Hours Risk and Wage Risk: Repercussions over the Life-Cycle

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School of Business & Economics
Discussion Paper
Economics
2020/2
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January 9, 2020

Abstract
We decompose permanent earnings risk into contributions from hours and wage shocks. To distinguish between hours shocks, modeled as innovations to the marginal disutility of work, and labor supply reactions to wage shocks we formulate a life-cycle model of consumption and labor supply. Both permanent wage and hours shocks are important to explain earnings risk, but wage shocks have greater relevance. Progressive taxation strongly attenuates cross-sectional earnings risk, its life-cycle insurance impact is much smaller. At the mean, a positive hours shock of one standard deviation raises life-time income by 10%, while a similar wage shock raises it by 12%.

Keywords Earnings Risk · Wage Risk · Labor Supply · Progressive Taxation · Consumption Insurance

JEL Classification D31 · J22 · J31
1 Introduction

What drives the riskiness of earnings? A glance at the recent literature on life-cycle consumption, saving and labor supply suggests an implicit consensus: shocks to wages are the central source of risk. In this paper, we re-open this discussion by starting from the natural decomposition of earnings into hours worked and wages. Our central contribution is a decomposition of earnings risk along these lines. We formulate a structural model of life-cycle labor supply that features earnings risk from both wage and hours shocks and assess the strength of their contributions to total earnings risk and life-time earnings. We find that both types of shocks are quantitatively important. Further, we introduce progressive taxation into the model and assess the insurance it offers. Progressive taxation strongly attenuates cross-sectional earnings risk; transitory shocks more so than permanent ones.

Knowing the extent to which hours and wage shocks contribute to total earnings risk is of general interest as it should inform future modeling decisions when estimating permanent earnings risk. In our model for example, we find that the estimate of the Marshallian labor supply elasticity is sensitive to accounting for hours shocks. More importantly, the relative contributions to risk should guide the evaluation of policy measures aimed at reducing earnings risk. For instance, if earnings risk was driven almost entirely by hours risk, focusing on policies to reduce the impact of wage shocks would not be expedient.

In our model individuals face idiosyncratic shocks to their productivity of market work, captured as wage shocks, as is standard. For instance, a promotion is a positive permanent wage shock and loss of human capital a negative one. Our concise extension of standard life-cycle models is to introduce hours shocks, which we model as innovations to the disutility of work. These shocks are conceptualized in an analogous fashion to wage shocks. As an illustration, consider the case of one’s elderly parent falling ill or being in need of around-the-clock care. This increases the opportunity cost of market work sharply. Depending on the nature of the illness, the shock is permanent or fades out. In terms of observed choices, one would then notice a shock to hours of work.

In most models wage shocks are the sole drivers of earnings risk. Conveniently, wage shocks can be recovered using the moments of residuals of a wage equation alone. This does not hold for hours shocks. In our setting, hours residuals contain hours shocks in addition to labor supply reactions to wage shocks. Net of Frisch labor supply adjustments, these reactions are governed by a single transmission parameter, which measures the impact of permanent income shocks on the marginal utility of wealth. Thus, without identifying this parameter separating the two shock types is impossible. We suggest a new method to estimate the transmission parameter without using consumption data, which are frequently employed for this purpose. The transmission parameter is linked to consumption insurance. The larger it is, the larger is the impact of permanent shocks on consumption, and the lower is the degree of insurance against risk. We show that the comovement of consumption and earnings implied by our estimate is in line with estimates in Blundell et al. (2008). The identification of the parameter serves an additional purpose. It enables us to calculate
the Marshallian labor supply elasticity foregoing consumption or asset data. Thus our approach offers a complementary view on existing estimates. Using similar considerations as in our study, the Marshallian elasticity has been estimated using the covariance of earnings and wages as well as asset data in Blundell et al. (2016, eq. A2.23). Heathcote et al. (2014) use the covariance of hours and consumption as well as of wages and consumption to estimate the Marshallian elasticity. In contrast, we rely only on hours and wage data.¹

We estimate our model using observations on married men in the US from the Panel Study of Income Dynamics (PSID). Our sample starts in 1970 and ends in 1997, when the survey frequency turned biennial. We focus on this group because the extensive labor supply margin plays a small role in their labor supply behavior and in order to allow for comparisons to the previous literature. Our labor supply estimation yields a tax-adjusted Frisch labor supply elasticity of 0.36 and our estimate of the average tax-adjusted Marshallian elasticity is -0.08, which is close to recent estimates in Blundell et al. (2016) and Heathcote et al. (2014). We find that the standard deviation of permanent wage shocks is larger than the standard deviation of transitory wage shocks. The same holds for hours shocks, where the standard deviation of permanent shocks is almost twice as large as that of transitory shocks. For most samples, the standard deviation of permanent hours shocks is larger than that of permanent wage shocks.

However, the respective impact on earnings risk cannot be inferred directly from this evidence. This is because the reaction to shocks depends on the degree of insurance. With the key components of earnings risk in hand we can pursue the variance decomposition that is the main contribution of the paper. We shut down each of the stochastic components except for one in order to quantify their respective contributions to overall earnings risk. At the sample mean of the transmission parameter, permanent wage shocks explain about 26% of earnings growth risk, while permanent hours shocks explain 17%. Transitory wage shocks dominate transitory hours shocks. Even though transitory shocks are responsible for the lion’s share of earnings growth risk, only permanent shocks have a substantial impact on life-time earnings. At the mean of the transmission parameter a positive permanent hours shock of one standard deviation increases life-time earnings by 10%, a positive permanent wage shock of one standard deviation increases life-time earnings by 12%. At age 30, for an individual with annual net earnings of 50 000 Dollar this corresponds to increases of 87 000 Dollar and 106 000 Dollar, respectively. Thus, both types of shocks play an important role for life-time earnings.

Progressive taxation lowers earnings risk by almost 40%. The contribution of transitory shocks is reduced by 45%, the contribution of permanent shocks only by 29%. Since it is more costly for individuals to self-insure against permanent shocks, this is an unfortunate finding. For transitory shocks the insurance through the tax system is of little value but of potentially large cost because it

¹Heathcote et al. (2014) also estimate a variant that does not rely on consumption data. Their approach differs because their specific island framework implies that the marginal utility of wealth is constant across individuals in the same age-year cell.
distorts labor supply decisions. In terms of life-time impact on earnings, the progressive tax system reduces the effect of permanent shocks by 16%.

To evaluate the importance of allowing for hours shocks, we consider a set of alternative models that resemble those applied in the literature. Crucially, a model abandoning transitory and permanent hours shocks fits the data worse and leads to a substantial overestimation—in absolute terms—of the Marshallian labor supply elasticity.

Finally, we show how our estimate of the transmission parameter can be used to calculate the pass-through of permanent wage shocks to consumption. Calibrating the parameter of relative risk aversion to two, we find that these pass-through parameters for different samples are roughly in line with those estimated in Blundell et al. (2016), who use consumption and earnings data to estimate this parameter. For the full sample this calculation implies that—on average—a permanent increase in wages by 1% leads to an increase in consumption by 0.62%.

Our paper is related to studies that decompose total income risk into persistent and transitory components, which derive from ideas by Friedman (1957) and Hall (1978) (see MacCurdy 1982; Abowd and Card 1989; Meghir and Pistaferri 2004; Guvenen 2007; Blundell et al. 2008; Guvenen 2009; Hryshko 2012; Heathcote et al. 2014; Blundell et al. 2016). Abowd and Card (1989) pioneered the analysis of the covariance structure of earnings and hours of work. They find that most of the idiosyncratic covariation of earnings and hours of work occurs at fixed wage rates.

In contrast, more recent papers have focused on insurance mechanisms rather than shock sources and restrict the source of risk to wage shocks. In a rich model of family labor supply and consumption, Blundell et al. (2016) estimate the Marshallian and Frisch consumption and labor supply elasticities using hours, income, asset, and consumption data. Similar to them, we allow for partial insurance of permanent wage shocks, but we depart from their approach by introducing hours shocks and using hours and earnings data alone. Blundell et al. (2018) build on their work in 2016 using dynamic programming and put the focus on time allocation between market work, leisure, and child-rearing. They determine the impact of permanent and transitory wage shocks on consumption and time allocation. Another related study is Wu and Krueger (2019), which shows in a calibrated model along the lines of Blundell et al. (2016) the optimal tax progressivity to be much lower in the two-earner case compared to the one-earner case.

With a similar focus, Heathcote et al. (2014) analyze the transmission of wage shocks to hours in a setting where shocks are either fully insurable or not insurable at all (island framework). They derive second hours-wage moments from which they identify variances of shocks, the Frisch elasticity of labor supply, and the coefficient of relative risk aversion. Our study differs in two important aspects: First, we assume that shocks are partially insurable as indicated by a consumption insurance parameter similar to Blundell et al. (2008, 2013, 2016). This parameter may differ between individuals. Second, we introduce hours shocks and estimate their variance. While Heathcote et al. (2014) allow for initial heterogeneity between agents in the disutility of work, they hold this parameter constant over the life-cycle.
Some microeconometric papers do focus on shock sources more explicitly and employ dynamic programming techniques for this purpose. Low et al. (2010) quantify the contributions of productivity shocks, job losses, and job offers to overall earnings risk. They find that wage risk is much more important than job destruction risk due to the transitory nature of the latter. They model labor supply as a discrete decision with fixed hours of work and the possibility of job loss, while we focus on the intensive margin of work hours and allow for hours adjustment and permanent as well as transitory shocks to hours. Similarly, Kaplan (2012) models consumption and hours of work and allows for involuntary unemployment shocks. These shocks along with nonseparable hours preferences on the extensive and intensive margin aid in the modeling of the declining inequality in hours worked over the first half of the life-cycle. Altonji et al. (2013) quantify the earnings variance contributions of i.i.d. wage and hours shocks in addition to employment and job changes, but they do not allow for individual-specific permanent hours shocks. Moreover, they do not work with a structural model, but rather approximate economic decisions of agents in their account of the dynamics of earnings and wage profiles. We are the first to model individual-specific hours shocks as a combination of permanent and transitory shocks.

In macroeconomics modeling shocks to hours is not uncommon. In Dynamic Stochastic General Equilibrium models these labor supply shocks, modeled as AR(1)-processes, are important determinants of output and real wage fluctuations (Smets and Wouters 2003; Adolfson et al. 2007; Galí et al. 2012; Justiniano et al. 2013; Foroni et al. 2018). However, in those models the shocks are to aggregate labor supply, while ours are individual-specific.

The next section outlines the life-cycle model of labor supply and consumption, Section 3 describes how we estimate shock variances and labor supply elasticities. In Section 4 we present results for the parameters of wage and hours processes and the Frisch and Marshallian labor supply elasticities. Then we offer decompositions of residual earnings growth variance and risk, which quantify the importance of wage and hours shocks as well as the impact of progressive taxation. Further we calculate the influence of the two shock types on life-time earnings. In Section 5 we give a characterization of permanent hours shocks, show results when varying the modeling assumptions, discuss the model fit and benchmark our results by relating them to consumption insurance estimates in the literature. Section 6 concludes.

