Signaling versus Costly Information Acquisition

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July 25, 2018

Abstract

In Spence's (1973) signaling by education model and in many of its extensions, firms can only infer workers' productivities from their education choices. In reality, firms also use sophisticated pre–employment auditing to learn workers' productivities. We characterize the trade–offs between signaling by workers and costly information acquisition by firms. Information acquisition is always associated with (partial) pooling of worker types, and education is used as a signal only if relatively few workers have low productivity. Our analysis applies also to other signaling problems, e.g. the financial structure of firms, warranties, and initial public offerings.

JEL classifications: D82, D86
Keywords: Signaling, Information Acquisition, Auditing

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1 Introduction

We investigate signaling in a market where the uninformed side of the market relies not only on informative signaling by the other side, but may itself acquire information by performing costly audits. Agents on the informed side of the market privately know their types and can choose publicly observable actions to signal their types. The uninformed agents make offers based on observed actions. But these offers are non–binding in the sense that an application may be rejected if an audit discovers unfavorable information. Our model thus extends the canonical model of Spence (1973), in which signaling is the only source of information provision. We analyze the trade–offs between signaling by informed agents and information acquisition by uninformed agents. These trade-offs occur because in our model information acquisition is endogenously determined as part of the equilibrium.[1] We fully characterize the set of equilibria and point out novel features of signaling in markets where the uninformed side has the option to invest in information acquisition.

For our analysis, we use the framing of the labor market setting by Spence (1973), who first proposed education as a signaling device in labor markets: workers have private information about their own productivity, education is more costly for low than for high productivity workers and therefore can be used to signal productivities. Hence, firms can only infer workers' productivities from their education choices. In reality, besides looking at the workers' education, firms use sophisticated testing, assessment centers and other instruments of auditing to learn workers' productivities.[2]

We combine these two features: workers have the option of signaling through education and firms have the option of conducting costly information acquisition by auditing workers. In particular, workers choose their education level and then firms announce wages as in Spence's (1973) model. After a worker applies for a wage for some education level, firms choose whether or not to audit the applicant and then decide on hiring. We assume that an audit reveals a worker's type to the firm; but auditing is non–verifiable and firms cannot pre–commit to auditing. We are interested in the strategic interaction between workers' signaling incentives and firms' auditing incentives. Although we frame our model and results in terms of labor market signaling, our analysis also applies to other

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[1]This distinguishes our analysis from models (see the related literature review below) where in addition to signaling some *exogenous* information is available.

[2]According to Dessler (2017, p. 210), more than 67 percent of employers tested applicants for various skills. There is a huge literature about common testing and auditing procedures for applicants. See, for example, Noe et al. (2018, Ch. 6), Armstrong and Taylor (2017, p. 254-263), Dessler (2017, Ch. 6), or Martin (2012, p. 207-208, p. 216-219).

signaling environments, as we point out below.

When the option of auditing is unavailable, the most prominent equilibrium is the least–cost separating equilibrium in which workers with low and high productivity choose different education levels. When firms can audit applicants, firms acquire information only when there is enough uncertainty and they have diffuse beliefs about workers' types. Given degenerate beliefs, firms have no incentives to audit workers. As a result, the least–cost separating equilibrium is also supported when firms have the option to audit, and auditing does not occur in this equilibrium. However, this may not be a plausible outcome when auditing costs are small and it is more efficient for firms to incur the auditing costs than for workers to signal through education. We adopt an extension by Bester and Ritzberger (2001) of the intuitive criterion of Cho and Kreps (1987) to refine beliefs and rule out such counterintuitive equilibria.[3] Then, we show that there is a unique equilibrium. When auditing costs are large, the least–cost separating equilibrium is the unique outcome, different types of workers choose different education levels and auditing does not occur in equilibrium; when auditing costs are small, the unique equilibrium outcome is a partial pooling one, and firms audit with positive probability.

The equilibrium results are intuitive. In a competitive market, firms' expected profits are driven down to zero in equilibrium. Workers with high productivity benefit from information revelation by being recognized as high types, and receive a wage equal to their productivity subtracting expected costs of information acquisition. Information about workers' types is revealed either through workers' costly education or through the firms' costly auditing. With either channel, high types effectively bear the expected costs. For large auditing cost, information revelation is relatively cheap through signaling; hence, high productivity workers have an incentive to signal their type through education; when auditing costs are small, expected auditing costs passed through to high types are relatively small, and it is beneficial for them to refrain from signaling and to rely on firms' audits instead.

The equilibrium has interesting features. When auditing costs are relatively small, the least–cost separating equilibrium cannot be supported, and the more plausible partial pooling equilibrium uniquely survives our belief refinement. When there are many high–productivity workers, our equilibrium is a partial pooling one in which some high–productivity workers signal through education while the remaining pool with low types on zero education, and firms audit some of the workers with zero education. High–productivity workers' education decreases as their education costs increase or as auditing costs decrease. For sufficiently small auditing costs, workers pool at zero education and

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[3]See Section 4 for more details and a discussion.

no worker uses education as a signal in equilibrium. With vanishing auditing costs, the pooling equilibrium becomes more and more informative and converges to the complete information outcome: workers' expected payoffs converge to their productivities. Furthermore, both types' payoffs are (sometimes strictly) higher in the pooling equilibrium with auditing than their respective payoffs in the separating equilibrium without auditing. Therefore, when firms can audit rather cheaply, workers indeed prefer not to signal.

Our contribution can be helpful for analyzing other environments that involve strategic interactions between signaling and costly information acquisition. Consider, for example, the model of Leland and Pyle (1977) in which entrepreneurs seek to sell their projects to investors. Each entrepreneur has private information about the future revenues of his project. As Leland and Pyle (1977) show, the equity participation of the entrepreneur can then be a signal of the project's quality. This is so because high–quality entrepreneurs have a higher incentive to retain a share of the revenues than low–quality entrepreneurs. Suppose now that investors can obtain information not only from observing the entrepreneur's equity share but also by auditing project quality. Our findings then indicate that in an equilibrium where investors invest in auditing, signaling plays a role only if the market share of high–quality projects is sufficiently high.

As another application, consider warranties. Sellers are privately informed about product quality, but they can offer warranties to signal the quality of their product. Offering warranties is costly, as products break down (Spence, 1977) or as they induce moral hazard on behalf of buyers (Lutz, 1989). In each case, offering a warranty is more expensive for sellers of low-quality products. Buyers observe the warranties and compete by placing bids. Suppose now that buyers have the option to inspect the product's quality in addition to inferring it from the warranties. Our results then suggest that in any equilibrium with inspection sellers use less warranties as a signaling device than classical signaling models without inspections would imply. This is in line with the "mixed conclusions" of the empirical literature about warranties as signals of product quality (Riley, 2001, p. 455).

Finally, consider initial public offerings (IPOs). In the IPO process, the choice of investment bankers and board members may be a signal of firm value to potential investors (see, e.g., Titman and Trueman (1986) and Certo, Daily, and Dalton (2001) for theoretical analysis and empirical evidence). Prestigious investment bankers and board members, who are more accurate at evaluating information about the firm, are more costly. But, owners of high–value firms are willing to pay a premium for hiring them to avoid underpricing. Our analysis suggests that for those IPOs where potential investors can learn the true firm value at relatively low cost, the firms may reduce their usage of high–cost agents

to signal firm value.

Related Literature

Our work contributes to the signaling literature by incorporating the option of costly information acquisition into the otherwise standard setup.[4] Costly information acquisition is a very natural scenario in the recruiting and hiring process in the labor market, and it has been studied extensively but in isolation of the signaling aspect so far.[5] In the combined model, we are able to offer some interesting observations on the trade–offs between workers' signaling incentives and firms' auditing incentives.

The paper is related to the literature on job market signaling when firms have some additional information about workers' productivities. Alos–Ferrer and Prat (2012) analyze a dynamic model in which the firm is able to extract information from noisy realizations of the worker's productivity *after* the worker is hired. In contrast, in our model firms can audit the worker's type *before* hiring. Feltovich, Harbaugh and To (2002) consider a model where, prior to making job offers, employers have access to grades and other information that is correlated with workers' productivity in addition to observing the education choices by workers. They point out countersignaling equilibria: only workers with intermediate productivity signal via education, while low and high productivity workers pool at zero education. Daley and Green (2014) fully characterize the equilibria of such a model and extend it in various directions. Kurlat and Scheuer (2018) study a competitive equilibrium model with firms that are heterogeneous in their precision of evaluating additional information. In these papers, firms' information is exogenous and non–strategic, while in our model, information acquisition is strategic and firms have the option whether to conduct costly audits or not.[6]

Our paper complements other applications of signaling games with information acquisition. Banks (1992) analyzes a setup where the monopolist knows ex ante its true marginal cost of production, the regulator observes the market price proposed by the monopolist and decides whether to verify the monopolist's marginal cost and impose a regulatory price for the monopoly product. Bester and Ritzberger (2001) study the environment where a monopolist uses pricing to signal product quality and consumers may

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[4]See Kreps and Sobel (1994) and Riley (2001) for a review on the role of signaling and its applications in different fields.

[5]See, for example, Guasch and Weiss (1981). In their model, there is no signaling, auditing is contractible and is used together with the wage scheme as a self-selection device. We differ from them by examining a model with signaling by education and unobservable and thus non–contractible auditing. See also the references in footnote 2.

[6]Prat (2002) considers signaling with additional exogenous information in a voting model.

acquire additional information about product quality.[7] Mayzlin and Shin (2011) consider a setting where a firm strategically chooses the message content of costless advertisement to (not) reveal aspects of the product's quality, and consumers can acquire additional information about the product's quality before purchasing. Stahl and Strausz (2017) analyze a setting in which buyers decide whether to acquire costly third party certification to verify the quality of the product on observing the prices posted by the seller. Garfagnini (2017) analyses a setup in which the career-concerned worker signals his type through overtime at work and the firm exercises oversight to identify low ability workers. These applications typically have very different features from the competitive job market signaling environment we focus on.

