Revisiting the Progressive Consumption Tax: a Business Cycle Perspective

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Abstract

This paper revisits the personal expenditure tax (PET), the most prominent version of a progressive consumption tax. The PET has a long intellectual tradition in economics, and the merits and demerits of this alternative to the personal income tax have been discussed at length. What has been missing in the literature so far, however, is a systematic account of its effect on the business cycle. This paper therefore seeks to add to the theoretical literature on the PET and the wider literature on automatic fiscal stabilizers by analyzing the PET’s macroeconomic properties in a modern business cycle model. To this effect, the paper introduces a highly stylized PET into a standard New Keynesian DSGE model, derives a log-linear version of the model, and draws a comparison with the existing income tax. The model simulations show that the two tax systems lead to quite different macroeconomic dynamics. Furthermore, it is found that the PET yields welfare gains, relative to the income tax, for all the demand shocks considered. The PET yields welfare losses, however, under a supply shock.

Keywords: Progressive Taxation, Consumption Taxation, Business Cycles, DSGE Model, Welfare Analysis

JEL classification: E2, E3, E32, E62, E52

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1 Introduction

“... the Equality of Imposition consisteth rather in the Equality of that which is consumed, than of the riches of the persons that consume the same. For what reason is there, that he which laboureth much, and sparing the fruits of his labour, consumeth little, should be more charged, than he that living idly getteth little, and spendeth all he gets: seeing the one hath no more protection from the Common-wealth than the other? But when the Impositions are layd upon those things which men consume, every man payeth Equally for what he useth: Nor is the Common-wealth defrauded by the luxurious waste of private men.”

- Hobbes (1651, p.181)

“Such a tax policy would discourage mansions and encourage factories. When rich men are an offense in the eyes of the relatively poor, it is because of their big domestic establishments and their big spendings, not because of their big savings and big industrial plants. Snobbery goes with the idle and extravagant way of living—with diamonds and retinues of servants; but snobbery is seldom seen in a big factory where the owner himself works. In fact, few workers in democratic America object to the rich man who lives and works like a poor man—who puts his gains into instruments of production, not into instruments of consumption.”

- Fisher and Fisher (1942, p.94)

“It is only by spending, not by earning or saving, that an individual imposes a burden on the rest of the community in attaining his own ends.”

- Kaldor (1955, p.53)

The personal expenditure tax (PET) has a long intellectual tradition in economics. Famous proponents of this, largely untested\(^1\), alternative to the personal income tax have been, amongst others, John Stuart Mill, Alfred Marshall, Arthur Pigou, Irving Fisher, Nicholas Kaldor, and James Meade.\(^2\) The main idea behind the

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\(^1\) According to Goode (1980), the only countries that briefly experimented with a PET are India and Sri Lanka (in the 1970s). More recent experiments are unknown to the author of this article.

\(^2\) See e.g. Mill (1884, Book V, Chapter 1), Marshall (1925), Pigou (1928, Part II, Chapter 10), Fisher (1939, 1942), Fisher and Fisher (1942), Kaldor (1955), and Institute for Fiscal Studies (1978). Before Irving Fisher showed that a PET could be implemented via a relatively simple set of accounting rules, the practicality of such a tax was generally questioned, however. Accordingly, Mill, Marshall, and Pigou were convinced of the theoretical merits of a PET but had doubts about its practical implementation. John Maynard Keynes, in a similar vein, declared before the Committee on National Debt and Taxation (Colwyn Committee) that whereas the tax is “perhaps theoretically sound, it is practically impossible” (quoted in Kaldor, 1955, p.12).
PET, put forward most prominently in Fisher and Fisher (1942) and Kaldor (1955), is quite simple: individuals (or households) report, in a first step, their income to the tax authority, and deduct, in a second step, all (net) savings. The resulting tax base equals personal consumption expenditure, to which, as under most conventional systems of personal income taxation, a set of graduated tax rates is finally applied. The PET, or at least its common formulation with graduated tax rates, is thus a progressive consumption tax. Other less familiar versions of a progressive consumption tax (not covered here for the sake of brevity), e.g. David Bradford’s more recent “X-Tax” (see e.g. Bradford, 1986; Viard and Carroll, 2012), may differ in terms of the details of implementation, but have two key features with the PET in common: firstly, savings (or investments) are, one way or another, exempted from the tax base; and secondly, the tax is imposed (at least in part) on individuals, thus implying that the tax structure can easily be made progressive. The first point clearly differentiates a PET-type system from the existing income tax, the second from existing sales or value-added taxes.

The case for the PET has been made on several grounds. Proponents argue that the PET would allow to retain the basic progressivity of the personal income tax (in contrast to a VAT or sales tax) but be superior to the latter—by virtue of having a consumption tax base—on grounds of equity, economic efficiency, and administrative simplicity.

The equity argument in favor of taxing consumption is straightforward and can be traced back to at least Thomas Hobbes’ Leviathan. Individuals (or households), it is claimed, should not be taxed according to what they contribute to a society’s

\[3\] The PET could be implemented in practice through, e.g., the use of so-called “qualified accounts”. For the sake of brevity, we cannot deal with this important issue here. We refer the reader to U.S. Treasury (1977), Institute for Fiscal Studies (1978), or Graetz (1979) for an extensive discussion of the implementation issues regarding the PET.

\[4\] To be more exact, a consumption tax is according to definition progressive when the average tax rate increases in the amount of consumption. A flat tax rate with only an allowance (e.g. the “Flat Tax” proposed by Hall and Rabushka, 1985) also satisfies this condition. Most formulations of a progressive consumption tax resort to a set of graduated tax rates (in addition to an allowance), however (see e.g. Fisher and Fisher, 1942; Kaldor, 1955; U.S. Treasury, 1977; Institute for Fiscal Studies, 1978).

\[5\] An incomplete list of other contemporary economists that have endorsed some version of a progressive consumption tax includes Kenneth Arrow (2015), Samuel Bowles (Bowles and Park, 2005), The Economist (2010), Martin Feldstein (1978), Robert Frank (2010, 2011, 2008), Kenneth Rogoff (2014, 2016), Laurence Seidman (1997), and John Whalley (Fullerton et al., 1983; Shoven and Whalley, 2005).

\[6\] It should be noted that the (income) tax system of many countries has some overlap with the PET. Pension plans (e.g. individual retirement accounts in the U.S.) often allow tax-deductible contributions and earnings to accumulate tax-free. Taxation only occurs at withdrawal. Tax-free contributions to pensions plans are usually limited in size, however, and early withdrawal is impractical or penalized.

\[7\] The PET and a VAT or sales tax further differ with respect to the incidence of taxation. See e.g. Kaldor (1955, Chapter 1) for an early reference on this point.

\[8\] It is not possible to give a comprehensive review of the literature on the PET, or consumption versus income taxation more generally, in this article. The reader may refer to U.S. Treasury (1977), Institute for Fiscal Studies (1978), or Pechman (1980) for a very thorough comparison between the PET and the income tax.
common pool of goods and services (through supplying labor or capital); instead, they should be taxed according to what they take out of the common pool (through their consumption). In other words, actual spending, and not spending power (i.e. income, or wealth), should be the basis for taxation (Kaldor, 1955, Chapter 1).

On grounds of economic efficiency, Irving Fisher has made a number of early contributions in favor of a consumption tax (Fisher, 1937, 1939, 1942). According to Fisher, taxing the income saved as well as the income from saving under an income tax amounts to “double taxation”, discriminating against saving and discouraging capital accumulation (and therefore also reducing consumption in the long-run). Expressed somewhat differently, income taxes are not neutral with respect to spending and saving, or, what amounts to the same thing, current and future consumption (Kaldor, 1955, Chapter 2). They change the slope of the intertemporal budget constraint by depressing the rate of return to the saver below the rate of return of the underlying investment, thus distorting the intertemporal consumption choice (U.S. Treasury, 1977, Chapter 2; Institute for Fiscal Studies, 1978, Chapter 3). A consumption tax, in contrast, does not give rise to this intertemporal distortion.

Lastly, and even more briefly, income taxes have been criticized on administrative grounds for necessitating complex rules concerning the measurement or imputation of income. A transition to a pure consumption tax would, for instance, allow to abolish tax regulations regarding capital gains, depreciation, and corporate profits.

1979; Pechman, 1980; Fullerton et al., 1983), with the most comprehensive accounts
being the U.S. Treasury’s Blueprints for Basic Tax Reform (1977) and the Institute
recently, there has been renewed interest in the subject. Particularly in the context
of the inequality debate, peaking with the publication of Thomas Piketty’s Capital in
the Twenty-First Century (Piketty and Goldhammer, 2014), some economists have
argued (see e.g. The Economist, 2010; Frank, 2011b; Rogoff, 2014, 2016; Arrow,
2015) that a PET, or some other version of a progressive consumption tax, would
allow to address the growing problem of economic inequality more efficiently, i.e.
with less harmful effects on for instance savings or work incentives, and in a more
targeted way (since we should ultimately care most about consumption inequality)
than measures based on the taxation of income (e.g. a significant increase in top
income tax rates; see Piketty et al., 2011) or wealth (e.g. the introduction of a global
wealth tax; see Piketty and Goldhammer, 2014). With some countries rapidly
moving towards a cashless economy, and others at least entertaining the idea of
abolishing cash, this and other debates around the PET might grow in importance
since one of the major obstacles to the PET’s implementation—tax evasion through
cash hoarding in the transitional period (see e.g. Graetz, 1980; Seidman, 1997)—
would disappear in such an economy.

Against this backdrop and given the growing academic interest on the role of fis-
cal policy in the macroeconomy following the financial crisis of 2007-08, this paper
seeks to add to the existing literature on the merits and demerits of the PET by
shedding light on a so far rather neglected issue: the PET’s effect on the business
cycle. It is by now a well-established result in macroeconomics that the design of
the tax and transfer system affects the cyclical properties of the economy; the liter-
ature on automatic fiscal stabilizers has explored how government policies like e.g.
progressive income taxes or unemployment benefits—policies enacted to promote
redistributive or social goals rather than macroeconomic goals—help mitigate the
impact of shocks on the real economy. To name but two studies that rely on micro-
simulations, Auerbach and Feenberg (2000), for instance, find that the U.S. income
and payroll tax alone offsets roughly 8 percent of a shock to GDP. More recently,
Dolls et al. (2012) find that automatic stabilizers absorb 32% (38%) of a propor-
tional shock to household income and 34% (47%) of an unemployment shock in the
U.S. (EU). There still remains uncertainty about the quantitative significance of

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15The reports of the U.S. Treasury and the UK-based Institute for Fiscal Studies were produced
under the guidance of David Bradford and James Meade, respectively. The UK report recommends
the adoption of a progressive consumption tax (given that transitional problems can be dealt with
satisfactorily), the U.S. report sees the tax as a promising alternative to the income tax.

16Two contributions, Bowles and Park (2005) and Frank (2008), also need to be mentioned
in this regard. Both make the case for a progressive consumption tax on grounds of positional
externalities in the consumption sphere.

