

# Aristotle vs. Plato: The Distributive Origins of the Cold War

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## Aristotle vs. Plato: The Distributive Origins of the Cold War<sup>1</sup>

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***Abstract:*** Competing definitions of justice in Plato's *Republic* and Aristotle's *Politics* indicate the existence of two distinct economic systems with different normative priorities. The three-class society of the Platonic economy (guardians, auxiliaries, producers) gives rise to guardians who by virtue are expected to enforce output targets on producers directly or through auxiliaries. The three-class society of the Aristotelian economy (rich, middle, poor) facilitates the emergence of different ruling coalitions and compensates efficiency losses of vertical production processes with political gains derived from representative governance. In the Aristotelian economy, the middle class is better off than in the Platonic economy (auxiliaries), because a just society (polity) is achieved under its rule. I argue that the equilibrium solutions of the Platonic and Aristotelian systems provide the normative foundations for the distinction between plan and market.

***Keywords:*** Plato, Aristotle, central planning, market mechanism, political regimes, economic systems

***JEL Codes :*** D63, P11, P14, P16, P21, P26, P52

### I. Introduction

The comparative study of capitalism and socialism has underpinned the role of informational asymmetries in the fulfillment of planned production targets and the centrality of principal-agent models in explaining the comparative advantages of the market mechanism over central planning (Weitzman, 1976, 1974). Moreover, resource allocation defined by impartiality, priority and solidarity may introduce a novel concept of fairness, where egalitarian distribution occurs in the space between resource- and outcome-equality corner solutions (Moreno-Ternerero and Roemer, 2006). Lawless societies reach competitive equilibrium solutions based on the initial distribution of power among their agents (jungle equilibrium as per Piccione and Rubinstein, 2007); the difference between an exchange and a jungle economy lies in the existence of involuntary exchange driven by coercion in the jungle (ibid.). In his integrated theory of justice and economic development, Roemer (2013) proposes the definition of an opportunity-equalizing economic development measured along two dimensions: the average income level of those more disadvantaged in society, and the effect of differential effort rather than circumstances on total income inequality. The political economy of egalitarianism underscores the difference between resource and welfare egalitarianism. While Dworkin's resource egalitarianism

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reflects randomly distributed characteristics and, for that matter, endowments, Roemer (1985a) suggests that it is morally efficient to implement welfare rather than resource egalitarianism.

Modeling the Athenian democracy as an economic system driven by interest representation and class conflict implies the relativity of institutional and growth differences between Cylon's tyranny and Solon's democracy (Fleck and Hanssen, 2012). Tyranny and democracy can be treated as complementary stages of a single balanced growth path, where tyranny involves transition dynamics from backwardness to the new steady state of sustainable growth, and democracy itself is the consolidating steady state (ibid.). The decentralized structure of the ancient Greek world that was organized in city-states (*poleis*) allowed the existence of multiple balanced growth paths in the same space and suggests that the speed of transition from tyranny to democracy can explain differential wealth levels throughout the 5<sup>th</sup> century BC. Furthermore, the inclusiveness of social, legal and economic institutions that allowed foreigners, resident aliens and sometimes slaves to seek judicial protection reveals that democracy is not the only explanatory variable for Athenian growth. The definition of Athens as a progressively open-access regime that facilitated prosperity implies that social stability and commitment to justice are equally important to democracy for economic development (Carugati, Ober and Weingast, 2015).

Communitarian and democratic critiques of capitalism rely on Aristotle's normative analysis of markets and labor (Katz, 1997). A comparative analysis of the *Republic* and *Politics* indicates the different focus of the two ideal forms of government. Plato treats oligarchy and democracy as equally detrimental for unity. Oligarchy divides society into poor and rich, while democracy leads to slavery in its extensive form of freedom (Grant, 2014). The three-class society of guardians, auxiliaries and producers proposed in the *Republic* preserves the hierarchical advantage of guardians and facilitates a fixed political unity without social mobility. In the Aristotelian economy, however, resource autonomy of economic units maximizes collective efficiency because it allows the emergence of a middle-class government, which is *ex-ante* excluded in the Platonic model (ibid.). In this paper, I model the Platonic economy as a three-player game, where guardians set production targets on producers directly or through auxiliaries. Furthermore, I model the Aristotelian economy as a three-player game, where there is no *ex-ante* hierarchy, but each of the three classes – rich, middle and poor – compete for leadership, with the leader in each round imposing production targets on the others directly. Aristotelian constitutional arrangements may lead to the emergence of three different regime types (kingship, aristocracy and polity) and its deviant forms (tyranny, oligarchy and democracy). Distributive efficiency in the Aristotelian economy is higher under polity than under any other regime

type, whereas it is higher in the Platonic economy when planned targets are set directly by the guardians rather than through delegation by auxiliaries.

The paper is structured as follows. In sections II and III, I solve the Platonic and Aristotelian economy games in their basic and egalitarian forms. In section IV, I derive the linkages between socialism and the Platonic economy on the one hand, and capitalism and the Aristotelian economy on the other. Section V concludes.

## II. The Platonic Economy

I assume a three-class society composed of guardians, auxiliaries and producers, with different levels of endowment and population share such that  $y^G > y^A > y^P$  and  $\alpha^P > \alpha^A > \alpha^G$ ,  $\sum_J \alpha^j = 1$ , where  $y^j$  is the average income of each class in society and  $\alpha^j$  is the population share of each of the three classes such that  $J = \{G, A, P\}$ ,  $j \in J$ .