## 2 The Model

Individuals maximize the discounted sum of utilities over their lifetime, which runs from $t_0$ to $T$. We omit individual-specific subscripts:

$$\max_{c_t, h_t} E_{t_0} \left[ \sum_{t=t_0}^{T} \rho^{t-t_0} v(c_t, h_t, b_t) \right],$$  

(1)
where $c_t$ is annual consumption and $h_t$ denotes annual hours of work, while $b_t$ contains taste shifters. $\rho$ denotes a discount factor and $v(\cdot)$ an in-period utility function.

The budget constraint is

$$\frac{a_{t+1}}{1 + r_t} = a_t + \chi (w_t h_t)^{1-\tau} + (1 - \tau N_t) N_t - c_t,$$

where $a_t$ represents assets, $r_t$ the real interest rate, and $N_t$ non-labor income. We obtain net labor income from the Feldstein tax approximation (Feldstein 1969) with progressivity parameter $1 - \tau$, while net non-labor income is determined by the net-of-tax rate $(1 - \tau N_t)$.

Instantaneous utility takes the additively-separable constant relative risk aversion (CRRA) form

$$v_t = \frac{c_t^{1-\vartheta}}{1-\vartheta} - b_t h_t^{1+\gamma}, \quad \vartheta \geq 0, \gamma \geq 0.$$  

We specify $b_t = \exp(\varsigma \Xi_t - \upsilon_t)$. $\Xi_t$ is a set of personal characteristics. $\upsilon_t$ is an idiosyncratic disturbance with mean zero, where the innovations to this term are the hours shocks. They capture unexpected changes in disutility of labor supply, e.g., childcare or spousal needs, sickness, and other unexpected changes in home production. Thus $b_t$ captures taste-shifters in a very broad sense. The set-up of the model is standard, but our approach differs from the previous literature by introducing dynamics to the taste shifter.

**Wage and hours shock processes** — Denote by $\Delta$ the first difference operator. Wage growth is determined by human capital related variables $X_t$, which contains $\Delta \Xi_t$, and an idiosyncratic error $\omega_t$:

$$\Delta \ln w_t = \alpha X_t + \Delta \omega_t$$

Idiosyncratic hours ($\upsilon_t$) and wage ($\omega_t$) components consist of permanent and transitory components, $P_t$ and $T_t$, that follow a random walk and an MA(1)-process, respectively. For $x \in \{\upsilon, \omega\}$ these are given by:

$$x_t = P^x_t + T^x_t$$

$$P^x_t = P^x_{t-1} + \xi^x_t$$

$$T^x_t = \theta_x \epsilon^x_{t-1} + \epsilon^x_t$$

$$E \left[ \xi^x_t \xi^x_{t-l} \right] = 0, \quad E \left[ \epsilon^x_t \epsilon^x_{t-l} \right] = 0 \quad \forall l \in \mathbb{Z}_{\neq 0}.$$

We use the Feldstein approximation for labor income taxation as it allows to model a progressive tax system, while staying tractable in the structural equations we derive for estimation. Related studies using this approximation are Blundell et al. (2016) and Heathcote et al. (2017). These studies show that the approximation fits the data well.
Permanent ($\zeta_t$) and transitory shocks ($\epsilon_t$) have mean zero and variances $\sigma^2_{\zeta,t}$ and $\sigma^2_{\epsilon,t}$, respectively. Permanent and transitory hours and wage shocks are uncorrelated. Note that hours and wage shocks contain only individual-specific shocks as $X_t$ and $\Xi_t$ contain year dummies.

**Labor supply** — We derive the intertemporal labor supply equation by approximating the first order condition of the optimization problem with respect to consumption (see MaCurdy (1981), Altonji (1986), and Appendix A):

$$\Delta \ln h_t = \frac{1}{\gamma + \tau} \left[ -\ln(1 + r_{t-1}) - \ln \rho + (1 - \tau) \Delta \ln w_t - \zeta \Delta \Xi_t + \eta_t + \Delta \nu_t \right],$$

where $\frac{1}{\gamma}$ is the Frisch elasticity of labor supply, $\frac{1 - \tau}{\gamma + \tau}$ is the tax-adjusted Frisch elasticity, and $\eta_t$ is a function of the expectation error in the marginal utility of wealth. $\frac{1 - \tau}{\gamma + \tau}$ is identified by estimating equation (5) using instrumental variables for $\Delta \ln w_t$.

In what follows, we want to estimate the variances of the shocks contained in $\nu_t$ and $\eta_t$, where the latter contains adjustments to wage shocks. To this end, we first make explicit how wage and hours shocks transmit into changes in permanent income, which, in turn, result in changes in the marginal utility of wealth.

Denote by $\Delta x$ idiosyncratic changes in $x$. Then $\Delta \ln y_t$ are changes in log net earnings that result from wage and hours shocks, where $y_t = \chi(w_t h_t)^{1-\tau}$. It is useful to group these into transitory and permanent changes, distinguished by the superscripts $T$ and $P$, respectively:

$$\Delta \ln y_t = (1 - \tau) \left[ \Delta \ln w^T_t + \Delta \ln w^P_t + \Delta \ln h^P_t + \Delta \ln h^T_t \right]$$

The expressions for transitory and permanent wage changes in terms of shocks are obtained directly from the wage process:

$$\Delta \ln w^T_t = \epsilon_t^\omega + (\theta - 1) \epsilon_{t-1}^\omega - \theta \epsilon_{t-2}^\omega$$

$$\Delta \ln w^P_t = \xi_t^\omega.$$  

(Note that in the case of transitory wage changes, everything apart from $\epsilon_t^\omega$ is known to the agent at $t - 1$. In contrast, the idiosyncratic wage change due to permanent shocks is entirely surprising. Write idiosyncratic hours changes as

$$\Delta \ln h_t = \Delta \ln h^P_t + \Delta \ln h^T_t = \frac{1}{\gamma + \tau} \left[ (1 - \tau) \Delta \ln w_t + \eta_t + \Delta \nu_t \right].$$

$^3\eta_t = \frac{\epsilon_t}{\lambda_t} - O\left(-1/2(\epsilon_t/\lambda_t)^2\right)$, i.e., it contains the expectation error of the marginal utility of wealth and the approximation error.
We make the simplifying assumption that transitory shocks do not impact $\eta_t$.\(^4\) Thus, the expressions for transitory hours changes in terms of shocks follow immediately from the stochastic processes of transitory shock components and the Frisch labor supply equation (9):

$$\Delta \ln h^/_t = \frac{1}{\tau + \gamma} \left( \epsilon^/_t + (\theta^/_u - 1)\epsilon^/_u - \theta^/_u \epsilon^/_u + (1 - \tau) (\epsilon^/_o + \theta^/_o - 1) \epsilon^/_o - \theta^/_o \epsilon^/_o - 2 + (1 - \tau) (\epsilon^/_w + (\theta^/_w - 1) \epsilon^/_w - 1 - \theta^/_w \epsilon^/_w) \right). \quad (10)$$

In our model the expectation error $\eta_t$ is a linear function of unexpected permanent changes to gross income. We derive the expression in Appendix B by approximating the life-time budget constraint as in Blundell et al. (2013, 2016).

$$\eta_t = -\phi^/\lambda_t (1 - \tau) \left( \Delta \ln w^/P_t + \Delta \ln h^/P_t \right), \quad \ln \phi^/\lambda_t \sim N \left( \mu^/\phi, \sigma^2/\phi \right). \quad (11)$$

The formula is intuitive: a change in gross income, given by $\Delta \ln w^/P_t + \Delta \ln h^/P_t$, is transformed into a change in net income by the factor $(1 - \tau)$, and this change in net income impacts the marginal utility of wealth with the factor $\phi^/\lambda_t$. The transmission parameter $\phi^/\lambda_t$ is a measure of consumption insurance since consumption is adjusted by $-\eta_t/\theta$. This can be seen by taking logs of the first derivative of equation (3) with respect to $c_t$, see equation (13) below. Under the assumptions for the proof in Appendix B, the transmission parameter $\phi^/\lambda_t$ is determined by preference parameter $\theta$ and the ratio of human wealth to total wealth. For individuals who have accumulated substantial assets, remaining life-time earnings only play a relatively small role in their total wealth. These individuals do not adjust their consumption by much in response to a shock.

Under more general assumptions, the degree of consumption insurance may be determined by other factors. For instance, parents might help their children out in case of a negative wage shock. In this case, our estimate of $\phi^/\lambda_t$ still yields the degree of consumption insurance, but then insurance goes beyond self-insurance. It is reasonable to expect that there is at least some degree of insurance, which implies that the estimate of $E[\phi^/\lambda_t]$ is a lower bound for the average degree of relative risk aversion $\theta$.

Positive income shocks lead to a decrease in the marginal utility of wealth, therefore $\phi^/\lambda_t$ is positive and should follow a distribution with no support on negative values. Hence, we estimate the model under the assumption that $\phi^/\lambda_t$ is lognormally distributed.

Note that the change in hours worked in equation (11) is an endogenous choice. Inserting equation (8) into (11) and subsequently (11) and (10) into (9) and solving for $\Delta \ln h^/P_t$ yields the expression for idiosyncratic permanent changes in hours of work:

\(^4\)A long time horizon implies that transitory shocks have a negligible impact on the marginal utility of wealth. Blundell et al. (2008) show that this result holds empirically for PSID data.
\[
\Delta \ln h_t^p = \frac{(1 - \tau) - (1 - \tau)\phi_t^h \xi_t^w}{\gamma + \tau + (1 - \tau)\phi_t^h \xi_t^w} + \frac{1}{\gamma + \tau + (1 - \tau)\phi_t^h \xi_t^w}
\] (12)

The term \( \kappa = \frac{(1 - \tau) - (1 - \tau)\phi_t^h \xi_t^w}{\gamma + \tau + (1 - \tau)\phi_t^h \xi_t^w} \) gives the uncompensated reaction to a gross permanent wage change, the tax-adjusted Marshallian labor supply elasticity. If \( \tau = 0 \), i.e. if the tax system is proportional, this reduces to the well-known expression \( \frac{1 - \phi_t^h}{\gamma + \phi_t^h} \), see for example Keane (2011). If \( \phi_t^h = 0 \), the case of perfect insurance, the Marshallian collapses to the tax-adjusted Frisch elasticity, the reaction to a transitory shock. The transmission coefficient for a permanent hours shock, \( \frac{1}{\gamma + \tau + (1 - \tau)\phi_t^h \xi_t^w} \), has the same property. The higher \( \phi_t^h \), the more cushioned are hours shocks. Regarding wage shocks, an increase in \( \phi_t^h \) leads to a decrease in the Marshallian. Thus, for positive Marshallian elasticities, \( \phi_t^h \) dampens the labor supply reaction to wage shocks, while it amplifies the negative labor supply reaction for negative Marshallians.

Equations (5)-(12) describe how wage and hours shocks affect labor supply and in turn income. These equations serve to derive the moment conditions, which are used for the estimation and stated in Appendix C. Next, we show how the shocks affect consumption.

**Consumption** — The equation for consumption growth can be obtained analogously to equation (5) (see, e.g., Altonji 1986):

\[
\Delta \ln c_t = \frac{1}{\theta} \left[ \ln(1 + r_{t-1}) + \ln \rho - \eta_t \right]
\] (13)

Thus income shocks are directly related to consumption growth by \( -\eta_t/\theta \). The direct estimation of equation (13) using consumption data is beyond the scope of this study. Nonetheless, we shall benchmark our results in Section 5 by calculating the reaction of consumption to a permanent wage shock by calibrating \( \theta \).