17Mattesini and Rossi (2012) and McKay and Reis (2016a,b) are other recent contributions on
automatic stabilizers.
the automatic stabilizers (see e.g. Veld et al., 2013) and the relative importance of the various stabilization channels (see e.g. McKay and Reis, 2016b), but a key take-away of the literature is that the design of the tax and transfer system matters for macroeconomic fluctuations.\textsuperscript{18}

To the best of our knowledge, Kaldor (1955, Chapter 6) is the only scholar that explicitly discusses the role of the PET in a business cycle context. Kaldor argues that discretionary tax changes are a more efficient instrument of macroeconomic control under a PET than under an income tax because the PET allows the policymaker to operate directly on aggregate demand.\textsuperscript{19} Kaldor, yet, does not discuss the built-in stabilization properties of the PET, i.e. its potential role as an automatic stabilizer. Seidman (1997, Chapter 4) solely addresses the possible short-term macroeconomic problems when transitioning to a PET. The two most exhaustive accounts of the PET, U.S. Treasury (1977) and Institute for Fiscal Studies (1978), do not touch on business cycle issues at all.\textsuperscript{20}

In this paper, we contribute to the literature on the PET and the wider literature on automatic fiscal stabilizers by analyzing the PET’s macroeconomic properties in a modern business cycle model. More specifically, we propose a simple way to model a PET and introduce the latter into an otherwise standard, closed-economy New Keynesian DSGE model. Mattesini and Rossi (2012) have shown, using the same baseline model, that a corresponding progressive tax on (wage) income considerably changes the economy’s response to shocks (relative to a flat tax). We investigate, instead, how the PET affects this response. The main aim of the paper is thus to help understand how a move to a different tax system, one that relies on the progressive taxation of consumption expenditure as opposed to income, affects macroeconomic fluctuations, and consequently, economic welfare.

\textsuperscript{18}McKay and Reis (2016b) also provide an excellent review of the literature.

\textsuperscript{19}To quote Kaldor at length: “Thus from the point of view of the efficient conduct and control of the economy it seems pointless to have taxes of any other kind than taxes (or subsidies) on expenditure. Income taxes, or taxes on business savings, are blunt, cumbrous, and ineffective as instruments of control—they operate in a round-about manner with uncertain effect except in those cases (like the taxation of the working classes) where income and expenditure, for lack of a cushion, are closely and rigidly linked so that the tax on the one has much the same influence as the tax on the other. But in all other cases income taxes, whether personal or business taxes, are peculiarly inappropriate as instruments of short-term or ‘anti-cyclical’ fiscal policy simply because their short-run effect on conduct is both less significant and less predictable than their long-run effect. If a change in the tax is introduced which appears to be associated with economic motives (and it would be difficult for a Chancellor to hide his true motives in such eventualities) the taxpayers (whether individuals or businesses) will expect it to be a temporary charge—which is just what it is intended to be—and react to it in much the same way as if it were a capital tax; [...] a purely short term change in income tax may be entirely at the expense of savings.” More recently, Frank (2011, Chapter 5) reasserts this point. He argues that temporary income tax cuts provide not much stimulus in a recession because they tend to be saved by consumers. In contrast, a temporary tax cut under a PET would provide a strong stimulus because consumers can only benefit from the cut by increasing their expenditures immediately.

\textsuperscript{20}Institute for Fiscal Studies (1978, Chapter 1) explicitly states: “We have not examined the special problems of the taxation of oil revenues or of land and development values. We have not investigated the tax problems involved in short term demand management for the macroeconomic control of economic activity. We have no intention of denying the great importance of these topics.”
The key results of the paper are the following: Firstly, we find that the PET, just as the conventional progressive income tax, stabilizes output (relative to a flat tax) and thus acts as an automatic stabilizer for the economy. Yet, and secondly, the PET has a quantitatively different effect on the volatilities of most macroeconomic variables than the progressive income tax. Thirdly, we find that a transition from the existing progressive income tax to the PET would improve economic welfare under government spending, monetary policy, time preference, and taste shocks. Welfare would decline, however, under a technology shock.

The paper is structured as follows. In Section 2, the DSGE model is presented and a PET is introduced (along with a conventional progressive income tax). For ease of illustration, a linearized model version is derived. The model is calibrated in Section 3 and the model dynamics are analyzed using impulse response functions. Section 4 conducts a comparative welfare analysis. Section 5 concludes.

2 The Model

The employed model is a textbook New Keynesian DSGE model of a closed economy (Galí, 2008), augmented by government expenditure and a progressive tax system. The model features several types of shocks commonly considered in the DSGE literature. We compare the economy’s response to these shocks under a progressive tax on consumption (of the PET-type) with that under a conventional progressive tax on (wage) income (Mattesini and Rossi, 2012). The economy is populated by a representative household that maximizes lifetime utility with respect to consumption and hours worked subject to a lifetime budget constraint. There are two types of firms. A perfectly competitive retail firm utilizes the output of intermediate goods firms to assemble a final good, the latter being used for private and government consumption. Intermediate goods firms are many in number, produce a differentiated good using labor only, and set prices in a staggered manner as in Calvo (1983). Monetary policy follows a standard Taylor-type interest rate rule (Taylor, 1993), government expenditure an exogenous process.

\footnote{At the very outset, note that this model does not allow for savings in equilibrium. We will see that it still makes a difference in terms of economic stabilization whether the expenditure side or the revenue side of the household budget is “targeted” by the progressive tax system.}

\footnote{In what follows, letters without a time index \( t \) always represent the (non-stochastic) steady state value of the respective variable.}
2.1 The Household Sector

Expenditure Tax. We first consider the household problem under the PET. Our representative household seeks to maximize lifetime utility given by

\[ E_t \sum_{k=0}^\infty \beta^k e^{\psi_{t+k}} \left\{ e^{\xi_{t+k}} \frac{(C_{t+k})^{1-\sigma}}{1-\sigma} - \frac{(N_{t+k})^{1+\varphi}}{1+\varphi} \right\} \]  

subject to a sequence of flow budget constraints

\[ (1 + \tau^c_t) P_t C_t + B_t = R_{t-1} B_{t-1} + W_t N_t + \Pi_t - T_t \]  

where \( E_t \) is the rational expectation operator, \( C_t \) a consumption bundle (defined below), \( P_t \) the price index for final goods (also defined below), \( N_t \) hours worked, and \( W_t \) the (nominal) wage. Prices and wages are taken as given by the household. \( B_t \) is the amount of a risk-free one-period bond purchased at the beginning of period \( t \), \( R_t \) is the corresponding (gross nominal) interest rate. \( \Pi_t \) are the profits of the intermediate goods sector, transferred to the owner household in the form of dividends. The coefficients \( \sigma \) and \( \varphi \) determine the degree of relative risk aversion and labor disutility (inverse of the Frisch labor supply elasticity), respectively. \( \beta \) is the subjective discount factor, \( \psi_t \) a time preference shock, \( \xi_t \) a taste shock. Finally, the household faces the tax \( \tau^c_t \) on personal consumption expenditure as well as lump-sum taxes \( T_t \) (which are zero on average; see below).

Our modeling strategy for the personal expenditure tax \( \tau^c_t \) follows Guo and Lansing (1998) and Mattesini and Rossi (2012).23 We assume that the tax schedule \( \tau^c_t \) has the form

\[ \tau^c_t = \eta_c \left( \frac{C_t}{C} \right) \phi_c - 1 \]  

where \( C \) is steady state consumption and the reference value for taxation, and where \( \eta_c > 1 \) pins down the level of the consumption tax schedule (the average tax rate), while \( \phi_c \geq 0 \) determines the progressivity of the consumption tax schedule.24

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23Guo and Lansing (1998) and Mattesini and Rossi (2012) also use a representative agent model. The former consider a progressive tax on capital and labor income in a Real Business Cycle model, the latter a progressive tax on labor income in a standard New Keynesian model.

24Notice that an income tax usually applies the relevant tax rate to a tax base that includes the tax payment itself, whereas consumption taxes usually apply the tax rate to a base that excludes the tax payment. Income tax rates are thus stated in what is called a tax-inclusive form, consumption taxes in a tax-exclusive form. To not confuse the reader, we follow the convention and also quote the personal expenditure tax in tax-exclusive form. The drawback is that the average tax rate and the progression coefficient have to be chosen and interpreted with care in order to make a valid comparison with the progressive income tax (e.g. holding the tax burden constant, tax-exclusive rates \( \tau^{cx} \) appear higher than tax-inclusive rates: \( \tau^{in} < \tau^{cx} = \frac{\tau^{in}}{1-\tau^{in}} \)). The results of this paper, however, are not affected by the modeling strategy. To be more concrete, we also checked a tax-inclusive schedule \( \tau^c_t = 1 - \eta_c \left( \frac{C^{in}_t}{C^{in}} \right) \phi_c \) with \( \eta_c \in (0,1], \phi_c \geq 0 \), and where \( C^{in} \) corresponds to before-tax or tax-inclusive consumption. This is the schedule employed below for the progressive
To understand the tax schedule, first assume that \( \phi_c = 0 \) holds. In this case, the tax rate on personal consumption expenditure \( \tau_c = \eta_c - 1 \) is constant and we speak of a “flat” consumption tax.\(^{25}\) In contrast, when \( \phi_c > 0 \) holds, the average tax rate \( \tau_c \) will be above (below) the steady state tax rate \( \tau_c \) whenever the tax base \( C_t \) is above (below) the reference value \( C \), with larger deviations leading to larger rate adjustments. In this case, it is appropriate to speak of a “progressive” consumption tax (this is the typical version of the PET).

To see this last point more formally, notice that the following relationship between the marginal tax rate \( \tau_{c,m} = \frac{\partial (\tau_c C_t)}{\partial C_t} \) and the average tax rate \( \tau_c \) holds:

\[
\tau_{c,m} = \tau_c + \eta_c \phi_c \left( \frac{C_t}{C} \right)^{\phi_c}.
\]

Accordingly, whenever \( \phi_c > 0 \), the marginal tax rate is higher than the average tax rate, or, what amounts to the same thing, the average tax rate increases in the tax base.

Under this setup, the representative household’s first order conditions are then given by

\[
1 = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma - \phi_c} \frac{P_t}{P_{t+1}} R_t e^{\psi_{t+1} \xi_{t+1}} \right\}
\]

\[
e^{\xi_t} \frac{W_t}{P_t} C_t^{-\sigma - \phi_c} = N_t \eta_c C^{-\phi_c} (1 + \phi_c)
\]

where the first condition is a consumption Euler equation and where the second condition determines the household’s labor supply. Apparently, the progressive consumption tax has a similar effect on the household’s intertemporal consumption choice as an increase in the concavity of the household’s consumption utility function (an increase in the coefficient \( \sigma \)). That is, all other things equal, the household income tax and the one also used by Guo and Lansing (1998) and Mattesini and Rossi (2012). In this case, the household’s budget would read (1 - \( \tau_c \))\( P_t C_t + B_t = R_{t-1} B_{t-1} + W_t N_t + \Pi_t - T_t \), or equivalently, \( \left( \frac{1}{\eta_c (C_{t}^{ex})^{\phi_c}} \right)^{\frac{1}{\phi_c}} P_t + B_t = R_{t-1} B_{t-1} + W_t N_t + \Pi_t - T_t \) where the first term in brackets on the left-hand side is equal to \( C_t^{in} \) and where \( C_t^{ex} \) is after-tax or tax-exclusive consumption (the consumption entering the utility function). This modeling strategy seems not very intuitive and requires to keep track of both \( C_t^{ex} \) and \( C_t^{in} \). Most importantly, the exact same results can be replicated with the tax-exclusive schedule (3) when adjusting the coefficients \( \eta_c \) and \( \phi_c \) properly. See also Section 3.1 for more details on this issue.