This paper is organized as follows. Section 2 contains the setup combining job-market signaling and costly auditing. In Section 3, we characterize the relation between firms' beliefs, the equilibrium wage and firms' incentive to audit for different auditing costs. In Section 4, we present the extension of the intuitive criterion that we will use to refine the equilibria. Section 5 and 6 fully characterize the set of equilibria and show uniqueness of the separating and the pooling equilibrium. Discussions and extensions can be found in Section 7. Section 8 contains the concluding remarks. All formal proofs are relegated to an appendix.

2 The Model

We consider the following adaptation of the Spence (1973) signaling model, in which workers choose education as a signal and then firms compete for workers. There is a unit mass of workers, who differ in their innate productivity. We restrict attention to the case of two types, $i \in \{L, H\}$, of workers. A fraction $1 - \lambda \in (0, 1)$ of workers has type L and productivity $X_L > 0$; the remaining fraction λ has type H and productivity $X_H > X_L$. Each worker's productivity is private information.

Before entering the job market, workers can choose an education level $y \geq 0$, which is publicly observable. The cost of education $c_i(y)$ is type dependent. We follow the consensus in the literature and assume that

$$c_L(0) = c_H(0) = 0, \quad c_L'(y) > c_H'(y) > 0, \tag{1}$$

for all $y > 0$. Thus, for any $y > 0$, type L has higher education costs than type H. If type i with education choice y is employed at the wage w, his utility is $w - c_i(y)$.

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[7]Note that in Bester and Ritzberger (2001) the informed seller makes price offers, whereas in our setting the uninformed firms offer wages.

There are at least two firms with constant returns to scale so that each firm faces no restrictions on the mass of workers that it can employ. Firms *a priori* only observe a worker's education choice y, but not his type. They, however, can choose to learn the type of a job applicant at the cost $k > 0$, and then make their hiring decision contingent on the observed type.[8] Information acquisition is not observable: whether a firm audits an applicant's type and if so which information it obtains is the firm's private information.[9] Therefore, firms cannot commit to pre–employment audits and they cannot make their wage offers contingent on their auditing choice and observation. Also, workers cannot pay firms for being audited to receive a public certification of their productivity. When a firm employs a worker of type i at the wage w, its profit is $X_i - w - k$ if it has performed an audit before hiring, and $X_i - w$ without an audit. If after auditing the firm decides not to hire the applicant, it incurs the loss $-k$.

Firms compete by making wage offers and so in equilibrium each firm will earn zero *expected* profit. Therefore, it is always optimal for a firm to offer the 'default' wage $w_L \equiv X_L$ and to hire all applicants, independently of their education, at w_L without an audit.[10] Obviously, if a worker decides to apply for the default wage, it is not optimal to acquire education. In contrast to the default wage, we assume that all wage offers $w > X_L$ are non–binding in the sense that the firm remains entitled to reject any application at–will.[11] For any offer $w \in (X_L, X_H]$, therefore, if the firm decides not to perform an audit, it optimally hires the applicant as long as his expected productivity is not below the wage w. In contrast, with auditing a firm will hire type H and reject type L. It is publicly observable when a worker's application for $w \in (X_L, X_H]$ is rejected. All firms then believe that this worker is of type L, and therefore he can apply anew only for the default wage w_L.[12] But, for the renewed application the worker pays a delay or switching cost $s \in (0, X_L)$.[13]

Workers who decide not to choose the default wage, select some education $y \geq 0$. After observing education choices, firms compete by offering a wage $w(y)$ for workers with education y. Obviously, their offers will depend on the belief about the average productivity of these workers. Also, as we show in the next section, their belief is important

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[8]For example, Guasch and Weiss (1981, p. 275) write that a "common practice is for firms to offer a wage for a given job classification, and to test applicants."

[9]For a model where testing job applications is contractible, see e.g. Guasch and Weiss (1981).

[10]As we discuss in Section 7, the availability of the default wage simplifies the specification of the firms' beliefs in a perfect Bayesian equilibrium.

[11]In Section 7, we show that our results remain robust when the firms can also make binding offers to hire all applicants.

[12]Of course, these beliefs will turn out to be consistent with the equilibrium outcome.

[13]If workers discount future wages by the factor $\delta \in (0, 1)$, then $s = (1 - \delta)X_L$ for being hired at $w_L = X_L$ at date 2 and not being hired immediately at date 1.

for the firms' decisions on information acquisition. We denote by $\mu(y) \in [0,1]$ the firms' belief that a fraction $\mu(y)$ has type H among all workers who have chosen education y and apply for $w(y)$.

The signaling and auditing game has the following sequence of events:

(i) Workers privately observe their type, L or H.

(ii) Workers either choose some education $y \geq 0$ or opt for the default wage $w_L = X_L$.

(iii) Firms compete by offering wages $w(y)$ for each education y chosen by some workers.

(iv) Workers with education y choose at which offer $w(y)$ to apply.

(v) Firms take auditing decisions based on their belief $\mu(y)$ and then decide on hiring.

(vi) After a worker is rejected by a firm, he receives the wage w_L, but incurs the cost s.

In this setting, firms combine job advertisements with wage offers and then decide on auditing job applicants. This sequence of events looks quite natural in many labor markets. Indeed, in their handbook on personnel management Armstrong and Taylor (2017, p. 248) point out that the first step of the recruiting process is to define job requirements including "terms and conditions (pay, benefits, …)".[14] After attracting candidates the next steps are sifting applications, interviewing, testing, assessing candidates, obtaining references and checking applications.

In what follows, we analyze the perfect Bayesian equilibrium of this game. To simplify the exposition, we assume that all firms have identical out–of–equilibrium beliefs. Further, whenever a wage offer attracts workers, the average productivity of applicants for this wage is the same for all firms that make this offer. We say that an equilibrium is *unique* if the workers' education choices and all wage offers that attract a positive mass of workers are uniquely determined.

3   Wages and Information Acquisition

We first analyze how the firms' equilibrium wage offers and their decisions on information acquisition depend on the belief $\mu(y)$ about workers with education y. Let $\rho(y) \in [0,1]$ denote the probability that a worker applying for $w(y)$ will be audited. Obviously, if the

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[14]Martin (2012, p. 200-201) confirms that the "job announcement should include …the salary and benefits attached to the position."

cost of information acquisition is too high, auditing cannot occur in equilibrium. Indeed, the following result shows that this is the case if the auditing cost k is above the critical level

$$\tilde{k} \equiv \frac{X_H - X_L}{4}. \tag{2}$$

With this definition, we obtain the following observation on firms' equilibrium wage offer and auditing decision for high auditing cost:

Lemma 1 *Let $k > \tilde{k}$. Then in equilibrium, firms offer $w(y) = X_L + \mu(y)(X_H - X_L)$ and choose $\rho(y) = 0$ after observing education y.*

By offering a wage $w(y) > w_L = X_L$ firms compete for high productivity workers. Given their belief $\mu(y)$, they face the trade–off of incurring auditing costs and hiring type H only, or saving the auditing cost and hiring both types. With auditing, the competitive wage, at which firms make zero profits, equals $X_H - k/\mu(y)$ because only a fraction $\mu(y)$ of all workers is expected to have type H and the auditing cost is passed through to these workers. Without auditing, the competitive wage equals the average productivity $X_L + \mu(y)(X_H - X_L)$ because both types are hired. As long as $k > \tilde{k}$, the average productivity is higher than the wage for type H with auditing irrespective of the belief $\mu(y)$. Therefore, competition precludes that firms invest in information acquisition.

Now consider the case $k \leq \tilde{k}$. In this case, the equation $k = \mu(1-\mu)(X_H - X_L)$ has two solutions if $k < \tilde{k}$, and in the limit $k = \tilde{k}$ these solutions coincide:

$$\mu_1(k) \equiv \frac{1}{2} - \frac{1}{2}\left(\frac{X_H - X_L - 4k}{X_H - X_L}\right)^{1/2}, \quad \mu_2(k) \equiv \frac{1}{2} + \frac{1}{2}\left(\frac{X_H - X_L - 4k}{X_H - X_L}\right)^{1/2}. \tag{3}$$

Note that $\mu_1(\tilde{k}) = \mu_2(\tilde{k}) = 1/2$, and $0 < \mu_1(k) < 1/2 < \mu_2(k) < 1$ if $k < \tilde{k}$. Further, for $k \in (0, \tilde{k})$

$$\mu_1'(k) > 0, \quad \mu_1(0) = 0, \quad \mu_2'(k) < 0, \quad \mu_2(0) = 1. \tag{4}$$

The following Lemma describes how the firms' equilibrium behavior depends on their beliefs when auditing costs are sufficiently small.

Lemma 2 *Let $k \leq \tilde{k}$. Then, after observing education y, in equilibrium:*

(i) if $\mu(y) \notin [\mu_1(k), \mu_2(k)]$, firms offer $w(y) = X_L + \mu(y)(X_H - X_L)$ and choose $\rho(y) = 0$.

(ii) if $\mu(y) \in (\mu_1(k), \mu_2(k))$, firms offer $w(y) = X_H - k/\mu(y)$ and choose $\rho(y) = 1$.