The guardian cares both about his individual welfare and the collective welfare of the city. His expected payoff is  $EU^G(y^G; g) = \rho \left[ \frac{y^G}{\alpha^G} + H(g) - C(g, \varepsilon) \right] + (1 - \rho) \left[ \frac{y^G - C(g, \varepsilon)}{\alpha^G} + H(g) \right]$  where  $y$  denotes the average income in society,  $\rho$  denotes the probability of direct monitoring of the guardian on the producer whereas  $1 - \rho$  suggests that the guardian monitors the producer through the auxiliary,  $C(g, \varepsilon)$  denotes the cost of providing the public good, which depends on the level of monitoring the producer,  $\varepsilon \in (0, 1)$  is the degree of monitoring over the auxiliary and the producer,  $g$  denotes the public good and  $H(g)$  is the utility from the provision of the public good in society.  $C(g, \varepsilon)$  is monotonically increasing in  $\varepsilon$  and a concave function such that  $\frac{\partial C}{\partial \varepsilon}, \frac{\partial C}{\partial g} > 0$  and  $\frac{\partial^2 C}{\partial \varepsilon^2}, \frac{\partial^2 C}{\partial g^2} < 0$ . The competing concepts of justice proposed by Polemarchus, Thrasymachus and Socrates in the *Republic* suggest three different normative concepts of economic efficiency. What Polemarchus defines as justice is the enforcement capacity of the ruler, who must be able to meet his obligations vis-à-vis his friends and punish his enemies (Book I, 332c-335d). Thrasymachus suggests that justice is a welfare-maximization mechanism for the guardians (Book I, 336b-354c). Socrates responds that justice is a residual concept and at the same time a synthesis of three human qualities: wisdom, moderation and courage (Book IV, 427d-433b). Hence, the constraints on the guardian's payoff are the following:

$$y^G \leq \frac{g + \eta + \theta}{\tau}$$

$$\theta = \iint_{\sigma^G, \sigma^L} \Delta(\varepsilon, \tau) \varphi(\sigma^G) f(\sigma^L) d\sigma^G d\sigma^L$$

where  $\tau \in (0, 1]$  is the tax imposed by the guardian on the auxiliary and the producer,  $\theta$  is the individual quality of the guardians as commitment to the common good,  $\eta$  denotes the output offered by the producer,  $\Delta(\varepsilon, \tau)$  denotes the common good payoff,  $\sigma^L \in [0, \bar{\sigma}^L]$  denotes the competence level (skills) of auxiliaries and producers in society and  $\sigma^G \in [\bar{\sigma}^L, 1]$  denotes the competence level (skills) of guardians, whereas  $\varphi(\sigma^G), f(\sigma^L)$  are the respective probability distribution functions and  $\bar{\sigma}^L$  the upper bound of the closed set of competence level (skills) for auxiliaries and producers in society. As Roemer (1993) points out, the egalitarian planner can only find a second-best approach in his optimization problem because there is no tax policy that leads to equality of opportunity for all types of citizens. This is why the planner will only maximin the expected welfare of those citizen types who showed the median level of responsibility and education.

To complete the optimization problem of the guardian, it also essential to define the payoffs of auxiliaries and producers, where  $y^A = y^P = y \leq g + \eta$ . The expected payoff of the auxiliary is  $EU^A(y^A; g) = \rho \frac{(1-\tau)y}{\alpha^A} + (1-\rho) \frac{(1-\tau)y - C(g, \varepsilon)}{\alpha^A}$ . Moreover, the producer will implement the plan proposed, either monitored directly by the guardian, or monitored indirectly by the guardian and directly by the auxiliary such that:

$$EU^P = \rho \left[ \frac{(1-\tau)y}{\alpha^P} + \frac{I}{\alpha^G} - \alpha^G J(\eta, e) \right] + (1-\rho) \left[ \frac{(1-\tau)y}{\alpha^P} + \frac{I}{\alpha^G + \alpha^A} - (\alpha^G + \alpha^A) J^2(\eta, e) \right], \text{ where}$$

$e \in (0, 1)$  denotes effort,  $J(\eta, e)$  the cost of production such that  $J_e, J_\eta > 0$  and  $J_{ee}, J_{\eta\eta} < 0$ , and  $I$  is the pairwise property (incentive) structure as per Weitzman (1976), such that:

$$B(e) = \begin{cases} \eta(e) + \beta[\eta(e) - \eta^*], & \text{if } \eta^* \leq \eta(e) \\ \eta(e) - \gamma[\eta^* - \eta(e)], & \text{if } \eta^* > \eta(e) \end{cases}$$

where  $\eta^*$  is the planned target proposed by the guardian and  $\beta, \gamma \in (0, 1)$  are the parameters of reward and punishment for over- and underfulfillment of the proposed plan, respectively. The timing of the game between the guardian, the auxiliary and the producer is the following:

1. The guardian sets the planned target  $\eta^*$  and decides whether to monitor its implementation directly or through the auxiliary. This is a binary decision denoted by  $\rho \in \{0,1\}$ .
2. The producer decides how much effort to provide in order to fulfill the plan and delivers the output  $\eta$ .

Moreover, for continuous levels of effort  $e$ , the producer's optimization problem can be written as follows:

$$\begin{aligned} \max_e EU^P &= \max_e \rho \left[ \int_0^{\bar{e}} \frac{(1-\tau)y}{\alpha^P} + \frac{\eta(e) - \gamma[\eta^* - \eta(e)]}{\alpha^G} - \alpha^G J(\eta, e) f(e) de + \int_e^1 \frac{(1-\tau)y}{\alpha^P} + \frac{\eta(e) + \beta[\eta(e) - \eta^*]}{\alpha^G} - \alpha^G J(\eta, e) f(e) de \right] + \\ (1-\rho) &\left[ \int_0^{\bar{e}} \frac{(1-\tau)y}{\alpha^P} + \frac{\eta(e) - \gamma[\eta^* - \eta(e)]}{\alpha^G + \alpha^A} - (\alpha^G + \alpha^A) J^2(\eta, e) f(e) de + \int_e^1 \frac{(1-\tau)y}{\alpha^P} + \frac{\eta(e) + \beta[\eta(e) - \eta^*]}{\alpha^G + \alpha^A} - (\alpha^G + \alpha^A) J^2(\eta, e) f(e) de \right] \Rightarrow \\ \max_e EU^P &= \max_e \rho \left[ \int_0^{\bar{e}} \frac{(1-\tau)y}{\alpha^P} + \frac{\eta(e)(1+\gamma) - \gamma\eta^*}{\alpha^G} - \alpha^G J(\eta, e) f(e) de + \int_e^1 \frac{(1-\tau)y}{\alpha^P} + \frac{\eta(e)(1+\beta) - \beta\eta^*}{\alpha^G} - \alpha^G J(\eta, e) f(e) de \right] + \\ (1-\rho) &\left[ \int_0^{\bar{e}} \frac{(1-\tau)y}{\alpha^P} + \frac{\eta(e)(1+\gamma) - \gamma\eta^*}{\alpha^G + \alpha^A} - (\alpha^G + \alpha^A) J^2(\eta, e) f(e) de + \int_e^1 \frac{(1-\tau)y}{\alpha^P} + \frac{\eta(e)(1+\beta) - \beta\eta^*}{\alpha^G + \alpha^A} - (\alpha^G + \alpha^A) J^2(\eta, e) f(e) de \right] \Rightarrow \end{aligned}$$