Figure 1: Transmission of Permanent Wage Shock

\[ \text{wage shock} \rightarrow \text{wage} \rightarrow \text{income} \rightarrow \text{consumption} \]

\[ \text{hours} \]

Note: Labels of arrows indicate corresponding equations.
Overview — Figures 1 and 2 show how each type of permanent shock propagates through the various quantities of interest. The major distinction for the two types is that wage shocks do not only have a direct effect on income, but also affect the choice of hours through the Marshallian elasticity.

3 Methodology and Data

In this section we sketch how the labor supply elasticities as well as the standard deviations of permanent and transitory parts of idiosyncratic wage components, \( \omega_t \), and hours components, \( \upsilon_t \), are recovered in the estimation. We give full details on the procedure in Appendix C. After the sketch, we present the procedure to correct for measurement errors and describe the data.

Identification strategy — We proceed through four stages:

1. We regress the log of post-government income on the log of pre-government income to obtain an estimate of the tax progressivity parameter \( \tau \). The result of the estimation is found in Table 9 in Appendix F.

2. We use OLS to obtain residuals from the wage equation (4) and IV to obtain residuals from the hours equation (5) as well as an estimate for the tax-adjusted Frisch labor supply elasticity. Using the estimate of \( \tau \) we then calculate the unadjusted Frisch elasticity \( 1/\gamma \).

3. We estimate the variances of transitory and permanent shocks and the persistence of transitory shocks to wages by fitting the theoretical variance as well as the first and second autocovariance of residual wage growth to their empirical counterparts using the method of moments.

4. To estimate hours shock variances we fit the corresponding autocovariance moments for the hours residuals to the data. The hours residuals contain \( \eta_t \) which depends on the labor supply reactions to permanent wage shocks, governed by \( \phi^A \). Therefore, a fourth moment, namely the covariance of residual hours and wage growth, is used to identify the mean of \( \phi^A \). The variance of hours residuals contains both the variance of permanent hours shocks and the variance of \( \phi^A \) as unknown parameters. An equivalent transmission parameter for income shocks to the marginal utility of wealth is estimated in Alan et al. (2018). Alan et al. (2018) assume a lognormal distribution for this parameter. We use their value to calibrate the variance of \( \phi^A \).
Step 4 builds on steps 1, 2, and 3 as it uses estimates for the wage process parameters and the Frisch elasticity. The estimated parameters allow us to calculate the tax-adjusted Marshallian labor supply elasticity.

**Measurement errors** — In Appendix C we state the variance-covariance moments with measurement error in hours and wages. Measurement error is modeled as having no intertemporal and cross-sectional correlation, but we do allow for correlation between the types of measurement error. Denote by

\[ \ln \tilde{x}_t = \ln x_t + me_{x,t} \]  

the observed value for the log of variable \( x \), where \( me_{x,t} \) is the mean zero measurement error with variance \( \sigma^2_{me,x} \). The variances encountered in the moment conditions are \( \sigma^2_{me,h}, \sigma^2_{me,w}, \) and \( \sigma^2_{me,h,w} \), which are the variances of measurement errors in log hours, log wages, and their covariance.

Following Meghir and Pistaferri (2004) and Blundell et al. (2016) we use estimates from the validation study by Bound et al. (1994) for the signal-to-noise ratios of wages, hours, and earnings. As in Blundell et al. (2016), we assume that the variance of the measurement error of hours is \( \sigma^2_{me,h} = 0.23 \text{var}(\ln h) \), the variance of the measurement error of wages is \( \sigma^2_{me,w} = 0.13 \text{var}(\ln w) \), and the variance of the measurement error of earnings is \( \sigma^2_{me,y} = 0.04 \text{var}(\ln y) \), where \( \text{var}(\ln h), \text{var}(\ln w), \) and \( \text{var}(\ln y) \) denote the variances of the levels of log wages, log hours, and log earnings. The covariance of the measurement errors of log wages and hours is given by \( \sigma^2_{me,h,w} = (\sigma^2_{me,y} - \sigma^2_{me,w} - \sigma^2_{me,h})/2 \). We correct the theoretical moments using these estimates for the parts that are attributable to error.

**The data** — We use annual data from the PSID for the survey years 1970 to 1997, which gives 27 years usable for first-differenced estimations. After this point in time the PSID is biennial. In total we have 46,340 observations across individuals and years. Annual hours of work and earnings refer to the previous calendar year. Hours in the PSID are calculated as weeks worked times usual hours of work per week. Earnings consist of wages and salaries from all jobs and include tips, bonuses, and overtime. We calculate the hourly wage by dividing gross earnings through hours of work. As hours and earnings are measured with error, a negative correlation between measured hours and wages is induced, which we correct for as described in the previous paragraph. Our sample consists of working, married males aged 28 to 60, who are the primary earners of their respective households. Table 1 shows summary statistics of the main sample. Monetary variables are adjusted to 2005 real values using the CPI-U.
Table 1: Descriptives

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>s.d.</th>
</tr>
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<tbody>
<tr>
<td>Age</td>
<td>41.35</td>
<td>8.66</td>
</tr>
<tr>
<td>Annual hours of work</td>
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<td>530.11</td>
</tr>
<tr>
<td>Hourly wage</td>
<td>26.86</td>
<td>22.83</td>
</tr>
<tr>
<td>Number of kids in household</td>
<td>1.64</td>
<td>1.39</td>
</tr>
<tr>
<td>N</td>
<td>46340</td>
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Note: Monetary values inflated to 2005 real dollars.

4 Main Results

In this section we show the results of our estimation procedure as well as the central exercise of this paper: the decomposition of earnings risk into contributions from wage and hours shocks. When we present the estimation results we show estimates for the main sample and for three sub-samples. We have constructed these sub-samples because they might differ with respect to their labor supply behavior and exposure to shocks and because they enable consistency checks with the related literature. The first sub-sample excludes young workers under 40, the second consists of individuals with more than high school education, and the third comprises individuals without children younger than seven years in the household.

Standard deviations of wage shocks — Table 2 reports the standard deviations of permanent and transitory wage shocks as well as the parameter of transitory shock persistence. First, while the magnitude of the standard deviation of permanent shocks ($\sigma_{\zeta,\omega}$) is similar in the four samples, excluding young workers leads to a decline of this figure. This is in line with the finding of slightly higher variances of permanent wage shocks at younger ages as in Blundell et al. (2016) and Meghir and Pistaferri (2004). Second, for all samples the standard deviation of transitory shocks ($\sigma_{\epsilon,\omega}$) is smaller than that of permanent shocks. Third, the highly educated face a substantially lower standard deviation of transitory shocks than the full sample. Lastly, for those without young children the standard deviations of permanent and transitory shocks are slightly lower than for the full sample.

Standard deviations of hours shocks — The first three rows in Table 3 show the parameters of the process of shocks to the disutility of work. For ease of interpretation and to compare them to the wage shocks, the parameters are reported as they enter the hours equation (5), i.e. multiplied with $1/(\gamma + \tau)$. The estimates for the hours process are of comparable size to those of the wage process. The standard deviation of permanent hours shocks drops when we consider only the highly educated. Otherwise, permanent shocks to the disutility of work are of a fairly consistent size across the samples. For all three subsamples the standard deviation of transitory shocks is lower than in the
full sample and persistence of the shocks also differs across samples. The magnitude of the standard deviation of permanent hours shocks is a first indicator that these shocks play a significant role for overall earnings risk. However, as described in Section 2, the effect of innovations in the marginal disutility of work on earnings depends on the degree of consumption insurance.

Table 2: Wage Process Parameters

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<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full sample</td>
<td>Age≥40</td>
<td>High educ</td>
<td>No children &lt;7</td>
</tr>
<tr>
<td>( \theta_\omega ) persistence parameter</td>
<td>0.2701 (0.0090)</td>
<td>0.3450 (0.0241)</td>
<td>0.2737 (0.0325)</td>
<td>0.1832 (0.0212)</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,\omega} ) SD transitory shocks</td>
<td>0.1337 (0.0017)</td>
<td>0.1382 (0.0030)</td>
<td>0.0772 (0.0015)</td>
<td>0.1166 (0.0025)</td>
</tr>
<tr>
<td>( \sigma_{\xi,\omega} ) SD permanent shocks</td>
<td>0.1770 (0.0009)</td>
<td>0.1554 (0.0014)</td>
<td>0.1765 (0.0007)</td>
<td>0.1639 (0.0011)</td>
</tr>
<tr>
<td>( N )</td>
<td>46340</td>
<td>20607</td>
<td>19831</td>
<td>24547</td>
</tr>
</tbody>
</table>

*Note:* Bootstrapped standard errors in parentheses (200 replicates).

Table 3: Hours Process Parameters and Labor Supply Elasticities

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full sample</td>
<td>Age≥40</td>
<td>High educ</td>
<td>No children &lt;7</td>
</tr>
<tr>
<td>( \theta_\nu/(\gamma + \tau) ) persistence parameter</td>
<td>0.1875 (0.0049)</td>
<td>0.4966 (0.0074)</td>
<td>0.1406 (0.0013)</td>
<td>0.3074 (0.0014)</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,\nu}/(\gamma + \tau) ) SD transitory shocks</td>
<td>0.1113 (0.0011)</td>
<td>0.0730 (0.0013)</td>
<td>0.0710 (0.0014)</td>
<td>0.0787 (0.0012)</td>
</tr>
<tr>
<td>( \sigma_{\xi,\nu}/(\gamma + \tau) ) SD permanent shocks</td>
<td>0.1990 (0.0010)</td>
<td>0.2102 (0.1325)</td>
<td>0.1647 (0.0010)</td>
<td>0.1915 (0.0035)</td>
</tr>
<tr>
<td>( (1 - \tau)/(\gamma + \tau) ) tax-adjusted Frisch elasticity</td>
<td>0.3614 (0.0856)</td>
<td>0.4020 (0.3778)</td>
<td>0.2851 (0.0975)</td>
<td>0.3148 (0.1080)</td>
</tr>
<tr>
<td>( E[\phi_t^t] ) transmission parameter</td>
<td>1.8917 (0.1119)</td>
<td>1.4084 (371.2670)</td>
<td>0.5669 (0.0437)</td>
<td>0.9564 (4.1316)</td>
</tr>
<tr>
<td>( E[\kappa] ) tax-adjusted Marshallian elasticity</td>
<td>-0.0767 (0.0105)</td>
<td>-0.0023 (0.0253)</td>
<td>0.1302 (0.0079)</td>
<td>0.0631 (0.0165)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note:* Clustered standard errors for \( (1 - \tau)/(\gamma + \tau) \), bootstrapped standard errors for other coefficients in parentheses (200 replicates).

**Frisch elasticity** — Row 4 in Table 3 reports the estimates of the tax-adjusted Frisch elasticity. In contrast to the most closely related papers (Blundell et al. 2016; Heathcote et al. 2014), we obtain the Frisch elasticity directly through IV estimation and not through covariance moments.\(^5\) The estimated tax-adjusted Frisch elasticity for the main sample is 0.36, which is in line with the literature.

\(^5\)Table 10 in the appendix additionally displays the Kleibergen and Paap (2006) F statistic, indicating that only sample II might suffer from weak instrument bias and should therefore be interpreted with caution.
The point estimate of this elasticity increases when excluding younger individuals. This result is expected as younger individuals could be less willing to reduce their hours of work in the case of a decrease in the hourly wage because the accumulation of human capital impacts their opportunity costs of time (Imai and Keane 2004). Similarly, human capital considerations are more important for the highly educated, where the Frisch elasticity is lower relatively low. The estimate for those without young children is just slightly smaller than that of the main sample.

