Quantum Estimation Methods for Quantum Illumination

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Quantum illumination consists in shining quantum light on a target region immersed in a bright thermal bath with the aim of detecting the presence of a possible low-reflective object. If the signal is entangled with the receiver, then a suitable choice of the measurement offers a gain with respect to the optimal classical protocol employing coherent states. Here, we tackle this detection problem by using quantum estimation techniques to measure the reflectivity parameter of the object, showing an enhancement in the signal-to-noise ratio up to 3 dB with respect to the classical case when implementing only local measurements. Our approach employs the quantum Fisher information to provide an upper bound for the error probability, supplies the concrete estimator saturating the bound, and extends the quantum illumination protocol to non-Gaussian states. As an example, we show how Schrödinger's cat states may be used for quantum illumination.

Introduction.—Entanglement is a necessary requirement for a number of quantum protocols, including quantum teleportation [1,2], superdense coding [3], and quantum computation [4,5], among others. In his pioneering work [6], S. Lloyd showed how suitable entangled states can be used to detect the presence of a low-reflectivity object embedded in a bright environment more efficiently than by using classical resources. This protocol, called quantum illumination (QI), consists of irradiating the target region by using a signal entangled with an ancilla, and optimally measuring the reflected signal together with the ancilla. Surprisingly, even if the final state is not entangled [7], the initial nonclassical correlations have a positive role in hypothesis testing performances. Lloyd's results, initially limited on a specific background noise, were extended considering a more general noise model [8]. An alternative attempt was given by S.-H. Tan *et al.* [9], in which the authors prove the advantage of an entangled Gaussian state in the QI performance. In this case, a part of a two-mode squeezed state is sent to the target region, while the other copy remains in the lab. Then, an optimal joint measurement on the copies of the received signal and the ancillary modes generates a gain of at least 6 dB in the error probability decaying rate. Because of technical difficulties, they found only a lower bound for the decaying rate of the optimal error probability, namely Bhattacharyya bound [10,11], by using tools specifically developed for Gaussian states [12]. Although this proves the existence of a QI protocol showing certain gain with respect the classical case, the estimator achieving this is highly nontrivial, as it requires the implementation of a quantum Schur transform [11]. In this sense, a local protocol is a desiderata, since it is simpler to be experimentally implemented. Finally, a protocol consisting of separate measurements of the single copies of the reflected signal and the

ancilla was found, showing a more modest 3 dB gain in the low photon regime [13]. These results paved the way for relevant experimental applications within the purview of quantum radar [14,15], quantum communication [16,17], and quantum phase estimation [18,19], in which the unavoidable noise plays a crucial role.

In this Letter, we show that, with a fixed number of photons, an entangled transmitter can improve the optimal estimation of the reflectivity parameter up to 3 dB with respect to a coherent state transmitter in the low-reflectivity limit. The optimal gain is achieved in the low-photon regime, and decays at least as the inverse of the signal photon number. This is proven by bounding the quantum Fisher information (QFI) for a family of states labeled by the reflectivity parameter. We relate these results to the QI protocol, discussing a strategy based on the quantum estimation of the reflectivity parameter. We show that the QFI provides a computable nontrivial upper bound on the optimal error probability, extending the QI protocol to non-Gaussian states. Our results are not limited by the usual QI constraints, since they can be applied to any number of photons in both the bath and the signal. Furthermore, this approach explicitly provides the concrete estimator attaining the proposed bound. The Letter is structured in the following way. First, we introduce the quantum estimation problem and compute the QFI. Then, we discuss our strategy for QI, providing the error probability bounds based on the QFI. Finally, we discuss two examples with Gaussian states and Schrödinger's cat states, showing that these states are also useful in QI.

