Freie Universität Berlin

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School of Business & Economics
Discussion Paper

Economics

2019/1

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December 19, 2018

Abstract

Evidence suggests that people evaluate outcomes relative to expectations. I analyze this expectation-based loss aversion [Kőszegi and Rabin (2006, 2009)] in the context of dynamic and static auctions, where the reference point is given by the (endogenous) equilibrium outcome. If agents update their reference point during the auction, the arrival of information crucially affects equilibrium behavior. Consequently, I show that—even with independent private values—the Vickrey auction yields strictly higher revenue than the English auction, violating the well known revenue equivalence. Thus, dynamic loss aversion offers a novel explanation for empirically observed differences between these auction formats.

Keywords: Vickrey auction, English auction, expectation-based loss aversion, revenue equivalence, dynamic loss aversion, personal equilibrium
JEL classification: D03, D44

*I thank Yves Breitmoser, Françoise Forges, Matthias Hammer, Paul Heidhues, Radosveta Ivanova-Stenzel, Johannes Johnen, Thomas Mariotti, Antonio Rosato, Thomas Schacherer, Ran Spiegler, Roland Strausz, and Georg Weizsäcker for helpful comments, as well as participants at the 11th World Congress of the Econometric Society (Montreal), the 2015 EEA conference (Mannheim), the Applied Theory Workshop at Toulouse School of Economics, the 2016 EARIE conference (Lisbon), the CRC conference in Berlin, the ZEW Research Seminar, and the 2017 Annual Conference of the Verein für Socialpolitik (Vienna). I gratefully acknowledge financial support of the German Research Foundation through CRC TRR 190 and RTG 1659.

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1 Introduction

Auctions are a universal tool to organize sales in markets. At the core of auction theory stand the well known revenue equivalence results. Vickrey (1961) notes the strategic equivalence between the dynamic English and the static Vickrey auction: if values are independent and private, there is no effect of sequential information and it is a weakly dominant strategy to bid (up to) one's private valuation in both formats.[1] These powerful theoretical predictions, however, stand in contrast to the experimental literature, which mostly finds lower revenues for the English auction.[2] I identify endogenous preferences in the form of expectation-based loss aversion as a possible explanation for this phenomenon.

In my model, bidders evaluate any auction outcome relative to their reference point, formed by rational expectations. Consequently, it is no longer optimal in neither the second-price (Vickrey), nor in the ascending-clock (English) auction to bid (up to) one's intrinsic valuation. Concretely, loss aversion leads to strong overbidding for high types in the Vickrey auction. Moreover, if an agent updates her reference point based on new information, opponents' behavior influences the agent's reference point, and thus her endogenous preferences. Hence, even if valuations for the object are entirely private, sequential information revelation affects bidding behavior. Consequently, the English and the Vickrey auction are no longer strategically equivalent. I demonstrate that, consistent with most of the experimental evidence, the English auction yields a lower revenue. I establish that this effect is driven by a time-inconsistency problem, which dynamic expectation-based loss-averse bidders face when forming their bidding strategy.

Following the concept of loss aversion by Kőszegi and Rabin (2006, 2009), I assume that, in addition to classical utility, bidders experience gain-loss utility from comparing the outcome to their expectations. Further, I assume that bidders bracket narrowly, meaning that they assign gains and losses separately to the money dimension and the good dimension. For the ease of exposition, the main part of this paper considers bidders who are exclusively loss averse with respect to the object.[3] If they win the auction, they experience a feeling of elation, proportional to how unexpected it was for them to win. Similarly, they perceive

[1]Myerson (1981) extends the results to show that all main auction formats give rise to the same expected revenue.

[2]For a summary of the experimental literature, see Kagel (1995).

[3]I show in section 7.1 that the main insights generalize to cases where bidders assign gains and losses separately to the money and good dimension.

a feeling of loss if they lose, proportional to their expectation of winning. Taking that into account, bidders will overbid their intrinsic valuation. Since losses with respect to expectations are weighted stronger than gains, high types—who expect to win more often—overbid more aggressively than low types in the symmetric equilibrium of the Vickrey auction.

To model the impact of dynamic information on an agent's reference point in the dynamic English auction, I take the continuous-time limit of Kőszegi and Rabin (2009): at each clock increment bidders observe whether opponents drop out of the auction. This information permanently updates bidders' expectations about winning the auction and about how much they have to pay. If the changes in beliefs immediately update the bidders' reference points, they instantaneously perceive gain-loss utility, which means that they assign gains and losses to changes in the reference distribution.

I consider the two most extreme cases as benchmarks: if the reference-point updating lags sufficiently behind changes in beliefs, there is no updating during the auction process and therefore no impact of sequential information. In that case, the English auction remains equivalent to the Vickrey auction. If the new information immediately updates the reference point, however, bidders' utility depends on the observed signals about opponents' bidding strategies during the auction process, even though values are private.

Kőszegi and Rabin interpret an agent's reference point as her lagged beliefs. Recent experimental findings, however, suggest that the reference point adjusts quickly to new information. Whether instantaneous reference-point updating is a realistic approximation may depend on the exact auction environment, e.g. the speed at which the price augments, which can differ immensely across different English auctions. In any case, it is clear that instantaneous updating constitutes a natural and important benchmark.

Since losses are weighted more strongly than gains, expected gain-loss utility is always negative. Consequently, bidders dislike fluctuation in beliefs. As bidders are forward looking, they will account for these costs when they form their bidding strategy. In principle, an aggressive bid would to some extent insure against belief fluctuations during the auction process. However, if the auction goes on and opponents prevail, bidders' belief to win the auction eventually declines. Bidders become less attached to the auctioned object, and, at the point at which they would have to bid aggressively, it becomes time inconsistent to do so. They eventually perceive themselves as a low type with respect to the remaining bidders. This leads to only moderate overbidding - mirroring low types in the Vickrey auction. Therefore, bidding is less aggressive in an English auction with reference point updating.

Since bidders dislike belief fluctuations, they would prefer not to observe the auction process and would rather use proxies to bid on their behalf. This logic is related to Benartzi et al. (1995) and Pagel (2016), who explain the equity premium puzzle using loss aversion: since stock prices fluctuate, an investor who regularly checks her portfolio will experience negative expected gain-loss utility. This disutility makes stocks less attractive relative to bonds.

Lange and Ratan (2010) highlight that in the presence of loss aversion in hedonic dimensions most laboratory results may not be transferable to the field: the effects of loss aversion are mainly driven by the assumption that bidders account losses and gains separately in the money and the good dimension (narrow bracketing). In order to control for private values, most auction experiments, however, use auction tokens, which can be interchanged for money at the end of the experiment. In context of these induced value experiments, bidders might not assign gains and losses separately to tokens and money, as they are in fact both money.[4] Since I assume narrow bracketing throughout this paper, my results are more likely to apply to real commodity auctions, rather than to experiments on induced value auctions. Therefore, my results can only explain the revenue gap between Vickrey auctions and English auctions in the experimental literature on induced values if we assume that bidders don't perceive the tokens as money.

Recent experiments find support for expectation-based loss aversion in auctions.[5] There is surprisingly little experimental literature that compares revenues of real commodity English auctions and Vickrey auctions.[6] The only laboratory-controlled experiment that I am aware of, is conducted by Schindler (2003). She reports 14 percent lower revenues in English auctions, therefore confirming the findings of the induced-value literature as well as my theoretical predictions.

The contribution of my paper is twofold. First, it provides a novel rationale to explain the observed revenue gap between the two auction formats. Second,

[4]Indeed, Shogren et al. (1994) run Vickrey auctions to sell different goods and show that an endowment effect is strongest for non-market goods with imperfect substitutes.

[5]See Banerji and Gupta (2014), Rosato and Tymula (2016), and the discussion in the Literature Section.

[6]The only field experiment I am aware of is conducted by Lucking-Reiley (1999), who trades collectable cards on an internet auction platform. He finds no significant difference in revenues, though he admits himself that he cannot entirely control for a potential selection bias and endogenous entry.

it contributes to the small body of literature on strategic interaction between loss-averse agents. To my best knowledge, this paper is the first to analyze such interaction in a dynamic game with more than two periods.

The remainder of the paper is structured as follows: Section 2 discusses the related literature, Section 3 introduces the model. Section 4 analyzes equilibrium behavior in the Vickrey auction, while Section 5 analyzes equilibrium behavior in the English auction with two loss-averse bidders. Section 6 discusses the revenue comparison of both auction formats. Section 7 analyzes several extensions, while Section 8 concludes.

2 Related Literature

Kahneman et al. (1990) establish the *endowment effect*: the robust empirical observation that agents' valuation for goods increase with ownership. It has since been experimentally replicated under many different circumstances, for summaries see Camerer (1995) and Horowitz and McConnell (2002). Tversky and Kahneman (1991) propose loss aversion with respect to the status quo to explain the endowment effect.

Kőszegi and Rabin (2006) suggest recent rational expectations as reference point. The hypothesis that expectations play a role in individual's preferences have been supported in recent experiments (Ericson and Fuster (2011) and Abeler et al. (2011)), as well as challenged (Heffetz and List (2014)).[7] In the context of auctions, Banerji and Gupta (2014) and Rosato and Tymula (2016) provide evidence that expectation-based loss aversion affects bidding behavior. Banerji and Gupta (2014) manipulate rational expectations by changing the support of the opponents' draw in a BDM auction, whereas Rosato and Tymula (2016) vary the number of bidders in a Vickrey auction between treatments. Both experiments find that bids significantly increase in the induced expectations to win, as predicted by the model of Kőszegi and Rabin (2006).

The idea that the reference point is determined by recent beliefs leads to the natural question of the speed of reference-point adjustment. Strahilevitz and Loewenstein (1998) provide early evidence that the time span for which individuals hold beliefs has an impact on the reference point. Gill and Prowse (2012) use a real effort task to measure loss aversion and find that in their framework "the

[7] For a literature revue on related evidence, see Ericson and Fuster (2014).

adjustment process is essentially instantaneous". Smith (2012) induces different probabilities of winning an item across groups of individuals. After the uncertainty resolves, he measures the willingness to pay for the item among bidders who have not won. In contrast to Ericson and Fuster (2011), who elicit valuations *before* the uncertainty resolves, Smith finds no significant difference between different groups, which suggests that the reference point is not so much determined by lagged beliefs, but rather adjusts quickly to the new information.[8]

For static environments Kőszegi and Rabin (2006) has arguably become the standard model of reference-dependent preferences, and been successfully applied to various fields, like mechanism design (Eisenhuth (2018)), contract theory (Herweg et al. (2010)), industrial organization (for instance Heidhues and Kőszegi (2008), Herweg and Mierendorff (2013), Karle and Peitz (2014), Rosato (2016)), and labor markets (Eliaz and Spiegler (2014)). Heidhues and Kőszegi (2014) show that buyers in monopolistic markets may face a similar form of time inconsistency as I establish for bidders in the English auction: ex ante they would like to commit not to buy. If the seller induces low prices with some probability, this plan, however, is time inconsistent. As a result, the consumer ends up buying for a high prices as well.

There is a small, but growing, body of literature concerning strategic interaction between multiple loss-averse players. Dato et al. (2017) extend the equilibrium concepts of Kőszegi and Rabin (2006) to strategic interaction. Mermer (2017) analyzes contests with loss-averse agents. Similar to my results in the Vickrey auction, she finds that the willingness to invest is increasing in the winning probability.

In the context of auctions, Lange and Ratan (2010) point out that loss-averse bidders may behave differently in laboratory experiments than in the field; bidders may not bracket narrowly in induced-value experiments. Further, they calculate the equilibrium bidding function of loss averse bidders in the first-price auction and Vickrey auction for a different equilibrium concept than I use in this paper. (For a more detailed discussion of the equilibrium concepts see Section 4.)

Ehrhart and Ott (2014) introduce a model of the Dutch and English auction, where sequential information updates the reference point, but—in contrast to Kőszegi and Rabin (2009)—does not induce gain-loss utility. As a result, in equilibrium there is never any feeling of loss in the English auction, since by the

[8]Smith's confidence intervals are, however, rather wide.

time a bidder drops out she expects to lose. Eisenhuth and Grunewald (2018) show that the all-pay auction yields higher payoffs than the first-price auction for narrow-bracketing bidders, since loss-averse bidders dislike payment uncertainty.

For dynamic environments Kőszegi and Rabin (2009) propose a model of dynamic loss aversion, where updates of expectations carry reference-dependent utility. This model has so far only been applied sparsely. First applications nevertheless seem promising. Rosato (2014) uses a two-period dynamic model to show that revenues are decreasing in sequential auctions with loss-averse bidders, due to a discouragement effect. Macera (2018) shows for a two-period moral hazard model with loss-averse agents that for the optimal contract wages are fixed and incentives are deferred into the future. To my best knowledge, Pagel is the first to rigorously apply Kőszegi and Rabin (2009) to dynamic problems with a longer time horizon. Pagel (2016) shows that dynamic reference-dependent preferences can explain the historical levels of equity premiums and premium volatility in asset prices. Related to the logic in the English auction, loss-averse agents dislike price fluctuations, which makes assets relatively unattractive. Pagel (2017) shows that dynamic reference-dependent preferences can explain empirical observations about saving schemes for life-cycle consumption.

To my best knowledge, my model is the first to analyze strategic interaction of loss-averse players in a dynamic game with more than two periods.

3 The Model

3.1 Auction Rules

There are n loss-averse bidders participating in an auction for a non-divisible good. Bidder i's intrinsic valuation θ_i is privately observed and independently drawn from a common distribution

$$\theta_i \sim G,$$

where G has a strictly positive, differentiable density g on support $[\theta^{\min}, \theta^{\max}]$, with $0 \leq \theta^{\min} < \theta^{\max}$.

For the Vickrey (second-price) auction, every bidder submits a sealed bid after learning her private valuation. Then the auction is resolved: the bidder with the highest bid receives the object and has to pay the amount of the second-highest bid.

For the English auction, I am considering a format sometimes referred to as the "Ascending Clock Auction" or the "Japanese Auction". Bidding starts at a fixed price and is raised incrementally by the auctioneer each time period. Each bidder signals—for example by raising or dropping her hand—when she wishes to drop out of the auction. Once a bidder dropped out she cannot bid again. The auction ends if there is only one active bidder left. This bidder has to pay the price at which the last of her opponents dropped out.

For simplicity, I assume that there is no reservation price in either auction.

3.2 Preferences

I follow Kőszegi and Rabin (2009) in how to model dynamic loss aversion: given their own bidding strategy, agents hold rational beliefs about winning the auction and the respective transfers made after the auction. These beliefs determine their initial reference point. For most of the exposition, I assume that bidders update their reference point instantaneously when they update their beliefs with respect to new information. In the Vickrey auction, the only information update takes place when the auction is resolved and the bidder learns whether she won and at which price. In contrast, in the English auction a bidder observes each period, whether any opponents drop out, and thus receives an information signal about the final outcome.

Let us denote by F_t^k a bidder's rational beliefs over final payoffs in $k \in \{money, good\}$, as anticipated at time t. When information revelation at any time t updates the reference point with respect to the auction outcome from F_{t-1}^k to F_t^k, the bidder experiences feelings of gains or losses. This psychological utility, called gain-loss utility, is denoted by $N(F_t^k|F_{t-1}^k)$. We assume throughout the paper that bidders are bracketing narrowly: they perceive gain-loss utility additively separated with respect to belief changes in money and good.