(iii) if $\mu(y) \in \{\mu_1(k), \mu_2(k)\}$, firms offer $w(y) = X_L + \mu(y)(X_H - X_L) = X_H - k/\mu(y)$ and any $\rho(y) \in [0, 1]$ is optimal for them.
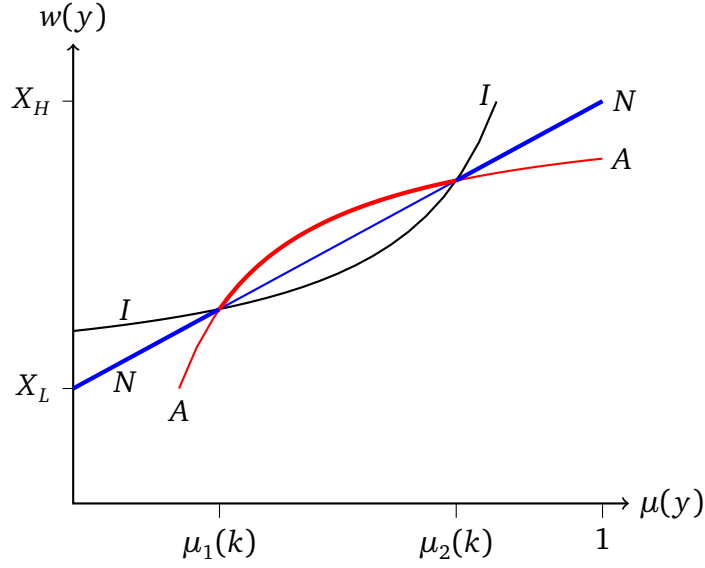
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Figure 1: *Illustration of Lemma 2*

Figure 1 illustrates Lemma 2. Firms are indifferent between information acquisition and non–acquisition for all combinations of wage w and belief $\mu(y)$ on the I–I line; above the line they optimally choose information acquisition with probability $\rho = 1$ and below the line with probability $\rho = 0$. Along the N–N line the expected profit from hiring an applicant without information acquisition is zero; the expected profit is positive below the line. Along the A–A line the expected profit from investing in information and hiring only type H is zero; it is positive below the line. The upper envelope of the N–N and the A–A line is the wage $w(y)$ for education y offered under competition. This wage is strictly increasing in $\mu(y)$. At this wage, not investing in information is strictly optimal for the firms if $\mu(y) < \mu_1(k)$ or $\mu(y) > \mu_2(k)$ because their belief is relatively precise and workers are highly likely to be either type H or L. Only for more diffuse beliefs $\mu(y) \in (\mu_1(k), \mu_2(k))$ there is a high uncertainty regarding the workers' types, so that firms are forced under competition to audit their applicants. If $\mu(y) = \mu_1(k)$ or $\mu(y) = \mu_2(k)$, the firms are indifferent between auditing or not at the competitive wage. In this situation they optimally audit some arbitrary fraction $\rho(y) \in [0, 1]$ of applicants.

Lemmas 1 and 2 allow us to derive the utility of workers from choosing education y, given the belief $\mu(\cdot)$ of firms. First consider type H. This type will never be rejected and so he always receives the wage $w(y)$ stated in the two Lemmas. Therefore, his utility is

$$U_H(y|\mu(y)) \equiv \begin{cases} X_H - k/\mu(y) - c_H(y) & \text{if } k \leq \tilde{k} \text{ and } \mu(y) \in [\mu_1(k), \mu_2(k)], \\ X_L + \mu(y)(X_H - X_L) - c_H(y) & \text{otherwise.} \end{cases} \quad (5)$$

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Note that the utility of type H is strictly increasing in the belief $\mu(y)$.

In contrast, type L gets the wage $w(y)$ only if he is not audited. After an audit he is rejected and only gets $X_L - s$. Therefore, his expected utility depends on the audit probability $\rho(y)$. This probability, however, is arbitrary when the parameter combination in part (iii) of Lemma 2 applies. For our purposes, however, it is sufficient that for all other parameter combinations the utility of type L is well–defined by

$$U_L(y|\mu(y)) \equiv \begin{cases} X_L - s - c_L(y) & \text{if } k \leq \tilde{k} \text{ and } \mu(y) \in (\mu_1(k), \mu_2(k)), \\ X_L + \mu(y)(X_H - X_L) - c_L(y) & \text{if } k \leq \tilde{k} \text{ and } \mu(y) \notin [\mu_1(k), \mu_2(k)], \quad (6) \\ X_L + \mu(y)(X_H - X_L) - c_L(y) & \text{if } k > \tilde{k}. \end{cases}$$

4 Belief Refinements

As is well–known, signaling games have a disconcerting multiplicity of equilibria. The reason is that the perfect Bayesian equilibrium in our context pins down the firms' beliefs only for equilibrium education choices. Therefore, multiple outcomes can be supported by out–of–equilibrium beliefs that deter any deviating education choice by interpreting it as a signal of low productivity. To rule out counterintuitive equilibria driven by such overly pessimistic beliefs, the literature has adopted belief refinements that impose plausible restrictions on out–of–equilibrium beliefs.

The standard refinement for Spence's (1973) model of education signaling with two types is the *intuitive criterion* of Cho and Kreps (1987). It yields a unique prediction in the model of two worker types by ruling out pessimistic beliefs about their productivity for certain out–of–equilibrium education choices. Let U_L^* and U_H^* denote the equilibrium utility of type L and H, respectively. The idea of the intuitive criterion is that an out–of–equilibrium education choice y should be considered as a signal of type H if – given this belief – only type H has an incentive to deviate to y:

Condition A *For any out–of–equilibrium education y, if*

$$U_H(y|1) > U_H^* \text{ and } U_L(y|1) < U_L^* \quad (7)$$

then $\mu(y) = 1$.

By the first inequality in (7), type H gains by choosing y if education y is interpreted as a signal of productivity X_H, whereas by the second inequality type L loses by choosing y even when he is considered to have productivity X_H. The intuitive criterion stipulates

that in this situation education y is a convincing signal of productivity X_H. Thus, whenever (7) holds for some out–of–equilibrium education y, the equilibrium does not satisfy Condition A: type H would gain by deviating to y because $U_H(y|\mu(y)) = U_H(y|1) > U_H^*$.

The intuitive criterion is designed for signaling games without alternative sources of information. As Bester and Ritzberger (2001) argue, it has the drawback that it fails to generate incentives for information acquisition. This is so because it specifies deterministic beliefs restrictions. Yet, as we have seen in the previous section, firms will invest in auditing only if their belief is sufficiently diffuse. Therefore, even with arbitrarily small auditing costs, the intuitive criterion cannot induce information acquisition as a response to a deviating education choice.[15] To provide a more effective role for information acquisition, Bester and Ritzberger (2001) propose an extension of the intuitive criterion.[16] We apply a slight modification, because in (6) the type L's utility $U_L(\cdot)$ is not defined for $\mu \in \{\mu_1, \mu_2\}$:[17]

Condition B *For any $\delta \in [0,1] \backslash \{\mu_1, \mu_2\}$ and for any out–of–equilibrium education y, if*

$$U_H(y|\delta) > U_H^* \ and \ U_L(y|\delta) < U_L^* \tag{8}$$

then $\mu(y) \geq \delta$.

Condition B contains the intuitive criterion as the special case $\delta = 1$. It extends the idea of this criterion to a situation where a deviation to y is profitable only for type H when the firms believe that the deviation originates from type L with probability δ. This belief is already rather pessimistic, because it actually gives no incentive to type L to deviate to y. In such a case Condition B requires that the firms' belief should not be even more pessimistic than δ. Thus, whenever (8) holds, the equilibrium violates Condition B, because $U_H(y|\mu(y)) \geq U_H(y|\delta) > U_H^*$ implies that type H would rather choose y than the supposed equilibrium education.

5 Separating Equilibrium

An equilibrium is called *separating* if education choices reveal a worker's type. Thus, the firms' beliefs for any education chosen in equilibrium are either zero or one, $\mu(y) \in \{0, 1\}$

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[15]Alternative refinements such as universal divinity in Banks (1992) and Condition $D1$ in Daley and Green (2014), require off–equilibrium beliefs to put positive probability only on the type that is most likely to defect from a given deviation. Like the intuitive criterion, such off–equilibrium beliefs are deterministic and therefore cannot induce information acquisition.

[16]For a recent application, see Stahl and Strausz (2017).

[17]Note that this makes Condition B less restrictive.

for all y contained in the support of workers' equilibrium education choices. As shown by Cho and Kreps (1987), the Spence (1973) model with two worker types has a unique equilibrium that satisfies the intuitive criterion of Condition A. It is separating and has the following properties: type L workers receive the wage X_L and choose zero education; type H workers receive the wage X_H and choose education y_H^* defined by

$$c_L(y_H^*) = X_H - X_L. \tag{9}$$

This is the least–cost separating equilibrium in the sense that y_H^* is the lowest level of education such that type L workers cannot gain from imitating the type H workers' education choice to receive the wage X_H.

To investigate whether this outcome remains an equilibrium in our extension of Spence's (1973) setting, we define a critical audit cost $\bar{k}$ by the condition that

$$\bar{k} = \tilde{k} \quad \text{if} \quad c_H(y_H^*) \geq 2\tilde{k}, \quad \frac{\bar{k}}{\mu_2(\bar{k})} = c_H(y_H^*) \quad \text{if} \quad c_H(y_H^*) < 2\tilde{k}, \tag{10}$$

with $\tilde{k}$ given by (2). Note that $\bar{k}$ is uniquely defined as $k/\mu_2(k)$ strictly increases in $k \in [0, \tilde{k}]$, is equal to zero for $k = 0$ and $\tilde{k}/\mu_2(\tilde{k}) = 2\tilde{k}$. Further, $\bar{k} < \tilde{k}$ if $c_H(y_H^*) < 2\tilde{k}$.

To understand the role of $\bar{k}$ for the equilibrium outcome, recall from the discussion of Lemma 1 that $k/\mu(y)$ is the auditing cost passed through to type H workers if the auditing cost is k and the firms' belief is $\mu(y)$. Since $\mu_2(k)$ is the largest possible belief that induces an audit, $k/\mu_2(k)$ is the minimum auditing cost borne by type H workers if an audit takes place. In the second part of (10), $\bar{k}$ is the critical value of k at which this minimum auditing cost is exactly the same as the minimum education cost a type H worker has to incur for the separating education y_H^*. For any $k < \bar{k}$, type H workers are better off by bearing the expected audit cost than signaling by education y_H^*, because $k/\mu_2(k) < c_H(y_H^*)$.

As the following proposition shows, the parameter $\bar{k}$ is indeed critical for the existence of a separating equilibrium under Condition B.

Proposition 1 *(i) For all k, there exists a unique separating equilibrium satisfying condition A: type L workers receive a wage of X_L and do not invest in education; type H workers receive a wage of X_H and choose education y_H^*. Auditing does not occur in equilibrium. (ii) The separating equilibrium satisfies Condition B if and only if $k \geq \bar{k}$.*

As indicated above, the statement in part (i) of the proposition is shown already in Cho and Kreps (1987) for a setting where the option of information acquisition is unavailable.