Quantum estimation.—Let us consider a general bipartite pure state representation of the signal-idler (SI) system written in the Schmidt decomposition form
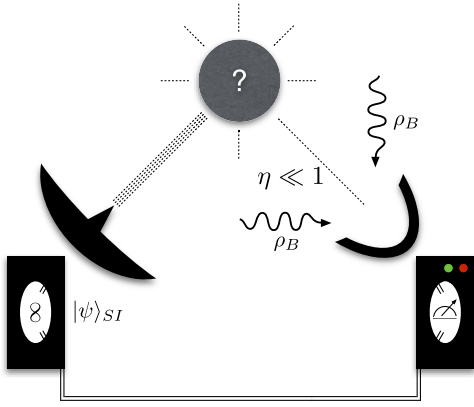
FIG. 1. Scheme of the quantum illumination protocol. (a) An entangled state, e.g., a two-mode squeezed state, is generated in the lab. The idler beam stays in a controlled transmission line while the signal is emitted toward the object we want to detect. Since its reflectivity is small, $\eta \ll 1$, most of the light captured by the receiver is thermal noise. By measuring the correlations between the signal and the idler beams, it is possible to detect the presence of an object with a smaller error probability than protocols involving classical light, with a gain up to 3 dB in the error probability exponent.

$$|\psi\rangle_{SI} = \sum_\alpha \sqrt{p_\alpha}\,|w_\alpha\rangle_S |v_\alpha\rangle_I, \qquad (1)$$

where $\langle w_\alpha|w_{\alpha'}\rangle = \langle v_\alpha|v_{\alpha'}\rangle = \delta_{\alpha,\alpha'}$. In the QI protocol, the signal modes of the M copies of $|\psi\rangle_{SI}$ are sent to the target region embedded in a bright thermal noise, in which there could possibly be an object. Then, we receive back M copies of the state $\rho_\eta = \mathrm{Tr}_S(U_\eta |\psi\rangle_{SI}\langle\psi| \otimes \rho_B U_\eta^\dagger)$. Here, $U_\eta = \exp[\sin^{-1}(\eta)(s^\dagger b - s b^\dagger)] \simeq \exp[\eta(s^\dagger b - s b^\dagger)]$ is the signal-object interaction, modeled as a beam splitter with amplitude reflectivity $\eta \ll 1$, and $\rho_B = \sum_n [N_B^n/(1 + N_B)^{1+n}]|n\rangle\langle n|$ is a thermal state with mean photon number N_B, as depicted in Fig. 1. In this framework, the case $\eta = 0$ corresponds to the absence of the object in the target region. In the following, we emphasize the case $N_B \gg 1$, corresponding to the typical regime where QI shows a gain with respect to the classical case. However, our treatment is completely general, and it holds for any value of N_B. In order to optimally estimate η, the QFI [20,21] is a paradigmatic tool. This is due to the Cramér-Rao bound [22], asserting the limits on the achievable precision of an unbiased estimator $\hat{\eta}$

$$\Delta\hat{\eta}^2 \geq \frac{1}{MH}, \qquad (2)$$

where $H = 2\sum_{mn}[|\langle\phi_m|(\partial_\eta\rho_\eta)|_{\eta=0}|\phi_n\rangle|^2/(\lambda_m + \lambda_n)]$ is the QFI for the family of states ρ_η, λ_n is the eigenvalue of $\rho_{\eta=0}$ corresponding to the eigenstate $|\phi_n\rangle$, and the derivative is evaluated at $\eta = 0$. A large value of the QFI means a high precision in the quantum estimation of the parameter η, provided that we choose the right measurement. Notice that

the optimal value of the QFI is achieved by a pure state, due to its convexity [21]. A possible estimator saturating Eq. (2) is given by the mean of the M single-copy measurement outcomes of the observable $\hat{O} = (\hat{L}/H)$, where $\hat{L} = 2\sum_{mn}[\langle\phi_m|(\partial_\eta\rho_\eta)|_{\eta=0}|\phi_n\rangle/(\lambda_m + \lambda_n)]|\phi_m\rangle\langle\phi_n|$ is the symmetric logarithmic derivative of ρ_η computed at $\eta = 0$ [20]. This estimator is optimal for evaluating the reflective parameter in the assumed neighborhood of zero.