\(^{25}\) As a side note, this case is identical to the conventional approach to model value-added taxes in the DSGE literature. That is, the literature assumes, unrealistically, that the VAT liability is transferred to the government by the consumer (i.e. a flat PET is actually assumed). In a business cycle context, this assumption seems innocuous as long as the (alleged) VAT rate remains unchanged. Voigt (2017) convincingly argues, however, that this modeling approach leads to erroneous conclusions about the macroeconomic effects of discretionary changes in the tax rate because instantaneous pass-through to consumers is implicitly assumed, contradicting a wealth of empirical evidence and being inconsistent with the sticky-price assumption in DSGE models.
seeks a smoother consumption path over time. The tax’s effect on labor supply is less apparent but also resembles that of an increase in $\sigma$. This will become more obvious when we look at a linearized version of equation (6) later.

In summary, with the PET we have introduced a countercyclical device (at least insofar as consumption and output move together) on the expenditure side of the household’s budget. Unsurprisingly, we will see below that this device reduces output fluctuations in general equilibrium, i.e., it acts as an automatic fiscal stabilizer for the economy.

**Income Tax.** We next consider the household problem under the income tax. The representative household maximizes lifetime utility (1) subject to a sequence of flow budget constraints

$$P_tC_t + B_t = (R_{t-1} - \tau^{int}(R_{t-1} - 1))B_{t-1} + W_t N_t(1 - \tau^n_t) + \Pi_t(1 - \tau^{div}) - T_t$$

(7)

where $\tau^{int}$ and $\tau^{div}$ are “flat” tax rates on interest income and dividend income, respectively, and where $\tau^n_t$ is a wage tax schedule given by (see Mattesini and Rossi, 2012)

$$\tau^n_t = 1 - \eta_n \left( \frac{Y_n}{Y_{n,t}} \right)^{\phi_n},$$

(8)

with $Y_{n,t} \equiv \frac{W_t N_t}{P_t}$ denoting current period real wage income, and with the corresponding steady state value $Y_n \equiv \frac{W N}{P}$ serving as the reference value for taxation.\(^{26}\) The coefficient $\eta_n \in (0, 1]$ determines the level of the tax schedule (the average tax rate), the coefficient $\phi_n \in [0, 1)$ its progressivity.\(^{27}\)

It is again straightforward to show that the following relationship between the marginal tax rate (on wage income) $\tau^{n,m}_t = \frac{\partial (\tau^n_t Y_{n,t})}{\partial Y_{n,t}}$ and the average tax rate $\tau^n_t$ holds:

$$\tau^{n,m}_t = \tau^n_t + \eta_n \phi_n \left( \frac{Y_n}{Y_{n,t}} \right)^{\phi_n}.$$

(9)

We thus speak of a “progressive” ("flat") wage tax schedule when $\phi_n > 0$ ($\phi_n = 0$) holds.

Under this tax regime, the household’s consumption Euler equation and the

\(^{26}\)Note that in contrast to most of the DSGE literature, and to obtain a maximum distinction between a tax on consumption expenditure only and an income tax, we allow for a “comprehensive” version of the latter and thus also consider a tax on household interest income. The tax can of course (and will) be “switched off” later to draw a proper comparison between the PET introduced above and the relevant existing literature on the progressive income tax (Mattesini and Rossi, 2012).

\(^{27}\)The mechanics underlying this tax schedule correspond to those of the consumption tax schedule introduced above. The wage tax is quoted in tax-inclusive form, however.
optimality condition for its labor supply, in turn, are given by

\[
1 = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} (R_t - \tau^{int}(R_t - 1)) e^{\Delta \psi_{t+1}} e^{\Delta \xi_{t+1}} \right\} 
\]

(10)

\[
e^{\xi_t} \left( \frac{W_t}{P_t} \right)^{1-\phi_n} C_t^{-\sigma} \eta_n (1 - \phi_n) \left( \frac{WN}{P} \right)^{\phi_n} = N_t^{\phi_n}. 
\]

(11)

The consumption Euler equation is standard, except for the fact that the tax on interest income \( \tau^{int} \) depresses the household’s rate of return on saving. Regarding labor supply, note that as the progressivity of the wage tax schedule increases, the quantity of hours worked becomes less responsive to a change in the real wage (holding consumption constant), or, to put it another way, the labor supply curve becomes steeper. That is, to induce a given increase in hours worked, a larger increase in the real wage is necessary (when the tax system is progressive) since a growing fraction of the latter is taxed away.

We will see below that through this supply-side effect, the progressive (wage) income tax reduces output fluctuations and thus acts as an automatic fiscal stabilizer (we refer to Mattesini and Rossi (2012) for a detailed account on this point).\(^{28}\)

2.2 The Government

2.2.1 Fiscal Policy

Depending on the tax regime in place, the fiscal authority finances an exogenous stream of government consumption \( G_t \) through either a tax on household consumption expenditure or household income.\(^{29}\) Across regimes, the government imposes a lump-sum tax \( T_t \) (which is zero on average) to balance the budget in each period.\(^{30}\)

**Expenditure Tax.** Under the PET, the period budget constraint of the government is given by

\[
P_t G_t = \tau_c^c P_tC_t + T_t. 
\]

(12)

**Income Tax.** Under the income tax, in contrast, the period budget constraint of the government is given by

\[
P_t G_t = \tau_n^nh_t N_t + \tau^{div} \Pi_t + \tau^{int}(R_t - 1) B_{t-1} + T_t. 
\]

(13)

\(^{28}\)Auerbach and Feenberg (2000) also stress this supply-side stabilization effect of the progressive income tax system: In the presence of labor demand fluctuations, a steeper labor supply curve reduces fluctuations in employment and ceteris paribus also output.

\(^{29}\)As will be clear below, \( G_t \) is defined analogously to the private consumption bundle \( C_t \).

\(^{30}\)Allowing for government debt would not change our results since Ricardian equivalence holds in the model economy.
2.2.2 Monetary Policy

Monetary policy follows a standard Taylor-type interest rate rule (Taylor, 1993). The rule targets price inflation only and is given by

\[ R_t = R \left( \frac{P_t}{P_{t-1}} \right)^{\phi_{\pi}} e^{\nu_t} \]  

(14)

where \( R \) is the steady state interest rate, \( \phi_{\pi} > 1 \) the Taylor inflation coefficient, and where \( v_t \) is a monetary policy shock.\(^{31}\)

2.3 The Firm Sector

2.3.1 Final Goods Producer

The representative, perfectly competitive final goods producer assembles the final good \( Y_t \) according to the following constant returns to scale technology

\[ Y_t = \left( \int_0^1 X_t(i)^{1 - \frac{1}{\epsilon_p}} di \right)^{-\epsilon_p} \]  

(15)

where \( X_t(i) \) is the amount of intermediate good \( i \), with \( i \in [0, 1] \), and where \( \epsilon_p \) is the elasticity of substitution (between intermediate goods). The firm takes the prices of the intermediate goods \( P_t(i) \) as well as the price of the final good \( P_t \) as given. Profit maximization results in standard demand functions for the intermediate goods \( i \)

\[ X_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon_p} Y_t \]  

(16)

with the price of the final good \( P_t \) given by

\[ P_t = \left( \int_0^1 P_t(i)^{1 - \epsilon_p} di \right)^{\frac{1}{1 - \epsilon_p}}. \]  

(17)

2.3.2 Intermediate Goods Producers

There is a continuum of monopolistically competitive intermediate goods firms, indexed by \( i \in [0, 1] \). Firm \( i \) produces differentiated good \( Y_t(i) \) according to

\[ Y_t(i) = A_t N_t(i) \]  

(18)

where \( N_t(i) \) is the amount of labor employed by firm \( i \) and \( A_t \) the (stochastic) level of technology common to all firms. The production function implies that real marginal

\(^{31}\)In the common case of \( \tau^{int} = 0 \), the steady state interest rate is given by \( R = \beta^{-1} \). Otherwise, we have \( R = \frac{1 - \beta^{\tau^{int}}}{\beta(1 - \beta^{\tau^{int}})} \). Note that these results follow from the Euler equations.
costs \( MC_t \) are equalized across firms, i.e.

\[
MC_t(i) = MC_t = \frac{W_t}{P_t} A_t^{-1}. \tag{19}
\]

We assume that intermediate goods firms set prices in a staggered fashion as in Calvo (1983). Each period \( t \), a randomly drawn fraction of firms \( 1 - \theta_p \), for some \( 0 < \theta_p < 1 \), is able to reset their prices, whereas the remaining fraction of firms \( \theta_p \) is not able to do so. Resetting firms take the demand functions for their good (16) as given. Their first-order condition with respect to the newly set price \( P_t^o \) is standard and given by

\[
\sum_{k=0}^{\infty} \theta_p^k E_t \left\{ Q_{t,t+k} \left( \frac{P_t^o}{P_{t+k}} \right)^{-\epsilon-1} Y_{t+k} \left[ \frac{P_t^o}{P_{t+k}} - \frac{\epsilon}{\epsilon - 1} MC_{t+k} \right] \right\} = 0 \tag{20}
\]

where \( Q_{t,t+k} \) is the household’s stochastic discount factor.\(^{32}\)

### 2.4 Exogenous Processes

We have five exogenous variables in our model: a productivity shock \( A_t \), government spending \( G_t \), a monetary policy shock \( v_t \), a time preference shock \( \psi_t \), and a taste shock \( \xi_t \). Let us define \( a_t = \ln(A_t) \) and \( \hat{g}_t \equiv \ln \left( \frac{G_t}{Q_t} \right) \). As is standard in the literature, we assume stationary AR(1) processes for all shocks, i.e.

\[
a_t = \rho_a a_{t-1} + \epsilon_{a,t} \tag{21}
\]

\[
\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t} \tag{22}
\]

\[
v_t = \rho_v v_{t-1} + \epsilon_{v,t} \tag{23}
\]

\[
\psi_t = \rho_\psi \psi_{t-1} + \epsilon_{\psi,t} \tag{24}
\]

\[
\xi_t = \rho_\xi \xi_{t-1} + \epsilon_{\xi,t} \tag{25}
\]

with \( 0 < \rho < 1 \) and innovation \( \epsilon \) drawn from a standard normal distribution.

### 2.5 Market Clearing and Aggregation

In a representative agent model such as the one at hand, bond market clearing implies \( B_t = 0 \) for all periods \( t \).