To see that this equilibrium also persists for all $k \geq 0$ if information acquisition is feasible, simply define the firms' beliefs as follows: $\mu(y) = 0$ for all $y < y_H^*$, and $\mu(y) = 1$ for all $y \geq y_H^*$. These beliefs are consistent with the equilibrium outcome and it is easily verified that they satisfy Condition A. Also, with these beliefs it follows directly from Lemma 2 that auditing occurs neither in equilibrium nor after out–of–equilibrium choices of y. Thus, the equilibrium selected by Condition A is not affected by the feasibility of auditing. Even when the auditing cost k is arbitrarily small, type H workers have to invest $c_H(y_H^*)$ to distinguish themselves from type L.

To complete the proof of Proposition 1, we show part (ii) by the following lemma:

Lemma 3 *Suppose there is an equilibrium such that some type H workers choose education y_H^* and receive a wage of X_H. (i) For all $k < \bar{k}$, this equilibrium does not satisfy Condition B. (ii) For all $k \geq \bar{k}$, this equilibrium satisfies Condition B if type L workers receive equilibrium utility $U_L^* = X_L$.*

Clearly, the equilibrium described in part (i) of Proposition 1 has all the properties required in Lemma 3, and so this proves part (ii) of the proposition.

By part (i) of Lemma 3, the separating equilibrium does not survive Condition B if $k < \bar{k}$. The idea is that for some small out–of-equilibrium choice of education, there always exists some belief δ that induces auditing, and given this belief, type H benefits from such a deviation while type L loses from it due to auditing. By Condition B, such deviation should be interpreted as originating from type H with probability no smaller than δ. Given this belief, type H prefers such an audit-inducing deviation over y_H^* because the auditing cost passed through to him is relatively low in comparison to the signaling cost associated with y_H^*, thus destroying the separating equilibrium.

Part (ii) of Lemma 3 implies that if $k \geq \bar{k}$, the separating equilibrium satisfies Condition B. The reason is that if there is any deviation that does not induce auditing and is profitable for type H, such a deviation is also profitable for type L, and so there is no violation of Condition B. If, however, a deviation induces auditing, it is never profitable for type H because the audit cost passed through to him is too high in comparison to the signaling cost from education y_H^*. Thus, there exists no deviation and belief such that the two inequalities in Condition B hold simultaneously, and therefore the separating equilibrium cannot be eliminated by Condition B when $k \geq \bar{k}$.

Note that Lemma 3 not only applies to the separating equilibrium in Proposition 1, it also applies to any equilibrium where some type H chooses y_H^* and receives the wage X_H. We will use this insight later on in Lemma 6 for the analysis of pooling equilibria.

6 Pooling Equilibrium

In a *pooling* equilibrium, some fraction $\sigma_L > 0$ of type L workers and some fraction $\sigma_H > 0$ of type H workers choose the same education y_P^* and do not opt for the wage $w_L = X_L$. Thus, the choice of y_P^* does not fully reveal a worker's type. We say that *partial pooling* occurs at y_P^* if $\sigma_i < 1$ for some $i \in \{L, H\}$; otherwise, if $\sigma_L = \sigma_H = 1$, we have *full pooling*. In a pooling equilibrium, the firms' beliefs at y_P^* are determined by Bayes' rule as

$$\mu(y_P^*) = \frac{\sigma_H \lambda}{\sigma_L(1-\lambda) + \sigma_H \lambda} \in (0, 1), \tag{11}$$

because a fraction λ of all workers has type H. Therefore, by Lemmas 1 and 2, firms offer a wage $w(y_P^*) \in (X_L, X_H)$ after observing education y_P^* chosen by workers who have not opted for the default wage $w_L = X_L$. To establish the conditions for existence of a pooling equilibrium, we first derive some necessary properties of such an equilibrium in the following Lemmas 4–6.

It is well–known from Cho and Kreps (1987) that pooling does not survive Condition A in an environment without auditing. Indeed, the proof of the following lemma uses their argument to show that auditing must take place at any pooling education y_P^* with positive probability.

Lemma 4 *Suppose pooling occurs at y_P^* in an equilibrium satisfying Condition A. Then $k \leq \tilde{k}$ and the auditing probability satisfies $\rho(y_P^*) \in (0, 1)$.*

The intuition for why Condition A does not rule out pooling at y_P^* is that type H receives the wage $w(y_P^*)$ with certainty, while type L is rejected with probability $\rho(y_P^*)$. Therefore, for type L the expected wage from education y_P^* is less than $w(y_P^*)$. This means that the wage increase from deviating to a signal y such that $\mu(y) = 1$ and $w(y) = X_H$ is higher for type L than for type H. The presence of auditing in a pooling equilibrium, therefore, makes it more costly for type H to distinguish himself by an education that type L is not willing to imitate. In fact, as our analysis of pooling below shows, this makes it unattractive for type H to deviate from y_P^* to an education y that satisfies the requirements of Condition A.

By Lemma 4 firms have to be indifferent between auditing and not auditing an applicant with education y_P^*. As this is implied already by the weaker Condition A, it is clearly also true under the stronger Condition B. Actually, the following result shows that Lemma 4 in combination with Condition B uniquely determines the firms' beliefs at y_P^*.

Lemma 5 *Let $k \leq \tilde{k}$ and suppose pooling occurs at y_P^* in an equilibrium satisfying Condition B. Then $\mu(y_P^*) = \mu_2(k)$ and so the firms offer the wage*

$$w(y_P^*) = X_L + \mu_2(k)(X_H - X_L) = X_H - k/\mu_2(k).$$

Recall from Lemma 2 that firms are indifferent between auditing or not only under the beliefs $\mu(y) \in \{\mu_1(k), \mu_2(k)\}$. Lemma 5 uses Condition B to rule out that firms hold the more pessimistic belief $\mu(y_P^*) = \mu_1(k) < 1/2$ in a pooling equilibrium. Indeed, for pessimistic beliefs with $\mu(y_P^*) < 1/2$, type H can benefit by deviating to some education level $y' > y_P^*$ which induces a more optimistic but also more diffuse belief. Under this belief firms audit with probability one so that only type H gains from deviating to y'. As a result, firms must hold the belief $\mu(y_P^*) = \mu_2(k) \geq 1/2$ in a pooling equilibrium.

Lemma 5 also implies that pooling cannot occur at more than one education choice. To see this, note that by the lemma the wage $w(y_P^*)$ is the same for any pooling education y_P^*. Since workers of type H are always hired after applying for $w(y_P^*)$, it cannot be optimal for them to choose different educations with different costs to receive the wage $w(y_P^*)$.

The following result restricts the choices of education that can occur in a pooling equilibrium:

Lemma 6 *Suppose pooling occurs at y_P^* in an equilibrium satisfying Condition B. (i) Then there is no education $y > 0$ chosen only by type L workers and at most one education $\hat{y}_H > 0$ chosen only by type H workers. (ii) For all $k < \bar{k}$, $y_P^* = 0$ and $\sigma_i = 1$ for at least one type $i \in \{L, H\}$.*

For $k < \bar{k}$, Lemma 6 rules out pooling equilibria that satisfy Condition B and involve partial pooling by both types. As a consequence, there remain three categories as candidates for a pooling equilibrium. First, in an equilibrium with $\sigma_L \in (0, 1)$ and $\sigma_H = 1$ there is partial pooling only by type L workers. These have to be indifferent between opting for the wage $w_L = X_L$ and applying for $w(y_P^*)$, where they are rejected with probability $\rho(y_P^*)$:

$$X_L = (1 - \rho(y_P^*)) w(y_P^*) + \rho(y_P^*)(X_L - s). \tag{12}$$

As $w(y_P^*)$ is given by Lemma 5, this equation uniquely determines the likelihood $\rho(y_P^*) \in (0, 1)$ of being audited when applying for $w(y_P^*)$. Further, as $\mu(y_P^*) = \mu_2(k)$, Bayes' rule in (11) determines σ_L.

Second, there is partial pooling only by type H workers if $\sigma_H \in (0, 1)$ and $\sigma_L = 1$. This means that type L at least weakly prefers applying for $w(y_P^*)$, and being audited with

probability $\rho(y_P^*)$, over opting for $w_L = X_L$. Also, type H must be indifferent between receiving $w(y_P^*)$ and choosing education $\hat{y}_H$ to receive X_H:

$$X_L \leq \big(1 - \rho(y_P^*)\big) w(y_P^*) + \rho(y_P^*)(X_L - s), \quad w(y_P^*) = X_H - c_H(\hat{y}_H). \tag{13}$$

Note that the second condition uniquely determines the education level $\hat{y}_H$. As an additional equilibrium requirement, type L should have no incentive to choose $\hat{y}_H$. In fact, by Condition A, it is easy to see that he has to be indifferent between applying for $w(y_P^*)$ and choosing $\hat{y}_H$ to obtain the wage X_H: if type L strictly preferred applying for $w(y_P^*)$ over choosing $\hat{y}_H$, then type H could appeal to the intuitive criterion of Condition A that also an education choice slightly below $\hat{y}_H$ should be considered as a convincing signal of high productivity. To rule out that this destroys the equilibrium, it must be the case that

$$\big(1 - \rho(y_P^*)\big) w(y_P^*) + \rho(y_P^*)(X_L - s) = X_H - c_L(\hat{y}_H). \tag{14}$$

This equation determines $\rho(y_P^*)$. Again, we can use Bayes' rule to derive the equilibrium value of σ_H.

Third, $\sigma_L = \sigma_H = 1$ in an equilibrium with full pooling. Actually, full pooling can occur only under non–generic parameter combinations: by Bayes' rule and Lemma 5, full pooling requires that $\mu_2(k) = \lambda$. Thus, for a given audit cost k, full pooling is possible at most for a single value of λ. Full pooling can be viewed as a limit case $\sigma_L \to 1$ of the first category so that the auditing probability is determined by (12). Alternatively, it can be viewed as the limiting case $\sigma_H \to 1$ of the second equilibrium category such that (13) and (14) hold, but in fact $\hat{y}_H$ is not chosen by type H.[18]

The following proposition shows how the category of pooling equilibrium depends on the cost k of information acquisition and the fraction λ of type H workers in the population of workers, as illustrated in Figure 2.

Proposition 2 *Let $k < \bar{k}$. Then there exists a unique pooling equilibrium satisfying Condition B and, therefore, also Condition A.[19] More specifically:*

(i) if $\lambda > \mu_2(k)$, then some fraction $\sigma_H \in (0,1)$ of type H workers chooses $y_P^* = 0$ and the remaining fraction chooses an education $\hat{y}_H \in (0, y_H^*)$. All type L workers choose y_P^*.