By using the fact that the derivative computed at $\eta = 0$ is given by the trace of the commutator $(\partial_\eta\rho_\eta)|_{\eta=0} = \mathrm{Tr}_S[s^\dagger b - s b^\dagger, |\psi\rangle_{SI}\langle\psi| \otimes \rho_B]$, and that $\rho_{\eta=0} = \sum_\alpha p_\alpha|v_\alpha\rangle\langle v_\alpha| \otimes \rho_B$ has a simple diagonal form, we can infer the following general formula for the QFI [23]:

$$H = \frac{4}{1 + N_B} \sum_{\alpha\alpha'} \frac{p_\alpha p_{\alpha'}}{p_{\alpha'} + p_\alpha \frac{N_B}{N_B+1}} |\langle w_{\alpha'}|s|w_\alpha\rangle|^2. \qquad (3)$$

Equation (3) relates the QFI to the Schmidt vectors of the signal, and it allows us to upper bound the maximal achievable precision. First, by implementing the inequality $p_{\alpha'}/[p_{\alpha'} + p_\alpha N_B/(N_B + 1)] \leq 1$, and by using the relations $\sum_\alpha |w_\alpha\rangle\langle w_\alpha| = \mathbb{1}$ and $N_S = \sum_\alpha p_\alpha\langle w_\alpha|s^\dagger s|w_\alpha\rangle$, we obtain $H \leq 4N_S/(1 + N_B) \equiv H_Q^{(1)}$. Notice that the completeness relation can be assumed by adding zero probability terms in the Schmidt decomposition. The bound $H_Q^{(1)}$ is saturated, for instance, by a two-mode squeezed state in the limit of zero photons (see the examples below). Instead, the inequality between arithmetic and geometric means $[p_\alpha p_{\alpha'}/(p_{\alpha'} + p_\alpha N_B/(N_B + 1))] \leq \sqrt{(N_B + 1)/N_B}(p_\alpha + p_{\alpha'})/4$ yields $H \leq (2N_S + 1)/N_B \equiv H_Q^{(2)}$ by the same argument as above. The bound $H_Q^{(2)}$ is particularly useful for the usual QI situation, in which we have a bright environment $N_B \gg 1$. In this case, $H_Q^{(2)}$ is attained by a classical coherent state with a large number of signal photons, as we will see soon. In the following, we will denote the bound on the QFI by $H_Q \equiv \min\{H_Q^{(1)}, H_Q^{(2)}\}$.

In order to quantify the maximum possible gain achieved by an entangled state, we optimize Eq. (3) for nonentangled states. In this case, if $|\psi\rangle$ is the quantum state of the signal, then $H = 4|\langle\psi|s|\psi\rangle|^2/(1 + 2N_B)$. The last expression is maximal when $|\psi\rangle$ is an eigenstate of the annihilation operator, i.e., a coherent state. Therefore, we obtain $H = 4N_S/(1 + 2N_B) \equiv H_C$, that saturates the bound H_Q on the QFI for large N_S. Notice that, in the case when $N_B \ll 1$, the QFI upper bound is saturated by a coherent state for any N_S, meaning that we can have a gain using nonclassical resources only when the target region is embedded in a thermal bath with a nonzero mean number of photons. From the previous inequalities, we conclude that $H/H_C \leq 2$ for low N_S, and $H/H_C \leq 1 + (1/2N_S)$ for large N_S. The derived bounds imply that no structured

optical device is able to go beyond 3 dB in the reflectivity estimation problem. As shown in the examples below, this gain is saturated at least by Gaussian states and Schrödinger's cat states in the limit of zero signal photons. Last, we notice that not all entangled states are useful for estimating the reflectivity parameter. A paradigmatic case is $|\psi\rangle_{SI} = (1/\sqrt{d}) \sum_{n=0}^{d-1} |n\rangle_S |n\rangle_I$, a maximally entangled state which has the same QFI as the coherent state, as one can straightforwardly check by using Eq. (3).

Quantum illumination.—In the QI protocol, we may consider a strategy based on the evaluation of the parameter η with the estimator $\hat{\eta} = (1/M) \sum_{i=1}^{M} O_i$, where O_i are the outcomes of the observable optimizing the QFI. The figure of merit used in Bayesian hypothesis testing is given by the error probability