We then derive the first-order condition with respect to effort  $e$ :

$$\begin{aligned} \frac{\partial EU^P}{\partial e} &= \rho \left[ P(\eta^* > \eta(e)) \left[ \frac{(1+\gamma)\eta_e}{\alpha^G} - \alpha^G J_e \right] + P(\eta^* \leq \eta(e)) \left[ \frac{(1+\beta)\eta_e}{\alpha^G} - \alpha^G J_e \right] \right] + \\ (1-\rho) &\left[ P(\eta^* > \eta(e)) \left[ \frac{(1+\gamma)\eta_e}{\alpha^G + \alpha^A} - 2(\alpha^G + \alpha^A) J_e \right] + P(\eta^* \leq \eta(e)) \left[ \frac{(1+\beta)\eta_e}{\alpha^G + \alpha^A} - 2(\alpha^G + \alpha^A) J_e \right] \right] = 0 \Rightarrow \\ P(\eta^* > \eta(e))(1+\gamma)\eta_e &\left[ \frac{2\alpha^G + \alpha^A}{\alpha^G(\alpha^G + \alpha^A)} \right] + P(\eta^* \leq \eta(e))(1+\beta)\eta_e \left[ \frac{2\alpha^G + \alpha^A}{\alpha^G(\alpha^G + \alpha^A)} \right] - \rho\alpha^G J_e - 2(1-\rho)(\alpha^G + \alpha^A) J_e = 0 \Rightarrow \\ \eta_e &\left[ \frac{2\alpha^G + \alpha^A}{\alpha^G(\alpha^G + \alpha^A)} \right] [P(\eta^* > \eta(e))(1+\gamma) + P(\eta^* \leq \eta(e))(1+\beta)] = J_e [\rho\alpha^G + 2(1-\rho)(\alpha^G + \alpha^A)] \Rightarrow \end{aligned}$$

$$\begin{aligned}
\eta_e &= J_e \frac{\rho\alpha^G + 2(1-\rho)(\alpha^G + \alpha^A)}{\left[ \frac{2\alpha^G + \alpha^A}{\alpha^G(\alpha^G + \alpha^A)} \right] [P(\eta^* > \eta(e))(1+\gamma) + P(\eta^* \leq \eta(e))(1+\beta)]} \Rightarrow \\
\eta_e &= J_e \frac{\alpha^G(2-\rho) + 2(1-\rho)\alpha^A}{\left[ \frac{2\alpha^G + \alpha^A}{\alpha^G(\alpha^G + \alpha^A)} \right] [1 + P(\eta^* > \eta(e))\gamma + P(\eta^* \leq \eta(e))\beta]} = \\
J_e &\frac{\left[ \alpha^G(\alpha^G + \alpha^A) \right] [\alpha^G(2-\rho) + 2(1-\rho)\alpha^A]}{\left[ 2\alpha^G + \alpha^A \right] [1 + P(\eta^* > \eta(e))\gamma + P(\eta^* \leq \eta(e))\beta]}, \text{ which solves for } e^* \Rightarrow \\
e^* &= \eta_e^{-1} \left( J_e \frac{\left[ \alpha^G(\alpha^G + \alpha^A) \right] [\alpha^G(2-\rho) + 2(1-\rho)\alpha^A]}{\left[ 2\alpha^G + \alpha^A \right] [1 + P(\eta^* > \eta(e))\gamma + P(\eta^* \leq \eta(e))\beta]} \right)
\end{aligned}$$

We find that  $\eta_e > 0$ , which suggests that output  $\eta(e)$  is monotonically increasing in effort  $e$ , an unsurprising result. We also observe that the existence of auxiliaries increases the marginal output provided by producers and, therefore, the overall efficiency of the economy. The larger the share of guardians in the economy, the less the effort the producers will deliver toward the optimal output level. Hence, Plato's argument about a just society ruled by the charismatic few is also confirmed in this model. A relatively higher share of auxiliaries has a smaller negative effect on the producer's effort than a relatively higher share of guardians.

As the *Republic* primarily concentrates on the justice-bound accountability of the guardians and the commitment of producers to output, the auxiliary does not have any decision-making capacity in my model. The wealth of the Platonic economy depends primarily on guardians and producers since the presence of auxiliaries reduces the cost of monitoring for guardians and increases the effort provided by producers. However, it is important also to consider the moral environment and the differential justice commitments of guardians and producers (Shokkaert and Overlaet, 1989).

Following the Leibniz rule, optimal output is given by  $\tilde{\eta} = \int_e \eta_e f(e) de$ , which offers the basis for the optimization problem of the guardian and his decision to delegate the monitoring of output production to the auxiliary or not. The indirect utility function of the guardian develops as follows:



$$\begin{aligned}
EU^G(y^G; g, \theta) &= \rho \left[ \frac{y}{\alpha^G} + H(g) - C(g, \theta, \varepsilon) \right] + (1-\rho) \left[ \frac{y - C(g, \theta, \varepsilon)}{\alpha^G} + H(g) \right] \Rightarrow \\
EU^G(y^G; g, \theta) &= \rho \left[ \frac{g + \tilde{\eta} + \theta}{\tau \alpha^G} + H(g) - C(g, \theta, \varepsilon) \right] + (1-\rho) \left[ \frac{\frac{g + \tilde{\eta} + \theta}{\tau} - C(g, \theta, \varepsilon)}{\alpha^G} + H(g) \right] \Rightarrow \\
EU^G(y^G; g, \theta) &= \rho \left[ \frac{g + \tilde{\eta} + \theta}{\tau \alpha^G} + H(g) - C(g, \theta, \varepsilon) \right] + (1-\rho) \left[ \frac{g + \tilde{\eta} + \theta - \tau C(g, \theta, \varepsilon)}{\tau \alpha^G} + H(g) \right]
\end{aligned}$$