(ii) if $\lambda = \mu_2(k)$, then all workers of both types choose $y_P^* = 0$.

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[18]Since (12) and (14) have different solutions for $\rho(y_P^*)$, the auditing rate is not uniquely determined under full pooling. Any $\rho(y_P^*)$ between the solutions of (12) and (14) can support full pooling.

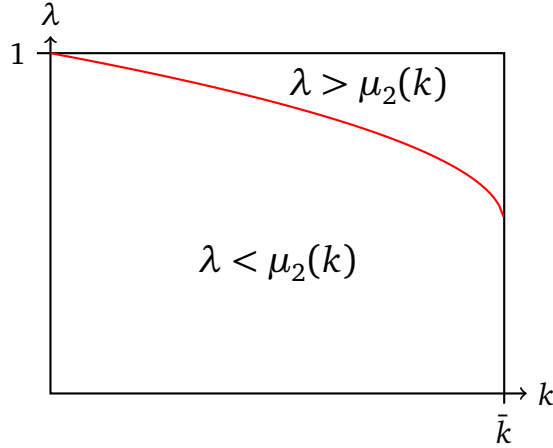[19]Recall that Condition B implies Condition A.

Figure 2: *Illustration of Proposition 2*

(iii) if $\lambda < \mu_2(k)$, then some fraction $\sigma_L \in (0,1)$ of type L workers chooses $y_P^* = 0$ and the remaining fraction applies for w_L. All type H workers choose $y_P^* = 0$.

For all $k > \bar{k}$ a pooling equilibrium satisfying Condition B does not exist.

As Figure 2 shows, parts (i) and (ii) of the proposition only apply for λ sufficiently large. Indeed, $\mu_2(.)$ is a decreasing function with $\mu_2(0) = 1$ and $\mu_2(k) \geq 1/2$ for all $k \leq \bar{k}$. Thus, if k is small enough or if $\lambda \leq 1/2$, the equilibrium is always as described in part (iii).

The refinement of Condition B selects pooling as the unique equilibrium for $k < \bar{k}$. For these values of k, the wage $w(y_P^*) = X_H - k/\mu_2(k)$ is more attractive for type H workers than their payoff $X_H - c_H(y_H^*)$ in the separating equilibrium.[20] Therefore, competition among the firms induces pooling of workers at the wage offer $w(y_P^*)$. At the same time, firms have to hold the belief $\mu(y_P^*) = \mu_2(k)$ to be willing to audit applicants with some probability $\rho(y_P^*) \in (0,1)$. Interestingly, as $y_P^* = 0$, education as a signal is not used in equilibrium if $\lambda \leq \mu_2(k)$, i.e. if k is small enough. Low costs of information acquisition eliminate signaling by education.

The requirement $\mu(y_P^*) = \mu_2(k)$ determines the equilibrium fraction σ_i of type i workers who apply for $w(y_P^*)$. Full pooling is consistent with the belief $\mu(y_P^*) = \mu_2(k)$ only in the boundary case $\lambda = \mu_2(k)$. Otherwise, there can be only partial pooling. In particular, if $\lambda > \mu_2(k)$, it cannot be the case that all type H workers apply for $w(y_P^*)$ because then Bayes' rule would imply that $\mu(y_P^*) > \mu_2(k)$ and firms would not invest in information acquisition. Therefore, some fraction of type H workers has to engage in signaling

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[20]In addition, k is small enough for $\mu_2(k)$ to be well–defined.

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by education $\hat{y}_H$. Conversely, if $\lambda < \mu_2(k)$, there cannot be an equilibrium such that all type L workers apply for $w(y_P^*)$, because then $\mu(y_P^*) < \mu_2(k)$ by Bayes' rule. In this case, therefore, some type L workers have to opt for the default wage w_L.

The following proposition summarizes the comparative statics properties of the pooling equilibrium.

Proposition 3 *Let $k < \bar{k}$. Then the pooling equilibrium in Proposition 2 has the following properties:*

(i) *if $\lambda > \mu_2(k)$, then $\sigma_L = 1$, σ_H is decreasing and $\rho(y_P^*)$ is increasing in k. Further, $\hat{y}_H$ increases in k and decreases in type H workers' education costs.[21]*

(ii) *if $\lambda < \mu_2(k)$, then $\sigma_H = 1$, σ_L is increasing and $\rho(y_P^*)$ is decreasing in k. Further, $\lim_{k \to 0} \sigma_L = 0$ and $\lim_{k \to 0} \rho(y_P^*) = (X_H - X_L)/(X_H - X_L + s) \in (0, 1)$.*

In the pooling equilibrium, the firms' beliefs have to satisfy $\mu(y_P^*) = \mu_2(k)$, which is decreasing in k. For $\lambda > \mu_2(k)$, therefore, a smaller fraction of type H workers needs to pool at y_P^* to keep the belief consistent with Bayes' rule when k increases. Analogously, for $\lambda < \mu_2(k)$, a larger fraction of type L is required to pool at y_P^* to keep the firms' belief consistent. As $\mu_2(0) = 1$, in the limit $k \to 0$ applications by type L for $w(y_P^*)$ tend to zero. Obviously, this requires the auditing rate to be bounded away from zero in this limit. Indeed, the auditing rate tends to unity for $k \to 0$ if the cost of a renewed application s becomes negligible. This keeps type L workers from applying for the pooling wage $w(y_P^*) > w_L = X_L$ for $k \to 0$ and $s \to 0$.

The comparative statics of the auditing rate depend on the equilibrium category. The reason is that the indifference condition for type L differs between the categories. For $\lambda > \mu_2(k)$, (14) requires type L to be indifferent between applying for $w(y_p^*)$ and imitating the type H's education $\hat{y}_H$. As k increases, $X_H - c_L(\hat{y}_P)$ decreases faster than $w(y_P^*)$, and so $\rho(y_P^*)$ is increasing in k. In contrast, for $\lambda < \mu_2(k)$, the firms' auditing rate in (12) has to make type L indifferent between applying for $w(y_P^*)$ and opting for the default wage X_L. If k increases, $w(y_P^*)$ decreases, and type L workers need to be audited less frequently to stay indifferent.

Interestingly, the equilibrium education $\hat{y}_H$ in part (i) of the proposition depends on the education costs of type H workers. This contrasts with the standard result in (9) that without auditing type H workers' education costs do not matter for their equilibrium education. With auditing, however, more expensive education lowers the education level

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[21]We say that type H workers' education costs increase if the new education costs are larger than the initial education costs for all education levels: $c_H^{\text{New}}(y) > c_H(y)$ for all $y > 0$.

necessary to make type *H* workers indifferent between the lower wage without education and the higher separation wage with education. Type *L* workers remain in the pooling contract as their utilities increase due to a lower auditing probability. If instead the auditing costs decrease, less type *H* workers obtain an education and the amount of education obtained decreases. Hence, the lower the auditing costs, the less signaling by education occurs. The reason is that lower auditing costs make auditing more attractive for firms. Therefore, more type *H* workers choose pooling and forgo any education without reducing firms' incentives to audit. At the same time, lower auditing costs imply higher wages for any contracts involving audits. Thus, less education is required to make type *H* workers indifferent between the higher separation wage with that education and the lower wage without education.

7   Discussion

In this section we discuss some implications and extensions of our analysis.

The limit $k \to 0$ Under perfect information about workers' productivities, each worker receives a wage equal to his productivity without signaling by education or costly auditing. It seems a reasonable conjecture that the equilibrium under asymmetric information resembles the perfect information outcome for sufficiently small costs of information acquisition. Yet, as part (i) of Proposition 1 shows, this is not true if merely Condition A is assumed. We now show, however, that the conjecture holds under the stronger Condition B. Recall that for $k < \bar{k}$, a separating equilibrium satisfying Condition B does not exist, while the pooling equilibrium as described in Proposition 2 survives. The following result shows that workers' equilibrium payoffs approach the outcome under perfect information when auditing costs become negligible.

Proposition 4 *Let $k < \bar{k}$. Then the pooling equilibrium in Proposition 2 has the following property: in the limit $k \to 0$, the belief $\mu(y_P^*)$ tends to unity and the payoffs of type H and type L workers become identical to their payoffs under perfect information. That is $\lim_{k \to 0} U_H^* = X_H$ and $\lim_{k \to 0} U_L^* = X_L$.*

In the separating equilibrium under Condition A, workers of type *H* invest $c_H(y_H^*)$ in education independently of k. Condition B yields the more plausible implication that in the limit $k \to 0$ the perfect information equilibrium is obtained. In this limit, auditing occurs with positive probability by part (ii) of Proposition 3; but this does not reduce welfare because auditing costs are zero in the limit.

Welfare Next, we address the welfare implications of information acquisition. If auditing is impossible, or equivalently, if it is too costly as $k > \bar{k}$, then Proposition 1 describes the outcome under Condition B: in the separating equilibrium without audits, workers of type H get the utility $X_H - c_H(y_H^*)$ and workers of type L get X_L. If, however, auditing is feasible and $k < \bar{k}$, the equilibrium selected by Condition B is the pooling equilibrium in Proposition 2. As the following proposition shows, for all $k < \bar{k}$ this equilibrium yields a Pareto–improvement relative to the separating equilibrium.

Proposition 5 *Let $k < \bar{k}$. Then in the pooling equilibrium in Proposition 2 the utilities of workers, U_H^* and U_L^*, have the following properties:*

(i) if $\lambda > \mu_2(k)$, then $U_H^* > X_H - c_H(y_H^*)$ and $U_L^* > X_L$. Further, U_H^* and U_L^* are decreasing in k.

(ii) if $\lambda < \mu_2(k)$, then $U_H^* > X_H - c_H(y_H^*)$ and $U_L^* = X_L$. Further, U_H^* is decreasing in k.

Interestingly, as part (i) of the proposition shows, not only type H but also type L workers may gain from the feasibility of information acquisition: if $\lambda > \mu_2(k)$, high productivity workers subsidize workers of low productivity. This happens in the pooling equilibrium because for these parameter values some fraction $1 - \sigma_H$ of type H workers does not pool with type L but chooses some education $\hat{y}_H < y_H^*$. Type L's payoff from choosing education $\hat{y}_H$ and getting the wage X_H is higher than X_L. Therefore, to keep him from deviating to $\hat{y}_H$, his expected payoff from pooling must be higher than X_L.