For the evaluation of gain-loss utility, bidders assign gains and losses to changes in the respective quantiles of the distribution function. Intuitively, they rank possible outcomes from worst to best and then evaluate changes to the worst, the second worst ,..., until the best outcome. Let us denote with $c_{F_t^k}$ the quantile function of F_t^k, which is mathematically the left-continuous inverse of

F_t^k. Then

$$N(F_t^k|F_{t-1}^k) = \int_0^1 \mu_k(c_{F_t^k}(p) - c_{F_{t-1}^k}(p))dp,$$

where the function μ_k measures feelings of gain and loss for respective belief changes. As a key feature, loss-averse bidders weight losses with respect to their reference distribution stronger than gains. Following Section IV in Kőszegi and Rabin (2006) and most of the literature, I take μ_k to be piecewise linear,

$$\mu_k(y) = \begin{cases} \eta_k y & y \geq 0, \\ \lambda_k \eta_k y & y < 0, \end{cases}$$

where $\eta_k > 0$, $\lambda_k > 1$. Moreover, I assume $\Lambda_k := \lambda_k\eta_k - \eta_k < 1$ for $k \in \{m, g\}$.[9]

The total utility from the auction is given by the classical utility from trade if the auction is won, and the accumulated gain-loss utility during the auction process. Suppose the English auction runs for at most T increments. For a fixed bidding strategy and induced initial beliefs F_0^m and F_0^g bidder i's total utility reads

$$u_i = \sum_{t=1}^{T} \big(N(F_t^m|F_{t-1}^m) + N(F_t^g|F_{t-1}^g)\big) + (\theta_i - x),$$

if bidder i wins the auction at a price of x.[10] In this case, the distribution F_T^m has unit mass on $-x$, whereas F_T^g has unit mass on θ. If bidder i loses the auction then her total utility reads

$$u_i = \sum_{t=1}^{T} \big(N(F_t^m|F_{t-1}^m) + N(F_t^g|F_{t-1}^g)\big),$$

where F_T^m and F_T^g both have unit mass on zero.

In contrast, in the Vickrey auction there is no updating before the auction is resolved. For the same fixed bidding strategy and initial belief as in the English auction, gain-loss utility consists only of the update from F_0^k to the auction

[9]The condition $\Lambda < 1$ is referred to as "no dominance of gain-loss utility" by Herweg et al. (2010) It ensures that the dislike for uncertainty isn't too strong. If $\Lambda > 1$ a bidder could potentially prefer a strictly dominated safe outcome to a lottery.

[10]The upper bound of T in the sum is without loss of generality; if the auction terminates early, all subsequent periods can be regarded as uninformative, and carry no further reference-dependent utility.

outcome F_T^k. Consequently, total utility is given by

$$u_i = N(F_T^m|F_0^m) + N(F_T^g|F_0^g)\big) + (\theta_i - x),$$

if the auction is won, and

$$u_i = N(F_T^m|F_0^m) + N(F_T^g|F_0^g)\big),$$

otherwise.

Before I define the appropriate equilibrium concept and derive optimal bidding strategies, the following example illustrates how gain-loss utility is formed during the auction process. It shows why loss-averse bidders prefer a Vickrey auction to an English auction—taken behavior of opponents as given.

Example 1. Consider an auction with two bidders. Bidder 1 (called the bidder) has a valuation of θ for the object. Suppose the bidder plans to drop out at a price of 8 and believes that the drop-out price of her opponent is ex ante uniformly distributed on $[0, 10]$ (we do not consider here, under which circumstances this behavior would be optimal). Ex ante, the bidder has a probability of 0.8 to win the auction and to have a payoff of θ in the good dimension. Thus, the ex ante quantile function for the good dimension is given by

$$c_{F_0^g}(p) = \begin{cases} 0 & p \leq 0.2, \\ \theta & p > 0.2. \end{cases}$$

Let us first consider the English auction. With every increase of the clock price, the bidder updates her beliefs in a Bayesian way. For each increment where her opponent does not drop out, her belief to win the auction decreases. If we use for the distribution subscript the current clock price rather than the time period, the updated quantile function in the good dimension for a clock price of y is given by

$$c_{F_y^g}(p) = \begin{cases} 0 & p \leq \frac{2}{10-y}, \\ \theta & p > \frac{2}{10-y}. \end{cases}$$

Suppose the opponent drops out at a price of 6. Figure 1 shows the quantile functions before the auction begins (dotted), at a clock price of 4 (dashed), an arbitrary small increment before 6 (solid), and after the drop out at 6 (solid constant function).

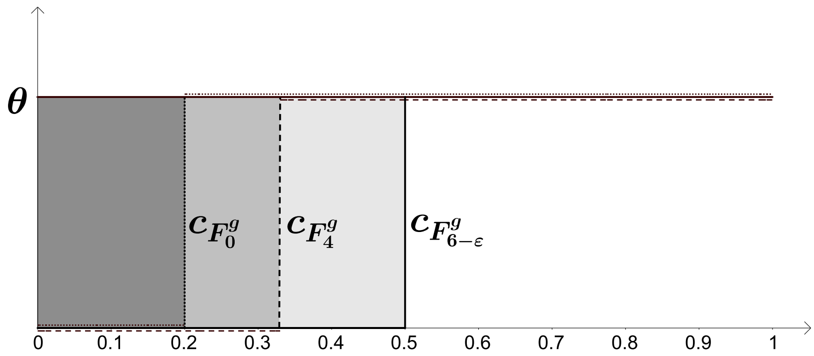

Figure 1: Updating in the English Auction

At a price of 4 the bidder's belief to lose has already increased from 0.2 to 1/3. The medium grey shaded area is proportional to the loss the bidder has accumulated up to the price of 4 as the integrated difference between the initial and current quantile function. Just before the opponent drops out at 6, the bidder's belief to lose has further increase to almost 0.5. The light shaded area shows the additional loss just before a price of 6 is announced. The losses are weighted with a factor of $\lambda\eta$.

The moment the price increases to 6, the opponent drops out and the bidder wins with certainty. The quantile function jumps to the constant function $c_{F_6^g} = \theta$, inducing a feeling of gain of η times the three combined shaded areas. Thus, the net gain-loss utility in the good dimension for the English auction is $(0.2\eta + 0.3(\eta - \lambda\eta))\theta = (0.2\eta - 0.3\Lambda)\theta$.

We now compare this gain-loss utility to the one from a Vickrey auction with the same valuation and the same bidding strategies. While the ex-ante belief is identically given by $c_{F_0^g}$, the only update takes place after the auction is resolved, and the belief jumps to the constant quantile function $c_{F_6^g}$. Thus, total gain-loss utility is given by $0.2\eta\theta$, hence proportional to only the dark grey area.

Intuitively, since losses are weighted stronger than gains, the fluctuation of beliefs in the English auction generates a net loss of $-0.3\Lambda\theta$ compared to the Vickrey auction. If the bidder could use a bidding proxy that enabled her to ignore new information in the English auction until the auction was over, she would forgo this unpleasant variation in beliefs, and receive the same utility as in a Vickrey auction. This logic is due to Kőszegi and Rabin (2009), who formally show in their Proposition 1 that, ceteris paribus, any collapse of information signals weakly increases agents' utility. The result that bidders in the English auction would prefer proxies to bid on their behalf is a testable prediction.

The updating with respect to money is a bit more complex than the updating in the good dimension: if an opponent does not drop out at some price, the probability of losing and paying nothing increases as well as the probability of paying a high price. Nevertheless the same intuition applies: fluctuations in beliefs are costly, and loss-averse bidders would prefer to get all information at once. The following Corollary summarizes the findings of the example. Formally, it is an immediate consequence of Proposition 1 in Kőszegi and Rabin (2009).

Corollary 1. *Loss-averse agents would prefer the use of proxies to bid on their behalf in the English auction. For a given set of bidders' maximal bids, any loss-averse bidder receives weakly higher utility in a Vickrey auction than in an English auction.*

3.3 Strategies and Equilibrium Concept

I apply the equilibrium concept of Kőszegi and Rabin (2009) to the auction framework. For full details and a psychological justification of the specific dynamic modeling choices, I refer to their paper.

Definition 1. A **bidding plan** b specifies an available action for every point in time and every possible history of information revelation. A **bidding strategy** $b(\theta)$ assigns to each possible type θ a bidding plan.

Consequently, in the English auction a bidding plan is a history-contingent plan for each increment to either remain or drop out, depending on the opponent drop-out history at that time. Note that for a two-bidder English auction such a plan can be described by a maximum bid $b \in \mathbb{R}_+$ up to which a bidder will decide to remain.[11] This is very similar to the Vickrey auction, where a bidding plan simply prescribes a sealed bid $b \in \mathbb{R}_+$ at the beginning of the auction.

I take the interim approach in the sense that first each bidder learns her valuation θ, and forms a bidding plan $b(\theta)$. Then, rational beliefs H_0 about the opponents' bidding plans define the bidder's initial reference point over payoffs as functions $F_0^k \equiv F_0^k(b, \theta, H_0)$. Then, the auction takes place.

[11] I relegate the more general formal description of a bidding plan in the English auction to Section 7.3, where I analyze the English auction for more than two bidders.

The Vickrey auction

To capture the static character of the Vickrey action, I assume there is no reference point updating until the auction is resolved. To keep notational similarity with the English auction, denote for a given maximal opponent bid x with $F_T^k(b,\theta)$ final payoffs from a bid b. Then, if a bidder forms the bidding plan to bid b^* but deviates to bid b instead, her utility is

$$u_0(b,\theta|b^*) = \underbrace{\sum_{k\in\{m,g\}} N(F_T^k(b,\theta)|F_0^k(b^*,\theta,H_0))}_{\text{gain-loss utility}} + \underbrace{\mathbb{1}_{b>x}(\theta - x)}_{\text{classical utility}}. \tag{1}$$

The English auction

In the English auction, a bidder not only updates the reference point with respect to new information at each increment, but also faces the opportunity to deviate to another bidding plan.

- At each period t, before the clock increases, each bidder may change from her bidding plan b^* to another plan b for the remaining of the auction. Such a deviation instantaneously changes rational beliefs about final payoffs. The belief change instantaneously induces a reference point update with gain-loss utility $N(F_t^k(b,\theta,H_t)|F_t^k(b^*,\theta,H_t))$ in both commodity dimensions.
- At each price increment between t and $t+1$ each bidder observes whether opponents drop out. Given a bidder's plan b, the respective update in their belief and their reference point instantaneously induces gain-loss utility $N(F_{t+1}^k(b,\theta,H_{t+1})|F_t^k(b,\theta,H_t))$ for both commodity dimensions.

After the auction is terminated, transfers are made according to the auction rules.[12]

In the following, we denote with

$$l_t(b,\theta,H_t) = \sum_{k\in\{money,good\}} \left(\sum_{s=t+1}^{T} N(F_s^k(b,\theta,H_s)|F_{s-1}^k(b,\theta,H_{s-1})) \right) \tag{2}$$

[12]For mathematical convenience, I abstract from tie breaking rules and assume that the good is not sold, if the remaining bidders drop out simultaneously. With our assumption of continuous density of types, as we let the increment size go to zero, this becomes equivalent to a tie breaking rule by coin-flip.

total gain-loss utility induced by plan b from time t onwards. Then total utility $u_t(b, \theta|b^*)$ from time t onwards if a bidder deviates at time t from plan b^* to plan b is given by

$$u_t(b, \theta|b^*) = u_c + \sum_{k \in \{money, good\}} N(F_t^k(b, \theta, H_t)|F_t^k(b^*, \theta, H_t)) + l_t(b, \theta, H_t), \quad (3)$$

where u_c is classical utility $\theta - x$ if the auction is won at some price x and zero otherwise.

We are ready to define the equilibrium concept used in both auction formats. Intuitively, a bidding plan constitutes a personal equilibrium if, given the reference point resulting from the plan, it maximizes expected utility at any point in time among all credible plans. In the following we denote with

$$U_t(b, \theta|b^*) \equiv \mathbb{E}_{H_t} u_t(b, \theta|b^*) \quad (4)$$

and

$$L_t(b, \theta|b^*) \equiv \mathbb{E}_{H_t} l_t(b, \theta|b^*) \quad (5)$$

the respective expected utilities at time t.

Definition 2. A bidding plan b^* constitutes a **personal equilibrium** (PE) for a bidder of type θ, if — given rational expectations derived from the plan — at all times t and all possible information revelations

$$U_t(b^*, \theta|b^*) \geq U_t(b, \theta|b^*), \quad (6)$$

for all credible bidding plans b that the bidder wants to carry through. A personal equilibrium is a **preferred personal equilibrium** (PPE) if it maximizes utility at time zero among all personal equilibria.

In practice, the set of credible plans for a given belief must be determined by thinking backwards. Crucially, the equilibrium concept implies that bidders don't have commitment power towards their future selfs in the sense that they cannot commit to bidding plans that they don't want to carry through at the time of actions. As we will see, committing to such unfavorable actions could be profitable, because it would alter beliefs, and therefore change gain-loss utility received during the auction.

The set of personal equilibria depends on the belief about other players' actions. To analyze the strategic interaction between multiple bidders, we focus on symmetric personal equilibria.

Definition 3. A bidding strategy **strategy** $b(\theta)$ constitutes a **(preferred) symmetric equilibrium** if for each type θ and the belief that all opponents bid according to strategy $b(\theta)$, the bidding plan $b(\theta)$ constitutes a (preferred) personal equilibrium.

4 The Vickrey Auction

In the Vickrey auction, the only decision is made at $t = 0$ where each bidder submits a bid $b^* \in \mathbb{R}_+$. By Definition 2, such a bid is a personal equilibrium if

$$U_0(b^*, \theta|b^*) \geq U_0(b, \theta|b^*)$$

for all $b \in \mathbb{R}_+$. This definition of a personal equilibrium for the special case of a single individual decision under uncertainty exactly coincides with the definition of an *unacclimating personal equilibrium (UPE)* in Kőszegi and Rabin (2007), as I show in the Appendix. It contrasts their concept of a *choice-acclimating personal equilibrium (CPE)*, which requires

$$U_0(b^*, \theta|b^*) \geq U_0(b, \theta|b)$$

for all $b \in \mathbb{R}_+$. Thus, in contrast to the UPE-bidder, a CPE-bidder—which is analyzed in Lange and Ratan (2010)—already internalizes the effects of her deviation on the reference point. While both concepts are frequently used in the literature, I focus on the UPE as a special case of the dynamic PE, in order to draw a clear comparison between the Vickrey auction and the English auction, and isolate the effect of dynamic information revelation. Moreover, it seems that a Vickrey auction, where bidders may form beliefs and bidding plans long before the auction starts, is a situation in which a bidder "anticipates the decision she faces but cannot commit to a choice until shortly before the outcome" as suggested for a UPE by Kőszegi and Rabin (2007).