The labor market is in equilibrium when household labor supply equals aggregate

\[^{32}\text{Because firms are owned by households they also use the same discount factor as households. Under the PET, the stochastic discount factor is given by } Q_{t,t+k} = \beta E_t \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma - \phi_c} \frac{P_{t+k}}{P_{t}} e^{\Delta \psi_{t+1} + \Delta \xi_{t+1}}. \text{ Under the income tax, we have } Q_{t,t+k} = \beta E_t \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_{t+k}}{P_{t}} e^{\Delta \psi_{t+1} + \Delta \xi_{t+1}} \text{ instead.} \]
labor demand by (intermediate goods) firms, i.e.

\[ N_t = \int_0^1 N_t(i) \, di. \]  

(26)

The intermediate goods market is in equilibrium when supply equals demand for all intermediate goods \( i \in [0, 1] \), i.e.

\[ Y_t(i) = X_t(i). \]  

(27)

In turn, the final goods market is in equilibrium when aggregate supply or real GDP equals the sum of private and government consumption demand, i.e.

\[ Y_t = C_t + G_t. \]  

(28)

Following Schmitt-Grohé and Uribe (2006), the aggregate production function of our economy is given by

\[ Y_t = s_t^{-1} A_t N_t \]  

(29)

where \( s_t \geq 1 \) is determined by the difference equation

\[ s_t = (1 - \theta_p)(\tilde{p}_t)^{-\epsilon_p} + \theta_p(1 + \pi_t)^{\epsilon_p} s_{t-1} \]  

(30)

with \( \tilde{p}_t \equiv \frac{P_o}{P_t} \) and where \( \pi_t \) denotes (final goods) price inflation. The variable \( s_t \) represents the resource cost from inefficient price dispersion across intermediate goods firms when the value exceeds one.\(^{33}\)

Finally, in the Calvo pricing model, the evolution of aggregate or final goods prices is given by the law of motion

\[ 1 = \theta_p(1 + \pi_t)^{-1+\epsilon_p} + (1 - \theta_p)\tilde{p}_t^{1-\epsilon_p}. \]  

(31)

### 2.6 Steady State

In the next section, we will employ a (log-)linear approximation of the model around the (non-stochastic) steady state. It will thus be useful to briefly characterize this steady state.

We first assume that aggregate price inflation is zero in the steady state. To find aggregate output or activity next, we combine the household’s labor supply first order condition with the steady state relations \( \frac{W}{P} = \frac{s_t^{-1}}{\epsilon_p} \) (from (19) and (20)) and \( Y = N \) and make use of the household’s budget constraint.

\(^{33}\)Since there is no price dispersion under flexible prices, \( s_t = 1 \) holds for all \( t \) in this case.
Expenditure Tax. Accordingly, under the consumption tax, steady state output is given by

\[ Y = \left( \frac{\epsilon_p - 1}{\epsilon_p} \left( \frac{1}{\eta_c} \right)^{1-\sigma} \frac{1}{1 + \phi_c} \right)^{\frac{1}{1-\sigma}}. \] (32)

Income Tax. In comparison, under the income tax, steady state output is given by\(^{34}\)

\[ Y = \left( \frac{\epsilon_p - 1}{\epsilon_p} \eta_n^{1-\sigma} (1 - \phi_n) \right)^{\frac{1}{1-\sigma}}. \] (33)

We see that the steady state output depends on both the average level of taxation (\(\eta\)) and the degree of tax progressivity (\(\phi\)). Our model calibration below will ensure that the steady state level of output is the same for both tax systems (i.e. the incentives to supply labor are equalized in the steady state).

2.7 Linearization

To make the model more tractable, we now employ a (log-)linear approximation of the model equations around the (non-stochastic) steady state. This also allows us to condense the model into three familiar equations: a Phillips curve, an IS curve, and a monetary policy rule. In the following, a small variable with a hat denotes the log-deviation of the respective variable from its steady state value, i.e.

\[ \hat{z}_t = \ln \left( \frac{Z_t}{Z}\right) - \ln \left( \frac{Z_t}{Z}\right) = \ln \left( 1 + \frac{Z_t - Z}{Z}\right) \approx \frac{Z_t - Z}{Z}, \]

where the last approximation holds for “small” percentage deviations of \(Z_t\) from \(Z\). The subsequent account will be rather brief but we will summarize our main findings at the end of this section.

2.7.1 The Phillips Curve

Expenditure Tax. After linearizing the price setting first order condition (20) and the law of motion of the aggregate price index (31), we combine the resulting equations to obtain the following standard forward-looking inflation equation

\[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \lambda \hat{m}_t \] (34)

where \(\lambda \equiv \frac{(1-\theta_p)(1-\beta \theta_p)}{\theta_p}. \)\(^{35}\)

We next derive linear expressions for the labor supply first-order condition (6), marginal cost (19), the aggregate production function (29), and the definition of real

\(^{34}\)To get this expression, we assumed a uniform tax rate for household labor and dividend income.

\(^{35}\)See e.g. Gali (2008, Chapter 3). Further note that the time preference shock \(\psi_t\) and the taste shock \(\xi_t\) have no first-order effect on the relationship between inflation and marginal cost.
GDP (28), respectively:

\[ \hat{\omega}_t + \xi_t = (\sigma + \phi_c)\hat{c}_t + \varphi \hat{n}_t \]  
\[ \hat{mc}_t = \hat{\omega}_t - a_t \]  
\[ \hat{y}_t = a_t + \hat{n}_t \]  
\[ \hat{y}_t = \gamma_c\hat{c}_t + (1 - \gamma_c)\hat{g}_t \]  

with \( \gamma_c \equiv \frac{C}{\hat{r}} \) and where \( \hat{\omega}_t \equiv \hat{\omega}_t - \hat{p}_t \). As indicated above, it becomes obvious from equation (35) that the PET has a similar effect on the labor supply decision as an increase in the coefficient \( \sigma \). We will refer to the resulting general equilibrium effects below.

Combining the previous equations allows us to express marginal cost in terms of aggregate output and the exogenous processes \( a_t, \hat{g}_t, \) and \( \xi_t \):

\[ \hat{mc}_t = \frac{\sigma + \varphi \gamma_c + \phi_c \gamma_c}{\gamma_c} \hat{y}_t - (1 + \varphi) a_t - \frac{(\sigma + \phi_c)(1 - \gamma_c)}{\gamma_c} \hat{g}_t - \xi_t. \]  

(39)

Since \( \hat{mc}_t = 0 \) holds under flexible prices (i.e. the price markup is constant), we also have

\[ 0 = \frac{\sigma + \varphi \gamma_c + \phi_c \gamma_c}{\gamma_c} \hat{y}_f - (1 + \varphi) a_t - \frac{(\sigma + \phi_c)(1 - \gamma_c)}{\gamma_c} \hat{g}_t - \xi_t \]  

(40)

where \( \hat{y}_f \) denotes the flexible price or “natural” output. Subtracting (40) from (39) then yields

\[ \hat{mc}_t = \frac{\sigma + \varphi \gamma_c + \phi_c \gamma_c}{\gamma_c} (\hat{y}_t - \hat{y}_f) \]  

(41)

where the flexible price output is given by

\[ \hat{y}_f = \frac{(1 + \varphi) \gamma_c}{\sigma + \varphi \gamma_c + \phi_c} a_t + \frac{(\sigma + \phi_c)(1 - \gamma_c)}{\sigma + \varphi \gamma_c + \phi_c} \hat{g}_t + \frac{\gamma_c}{\sigma + \varphi \gamma_c + \phi_c} \xi_t. \]  

(42)

Finally, by substituting (41) into (34), we obtain the New Keynesian Phillips curve under the PET

\[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa_c \tilde{y}_t \]  

(43)

where the slope of the Phillips curve is given by

\[ \kappa_c \equiv \lambda \frac{\sigma + \varphi \gamma_c + \phi_c}{\gamma_c} \]  

(44)

and where \( \tilde{y}_t \equiv \hat{y}_t - \hat{y}_f \) is the output gap.
**Income Tax.** The linearized version of the labor supply first-order condition (11) is given by

\[(1 - \phi_n)\hat{\omega}t + \xi t = \sigma\hat{c}t + (\varphi + \phi_n)\hat{n}t.\] (45)

Using (45) instead of (35) and repeating the steps taken above, we obtain the following New Keynesian Phillips curve under the progressive income tax (see Matthesini and Rossi, 2012)

\[\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa_n \tilde{y}_t\] (46)

where the slope of the Phillips curve is given by

\[\kappa_n \equiv \lambda \frac{\sigma + \gamma_c (\varphi + \phi_n)}{\gamma_c (1 - \phi_n)}.\] (47)

The variable \(\tilde{y}_t \equiv \hat{y}_t - \hat{y}^f_t\) represents the output gap under the income tax, and the corresponding natural output is given by

\[\hat{y}^f_t = \frac{(1 + \varphi)\gamma_c}{\sigma + (\varphi + \phi_n)\gamma_c} a_t + \frac{\sigma (1 - \gamma_c)}{\sigma + (\varphi + \phi_n)\gamma_c} \hat{y}_t + \frac{\gamma_c}{\sigma + (\varphi + \phi_n)\gamma_c} \xi t.\] (48)

### 2.7.2 The IS Curve

**Expenditure Tax.** Linearizing the consumption Euler equation (5) gives

\[\hat{c}_t = E_t \{\hat{c}_{t+1}\} - \frac{1}{\sigma + \phi_c} (\hat{r}_t - E_t \{\pi_{t+1}\} + E_t \{\Delta \psi_{t+1} + \Delta \xi_{t+1}\}).\] (49)

Combining the last equation with the linear expression for real GDP (38) yields the model’s IS curve, expressed in terms of aggregate output, under the PET:

\[\hat{y}_t = E_t \{\hat{y}_{t+1}\} - (1 - \gamma_c) E_t \{\Delta \hat{y}_{t+1}\} - \frac{\gamma_c}{\sigma + \phi_c} (\hat{r}_t - E_t \{\pi_{t+1}\} + E_t \{\Delta \psi_{t+1} + \Delta \xi_{t+1}\}) .\] (50)

Expressed in terms of the output gap, the IS curve reads

\[\tilde{y}_t = E_t \{\tilde{y}_{t+1}\} - \frac{\gamma_c}{\sigma + \phi_c} \left(\hat{r}_t - E_t \{\pi_{t+1}\} - \hat{r}^f_t\right)\] (51)

where \(\hat{r}^f_t \equiv \hat{r}_t - E_t \{\pi^f_{t+1}\}\) is the real interest rate under flexible prices, often
denoted as the “natural” real rate, and given by

\[ \hat{r}_t^f = \frac{(1 + \varphi)(\sigma + \phi_c)}{\sigma + \varphi \gamma_c + \phi_c} E_t \{ \Delta a_{t+1} \} - \frac{(\sigma + \phi_c)(1 - \gamma_c)\varphi}{\sigma + \varphi \gamma_c + \phi_c} E_t \{ \Delta \hat{y}_{t+1} \} \]

\[ - \frac{\varphi \gamma_c}{\sigma + \varphi \gamma_c + \phi_c} E_t \{ \Delta \xi_{t+1} \} - E_t \{ \Delta \psi_{t+1} \}. \]  

(52)

**Income Tax.** Linearizing the consumption Euler equation (10) yields

\[ \hat{c}_t = E_t \{ \hat{c}_{t+1} \} - \frac{1}{\sigma} ((1 - \beta \tau^{int})\hat{r}_t - E_t \{ \pi_{t+1} \} + E_t \{ \Delta \psi_{t+1} + \Delta \xi_{t+1} \}). \]  

(53)

The IS curve, expressed in terms of aggregate output, is then given by

\[ \hat{y}_t = E_t \{ \hat{y}_{t+1} \} - (1 - \gamma_c)E_t \{ \Delta \hat{g}_{t+1} \} - \frac{\gamma_c}{\sigma} ((1 - \beta \tau^{int})\hat{r}_t - E_t \{ \pi_{t+1} \} \]

\[ + E_t \{ \Delta \psi_{t+1} + \Delta \xi_{t+1} \}). \]  