Similarly, the guardian's optimization problem with respect to  $g$  :

$$\begin{aligned}
\frac{\partial EU^G}{\partial g} &= \rho \left[ \frac{1}{\tau \alpha^G} + H_g - C_g \right] + (1-\rho) \left[ \frac{g - \tau C_g}{\tau \alpha^G} + H_g \right] = 0 \Rightarrow \\
\frac{\partial EU^G}{\partial g} &= \frac{1}{\tau \alpha^G} + H_g - \rho C_g - (1-\rho) \frac{C_g}{\alpha^G} = 0 \Rightarrow \frac{1}{\tau \alpha^G} + H_g = \rho C_g + (1-\rho) \frac{C_g}{\alpha^G} \Rightarrow \\
\frac{1}{\tau \alpha^G} + H_g &= \frac{\alpha^G \rho C_g + (1-\rho) C_g}{\alpha^G} \Rightarrow H_g = \frac{\alpha^G \rho C_g + (1-\rho) C_g}{\alpha^G} - \frac{1}{\tau \alpha^G} \Rightarrow \\
H_g &= \frac{\alpha^G \rho C_g + (1-\rho) C_g}{\alpha^G} - \frac{1}{\tau \alpha^G} = \frac{C_g [1 + \rho(\alpha^G - 1)]}{\alpha^G} - \frac{1}{\tau \alpha^G} \Rightarrow \\
H_g &= C_g \frac{\tau [1 + \rho(\alpha^G - 1)] - 1}{\tau \alpha^G}, \text{ which solves for } g^* \Rightarrow g^* = H_g^{-1} \left( C_g \frac{\tau [1 + \rho(\alpha^G - 1)] - 1}{\tau \alpha^G} \right)
\end{aligned}$$

The marginal utility from the provision of the public good is negative for the guardian, resulting in  $H_g < 0$ . The optimal level of public good  $g^*$  declines with  $\tau$ , which implies that a more redistributive guardian will provide a lower amount of public goods. In the Platonic economy, there is a negative correlation between taxation and public goods provision. Taxation does not mean redistribution and cannot be part of the justice schedule as its main purpose is to facilitate an increase in the guardian's income. This is a major difference observed between Platonic and Aristotelian economies. In the *Republic*, the concept of justice is twofold. First, it is linked to the provision of public goods by guardians, and, second, as the individual commitment of the guardians to the common good. Therefore, we also take the first-order condition with respect to  $\theta$  :

$$EU^G(y^G, g, \theta) = \rho \left[ \frac{g + \tilde{\eta} + \theta}{\tau \alpha^G} + H(g) - C(g, \theta, \varepsilon) \right] + (1-\rho) \left[ \frac{g + \tilde{\eta} + \theta - \tau C(g, \theta, \varepsilon)}{\tau \alpha^G} + H(g) \right] \Rightarrow$$

$$EU^G = \rho \left[ \frac{g + \tilde{\eta} + \iint_{\sigma^G, \sigma^L} \Delta(\varepsilon, \tau) \varphi(\sigma^G) f(\sigma^L) d\sigma^G d\sigma^L}{\tau \alpha^G} + H(g) - C(g, \theta, \varepsilon) \right] + (1 - \rho) \left[ \frac{g + \tilde{\eta} + \iint_{\sigma^G, \sigma^L} \Delta(\varepsilon, \tau) \varphi(\sigma^G) f(\sigma^L) d\sigma^G d\sigma^L - \tau C(g, \theta, \varepsilon)}{\tau \alpha^G} + H(g) \right]$$

Given that  $\theta$  is a function of  $\varepsilon$ , we implement the implicit function theorem, yielding:

$$\frac{\partial EU^G}{\partial \theta} = \rho \left[ \frac{1}{\tau \alpha^G} - C_\theta \right] + (1 - \rho) \left[ \frac{1 - \tau C_\theta}{\tau \alpha^G} \right] = 0 \Rightarrow \frac{1}{\tau \alpha^G} - \rho C_\theta - (1 - \rho) C_\theta \frac{1}{\alpha^G} = 0 \Rightarrow \frac{1}{\tau} - \alpha^G \rho C_\theta - (1 - \rho) C_\theta = 0 \Rightarrow \frac{1}{\tau} - C_\theta [\alpha^G \rho + 1 - \rho] = 0 \Rightarrow$$

$$\frac{\partial EU^G}{\partial \theta} = \frac{1}{\tau} - C_\theta [1 - \rho(1 - \alpha^G)] = 0 \Rightarrow C_\theta = \frac{1}{\tau [1 - \rho(1 - \alpha^G)]} \Rightarrow \theta^* = C_\theta^{-1} \left( \frac{1}{\tau [1 - \rho(1 - \alpha^G)]} \right)$$

$$\frac{\partial EU^G}{\partial \varepsilon} = \rho \left[ \frac{\frac{\partial}{\partial \varepsilon} \iint_{\sigma^G, \sigma^L} \Delta(\varepsilon, \tau) \varphi(\sigma^G) f(\sigma^L) d\sigma^G d\sigma^L}{\tau \alpha^G} - C_\varepsilon \right] + (1 - \rho) \left[ \frac{\frac{\partial}{\partial \varepsilon} \iint_{\sigma^G, \sigma^L} \Delta(\varepsilon, \tau) \varphi(\sigma^G) f(\sigma^L) d\sigma^G d\sigma^L - \tau C_\varepsilon}{\tau \alpha^G} \right] \Rightarrow$$

$$\frac{\partial EU^G}{\partial \varepsilon} = \rho \left[ \frac{F(\sigma^L) \Phi(\sigma^G) \Delta_\varepsilon - C_\varepsilon}{\tau \alpha^G} \right] + (1 - \rho) \left[ \frac{F(\sigma^L) \Phi(\sigma^G) \Delta_\varepsilon - \tau C_\varepsilon}{\tau \alpha^G} \right] \Rightarrow$$

$$\frac{\partial EU^G}{\partial \varepsilon} = \rho \left[ \frac{F(\sigma^L) [1 - F(\sigma^L)] \Delta_\varepsilon}{\tau \alpha^G} - C_\varepsilon \right] + (1 - \rho) \left[ \frac{F(\sigma^L) [1 - F(\sigma^L)] \Delta_\varepsilon - \tau C_\varepsilon}{\tau \alpha^G} \right] = 0 \Rightarrow$$