Binding wage offers In our model, firms make non–binding wage announcements at stage (iii). In particular, they can reject any application at stage (v). Now suppose that firms can make also binding wage offer at stage (iii) in addition to non–binding wage announcements. Thus, they have to hire any applicant who applies for such a binding wage offer. Notice that a firm will never audit applicants for a binding wage offer, because it cannot make use of the acquired information. Does this modification of the model affect the equilibria determined in Propositions 1 and 2?

Proposition 6 *Suppose firms can make binding wage offers. Then Propositions 1 and 2 are still valid.*

Binding wage offers do not change the equilibria we discussed above. Further, binding wage offers cannot create new equilibria. This follows directly from Lemma 4, which shows that the intuitive criterion does not allow for any equilibrium contracts with pooling and no audits.

Default wage In our model, firms offer the default wage $w_L = X_L$ even before workers take education decisions. This may look a bit unusual from the perspective of the standard signaling by education model, where *all* wage offers are made after education choices. But this only serves to simplify the specification of the firms' beliefs. Without this simplification we would have to make the firms' beliefs contingent not only on the observed education choice but also on the wage for which a worker applies. Recall that in part (iii) of Proposition 2 some fraction of type L workers pools with type H at education $y_P^* = 0$ and applies for $w(y_P^*) > w_L = X_L$, while the remaining fraction applies for the default wage $w_L = X_L$. In this equilibrium, workers distinguish themselves not by education but only by the wage they apply for. To support this outcome as an equilibrium when *all* wage offers are made after education choices, one can specify beliefs in the following way: Irrespective of the chosen education, firms believe that a worker has type L if he applies for a wage offer $w \leq X_L$. Clearly, it is not optimal for the firms to audit applicants for $w \leq X_L$. If a worker applies for a wage offer $w(y) > X_L$ beliefs are specified as in the above analysis. These beliefs effectively replicate the availability of the default wage w_L: in equilibrium firms compete for low productivity workers by offering the wage X_L and they accept all applicants for this wage.

8 Conclusions

In signaling models à la Spence (1973), firms can only infer workers' productivities from their education choices. In reality, firms additionally use costly auditing to learn workers' productivities. We characterize the trade–offs between workers' signaling incentives and firms' information acquisition incentives. Under an extension of the intuitive criterion, there exists a unique equilibrium: it involves separation for large auditing costs and partial pooling for small auditing costs. For sufficiently small auditing costs, all workers pool at zero education. Thus, education as a signal is not used in equilibrium for low costs of information acquisition. With vanishing costs the partial pooling equilibrium becomes more and more informative and approaches the perfect-information outcome.

Our analysis considers the role of auditing in a signaling environment, in which the informed agents move first by choosing a signal. An alternative setting is the screening model developed by Rothschild and Stiglitz (1976). In this setting the uninformed players first announce offers, and then each informed type chooses among offers. In the labor market context of our paper this would mean that firms compete by making education contingent wage offers. Then each worker chooses the offer which is best for him and acquires the required education. Our analysis of information acquisition could be applied

to such an environment by assuming that firms make offers but maintain the right to audit applicants before taking hiring decisions. It seems reasonable that auditing will occur also in a screening version of our model if audit costs are small enough. But, it is an open question how this affects screening by education in equilibrium.

Appendix

This Appendix contains the proofs of Lemmas 1–6 and of Propositions 2–6. The proof of Proposition 1 is substantiated in the main text.

Proof of Lemma 1: Obviously, no firm offers a wage $w > X_H$ to employ workers at this wage. Consider any wage $w \in (X_L, X_H]$. If the firm audits an applicant, its profit is $\mu(y)(X_H - w) - k$, because after the audit it will optimally reject type L and hire type H. Hiring an applicant without an audit gives the profit $\mu(y)X_H + (1 - \mu(y))X_L - w$. Therefore the firm's profit is

$$\Pi \equiv \max[\mu(y)(X_H - w) - k, \mu(y)X_H + (1 - \mu(y))X_L - w]. \tag{15}$$

Bertrand competition implies that $\Pi = 0$. Therefore,

$$w(y) = \max[X_H - k/\mu(y), \mu(y)X_H + (1 - \mu(y))X_L]. \tag{16}$$

Suppose that $\rho(y) > 0$. Then $X_H - k/\mu(y) \geq \mu(y)X_H + (1 - \mu(y))X_L$, which implies

$$k \leq \mu(y)(1 - \mu(y)(X_H - X_L) \leq \tilde{k}, \tag{17}$$

where the second inequality holds because $\mu(y)(1 - \mu(y)$ attains its maximum at $\mu(y) = 1/2$. Since (17) yields a contradiction to $k > \tilde{k}$, this proves Lemma 1.

Finally, at any wage $w \leq X_L$ firms optimally accept all applicants without an audit. Therefore, Bertrand competition implies $w(y) = \mu(y)X_H + (1 - \mu(y))X_L \geq X_L$. This is only consistent with $w \leq X_L$ for $\mu(y) = 0$ and $w(y) = X_L$. \hfill Q.E.D.

Proof of Lemma 2: By the proof of Lemma 1, the equilibrium wage offer is given by (16). Further, the argument in Lemma 1 shows that $\rho(y) = 0$ if the first inequality in (17) is violated. Since $\mu(y)(1 - \mu(y))$ is strictly concave in $\mu(y)$, $k > \mu(y)(1 - \mu(y))(X_H - X_L)$ is equivalent to $\mu(y) \notin [\mu_1(k), \mu_2(k)]$. This proves part (i).

Part (ii) follows by an analogous argument, because acquisition of information is optimal if $k < \mu(y)(1 - \mu(y))(X_H - X_L)$, which is equivalent to $\mu(y) \in (\mu_1(k), \mu_2(k))$. Finally, part (iii) holds because firms are indifferent between acquiring and not acquiring information if $k = \mu(y)(1 - \mu(y))(X_H - X_L)$, which is equivalent to $\mu(y) \in \{\mu_1(k), \mu_2(k)\}$. Q.E.D.

Proof of Lemma 3: (i) Suppose that for $k < \bar{k}$ there is an equilibrium with the properties stated in the lemma that satisfies Condition B. Then, by definition of $\bar{k}$ in (10), $c_H(y_H^*) - k/\mu_2(k) > 0$ because $k/\mu_2(k)$ is increasing in k and hence $k/\mu_2(k) < \bar{k}/\mu_2(\bar{k})$. Therefore, there must exist an out–of–equilibrium education $y' > 0$ close enough to zero such that

$$c_H(y') < c_H(y_H^*) - k/\mu_2(k). \tag{18}$$

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Suppose a worker chooses education y' and firms' beliefs are $\delta = \mu_2(k) - \epsilon'$ with $\epsilon' > 0$. Then, for ϵ' sufficiently small, $\delta \in (\mu_1(k), \mu_2(k))$. Therefore, by (5) and (6),

$$U_H(y'|\delta) = X_H - \frac{k}{\mu_2(k) - \epsilon'} - c_H(y'), \quad U_L(y'|\delta) = X_L - s - c_L(y'). \tag{19}$$

Because in equilibrium type H chooses y_H^* and receives the wage X_H and type L can always apply for $w_L = X_L$, we have

$$U_H^* = X_H - c_H(y_H^*), \quad U_L^* \geq X_L. \tag{20}$$

For ϵ' sufficiently small, therefore (18)–(20) imply that

$$U_H(y'|\delta) > U_H^*, \quad U_L(y'|\delta) < U_L^*. \tag{21}$$

By Condition B, therefore, $\mu(y') \geq \delta$. Since $U_H(y'|\mu)$ is increasing in μ, we thus obtain by (21) that $U_H(y'|\mu(y')) \geq U_H(y'|\delta) > U_H^*$, a contradiction to type H workers optimally choosing y_H^* in equilibrium.

(ii) We show that (8) cannot hold if $k \geq \bar{k}$, which means that Condition B does not apply. Since this condition imposes no restrictions for beliefs $\delta \in \{\mu_1(k), \mu_2(k)\}$, we consider only beliefs $\delta \notin \{\mu_1(k), \mu_2(k)\}$. This implies that $\rho(y') \in \{0, 1\}$ for an out–of–equilibrium choice y'. First consider the case $\rho(y') = 1$. By Lemmas 1 and 2 this requires that $k \in [\bar{k}, \tilde{k}]$ and $\delta \in (\mu_1(k), \mu_2(k))$. Note that, for any education y' and any $k \in [\bar{k}, \tilde{k}]$

$$c_H(y') \geq 0 \geq c_H(y_H^*) - \frac{k}{\mu_2(k)}. \tag{22}$$

As $\delta < \mu_2(k)$, we obtain by (5) that

$$U_H(y'|\delta) = X_H - \frac{k}{\delta} - c_H(y') < X_H - \frac{k}{\mu_2(k)} - c_H(y') \leq X_H - c_H(y_H^*) = U_H^*, \tag{23}$$

where the last inequality follows from (22). This shows that the first inequality in (8) cannot hold.

Now consider the case where $\rho(y') = 0$ given the belief δ after observing y'. Then, by Lemmas 1 and 2, the first inequality in (8) is equivalent to

$$U_H(y'|\delta) = X_L + \delta(X_H - X_L) - c_H(y') > U_H^* = X_H - c_H(y_H^*). \tag{24}$$

Further, by $U_L^* = X_L$ and the definition of y_H^* in (9), the second inequality in (8) requires that

$$U_L(y'|\delta) = X_L + \delta(X_H - X_L) - c_L(y') < U_L^* = X_L = X_H - c_L(y_H^*). \tag{25}$$

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Therefore,

$$c_H(y_H^*) - c_H(y') > (1-\delta)(X_H - X_L) > c_L(y_H^*) - c_L(y'). \qquad (26)$$

As $(1-\delta)(X_H - X_L) \geq 0$, $y_H^* > y'$ by the first inequality. Therefore, (26) implies that

$$c_H(y_H^*) - c_H(y') = \int_{y'}^{y_H^*} c_H'(y) \, dy > c_L(y_H^*) - c_L(y') = \int_{y'}^{y_H^*} c_L'(y) \, dy. \qquad (27)$$

Since this contradicts the assumption in (1) that $c_L'(y) > c_H'(y) > 0$, this shows that (8) in Condition B cannot hold. $\hfill$ Q.E.D.