$$\mathrm{Pr}_{\mathrm{err}} = \pi_0 \Pr(1|H_0) + \pi_1 \Pr(0|H_1), \qquad (4)$$

where π_0 (π_1) is the *a priori* probability of the absence (presence) of the object, while $\Pr(1|H_0)$ [$\Pr(0|H_1)$] is the probability to have a false positive (false negative), denoted as a type I (II) error. First, we remind that $\hat{\eta}$ is unbiased in a neighborhood of $\eta = 0$, having $\mathrm{Tr}(\rho_\eta \hat{O}) = \eta + O(\eta^2)$. Thus, it is natural to define a test as follows: we declare the presence of the object whether $\hat{\eta} > \xi\eta$ for some $0 < \xi < 1$, and its absence, otherwise. In this case, we have that $\Pr(1|H_0) = \Pr[\hat{\eta} > \xi\eta|H_0] \equiv P_I$ and $\Pr(0|H_1) = \Pr[\hat{\eta} - \eta < -(1-\xi)\eta|H_1] \equiv P_{II}$. Eventually, one should choose ξ in order to minimize the error probability $\mathrm{Pr}_{\mathrm{err}}$. If we transmit a signal in a coherent state $|\alpha\rangle$, with $\alpha = \sqrt{N_S}e^{i\phi}$, the error probability optimized upon global measurements is given by $\mathrm{Pr}_{\mathrm{err}}^C \sim e^{-\eta^2 N_S(\sqrt{N_B+1} - \sqrt{N_B})^2 M}$ [24]. In the $N_B \gg 1$ limit, the same decaying rate is reached by the measurement that optimized the QFI, i.e., $\hat{O}_C = (e^{-i\phi}b + e^{i\phi}b^\dagger)/2\sqrt{N_S}$, which is a quadrature operator up to a normalization factor. In fact, by using that the measured state is Gaussian for any η, we can deduce the classical type I and II error probabilities

$$P_{I,II} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\eta_{I,II}^2 H_C M}{2}}\right) \sim \exp\left(-\frac{\eta_{I,II}^2 H_C M}{2}\right), \qquad (5)$$

where $\eta_I = \xi\eta$ and $\eta_{II} = (1-\xi)\eta$. Both types of error decay exponentially with a rate $\eta_{I,II}^2 H_C M/2$, and the optimal decaying rate of the error probability is obtained for $\xi = \frac{1}{2}$. In the $N_B \gg 1$ limit, the optimal error probability approximates to $\mathrm{Pr}_{\mathrm{err}}^C \approx e^{-M\eta^2 N_S/4N_B}$, whose decaying rate is the same as the one of P_I and P_{II} found in Eq. (5) if we set $\xi = \frac{1}{2}$. In the following, we will compare a suboptimal error probability for entangled states with the optimal one for coherent states in order to show the quantum enhancement case by case. The aim of using nonclassical resources is to find a better convergence rate for the error probability $\mathrm{Pr}_{\mathrm{err}}$,

or to minimize P_{II} by keeping bounded P_I (see Refs. [25,26] for this analysis). We will show that both types of error decay faster if an entangled state and the optimal measurement given by the QFI are used. This is proven by applying the classical Cramér-Chernoff theorem [27] to the distribution of the measurement outcomes. In the following, we consider the maximally entangled states of the form $|\psi\rangle_{SI} = \sum_n \sqrt{p_n} |n\rangle_S |v_n\rangle_I$.

Theorem 1: Type I and II error probabilities.—Let $|\psi\rangle_{SI} = \sum_n \sqrt{p_n} |n\rangle_S |v_n\rangle_I$ be the Schmidt decomposition of the signal-idler state, and denote ρ_S the state of the signal. Then $P_{I,II} \sim \exp[-(\eta_{I,II}^2 H M/2)]$ provided that $\exists C > 0$ s.t. $\langle s^k s^{\dagger k}\rangle_{\rho_S} \leq k! C^k \forall k \in \mathbb{N}$.

Proof of Theorem 1.—In the Supplemental Material [23], we prove that the moment generating function $M_\eta(t) = \mathrm{Tr}\rho_\eta e^{t(\hat{O} - \mathrm{Tr}\rho_\eta \hat{O})}$ is finite in the interval $t \in [0, \sqrt{H^2 N_B/C}]$, provided that $\langle s^k s^{\dagger k}\rangle_{\rho_S} \leq k! C^k$. The proof is based on bounding the moments of the outcome distribution in a neighborhood of $\eta = 0$. Moreover, the expression $M_\eta(t) = 1 + [(1/2H) + O(\eta)]t^2 + O(t^3)$ holds as $t \to 0$. Now, the classical Cramér-Chernoff theorem says that $-(1/M)\log P_I \sim \sup_t[\eta_I t - \log M_{\eta=0}(t)]$. The supremum is achieved for $t = t^* \simeq \eta_I H$, as $\log M_{\eta=0}(t) \simeq t^2/2H$ for small t. Therefore, we have $P_I \sim \exp[-\eta_I^2 H M/2]$. Similarly, one can show that $P_{II} \sim \exp[-\eta_{II}^2 H M/2]$.