$$\frac{F(\sigma^L) [1 - F(\sigma^L)]}{\tau \alpha^G} \Delta_\varepsilon - C_\varepsilon \left[ \frac{\alpha^G \rho + 1 - \rho}{\alpha^G} \right] = 0 \Rightarrow \frac{F(\sigma^L) [1 - F(\sigma^L)]}{\tau \alpha^G} \Delta_\varepsilon = C_\varepsilon \left[ \frac{\alpha^G \rho + 1 - \rho}{\alpha^G} \right] \Rightarrow$$

$$\frac{F(\sigma^L) [1 - F(\sigma^L)]}{\tau} \Delta_\varepsilon = C_\varepsilon [1 - \rho(1 - \alpha^G)] \Rightarrow \Delta_\varepsilon = C_\varepsilon \frac{\tau [1 - \rho(1 - \alpha^G)]}{F(\sigma^L) [1 - F(\sigma^L)]}$$

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Hence, completing the implicit function theorem:

$$\frac{\partial \theta}{\partial \varepsilon} = - \frac{\frac{1}{\tau} - C_\theta [1 - \rho(1 - \alpha^G)]}{\frac{F(\bar{\sigma}^L) [1 - F(\bar{\sigma}^L)]}{\tau} \Delta_\varepsilon - C_\varepsilon [1 - \rho(1 - \alpha^G)]} \Rightarrow \frac{\partial \theta}{\partial \varepsilon} = - \frac{1 - \tau C_\theta [1 - \rho(1 - \alpha^G)]}{F(\bar{\sigma}^L) [1 - F(\bar{\sigma}^L)] \Delta_\varepsilon - \tau C_\varepsilon [1 - \rho(1 - \alpha^G)]}$$

**Proposition 1 (The Platonic Equilibrium)**

There is a unique subgame perfect equilibrium of the Platonic economy game that has the following form:

1. If  $e < \alpha^G$  or  $e \leq \alpha^G + \alpha^A$ , then the producer does not deliver  $\tilde{\eta}$  and a just society collapses in favor of democracy.
2. If  $|e - \beta| \leq |\gamma - e|$  and  $e < \alpha^G$  or  $e \leq \alpha^G + \alpha^A$ , then the producer does not deliver  $\tilde{\eta}$  and a just society collapses in favor of tyranny.
3. If  $e \geq \alpha^G$  or  $e > \alpha^G + \alpha^A$ , then the producer delivers  $\tilde{\eta}$  and the following solutions are observed:
  - a. If  $\theta \leq \varepsilon$  and  $\bar{\sigma}^L > \frac{1}{2}$ , then the guardian underprovides the public good  $g$  and a just society (*politeia*) collapses in favor of timocracy.
  - b. If  $\theta \leq \varepsilon$  and  $\bar{\sigma}^L \leq \frac{1}{2}$ , then the guardian underprovides the public good  $g$  and a just society (*politeia*) collapses in favor of oligarchy.
  - c. If  $\theta > \varepsilon$ , then the guardian provides the public good  $g$  at least at  $g^*$  and a just society (*politeia*) is preserved.

The idea of a just society (*politeia*) is preserved when the guardian's individual commitment to the common good is higher than the combined skills of the auxiliary and the producer. However, when the share of skills possessed by the auxiliary and the producer is higher than the distributive commitment of the guardian, then two possible regime equilibria are observed, *timocracy* and *oligarchy*. When the marginal cost of individual commitment to the common good is lower than the marginal cost of effort toward its delivery and lower than the marginal benefit from the common good, then timocracy emerges. In the Platonic economy, this regime type advances the significance of honor through power and of military capacity through the accumulation of resources (Book VIII, 543a-550c). Oligarchy is the regime type that relies on the power of the wealthy, whose actions are constrained by some public standards for values, which do not exist in democracy (Book VIII, 550c-555d). In oligarchy, the guardians are extractive, as they are under timocracy, but the objective of wealth accumulation is private rather than public. In the Platonic economy, taxation is no guarantee for the emergence of

*politeia* because it offers the resource foundations for its competing equilibrium solutions, *timocracy* and *oligarchy*.

Fleck and Hansen (2006) offer an interesting comparison of Greek city-states by concentrating not only on Athens and Sparta as the key competing paradigms but also on Argos, Thebes and Corinth as well as Thessaly, the most fertile area of the classical Greek world. They argue that democracy is more likely to emerge when political institutions can resolve time-inconsistency problems and facilitate investment. Exogenous variation in potential returns to agricultural investment explains - from an elite's perspective - the value of granting secure land property to *demos* and therefore the likelihood of democracy (ibid.). In the Platonic economy, the distributive commitment of the guardian to the provision of the public good shapes the emergence of *politeia*, whereas in the economy of Greek city-states as per Fleck and Hansen (2006) it is the monitoring of returns to agricultural investment that does this.

The rise of both democracy and tyranny are conditioned on the non-fulfillment of the optimal plan by the producer. If the effort delivered by the producer is strictly dominated by the share of guardians in society, then the enforcement capacity of the guardian is weakened and the *politeia* collapses toward democracy. The Platonic definition of democracy implies a distributive anarchy, where producers do not perform based on the joint monitoring of guardians and auxiliaries, but maximize their utility without hierarchical constraints. This is particularly the case in societies with a large share of guardians and auxiliaries, which renders optimal effort extremely high for producers and therefore makes the option of democracy a preferred outcome. The difference between democracy and tyranny in that respect lies in the ability of the guardians to provide a system of rewards and punishments as per Weitzman (1976) that they can implement in such a way that punishment for underfulfillment of the proposed plan schedule is costlier than its exact or overfulfillment. The relative difference between the consequences of underfulfillment and exact fulfillment or overfulfillment - their asymmetry - offers an additional incentive to producers not only to radicalize in the direction of democracy, but also to consolidate their rule with the appointment of a tyrant, who expropriates the resources not only of the guardians and the auxiliaries, but also of some of the producers. While both democracy and tyranny are deviations from *politeia*, tyranny complements distributive anarchy with centralized rule.