(54)

Expressed in terms of the output gap, the IS curve reads

\[ \tilde{y}_t = E_t \{ \tilde{y}_{t+1} \} - \frac{\gamma_c}{\sigma} ((1 - \beta \tau^{int})\tilde{r}_t - E_t \{ \pi_{t+1} \} - \tilde{r}_t^f) \]  

(55)

where \( \tilde{r}_t^f \equiv (1 - \beta \tau^{int})\hat{r}_t^f - E_t \{ \pi_{t+1}^f \} \) is the (after-tax) real interest rate under flexible prices and given by

\[ \tilde{r}_t^f = \frac{(1 + \varphi)\sigma}{\sigma + (\varphi + \phi_n)\gamma_c} E_t \{ \Delta a_{t+1} \} - \frac{\sigma(1 - \gamma_c)(\varphi + \phi_n)}{\sigma + (\varphi + \phi_n)\gamma_c} E_t \{ \Delta \hat{g}_{t+1} \} \]

\[ - \frac{(\varphi + \phi_n)\gamma_c}{\sigma + (\varphi + \phi_n)\gamma_c} E_t \{ \Delta \xi_{t+1} \} - E_t \{ \Delta \psi_{t+1} \}. \]  

(56)

**2.7.3 Government Policy**

Finally, to close the model, the linearized version of the interest rate rule (14) is given by

\[ \hat{r}_t = \phi_{\pi} \pi_t + v_t. \]  

(57)

**Expenditure Tax.** For the sake of completeness, the linearized version of the consumption tax schedule (3) is

\[ \hat{\tau}_t^c = \left( \frac{\eta_c}{\eta_c - 1} \right) \phi_{\pi} \hat{c}_t. \]  

(58)

---

36To obtain the natural real interest rate, insert the equation for natural output (42) into the IS curve (50) and solve for the real interest rate.
Similarly, the linearized version of the wage tax schedule (8) is

\[ \hat{\tau}_t^n = \left( \frac{\eta_n}{1 - \eta_n} \right) \phi_n(\hat{\omega}_t + \hat{n}_t). \]  

(59)
Table 1: Summary: Linearized Model

<table>
<thead>
<tr>
<th>Equation Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler equation [ET]</td>
<td>( \ddot{c}<em>t = E_t { \ddot{c}</em>{t+1} } - \frac{1}{\sigma + \phi_c} (\ddot{r}<em>t - E_t { \pi</em>{t+1} } + E_t { \Delta \psi_{t+1} + \Delta \xi_{t+1} }) )</td>
</tr>
<tr>
<td>Euler equation [IT]</td>
<td>( \ddot{c}<em>t = E_t { \ddot{c}</em>{t+1} } - \frac{1}{\sigma} ((1 - \beta \tau^{int}) \ddot{r}<em>t - E_t { \pi</em>{t+1} } + E_t { \Delta \psi_{t+1} + \Delta \xi_{t+1} }) )</td>
</tr>
<tr>
<td>IS curve [ET]</td>
<td>( \ddot{y}<em>t = E_t { \ddot{y}</em>{t+1} } - \gamma_c E_t { \Delta \ddot{y}<em>{t+1} } - \frac{\gamma_c}{\sigma + \phi_c} (\ddot{r}<em>t - E_t { \pi</em>{t+1} } + E_t { \Delta \psi</em>{t+1} + \Delta \xi_{t+1} }) )</td>
</tr>
<tr>
<td>IS curve [IT]</td>
<td>( \ddot{y}<em>t = E_t { \ddot{y}</em>{t+1} } - (1 - \gamma_c) E_t { \Delta \ddot{y}<em>{t+1} } - \frac{\gamma_c}{\sigma} ((1 - \beta \tau^{int}) \ddot{r}<em>t - E_t { \pi</em>{t+1} } + E_t { \Delta \psi</em>{t+1} + \Delta \xi_{t+1} }) )</td>
</tr>
<tr>
<td>Phillips curve [ET]</td>
<td>( \pi_t = \beta E_t { \pi_{t+1} } + \lambda \frac{\sigma + \phi_c}{\gamma \gamma_c} (\ddot{g}_t - \ddot{g}_t^f) )</td>
</tr>
<tr>
<td>Phillips curve [IT]</td>
<td>( \pi_t = \beta E_t { \pi_{t+1} } + \lambda \frac{\sigma + \phi_c}{\gamma \gamma_c} (\ddot{g}_t - \ddot{g}_t^f) )</td>
</tr>
<tr>
<td>Natural output [ET]</td>
<td>( \ddot{y}_t^f = \frac{(1 + \gamma_c) \gamma_c - \phi_c}{\sigma + \phi_c} \alpha_t + \frac{\phi_c (1 - \gamma_c)}{\sigma + \phi_c} \ddot{g}_t + \frac{\gamma_c}{\sigma + \phi_c} \xi_t )</td>
</tr>
<tr>
<td>Natural output [IT]</td>
<td>( \ddot{y}_t^f = \frac{(1 + \gamma_c) \gamma_c - \phi_c}{\sigma + \phi_c} \alpha_t + \frac{\phi_c (1 - \gamma_c)}{\sigma + \phi_c} \ddot{g}_t + \frac{\gamma_c}{\sigma + \phi_c} \xi_t )</td>
</tr>
<tr>
<td>Natural rate [ET]</td>
<td>( \ddot{r}<em>t^f = \frac{(1 + \gamma_c) \gamma_c - \phi_c}{\sigma + \phi_c} E_t { \Delta a</em>{t+1} } - \frac{\phi_c (1 - \gamma_c) \gamma_c}{\sigma + \phi_c} E_t { \Delta \ddot{y}<em>{t+1} } - \frac{\gamma_c}{\sigma + \phi_c} E_t { \Delta \xi</em>{t+1} } - E_t { \Delta \psi_{t+1} } )</td>
</tr>
<tr>
<td>Natural rate [IT]</td>
<td>( \ddot{r}<em>t^f = \frac{(1 + \gamma_c) \gamma_c - \phi_c}{\sigma + \phi_c} E_t { \Delta a</em>{t+1} } - \frac{\phi_c (1 - \gamma_c) \gamma_c}{\sigma + \phi_c} E_t { \Delta \ddot{y}<em>{t+1} } - \frac{\gamma_c}{\sigma + \phi_c} E_t { \Delta \xi</em>{t+1} } - E_t { \Delta \psi_{t+1} } )</td>
</tr>
<tr>
<td>Labor supply [ET]</td>
<td>( (\sigma + \phi_n) \ddot{c}_t + \phi_n \ddot{a}_t = \ddot{w}_t + \xi_t )</td>
</tr>
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<td>Labor supply [IT]</td>
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</tr>
<tr>
<td>Tax schedule [ET]</td>
<td>( \ddot{a}_t = \left( \frac{n_t}{1 - \phi_c} \right) \phi_c (\ddot{w}_t + \ddot{\pi}_t) )</td>
</tr>
<tr>
<td>Tax schedule [IT]</td>
<td>( \ddot{a}_t = \left( \frac{n_t}{1 - \phi_c} \right) \phi_c (\ddot{w}_t + \ddot{\pi}_t) )</td>
</tr>
<tr>
<td>Production function</td>
<td>( \ddot{y}_t = a_t + \ddot{\pi}_t )</td>
</tr>
<tr>
<td>Aggregate demand</td>
<td>( \ddot{y}_t = \gamma_c \ddot{c}_t + (1 - \gamma_c) \ddot{g}_t )</td>
</tr>
<tr>
<td>Output gap</td>
<td>( \ddot{y}_t = \ddot{y}_t - \ddot{y}_t^f )</td>
</tr>
<tr>
<td>Real marginal cost</td>
<td>( \ddot{m}_t = \ddot{w}_t - \alpha_t )</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>( \ddot{r}_t = \phi_n \pi_t + v_t )</td>
</tr>
</tbody>
</table>

Notes: ET (IT) denotes the model with progressive consumption (income) taxation. Equations without specification apply to both model versions. Note that some of the equations are redundant but are shown nonetheless for comparative purposes.
2.7.4 Model Summary

In conclusion, Table 1 contrasts both tax regimes. The linearized equations depicted summarize the equilibrium dynamics of all the model variables. The model dynamics, however, can also be expressed more compactly in terms of the New Keynesian Phillips curve, the IS curve, and the interest rate rule only.

**Expenditure Tax.** Under the PET, the model’s linearized equilibrium can be expressed compactly by the New Keynesian Phillips curve (43), the IS curve (51), and the interest rate rule (57). These three equations, together with the process for the natural rate of interest (52), fully describe the dynamics of inflation $\pi_t$, the output gap $\tilde{y}_t$, and the interest rate $\hat{r}_t$.

**Income Tax.** Equivalently, under the income tax, the New Keynesian Phillips curve (46), the IS curve (55), and the interest rate rule (57), together with the natural rate (56), completely determine the dynamics of inflation $\pi_t$, the output gap $\tilde{y}_t$, and the interest rate $\hat{r}_t$.

Before we simulate the model to illustrate the general equilibrium effects of the two tax systems, we need to discuss some of our previous findings.

Firstly, both model versions collapse into the same standard New Keynesian model when we set $\phi_c = \phi_n = 0$ (and set $\tau^{int} = 0$ under the income tax, as is common in the literature), i.e. when we assume a flat tax system. Income and consumption taxes are thus equivalent in this case.

Secondly, and as already suggested above, the equations for natural output show that both tax systems act as an automatic stabilizer for the flexible-price economy in the sense that the relevant shocks (technology $a_t$, government spending $g_t$, taste $\xi_t$) have a smaller impact on output (relative to the flat tax). The exception is the PET’s amplifying effect on output under government spending shocks (the derivative of the appropriate coefficient with respect to $\phi_c$ is positive). These effects will hold in the sticky-price economy as well (see the next section).

Thirdly, unlike the progressive tax on wage income, the progressive tax on consumption affects the household’s Euler equation and thus the economy’s IS curve. Unsurprisingly, all other things equal, the progressive consumption tax creates a greater incentive to smooth consumption (and thus output) over time, i.e., it makes the economy less responsive to “intertemporal disturbances” (shocks to $\psi_t$ and $\xi_t$) and interest rate fluctuations (see e.g. equation (50)).