**Transmission parameter** — Row 5 in Table 3 shows the estimated mean of the parameter that measures the transmission of shocks to the marginal utility of wealth, $E[\phi_t^1]$. The smaller this parameter, the more are individuals insured against shocks. A value of zero indicates that permanent shocks do not impact the marginal utility of wealth at all. We expect households with larger accumulation of assets relative to human wealth to exhibit smaller values of $E[\phi_t^1]$. The point estimate drops only slightly relative to the full sample when excluding young workers. The estimate is substantially smaller when focusing on those without young children and even smaller when focusing on the highly educated. These drops with respect to the main sample are expected.

**Marshallian elasticity** — Row 6 in Table 3 reports the average of the tax-adjusted Marshallian elasticity defined in equation (12) as the reaction to a permanent wage shock. The wealth effect outweighs the substitution effect, leading to a negative (but small) estimate for the main sample, in line with the recent literature. The negative Marshallian implies that hours move in the opposite direction of wages and thus function as a consumption smoothing device. When excluding younger workers, the estimate edges closer to zero, signifying virtually no long-term adjustment in hours for older workers. The smaller the average transmission parameter, the closer is the average Marshallian to the Frisch elasticity because the shock has a smaller effect on the marginal utility of wealth. The smaller wealth effect for older workers is expected because individuals close to the end of their life-cycle experience the same change to their marginal utility of wealth from either a transitory or a permanent shock. In the sample without young children in the household the estimate is positive, making the substitution effect the dominant force as the average transmission parameter is relatively small for this sample. The highly educated show the highest positive tax-adjusted Marshallian elasticity due to their very small average transmission parameter.

**Importance of hours and wage shocks** — Using our estimates for the variances of hours and wage shocks allows us to quantify their contribution to the cross-sectional variance of stochastic net

---

6 The extremely high standard error may be due to numerical instability of the numerical integrals we compute to estimate $E[\phi_t^1]$ in some bootstrap samples. See Appendix D.

7 We calculate $\kappa$ as the numerical expectation $E \left[ \frac{(1-\tau) - (1-\gamma)\phi_t^1}{1 + (1-\tau)\phi_t^1} \right]$.  

8 Blundell et al. (2016, p.414) and Heathcote et al. (2014) find Marshallian elasticities for men of -0.08 and -0.16, respectively. The latter number is obtained by inserting the obtained parameter estimates in the formula for the labor supply reaction to an uninsurable shock (Heathcote et al. 2014, p. 2120). Altonji et al. (2013) report a coefficient that determines "the response to a relatively permanent wage change" of -0.08.
earnings growth. The stochastic component without measurement error is given by the sum of hours and wage residuals plus the Frisch reactions to idiosyncratic wage changes, which we had removed from the hours residual by detrending with wages, see equation (5). Equation (44) in Appendix E describes how the variance of stochastic net earnings growth is calculated. Note that the variance of stochastic net earnings growth depends on the mean and the variance of the transmission parameter \( \phi^t_l \), which are known to individuals. Additionally the realization of transitory components of wage and hours growth are partially known in advance, see equation (7). Therefore this variance is not a measure of risk.

The first row of Table 4 shows this variance for the main sample. Rows 2-5 show the contributions of each shock component, i.e., the variance of stochastic net earnings growth when the variances of all other shock components are set to zero. Over the columns we vary the progressivity of the tax system: the first gives values computed with our estimate of \( \tau \) from Table 9, the second sets \( \tau \) equal to zero (proportional tax system), and the third shows the reduction in the variance due to progressivity of the tax system. First, we see that about 44% of the variance is due transitory wage shocks. Permanent wage shocks are the second biggest contributor driving about a quarter of the variance. Transitory and permanent hours shocks contribute equally to the variance; about 16%. Eliminating progressive taxation in column II increases the total variance by about 73%. The relative importance of transitory shocks increases slightly because the progressive tax system reduces the variance contribution of transitory shocks of either kind by 45% (Column III). In the case of permanent shocks this reduction only amounts to 37%. The reduction in total variance due to progressive taxation is 42%.

Table 4: Decomposition of Variance of Earnings Growth

<table>
<thead>
<tr>
<th>Contribution of</th>
<th>I ( \tau = 0.192 )</th>
<th>II ( \tau = 0 )</th>
<th>III % reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>trans. wage shocks ((\sigma_{e,\omega}))</td>
<td>0.0347</td>
<td>0.0637</td>
<td>45.44</td>
</tr>
<tr>
<td>perm. wage shocks ((\sigma_{z,\omega}))</td>
<td>0.0188</td>
<td>0.0296</td>
<td>36.52</td>
</tr>
<tr>
<td>trans. hours shocks ((\sigma_{e,\upsilon}))</td>
<td>0.0122</td>
<td>0.0224</td>
<td>45.44</td>
</tr>
<tr>
<td>perm. hours shocks ((\sigma_{z,\upsilon}))</td>
<td>0.0128</td>
<td>0.0202</td>
<td>36.52</td>
</tr>
<tr>
<td>( V (\Delta \ln y) )</td>
<td>0.0786</td>
<td>0.136</td>
<td>42.17</td>
</tr>
</tbody>
</table>

*Note:* First line: variance of \( \Delta \ln y \) given by equation (44). Lines 2-5 show the variance of \( \Delta \ln y \) when all other shock variances are set to zero. Column II shows variances without progressive taxation. Column III shows the variance reduction from column II to I.

The variance shown in Table 4 is not a measure of risk, i.e. the component of the variance demanding insurance. In Table 5 we repeat the exercise of Table 4 but using the measure of earnings growth risk. Note that permanent changes in income lead directly to changes in consumption,
only mediated through the consumption insurance parameter. Thus earnings risk translates to
consumption risk and has immediate welfare implications for risk averse individuals. Risk naturally
means unexpected earnings growth. Therefore, when evaluating the risk of idiosyncratic earnings
growth instead of its cross-sectional variance, everything that is known to the individual at $t - 1$
must be excluded from equation (44) and $\phi_t^\lambda$ must be treated as non-stochastic. Equation (45) in
Appendix E gives earnings growth risk conditional on the individual’s information set in $t - 1$,
denoted $I_{t-1}$. For the calculation in Table 5, we set $\phi_t^\lambda$ to the sample mean.

Risk, given in the first line, is roughly 66% of the net earnings growth variance. The proportions
of contributions of the respective shocks are quite similar to the proportions of contributions to the
variance. Strikingly, the percentage reduction of the contribution of permanent shocks to earnings
risk due to insurance through progressive taxation is now even lower than in the case of the variance.
It is less than two thirds the reduction of transitory shocks.

While progressive taxation can sensibly be used to insure against permanent shocks, as first
discussed by Varian (1980), later in the case of labor supply by Eaton and Rosen (1980), and
recently quantified by Heathcote et al. (2017), the argument to use progressive taxation to insure
against transitory shocks is much weaker. Individuals may take several precautionary measures,
such as saving and labor supply, to self-insure. Ideally, progressive taxation would only act
upon the permanent component of earnings and leave the decisions following transitory shocks
undistorted. Yet, progressive taxation acts upon both components leading to undesirable distortions
of intertemporal labor supply decisions. In the case of transitory shocks the distortion is not
countervailed through valuable insurance.

Table 5: Decomposition of Earnings Risk at Mean

<table>
<thead>
<tr>
<th>Contribution of</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V(\tilde{\Delta} \ln y</td>
<td>I_{t-1})$</td>
<td>0.0522</td>
<td>0.086</td>
</tr>
<tr>
<td>trans. wage shocks ($\sigma_{e,\omega}$)</td>
<td>0.0216</td>
<td>0.0397</td>
<td>45.44</td>
</tr>
<tr>
<td>perm. wage shocks ($\sigma_{\zeta,\omega}$)</td>
<td>0.0134</td>
<td>0.0187</td>
<td>28.65</td>
</tr>
<tr>
<td>trans. hours shocks ($\sigma_{e,\upsilon}$)</td>
<td>0.0081</td>
<td>0.0148</td>
<td>45.44</td>
</tr>
<tr>
<td>perm. hours shocks ($\sigma_{\zeta,\upsilon}$)</td>
<td>0.0091</td>
<td>0.0128</td>
<td>28.64</td>
</tr>
</tbody>
</table>

Note: The first line shows total earnings risk given by equation (45)
with $\phi_t^\lambda$ set to the sample mean. Lines 2-5 show the risk when all
other shock variances are set to zero. Column II shows risk without
progressive taxation. Column III shows the reduction from column
II to I.

While transitory shocks are an important driver of the cross-sectional earnings growth variance,
only permanent shocks have a large impact on the present value of life-time earnings, which directly
relates to individuals’ consumption and thus utility. At the mean of the consumption insurance

16
parameter, a positive hours shock of one standard deviation raises earnings by 10%, while a similar wage shock raises earnings by 12%.\textsuperscript{9} A back-of-the-envelope calculation\textsuperscript{10} reveals that for an individual with an annual net labor income of 50 000 Dollar, aged 30 and retiring at 65, a typical positive permanent wage shock of one standard deviation increases the present value of life-time earnings by about 106 000 Dollar, while a typical positive permanent hours shock increases it by 87 000 Dollar.\textsuperscript{11} Typical permanent wage and hours shocks at age 50 for this individual increase life-time income by 65 000 and 54 000 Dollar, respectively. Thus, both types of permanent shocks have a substantial impact on life-time earnings.

Without progressive taxation, i.e. setting $\tau = 0$, a typical positive permanent wage shock for this individual at age 30 increases life-time income by 125 000, an hours shock increases life-time income by 103 000 Dollar. Thus, progressive taxation reduces the impact of permanent shocks over the life-cycle by 16%.

The impact of hours shocks depends largely on the consumption insurance parameter. In the benchmark case of full insurance with $\phi_t^I = 0$ individuals adjust their hours of work much more in response to a shock to the disutility of work. In this case and with progressive taxation, the impact of a typical permanent wage shock at age 30 is 178 000 Dollar because here the Frisch labor supply reaction amplifies the wage shock. The analogous impact of a typical hours shock is 147 000 Dollar. These figures correspond to increases by 20% and 16%, respectively. Clearly, the impact of a permanent shock on life-time income varies widely between individuals.

In sum, both permanent hours and wage shocks are significant drivers of both cross-sectional and life-time earnings risk. Progressive taxation is an important insurance mechanism to reduce the impact of these shocks.

\section{Discussion}

In the following discussion we will 1) investigate the sources of permanent hours shocks, 2) evaluate alternative model specifications with respect to data fit to determine whether hours shocks are important, 3) evaluate model fit over the life-cycle, and 4) calibrate the parameter of relative risk aversion to calculate partial consumption insurance.

\textbf{Hours shocks in alternative samples} — In order to investigate and understand the sources of permanent hours shocks, we estimate their standard deviation in alternative samples. Column I

\textsuperscript{9}These values are obtained from $y(1 - \tau)(1 + (1 - \tau - (1 - \tau))(E[\phi_t^I]) / (\gamma + \tau + (1 - \tau)E[\phi_t^I])) \sigma_{\zeta, \omega}$ for a wage shock and $y(1 - \tau)(1/(\gamma + \tau + (1 - \tau)E[\phi_t^I]))$ for an hours shock.