#### *Comparative Statics*

In the analysis of equilibrium comparative statics, I identify the relationship between the distributive commitment of the guardian to the provision of the public good and his degree of monitoring over the

auxiliary and the producer, the relationship between public goods provision and the share of guardians in the Platonic economy, and the relationship between effort delivered by the producers and the joint

share of guardians and auxiliaries. I find that  $\frac{\partial \theta}{\partial \varepsilon} = -\frac{1 - \tau C_\theta [1 - \rho(1 - \alpha^G)]}{F(\bar{\sigma}^L) [1 - F(\bar{\sigma}^L)] \Delta_\varepsilon - \tau C_\varepsilon [1 - \rho(1 - \alpha^G)]} > 0$ ,

which means that the distributive commitment of the guardian is monotonically increasing with the quality of monitoring institutions in the economy. The higher the degree of monitoring, the higher the distributive commitment of the guardian to the common good. Hence, there is an optimal threshold  $\varepsilon^*$ ,

which solves  $\frac{\partial \theta}{\partial \varepsilon}$  and is defined by the space  $(\underline{\varepsilon}, \bar{\varepsilon}]$  such that  $\varepsilon \in (\underline{\varepsilon}, \bar{\varepsilon}]$ . Oligarchy occurs at low levels

of monitoring, such that  $\varepsilon \leq \underline{\varepsilon}$ , whereas timocracy occurs at intermediate levels of monitoring, such

that  $\underline{\varepsilon} < \varepsilon \leq \bar{\varepsilon}$ . Similarly, at high levels of monitoring *politeia* occurs such that  $\varepsilon > \bar{\varepsilon}$ . Monitoring

capacity is an exogenous parameter that may be related to the long-run quality of institutions in a city-

state. Contingent on the *ex-ante* quality of monitoring institutions in the city-state, the guardian

commits to the common good and therefore a just society is observed. However, in societies with an

initial low quality of institutions, there is a much higher inclination toward oligarchy or timocracy,

depending on the punitive nature of the guardian's proposed plan schedule.

### **Lemma 1**

*In the Platonic economy game, there is a positive monotonic relationship between the guardian's distributive commitment to the common good and the ex-ante quality of monitoring institutions. When  $\varepsilon \leq \underline{\varepsilon}$ , oligarchy is*

*observed, whereas for intermediate values of  $\varepsilon$  such that  $\underline{\varepsilon} < \varepsilon \leq \bar{\varepsilon}$  timocracy occurs. Politeia emerges at high*

*levels of ex-ante monitoring such that  $\varepsilon > \bar{\varepsilon}$ .*

The relationship between the provision of public goods and the share of guardians in the population of

the city-state is defined by  $g^* = H_g^{-1} \left( C_g \frac{\tau [1 + \rho(\alpha^G - 1)] - 1}{\tau \alpha^G} \right)$  such that

$\frac{\partial g}{\partial \alpha^G} = H_{g\alpha^G}^{-1} \left( C_g \frac{\tau [1 + \rho(\alpha^G - 1)] - 1}{\tau \alpha^G} \right) C_g \frac{\rho - 1}{\alpha^{G^2}} > 0$ . The higher the share of the guardian class in

society, the higher the provision of public goods. Because the guardian preserves his authority through

the provision of public goods, he is better able to do so where his share of influence in society is higher

rather than lower. A Platonic economy with a high share of guardians is more likely to become a

democracy because, in a populist pluralist regime, as Plato defines democracy, guardians can buy off

the producers' support with the provision of public goods while avoiding internal conflict. In contrast, if a Platonic economy has a low share of guardians, it is more likely to become a tyranny. In this case, a guardian has a lower opportunity cost and is faced with less internal competition toward the imposition of a repressive regime that underprovides public goods and relies less on the support of producers.

**Lemma 2**

*In the Platonic economy game, there is a positive monotonic relationship between the provision of the public good and the share of guardians. When  $\alpha^G \leq \bar{\alpha}^G$ , tyranny is observed, whereas for higher values of  $\alpha^G$  such that  $\alpha^G > \bar{\alpha}^G$  democracy occurs.*

Similarly,  $\frac{\partial e}{\partial \alpha^G} < 0$  and  $\frac{\partial e}{\partial \alpha^A} < 0$ , which suggests that the effort delivered by producers decreases with the share of elites and their direct agents. Hence, Plato is advocating an economic system where the monitoring capacity of the elites and their direct agents does not undermine output production and thus does not provide incentives for free riding by producers.

**Corollary 2A**

*In the Platonic economy game, there is a negative monotonic relationship between the effort delivered by producers and the joint share of guardians and auxiliaries. When  $\alpha^G + \alpha^A > \underline{\alpha}^G + \underline{\alpha}^A$ , then  $\tilde{\eta}$  is not delivered and tyranny or democracy emerges. When  $\alpha^G + \alpha^A \leq \underline{\alpha}^G + \underline{\alpha}^A$ , then  $\tilde{\eta}$  is delivered and politeia or timocracy or oligarchy emerges.*

In Plato's *Republic*, the distributive commitment of guardians drives the economy toward its self-realization as *politeia*. My results are driven by two sets of exogenous parameters, the *ex-ante* quality of monitoring institutions and the share of elites and their intermediaries in society. City-states with a higher level of prior institutional development are more likely to reach the equilibrium of *politeia* because guardians are less inclined to free ride against the common good. A more complex set of political and judicial institutions predicts higher levels of economic development and social peace. Similarly, an economy with an overwhelmingly large share of guardians and auxiliaries leads to the collapse of production processes and the generalization of anarchy in the direction of repressive (tyranny) or populist (democracy) solutions. Oligarchy and timocracy are suboptimal solutions with respect to *politeia*. Oligarchy transforms political office into private gain for the elites, whereas timocracy advances their public standing with the purpose of authority preservation.