Fourthly, due to their effect on the labor supply decision, both the progressive consumption tax and the progressive wage tax increase the slope of the Phillips curve (relative to the flat tax). The intuition for the wage tax is straightforward (see Mattesini and Rossi, 2012): a given increase in hours worked can only be induced by
offering higher real wages than in the flat tax case since a growing fraction of wages is taxed away. Consequently, increasing output above its natural level (through hiring more labor, at least compared to the flexible price scenario) is more costly for firms and creates more inflationary pressure. The intuition for the consumption tax is not too dissimilar: households work to (eventually) consume. To induce a given increase in hours, higher real wages than in the flat tax case have to be offered because the accompanying consumption increase is taxed at increasing rates. We thus observe more inflationary pressure when raising output above its natural level.\footnote{As already referred to above, note that an increase in the concavity of the household’s consumption utility function (a larger $\sigma$) would have the same effect as the progressive consumption tax in this regard. In this case, when increasing output above its natural level, higher real wages have to be offered (relative to the case of a smaller $\sigma$) due to a more rapidly diminishing marginal utility of consumption.}

However, notice that the Phillips curve is steeper under the progressive wage tax for all (plausible) parameter values.\footnote{To obtain this condition, simply rearrange the expressions for $\kappa_n$ and $\kappa_c$ and assume that $\phi_c = \frac{\phi_n}{1 - \phi_n}$. The latter assumption equalizes steady state employment and output (for the same relative size of the government) and implies a comparable degree of tax progressivity across the two tax systems. See the subsequent section 3.1 on the model’s calibration for details regarding this point.}

$$\kappa_n > \kappa_c \iff \sigma + \gamma_c (1 + \phi) > 1.$$ 

(60)

### 3 Equilibrium Dynamics

In this section, we examine whether the structural differences between the PET and the income tax identified above also lead to quantitatively significant differences in general equilibrium. To this effect, we compute impulse response functions; these will graphically illustrate how the tax system affects our (linearized) model economy’s cyclical behavior.\footnote{We also employed a second-order approximation to the original, non-linear model equations. The order of approximation does not affect the qualitative nature of the impulse responses.}

We depict the dynamic responses for the progressive consumption tax, the progressive income tax, and, for comparative purposes, a flat tax. The program Dynare (Adjemian et al., 2011) is used for this exercise.\footnote{The linearized model is simple enough to be also solved by “pen and paper”. We used e.g. the method of undetermined coefficients to derive closed-form solutions for inflation and the output gap for the PET and an income tax with $r^{int} = 0$ (Mattesini and Rossi, 2012). See Appendix A.1 for the results. This approach, however, becomes quite cumbersome if one is interested in the responses of the remaining model variables as well.}

As a complement, we also present business cycle statistics of the simulated model (for the progressive tax systems only).

#### 3.1 Calibration

The calibration we employ for our model simulations is based on the assumption that the relevant time period is one quarter. Our parametrization looks as follows:
the household’s subjective discount factor $\beta$ is set to 0.99, consistent with a steady state value of the real interest rate of approximately 4 percent. The values $\sigma = 1$ (log utility of consumption) and $\varphi = 1$ (unitary Frisch elasticity of labor supply) for the household’s utility function are standard in the literature. The elasticity of substitution between goods $\epsilon_p$ takes a value of 6, implying a steady state gross price markup of size 1.2 (for intermediate goods producers). The degree of price rigidity is given by $\theta_p = 2/3$, i.e. the average duration of (intermediate goods) prices is assumed to be 3 quarters. These last two parametrizations are also commonly used in the business cycle literature (see e.g. Galí, 2008).

Turning to the fiscal and monetary policy parameters, we first have $\phi_{\pi} = 1.5$, a standard value for the Taylor inflation coefficient. For the (progressive) income tax, we set $\tau^{\text{div}} = 0.2$ and $\tau^n = 0.2$ (i.e. $\eta_n = 0.8$), consistent with a government spending share in GDP of 20% ($1 - \gamma_c = 0.2$). The wage income tax progressivity parameter is set equal to the observed, GDP-weighted average value for the EA-12 member countries: $\phi_n = 0.34$ (based on the computations of Mattesini and Rossi, 2012).\footnote{The qualitative nature of our results does not depend on the size of this parameter. We choose the EA-12 value because it is somewhat higher than e.g. the respective U.S. value (0.18) and thus more convenient for illustrative purposes.} As a baseline, we set $\tau^{\text{int}} = 0$, thereby replicating the income tax system in Mattesini and Rossi (2012). To also implement a “comprehensive” income tax, we set $\tau^{\text{int}} = 0.2$. For the (progressive) consumption tax, we assume that $\eta_c = 1.25$ holds, amounting to an average tax rate on consumption of 25% ($\tau^c = 0.25$).\footnote{Recall that we express the income tax in tax-inclusive form, the consumption tax in tax-exclusive form, however. The tax rate on income thus only appears to be lower.} This again yields a government spending share in GDP of 20%. The value of the consumption tax progressivity parameter is set to $\phi_c = \frac{\phi_n}{1 - \phi_n} = 0.51$, a value that aligns the steady state work incentives (and therefore the employment and output levels) under the PET with those under the income tax (with $\phi_n = 0.34$). This last parametrization thus ensures that the two tax systems are equally “progressive”.\footnote{The formulas for steady state output (32) and (33) show that given our choice of $\eta_n$ and $\eta_c$ (our numbers imply the same relative size of the government), steady state output, and by definition employment, is equalized across the two tax regimes when $\phi_c = \frac{\phi_n}{1 - \phi_n} = 0.34$ holds, implying identical incentives to supply labor in the steady state. This last point can also be illustrated by evaluating the marginal tax rates, given by (4) and (9), at the steady state. For our parametrization, this yields $\tau^{m,c} \approx 0.89$ and $\tau^{m,n} \approx 0.47$, respectively. Hence, under the PET, and starting in the steady state, 1.89 additional units of real income are required (obtainable by supplying more labor) to increase consumption by one unit. Likewise, under the income tax, one additional unit of real income allows to increase consumption by 0.53 units. In other words, 1.89 additional units of real income are required to increase consumption by one unit. We thus have the same “rate of conversion” between labor and consumption across tax regimes. Finally, and as mentioned earlier, notice that we could have expressed the consumption tax in tax-inclusive form instead. In this case, the tax progressivity coefficients $\phi$ would have been directly comparable across tax regimes (i.e. we would have chosen $\phi_c = 0.34$ to guarantee the same degree of progressivity). As explained above, our modeling strategy does not affect the results, i.e. the more intuitive tax-exclusive formulation with $\phi_c = \frac{\phi_n}{1 - \phi_n} = 0.51$ is equivalent to the tax-inclusive formulation with $\phi_c = 0.34$.}

As mentioned, we also consider a flat tax for comparative purposes below. Our
model allows for two different versions of a flat tax: a flat income tax \((\phi_n = 0)\) with interest rate taxation \((\tau^{int} = 0.2)\); and a flat tax without interest rate taxation \((\tau^{int} = 0)\). In the latter case, the income tax \((\phi_n = 0)\) corresponds to the consumption tax \((\phi_c = 0)\). See Section 2.7.

Finally, note that since we consider each shock type separately in the following and are only interested in the qualitative nature of the subsequent results (given the simplicity of our model economy), we will not make an effort to calibrate the shock processes so as to match observable business cycle statistics. A calibration exercise of this sort would also be quite cumbersome as we allow for five different shock types. The autocorrelation coefficients \(\rho\) of the shock processes are thus all set to the standard textbook value 0.9. The standard deviations of the innovations \(\epsilon\) are all set to the standard value 0.01.\(^{44}\)

Before we turn to the model simulations, notice that our parametrization implies \(\kappa_c \approx 0.49\) and \(\kappa_n \approx 0.67\) (the corresponding value for the flat tax is roughly 0.38). The progressive consumption tax thus indeed features a “flatter” Phillips curve than the progressive income tax.

### 3.2 Model Simulations

Figures 1 to 5 in Appendix A.2 show the impulse response functions (for the main model variables) to a technology, government spending, monetary policy, time preference, and taste shock, respectively.\(^{45}\) The figures show the responses of five different tax systems: the progressive consumption tax (PET), a progressive income tax (IT) as in Mattesini and Rossi (2012) where \(\tau^{int} = 0\) holds, a “comprehensive” or “full” progressive income tax (full IT) where \(\tau^{int} > 0\) holds in addition to the previous system, a flat tax (FL) on either consumption or income (where \(\tau^{int} = 0\) holds under the income tax), and a “comprehensive” or “full” flat income tax (full FL) where \(\tau^{int} > 0\) holds. In what follows, and for obvious reasons, we are mostly interested in how the PET performs relative to the progressive income tax. The flat tax, however, also serves as a useful benchmark. As already mentioned above, the assumption \(\tau^{int} > 0\) is rather uncommon in the DSGE literature. We will therefore refer to the two “full” income tax systems only in passing in the following. Finally, notice that the main purpose of the subsequent account is to only give a brief, first impression of the simulation results; to show that there are—for a wide range of shocks—important quantitative (and sometimes also qualitative) differences between the PET and the other tax systems. Especially since we are dealing with five different shock types, a comprehensive analysis of the deeper economic mechanisms driving our results—in particular some of the more subtle differences between the PET and the progressive income tax (IT)—is not within the scope of this paper and

\(^{44}\)We checked that our results are unaffected by these choices.

\(^{45}\)The results are robust to changes in the model parameters.
will be therefore left for future research.

To illustrate the role of the PET in the business cycle, it will be best to first draw a comparison with the “naked” flat tax (FL). A quick glance at the impulse response functions reveals that there are noticeable quantitative differences between the two tax systems. Not unexpectedly, but crucially, the responses show that for all shock types considered, the PET leads, relative to the flat tax, to a significant stabilization of household consumption demand. The latter, in turn, brings about, with the exception of the government spending shock, a stabilization of aggregate output. A first important result of this simulation exercise is thus that the PET—just as the conventional progressive income tax (see the impulse responses for the tax system IT)—acts as an automatic fiscal stabilizer for the economy.

Consider, for example, the responses to a (positive) technology shock. Consumption and output increase, but the responses are significantly dampened under the PET (still compared to FL). The intuition is straightforward: as consumption rises above its steady state value, the average tax rate on household consumption expenditure (automatically) rises as well; this mitigates the increase in consumption demand and therefore output. Over the business cycle, there is thus a greater incentive for households to smooth their consumption, the latter also mitigating output fluctuations. For the time preference, taste, and monetary policy shocks, the economic intuition behind the PET’s stabilizing effect on output is similar. Since consumption and output move inversely under government spending shocks (in contrast to the other shock types), however, the PET’s stabilizing effect on consumption in fact increases output fluctuations in this case (as already indicated above).\(^46\)

Furthermore, our simulations show that the PET likewise reduces fluctuations in the output gap relative to the flat tax. As the PET also increases the slope of the Phillips curve (see Section 2.7), the latter, however, does not automatically translate into a more stable inflation rate. We indeed observe a higher volatility of inflation in the presence of technology and government spending shocks (see the amplitude of the impulse responses).

Lastly, note that the PET’s performance relative to the flat tax with interest rate taxation (full FL) is qualitatively rather similar. Yet, it becomes quite apparent that the latter system is less successful in terms of macroeconomic stabilization than the “naked” flat tax, especially with regards to output gap and inflation stabilization. The reason is that a system of interest rate taxation reduces the effectiveness of monetary policy (see the relevant IS curve).

Before we compare the PET with the progressive income tax, recall that the PET exerts the just described general equilibrium effects in the sticky-price economy

\(^46\)The PET’s effect on employment (relative to FL) also depends on whether consumption and employment move together or in opposite directions after a shock hits the economy. Thus the PET’s stabilizing (destabilizing) effect on employment in the presence of monetary policy, time preference, and taste shocks (technology and government spending shocks).
through, firstly, affecting the household’s intratemporal choice (labor supply), and secondly, its intertemporal choice (Euler equation). As already referred to above, an increase in the concavity of the household’s consumption utility function would have a similar effect in general equilibrium (just set $\phi_c = 0$ and imagine a higher $\sigma$ in the equations depicted in Table 1). The PET reduces consumption (and output) volatility due to an automatic adjustment of tax rates over the business cycle; a higher $\sigma$ due to a more rapidly declining marginal utility of consumption.