As a consequence, a gain in the QFI implies the same gain in the exponent of both types of error probability in the $N_B \gg 1$ limit. In addition, this approach also holds for discriminating between two values of the reflectivity parameter which are small, but different from zero. In this case, the optimal estimator does not change, and the η in the error probability exponent should be replaced by the numerical difference of the two values of interest. The possible quantum advantage is also kept in this scenario. Let us note that examples of strategies based on the estimation of the parameter η have been investigated both theoretically [13] and experimentally [14,17,18] for the case of Gaussian states in the limit of small signal photons. Here, we have extended the analysis to non-Gaussian quantum states and to any number of photons in the bath and the signal. Additionally, our analysis provides a computable upper bound for the error probability optimized upon all local quantum measurements (local strategies). Finally, a relevant advantage of our approach consists in providing explicitly the estimator which attains the aforementioned bound.

Examples.—Let us now illustrate our techniques by introducing a couple of paradigmatic examples achieving the maximum gain for the QI protocol in the low-photon regime, which shows that our upper bound to the optimal error probability is nontrivial. As discussed, it is sufficient to show the gain in the QFI and use the aforementioned estimator.

Gaussian states.—Regarding the two-mode squeezed state case, the QFI can be analytically computed, due

to the easy Schmidt decomposition, i.e., $|\psi\rangle_{SI} = \sum_{n=0}^{\infty} \sqrt{[N_S^n/(1+N_S)^{1+n}]}\,|n\rangle_S|n\rangle_I$. By using Eq. (3), one can find that [23]

$$H_{\text{Gauss}} = \frac{4N_S}{1+N_B}\frac{1}{1+\frac{N_S}{1+N_S}\frac{N_B}{1+N_B}}. \qquad (6)$$

Notice that $H_{\text{Gauss}}/H_C \simeq 1 + 1/(1+2N_S)$ for large N_B, so the gain achieved in this strategy decreases when N_S increases. We also notice that, for $N_B \ll 1$, this strategy performs worse than the optimal receiver for coherent states. This fact agrees with the previous bounds, which state that, in this regime, the QFI of any entangled states cannot be larger than the one corresponding to a coherent state. The optimal observable is $\hat{O}_{\text{Gauss}} = ab + a^\dagger b^\dagger$, up to a normalization factor, where a is the idler mode, and b is the incoming signal mode. This measurement can be realized with linear optical elements and photon counting measurements. An optical circuit implementing this measurement has been proposed in Guha *et al.* [13] in the limit of small N_S.