\textsuperscript{10}The relative impacts of wage and hours shocks are multiplied with the geometric series for the remaining life-time earnings $(1 - 1/r, ^{65-\text{age}})/(1 - 1/r)$, where the real interest rate $r$ is 1.0448 based on World Bank figures for our period. This abstracts from deterministic earnings growth, i.e. it makes the simplifying assumption that earnings would remain constant without shocks.

\textsuperscript{11}Note that the ratio of the impacts of typical permanent hours and wage shocks on lifetime earnings equals the square root of the corresponding ratio of contributions to earnings risk reported in Table 5.
in Table 6 reports the estimate for the full sample. Column II contains results for a sample of individuals in “blue collar” industries. Individuals in advanced technical sectors like electrical and mechanical engineering or skilled service jobs like legal or medical services are excluded. One could expect that the demand for these more routine jobs only allows for very limited variation in hours. However, this does not seem to be the case, as the estimate of the permanent shocks hardly changes. In column III we exclude the years 1981 and 1982, when a global recession hit the US. There is only a modest change to the estimate of the standard deviation of permanent hours shocks, which shows that the results are not driven by the crisis. Finally, in column IV the sample is restricted to individuals who have stayed in their current job for at least 3 years consecutively. Given that the estimate for the permanent hours shock variance is very close to that of the main sample, we can infer that permanent hours shocks do not merely reflect changes in occupation or job instability. In contrast, the variance of transitory hours shocks in the stayers sample drops sharply \((\sigma_{\epsilon,\upsilon}/(\gamma + \tau) = 0.0492, \text{s.e.: } 0.0011)\), attesting to the transitory nature of the effect of job changes on hours. The upshot of all of these results is that permanent hours shocks play an important role throughout all samples and are not restricted to very specific adjustments or at-risk groups. The fact that hours shocks do not seem to be driven by job changes or possibly unwanted changes in hours of work during crises suggests an interpretation of permanent hours shocks as shocks to the marginal disutility of work, caused, e.g., by shocks to home production.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Main</td>
<td>Blue collar</td>
<td>Exclude years 81-82</td>
<td>Only stayers</td>
</tr>
<tr>
<td>(\sigma_{\epsilon,\upsilon}/(\gamma + \tau))</td>
<td>0.1990</td>
<td>0.2066</td>
<td>0.2064</td>
<td>0.1918</td>
</tr>
<tr>
<td></td>
<td>(0.0010)</td>
<td>(0.0022)</td>
<td>(0.0023)</td>
<td>(0.0016)</td>
</tr>
<tr>
<td>(N)</td>
<td>46340</td>
<td>32313</td>
<td>40999</td>
<td>29211</td>
</tr>
</tbody>
</table>

Note: Bootstrapped standard errors in parentheses.

**Hours shocks and transmission in alternative models** — In Table 7 we report the parameters of the hours shock process and the transmission parameter as well as the implied tax-adjusted Marshallian elasticity for the main sample under various restrictions of parameters or alternative assumptions. Further we display a measure of overall fit of these alternative models, namely the value of the distance function \(DF(\Theta)\) as a measure of how well the model fits the data. The estimates of the main model are repeated for comparison in column I.

---

12In particular, individuals in education, sport, legal, health, and other services including service industries, mechanical and electrical engineering, financial institutions, and insurance as well as public administration, social security, private households, volunteering and churches are excluded.

13Note that this means that all moments rely only on information from the current job.
In column II, the variance of \( \ln(\phi_t^\lambda) \) is calibrated to half the value in our main specification. All estimated coefficients except for the standard deviation of permanent hours shocks and the mean of the transmission parameter are unchanged. The standard deviation of the permanent hours shocks is slightly larger. The reason is that the variance of the transmission parameter interacts with the variance of hours shocks in explaining the variance of hours growth (equation (37) in Appendix C). As the model is exactly identified and the variance of permanent hours shocks can freely adjust, the fit is virtually unchanged.\(^{14}\) The exercise demonstrates that the results only depend to a small degree on this calibration.

Columns III and IV illustrate the importance of allowing for hours shocks. In column III the variance of permanent hours shocks is set to zero. While the estimated variance of transitory hours shocks increases only slightly, the estimated mean of the transmission parameter increases to roughly 2.47. The fit of this model is substantially worse. The implied Marshallian elasticity doubles. In column IV both transitory and permanent hours shocks are restricted to zero. In this case the estimated average transmission parameter increases to roughly 20.9 and the implied Marshallian elasticity is about -0.7. These extreme estimates are caused by the fact that the transmission of wage shocks is now the only channel to explain hours variance. Naturally, the fit takes another hit from this restriction.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{\upsilon}/(\gamma + \tau) )</td>
<td>persistence</td>
<td>0.1875</td>
<td>0.1875</td>
<td>0.1798</td>
</tr>
<tr>
<td></td>
<td>parameter</td>
<td>(0.0049)</td>
<td>(0.0049)</td>
<td>(0.0016)</td>
</tr>
<tr>
<td>( \sigma_{\epsilon,\upsilon}/(\gamma + \tau) )</td>
<td>SD transitory shocks</td>
<td>0.1113</td>
<td>0.1114</td>
<td>0.1501</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0011)</td>
<td>(0.0011)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>( \sigma_{\zeta,\upsilon}/(\gamma + \tau) )</td>
<td>SD permanent shocks</td>
<td>0.1990</td>
<td>0.2116</td>
<td>(0.0033)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0010)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E[\phi_t^\lambda] )</td>
<td>transmission parameter</td>
<td>1.8917</td>
<td>1.4316</td>
<td>2.4692</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1119)</td>
<td>(0.0691)</td>
<td>(0.1782)</td>
</tr>
<tr>
<td>( E[\kappa] )</td>
<td>tax-adjusted Marshallian elasticity</td>
<td>-0.0767</td>
<td>-0.0767</td>
<td>-0.1448</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0105)</td>
<td>(0.0105)</td>
<td>(0.0111)</td>
</tr>
<tr>
<td>( DF(\Theta) )</td>
<td>distance</td>
<td>( 1.9398 \times 10^{-11} )</td>
<td>( 3.0847 \times 10^{-11} )</td>
<td>( 3.8247 \times 10^{-05} )</td>
</tr>
</tbody>
</table>

Note: Bootstrapped standard errors in parentheses (200 replicates).

Model Fit — Figures 3a, 3b and 3c show the hours and wage residual variance series and the covariance over age. The current model does not allow for variation in these targeted variances over age.

\(^{14}\)Note that the estimate for the tax-adjusted Marshallian elasticity is unchanged as well. The reason is that the model is exactly identified and \( \mu_{\phi} \) adjusts such that \( E[\kappa] \), which results directly from the covariance of hours and wage residuals, is unchanged, see Appendix C.
Figure 3: Fit of Variance and Covariance Moments over the Life-Cycle

(a) Variance of Hours Residuals
(b) Variance of Wage Residuals
(c) Covariance of Hours and Wage Residuals

Note: Empirical and theoretical variance and covariance moments of residuals obtained from the estimation of equations (4) and (5) for the main sample with bootstrapped 95% confidence interval.

age groups and thus imposes that their pattern is essentially flat over the life-cycle. The figures show that these variances do not vary substantially over the life-cycle.

Partial consumption insurance — The parameter $\phi^t_l$ is directly related to consumption growth, see equation (13). In our model with endogenous labor supply, permanent wage shocks translate into changes in consumption by \( \frac{\phi^t_l(1-\tau)}{\theta} \times \frac{1+\gamma}{\gamma+\tau+(1-\tau)\phi^t_l} \). We set $\theta = 2$, which is close to the estimates of related papers\(^{15}\) and calculate the resulting pass-through at mean values of $\phi^t_l$, reported in Table 8. For the full sample we find that on average a positive permanent wage shock of 1% leads to an increase in consumption by 0.62%. This figure can be compared to studies that use consumption data to obtain similar parameters. Blundell et al. (2016) use 1999-2009 PSID data and find that the

\(^{15}\)Blundell et al. (2016) estimate a parameter of relative risk aversion of 2.4 and Heathcote et al. (2014) estimate 1.7.
Marshallian response of consumption to the male’s wage shock is 0.58, when female labor supply is held constant. We obtain a slightly smaller pass-through parameter for the older sample than for the main sample, but find a substantially smaller pass-through for the highly educated, for whom a permanent wage increase by 1% leads to an increase in consumption of just 0.25%. Using a similar data set to ours, 1978-1992 PSID data, Blundell et al. (2008) estimate the pass-through of permanent income shocks to consumption, which is given by \((1 - \tau)\phi_\lambda^t / \varphi\) in our model. With a Marshallian labor supply elasticity close to zero—as the one we have estimated—this parameter comes close to the pass-through of permanent wage shocks. Their estimate for the full sample is 0.64 and the estimate for their college sample is 0.42. While our estimate for this group is somewhat smaller, we confirm that the highly educated are better insured against income shocks than the whole population.

### Table 8: Pass-through of Permanent Wage Shocks to Consumption

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau = 0.192)</td>
<td>0.618</td>
<td>0.7316</td>
<td>15.52</td>
</tr>
<tr>
<td>(\tau = 0)</td>
<td>0.5094</td>
<td>0.615</td>
<td>17.18</td>
</tr>
<tr>
<td>Age&gt;=40</td>
<td>0.2534</td>
<td>0.3217</td>
<td>21.24</td>
</tr>
<tr>
<td>High educ</td>
<td>0.3905</td>
<td>0.4845</td>
<td>19.4</td>
</tr>
</tbody>
</table>

*Note: The pass-through is \(\frac{E[\phi_\lambda^t|(1-\tau)\phi_\lambda^t]}{\varphi} \times \frac{1+\gamma}{\gamma+\tau+(1-\tau)E[\phi_\lambda^t]}\) and we set \(\varphi=2\).*

In columns II and III we conduct another tax experiment by setting \(\tau\) to zero. For the main sample the reduction in pass-through is about 16% in line with the reduction of the impact of permanent shocks on life-time income. For the highly educated the relative reduction is the strongest with about 21%, while the absolute reduction is the strongest for the full sample.

Much of the literature on consumption insurance makes use of moment conditions involving consumption data. We obtain comparable estimates from labor supply and earnings data alone. Similarly, Heathcote et al. (2014) estimate their model with and without moment conditions of consumption. This does not lead to large changes in their estimates. A back-of-the-envelope calculation based on our results for the pass-through parameter to the marginal utility of wealth yields consumption insurance parameters that are broadly comparable to those obtained in previous papers using consumption data. This adds to the notion that much can be learned about consumption insurance from earnings and labor supply data alone.

### 6 Conclusion

At the outset we asked a simple question: What are the drivers of earnings risk? To get at the answer, we have decomposed idiosyncratic earnings risk into contributions of transitory and permanent wage
and hours shocks. This is a departure from previous work, where unexplained earnings volatility is entirely due to wage shocks. In order to separate hours shocks from labor supply reactions to wage shocks, we build on a life-cycle model of labor supply and consumption and estimate a transmission parameter that captures the impact of shocks on the marginal utility of wealth and varies between individuals. This parameter is directly related to consumption insurance. We find that both wages and hours are subject to permanent shocks. At the mean, permanent wage shocks have a stronger impact on life-time earnings. Using the mean of the transmission parameter, a positive permanent wage shock of one standard-deviation for an individual with annual net labor earnings of 50,000 Dollar at age 30 increases life-time earnings by 106,000 Dollar, while the effect of a permanent hours shock of one standard-deviation is 87,000 Dollar. Both permanent hours and wage shocks are important sources of cross-sectional earnings growth risk. Ergo, the data tell a story beyond wage risk.

