$$a - \sigma - Z^{-1}(x) - xZ^{-1\prime}(x) \geq 0.$$

The left-hand side is decreasing in σ. As

$$\frac{d}{dx}\left(Z^{-1}(x) + xZ^{-1\prime}(x)\right) = \frac{1}{z(Z^{-1}(x))}\left(2 - \frac{z'(Z^{-1}(x))x}{z(Z^{-1}(x))^2}\right) \geq 0 \Leftrightarrow 2 \geq \frac{z'(\varepsilon)Z(\varepsilon)}{z(\varepsilon)^2},$$

logconcavity of $Z(\varepsilon)$ implies that $x_a(\sigma)$ is decreasing. Thus, condition i) of Proposition 6 is satisfied. Next, $\min Supp\{G(p|\overline{\sigma})\} \neq p^* \Leftrightarrow \overline{\sigma} - \underline{\varepsilon} \neq p^*$ implies that condition ii) is satisfied. Finally, note that $p^* \in (\underline{\theta}, \overline{\theta})$ implies that some of the first-order conditions above are satisfied with equality. Logconcavity of $Z(\varepsilon)$ then implies that $x_a(\sigma)$ is strictly decreasing whenever $x_a(\sigma) \in (0, 1)$. Thus, condition iii) of Proposition 6 is satisfied.

Consider a partitional information structure. Note that

$$G(p|\sigma) = \frac{F(p) - F(l(\sigma))}{F(h(\sigma)) - F(l(\sigma))}.$$

The problem $\max_p (a-p)(F(p)-F(l(\sigma)))$ has the first-order condition

$$a - p - \frac{F(p) - F(l(\sigma))}{f(p)} \geq 0.$$

By logconcavity of $F(\theta)$, the left-hand side is decreasing in p and increasing in σ. Thus, $p_a(\sigma)$ is increasing in σ. The problem above can be rewritten as

$$\max_x \left(a - F^{-1}(F(l(\sigma)) + (F(h(\sigma)) - F(l(\sigma)))x)\right) x.$$

For $y = F(l(\sigma)) + (F(h(\sigma)) - F(l(\sigma))x$, the first-order condition is given by

$$a - F^{-1}(y) - F^{-1\prime}(y)(F(h(\sigma)) - F(l(\sigma)))x \geq 0. \tag{13}$$

First, I show that the left-hand side is increasing in σ. If $F(h(\sigma)) - F(l(\sigma))$ is increasing in σ, this holds if $F^{-1}(y) + F^{-1\prime}(y)(F(h(\sigma)) - F(l(\sigma)))x$ is increasing in y as $y = F(l(\sigma)) + (F(h(\sigma)) - F(l(\sigma))x$ is increasing in σ. Note that

$$\frac{d}{dy}\left(F^{-1}(y) + F^{-1\prime}(y)(F(h(\sigma)) - F(l(\sigma)))x\right) \geq 0$$
$$\Leftrightarrow \frac{1}{f(F^{-1}(y))} - \frac{f'(F^{-1}(y))(F(h(\sigma)) - F(l(\sigma)))x}{f(F^{-1}(y))^3} \geq 0.$$

Using $\theta = F^{-1}(y)$, this is equivalent to $1 - \frac{f'(\theta)(F(\theta)-F(l(\sigma))}{f(\theta)^2} \geq 0$ which follows from logconcavity of $F(\theta)$. Second, I show that the left-hand side of (13) is increasing in x. Differentiating the left-hand side with respect to x gives

$$2F^{-1\prime}(y) + F^{-1\prime\prime}(y)m(\sigma)x \geq 0$$

for $y = F(l(\sigma)) + (F(h(\sigma)) - F(l(\sigma))x$. This condition can be rewritten as $2 - \frac{f'(\theta)(F(\theta)-F(l(\sigma))}{f(\theta)^2} \geq 0$ which follows from logconcavity of $F(\theta)$. Thus, condition i) is satisfied. Next, $l(\sigma) \neq p^*$ and $h(\sigma) \neq p^*, \forall \sigma \in \Sigma$ implies condition ii) of Proposition 6. Finally, $p^* \in (\underline{\theta}, \overline{\theta})$ implies that the first-order conditions above are satisfied with equality for some signal realizations. Log-concavity of $F(\theta)$ implies that $x_a(\sigma) < 1$ for some signal realizations and that $x_a(\sigma)$ is strictly decreasing whenever $x_a(\sigma) \in (0, 1)$. □

Proof of Corollary 5

Consider a grand contract that is feasible under delegation and ex-ante collusion-proof. Let x be the ex-ante production probability. Point ii) of Lemma 2 implies that there exists a signal $\sigma \in \Sigma$ with $t^1_A(\sigma) \geq \overline{\theta}(x)$. Together with point v) of Lemma 2, this implies $t^1 \geq \overline{\theta}(x)$. Ex-ante collusion-proofness implies $t^0 \geq 0$. Thus, P's payoff satisfies $x(v - t^1) - (1-x)t^0 \leq x(v - \overline{\theta}(x)) \leq \max_x x(v - \overline{\theta}(x)) = \underline{W}$. □

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