We next draw a brief comparison between the PET and the conventional progressive income tax (IT). The impulse response functions reveal that for all five shock types considered, there are significant quantitative differences between the two progressive tax systems. These differences are also summarized in Table 2, which depicts the change in the standard deviation of the main model variables when moving from the income tax to the PET (for each shock type in isolation). A second important result of this simulation exercise is thus that a progressive tax on consumption expenditure produces quite different macroeconomic dynamics than a progressive tax on wage income.

Compared to the flat tax, the macroeconomic differences between the PET and the income tax are less clear-cut and more difficult to pin down, however. This is not too surprising since the income tax also exerts, through affecting the household’s intratemporal choice (labor supply), a stabilizing influence on the economy (Mattesini and Rossi, 2012). We will therefore only highlight the most salient results and leave a thorough interpretation of these results for future research.

The impulse response functions first reveal that for the shocks that affect the flexible-price allocation (technology $a_t$, government spending $g_t$, taste $\xi_t$), the (quite intuitive) results of the previous comparison with the flat tax largely carry over, at least in qualitative terms. For all three shock types, we observe that the PET stabilizes consumption relative to the income tax. As before, with again the exception

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<td>-49.0</td>
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<tr>
<td>Full IT</td>
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Notes: Results denote the percentage change in the standard deviation of the model variable when moving from the respective tax regime to the PET.

| $sd(\pi)$ | +31.1 | -31.7 | -2.1 | -49.0 | -3.4 | -49.6 | -35.4 | -66.3 |
| $sd(c)$    | -13.5 | -10.7 | -35.5 | -33.4 | +31.1 | -14.8 | +31.1 | -31.6 | -10.6 | -13.3 |
| $sd(n)$    | +39.8 | +28.0 | +35.5 | +31.4 | +31.0 | -14.8 | +31.1 | -31.6 | -10.6 | -13.3 |
| $sd(y)$    | -13.5 | -10.7 | +35.5 | +31.4 | +31.0 | -14.8 | +31.1 | -31.6 | -10.6 | -13.3 |
| $sd(\tilde{y})$ | +77.7 | -7.4 | +33.3 | -30.7 | +31.0 | -14.8 | +31.1 | -31.6 | -12.5 | -54.3 |
of the government spending shock, this stabilizes output.\textsuperscript{47}

One of the most noticeable and interesting differences between the two tax systems indeed occurs under the government spending shock. The reason is that this shock has an opposite effect on the tax bases of the two tax systems: consumption and real wage income. A positive government spending shock increases aggregate demand and thus real wages and employment but crowds out household consumption demand. Due to the automatic reduction in the tax rate on consumption expenditure, the latter effect is attenuated under the PET, however. Instead, under the progressive income tax, the negative effect on consumption demand is amplified by an automatic increase in the tax rate on wages. Since consumption and output move in opposite directions, we thus observe a bigger output response under the PET.

Interestingly, and now in contrast to the flat tax, for both shock types that do not affect the flexible-price allocation (monetary policy \(v_t\), time preference \(\psi_t\)), the income tax outperforms the PET in terms of consumption and output or employment stabilization. This is somewhat surprising at first sight since all other things equal, the PET makes the economy less responsive to intertemporal disturbances or (exogenous) interest rate fluctuations (compare the IS curves in Table 1). The general equilibrium effect of these shocks, however, also depends on the endogenous monetary policy response to inflation and its interaction with the (slope of the) Phillips curve and is therefore difficult to work out beforehand.\textsuperscript{48} The conclusion to be drawn from the impulse response functions is that the steeper slope of the Phillips curve under the income tax is the decisive factor that reduces consumption fluctuations, relative to the PET, in general equilibrium.\textsuperscript{49}

At this point, it will also be useful to briefly highlight the differential effect of the two tax systems on the volatility of inflation. The impulse response functions show that the PET generates larger fluctuations in the inflation rate under the technology shock, but smaller fluctuations under the government spending, monetary policy, time preference, and taste shock (see also Table 2 for a numerical comparison). Even though the income tax generally speaking leads to smaller output gap fluctuations than the PET, this effect seems to be overcompensated for by the steeper slope of the Phillips curve under the income tax.

Finally, notice that equivalent to the flat tax case considered above, the progressive income tax with interest rate taxation (full IT) has inferior macroeconomic

\textsuperscript{47}The PET's relative effect on employment then again follows from these results.

\textsuperscript{48}Consider, for instance, the "first round" under the (positive) monetary policy shock (not visible in the impulse responses). The larger initial impact of the shock on output, and by definition the output gap, under the income tax (see the IS curve) has an even more pronounced deflationary impact due to the steeper Phillips curve. The latter creates a stronger (endogenous) monetary policy reversal than under the PET. This reversal then has a bigger impact on output according to the IS curve and so forth. The net effect (not even taking expectations into account) seems unclear.

\textsuperscript{49}This seems to be a robust outcome. We also checked this result using the closed-form solutions in Appendix A.1.
stabilization properties (relative to both the conventional progressive income tax and the PET; see also Table 2). Especially the output gap and thus inflation again display rather large fluctuations.

4 Welfare

The last section has shown that the PET and the progressive income tax lead to quite different macroeconomic dynamics. In this section, we will briefly consider the resulting welfare implications. To this effect, we return to the original, non-linearized model equations and employ a second-order approximation to the latter as well as the household’s (expected) lifetime utility function.\footnote{It is in principle possible to conduct the welfare analysis using a linear-quadratic approach. This approach is very cumbersome and prone to error, however. See e.g. Kim and Kim (2003).} The program Dynare (Adjemian et al., 2011) is again used for this exercise. Subsequently, we can compare household welfare across tax regimes. More precisely, for both regimes, we convert our welfare measure into a consumption loss equivalent à la Lucas (1987). That is, we compute the variable $\zeta^{\text{tax}}$ of the following equation:

$$
E_t \sum_{k=0}^{\infty} \beta^k U \left( C(1 - \zeta^{\text{tax}}), N \right) = E_t \sum_{k=0}^{\infty} \beta^k U \left( C_{t+k}, N_{t+k} \right).
$$

(61)

$\zeta^{\text{tax}}$ is the percentage reduction in average steady state consumption that makes the household indifferent between living in the (policy invariant) steady state environment (with reduced average consumption) and the stochastic environment under a particular tax regime.

For our model parametrization, the consumption loss equivalent is given by

$$
\zeta^{\text{tax}} = 100 \left( 1 - \exp \left( (W^{\text{tax}} - W)(1 - \beta) \right) \right)
$$

(62)

where $W^{\text{tax}}$ ($W$) is welfare in the stochastic (steady state) environment.

We compute $\zeta^{\text{tax}}$ for each shock type separately. Since the absolute values of $\zeta^{\text{tax}}$ are of less concern here (recall that we have chosen arbitrary values for the autocorrelation coefficients and the standard deviations of the shock processes), we only report the percentage change in $\zeta^{\text{tax}}$ when moving from the progressive income tax to the PET.\footnote{We checked that the results below do not depend on our particular shock calibration.} For the conventional income tax (IT), the results are as follows and seem quantitatively significant: The consumption loss equivalent increases by roughly 55% under technology shocks, but decreases by roughly 13% under government spending shocks, 12% under monetary policy shocks, and 7% under time preference shocks. Under taste shocks, welfare is higher under the PET as well. Since our computations reveal that welfare in the stochastic environment under the PET (marginally) exceeds steady state welfare, we are not able to compute...
Table 3: Welfare and Model Parameters

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Notes: Results show the change in welfare when moving from the respective tax regime to the PET. A + (−) sign thus implies a higher (lower) level of welfare under the PET. The first row shows the results for the baseline calibration. For the remaining rows, except for the parameter explicitly stated, all other parameters are at their baseline value.

In summary, moving to the PET increases welfare in the presence of all the demand shocks, but decreases welfare in the presence of the supply shock. From a welfare perspective, at least through the lens of our simple New Keynesian model, the desirability of the PET thus crucially depends on whether shocks originate from the demand-side or the supply-side of the economy. Furthermore, notice that the PET’s performance relative to the progressive income tax with interest rate taxation (full IT) is rather similar (no numbers shown). The welfare gains for the demand shocks are somewhat higher, however. Furthermore, there is now a welfare gain for the technology shock as well. Table 3 at last confirms that the previous results are also quite robust across a set of different parameter values.

It is not within the scope of this paper to thoroughly analyze the drivers behind these results. The model simulations suggest a common theme, however. When comparing the PET with the income tax (IT), it becomes apparent that the former increases (decreases) welfare whenever it decreases (increases) the volatility of inflation relative to the latter (this effect on inflation is also captured in the linear model above; see the respective numbers in Table 2). Furthermore, the simulations reveal that each welfare increase (decrease) is associated with a higher (lower) consump-

\[52\] This is a rather rare but not necessarily illogical case. See e.g. Lester et al. (2014).

\[53\] Under supply (demand) shocks, output and prices move in the opposite (same) direction in our model economy.
tion level (not shown; note that this effect is not captured in the linear model). The link between the volatility of inflation and the consumption level seems obvious: a lower volatility reduces inefficient price dispersion between firms (see equation (30)); this increases the economy’s productivity (see equation (29)) and ceteris paribus affords more output and thus consumption. The previous results therefore seem to again confirm the importance of price stability in sticky price models. Lastly, and as already indicated above, note that one possible reason for the PET’s relative superiority with respect to inflation stabilization (at least as far as demand shocks are concerned) might be the smaller slope of the Phillips curve under the PET (which, all other things equal, implies a lower inflation volatility). However, more research has to be conducted to understand this link as well as the other possible drivers of welfare.

5 Conclusion

This paper was a first attempt to examine the business cycle properties (and the resulting welfare implications) of the personal expenditure tax (PET), an age-old yet largely untested alternative to the personal income tax. The main contribution of the paper was to propose a simple way to model a PET, to introduce the latter into an otherwise standard New Keynesian DSGE model (augmented by government expenditure), to derive a log-linear version of the model, and to draw a comparison with the existing income tax (Mattesini and Rossi, 2012). The model simulations have shown three things: Firstly, the PET, just as the progressive income tax, acts as an automatic stabilizer for the economy. Yet, and secondly, the PET has a quantitatively quite different effect on the volatilities of the main macroeconomic variables than the income tax. Thirdly, the PET yields welfare gains, relative to the income tax, for all the demand shocks considered; there are welfare losses, however, under a technology shock. Overall, the simulation results suggest that there is ample room for future research on the role of the PET in the business cycle.

The most interesting and natural extensions of the model at hand would be to include an open economy dimension and/or real investment and capital accumulation. Both extensions would e.g. allow for more situations where (wage) income and consumption move in opposite directions after shocks (in our model, this holds only for government spending shocks) and where the PET thus clearly differentiates itself from the income tax in terms of the direction of tax rate adjustments.

In a somewhat different and elaborate model framework, our analysis of the business cycle characteristics of the PET could be extended in a number of other

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54 The same line of reasoning also works for the progressive income tax with interest rate taxation.