Progressive income taxation moderates the impact of a permanent shock on life-time income by 16%. It reduces the contribution of transitory shocks to earnings growth risk by 45% and the contribution of transitory shocks by 29%. Since it is more costly for individuals to self-insure against permanent shocks, this is an unfortunate finding. For transitory shocks insurance through the tax system is of little value but of potentially large cost because it distorts labor supply decisions.

Along the way to these results, we have shown an alternative way to calculate the tax-adjusted Marshallian elasticity of labor supply, which we find to be negative, but small, at -0.08. There is more insurance against permanent wage shocks among the highly educated, for whom we estimate a small positive Marshallian elasticity.

Our investigation of the sources of permanent hours shocks shows that they are a pervasive phenomenon. We rule out two potential major sources of hours shocks. Permanent hours shocks persist as a phenomenon when restricting the sample to individuals who stay in their respective jobs and when excluding the years 1981-82, during which a major economic crisis hit the US. These tests, along with the results from our four main samples, strongly suggest that hours shocks are a phenomenon that is not restricted to specific, one-off adjustments or only relevant for narrowly defined groups.

Calibrating the coefficient of relative risk aversion, we calculate the pass-through of permanent wage shocks to consumption and find reasonable figures in the same range as those reported in Blundell et al. (2008, 2016). These results are encouraging as they show that comparable estimates of consumption insurance can be obtained using either consumption or labor supply data.

Natural extensions of our framework include modeling family labor supply and the extensive labor supply margin. Moreover, the sources of hours shocks merit further research. One promising avenue would be to explicitly model and then separate out shocks to home production from other restrictions to labor supply.
Appendix

A Derivation of the Labor Supply Equation

The residual in the labor supply equation consists of in-period shocks and expectations corrections in the marginal utility of wealth due both to wage and hours shocks.

The first order condition of the consumer’s problem w.r.t. $h_t$ is:

$$\frac{\partial L}{\partial h_t} = E_t \left[ \left( -b_t h_t^\gamma \right) + \lambda_t \left( \chi (1 - \tau) h_t^{-\tau} w_t^{1-\tau} \right) \right] = 0, \quad (15)$$

where $\lambda_t = \frac{\partial u(c_t, h_t, b_t)}{\partial c_t}$ denotes the marginal utility of wealth. The Euler equation of consumption is given by

$$\frac{1}{\rho(1 + r_t)} \lambda_t = E_t[\lambda_{t+1}]. \quad (16)$$

Expectations are rational, i.e., $\lambda_{t+1} = E_t[\lambda_{t+1}] + \varepsilon_{\lambda_{t+1}}$, where $\varepsilon_{\lambda_{t+1}}$ denotes the mean-zero expectation correction of $E_t[\lambda_{t+1}]$ performed in period $t + 1$. Expectation errors are caused by innovations in the hourly wage residual $\omega_{t+1}$ and innovations in hours shocks $\upsilon_{t+1}$, which, as implied by rational expectations, are uncorrelated with $E_t[\lambda_{t+1}]$. Rational expectations imply that $\varepsilon_{\lambda_{t+1}}$ is uncorrelated over time, so that regardless of the autocorrelative structure of the shock terms, $\varepsilon_{\lambda_{t+1}}$ will only be correlated with the innovations of the shock processes.

Resolving the expectation operator in equation (15) yields

$$b_t h_t^\gamma = \lambda_t \left( \chi (1 - \tau) h_t^{-\tau} w_t^{1-\tau} \right). \quad (17)$$

Taking logs of both sides we arrive at the structural labor supply equation

$$\ln h_t = \frac{1}{\gamma + \tau} \left( \ln \lambda_t + \ln (1 - \tau) + \ln \chi + (1 - \tau) \ln w_t - \ln b_t \right). \quad (18)$$

To find an estimable form for $\ln h_t$, we take logs of (16) and resolve the expectation:

$$\ln \lambda_t = \ln (1 + r_t) + \ln \rho + \ln \left( \lambda_{t+1} - \varepsilon_{\lambda_{t+1}} \right)$$

A first order Taylor-expansion of $\ln \left( \lambda_{t+1} - \varepsilon_{\lambda_{t+1}} \right)$ gives $\ln \left( \lambda_{t+1} \right) - \frac{\varepsilon_{\lambda_{t+1}}}{\lambda_{t+1}}$, leading to the expression

$$\ln \lambda_t = \ln (1 + r_t) + \ln \rho + \ln \left( \lambda_{t+1} \right) - \frac{\varepsilon_{\lambda_{t+1}}}{\lambda_{t+1}} + O \left( \frac{1}{2} \left( \frac{\varepsilon_{\lambda_{t+1}}}{\lambda_{t+1}} \right)^2 \right). \quad (19)$$

Accordingly, when we backdate (19), we can insert it in (18) and remove $\ln \lambda_t$ by first differencing.
B Approximation of the Life-Time Budget Constraint

To relate shocks and innovations in the marginal utility of wealth, we need to approximate the life-time budget constraint. The life-time budget constraint at any given point in time \( t \) is

\[
\sum_{k=0}^{T-t} \frac{c_{t+k}}{(1+r)^k} = \sum_{k=0}^{T-t} \frac{\chi(w_{t+k} h_{t+k})^{1-\tau} + (1-\tau_N) N_{t+k}}{(1+r)^k} + a_t, \quad (20)
\]

A series \( \kappa_k \) can be approximated in the following way up to first order around \( \kappa_k^0 \) (Blundell et al. 2013, 2016),

\[
E_t \left[ \ln \sum_{k=0}^{T-t} \exp \kappa_k \right] \approx \ln \sum_{k=0}^{T-t} \exp \kappa_k^0 + \sum_{l=0}^{T-t} \frac{\exp \kappa_k^0}{\sum_{k=0}^{T-t} \exp \kappa_k^0} \left( E_t \kappa_l - \kappa_l^0 \right), \quad (21)
\]

where \( l \) is an arbitrary information set. We start with the left-hand side of the life-time budget constraint,

\[
E_t \left[ \ln \sum_{k=0}^{T-t} \frac{c_{t+k}}{(1+r)^k} \right] \approx \ln \sum_{k=0}^{T-t} \exp \left( E_{t-1} \left[ \ln c_{t+k} - k \ln(1+r) \right] \right) + \sum_{k=0}^{T-t} \pi_{t+k} \left( E_t \left[ \ln c_{t+k} \right] - E_{t-1} \left[ \ln c_{t+k} \right] \right), \quad (22)
\]

\[
\pi_{t+k} = \frac{\exp \left( E_{t-1} \left[ \ln c_{t+k} - k \ln(1+r) \right] \right)}{\sum_{j=0}^{T-t} \exp \left( E_{t-1} \left[ \ln c_{t+j} - j \ln(1+r) \right] \right)}.
\]

Dating the approximation to \( E_{t-1} \) and \( E_t \), we obtain

\[
E_{t-1} \left[ \ln \sum_{k=0}^{T-t} \frac{c_{t+k}}{(1+r)^k} \right] \approx \ln \sum_{k=0}^{T-t} \exp \left( E_{t-1} \left[ \ln c_{t+k} - k \ln(1+r) \right] \right), \quad (23)
\]

\[
E_t \left[ \ln \sum_{k=0}^{T-t} \frac{c_{t+k}}{(1+r)^k} \right] \approx \ln \sum_{k=0}^{T-t} \exp \left( E_{t-1} \left[ \ln c_{t+k} - k \ln(1+r) \right] \right)
\]

\[
+ \sum_{k=0}^{T-t} \pi_{t+k} \left( E_t \left[ \ln c_{t+k} \right] - E_{t-1} \left[ \ln c_{t+k} \right] \right).
\]

The additional term in the \( t \)-dated expectation is resolved through the approximated Euler equation (19).

\[
E_t \left[ \ln c_{t+k} \right] - E_{t-1} \left[ \ln c_{t+k} \right] \approx -\frac{1}{\theta} \eta_t, \quad (24)
\]

Thus the change in the expectation of life-time consumption after one period is determined by the innovation to the marginal utility of wealth. Since \( \sum_{k=0}^{T-t} \pi_{t+k} = 1 \), it follows that
\[
E_t \left[ \ln \sum_{k=0}^{T-t} \frac{c_{t+k}}{(1+r)^k} \right] - E_{t-1} \left[ \ln \sum_{k=0}^{T-t} \frac{c_{t+k}}{(1+r)^k} \right] \approx -\frac{1}{\theta} \eta_t \tag{25}
\]

The right-hand side of the budget constraint is approximated in the same fashion. We define

\[
P_1 = \sum_{k=0}^{T-t} \exp \left( E_{t-1} \left[ \ln \left( \chi \left( w_{t+k} h_{t+k} \right) \right)^{1-\tau} - k \ln(1+r) \right] \right),
\]

\[
P_2 = \exp \left( E_{t-1} \left[ \ln a_t \right] \right),
\]

\[
P_3 = \sum_{k=0}^{T-t} \exp \left( E_{t-1} \left[ \ln \left( (1 - \tau N_t) N_{t+k} \right) - k \ln(1+r) \right] \right).
\]

The expectation at period \( t \) of remaining human wealth plus assets is approximated as follows

\[
E_t \left[ \ln \left( \sum_{k=0}^{T-t} \chi \left( w_{t+k} h_{t+k} \right)^{1-\tau} \left( 1 - \tau N_t \right) N_{t+k} + a_t \right) \right] \approx \ln \left( P_1 + P_2 + P_3 \right) \tag{27}
\]

\[
+ (1-\tau) \sum_{k=0}^{T-t} \frac{\exp \left( E_{t-1} \left[ \ln \left( \chi \left( w_{t+k} h_{t+k} \right)^{1-\tau} - k \ln(1+r) \right] \right) \right)}{P_1 + P_2 + P_3} \left( E_t \left[ \ln \left( w_{t+k} h_{t+k} \right) \right] - E_{t-1} \left[ \ln \left( w_{t+k} h_{t+k} \right) \right] \right)
\]

\[
+ \frac{P_2}{P_1 + P_2 + P_3} \left( E_t \left[ \ln \left( a_t \right) \right] - E_{t-1} \left[ \ln \left( a_t \right) \right] \right),
\]

\[
+ \sum_{k=0}^{T-t} \frac{\exp \left( E_{t-1} \left[ \ln \left( (1 - \tau N_t) N_{t+k} \right) - k \ln(1+r) \right] \right)}{P_1 + P_2 + P_3} \left( E_t \left[ \ln \left( N_{t+k} \right) \right] - E_{t-1} \left[ \ln \left( N_{t+k} \right) \right] \right)
\]

Since \( a_t \) is determined in period \( t - 1 \), the last term on the r.h.s., further innovations to non-labor income, is assumed to be negligible, so the last line is approximately zero.