### III. The Aristotelian Economy

Deriving the proposed social structure from Aristotle's *Politics*, I assume a three-class society composed of rich, middle and poor with different income levels and population share such that  $y^R > y^M > y^\Phi$  and  $\alpha^R < \alpha^M < \alpha^\Phi$ ,  $\sum_{j \in J} \alpha^j = 1$ , where  $y^j$  corresponds to the representative income of each class in society and  $\alpha^j$  denotes the population share for each social class such that  $J = \{R, M, \Phi\}$ ,  $j \in J$ . We treat the rich class as the incumbent whose utility depends on their income and the provision level of public goods such that:

$$EU^R(y^R; g) = \psi \left[ \frac{y^R}{\alpha^R + \alpha^M} + H(g) + \kappa r - (\alpha^R + \alpha^M) C(g, \varepsilon) \right] + (1 - \psi) \left[ \lambda \left( \frac{y^R}{\alpha^R + \alpha^\Phi} + H(g) + (1 + \kappa)r - (\alpha^R + \alpha^\Phi) C(g, \varepsilon) \right) + \frac{(1 - \lambda)y^R}{\alpha^R} \right]$$

where  $\psi$  denotes the probability of a coalition with the middle class in the state legislature (*ecclesia of demos*),  $\lambda$  the probability of a coalition with the poor,  $r > 0$  political rents and  $\kappa \in (0, 1)$  an expropriation rate. For  $C(g, \varepsilon)$  and  $H(g)$ , the same properties as in the Platonic economy hold.

A key difference between the Platonic and the Aristotelian economies is the permissibility of rents, which allows the constituent members of a winning coalition to derive private gains from politics. Rents are modeled as a cost carried by the class that finds itself in the minority condition and are inversely related to the provision of public goods and the utility accumulated therefrom. The three-class society of the Aristotelian economy (rich, middle, poor) concentrates on the emergence of governing majorities that are based on interclass alliances and thus facilitate efficient constitutional arrangements. Aristotle in *Politics* defines monarchy, aristocracy and polity as optimal forms of political regimes depending on the number of people ruling: one in monarchy, a few in aristocracy and many in polity (Book III, Chapter 7). Similarly, their respective deviations are tyranny, oligarchy and democracy (ibid.: 1289a38). The underlying difference between the optimal regime forms and their deviations is the ruler's commitment to the common good (ibid.).

We assume that monarchy or tyranny emerges when there is a coalition between the rich and the middle. Aristocracy or oligarchy emerges when there is a coalition between the rich and the poor. Similarly, polity or democracy emerges when there is a coalition between the middle and the poor. In the Aristotelian concept of political competition, distributive justice is linked to the government's

ability to serve the common good and at the same time deliver a minimum level of public good to the class that is not part of the governing coalition. Hence, the resource and policy constraints to the ruler's payoff are the following:

$$y^R \leq \frac{g + r + \theta^R + \omega}{\tau}$$

$$\theta^R = \frac{\psi}{\alpha^R} \iint_{\xi^R, \xi^M} \Gamma(r, \varepsilon) \chi(\xi^R) h(\xi^M) d\xi^R d\xi^M + \frac{(1-\psi)\lambda}{\alpha^R} \iint_{\xi^R, \xi^\Phi} \Gamma(r, \varepsilon) \chi(\xi^R) q(\xi^\Phi) d\xi^R d\xi^\Phi$$

where  $\tau \in (0,1]$  is the tax imposed by the rich on the other two classes in society,  $\theta^R$  the individual quality of the rich as commitment to the common good,  $\Gamma(r, \varepsilon)$  the common good payoff,  $\xi^M \in [0, \bar{\xi}^M]$  the competence level (skills) of the middle class,  $\xi^\Phi \in [0, \bar{\xi}^\Phi]$  the competence level (skills) of the poor class and  $\xi^R \in [\bar{\xi}^M, 1]$  the competence level (skills) of the rich. Moreover,  $\chi(\xi^R), h(\xi^M)$  and  $q(\xi^\Phi)$  are the respective probability distribution functions and  $\bar{\xi}^M$  the upper bound of the closed set of competence level (skills) for the middle class where  $\bar{\xi}^M > \bar{\xi}^\Phi$ . Unlike in the Platonic economy, in the Aristotelian economy there is no fixed hierarchy *ex-ante*. However, the rich class has an initial incumbency advantage. The timing of the game between the rich, the middle and the poor is the following:

1. The rich class selects the planned output target  $\omega^*$  and its optimal level of rents  $r^*$  and decides whether to form a coalition government with the middle; this is a binary decision denoted by  $\psi \in \{0,1\}$ .
2. The poor observe the decision of the rich and decide whether to form a coalition with the middle or accept the incumbent's decision and produce  $\omega$ . This is a binary decision denoted by  $\lambda \in \{0,1\}$ .

The payoff and constraints for the middle class are the following:

$$EU^M(y^M; g) = \psi \left[ \frac{(1-\tau)y^M - C(g, \varepsilon)}{\alpha^R + \alpha^M} + (1-\kappa)r \right] + (1-\psi) \left[ \lambda \frac{(1-\tau)y^M}{\alpha^M} + (1-\lambda) \left( \frac{(1-\tau)y^M - C(g, \varepsilon)}{\alpha^M + \alpha^\Phi} + \kappa r \right) \right]$$

$$y^M \leq g + r + \theta^M + \omega$$

$$\theta^M = \frac{\psi}{\alpha^M} \iint_{\xi^R, \xi^M} \Gamma(\varepsilon, r) \chi(\xi^R) h(\xi^M) d\xi^R d\xi^M + \frac{(1-\psi)(1-\lambda)}{\alpha^M} \iint_{\xi^M, \xi^\Phi} \Gamma(\varepsilon, r) h(\xi^R) q(\xi^\Phi) d\xi^M d\xi^\Phi$$