Because it suffices for demonstrating the novel economic effect of information revelation and allows for a significantly simpler exposition, I first focus on the case in which bidders are loss averse in the good dimension only, i.e. $\eta_m = 0$. In

Section 7.1 I show that my results generalize to the case where we allow for loss aversion in the money dimension as well.[13]

Rational beliefs about winning the auction and the respective price are determined by the distribution over the maximal opponent bid. Fixing a bidder of type θ, we let $H(b)$ be the distribution over the maximal opponent bid. Since the bidder receives a payoff of θ if and only if she wins the auction, the distribution $F_0^g(b^*, \theta, H)$ has an atom of $H(b^*)$ at θ and an atom of $1 - H(b^*)$ at zero. For a given maximal opponent bid x, the gain-loss utility associated with bid b then reads

$$N(F_T^g(b,\theta)|F_0^g(b^*,\theta,H)) = \begin{cases} H(b^*)\mu(-\theta) & b \leq x, \\ (1-H(b^*))\mu(\theta) & b > x, \end{cases}$$

where the first line describes the feeling of loss if the agent loses the auction, and the second line describes the feeling of gain if she wins. Using (4) and (1) we obtain

$$\begin{aligned} U_0(b,\theta|b^*) &= \mathbb{E}_H\Big(H(b^*)\mu(-\theta)\mathbb{1}_{x\geq b} + (1-H(b^*))\mu(\theta)\mathbb{1}_{x<b} + \mathbb{1}_{x<b}(\theta - x)\Big) \\ &= \underbrace{(1-H(b))}_{\text{Prob to lose}}\underbrace{H(b^*)\mu(-\theta)}_{\text{feeling of loss}} + \underbrace{H(b)}_{\text{Prob to win}}\underbrace{(1-H(b^*))\mu(\theta)}_{\text{feeling of gain}} + \underbrace{\int_0^b (\theta - s)dH(s)}_{\text{classical utility}}. \end{aligned}$$

By definition bid b^* is a personal equilibrium if it maximizes $U_0(b,\theta|b^*)$ among all bids $b \in \mathbb{R}_+$. In a symmetric equilibrium the rational belief H is determined by the symmetric equilibrium bidding function. Thus, for any symmetric increasing equilibrium bidding function $b(\theta)$ the distribution of the maximal opponent is given by the first order statistic statistic of opponents' types, i.e.

$$H(b(\theta)) = G^{n-1}(\theta).$$

Proposition 1. *The unique symmetric increasing continuously differentiable PE in the Vickrey auction with n bidders who are loss averse with respect to the good*

[13] Horowitz and McConnell (2002) conclude in their summary that the endowment effect is "highest for non-market goods, next highest for ordinary private goods, and lowest for experiments involving forms of money". In this sense it may be plausible that loss aversion mainly applies to the good dimension.

is given by

$$b(\theta) = \big(1 + \eta(1 - G^{n-1}(\theta)) + \lambda\eta G^{n-1}(\theta)\big)\theta.$$

Note that all types overbid with respect to their intrinsic valuation θ. This should not be too surprising since we have assigned loss aversion only to the good dimension, and therefore made the good relatively more important, compared to money. More interestingly, the degree of overbidding is increasing in the type. The lowest type moderately overbids by

$$b(\theta^{\min}) = (1 + \eta)\theta^{\min},$$

while the highest type aggressively overbids by

$$b(\theta^{\max}) = (1 + \lambda\eta)\theta^{\max}.$$

The reason is the so called attachment effect: high types believe to win. Not winning would create a feeling of loss, which they try to prevent by placing an aggressive bid. As we will see in Section 7.1, this intuition remains intact, if we allow for loss aversion in money as well.

5 The English Auction for Two Bidders

I analyze the set of symmetric equilibria in the English auction with two bidders, as the increment size goes to zero. Again, for ease of exposition, I first restrict attention to loss aversion in the good dimension, and relegate the case of loss aversion in both dimensions to Section 7.1 . In Section 7.3, I show that the main insights generalize to the n bidder auction. While the history-dependent strategy space in an n-bidder English auction is huge, it is fairly simple in a two-bidder game. Given type θ, a bidding plan prescribes the price at which the bidder plans to drop out, provided that the opponent is still active.

Each period the bidder observes whether her opponent remains in the auction. This information permanently updates her reference point, which induces gain-loss utility in each increment. An optimal bidding strategy will take the expected gain-loss utility from news into account.

For calculating the ex-ante expected gain-loss utility, it is more convenient to work with distribution functions rather than with quantile functions. This is possible, since they are generalized inverses of each other, and the integral between functions equals the integral between their inverses up to the sign:

Lemma 1. *Let F_1 and F_2 be distributions on some interval $[a, b]$ and let c_{F_1}, c_{F_2} be the respective quantile functions. Then*

$$\int_a^b \mu(F_1(x) - F_2(x))dx = \int_0^1 \mu(c_{F_2}(p) - c_{F_1}(p))dp.$$

With this result, one can look at the expected disutility from news.

Lemma 2. *Suppose that a loss-averse agent's payoff is distributed according to F_1 with probability Δ, and according to F_2 with probability $1 - \Delta$. Let $[a, b]$ contain the supports of F_1 and F_2. Denote with $F = \Delta F_1 + (1 - \Delta)F_2$ the ex ante distribution of payoffs. Then the ex ante expected reference-dependent utility from learning, whether the distribution is F_1 or F_2, is given by*

$$\mathbb{E}(N(F_i|F)) = -\Delta\Lambda \int_a^b |F(x) - F_1(x)|dx.$$

The intuition for the result is as follows: on average, there is "as much good news as bad news". If gains and losses were weighted equally, one would have zero gain-loss utility in expectation. Since losses loom larger than gains, news will generate negative utility in expectation where the amount of negative utility is proportional to the expected variation and the loss dominance parameter Λ.

With this result we can calculate the accumulated expected loss due to gain-loss utility, as the increment size goes to zero. In the following, it is convenient to use the time subscript t for the current clock price rather than the number of increments. Let us denote with F the distribution of the opponent's drop-out price, in the sense that an opponent with drop-out price y remains in the auction at any clock price $t < y$, and drops out at prices $t \geq y$.

Proposition 2. *Consider a loss-averse bidder of type θ in the English auction with one opponent and increments of ε. Let the opponent's drop-out price be distributed according to distribution F with density f. Suppose the bidder plans to drop out at b, and the opponent hasn't dropped out until price $t < b$. Then, for ε going to zero, in the limit the ex ante expected marginal gain-loss utility at* *price t is given by*

$$\ell_t(b,\theta,F) = \frac{-f(t)}{(1-F(t))^2}(1-F(b))\Lambda\theta.$$

Expected gain-loss utility for the remaining of the auction at time t is in the limit given by

$$L_t(b,\theta,F|b) = \ln\left(\frac{1-F(b)}{1-F(t)}\right)\frac{1-F(b)}{1-F(t)}\Lambda\theta.$$

Note that the amount of marginal disutility from expected gain-loss utility is decreasing in b: an aggressive strategy induces less belief fluctuation at each information update, and thus partly insures against high gain-loss disutility in each increment. There is, however, a countervailing effect on total gain-loss disutility: the higher bidder's drop-out price, the longer she may stay in the auction and be exposed to gain-loss disutility. Figure 2 shows total expected gain-loss disutility at the beginning of the auction for $F \sim U[0,1]$. We see that losses are the strongest for intermediate bids, which induce the highest uncertainty. Bidding 0 or 1 induces no uncertainty, and therefore no gain-loss utility.

In the following, we refer to the limit result as we let the increment size go to zero as the *continuous English auction.*[14]

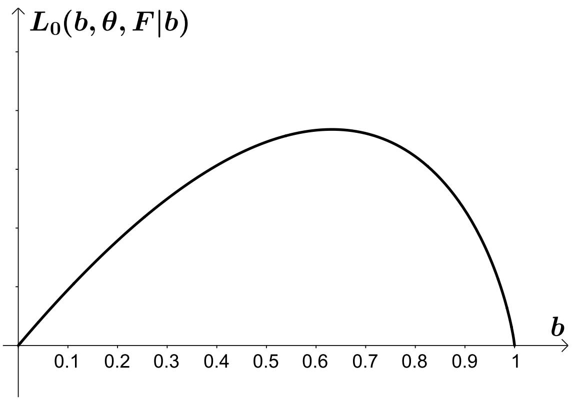

Figure 2: Total Expected Loss for $F \sim U[0,1]$

With $L_t(b^*,\theta|b^*)$ we have established the expected gain-loss utility *on equi-*

[14]This notion does not intend to refer to the concept of *continuous games* by Simon and Stinchcombe (1989). One should still regard the game as one with discrete increments on the clock which are, however, arbitrarily small.

librium path for an equilibrium strategy b^*. We now calculate the instantaneous gain-loss utility that the bidder perceives, if she decides to *deviate* from strategy b^* to strategy b at some point in time:

Lemma 3. *If at time t the bidder changes her strategy from dropping out at $b^* \geq t$ to dropping out at $b \geq t$, this deviation induces an instantaneous gain-loss utility of*

$$N(F_t(b,\theta,F)|F_t(b^*,\theta,F)) = \frac{\mu(F(b)-F(b^*))}{1-F(t)}\theta.$$

Summarizing Proposition 2 and Lemma 3 we obtain that for the continuous English auction expected utility from time t onwards is given by

$$U_t(b,\theta,F|b^*) = \underbrace{\frac{\int_t^b(\theta-s)dF(s)}{1-F(t)}}_{\text{classical utility}} + \underbrace{\frac{\mu(F(b)-F(b^*))}{1-F(t)}\theta}_{\substack{\text{one-time gain/loss}\\ \text{from deviating}}} + \underbrace{L_t(b,\theta,F|b).}_{\substack{\text{expected gain-loss utility}\\ \text{of remaining auction}}} \tag{7}$$

All three terms change if a bidder deviates from b^* to b at some time. Note that the deviation utility is non-differentiable at $b = b^*$, since μ has a kink at zero.

Recall that a plan b^* is a personal equilibrium if

$$b^* = \arg\max_b U_t(b,\theta,F|b^*)$$

at all times t. In particular, bidding up to b^* it must be optimal as t approaches b^*. This leads to the following constraint on time consistent plans.

Lemma 4. *Let the opponent's drop-out price be distributed according to distribution F with non-zero density f on some positive support $[a,c]$. Then, for the continuous English auction any time consistent bidding strategy $b \in (a,c)$ satisfies*

$$b \leq (1+\eta)\theta.$$

To understand the significance of this result, it is insightful to look at plans the bidder would choose if she could commit to a bidding strategy before the auction starts. She would not like to deviate from a strategy ex ante if and only if

$$U_0(b,\theta,F|b^*) \leq U_0(b^*,\theta,F|b^*)$$

for all b.

Lemma 5. *If two loss averse bidders could commit ex ante to a bidding strategy in the continuous English auction, the lowest symmetric increasing differentiable equilibrium would satisfy*

$$b(\theta) = \big(1 + \eta - \Lambda(1 + \ln(1 - G(\theta)))\big)\theta.$$

Figure 3 shows the ex ante optimal strategy (solid function) and the boundary of time-consistent strategies (dashed line) for two loss averse bidders.

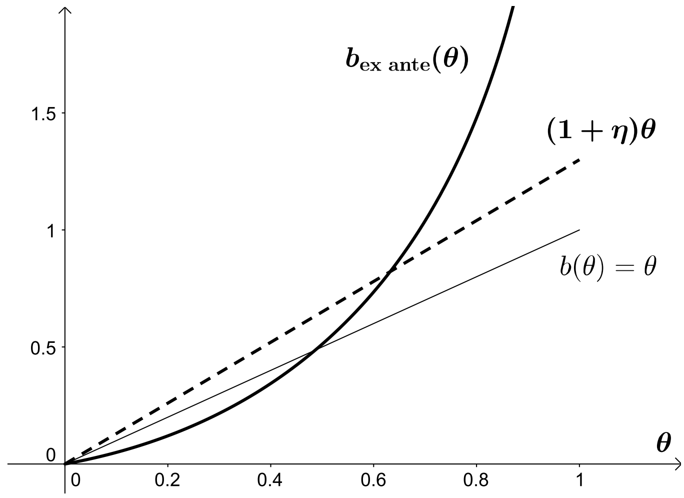

Figure 3: $G(\theta) \sim U[0,1]$, $\eta = 0.3$, $\lambda = 4$

We see that low types ex ante may wish to underbid, while high types wish to strongly overbid. The intuition here is the same as in the Vickrey auction: bidders want to reduce expected gain-loss utility, and therefore try to reduce the uncertainty about winning. Moreover, high types would wish to insure with an aggressive bid against belief fluctuations during the auction process.

However, it is time-inconsistent to bid above $b = (1 + \eta)\theta$. Even though a bidder with a high valuation would ex ante like to commit to an aggressive bidding strategy, at the time she has to do so, she is not any more willing to carry that action through: as the auction proceeds, the winning chances for the bidder gradually decline. Thus, she gradually becomes a low type with respect to the remaining auction, and therefore her initial strategy of overbidding becomes less appealing. Just one increment before the bidder's drop out, she perceives the remaining auction similarly as a Vickrey auction, where she has the lowest possible type. Hence, at that point in time, her optimal bidding strategy resembles that of the lowest type in the Vickrey auction, i.e. she bids no more than $b = (1+\eta)\theta$.

We have so far only considered constraints on equilibrium behavior at time 0 and at time b. It turns out that these are the binding constraints.

Lemma 6. *Consider a loss-averse bidder of type θ in the continuous English auction with one opponent. Let the opponent's drop-out price be distributed according to distribution F with non-zero density f on some positive support $[a, c]$. Then a strategy $b^* \in (a, c)$ is a PE if and only if*

1. $b^* \leq (1+\eta)\theta$;
2. *for any* $b \in [b^*, (1+\eta)\theta]$ *we have* $U_0(b^*, \theta, F|b^*) \geq U_0(b, \theta, F|b^*)$.

With this result we can now characterize the symmetric equilibria of the English auction.

Proposition 3. *An increasing, almost everywhere differentiable function $b(\theta)$ is a symmetric equilibrium in the continuous English auction with two loss averse bidders if and only if for all θ*

1. $b(\theta) \leq (1+\eta)\theta$;
2. $b(\theta) \geq \min\{(1+\eta)\theta\ ;\ \big(1+\eta-\Lambda(1+\ln(1-G(\theta)))\big)\theta\ \}$.

Thus, any increasing smooth function in the the gray shaded area of Figure 4 constitutes a symmetric equilibrium.

The thick line indicates the preferred symmetric equilibrium (PPE). Point A, where the PPE hits the boundary of time consistent strategies can be easily determined:

$$\big(1+\eta-\Lambda(1+\ln(1-G(\theta)))\big)\theta = (1+\eta)\theta$$

if and only if $G(\theta) = 1 - 1/e \approx 0.632$.

Note that the PPE is tangent to $(1+\eta-\Lambda)\theta$ at the lowest type. Hence there is underbidding for low types if and only if $\eta - \Lambda < 0$, thus if and only if $\lambda > 2$.

Corollary 2. *The symmetric PPE in the continuous English auction with two loss averse bidders is given by*

$$b_{PPE}(\theta) = \begin{cases} \big(1+\eta-\Lambda(1+\ln(1-G(\theta)))\big)\theta & G(\theta) \leq 1-1/e, \\ (1+\eta)\theta & G(\theta) > 1-1/e. \end{cases}$$

Low types underbid their intrinsic valuation θ in the PPE if and only if $\lambda > 2$.