Let

\[
\Omega_t = \frac{P_1}{P_1 + P_2 + P_3},
\]

\[
\alpha_{t+k} = \frac{\exp \left( E_{t-1} \left[ \ln \left( \chi \left( w_{t+k} h_{t+k} \right)^{1-\tau} - k \ln(1+r) \right] \right) \right)}{P_1},
\]

then in terms of changes in hours and wages
Unexpected wage and hours changes consist of the transitory and permanent shocks dated period $t$.

$$
\sum_{k=0}^{T-t} \alpha_{t+k} \left( E_t [\ln(w_{t+k})] - E_{t-1} [\ln(w_{t+k})] \right) = \alpha_t \epsilon_t^\omega + \alpha_{t+1} \theta^\omega \epsilon_t^\omega + \sum_{k=0}^{T-t} \alpha_{t+k} \zeta_t^\omega,
$$

$$
\sum_{k=0}^{T-t} \alpha_{t+k} \left( E_t [\ln(h_{t+k})] - E_{t-1} [\ln(h_{t+k})] \right) = \alpha_t \left( \frac{1}{\gamma + \tau} (\epsilon_t^\nu + (1 - \tau)\epsilon_t^\omega) \right) + \alpha_{t+1} \left( \frac{1}{\gamma + \tau} (\theta^\omega \epsilon_t^\nu + (1 - \tau)\theta^\omega \epsilon_t^\omega) \right) + \sum_{k=0}^{T-t} \alpha_{t+k} \left( \frac{1}{\gamma + \tau} ((1 - \tau)\zeta_t^\nu + \zeta_t^\nu + \eta_t) \right).
$$

Since $\alpha_t$ and $\alpha_{t+1}$ are small compared to human wealth, the first and second terms on the right-hand side of both expressions can be neglected. Thus, the expectation innovation on the right-hand side of the life-time budget constraint is

$$
E_t \left[ \ln \left( \sum_{k=0}^{T-t} \frac{\chi (w_{t+k} h_{t+k})^{1-\tau} + (1 - \tau N) N_{t+k}}{(1 + r)^k} + a_t \right) \right] \approx \ln (P_1 + P_2 + P_3) \quad (28)
$$

$$
+ (1 - \tau) \Omega_t \sum_{k=0}^{T-t} \alpha_{t+k} \left( E_t [\ln(w_{t+k})] - E_{t-1} [\ln(w_{t+k})] \right)
$$

$$
+ (1 - \tau) \Omega_t \sum_{k=0}^{T-t} \alpha_{t+k} \left( E_t [\ln(h_{t+k})] - E_{t-1} [\ln(h_{t+k})] \right).
$$

where the last line follows because $\sum_{k=0}^{T-t} \alpha_{t+k} = 1$.

Equating (24) and (29), we find

$$
E_t \left[ \ln \left( \sum_{k=0}^{T-t} \frac{\chi (w_{t+k} h_{t+k})^{1-\tau} + (1 - \tau N) N_{t+k}}{(1 + r)^k} + a_t \right) \right] \approx \ln (P_1 + P_2 + P_3) \quad (29)
$$
\[ \eta_t = -\phi_t^i(1 - \tau) \left( \frac{\zeta^\omega_t}{\Delta \ln \omega^P_t} + \frac{(1 - \tau)(1 - \tau)\phi_t^j}{\gamma + \tau + (1 - \tau)\phi_t^i} \zeta^\omega_t + \frac{1}{\gamma + (1 - \tau)\phi_t^i} \zeta^\omega_t \right), \tag{30} \]

where \( \phi_t^i = \Theta \Omega_t \).

## C Estimation procedure

**Step 1: Tax Progressivity** — To estimate the tax progressivity parameter \( \tau \) we estimate the following equation by OLS:

\[
\ln(\text{post-government income}_t) = \text{cons} + (1 - \tau) \ln(\text{pre-government income}_t) + e_t, \tag{31}
\]

where both post- and pre-government income are taken from the cross-national equivalence file of the PSID. This approach broadly follows Heathcote et al. (2017).

**Step 2: Frisch elasticity, hours residuals, and wage residuals** — The augmented empirical labor supply equation containing measurement errors is

\[
\Delta \ln \tilde{h}_t = \frac{1}{\gamma + \tau} \left[ -\ln(1 + r_{t-1}) - \ln \rho + (1 - \tau)\Delta \ln \tilde{w}_t - \zeta^\omega \Delta \Xi_t + \eta_t + \Delta \upsilon_t \right] - \frac{1 - \tau}{\gamma + \tau} \Delta \me_{w,t} + \Delta \me_{h,t}. \tag{32}
\]

The error term of equation (32) is correlated with differenced log wages because wage shocks impact the marginal utility of wealth and because of measurement error. To obtain the tax-adjusted Frisch elasticity \((1 - \tau)/(\gamma + \tau)\) from equation (32) we use human capital related instrumental variables for \( \Delta \ln \tilde{w}_t \) following MaCurdy (1981). These instruments predict the expected part of wage growth. Thus, the instruments are uncorrelated with innovations in the marginal utility of wealth and measurement error. Hours residuals \((\eta + \Delta \upsilon_t)/(\gamma + \tau) = (\eta + \Delta \upsilon_t - (1 - \tau)\Delta \me_{w,t})/(\gamma + \tau) + \Delta \me_{h,t} \)

are obtained by running IV on differenced log hours using differentiated year, child, disability and state dummies as covariates. The instruments for the differenced log wage are interactions of age and years of education, i.e., age, education, education^2, age \times education, age \times education^2, age^2 \times education and age^2 \times education^2. Wage residuals \( \Delta \tilde{w}_t = \Delta \omega_t + \Delta \me_{w,t} \) are obtained by estimating equation (4) augmented by an error term, i.e. regressing differenced log wages on the same exogenous regressors as in the hours equation as well as the excluded instruments.

The results of Step 2 are an estimate for \((1 - \tau)/(\gamma + \tau)\), which is easily transformed into \(1/\gamma\), a matrix of hours residuals, and a matrix of wage residuals. These are used in Steps 3 and 4.
Step 3: Wage shocks — After recovering $\Delta \tilde{\omega}_t$, all parameters of the autoregressive process, $(\theta, \sigma^2_{\epsilon,\omega}, \sigma^2_{\zeta,\omega})$, are identified through combinations of the autocovariance moments. Label the $k$-th autocovariance moment by $\Lambda_{\tilde{\omega},k}$:

\[
\Lambda_{\tilde{\omega},0} = E[(\Delta \tilde{\omega}_t)^2] = 2\left(1 - \theta_{\omega} + \theta_{\omega}^2\right)\sigma^2_{\epsilon,\omega} + \sigma^2_{\zeta,\omega} + 2\sigma^2_{me,w} \tag{33}
\]

\[
\Lambda_{\tilde{\omega},1} = E[\Delta \tilde{\omega}_t \Delta \tilde{\omega}_{t-1}] = - (\theta_{\omega} - 1)^2 \sigma^2_{\epsilon,\omega} - \sigma^2_{me,w} \tag{34}
\]

\[
\Lambda_{\tilde{\omega},2} = E[\Delta \tilde{\omega}_t \Delta \tilde{\omega}_{t-2}] = - \theta_{\omega} \sigma^2_{\epsilon,\omega} \tag{35}
\]

Net of $\sigma^2_{me,w}$, dividing $\Lambda_{\tilde{\omega},2}$ by $\Lambda_{\tilde{\omega},1}$ identifies the parameter $\theta_{\omega}$. Successively, the variance of the transitory shock is identified from $\Lambda_{\tilde{\omega},1}$ and the variance of the permanent shock from $\Lambda_{\tilde{\omega},0}$ (see Hryshko 2012).

Step 4: Hours shocks — The residual obtained from estimating the labor supply equation contains both hours shocks $\upsilon_t$ and the expectation error, $\eta_t$. The variance of the residual of the labor supply equation contains both the mean and the variance of $(1 - \tau)\phi^4_{\lambda}t$ and the variance of the permanent hours shocks. Since more parameters need to be identified than in the wage case, additional moments are required for estimation. We use the contemporaneous covariance of hours and wage residuals to identify the mean of $(1 - \gamma + \tau)\phi^4_{\lambda}t$. We write $(1 - \tau)\phi^4_{\lambda}t$ as $1 - \frac{\gamma + \tau}{\gamma + \tau + (1 - \tau)\phi^4_{\lambda}t}$ because the mean and variance of $\frac{\gamma + \tau}{\gamma + \tau + (1 - \tau)\phi^4_{\lambda}t}$ are more easily estimated. To arrive at the theoretical variance moment, substitute equations (8) and (12) into (11) and subsequently (11) into (32) to find the following expression for the hours residual

\[
\eta + \Delta \tilde{\upsilon}_t = \frac{1}{\gamma + \tau} \left[ - \frac{1}{\gamma + \tau + (1 - \tau)\phi^4_{\lambda}t} \xi_t^\upsilon - \frac{1}{\gamma + \tau} \right] \xi_t^\omega \\
- (1 + \gamma) \left(1 - \frac{1}{\gamma + \tau} \right) \left(1 - \frac{1}{\gamma + \tau + (1 - \tau)\phi^4_{\lambda}t} \right) \xi_t^\omega \\
+ \xi_t^\upsilon + \xi_t^\omega + (\theta \upsilon - 1)\xi_{t-1}^\upsilon - \theta \upsilon \xi_{t-2}^\upsilon \\
- \frac{1}{\gamma + \tau} \Delta me_{w,t} + \Delta me_{h,t}, \tag{36}
\]

where the first and second line on the right hand side equal $\eta_t/(\gamma + \tau)$, i.e. the labor supply reaction due to the impact of shocks on the marginal utility of wealth. These terms give the wealth
effect due to the income change caused by permanent shocks to the disutility of work or to the hourly wage, respectively. These income changes include labor supply adjustments. Note that in the case of full insurance ($\phi^1_t = 0$) these terms equal zero. The second line contains immediate, i.e. Frisch, reactions to shocks in the disutility of work. The third line contains the terms due to measurement error. The variance can be written as

$$\Lambda_{\tilde{\omega},0} = E\left[\left(\frac{\eta_1 + \Delta \tilde{\omega}_1}{\gamma + \tau}\right)^2\right] = \frac{1}{(\gamma + \tau)^2} \left(1 + \gamma\right)^2 \left(1 - 2(\gamma + \tau)M_1 + (\gamma + \tau)^2 M_2\right) \sigma^2_{\xi,\omega}$$

$$+ (\gamma + \tau)^2 M_2 \sigma^2_{\xi,u} + \frac{1}{(\gamma + \tau)^2} \left(\sigma^2_{\xi,u} + 2\left(\theta_\nu - \theta_u + 1\right)\sigma^2_{\epsilon,u}\right)$$

$$+ 2\sigma^2_{me,h} + \frac{2(1 - \tau)^2 \sigma^2_{me,w}}{(\gamma + \tau)^2} - \frac{4(1 - \tau) \sigma^2_{me,h,w}}{\gamma + \tau},$$

where $M_1$ and $M_2$ denote the first and second non-central moments of $1/(\gamma + \tau + (1 - \tau)\phi^1_t)$. As no analytical expression for these moments exists, we find them numerically as described in Appendix D. The interpretation is analogous to equation (36): the first term in parentheses captures the part of the variance that is due to marginal utility of wealth effects, while the second term captures the part of the variance due to direct labor supply reactions to hours shocks. The third line is due to measurement error.