Proof of Lemma 4: We first show that $\rho(y_P^*) \in (0,1)$. Suppose that $\rho(y_P^*) = 0$. Then the equilibrium utilities are

$$U_L^* = w(y_P^*) - c_L(y_P^*), \quad U_H^* = w(y_P^*) - c_H(y_P^*). \qquad (28)$$

Since $c_L'(y) > c_H'(y)$ and $w(y_P^*) < X_H$, there exists a $y' > y_P^*$ such that

$$U_H(y'|1) = X_H - c_H(y') > U_H^* \text{ and } U_L(y'|1) = X_H - c_L(y') < U_L^*. \qquad (29)$$

Therefore, Condition A implies that $\mu(y') = 1$. But then the first inequality in (29) implies that all H types would choose y' rather than y_P^*, a contradiction.

Suppose that $\rho(y_P^*) = 1$. Then $U_L^* = X_L - s - c_L(y_P^*)$, because type L is always detected. But, by applying for the default wage $w_L = X_L$, type L can ensure himself the utility $X_L > U_L^*$, a contradiction.

As $\rho(y_P^*) > 0$, it follows from Lemma 1 that $k \leq \tilde{k}$, because firms will never invest in information acquisition if $k > \tilde{k}$. $\hfill$ Q.E.D.

Proof of Lemma 5: By Lemmas 2 and 4, we obtain that $\mu(y_P^*) \in \{\mu_1(k), \mu_2(k)\}$. If $k = \tilde{k}$, then $\mu_1(k) = \mu_2(k)$, which proves the Lemma. Now consider the case $k < \tilde{k}$, where $\mu_1(k) < \mu_2(k)$, and suppose $\mu(y_P^*) = \mu_1(k)$. Then the equilibrium utilities satisfy

$$U_H^* = X_H - k/\mu_1(k) - c_H(y_P^*), \quad U_L^* \geq X_L. \qquad (30)$$

Consider out–of–equilibrium education $y_P^* + \epsilon$ and the belief $\delta = (\mu_1(k) + \mu_2(k))/2$. Since $\delta \in (\mu_1(k), \mu_2(k))$, with this belief the firms will choose $\rho = 1$ and so

$$U_H(y'|\delta) = X_H - \frac{k}{\delta} - c_H(y_P^* + \epsilon), \quad U_L(y'|\delta) = X_L - s - c_L(y_P^* + \epsilon). \qquad (31)$$

Since $\delta > \mu_1(k)$ we obtain for ϵ sufficiently small that (8) applies and so Condition B implies that $\mu(y') \geq \delta$. Thus, $U_H(y'|\mu(y')) \geq U_H(y'|\delta) > U_H^*$, a contradiction to the

equilibrium requirement that choosing y_P^* is optimal for type H. This implies that $\mu(y_P^*) = \mu_2(k)$.
<div align="right">Q.E.D.</div>

Proof of Lemma 6: (i) Suppose there is a $y > 0$ chosen only by type L workers. Then $w(y) = X_L$ and so their utility is $X_L - c_L(y) < X_L$. As type L can opt for $w_L = X_L$, this yields a contradiction. Suppose there are two education levels $0 < y' < y''$ chosen only by type H. Then $w(y') = w(y'') = X_H$ and so $c_H(y') < c_H(y'')$ yields a contradiction to y'' being optimal for type H.

(ii) Suppose $y_P^* > 0$. Then the equilibrium utilities satisfy

$$U_H^* = X_H - k/\mu_2(k) - c_H(y_P^*), \quad U_L^* \geq X_L. \tag{32}$$

Consider the out–of–equilibrium education $y' > 0$ and some belief $\delta \in (\mu_1(k), \mu_2(k))$ so that with this belief the firms will choose $\rho(y') = 1$. Then for δ sufficiently close to $\mu_2(k)$ and y' sufficiently close to zero we have

$$U_H(y'|\delta) = X_H - \frac{k}{\delta} - c_H(y') > U_H^*, \quad U_L(y'|\delta) = X_L - s - c_L(y') < X_L \leq U_L^*. \tag{33}$$

Therefore Condition B implies that $\mu(y') \geq \delta$. Thus, $U_H(y'|\mu(y')) \geq U_H(y'|\delta) > U_H^*$, a contradiction to the equilibrium requirement that choosing y_P^* is optimal for type H. This implies that $y_P^* = 0$.

Suppose $\sigma_L \in (0,1)$ and $\sigma_H \in (0,1)$. Then some fraction $1 - \sigma_L$ of type L workers apply for $w_L = X_L$ and some fraction $1 - \sigma_H$ of type H workers choose some $\hat{y}_H$ to receive the wage $w(\hat{y}_H) = X_H$. This requires that type L cannot gain from choosing $\hat{y}_H$:

$$X_L \geq X_H - c_L(\hat{y}_H). \tag{34}$$

Thus, $\hat{y}_H \geq y_H^*$ by (9). If $\hat{y}_H > y_H^*$, the inequality in (34) is strict. It is easily verified, that then $\hat{y}_H$ does not satisfy Condition A, which is implied by Condition B. Therefore, $\hat{y}_H = y_H^*$. For $k < \bar{k}$, this yields a contradiction to part (i) of Lemma 3. Thus, for all $k < \bar{k}$, $\sigma_L \in (0,1)$ and $\sigma_H \in (0,1)$ cannot hold simultaneously in a pooling equilibrium satisfying condition B.
<div align="right">Q.E.D.</div>

Proof of Proposition 2: (i) If $\lambda > \mu_2(k) = \mu(y_P^*)$, Bayes' rule in (11) implies that $\sigma_H < \sigma_L$. Therefore, by part (ii) of Lemma 6, $\sigma_H \in (0,1)$ and $\sigma_L = 1$. Thus, (13) and (14) have to hold. Since $w(y_P^*) = X_H - k/\mu_2(k)$ by Lemma 5, the second condition in (13) is equivalent to $c_H(\hat{y}_H) = k/\mu_2(k)$. Since $k/\mu_2(k)$ is increasing in k and $k < \bar{k}$, we obtain from (10) that $c_H(\hat{y}_H) < c_H(y_H^*)$. Therefore, $\hat{y}_H < y_H^*$. This implies by (9) that

$$X_H - c_L(\hat{y}_H) > X_H - c_L(y_H^*) = X_L. \tag{35}$$

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Therefore the first condition in (13) is implied by (14). Finally, as $w(y_P^*) = X_H - c_H(\hat{y}_H) > X_H - c_L(\hat{y}_H)$ and $X_L - s < X_L < X_H - c_L(\hat{y}_H)$, (14) has a unique solution $\rho(y_P^*) \in (0, 1)$. This proves that the equilibrium conditions (13) and (14) are satisfied for unique values of $w(y_P^*)$, $\hat{y}_H$ and $\rho(y_P^*)$.

The equilibrium can be supported by beliefs $\mu(y) = \mu_2(k)$ for $y < \hat{y}_H$, and $\mu(y) = 1$ for $y \geq \hat{y}_H$. To see that these beliefs satisfy Condition B, note that type H certainly cannot gain by choosing some $y' > \hat{y}_H$ because his equilibrium payoff is $X_H - c_H(\hat{y}_H)$. Suppose he could gain by deviating to some $y' \in (0, \hat{y}_H)$. Then the wage for workers with education y' has to satisfy $w(y') > w(y_P^*)$. This requires the belief $\mu(y') > \mu_2(k)$, which implies that $\rho(y') = 0$. This means that also type L will be hired at $w(y')$ after choosing y'. By (13) type H gains from choosing y' if $w(y') - c_H(y') > X_H - c_H(\hat{y})$. But, because by (1) $c_H(\hat{y}) - c_H(y') < c_L(\hat{y}) - c_L(y')$, this implies $w(y') - c_L(y') > X_H - c_L(\hat{y})$. Therefore, by (14) also type L would gain by choosing y'. This proves that the equilibrium survives Condition B.

(ii) If $\lambda = \mu_2(k) = \mu(y_P^*)$, Bayes' rule in (11) implies that $\sigma_H = \sigma_L$. Therefore, by part (ii) of Lemma 6, $\sigma_L = \sigma_H = 1$. This is simply the limiting case of (i) and the same arguments as above can be applied.

(iii) If $\lambda < \mu_2(k) = \mu(y_P^*)$, Bayes' rule in (11) implies that $\sigma_H > \sigma_L$. Therefore, by part (ii) of Lemma 6, $\sigma_L \in (0, 1)$ and $\sigma_H = 1$. Thus, (12) has to hold. This uniquely determines $\rho(y_P^*)$. The equilibrium can be supported by the belief $\mu(y) = \mu_2(k)$ for all y. To show that Condition B is not violated, suppose that type H could gain by deviating to some $y' > 0$. Then the wage for education y' has to satisfy $w(y') > w(y_P^*)$, which by the argument above implies also type L will be hired at $w(y')$ after choosing y'. For type H to gain, it has to be the case that

$$w(y') - c_H(y') > w(y_P^*) = X_H - \frac{k}{\mu_2(k)} \geq X_H - c_H(y_H^*), \tag{36}$$

where the last inequality holds by (10) because $k < \bar{k}$ and $k/\mu_2(k)$ is increasing in k. Note that this implies $y' < y_H^*$ because $w(y') \leq X_H$. For type L to lose from deviating to y', it has to be the case that

$$w(y') - c_L(y') < X_L = X_H - c_L(y_H^*), \tag{37}$$

where the equality holds by definition of y_H^* in (9). By (36) and (37), $c_H(y_H^*) - c_H(y') > c_L(y_H^*) - c_L(y')$, a contradiction to assumption (1). This proves that (8) in Condition B cannot hold.