Schrödinger's cat state.—In order to compare this result, we consider the Schrödinger's cat state $|\psi\rangle_{SI} = (1/\sqrt{d})\sum_{k=0}^{d-1}|\alpha_k\rangle_S|w_k\rangle_I$, where $|\alpha_k\rangle$ is a coherent state with amplitude $\alpha_k = \sqrt{N_S}e^{i(2\pi k/d)}$, $\langle w_k|w_{k'}\rangle = \delta_{kk'}$, and $d \geq 2$. These states can be generated, for instance, in a circuit QED platform. One may use the results in Ref. [28] to create a standard Schrödinger's cat ($d = 2$) in the transmon-resonator system and, then, use the fact that a transmon can simultaneously interact with additional resonators [29]. Regarding the computation of the QFI ($H_{\text{Schrö}}$), we need to find the eigenvectors and the eigenvalues of the separable state $\rho_I \otimes \rho_B$. As ρ_B has a simple diagonal form, let us focus on $\rho_I = (1/d)\sum_{k,k'}\langle \alpha_k|\alpha_{k'}\rangle |w_k\rangle\langle w_k|$. We notice that $[\hat{T}_d, \rho_I] = 0$, where $\hat{T}_d = \sum_{l=0}^{d-1}|w_l\rangle\langle w_{l+1}|$, with the convention $|w_d\rangle \equiv |w_0\rangle$, is the boost operator. It follows that ρ_I has the same eigenvectors of the nondegenerate operator $\hat{T}_d$, i.e., $|v_k\rangle = (1/\sqrt{d})\sum_{l=0}^{d-1}e^{i(2\pi kl/d)}|w_l\rangle$ $(k = 0,\ldots,d-1)$, with corresponding eigenvalues $\lambda_k = \sqrt{d}\langle w_0|\rho_I|v_k\rangle$. It can be easily shown that $H_{\text{Schrö}}/H_C \simeq 2$ for low N_S [23], while for finite N_S a numerical calculation is needed. This can be done by suitably truncating the Hilbert space of the received signal. One should expect a higher QFI for a larger dimension. For this reason, we have considered the limit of infinite d, corresponding to the state $|\psi\rangle_{SI} = \sum_{n=0}^{\infty}\sqrt{e^{-N_S}(N_S^n/n!)}\,|n\rangle_S|z_n\rangle_I$ with $\langle z_n|z_{n'}\rangle = \delta_{nn'}$ [23], whose form is the one considered in Theorem 1. In this case, the state is already written as its Schmidt decomposition, and the QFI is obtained by plugging the probabilities obtained from Schmidt coefficients in Eq. (3). The resulting series converges fast and can be computed efficiently up to an arbitrary small error. The results for
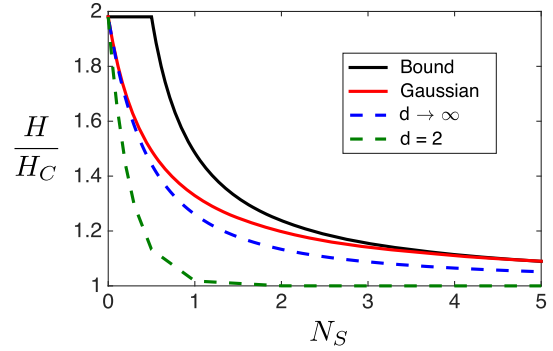

FIG. 2. We plot the gain in the quantum Fisher information H for the Gaussian state (red solid line) and Schrödinger's cat states (dashed lines), versus the classical case H_C corresponding to a coherent state transmitter. The lines corresponds to the case $N_B = 50$. Both Gaussian and Schrödinger's cat states achieve the maximum gain in the low photon regime, but the former are sizably more stable. The calculation for the Gaussian state is exact, see Eq. (6), while the QFI for the Schrödinger's cat states has been calculated numerically by truncating the Hilbert space of the received signal. This has been done by checking the convergence of the QFI for increasing Hilbert space dimension.

$d = 2$ and infinite d are depicted in Fig. 2, showing that these states perform worse than the two-mode squeezed state in the nonzero signal photon regime, but they are essentially the same in the low-photon regime. The optimal operator, in this case, is rather complicated, but for the case $d = 2$ in the low signal photon regime, it corresponds to the measurement in the degenerate Jaynes-Cumming basis, i.e., $\hat{O}_{\text{Schrö}} = \sigma^+b + \sigma^- b^\dagger$ up to a normalization factor. This measurement can be implemented, in principle, by adapting the ideas of spectroscopy of the Rabi model [30,31] to the Jaynes-Cumming case.

Conclusions.—We have considered the problem of optimally estimating the reflectivity parameter of an object embedded in an environment. Our analysis shows that, using entangled states as a resource, we can obtain an advantage up to 3 dB in the QFI with respect the optimal classical strategy. We have applied these results to the QI scenario, providing a nontrivial upper bound on the optimal error probability. This bound depends solely on the QFI of the signal-idler state, which is easily computable, and it allows us to extend the advantage of the QI protocol to a class of non-Gaussian states. Moreover, our results are not limited to bright environment ($N_B \gg 1$) and low signal photon ($N_S \ll 1$) cases, but they hold for any number of photons in the bath and the signal. In the examples, we have discussed both Gaussian states and multilevel Schrödinger's cat states, and have shown that the latter also perform optimally in the low-photon regime. Indeed, recent technological advances show that Schrödinger's cat states can be useful for quantum computation [32], and this makes them a possible alternative to the Gaussian states in the QI protocol.

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