The autocovariance moments of the hours residual $\Lambda_{\tilde{\omega},1}$ and $\Lambda_{\tilde{\omega},2}$ are analogous to their wage process counterparts:

$$\Lambda_{\tilde{\omega},1} = E\left[\frac{(\eta_1 + \Delta \tilde{\omega}_1)(\eta_{t-1} + \Delta \tilde{\omega}_{t-1})}{(\gamma + \tau)^2}\right] = \frac{(\theta_\nu - 1)^2 \sigma^2_{\epsilon,u}}{(\gamma + \tau)^2}$$

$$- \sigma^2_{me,h} - (1 - \tau)^2 \frac{\sigma^2_{me,w}}{(\gamma + \tau)^2} + \frac{2(1 - \tau) \sigma^2_{me,h,w}}{(\gamma + \tau)^2},$$

$$\Lambda_{\tilde{\omega},2} = E\left[\frac{(\eta_1 + \Delta \tilde{\omega}_1)(\eta_{t-2} + \Delta \tilde{\omega}_{t-2})}{(\gamma + \tau)^2}\right] = -\frac{\theta_\nu \sigma^2_{\epsilon,u}}{(\gamma + \tau)^2}.$$
This covariance is larger in absolute value the smaller $\gamma + \tau$ and the smaller $M_1$, which is due to a larger $E[\phi_t^4]$. When $\gamma + \tau$ goes to infinity, the effect of permanent wage shocks on income is only mechanical and not through labor supply reactions. It is important to stress that, given estimates for $\gamma + \tau$ and $\sigma^2_{\xi,\omega}$ only equation (40) is needed for the estimation of $E[\phi_t^4]$. The estimation of the parameter does not hinge on the identification of hours shocks.

$M_1$ contains $\mu_\phi$ and $M_2$ contains both $\mu_\phi$ and $\sigma_\phi$, the mean and standard deviation of the natural logarithm of $\phi_t^4$. To estimate the variance of permanent hours shocks, an estimate of $\sigma_\phi$ is needed to make the variance of hours residuals informative. Theoretically, $\sigma_\phi$ is identified through the cokurtosis moments of the wage and hours residuals. However, cokurtosis moments are very noisy, hence $\sigma_\phi$ can only be estimated to a reasonable degree of reliability when using several million observations. Therefore, we calibrate $\sigma_\phi$ to 1.023 based on results in Alan et al. (2018). In that paper, $\phi_t^4$ follows a log-normal distribution and is identified through a separate moment condition based on consumption data. Using this calibration, once $M_1$ is estimated, the mean of $\phi_t^4$, $E[\phi_t^4] = e^{\mu_\phi + 1/2 \sigma_\phi^2}$, can be recovered. In Section 5 we show the robustness of our results to an alternative value of this parameter. Halving the standard deviation $\sigma_\phi$ increases the variance of permanent hours shocks only slightly and the estimate for $E[\phi_t^4]$ is qualitatively the same.

To sum up, the parameters $\theta_{\omega}, \sigma_{e,\omega}, \sigma_{\xi,\omega}$, and $E[\phi_t^4]$ are identified through the moments $\Lambda_{\omega,0}, \Lambda_{\omega,1}, \Lambda_{\omega,2}$, and $\Lambda_{\omega,3,0}$.

**Marshallian elasticity** — The term multiplied with $\sigma_{\xi,\omega}^2$ in equation (40) can be rewritten as

$$E\left[\frac{1-\tau-(1-\tau)\phi_t^4}{y+\tau+(1-\tau)\phi_t^4}\right] = \frac{1-\tau}{y+\tau},$$

the average tax-adjusted Marshallian minus the tax-adjusted Frisch elasticity of labor supply. Thus, the Marshallian can directly be calculated using the parameter estimates. As long as the model is exactly identified, the Marshallian can be calculated directly from the covariance moment using estimates of $\gamma$ and $\tau$. The Marshallian elasticity is the uncompensated reaction to a permanent wage shock.

**Estimation** — We estimate the parameters of the autoregressive processes and the transition of wage shocks by fitting the theoretical moments $\{\Lambda_{\omega,k}, \Lambda_{\nu,k}, \Lambda_{\omega,\nu,k}\}$ to those of the data. The vector of parameters, denoted $\Theta$, is estimated using the method of minimum distance and an identity matrix serves as the weighting matrix. The distance function is given by

$$DF(\Theta) = [m(\Theta) - m^d]'I[m(\Theta) - m^d],$$

16 Simulations evidencing this are available upon request from the authors.
17 See Keane (2011, p.1008).
18 Altonji and Segal (1996) show that the identity weighting matrix is preferable for the estimation of autocovariance structures using micro data.
where \( m(\Theta) \) indicates theoretical moments and \( m \) empirical moments. An outline of the entire estimation procedure is detailed in Hryshko (2012). Standard errors are obtained by block bootstrap with 200 replicates.

D Distribution of the Shock Pass-Through on Hours

The moments of the term \( \frac{(1-\tau)\phi_t^I}{\gamma + \tau + (1-\tau)\phi_t^I} \) are not as tractable as the rest of the random variables in the variance moment estimation, since we assume \( \ln \phi_t^I \sim N(\mu_\phi, \sigma_\phi) \). We can refine the expression to find a more basic expression:

\[
\frac{(1-\tau)\phi_t^I}{\gamma + \tau + (1-\tau)\phi_t^I} = 1 - \frac{\gamma + \tau}{\gamma + \tau + (1-\tau)\phi_t^I}
\]

The only random term in this expression is \( \frac{1}{\gamma + \tau + (1-\tau)\phi_t^I} \). We can find its distribution by re-expressing its CDF in terms of the underlying normal distribution of \( \ln \phi_t^I \). Let

\[
\ln \phi_t^I = Z.
\]

Then

\[
P(Z \leq z) = P \left( \frac{1}{\gamma + \tau + (1-\tau)\phi_t^I} \leq z \right)
\]

\[
P \left( \ln \phi_t^I \leq \ln \left( \frac{1}{z} \frac{(1-\gamma + (1-\tau)}{\tau} \right) \right) = \int_{-\infty}^{\ln \left( \frac{1}{z} \frac{(1-\gamma + (1-\tau)}{\tau} \right)} \exp \left( -\frac{(x-\mu_\phi)^2}{2\sigma_\phi^2} \right) dx
\]

Integrating this CDF, we find the CDF for the random variable \( Z \).

\[
F(z) = 1/2 \left( 1 + \text{Erf} \left( \frac{\ln \left( \frac{1}{z} \frac{(1-\gamma + (1-\tau)}{\tau} \right) - \mu_\phi}{(2\sigma_\phi^2)^{1/2}} \right) \right)
\]

Here \( \text{Erf}(\cdot) \) is the Gaussian error function. To generate the first and second noncentral moments, we take the derivative to find the PDF of \( Z \).

\[
f(z) = \exp \left( -\frac{(\ln \left( \frac{1}{z} \frac{(1-\gamma + (1-\tau)}{\tau} \right) - \mu_\phi)^2}{2\sigma_\phi^2} \right) \sqrt{2\pi \sigma_\phi^2 z(z(\gamma + \tau) - 1)}
\]

The first and second noncentral moments are \( M_1 = \int_0^{1/(\gamma + \tau)} z f(z)dz \) and \( M_2 = \int_0^{1/(\gamma + \tau)} z^2 f(z)dz \). These are calculated via numerical integration, as there is no closed form solution. We implement these formulas in our moment conditions. In estimation we restrict the values of \( \mu_\phi \) not to exceed 10, as the moments of \( \frac{1}{\gamma + \tau + (1-\tau)\phi_t^I} \) asymptote beyond that point.
E Variance of Stochastic Earnings Growth and Earnings Growth Risk

The formula for the variance of stochastic earnings growth is given by

\[
E \left[ \left( \Delta \ln y \right)^2 \right] = \frac{1 - \tau}{(\gamma + \tau)^2} \left[ (\gamma + \tau)^2 M_2 \left( \sigma_{\xi,\nu}^2 + (\gamma + 1)^2 \sigma_{\xi,\omega}^2 \right) \right. \\
+ 2(\gamma + 1)^2((\theta_{\omega} - 1)\theta_{\omega} + 1)\sigma_{\xi,\omega}^2 + 2((\theta_{\nu} - 1)\theta_{\nu} + 1)\sigma_{\xi,\nu}^2 \Big] \\
+ 2(\gamma + 1)^2((\theta_{\nu} - 1)\theta_{\nu} + 1)\sigma_{\xi,\nu}^2 + 2((\theta_{\omega} - 1)\theta_{\omega} + 1)\sigma_{\xi,\omega}^2 \Big],
\]

where \( M_2 \) is obtained numerically using the estimates of the underlying parameters. Note that \( M_2 \) depends on the mean and the variance of the transmission parameter \( \phi_t \), which are known to individuals. Additionally the realization of transitory components of wage and hours growth are partially known in advance, see equation (7). Therefore this overall variance is not a measure of risk. The relevant measure of risk removes the shocks realized before time \( t \).

Denote by \( I_{t-1} \) the agent’s information set at \( t - 1 \). At that point in time the agent knows \( \phi_t \) and the realization of shocks in \( t - 1 \). Thus, \( E \left[ \Delta \ln y_t | I_{t-1} \right] \) includes the transitory components from the previous two periods. The resulting equation for earnings risk conditional on the information set in \( t - 1 \) is

\[
E \left[ \left( \Delta \ln y_t - \frac{E \left[ \Delta \ln y_t | I_{t-1} \right]}{I_{t-1}} \right)^2 \right] = (1 - \tau)^2 \left( \frac{\sigma_{\xi,\nu}^2 + (\gamma + 1)^2 \sigma_{\xi,\omega}^2}{(\gamma + \tau)^2} + \frac{\sigma_{\xi,\nu}^2 + (\gamma + 1)^2 \sigma_{\xi,\omega}^2}{\gamma + \tau + (1 - \tau)\phi_t^2} \right).
\]

Table 9: Tax Progressivity Parameter Estimation

<table>
<thead>
<tr>
<th></th>
<th>ln(post-government income)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(pre-government income)</td>
<td>0.8080 (0.0011)</td>
</tr>
<tr>
<td>constant</td>
<td>1.9616 (0.0127)</td>
</tr>
<tr>
<td>( N )</td>
<td>35504</td>
</tr>
</tbody>
</table>

*Note: Standard errors in parentheses.*
Table 10: Frisch Labor Supply Equation Estimation

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ ln(wage)</td>
<td>0.3614</td>
<td>0.4020</td>
<td>0.2851</td>
<td>0.3148</td>
</tr>
<tr>
<td></td>
<td>(0.0856)</td>
<td>(0.3778)</td>
<td>(0.0975)</td>
<td>(0.1080)</td>
</tr>
<tr>
<td>N</td>
<td>46340</td>
<td>20607</td>
<td>19831</td>
<td>24547</td>
</tr>
</tbody>
</table>

Note: Clustered standard errors in parentheses.

Acknowledgements

We thank Richard Blundell, Flora Budianto, Roland Döhrn, Steffen Elstner, Giulio Fella, Michael Graber, Thorben Korfhage, Jack Light, Maria Metzing, Ian Preston, Itay Saporta-Eksten, Carsten Schröder, Viktor Steiner, Alexandros Theloudis, Guglielmo Weber, seminar participants at the Freie Universität Berlin, the EALE conference in 2018 in Lyon, the ESWM 2018 in Naples, and the IAAE 2019 in Nikosia for valuable comments.

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