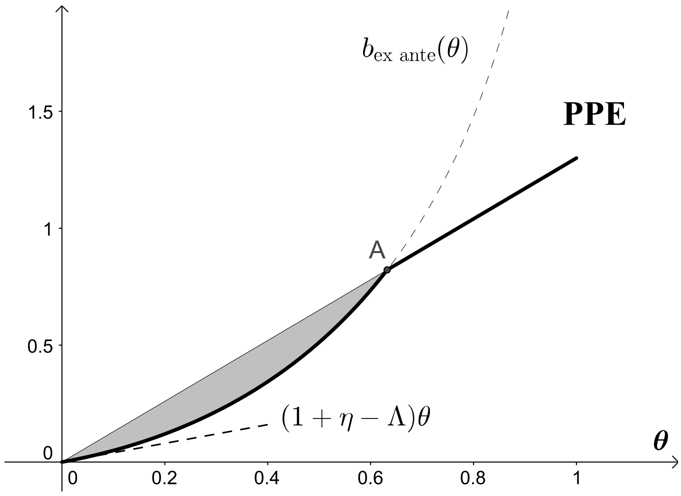

Figure 4: $G(\theta) \sim U[0,1]$, $\eta = 0.3$, $\lambda = 4$

6 Revenue Comparison

The equilibrium bidding function of an English auction with loss-averse bidders strongly depends on the question of how quickly new information is absorbed into the reference point.

If the reference point consists of lagged beliefs, and the lag is sufficiently high, new information during the auction process will have no impact on bidders' reference point. If values are private, there is therefore no impact of information gathered during the auction process. Each bidder will form her optimal decision with respect to the initial belief, and thus faces the same objective function as in the Vickrey auction—the strategic equivalence between English and Vickrey auction remains.

If bidders, however, update their reference point dynamically with respect to new information, loss-averse bidders bid at most $(1+\eta)\theta$.

The following figure shows the equilibrium bidding function for the Vickrey auction, $b_{\text{Vickrey}}(\theta)$, and the PPE of the English auction with dynamic reference point updating, $b_{\text{English}}(\theta)$. The shaded area indicates the potential other symmetric equilibria in the English auction, which are bounded by the line $(1+\eta)\theta$.

As we have seen in Section 4, overbidding with respect to θ is moderate for low types and strong for high types in the Vickrey auction. We can see that $b_{\text{Vickrey}}(\theta)$ at the lowest type is tangent to $(1+\eta)\theta$—the upper bound of equilibria in the

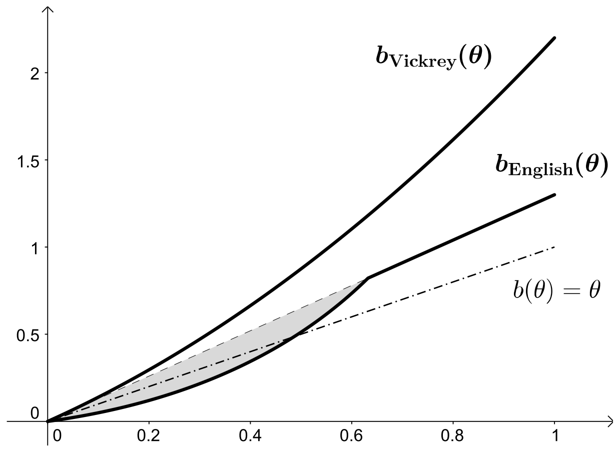

Figure 5: $G(\theta) \sim U[0,1]$, $\eta = 0.3$, $\lambda = 4$

English auction. The intuition is that for low types the decision problem in both auction formats becomes increasingly similar: since bidders in the English auction only learn whether there are opponents with lower valuation than their own, the information difference between the two auction formats at the time the bidder places her (maximal) bid is small for low types.

Since the bidding function in the Vickrey auction satisfies $b_{\text{Vickrey}}(\theta) > (1+\eta)\theta$ for all types $\theta > \theta^{\min}$, it is immediate that the Vickrey auction dominates the English auction with respect to revenue.

Proposition 4.
1. *If bidders are loss averse and do not update their reference point during the auction process, the Vickrey auction and the English auction are strategically equivalent: for a given continuous belief on the maximal opponent bid, a bid b is a personal equilibrium in the Vickrey auction if and only if bidding up to b is a personal equilibrium in the English auction.*

2. *If bidders are loss averse and update their reference point instantaneously during the auction process, equilibrium bids of the lowest type may coincide for both auction formats. For all other types, the Vickrey auction attains strictly higher revenue than the English auction.*

7 Extensions and Robustness

7.1 Loss Aversion in the Money Dimensions

We generalize the baseline model to the case where bidders are loss averse in both commodity dimensions—money and good.

The Vickrey Auction

Denote in short with $F_b \equiv F_0^m(b, \theta, H)$ the distribution of payments for a submitted bid b given the continuous distribution of the highest opponent bid H. Since with probability $1 - H(b)$ a bidder loses and pays nothing, the distribution F_b is given by

$$F_b(s) = \begin{cases} 1 - H(b) + H(s) & s \leq b, \\ 1 & s > x, \end{cases}$$

For a reference point induced by bid b^* a realized payment of x induces gain-loss utiltiy

$$\begin{aligned} \int_0^1 \mu_m(c_{F_{b^*}}(p) - x)dp &= \int_0^\infty \mu_m(s - x)dF_{b^*}(s) \\ &= (1 - H(b^*))\mu_m(-x) + \int_0^{b^*} \mu_m(s - x)dH(s), \end{aligned}$$

where for the first equality we used that integration by dF_{b^*} is the pushforward of Lebesgue integration under $c_{F_{b^*}} : (0, 1) \to \mathbb{R}^+$ (see, for instance, Theorem 1.104 in Klenke (2013)).

Hence, expected gain-loss utility in the money dimension from a bid b when the reference point is given by bid b^* is

$$\begin{aligned} L_0^m(b, \theta, H|b^*) &= \int_0^\infty \left((1 - H(b^*))\mu_m(-x) + \int_0^{b^*} \mu_m(s - x)dH(s) \right) dF_b(x) \\ &= \int_0^b \left((1 - H(b^*))\mu_m(-x) + \int_0^{b^*} \mu_m(s - x)dH(s) \right) dH(x) \\ &\quad + (1 - H(b)) \int_0^{b^*} \mu_m(s)dH(s), \end{aligned}$$

where we used that x is zero with probability $1 - H(b)$. Intuitively, the first summand is the loss from winning and paying unexpectedly, the second summand is gain-loss utility from winning at a price different than expected, and the third summand is the gain from losing unexpectedly and not paying. For total expected utility we plug together the derived gain-loss utility in the money dimension with classical utility and gain-loss utility in the good dimension as derived in Section 4.

$$\begin{aligned} U_0(b,\theta|b^*) = & \int_0^b (\theta - x)dH(x) + (1-H(b))H(b^*)\mu_g(-\theta) + H(b)(1-H(b^*))\mu_g(\theta) \\ & + (1-H(b^*)\int_0^b \mu_m(-x)dH(x) + \int_0^b \int_0^{b^*} \mu_m(s-x)dH(s)dH(x) \\ & + (1-H(b))\int_0^{b^*} \mu_m(s)dH(s). \end{aligned}$$

In equilibrium the order statistic H is again endogenously determined by the opponents' equilibrium bids $b(\theta_{-i})$. Using the opponents' response functions, it is straightforward to calculate the symmetric equilibrium bidding function:

Proposition 5. *The unique symmetric increasing continuously differentiable UPE for n loss averse bidders in the Vickrey auction for commodities is given by*

$$\begin{aligned} b(\theta) = & \frac{1+\eta_g+\Lambda_g G^{n-1}(\theta)}{1+\lambda_m\eta_m}\theta \\ & + \int_{\theta^{min}}^{\theta} \frac{\Lambda_m(1+\eta_g+\Lambda_g G^{n-1}(x))}{(1+\lambda_m\eta_m)^2} x \exp\left(\frac{\Lambda_m}{1+\lambda_m\eta_m}(G^{n-1}(\theta)-G^{n-1}(x))\right) dG(x). \end{aligned}$$

Note that

$$b(\theta^{\min}) = \frac{1+\eta_g}{1+\lambda_m\eta_m}\theta^{\min},$$

while for any $\theta > \theta^{\min}$

$$b(\theta) > \frac{1+\eta_g+\Lambda_g G^{n-1}(\theta)}{1+\lambda_m\eta_m}\theta > \frac{1+\eta_g}{1+\lambda_m\eta_m}\theta.$$

In particular, for equally weighted loss aversion in both dimensions, low types underbid, while

$$
\begin{aligned}
b(\theta^{\max}) &> \frac{1+\eta+\Lambda G^{n-1}(\theta^{\max})}{1+\lambda\eta}\theta^{\max} \\
&= \frac{1+\eta+\Lambda}{1+\lambda\eta}\theta^{\max} \\
&= \theta^{\max}
\end{aligned}
$$

shows that high types overbid their intrinsic valuation. The intuition is that low types don't expect to win and try to avoid unexpected losses in the money dimension. In contrast, high types expect to win and try to avoid unexpected losses in the good dimension.

The English Auction

We avoid to fully classify the set of symmetric PE again, but rather straightforwardly prove that the revenue ranking between the two auction formats remains intact.[15] The following Lemma parallels Lemma 4.

Lemma 7. *Consider a loss-averse bidder of type θ in the continuous English auction with one opponent. Let the opponent's drop-out price be distributed according to distribution F with nonzero density f on some positive support $[a, b]$. Then, any time consistent bidding strategy $x \in (a, b)$ satisfies*

$$x \leq \frac{1+\eta_g}{1+\lambda_m\eta_m}\theta.$$

Again, the bidders of high type ex ante like to commit to excessive bids, but they know that the plan to bid above the threshold of $\frac{1+\eta_g}{1+\lambda_m\eta_m}\theta$ is time-inconsistent. Just one increment before they drop out, their belief to win and pay is virtually zero and—similarly to the lowest type in the Vickrey auction—they trade off the unexpected gain of the good against the unexpected loss in money, which may both occur with very small probability. If loss aversion is equally pronounced in both dimensions, then bidders underbid their intrinsic value θ, since losses are weighted stronger than gains.

Revenue Comparison

[15]The full derivation of the symmetric equilibrium bidding functions is available on request.

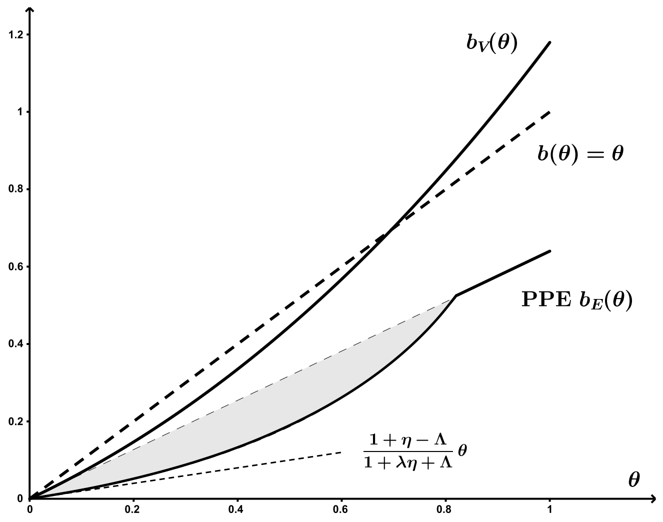

Figure 6: $G(\theta) \sim U[0,1]$, $\eta = 0.4$, $\lambda = 3$

Since in the Vickrey auction we have

$$b_{\text{Vickrey}}(\theta) \geq \frac{1+\eta_g}{1+\lambda_m\eta_m}\theta,$$

with equality only for $\theta^{\min}$, and in the English auction we have

$$b_{\text{English}}(\theta) \leq \frac{1+\eta_g}{1+\lambda_m\eta_m}\theta,$$

it is immediate that the Vickrey auction remains to dominate the English auction with respect to revenue. Figure 6 shows the gray shaded area of potential equilibria in the English auctions, together with its PPE, and the equilibrium in the Vickrey auction. If loss aversion is equally pronounced in both dimensions, there is unambiguously underbidding in the English auction, while in the Vickrey auction low types underbid and high types overbid.

7.2 False Beliefs or Heterogeneous Preferences

So far we have assumed that all participating bidders are loss averse and hold rational beliefs over opponents' behavior. This is not a crucial assumption. Loss-averse bidders will bid higher in the Vickrey auction than in the English auction for *any* continuous belief with full support that they hold over opponents' strategies.

Following the analysis of Section 4, Equation 10 in the proof of Proposition 1 states that for any such belief H the bidding function in the Vickrey auction is given by

$$b(\theta) = \big(1 + \eta(1 - H(b(\theta))) + \lambda\eta H(b(\theta))\big)\theta,$$

which shows that

$$b(\theta) > (1 + \eta)\theta$$

for all types, who win with positive probability. Contrary, in the English auction Lemma 4 shows that for any such belief

$$b(\theta) \leq (1 + \eta)\theta.$$

7.3 Generalization to n bidders

In auctions where bidders face more than one opponent, the set of possible bidding plans becomes very large. Recall that a bidding plan prescribes a consistent action for any history and any future contingency at any time. While in the two bidder case the history is rather simple—either the opponent dropped out and the auction is over, or we are still in the auction process—with more bidders the individual decision at each time may in principle depend on the exact timing at which opponents dropped out in the past.

Since each decision must be sequentially optimal, given expectations about the future, one might hope to be able to restrict to Markov perfect equilibria, in the sense that at time t the individual type θ_i and the number of currently active bidders is a sufficient statistic for the optimal decision of bidder i. However, this is not the case. While the set of personal equilibria starting at time t can be determined without looking into the past, the specific equilibrium path will depend on the evolution of beliefs up to time t.

In order to deal with strategies contingent on histories, we introduce the following notation:

Definition 4. For any n-bidder auction, define for all $k \in \{0, ..., n-2\}$

$$H_k = \{(t_1, ..., t_k) | 0 \leq t_1 \leq, ..., \leq t_k\}$$

as the set of histories / future contingencies with k drop outs at the respective prices $t_1, ..., t_k$, with the convention $H_0 = \{\emptyset\}$.

With this notation, a complete bidding plan prescribes for each history and future contingency the price at which a bidder of type θ plans to drop out:

Definition 5. A pure strategy bidding plan prescribes a bidding strategy

$$b : \bigcup_{0 \le k \le n-2} H_k \times [\theta^{\min}, \theta^{\max}] \to \mathbb{R}_+,$$

with the restriction that for any $(t_1, ..., t_k, \theta)$ we have

$$b(t_1, ..., t_k, \theta) > t_k,$$

The latter condition on the bidding function ensures that bidders cannot condition their drop out on events that happen after the drop out.

Again, we restrict attention to differentiable and increasing equilibrium bidding functions in the following sense:

Definition 6. A bidding strategy b in the English auction is differentiable and increasing if for all $(t_1, ..., t_k) \in \bigcup_{0 \le k \le n-2} H_k$ the function $b(t_1, ..., t_k, \theta)$ is differentiable and increasing in θ.

Example 2. Consider a continuous English auction with three loss-averse bidders. A complete strategy prescribes for every θ:

- A price $b(\theta)$ for which a bidder drops out if no opponent dropped out before
- For any opponent drop out at some price $t < b(\theta)$, a price $b(t, \theta)$ at which the bidder drops out in the subsequent two-bidder auction

The aim of the example is to illustrate why the optimal strategy $b(t, \theta)$ for the two-bidder auction following the first drop out depends on t. Suppose that all three bidders bid according to the same symmetric equilibrium bidding strategy $(b(\theta), b(t, \theta))$. Let us focus on the decision problem of a bidder, whose valuation θ is sufficiently high, such that $b(\theta) = (1 + \eta)\theta$ were the only time-consistent strategy in the two-bidder English auction.