The arguments in (i)–(iii) show that for all $k < \bar{k}$ there exists a unique pooling equilibrium satisfying Condition B. Suppose that an equilibrium with pooling at some education

y_P^* exists also for $k > \bar{k}$. By Lemma 4 this implies $\rho(y_P^*) \in (0, 1)$ and therefore, by Lemma 1 it has to be the case that $k \leq \tilde{k}$. For $k > \bar{k}$, $k/\mu_2(k) > c_H(y_H^*)$ by (10). Therefore, for $\epsilon > 0$ sufficiently small, the type H's equilibrium utility satisfies

$$U_H^* = X_H - k/\mu_2(k) < X_H - c_H(y_H^* + \epsilon) \tag{38}$$

by Lemma 5. Since type L can always apply for $w_L = X_L$, by (9) we have

$$U_L^* \geq X_L = X_H - c_L(y_H^*) > X_H - c_L(y_H^* + \epsilon). \tag{39}$$

Thus for the education $y_H^* + \epsilon$ the inequalities in (7) are satisfied, which by Condition A implies $\mu(y_H^* + \epsilon) = 1$. But then by (38), type H would deviate to $y_H^* + \epsilon$. This proves that for $k > \bar{k}$ already Condition A, which is weaker than Condition B, precludes the existence of a pooling equilibrium. Q.E.D.

Proof of Proposition 3: (i) If $\lambda > \mu_2(k) = \mu(y_P^*)$, $\sigma_L = 1$ by Proposition 2 (i) and so (11) yields

$$\sigma_H = (1 - \lambda)\mu_2(k)/\lambda(1 - \mu_2(k)). \tag{40}$$

As $\mu_2'(k) < 0$, this proves that $\partial\sigma_H/\partial k < 0$. The auditing rate $\rho(y_P^*)$ is determined by (14). By Lemma 5, this is equivalent to

$$c_L(\hat{y}_H) - \frac{k}{\mu_2(k)} = \rho(y_P^*)\left(X_H - X_L - \frac{k}{\mu_2(k)} + s\right). \tag{41}$$

The term in brackets on the right-hand side decreases in k. The derivative of the left-hand side is

$$\frac{\partial c_L(\hat{y}_H)}{\partial y}\frac{\partial \hat{y}_H}{\partial k} - \frac{\partial k/\mu_2(k)}{\partial k} > \frac{\partial c_H(\hat{y}_H)}{\partial y}\frac{\partial \hat{y}_H}{\partial k} - \frac{\partial k/\mu_2(k)}{\partial k} = 0, \tag{42}$$

because $c_H(\hat{y}_H) = k/\mu_2(k)$ by the definition of $\hat{y}_H$ in (13). Therefore the left-hand side increases in k. As both sides of the equation are positive, this implies that $\rho(y_P^*)$ increases in k.

The education $\hat{y}_H$ obtained by some type H workers is determined by the indifference condition $X_H - k/\mu_2(k) = X_H - c_H(\hat{y}_H)$. Hence, $\hat{y}_H$ decreases as type H workers' education costs increase as defined in footnote 21. In addition, $\hat{y}_H$ increases in k, because $k/\mu_2(k)$ increases in k. Hence, the amount of education $\hat{y}_H$ increases in auditing costs k.

(ii) If $\lambda < \mu_2(k) = \mu(y_P^*)$, $\sigma_H = 1$ by Proposition 2 (iii) and so (11) yields

$$\sigma_L = \lambda(1 - \mu_2(k))/(1 - \lambda)\mu_2(k) \tag{43}$$

As $\mu_2'(k) < 0$, this proves that $\partial\sigma_L/\partial k > 0$. Since $\lim_{k\to 0}\mu_2(k) = 1$, we obtain $\lim_{k\to 0}\sigma_L = 0$. By (12) and Lemma 5

$$\rho(y_P^*) = \frac{X_H - X_L - k/\mu_2(k)}{X_H - X_L - k/\mu_2(k) + s}. \tag{44}$$

Since $k/\mu_2(k)$ increases in k, $\rho(y_P^*)$ decreases in k. Moreover, the limit of $\rho(y_P^*)$ for $k \to 0$ is as stated in the proposition because $\lim_{k\to 0} k/\mu_2(k) = 0$. Q.E.D.

Proof of Proposition 4: First, notice that $\mu(y_P^*) = \mu_2(k)$ and that in the limit $\lim_{k\to 0} \mu_2(k) = 1$ and hence, $\lambda < \mu_2(k)$ for $k \to 0$. Proposition 2 (iii) shows $U_H^* = w(y_P^*) = X_H - k/\mu_2(k)$ and $U_L^* = X_L$ by equation (12). Therefore $\lim_{k\to 0} U_H^* = \lim_{k\to 0} X_H - k/\mu_2(k) = X_H$ and $\lim_{k\to 0} U_L^* = X_L$. Q.E.D.

Proof of Proposition 5: (i) Suppose $\lambda > \mu_2(k)$. Proposition 2 (i) proves that $U_H^* = X_H - c_H(\hat{y}_H)$ and $\hat{y}_H < y_H^*$. Therefore, $U_H^* = X_H - c_H(\hat{y}_H) > X_H - c_H(y_H^*)$. In addition, $\hat{y}_H$ is determined by $c_H(\hat{y}_H) = k/\mu_2(k)$ according to the proof of Proposition 2 (i). Since $\mu_2(k)$ decreases and $k/\mu_2(k)$ increases in k, $\hat{y}_H$ increases and $U_H^* = X_H - c_H(\hat{y}_H)$ decreases in k.

Proposition 2 (i) also proves that U_L^* is determined by (14) and, hence, $U_L^* = X_H - c_L(\hat{y}_H)$. Recall the definition of y_H^* with $X_L = X_H - c_L(y_H^*)$. Therefore, $\hat{y}_H < y_H^*$ implies $U_L^* = X_H - c_L(\hat{y}_H) > X_H - c_L(y_H^*) = X_L$ and U_L^* decreases in k, because $\hat{y}_H$ increases in k.

(ii) Suppose $\lambda < \mu_2(k)$. Proposition 2 (iii) proves $U_H^* = w(y_P^*) = X_H - k/\mu_2(k)$. The definition of $\bar{k}$ implies $\bar{k}/\mu_2(\bar{k}) \le c_H(y_H^*)$. Since $k/\mu_2(k)$ increases in k and $k < \bar{k}$, $U_H^* = X_H - k/\mu_2(k) > X_H - \bar{k}/\mu_2(\bar{k}) = X_H - c_H(y_H^*)$. Moreover, $U_H^* = X_H - k/\mu_2(k)$ decreases in k. Proposition 2 (iii) also proves that U_L^* is determined by (12). Therefore, $U_L^* = X_L$. Q.E.D.

Proof of Proposition 6: In the fully separating equilibrium of Proposition 1, there are no audits in equilibrium. Hence, for $k \ge \bar{k}$ it does not matter whether wage offers are binding or non–binding. The reasoning of Lemma 3 is also unaffected by additional binding wage offers.

Suppose for $k < \bar{k}$ there is an additional equilibrium besides the one in Proposition 2 with a binding offer $w(y')$ for some education y' inducing the belief $\mu(y')$. By Lemma 2 there are no audits even for non–binding wage offers if $\mu(y') \notin [\mu_1(k), \mu_2(k)]$. Thus, binding offers with $\mu(y') \notin [\mu_1(k), \mu_2(k)]$ are effectively already included in our analysis leading to Proposition 2. This implies that $\mu(y') \in [\mu_1(k), \mu_2(k)]$ and so $\mu(y') \in (0, 1)$. Yet, as Lemma 4 shows, pooling at some y' implies that auditing occurs with probability $\rho(y') > 0$. Because auditing is never optimal for a binding offer, this means that $w(y')$ cannot be an equilibrium offer. Consequently, for $k < \bar{k}$ there cannot be an additional equilibrium with a binding wage offer.

Next, we verify that the pooling equilibrium in Proposition 2 remains an equilibrium by arguing that the firm has no incentive to deviate to make a binding wage offer at stage (iii). Begin with the case $\lambda = \mu_2(k)$. All workers choose education $y_P = 0$ and beliefs are $\mu(y_P) = \mu_2(k)$. If the firm deviates by making a binding wage offer below

$X_H - k/\mu_2(k)$, the deviation attracts only type L workers, if it attracts any workers at all, and the deviation is unprofitable. If the binding wage offer is above or equal to $X_H - k/\mu_2(k)$, the deviation attracts all types of workers, but the deviation is unprofitable, because by the definition of $\mu_2(k)$

$$k = \mu_2(k)(1-\mu_2(k))(X_H - X_L) \Longleftrightarrow k/\mu_2(k) = (1-\mu_2(k))(X_H - X_L) \Longleftrightarrow$$
$$X_L + \mu_2(k)(X_H - X_L) = X_H - k/\mu_2(k)$$

Hence, the average productivity cannot exceed the deviation wage and the deviation is unprofitable for the firm.

Now turn to the case $\lambda < \mu_2(k)$. All workers choose the default contract or education $y_P = 0$ and beliefs are $\mu(y_P) = \mu_2(k)$. If the binding wage offer is below $X_H - k/\mu_2(k)$, the deviation attracts only type L workers, if it attracts any workers at all, and the deviation is unprofitable. If the binding wage offer is above or equal to $X_H - k/\mu_2(k)$, the deviation attracts all types of workers, but the deviation is unprofitable, due to the definition of $\mu_2(k)$ as above.

Finally, consider the case $\lambda > \mu_2(k)$. Some type H workers choose education $\hat{y}_H$, all remaining workers choose education $y_P = 0$ and beliefs are $\mu(y_P) = \mu_2(k)$. If a firm offers a binding wage for education $\hat{y}_H$, such a deviation is unprofitable, because a binding wage that is attractive for workers with $\hat{y}_H$ must attract type L workers as well. If a firm offers a binding wage for education y_P and the binding wage offer is below $X_H - k/\mu_2(k)$, the deviation attracts only type L workers, if it attracts any workers at all, and the deviation is unprofitable. If the binding wage offer for education y_P is above or equal to $X_H - k/\mu_2(k)$, the deviation attracts all types of workers with education y_P, but the deviation is unprofitable, due to the definition of $\mu_2(k)$ as above. The deviation does not attract any workers with education $\hat{y}_H$, as long as the wage is below X_H, because education costs are sunk. Q.E.D.

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