55 We briefly experimented with a model including capital. The results of the previous analysis did not change much. However, we did not yet examine shocks that can only be considered in this kind of model (e.g. investment shocks).
promising ways. Firstly, the zero lower bound (ZLB) on nominal interest rates could be incorporated into the model. This would allow us to analyze the effectiveness of discretionary fiscal policy in a depressed economy (where the ZLB binds and conventional monetary policy is thus impotent). One obvious exercise would be to look at the size of the government spending multiplier in this setting. Another interesting question to ask would be whether a temporary cut in tax rates would provide a bigger stimulus under the PET than under the existing income tax (as suggested by e.g. Kaldor, 1955; Frank, 2011). Secondly, to investigate how the PET affects the economy’s response to financial shocks, a model with a realistic financial sector (similar to e.g. Jakab and Kumhof, 2015) could be employed. For instance, it seems plausible at first sight that a progressive tax on consumption might be more successful in curbing economic fluctuations originating from volatile mortgage or consumer credit markets than a progressive tax on income. It might be worthwhile to check this intuition using a formal model. Thirdly, agent heterogeneity as in McKay and Reis (2016b) could be included into the model. This would allow us to study how the redistributive side of the PET interplays with its business cycle characteristics.
References


Voigts, S., 2017. Revisiting the effect of VAT changes on output: the importance of pass-through dynamics. Unpublished manuscript.
A Appendix

A.1 Closed-Form Solutions

For convenience, we only derived closed-form solutions for inflation $\pi_t$ and the output gap $\hat{y}_t$. The closed-form solutions for the remaining model variables could be obtained in a straightforward way, e.g. $\hat{y}_t = \tilde{y}_t + \gamma_t'$, $\hat{n}_t = \hat{y}_t - a_t$, and so forth. Note that when we “switch off” the progressivity ($\phi_c = \phi_n = 0$), both systems of taxation turn into one and the same flat tax. Also note that as in Mattesini and Rossi (2012), the income tax featured here refrains from taxing interest income (i.e. $\tau^{int} = 0$ is assumed).

For inflation $\pi_t$, the closed-form solution is given by

$$\pi_t = o_{\pi a}a_t + o_{\pi g}g_t + o_{\pi \xi}\xi_t + o_{\pi v}v_t + o_{\pi \psi}\psi_t$$

(63)

where

$$\begin{align*}
[ET] & \quad o_{\pi a} = -\frac{\gamma_c(1 + \varphi)(1 - \rho_a)(\sigma + \phi_c)\kappa_c}{(\sigma + \varphi\gamma_c + \phi_c)[(1 - \beta\rho_a)(1 - \rho_a)(\sigma + \phi_c) + \gamma_c(\phi_\pi - \rho_a)\kappa_c]} < 0 \\
[IT] & \quad o_{\pi a} = -\frac{\gamma_c(1 + \varphi)(1 - \rho_a)\sigma\kappa_n}{(\sigma + \varphi\gamma_c + \phi_c)[(1 - \beta\rho_a)(1 - \rho_a)\sigma + \gamma_c(\phi_\pi - \rho_a)\kappa_n]} < 0
\end{align*}$$

$$\begin{align*}
[ET] & \quad o_{\pi g} = \frac{\gamma_c(1 - \gamma_c)\varphi(1 - \rho_g)(\sigma + \phi_c)\kappa_c}{(\sigma + \varphi\gamma_c + \phi_c)[(1 - \beta\rho_g)(1 - \rho_g)(\sigma + \phi_c) + \gamma_c(\phi_\pi - \rho_g)\kappa_c]} > 0 \\
[IT] & \quad o_{\pi g} = \frac{\gamma_c(1 - \gamma_c)(\varphi + \phi_n)(1 - \rho_g)\sigma\kappa_n}{(\sigma + \varphi\gamma_c + \phi_c)[(1 - \beta\rho_g)(1 - \rho_g)\sigma + \gamma_c(\phi_\pi - \rho_g)\kappa_n]} > 0
\end{align*}$$

$$\begin{align*}
[ET] & \quad o_{\pi \xi} = \frac{\gamma_c^2(1 - \rho_\xi)\kappa_c}{(\sigma + \varphi\gamma_c + \phi_c)[(1 - \beta\rho_\xi)(1 - \rho_\xi)(\sigma + \phi_c) + \gamma_c(\phi_\pi - \rho_\xi)\kappa_c]} > 0 \\
[IT] & \quad o_{\pi \xi} = \frac{\gamma_c^2(\varphi + \phi_n)(1 - \rho_\xi)\kappa_n}{(\sigma + \varphi\gamma_c + \phi_c)[(1 - \beta\rho_\xi)(1 - \rho_\xi)\sigma + \gamma_c(\phi_\pi - \rho_\xi)\kappa_n]} > 0
\end{align*}$$

$$\begin{align*}
[ET] & \quad o_{\pi v} = -\frac{\gamma_c\kappa_c}{(1 - \beta\rho_v)(1 - \rho_v)(\sigma + \phi_c) + \gamma_c(\phi_\pi - \rho_v)\kappa_c} < 0 \\
[IT] & \quad o_{\pi v} = -\frac{\gamma_c\kappa_n}{(1 - \beta\rho_v)(1 - \rho_v)\sigma + \gamma_c(\phi_\pi - \rho_v)\kappa_n} < 0
\end{align*}$$

$$\begin{align*}
[ET] & \quad o_{\pi \psi} = \frac{\gamma_c(1 - \rho_\psi)\kappa_c}{(1 - \beta\rho_\psi)(1 - \rho_\psi)(\sigma + \phi_c) + \gamma_c(\phi_\pi - \rho_\psi)\kappa_c} > 0 \\
[IT] & \quad o_{\pi \psi} = \frac{\gamma_c(1 - \rho_\psi)\kappa_n}{(1 - \beta\rho_\psi)(1 - \rho_\psi)\sigma + \gamma_c(\phi_\pi - \rho_\psi)\kappa_n} > 0
\end{align*}$$
For the output gap $\bar{y}_t$, the closed form solution is given by

$$\bar{y}_t = o_{ya}a_t + o_{yg}g_t + o_{y\xi}\xi_t + o_{yv}v_t + o_{y\psi}\psi_t$$

(64)

where

\[ \begin{align*}
\text{[ET]} & \quad o_{ya} = - \frac{\gamma_c(1 + \varphi)(1 - \rho_a)(\sigma + \phi_c)(1 - \beta\rho_a)}{(\sigma + \varphi\gamma_c + \phi_c)[(1 - \beta\rho_a)(1 - \rho_a)(\sigma + \phi_c) + \gamma_c(\phi_\pi - \rho_a)\kappa_c]} < 0 \\
\text{[IT]} & \quad o_{ya} = - \frac{\gamma_c(1 + \varphi)(1 - \rho_a)\sigma(1 - \beta\rho_a)}{(\sigma + (\varphi + \phi_n)\gamma_c)[(1 - \beta\rho_a)(1 - \rho_a)\sigma + \gamma_c(\phi_\pi - \rho_a)\kappa_n]} < 0 \\
\text{[ET]} & \quad o_{yg} = \frac{\gamma_c(1 - \gamma_c)(1 - \rho_g)(\sigma + \phi_c)(1 - \beta\rho_g)}{(\sigma + \varphi\gamma_c + \phi_c)[(1 - \beta\rho_g)(1 - \rho_g)(\sigma + \phi_c) + \gamma_c(\phi_\pi - \rho_g)\kappa_c]} > 0 \\
\text{[IT]} & \quad o_{yg} = \frac{\gamma_c(1 - \gamma_c)(\varphi + \phi_n)(1 - \rho_g)(1 - \beta\rho_g)}{(\sigma + (\varphi + \phi_n)\gamma_c)[(1 - \beta\rho_g)(1 - \rho_g)\sigma + \gamma_c(\phi_\pi - \rho_g)\kappa_n]} > 0 \\
\text{[ET]} & \quad o_{y\xi} = \frac{\gamma_c^2\varphi(1 - \rho_\xi)(1 - \beta\rho_\xi)}{(\sigma + \varphi\gamma_c + \phi_c)[(1 - \beta\rho_\xi)(1 - \rho_\xi)(\sigma + \phi_c) + \gamma_c(\phi_\pi - \rho_\xi)\kappa_c]} > 0 \\
\text{[IT]} & \quad o_{y\xi} = \frac{\gamma_c^2(\varphi + \phi_n)(1 - \rho_\xi)(1 - \beta\rho_\xi)}{(\sigma + (\varphi + \phi_n)\gamma_c)[(1 - \beta\rho_\xi)(1 - \rho_\xi)\sigma + \gamma_c(\phi_\pi - \rho_\xi)\kappa_n]} > 0 \\
\text{[ET]} & \quad o_{yv} = - \frac{\gamma_c(1 - \beta\rho_v)}{(1 - \beta\rho_v)(1 - \rho_v)(\sigma + \phi_c) + \gamma_c(\phi_\pi - \rho_v)\kappa_c} < 0 \\
\text{[IT]} & \quad o_{yv} = - \frac{\gamma_c(1 - \beta\rho_v)}{(1 - \beta\rho_v)(1 - \rho_v)\sigma + \gamma_c(\phi_\pi - \rho_v)\kappa_n} < 0 \\
\text{[ET]} & \quad o_{y\psi} = \frac{\gamma_c(1 - \rho_\psi)(1 - \beta\rho_\psi)}{(1 - \beta\rho_\psi)(1 - \rho_\psi)(\sigma + \phi_c) + \gamma_c(\phi_\pi - \rho_\psi)\kappa_c} > 0 \\
\text{[IT]} & \quad o_{y\psi} = \frac{\gamma_c(1 - \rho_\psi)(1 - \beta\rho_\psi)}{(1 - \beta\rho_\psi)(1 - \rho_\psi)\sigma + \gamma_c(\phi_\pi - \rho_\psi)\kappa_n} > 0.
\end{align*} \]
A.2 Linearized Model: Impulse Response Functions

The figures on the next pages show the impulse response functions for the main model variables and five different tax systems. 56 “PET” denotes the progressive consumption tax, “IT” the conventional progressive income tax (where $\tau^{int} = 0$), “FL” the flat tax on either consumption or income (i.e. with $\tau^{int} = 0$ under the income tax), “full IT” the progressive income tax with $\tau^{int} = 0.2$, and “full FL” a flat tax on all income (i.e. $\tau^{int} = 0.2$). For each model variable depicted, the graph shows the log-deviation from the steady state after a positive realization of the relevant innovation $\epsilon$ of one standard deviation.

56 “Nom. Rate” denotes the nominal interest rate, “Real Rate” the (after-tax) real interest rate.
Figure 1: Impulse Responses to a Positive Technology Shock

- **Inflation**
- **Consumption**
- **Employment**
- **Output**
- **Output Gap**
- **Nominal Rate**
- **Real Rate**
- **Tax Rate**
- **Real Wage**
Figure 2: Impulse Responses to a Positive Government Spending Shock
Figure 3: Impulse Responses to a Positive Monetary Policy Shock

- Inflation
- Consumption
- Employment
- Output
- Output Gap
- Nominal Rate
- Real Rate
- Tax Rate
- Real Wage

Legend:
- PET
- IT
- FL
- full IT
- full FL
Figure 4: Impulse Responses to a Positive Time Preference Shock
Figure 5: Impulse Responses to a Positive Taste Shock
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