Suppose first that an opponent has a valuation of zero and drops out at $t = 0$. For the strategy $b(0, \theta)$ the bidder is now bound by the set of time-consistent strategies of the two-bidder auction, as outlined in Proposition 3. Since she has high beliefs to win, the only time-consistent strategy is $b(0,\theta) = (1+\eta)\theta$.

Next, we analyze optimal strategies $b(t,\theta)$ for t being smaller, but close to $b(\theta)$. Similar to the two-bidder auction, a bidder with a high winning probability would ex ante like to insure against belief fluctuations with an aggressive strategy. Any strategy for $b(t,\theta)$, however, must be time consistent in the sense that the bidder is willing to stick to it until t. Just before t the belief to win the auction has decreased considerably. The bidder trades off the expected gains from trade against the expected loss from news. The following Lemma states the expected loss at time t for the three bidder case.

Lemma 8. *Consider a continuous English auction with three loss-averse bidders. Assume all bidders follow a symmetric, differentiable, increasing bidding strategy $(b(\theta), b(t,\theta))$. Assume further that no bidder dropped out until $t \in [b(\theta^{min}), b(\theta^{max})]$. Let $\theta(t)$ be defined by $b(\theta) = t$. Then expected gain-loss utility at time t is given by*

$$L_t(\theta) = -\Lambda\theta \int_{\theta(t)}^{\theta} \frac{2g(s)(1-G(s))}{(1-G(\theta(t)))^2} \Bigg[\underbrace{\frac{G(\theta)-G(s)}{1-G(s)} - \left(\frac{G(\theta)-G(s)}{1-G(s)}\right)^2}_{A} \underbrace{- \ln\left(\frac{1-G(\theta)}{1-G(s)}\right) \frac{1-G(\theta)}{1-G(s)}}_{B} \Bigg] ds.$$

The terms of $L_t(\theta)$ are easy to interpret. At time t the conditional marginal probability that the first drop out is of type s is given by $\frac{2g(s)(1-G(s))}{(1-G(\theta(t)))^2}$. In this case, the bidder would update the winning probability from $\left(\frac{G(\theta)-G(s)}{1-G(s)}\right)^2$ to $\frac{G(\theta)-G(s)}{1-G(s)}$ (term A). Further, term B shows the expected loss for the following 2-bidder auction, as calculated in Proposition 2.

Term A indicates an additional source of expected gain-loss disutility, compared to the two bidder auction: even if a bidder loses after all, beliefs to win don't necessarily gradually decline to zero, but might temporarily increase due to one opponent dropping out. This effect leads to more belief fluctuations and worsens bidder's trade-off between expected news disutility and expected gains from trade. As a result, it is no longer time consistent to bid up to $b(t,\theta) = (1+\eta)\theta$ for all t.

Corollary 3. *In any symmetric, increasing, differentiable equilibrium $(b(\theta), b(t,\theta))$ of the English auction with three loss-averse bidders, expected news disutility for* *any* $\theta \in (\theta^{min}, \theta^{max})$ *satisfies*

$$\lim_{t \to b(\theta)} \frac{L_t(\theta)}{\left(\frac{G(\theta)-G(\theta(t))}{1-G(\theta(t))}\right)^2} = -2\Lambda\theta.$$

If $b(t,\theta)$ *is continuous in* t*, then—by time-consistency—*

$$\lim_{t \to b(\theta)} b(t,\theta) \leq (1+\eta-\Lambda)\theta.$$

Since we have argued above that $b(0,\theta) = (1+\eta)\theta$, the corollary illustrates that bidding behavior $b(t,\theta)$ in general depends on opponents' drop-out history t.

Even if the sales price depends on all type realizations, it is immediate that for n bidders the revenue ranking between the two auction format remains: since bidders generically don't share the same valuation, in any symmetric continuous increasing equilibrium they will drop out of the auction consecutively, in order of their types. Eventually, with probability one, the two bidders with the highest valuation will end up in the two-bidder subgame. Here they are bound to the constraints on time-consistent behavior, as analyzed in section 5. In particular by Lemma 4, any time-consistent strategy for the two-bidder auction satisfies $b(\theta) \leq (1+\eta)\theta$.

To summarize:

Corollary 4. *In a symmetric increasing equilibrium of the continuous English auction with* n *loss-averse bidders, the revenue may depend on all type realizations. For any opponent drop-out history, every bidder's maximal bid is bounded by* $b(\theta) \leq (1+\eta)\theta$*. Thus, with* n *loss-averse bidders, the English auction remains to yield lower revenues than the Vickrey auction.*

Even if the auction outcome for many bidders is similar to the one for two bidders, it is worth noting that individual bidders obtain less utility, compared to two-bidder auctions with the same sales price. To see this, consider—hypothetically—that bidders could choose not to observe individual drop outs, but rather learn in each period, whether *any* opponent is still in the game. The auction would then subjectively resemble an English auction with two bidders, where the opponent's type is drawn from the first order-statistic over all opponents. The key difference is that information is fluctuating much less. As already mentioned earlier and stated in generality in Proposition 1 of Kőszegi and Rabin (2009), the collapse of multiple signals into one will always weakly decrease gain-loss disutility.

8 Conclusion

I studied the effects of expectation-based preferences in dynamic environments with strategic interaction, comparing the dynamic English auction to the static Vickrey auction. If the reference point is static and doesn't respond to the arrival of new information, there is no strategic difference between the English auction and the Vickrey auction. If bidders update their reference point instantaneously with respect to new information, however, dynamic information in the English influences the bidders' endogenous preferences, and thus their bidding strategies. The classical strategic equivalence between the the two auction formats breaks down and the English auction attains strictly lower revenue than the Vickrey auction.

This difference highlights the importance of understanding the evolution of the reference point in dynamic environments. In particular, research about the speed of reference point adaptation with respect to new information is still in its infancy and deserves further study.

The non-equivalence of the two auction formats stands in sharp contrast to the revenue equivalence principles by Vickrey (1961) and Myerson (1981). Indeed, the powerful approach of mechanism design and the revelation principle relies on the assumption that agents' valuations are exogenously given and do not depend on the choice of mechanism. This assumption is violated if bidders have endogenous preferences that depend on expectations induced by the mechanism itself. In particular, if agents update their reference point with respect to new information in a multi-stage mechanism, such a mechanism cannot be replaced by a simple direct mechanism without changing agents' incentives. The failure of the revelation principle naturally leads to the question of optimal mechanism design in dynamic environments with expectation-based loss-averse agents. The study of optimal expectation management in these environments is an interesting question left for future research.

9 Appendix

Equivalence of the UPE and the PE for a Single Decision under Uncertainty

This paragraph formally derives how the concept of a personal equilibrium (PE) in Kőszegi and Rabin (2009) for our special case of a single decision under uncertainty coincides with the concept of an unacclimated personal equilibrium (UPE) for static decision problems in Kőszegi and Rabin (2007).

Since utility is additively separable across different commodity dimensions, it suffices to consider one dimension. For the framework of Kőszegi and Rabin (2009) suppose that a person in period 0 chooses an action from some choice set D. The action is characterized by its distribution G of payoffs in period 1. The distribution G determines the reference point for the payoffs. Utility from a payoff x in period 1 is then given by

$$u_1(x) = x + \int_0^1 \mu(x - c_G(p))dp.$$

Choosing some action with distribution F when the reference point is G therefore induces expected utility of

$$U(F|G) = \int_{-\infty}^{\infty} \left(x + \int_0^1 \mu(x - c_G(p))dp \right) dF(x). \tag{8}$$

By definition in Kőszegi and Rabin (2009) an action with distribution G is a personal equilibrium if it maximizes expected utility, given its induced beliefs, i.e. if $U(G|G) \geq U(F|G)$ for all $F \in D$. Similarly, by Equation (2) in Kőszegi and Rabin (2007) expected utility of a payoff distribution F when the reference point is G is given by

$$U(F|G) = \int_{-\infty}^{\infty} \left(x + \int_{-\infty}^{\infty} \mu(x - s)dG(s) \right) dF(x), \tag{9}$$

and G is a UPE if $U(G|G) \geq U(F|G)$ for all $F \in D$. It therefore remains to show that

$$\int_0^1 \mu(x - c_G(p))dp = \int_{-\infty}^{\infty} \mu(x - s)dG(s),$$

such that the definitions of $U(F|G)$ in equation (8) and (9) coincide. For continuously increasing distributions G this is a consequence of integration by substitution. For general distributions it follows from the fact that integration $\int \cdot\, dG(x)$ is the pushforward measure of the Lebesgue measure under $c_G : (0,1) \to \mathbb{R}$ (c.f. Theorem 1.104 in Klenke (2013)).

Proofs

Proof of Proposition 1. Suppose that all opponents bid according to some increasing, continuously differentiable bidding function $b(\theta)$. Since $G(\theta)$ is a distribution with strictly positive, continuous density g, it follows that the distribution of the maximal opponent bid, $H(x) = G^{n-1}(b^{-1}(x))$, is a differentiable distribution with positive, continuous density $h(x)$ on $[b(\theta^{\min}), b(\theta^{\max})]$ as well.

The bidding function $b(\theta)$ constitutes a PE if and only if the utility function $U_0(x, \theta|b(\theta))$ attains its maximum at $x = b(\theta)$ for all θ. Differentiation with respect to x yields

$$\frac{\partial U_0(x, \theta|b(\theta))}{\partial x} = h(x)(1 - H(b(\theta)))\mu(\theta) - h(x)H(b(\theta))\mu(-\theta) + (\theta - x)h(x).$$

By dividing by $h(x)$ and evaluating at $x = b(\theta)$ we obtain the first-order condition

$$0 = (1 - H(b(\theta)))\eta\theta + H(b(\theta))\lambda\eta\theta + (\theta - b(\theta)).$$

Rearranging yields

$$b(\theta) = \big(1 + \eta(1 - H(b(\theta))) + \lambda\eta H(b(\theta))\big)\theta. \tag{10}$$

Using that $H(b(\theta)) = G^{n-1}(\theta)$ we obtain

$$b(\theta) = \big(1 + \eta(1 - G^{n-1}(\theta)) + \lambda\eta G^{n-1}(\theta)\big)\theta$$

as the unique equilibrium candidate. For sufficiency note first that

$$h(b(\theta)) = \frac{(G^{n-1})'(\theta)}{b'(\theta)} = \frac{(n-1)G^{n-2}(\theta)g(\theta)}{(1 + \eta(1 - G^{n-1}(\theta)) + \lambda\eta G^{n-1}(\theta)) + \Lambda(n-1)G^{n-2}(\theta)g(\theta)\theta}$$

is differentiable since $g(\theta)$ is differentiable. Now it is immediate that

$$\frac{\partial^2 U_0(x,\theta|b(\theta))}{(\partial x)^2}\Big|_{x=b(\theta)} = -h(b(\theta)) + h'(b(\theta))\underbrace{[\theta - b(\theta) + (1-H(b(\theta)))\mu(\theta) - H(b(\theta))\mu(-\theta)]}_{=0}$$
$$< 0.$$

□

Proof of Lemma 1. Suppose first that F_1 and F_2 are invertible on $[a,b]$. By the theorem of the integral over inverse functions (e.g. Theorem 1 in Key (1994)), any invertible distribution F on $[a,b]$ satisfies

$$\int_a^b F(x)dx = bF(b) - aF(a) - \int_0^1 c_F(p)dp = b - \int_0^1 c_F(p)dp,$$

which implies

$$\int_a^b (F_1(x) - F_2(x))dx = (b-b) - \int_0^1 c_{F_1}(p)dp + \int_0^1 c_{F_2}(p)dp = \int_0^1 (c_{F_2}(p) - c_{F_1}(p))dp.$$

Define now

$$F_1^+(x) = \begin{cases} F_1(x) & F_1(x) > F_2(x), \\ F_2(x) & F_1(x) \leq F_2(x), \end{cases}$$

and similarly

$$F_1^-(x) = \begin{cases} F_1(x) & F_1(x) \leq F_2(x), \\ F_2(x) & F_1(x) > F_2(x). \end{cases}$$

By construction, F_1^+ and F_1^- are invertible and satisfy $F_1^+(x) \geq F_2(x) \geq F_1^-(x)$ for all $x \in [a,b]$, and moreover

$$c_{F_1^+}(p) = \begin{cases} c_{F_1}(p) & F_1(c_{F_1}(p)) > F_2(c_{F_1}(p)), \\ c_{F_2}(p) & F_1(c_{F_1}(p)) \leq F_2(c_{F_1}(p)), \end{cases}$$

and

$$c_{F_1^-}(p) = \begin{cases} c_{F_1}(p) & F_1(c_{F_1}(p)) \leq F_2(c_{F_1}(p)), \\ c_{F_2}(p) & F_1(c_{F_1}(p)) > F_2(c_{F_1}(p)), \end{cases}$$

for all $p \in [0,1]$. With these constructions we obtain

$$\begin{aligned}
&\int_a^b \mu(F_1(x) - F_2(x))dx \\
&= \int_a^b \eta(F_1^+(x) - F_2(x))dx + \int_a^b \lambda\eta(F_1^-(x) - F_2(x))dx \\
&= \int_0^1 \eta(c_{F_2}(p) - c_{F_1^+}(p))dp + \int_0^1 \lambda\eta(c_{F_2}(p) - c_{F_1^-}(p))dp \\
&= \int_0^1 \mu(c_{F_2}(p) - c_{F_1}(p))\mathbb{1}_{F_1(c_{F_1}(p))>F_2(c_{F_1}(p))}dp + \int_0^1 \mu(c_{F_2}(p) - c_{F_1}(p))\mathbb{1}_{F_1(c_{F_1}(p))\leq F_2(c_{F_1}(p))}dp \\
&= \int_0^1 \mu(c_{F_2}(p) - c_{F_1}(p))d(p),
\end{aligned}$$

which proves the lemma for invertible distributions. We now show the lemma for general distribution functions, when the quantile functions c_{F_i} are defined as usual by

$$c_{F_i}(p) = \inf\{x \in \mathbb{R} | p \leq F(x)\}.$$

Take two arbitrary sequences of continuously increasing distribution functions $(F_{1,n})$, $(F_{2,n})$ which converge pointwise $F_{i,n} \to F_i$ everywhere outside the null-set of discontinuity points of F_i for $i = 1, 2$.[16] By Theorem 1.1.1 in De Haan and Ferreira (2007), $\lim_{n\to\infty} F_{i,n}(x) = F_i(x)$ for all continuity points of F_i implies $\lim_{n\to\infty} c_{F_{i,n}}(p) = c_{F_i}(p)$ for all continuity points of $c_{F_i}(p)$. Using Lebesgue's dominated convergence theorem and that the set of discontinuity points is a null-set, we get

$$\begin{aligned}
\int_a^b \mu(F_1(x) - F_2(x))dx &= \int_a^b \lim_{n\to\infty} \mu(F_{1,n}(x) - F_{2,n}(x))dx \\
&= \lim_{n\to\infty} \int_a^b \mu(F_{1,n}(x) - F_{2,n}(x))dx \\
&= \lim_{n\to\infty} \int_0^1 \mu(c_{F_{2,n}}(p) - c_{F_{1,n}}(p))d(p) \\
&= \int_0^1 \lim_{n\to\infty} \mu(c_{F_{2,n}}(p) - c_{F_{1,n}}(p))d(p) \\
&= \int_0^1 \mu(c_{F_2}(p) - c_{F_1}(p))d(p),
\end{aligned}$$

[16] To see existence of such a sequence, take a positive sequence $\varepsilon_n \to 0$ and define $F_{i,n} = (1-\varepsilon_n)G_{i,n} + \varepsilon_n \frac{x-a}{b-a}$, where $G_{i,n}$ is the continuous function which is linear on ε_n-balls around any discontinuity point of F_i and coincides with F_i elsewhere.

which concludes the proof for arbitrary distributions. □

Proof of Lemma 2. By applying Lemma 1, and using the fact that μ is piecewise linear, we can write

$$\begin{aligned}
\mathbb{E}(N(F_i|F)) &= \Delta N(F_1|F) + (1-\Delta)N(F_2|F) \\
&= \Delta \int_0^1 \mu(c_{F_1}(p) - c_F(p))dp + (1-\Delta) \int_0^1 \mu(c_{F_2}(p) - c_F(p))dp \\
&= \Delta \int_a^b \mu(F(x) - F_1(x))dx + (1-\Delta) \int_a^b \mu(F(x) - F_2(x))dx \\
&= \Delta \int_a^b \mu(F(x) - F_1(x))dx + \int_a^b \mu((1-\Delta)F(x) - (1-\Delta)F_2(x))dx \\
&= \Delta \int_a^b \mu(F(x) - F_1(x))dx + \int_a^b \mu((1-\Delta)F(x) - (F(x) - \Delta F_1(x)))dx \\
&= \Delta \int_a^b \mu(F(x) - F_1(x))dx + \int_a^b \mu(-\Delta F(x) + \Delta F_1(x))dx \\
&= \Delta \int_a^b \mu(F(x) - F_1(x))dx + \Delta \int_a^b \mu(-F(x) + F_1(x))dx \\
&= \Delta(-\lambda\eta + \eta) \int_a^b |F(x) - F_1(x)|dx \\
&= -\Delta\Lambda \int_a^b |F(x) - F_1(x)|dx.
\end{aligned}$$

□

Proof of Proposition 2. Suppose the current clock price is t and the opponent hasn't dropped out yet. If the clock increases in increments of ε, then the conditional probability that the opponent drops out at the next increment is given by

$$\Delta_t := \frac{F(t+\varepsilon) - F(t)}{1 - F(t)}.$$

Given her strategy b and that the opponent hasn't dropped out at t, the bidder faces the conditional probability of $\frac{1-F(b)}{1-F(t)}$ to lose the auction. Thus, if F_t^b denotes the belief about payoffs in the good dimension at time t given strategy b, we have

$$F_t^b(z) = \begin{cases} \frac{1-F(b)}{1-F(t)} & z < \theta, \\ 1 & z \geq \theta. \end{cases}$$

If the bidder wins in the next increment, the belief will update to

$$F^b_{t+\varepsilon}(z) = \begin{cases} 0 & z < \theta, \\ 1 & z \geq \theta. \end{cases}$$

According to Lemma 2, expected gain-loss utility of the increment from t to $t+\varepsilon$ is then given by

$$\mathbb{E}(N(F^b_{t+\varepsilon}|F^b_t)) = -\Delta_t\Lambda \int |F^b_t(z) - F^b_{t+\varepsilon}(z)|dz = -\Delta_t\Lambda\frac{1-F(b)}{1-F(t)}\theta.$$

Now, the marginal loss at time t if ε goes to zero reads

$$\ell_t(b,\theta,F) = \lim_{\varepsilon\to 0}\frac{-\Delta_t\Lambda\frac{1-F(b)}{1-F(t)}\theta}{\varepsilon} = \frac{-f(t)}{(1-F(t))^2}(1-F(b))\Lambda\theta.$$

To calculate total expected gain-loss utility starting at time t, note that any information update at time $s > t$ is only informative and carries gain-loss utility if the opponent hasn't already dropped out between t and s, which holds true with the conditional probability $\frac{1-F(s)}{1-F(t)}$. Thus

$$\begin{aligned} L_t(b,\theta,F|b) &= \lim_{\varepsilon\to 0}\sum_{i=0}^{\lfloor\frac{b-t}{\varepsilon}\rfloor-1} N(F^b_{t+(i+1)\varepsilon}|F^b_{t+i\varepsilon}) \\ &= \lim_{\varepsilon\to 0}\sum_{i=0}^{\lfloor\frac{b-t}{\varepsilon}\rfloor-1} -\frac{1-F(t+i\varepsilon)}{1-F(t)}\Delta_{t+i\varepsilon}\Lambda\frac{1-F(b)}{1-F(t+i\varepsilon)}\theta \\ &= \int_t^b \frac{-f(s)}{1-F(s)}\frac{1-F(b)}{1-F(t)}\Lambda\theta ds \\ &= \big(\ln(1-F(b)) - \ln(1-F(t))\big)\frac{1-F(b)}{1-F(t)}\Lambda\theta \\ &= \ln\left(\frac{1-F(b)}{1-F(t)}\right)\frac{1-F(b)}{1-F(t)}\Lambda\theta. \end{aligned}$$

□

Proof of Lemma 3. At time t the winning probability is given by the probability that the opponent drops out between t and b^*, given she didn't drop out before

t, thus $\frac{F(b^*)-F(t)}{1-F(t)}$. The update changes the probability of getting θ by

$$\frac{F(b)-F(t)}{1-F(t)} - \frac{F(b^*)-F(t)}{1-F(t)} = \frac{F(b)-F(b^*)}{1-F(t)}.$$

Hence,

$$N(F_t^b|F_t^{b^*}) = \mu\left(\frac{F(b)-F(b^*)}{1-F(t)}\theta\right) = \frac{\mu(F(b)-F(b^*))}{1-F(t)}\theta.$$

□

Proof of Lemma 4. The bidder does not want do deviate to a lower strategy y at any time t, given plan b if and only if

$$U_t(y,\theta,F|b) \leq U_t(b,\theta,F|b)$$

for all $t \leq y \leq b$. In particular it is necessary that for all $t < b$ the derivative from the left satisfies

$$\begin{aligned} 0 &\leq \lim_{y\nearrow b} \frac{\partial U_t(y,\theta,F|b)}{\partial y} \\ &= \frac{f(b)}{1-F(t)}\left(\theta - b + \lambda\eta\theta - \Lambda\left(1+\ln\left(\frac{1-F(b)}{1-F(t)}\right)\right)\theta\right). \end{aligned}$$

This expression is well defined, since $F(t) < F(b) < 1$. Now, as t approaches b we get

$$\begin{aligned} 0 &\leq \lim_{t\to b} \frac{f(b)}{1-F(t)}\left(\theta - b + \lambda\eta\theta - \Lambda\left(1+\ln\left(\frac{1-F(b)}{1-F(t)}\right)\right)\theta\right) \\ &= \frac{f(b)}{1-F(b)}\left(\theta - b + \lambda\eta\theta - \Lambda\theta\right). \end{aligned}$$

Since, by assumption, $f(b) > 0$, this means that necessarily

$$b \leq (1+\lambda\eta-\Lambda)\theta = (1+\eta)\theta.$$

□

Proof of Lemma 5. Given opponent's strategy F and bidder's type θ, a bid $b(\theta)$ is a personal equilibrium in the auction with commitment if and only if

$$U_0(y,\theta,F|b(\theta)) \leq U_0(b(\theta),\theta,F|b(\theta))$$

for all y. In particular, it is necessary that

$$\lim_{y \searrow b(\theta)} \frac{\partial U_0(y, \theta, F|b(\theta))}{\partial y} \leq 0.$$

By Equation (7) the utility for $y > b(\theta)$ at time zero reads

$$U_0(y, \theta, F|b(\theta)) = \int_0^y (\theta - s) dF(s) + \eta(F(y) - F(b(\theta)))\theta + \ln(1 - F(y))(1 - F(y))\Lambda\theta.$$

Hence, the necessary condition is equivalent to

$$f(b(\theta))\big(\theta - b(\theta) + \eta\theta - \Lambda(1 + \ln(1 - F(b(\theta))))\theta\big) \leq 0.$$

In any symmetric equilibrium, the opponent bids according to $b(\theta)$ as well, and therefore we have $F(b(\theta)) = G(\theta)$. From $g(\theta) = f(b(\theta))b'(\theta)$ and the restriction that b is increasing it follows that $f(b(\theta)) > 0$. Hence we have

$$b(\theta) \geq \big(1 + \eta - \Lambda(1 + \ln(1 - G(\theta)))\big)\theta$$

for any equilibrium candidate. It remains to verify that

$$b(\theta) = \big(1 + \eta - \Lambda(1 + \ln(1 - G(\theta)))\big)\theta \tag{11}$$

is a personal equilibrium, given opponent's response $b(\theta)$. For this it is sufficient to show that

$$\frac{\partial U_0(y, \theta, F|b(\theta))}{\partial y} \leq 0$$

for all $y > b(\theta)$, and

$$\frac{\partial U_0(y, \theta, F|b(\theta))}{\partial y} \geq 0$$

for all $y < b(\theta)$. Note that we can without loss of generality restrict to $y \in [b(\theta^{\min}), b(\theta^{\max})]$.

For any such y there exists some $\tilde{\theta}$ with $y = b(\tilde{\theta})$, since the bidding function is continuous.

Consider first $y > b(\theta)$, thus $\tilde{\theta} > \theta$. Then

$$\begin{aligned}\frac{\partial U_0(y,\theta,F|b(\theta))}{\partial y}\Big|_{y=b(\tilde{\theta})} &= f(b(\tilde{\theta}))\big(\theta - b(\tilde{\theta}) + \eta\theta - \Lambda(1+\ln(1-F(b(\tilde{\theta}))))\theta\big) \\ &< f(b(\tilde{\theta}))\big(\tilde{\theta} - b(\tilde{\theta}) + \eta\tilde{\theta} - \Lambda(1+\ln(1-F(b(\tilde{\theta}))))\tilde{\theta}\big) \\ &= \lim_{y\searrow b(\tilde{\theta})} \frac{\partial U_0(y,\tilde{\theta},F|b(\tilde{\theta}))}{\partial y} \\ &= 0,\end{aligned}$$

where the last equality is due to equality in (11). Similarly, for $y < b(\theta)$, thus $\tilde{\theta} < \theta$ we have

$$\begin{aligned}\frac{\partial U_0(y,\theta,F|b(\theta))}{\partial y}\Big|_{y=b(\tilde{\theta})} &= f(b(\tilde{\theta}))\big(\theta - b(\tilde{\theta}) + \lambda\eta\theta - \Lambda(1+\ln(1-F(b(\tilde{\theta}))))\theta\big) \\ &> f(b(\tilde{\theta}))\big(\tilde{\theta} - b(\tilde{\theta}) + \eta\tilde{\theta} - \Lambda(1+\ln(1-F(b(\tilde{\theta}))))\tilde{\theta}\big) \\ &= \lim_{y\searrow b(\tilde{\theta})} \frac{\partial U_0(y,\tilde{\theta},F|b(\tilde{\theta}))}{\partial y} \\ &= 0.\end{aligned}$$

□

Proof of Lemma 6. Consider a bidding strategy b^*.

Claim 1: If and only if $b^* \leq (1+\eta)\theta$, it is at no time $t < b^*$ profitable to deviate to a lower strategy $b \in [t, b^*)$.

Proof: the "only if" has been proved in Lemma 4. For the "if", assume that $b^* \leq (1+\eta)\theta$. Consider a deviation at some time $t < b^*$ from b^* to $b \in [t, b^*)$. We first look at the change in expected gain-loss disutility: term A can be interpreted as the change due to different expectations at each time between t and b, while term B is forgone gain-loss disutility, since the auction necessarily ends at b:

$$\begin{aligned}
&L_t(b,\theta,F|b) - L_t(b^*,\theta,F|b^*)\\
=&\Lambda\theta\left(\ln\left(\frac{1-F(b)}{1-F(t)}\right)\frac{1-F(b)}{1-F(t)} - \ln\left(\frac{1-F(b^*)}{1-F(t)}\right)\frac{1-F(b^*)}{1-F(t)}\right)\\
=&\Lambda\theta\left(\int_t^b \frac{-f(s)}{1-F(s)}ds\frac{1-F(b)}{1-F(t)} - \int_t^{b^*}\frac{-f(s)}{1-F(s)}ds\frac{1-F(b^*)}{1-F(t)}\right)\\
=&\Lambda\theta\left(\int_t^b \frac{-f(s)}{1-F(s)}ds\frac{1-F(b)}{1-F(t)} - \int_t^{b}\frac{-f(s)}{1-F(s)}ds\frac{1-F(b^*)}{1-F(t)} - \int_b^{b^*}\frac{-f(s)}{1-F(s)}ds\frac{1-F(b^*)}{1-F(t)}\right)\\
=&\Lambda\theta\left(\underbrace{\int_t^b \frac{-f(s)}{1-F(s)}ds\frac{F(b^*)-F(b)}{1-F(t)}}_{A} - \underbrace{\int_b^{b^*}\frac{-f(s)}{1-F(s)}ds\frac{1-F(b^*)}{1-F(t)}}_{B}\right)\\
\leq&\Lambda\theta\int_b^{b^*}\frac{f(s)}{1-F(s)}ds\frac{1-F(b^*)}{1-F(t)}\\
<&\Lambda\theta\int_b^{b^*}f(s)ds\frac{1-F(b^*)}{(1-F(b^*))(1-F(t))}\\
=&\Lambda\theta\frac{F(b^*)-F(b)}{1-F(t)}.
\end{aligned}$$

Now we have

$$\begin{aligned}
&U_t(b,\theta,F|b^*) - U_t(b^*,\theta,F|b^*)\\
<&\frac{1}{1-F(t)}\left(-\int_b^{b^*}(\theta-s)dF(s) + \mu(F(b)-F(b^*))\theta + \Lambda\theta(F(b^*)-F(b))\right)\\
<&\frac{F(b^*)-F(b)}{1-F(t)}\left(-\theta + b^* - \lambda\eta\theta + \Lambda\theta\right)\\
=&\frac{F(b^*)-F(b)}{1-F(t)}\left(-(1+\eta)\theta + b^*\right)\\
\leq&0.
\end{aligned}$$

Thus, there is no profitable deviation to $b < b^*$ at any time, which concludes the proof of Claim 1.

Claim 1 directly shows the necessity of 1. for any PE. Certainly, 2. is necessary as well.

Claim 2: If it is not profitable to deviate to a strategy $b > b^*$ at time $t = 0$, then it is not profitable at any time $t \leq b^*$.
Proof: It is not profitable to deviate to a strategy $b > b^*$ at time t if and only if

$$0 \geq U_t(b, \theta, F|b^*) - U_t(b^*, \theta, F|b^*)$$

Now,

$$U_t(b, \theta, F|b^*) - U_t(b^*, \theta, F|b^*)$$

$$= \frac{1}{1-F(t)} \left(\int_{b^*}^{b} (\theta - s) dF(s) + \mu(F(b) - F(b^*))\theta \right)$$

$$+ \Lambda\theta \left(\frac{1-F(b)}{1-F(t)} \ln\left(\frac{1-F(b)}{1-F(t)}\right) - \frac{1-F(b^*)}{1-F(t)} \ln\left(\frac{1-F(b^*)}{1-F(t)}\right) \right)$$

$$= \frac{1}{1-F(t)} \left(\int_{b^*}^{b} (\theta - s) dF(s) + \mu(F(b) - F(b^*))\theta \; ... \right.$$

$$\left. ... + \Lambda\theta((1-F(b))\ln(1-F(b)) - (1-F(b^*))\ln(1-F(b^*)) + (F(b) - F(b^*))\ln(1-F(t))) \right).$$

Note that the expression in the big brackets is decreasing in t. Thus, if it is negative for $t = 0$, then it is as well negative for all $t > 0$. Hence, if

$$0 \geq U_0(b, \theta, F|b^*) - U_0(b^*, \theta, F|b^*)$$

then

$$0 \geq U_t(b, \theta, F|b^*) - U_t(b^*, \theta, F|b^*)$$

for all $t > 0$, which concludes the proof of Claim 2.

Now we are ready to show sufficiency: assume 1. and 2. hold. Then by Claim 1 it can't be profitable to deviate to a lower strategy at any time. To show that there is no profitable deviation to a higher strategy, take any time-consistent strategy $b \geq b^*$. By Claim 1 this necessarily means $b \in [b^*, (1+\eta)\theta]$. From 2. it follows that $U_0(b^*, \theta, F|b^*) \geq U_0(b, \theta, F|b^*)$. Then, by Claim 2, the agent does not want to deviate to a higher strategy at any time, and b^* is indeed a PE. $\square$

Proof of Proposition 3. Take some increasing equilibrium function. By Lemma 6, it satisfies $b(\theta) \leq (1+\eta)\theta$ for all $\theta \in (\theta^{\min}, \theta^{\max})$. If $b(\theta) < (1+\eta)\theta$ for some θ, then—again by Lemma 6—any $y \in [b(\theta), (1+\eta)\theta]$ satisfies $U_0(b(\theta), \theta, F|b(\theta)) \geq U_0(y, \theta, F|x)$. This means that

$$\lim_{y \searrow b(\theta)} \frac{\partial U_0(y, \theta, F|b(\theta))}{\partial y} \leq 0,$$

which—as we have seen in the proof of Lemma 5—straightforwardly solves to

$$b(\theta) \geq \big(1 + \eta - \Lambda(1 + \ln(1 - G(\theta)))\big)\theta$$

in equilibrium. This shows that any increasing equilibrium satisfies 1. and 2. for all $\theta \in (\theta^{\min}, \theta^{\max})$. By continuity it also holds for all $\theta \in [\theta^{\min}, \theta^{\max}]$. Conversely, assume that $b(\theta)$ satisfies 1. and 2. By Lemma 6 it only remains to show that for any

$$y \in [b(\theta), (1+\eta)\theta]$$

we have

$$U_0(b(\theta), \theta, F|b(\theta)) \geq U_0(y, \theta, F|b(\theta)).$$

This condition is trivially satisfied for any θ with $b(\theta) = (1+\eta)\theta$. Consider therefore θ with $b(\theta) < (1+\eta)\theta$. It suffices to show that

$$\frac{\partial U_0(y, \theta, F|b(\theta))}{\partial y} \leq 0$$

for all $y \in [b(\theta), (1+\eta)\theta]$. Let $\tilde{y}$ be any of such y. Since

$$b(\theta^{\max}) = (1+\eta)\theta^{\max} > (1+\eta)\theta \geq \tilde{y} \geq b(\theta),$$

and b is continuous, there exists some $\tilde{\theta} \geq \theta$ with $b(\tilde{\theta}) = \tilde{y}$. Since $(1+\eta)\theta > (1+\eta)\theta \geq \tilde{y} = b(\tilde{\theta})$, we know by 2. that

$$b(\theta) \geq \big(1 + \eta - \Lambda(1 + \ln(1 - G(\theta)))\big)\theta.$$

Now,

$$
\begin{aligned}
\frac{\partial U_0(y,\theta,F|b(\theta))}{\partial y}\Big|_{y=\tilde{y}} &= [(1+\eta)\theta - \tilde{y} - \Lambda\theta(1+\ln(1-F(\tilde{y})))]f(\tilde{y}) \\
&= [\underbrace{(1+\eta-\Lambda(1+\ln(1-F(b(\tilde{\theta})))))}_{>0}\theta - b(\tilde{\theta})]f(b(\tilde{\theta})) \\
&\leq [\underbrace{(1+\eta-\Lambda(1+\ln(1-F(b(\tilde{\theta})))))\tilde{\theta}}_{\leq b(\tilde{\theta})} - b(\tilde{\theta})]f(b(\tilde{\theta})) \\
&\leq 0.
\end{aligned}
$$

□

Proof of Corollary 2. We have

$$
\big(1+\eta-\Lambda(1+\ln(1-G(\theta)))\big)\theta \leq (1+\eta)\theta
$$

if and only if $-(1+\ln(1-G(\theta))) \leq 0$, which is equivalent to $G(\theta) \leq 1-1/e$. Therefore, by Proposition 3, a fuction $b(\theta)$ is a symmetric equilibrium if and only if

- $b(\theta) \in \big[(1+\eta-\Lambda(1+\ln(1-G(\theta))))\theta, (1+\eta)\theta\big]$ for $G(\theta) \leq 1-1/e$, and
- $b(\theta) = (1+\eta)\theta$ for $G(\theta) > 1-1/e$.

We determine the utility maximizing equilibrium on the interval where $G(\theta) \leq 1-1/e$. Bidder's expected utility of a bid b is

$$
\begin{aligned}
U_0(b,\theta,F|b) &= \int_0^b (\theta - s)dF(s) + L_t(b,\theta,F) \\
&= \int_0^b (\theta - s)dF(s) + \Lambda\theta\ln(1-F(b))(1-F(b)).
\end{aligned}
$$

Thus, for any $b \geq \big(1+\eta-\Lambda(1+\ln(1-G(\theta)))\big)\theta$

$$
\begin{aligned}
\frac{\partial U_0(b,\theta,F|b)}{\partial b} &= (\theta-b)f(b) - \Lambda\theta(1+\ln(1-F(b)))f(b) \\
&\leq (\theta - (1+\eta-\Lambda(1+\ln(1-G(\theta))))\theta)f(b) - \Lambda\theta f(b) \\
&\leq (\theta - (1+\eta-\Lambda)\theta)f(b) - \Lambda\theta f(b) \\
&= -f(b)\eta\theta \\
&< 0.
\end{aligned}
$$

This shows that the lowest b among all equilibrium strategies yields the highest utility.
Finally, since for the PPE

$$b(\theta^{\min}) = \big(1+\eta-\Lambda(1+\ln(1-G(\theta^{\min})))\big)\theta^{\min} = (1+\eta-\Lambda)\theta^{\min},$$

there is underbidding for low types in the PPE if and only if

$$0 > \eta - \Lambda = 2\eta - \lambda\eta,$$

hence if and only if $\lambda > 2$. □

Proof of Proposition 4. If bidders do not update their reference point during the English auction, but only once, after the auction is resolved, their utility is given by

$$U_0(b,\theta|b^*) = \underbrace{\sum_{k\in\{m,g\}} N(F_T^k(b,\theta)|F_0^k(b^*,\theta,H_0))}_{\text{gain-loss utility}} + \underbrace{\mathbb{1}_{b>x}(\theta-x)}_{\text{classical utility}},$$

as in the Vickrey auction. Since for a two-bidder English auction the set of bidding strategies is the same as in a Vickrey auction, this concludes the proof of the first claim for the two-bidder case.

For completeness, we also show the claim for n bidders. With multiple opponents in the English auction, a bidder can use history-dependent bidding strategies for both, the equilibrium bidding plan and any deviation. Such strategies may induce price distributions that are unavailable in the Vickrey auction.

Since the prior distribution of types and opponents' bidding strategies are continuous, the distribution H of the maximal opponent bid, given the bidder is still in the auction at that price, is continuous as well. Hence, by the intermediate value theorem there exists a bid $\bar{b}$ such that (history-independently) bidding up to $\bar{b}$ induces the same winning probability as b, and therefore the same utility in the good dimension.[17] In the following we show formally that, irrespective of the reference point, strategy $\bar{b}$ dominates strategy b as it induces lower payments.

For bidding strategy $\bar{b}$ the bidder wins with probability one, whenever the maximal opponent bid is below $\bar{b}$. For bidding strategy b denote with $p_b(x)$ the induced probability of winning at price x conditional on the maximal opponent bid being x. Hence, for F_b^m denoting the distribution of monetary transfers to

[17] If H were discontinuous we could obtain the same result by randomization at $\bar{b}$.

pay by the bidder if she bids according to b, we obtain

$$F_b^m(x) = (1 - H(\bar{b})) + \int_0^x p_b(s) dH(s),$$

$$F_{\bar{b}}^m(x) = (1 - H(\bar{b})) + \int_0^x 1 dH(s).$$

Thus, F_b^m weakly dominates $F_{\bar{b}}^m$ in the first order sense, strictly so if the outcome of b differs to $\bar{b}$ on more than a Null set. Since the integrand in the definition of expected utility in the money dimension

$$U_0^m(b, \theta | b^*) = \int \left(\int_0^1 \mu_m(-x + c_{F_{b^*}^m}(p) dp - x \right) dF_b^m(x)$$

is strictly decreasing in x, first order stochastic dominance implies that

$$\begin{aligned} U_0^m(b, \theta | b^*) &= \int \left(\int_0^1 \mu_m(-x + c_{F_{b^*}^m}(p) dp - x \right) dF_b^m(x) \\ &< \int \left(\int_0^1 \mu_m(-x + c_{F_{b^*}^m}(p) dp - x \right) dF_{\bar{b}}^m(x) \\ &= U_0^m(\bar{b}, \theta | b^*) \end{aligned}$$

for all b^* and all b that differ to $\bar{b}$ on more than a Null set. This implies in particular that any history-dependent strategy b^* which differs to the history-independent strategy $\bar{b}^*$ on more than a Null set cannot be a personal equilibrium in the English auction, since

$$U_0(\bar{b}^*, \theta | b^*) > U_0(b^*, \theta | b^*).$$

Moreover, for any strategy to bid up to $b^* \in \mathbb{R}_+$ we have $U_0(b, \theta | b^*) \leq U_0(b^*, \theta | b^*)$ for all b if and only if $U_0(\bar{b}, \theta | b^*) \leq U_0(b^*, \theta | b^*)$ for all history-independent strategies $\bar{b}$, which concludes that a strategy is an equilibrium in the English auction if and only if it is an equilibrium in the Vickrey auction.

For (2) note that by Proposition 1 the equilibrium bidding function for the Vickrey auction is given by

$$b_{\text{Vickrey}}(\theta) = (1 + \eta + \Lambda G^{n-1}(\theta))\theta,$$

whereas any equilibrium bidding function in the English auction with instantaneous reference point updating by Lemma 6 satisfies

$$b_{\text{English}}(\theta) \leq (1+\eta)\theta.$$

Since, by assumption, $G^{n-1}(\theta)$ is strictly increasing, we have $G^{n-1}(\theta) > 0$ for all $\theta > \theta^{\min}$, and the claim follows. □

Proof of Proposition 5. The structure of the proof is similar to the one of Proposition 3 in Lange and Ratan (2010). Suppose that all opponents bid according to some increasing, continuously differentiable bidding function $b(\theta)$. Since $G(\theta)$ is a distribution with strictly positive, continuous density g, the distribution of the maximal opponent bid $H(x) = G^{n-1}(b^{-1}(x))$ is a differentiable distribution with positive, continuous density $h(x)$ on $[b(\theta^{\min}), b(\theta^{\max})]$ as well. The bidding function $b(\theta)$ constitutes a PE if and only if $U_0(y, \theta|b(\theta))$ attains a maximum at $y = b(\theta)$ for all θ. Differentiation of the utility function with respect to y yields

$$\begin{aligned}\frac{\partial U_0(y,\theta|b(\theta))}{\partial y} =&(\theta - y)h(y) + h(y)H(b(\theta))\lambda_g\eta_g\theta + h(y)(1 - H(b(\theta)))\eta_g\theta \\ &+ (1 - H(b(\theta)))\mu_m(-y)h(y) + \int_0^{b(\theta)} \mu_m(s-y)h(y)dH(t) \\ &- h(y)\int_0^{b(\theta)} \mu_m(s)dH(s).\end{aligned}$$

By dividing by $h(y)$ and evaluating at $y = b(\theta)$, we obtain the first-order condition

$$\begin{aligned}0 =&(\theta - b(\theta)) + (1 - H(b(\theta)))\eta_g\theta + H(b(\theta))\lambda_g\eta_g\theta \\ &+ (1 - H(b(\theta)))\mu_m(-b(\theta)) + \int_0^{b(\theta)} \mu_m(s - b(\theta))dH(s) - \int_0^{b(\theta)} \mu_m(s)dH(s) \\ =&(\theta - b(\theta)) + (1 - H(b(\theta)))\eta_g\theta + H(b(\theta))\lambda_g\eta_g\theta \\ &+ (1 - H(b(\theta)))(-\lambda_m\eta_m b(\theta)) - \lambda_m\eta_m\int_0^{b(\theta)} (b(\theta) - s)dH(s) - \eta_m\int_0^{b(\theta)} sdH(s),\end{aligned}$$

which simplifies to

$$0 = (1+\eta_g)\theta - (1+\lambda_m\eta_m)b(\theta) + \Lambda_m\int_0^{b(\theta)} sdH(s) + \Lambda_g H(b(\theta))\theta. \tag{12}$$

Using that $H(b(\theta)) = G^{n-1}(\theta)$ we can rewrite this equation to

$$0 = (1+\eta_g)\theta - (1+\lambda_m\eta_m)b(\theta) + \Lambda_m \int_0^\theta b(s)dG^{n-1}(s) + \Lambda_g G^{n-1}(\theta)\theta.$$

Differentiation with respect to θ yields

$$0 = (1+\eta_g) - (1+\lambda_m\eta_m)b'(\theta) + \Lambda_m b(\theta)(G^{n-1})'(\theta) + \Lambda_g(G^{n-1}(\theta)\theta)'.$$

The rearranged equation

$$b'(\theta) = \frac{\Lambda_m (G^{n-1})'(\theta)}{1+\lambda_m\eta_m} b(\theta) + \frac{1+\eta_g+\Lambda_g(\theta G^{n-1}(\theta))'}{1+\lambda_m\eta_m}$$

is a first-order linear differential equation, which solves to

$$b(\theta) = \exp\left(\frac{\Lambda_m}{1+\lambda_m\eta_m} G^{n-1}(\theta)\right)\left(\int_0^\theta \frac{1+\eta_g+\Lambda_g(xG^{n-1}(x))'}{1+\lambda_m\eta_m} \exp\left(-\frac{\Lambda_m}{1+\lambda_m\eta_m} G^{n-1}(x)\right) dx + C\right),$$

where C is the constant of integration. Since $G(x) = 0$ for $x \leq \theta^{min}$, we have

$$b(\theta^{\min}) = \exp(0)\left(\int_0^{\theta^{\min}} \frac{1+\eta_g}{1+\lambda_m\eta_m} \exp(0)dx + C\right) = \frac{1+\eta_g}{1+\lambda_m\eta_m}\theta^{\min} + C.$$

To determine C, we insert $\theta^{\min}$ into equation (12) and obtain that

$$0 = (\theta^{\min} - b(\theta^{\min})) + (-\lambda_m\eta_m b(\theta^{\min})) + \eta_g\theta^{\min},$$

or equivalently

$$b(\theta^{\min}) = \frac{1+\eta_g}{1+\lambda_m\eta_m}\theta^{\min},$$

which shows that $C = 0$. Now we can use integration by parts in order to rewrite the solution into

$$b(\theta) = \frac{1+\eta_g+\Lambda_g G^{n-1}(\theta)}{1+\lambda_m\eta_m}\theta + \int_0^\theta \frac{\Lambda_m(1+\eta_g+\Lambda_g G^{n-1}(x))}{(1+\lambda_m\eta_m)^2} x \exp\left(\frac{\Lambda_m}{1+\lambda_m\eta_m}(G^{n-1}(\theta) - G^{n-1}(x))\right) dG(x).$$

Since $G(x) = 0$ for all $x \leq \theta^{\min}$, we finally have

$$b(\theta) = \frac{1+\eta_g+\Lambda_g G^{n-1}(\theta)}{1+\lambda_m\eta_m}\theta + \int_{\theta^{\min}}^\theta \frac{\Lambda_m(1+\eta_g+\Lambda_g G^{n-1}(x))}{(1+\lambda_m\eta_m)^2} x \exp\left(\frac{\Lambda_m}{1+\lambda_m\eta_m}(G^{n-1}(\theta) - G^{n-1}(x))\right) dG(x).$$

For sufficiency note first that $b'(\theta)$ is differentiable, since $g(\theta)$ is differentiable. It follows that

$$h(b(\theta)) = \frac{(G^{n-1})'(\theta)}{b'(\theta)}$$

is differentiable as well. Now it is immediate that

$$\begin{aligned}
&\left.\frac{\partial^2 U_0(x,\theta|b(\theta))}{(\partial x)^2}\right|_{x=b(\theta)} \\
=&\frac{\partial}{\partial x}\left(h(x)\frac{\partial U_0(x,\theta|b(\theta)/\partial x)}{h(x)}\right)\Bigg|_{x=b(\theta)} \\
=&h'(b(\theta))\underbrace{\left(\frac{\partial U_0(x,\theta|b(\theta)/\partial x)}{h(x)}\right)\Bigg|_{x=b(\theta)}}_{=0} \\
&+h(b(\theta))\underbrace{\left[-1+\int_0^{b(\theta)} -\lambda_m\eta_m dH(s) - \lambda_m\eta_m(1-H(b(\theta)))\right]}_{<0} \\
<& \, 0.
\end{aligned}$$

□

Proof of Lemma 7. Assume the clock increases in increments of ε and the bidder plans to bid up to $x \in (a,b)$. Assume the clock price is $x-\varepsilon$, and the opponent has not dropped out yet. We analyze bidders incentives to bid at x given her plan to do so.

Let $\Delta = \Delta(\varepsilon) = \frac{F(x)-F(x-\varepsilon)}{1-F(x-\varepsilon)}$ be the probability that the opponent drops out at x, given she is still in at $x-\varepsilon$. This means the bidder beliefs to win the auction and get a payoff of $(\theta, -(x-\varepsilon))$ with probability Δ. If the bidder bids at x she receives a utility of

$$\begin{aligned}
U_0(x,\theta,F|x) =& \underbrace{\Delta(\theta-(x-\varepsilon))}_{\text{classical utility}} + \Delta\underbrace{(1-\Delta)(\eta_g\theta - \lambda_m\eta_m(x-\varepsilon))}_{\text{gain-loss of winning the auction}} \\
&+(1-\Delta)\underbrace{\Delta(-\lambda_g\eta_g\theta + \eta_m(x-\varepsilon))}_{\text{gain-loss of losing the auction}}.
\end{aligned}$$

If she drops out before bidding x, she receives

$$U_0(x-\varepsilon,\theta,F|x) = \underbrace{\Delta(-\lambda_g\eta_g\theta + \eta_m(x-\varepsilon))}_{\text{gain-loss of losing the auction}}.$$

If bidding up to x is time consistent, then

$$U_0(x, \theta, F|x) \geq u(x - \varepsilon, \theta, F|x).$$

This is equivalent to

$$\Delta[\theta - (x - \varepsilon) + (1 - \Delta)(\eta_g\theta - \lambda_m\eta_m(x - \varepsilon)) - \Delta(-\lambda_g\eta_g\theta + \eta_m(x - \varepsilon))] \geq 0.$$

Since F has a positive density, we have $\Delta > 0$, and it follows

$$(1 + \eta_g)\theta - (1 + \lambda_m\eta_m)(x - \varepsilon) + \Delta(\Lambda_g\theta + \Lambda_m(x - \varepsilon)) \geq 0.$$

Since F has no atoms, $\lim_{\varepsilon\to 0} \Delta(\varepsilon) = 0$. Thus, in the limit as the increment size goes to zero, we obtain

$$(1 + \eta_g)\theta - (1 + \lambda_m\eta_m)x \geq 0,$$

or equivalently

$$x \leq \frac{1 + \eta_g}{1 + \lambda_m\eta_m}\theta.$$

□

Proof of Lemma 8. From the perspective of a representative bidder, we denote with $F(x)$ the distribution of prices, at which a particular opponent drops out, i.e. $F(b(\theta)) = G(\theta)$. Similarly we denote with $F_t(x)$ the distribution of drop-out prices of the remaining opponent, given the other opponent drops out at t. Since the remaining opponent j didn't drop out until t, her type θ_j necessarily satisfies $\theta_j > \theta(t)$, and therefore

$$F_t(b(t, \theta)) = Prob(\theta_j \leq \theta|\theta_j > \theta(t)) = \frac{G(\theta) - G(\theta(t))}{1 - G(\theta(t))}.$$

If we denote with $L_{2,t}$ expected gain-loss utility in the two-bidder subgame following an opponent's drop out at price t, then by Proposition 2

$$\begin{aligned} L_{2,t}(\theta) &= \ln\left(\frac{1-F_t(b(t,\theta))}{1-F_t(t)}\right)\frac{1-F_t(b(t,\theta))}{1-F_t(t)}\Lambda\theta \\ &= \ln(1-F_t(b(t,\theta)))(1-F_t(b(t,\theta)))\Lambda\theta \\ &= \ln\left(\frac{1-G(\theta)}{1-G(\theta(t))}\right)\frac{1-G(\theta)}{1-G(\theta(t))}. \end{aligned}$$

For the 3-bidder auction leading to the first drop out, consider first price increments of ε. Suppose the clock is at price s and both opponents are still remaining. Since we restrict to symmetric increasing bidding functions, a bidder of type θ wins the auction if and only if both opponents have a type lower that θ. Given that they didn't drop out until s, this holds true with probability $\left(\frac{G(\theta)-G(\theta(s))}{1-G(\theta(s))}\right)^2$.

The probability that a particular opponent j drops out at the next increment is

$$\Delta(s) = \frac{F(s+\varepsilon)-F(s)}{1-F(s)}.$$

At the next increment $s+\varepsilon$ there are three possibilities:

- With probability $(\Delta(s))^2$ both opponents drop out. The bidder wins with certainty, which induces a gain of $\left(1-\left(\frac{G(\theta)-G(\theta(s))}{1-G(\theta(s))}\right)^2\right)\eta\theta$.
- With probability $2\Delta(s)(1-\Delta(s))$ exactly one opponent drops out. The bidder updates her belief to win, which induces a gain of $\left(\frac{G(\theta)-G(\theta(s+\varepsilon))}{1-G(\theta(s+\varepsilon))}-\left(\frac{G(\theta)-G(\theta(s))}{1-G(\theta(s))}\right)^2\right)\eta\theta$.
- With probability $(1-\Delta(s))^2$ no opponent drops out, which induces a loss of $\left(\left(\frac{G(\theta)-G(\theta(s+\varepsilon))}{1-G(\theta(s+\varepsilon))}\right)^2-\left(\frac{G(\theta)-G(\theta(s))}{1-G(\theta(s))}\right)^2\right)\lambda\eta\theta$.

Since F is continuous, $\Delta(s)$ approaches zero, as the increment size goes to zero. Therefore, in the limit for the continuous English auction, the probability that both opponents drop out at the same time is of second order and has no impact on expected gain-loss utility. Applying Lemma 2, expected gain-loss utility in the increment from s to $s+\varepsilon$ for small ε with both opponents being active approaches

$$L_{s+\varepsilon}(\theta)-L_s(\theta) = -2\Delta(s)(1-\Delta(s))\left(\frac{G(\theta)-G(\theta(s+\varepsilon))}{1-G(\theta(s+\varepsilon))}-\left(\frac{G(\theta)-G(\theta(s))}{1-G(\theta(s))}\right)^2\right)\Lambda\theta.$$

As the increment size goes to zero, in the limit the marginal expected gain-loss utility with both opponents being active at time s is given by

$$\ell(s)(\theta) = \frac{-2f(s)}{1-F(s)}\left(\frac{G(\theta)-G(\theta(s))}{1-G(\theta(s))} - \left(\frac{G(\theta)-G(\theta(s))}{1-G(\theta(s))}\right)^2\right)\Lambda\theta.$$

At time t, the probability that time $s > t$ is reached without at least one opponent drop out is $\left(\frac{1-F(s)}{1-F(t)}\right)^2$. Consequently the marginal probability of a drop out at s—which triggers the 2-bidder auction with expected loss $L_{2,s}$—is

$$\frac{\partial}{\partial s}\left(\frac{(1-F(s))^2}{(1-F(t))^2}\right) = \frac{2f(s)(1-F(s))}{(1-F(t))^2}.$$

Putting the two sources of gain-loss utility together and integrating over s yields

$$\begin{aligned} L_t(\theta) =& \int_t^{b(\theta)} \left(\left(\frac{1-F(s)}{1-F(t)}\right)^2 \ell(s) + \frac{2f(s)(1-F(s))}{(1-F(t))^2} L_{2,s}(\theta)\right) ds \\ =& -\Lambda\theta \int_t^{b(\theta)} \frac{2f(s)(1-F(s))}{(1-F(t))^2}\left(\frac{G(\theta)-G(\theta(s))}{1-G((\theta(s))} - \left(\frac{G(\theta)-G(\theta(s))}{1-G(\theta(s))}\right)^2\right) ds \\ &+ \Lambda\theta \int_t^{b(\theta)} \frac{2f(s)(1-F(s))}{(1-F(t))^2} \ln\left(\frac{1-G(\theta)}{1-G(\theta(s))}\right)\frac{1-G(\theta)}{1-G(\theta(s))} ds. \end{aligned}$$

Since $F(s) = G(\theta(s))$ and consequently $f(s) = g(\theta(s))/b'(\theta(s))$, integration by substitution yields

$$L_t(\theta) = -\Lambda\theta \int_{\theta(t)}^{\theta} \frac{2g(s)(1-G(s))}{(1-G(\theta(t)))^2}\left[\frac{G(\theta)-G(s)}{1-G(s)} - \left(\frac{G(\theta)-G(s)}{1-G(s)}\right)^2 - \ln\left(\frac{1-G(\theta)}{1-G(s)}\right)\frac{1-G(\theta)}{1-G(s)}\right] ds.$$

□

Proof of Corollary 3. Define

$$\delta(s) = \frac{G(\theta)-G(s)}{1-G(s)}.$$

Since for $\theta < \theta^{\max}$ we have $\delta(s) < 1$, and we can use the power series of the logarithm to rewrite

$$L_t(\theta) = -\Lambda\theta \int_{\theta(t)}^{\theta} \frac{2g(s)(1-G(s))}{(1-G(\theta(t)))^2}\left[\delta(s) - (\delta(s))^2 - (-\delta(s) - \frac{\delta(s)^2}{2} - \frac{\delta(s)^3}{3} ...)(1-\delta(s))\right] ds.$$

Since $\lim_{s\to\theta}\delta(s)=0$, we have

$$
\begin{aligned}
&\lim_{t\to b(\theta)} \frac{L_t(\theta)}{\left(\frac{G(\theta)-G(\theta(t))}{1-G(\theta(t))}\right)^2}\\
=&\lim_{t\to b(\theta)} -\Lambda\theta \int_{\theta(t)}^{\theta} \frac{2g(s)(1-G(s))}{(G(\theta)-G(\theta(t)))^2}\big[\delta(s)-(\delta(s))^2-(-\delta(s)-\frac{\delta(s)^2}{2}...)(1-\delta(s))\big]ds\\
=&\lim_{(\theta(t)\to\theta} -\Lambda\theta \int_{\theta(t)}^{\theta} \frac{2g(s)(1-G(s))}{(G(\theta)-G(\theta(t)))^2}\big[\delta(s)-(\delta(s))^2-(-\delta(s)-\frac{\delta(s)^2}{2}...)(1-\delta(s))\big]ds\\
=&\lim_{\theta(t)\to\theta} -\Lambda\theta \int_{\theta(t)}^{\theta} \frac{2g(s)(1-G(s))}{(G(\theta)-G(\theta(t)))^2}2\delta(s)ds\\
=&\lim_{\theta(t)\to\theta} -2\Lambda\theta \int_{\theta(t)}^{\theta} \frac{2g(s)(G(\theta)-G(s))}{(G(\theta)-G(\theta(t)))^2}ds\\
=&\lim_{\theta(t)\to\theta} -2\Lambda\theta \left[\frac{-(G(\theta)-G(s))^2}{(G(\theta)-G(\theta(t)))^2}\right]_{\theta(t)}^{\theta}\\
=&\lim_{\theta(t)\to\theta} -2\Lambda\theta\\
=&-2\Lambda\theta.
\end{aligned}
$$

Now, since $b(t,\theta)$ is continuous in t, $\lim_{t\to b(\theta)} b(t,\theta)$ exists. We prove the threshold of time-consistent behavior for $(\theta^{\min},\theta^{\max})$ by contradiction. For the boundaries it follows by continuity. Assume that there is some $\overline{\theta}\in(\theta^{\min},\theta^{\max})$ with

$$\lim_{t\to b(\overline{\theta})} b(t,\overline{\theta}) > (1+\eta-\Lambda)\overline{\theta}.$$

Since $b(t,\theta)$ is continuous there is some $\hat{t}<b(\overline{\theta})$ and $\hat{\theta}\in[\theta(\hat{t}),\overline{\theta}]$, such that

$$b(t,\theta) > (1+\eta-\Lambda)\overline{\theta}$$

for all $t\in[\hat{t},b(\overline{\theta})]$, $\theta\in[\hat{\theta},\overline{\theta}]$. This implies that the sales price for the good exceeds $(1+\eta-\Lambda)\overline{\theta}$ if no bidder drops out until $\hat{t}$. If $b(t,\theta)$ is a time-consistent strategy, then at time $\hat{t}$ a bidder of type $\overline{\theta}$ must weakly prefer this strategy to an instantaneous drop out. Since at time $\hat{t}$ her belief to win is $\left(\frac{G(\overline{\theta})-G(\theta(\hat{t}))}{1-G(\theta(\hat{t}))}\right)^2$, this condition reads

$$-\lambda\eta\overline{\theta}\left(\frac{G(\overline{\theta})-G(\theta(\hat{t}))}{1-G(\theta(\hat{t}))}\right)^2 < \left(\frac{G(\overline{\theta})-G(\theta(\hat{t}))}{1-G(\theta(\hat{t}))}\right)^2(\overline{\theta}-(1+\eta-\Lambda)\overline{\theta})+L_{\hat{t}}(\theta),$$

with strict inequality since the price strictly exceeds $(1+\eta-\Lambda)\overline{\theta}$. This is equivalent to

$$L_{\hat{t}}(\overline{\theta}) > -2\Lambda\overline{\theta}\left(\frac{G(\overline{\theta})-G(\theta(\hat{t}))}{1-G(\theta(\hat{t}))}\right)^2,$$

a contradiction for $\hat{t}$ sufficiently close to $b(\overline{\theta})$. □

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