Similarly, the poor will either implement the plan proposed by the rich or coalesce with the middle such that:

$$EU^\Phi = \psi \left[ \frac{(1-\tau)y^\Phi + \Upsilon}{\alpha^\Phi} - \alpha^\Phi W(\omega, \kappa) \right] + (1-\psi)(1-\lambda) \left[ \frac{(1-\tau)y^\Phi}{\alpha^\Phi} + \frac{\Upsilon}{\alpha^M + \alpha^\Phi} - (\alpha^M + \alpha^\Phi) W^2(\omega, \kappa) \right] + (1-\psi)\lambda \left[ \frac{(1-\tau)y^\Phi}{\alpha^\Phi} + \frac{\Upsilon}{\alpha^R + \alpha^\Phi} - (\alpha^R + \alpha^\Phi) W^2(\omega, \kappa) \right],$$

where  $W(\omega, \kappa)$  denotes the cost of output delivery and  $\kappa$  the degree of expropriation such that  $W_\kappa < 0, W_\omega > 0$  and  $W_{\kappa\kappa}, W_{\omega\omega} < 0$ , and  $\Upsilon$  is the pairwise property (incentive) structure also as per Weitzman (1976):

$$\Upsilon(\kappa) = \begin{cases} \omega(\kappa) + \nu[\omega(\kappa) - \omega^*], & \text{if } \omega^* \leq \omega(\kappa) \\ \omega(\kappa) - \iota[\omega^* - \omega(\kappa)], & \text{if } \omega^* > \omega(\kappa) \end{cases},$$

where  $\nu, \iota \in (0, 1)$  are the parameters of reward and punishment such that  $\nu > \alpha$  and  $\iota < \beta$ , which implies that overfulfillment is rewarded more and underfulfillment is penalized less in the Aristotelian economy than in the Platonic.

Similarly, for continuous levels of expropriation  $\kappa$ , the optimization problem of the poor can be written as follows:

$$\begin{aligned}
\max_{\kappa} EU^{\Phi} &= \max_{\kappa} \psi \left[ \int_0^{\bar{\gamma}} \frac{(1-\tau)y^{\Phi} + \omega(\kappa) - l[\omega^* - \omega(\kappa)]}{\alpha^{\Phi}} - \alpha^{\Phi} W(\omega, \kappa) h(\kappa) d\kappa + \int_{\bar{\gamma}}^1 \frac{(1-\tau)y^{\Phi} + \omega(\kappa) + \nu[\omega(\kappa) - \omega^*]}{\alpha^{\Phi}} - \alpha^{\Phi} W(\omega, \kappa) h(\kappa) d\kappa \right] + \\
(1-\psi)(1-\lambda) &\left[ \int_0^{\bar{\gamma}} \frac{(1-\tau)y^{\Phi} + \omega(\kappa) - l[\omega^* - \omega(\kappa)]}{\alpha^{\Phi} + \alpha^M} - (\alpha^M + \alpha^{\Phi}) W^2(\omega, \kappa) h(\kappa) d\kappa + \int_{\bar{\gamma}}^1 \frac{(1-\tau)y^{\Phi} + \omega(\kappa) + \nu[\omega(\kappa) - \omega^*]}{\alpha^{\Phi} + \alpha^M} - (\alpha^M + \alpha^{\Phi}) W^2(\omega, \kappa) h(\kappa) d\kappa \right] + \\
(1-\psi)\lambda &\left[ \int_0^{\bar{\gamma}} \frac{(1-\tau)y^{\Phi} + \omega(\kappa) - l[\omega^* - \omega(\kappa)]}{\alpha^{\Phi} + \alpha^R} - (\alpha^R + \alpha^{\Phi}) W^2(\omega, \kappa) h(\kappa) d\kappa + \int_{\bar{\gamma}}^1 \frac{(1-\tau)y^{\Phi} + \omega(\kappa) + \nu[\omega(\kappa) - \omega^*]}{\alpha^{\Phi} + \alpha^R} - (\alpha^R + \alpha^{\Phi}) W^2(\omega, \kappa) h(\kappa) d\kappa \right] \Rightarrow \\
\max_{\kappa} EU^{\Phi} &= \max_{\kappa} \psi \left[ \int_0^{\bar{\gamma}} \frac{(1-\tau)y^{\Phi} + \omega(\kappa)(1+l) - l\omega^*}{\alpha^{\Phi}} - \alpha^{\Phi} W(\omega, \kappa) h(\kappa) d\kappa + \int_{\bar{\gamma}}^1 \frac{(1-\tau)y^{\Phi} + \omega(\kappa)(1+\nu) - \nu\omega^*}{\alpha^{\Phi}} - \alpha^{\Phi} W(\omega, \kappa) h(\kappa) d\kappa \right] + \\
(1-\psi)\lambda &\left[ \int_0^{\bar{\gamma}} \frac{(1-\tau)y^{\Phi} + \omega(\kappa)(1+l) - l\omega^*}{\alpha^{\Phi} + \alpha^M} - (\alpha^M + \alpha^{\Phi}) W^2(\omega, \kappa) h(\kappa) d\kappa + \int_{\bar{\gamma}}^1 \frac{(1-\tau)y^{\Phi} + \omega(\kappa)(1+\nu) - \nu\omega^*}{\alpha^{\Phi} + \alpha^M} - (\alpha^M + \alpha^{\Phi}) W^2(\omega, \kappa) h(\kappa) d\kappa \right] + \\
(1-\psi)(1-\lambda) &\left[ \int_0^{\bar{\gamma}} \frac{(1-\tau)y^{\Phi} + \omega(\kappa)(1+l) - l\omega^*}{\alpha^{\Phi} + \alpha^R} - (\alpha^R + \alpha^{\Phi}) W^2(\omega, \kappa) h(\kappa) d\kappa + \int_{\bar{\gamma}}^1 \frac{(1-\tau)y^{\Phi} + \omega(\kappa)(1+\nu) - \nu\omega^*}{\alpha^{\Phi} + \alpha^R} - (\alpha^R + \alpha^{\Phi}) W^2(\omega, \kappa) h(\kappa) d\kappa \right]
\end{aligned}$$

The first-order condition with respect to expropriation  $\kappa$  is the following:

$$\begin{aligned}
\frac{\partial EU^\Phi}{\partial \kappa} &= \psi \left[ P(\omega^* > \omega(\kappa)) \left[ \frac{(1+t)\omega_\kappa}{\alpha^\Phi} - \alpha^\Phi W_\kappa \right] + P(\omega^* \leq \omega(\kappa)) \left[ \frac{(1+v)\omega_\kappa}{\alpha^\Phi} - \alpha^\Phi W_\kappa \right] \right] + \\
(1-\psi)(1-\lambda) &\left[ P(\omega^* > \omega(\kappa)) \left[ \frac{(1+t)\omega_\kappa}{\alpha^M + \alpha^\Phi} - 2(\alpha^M + \alpha^\Phi)W_\kappa \right] + P(\omega^* \leq \omega(\kappa)) \left[ \frac{(1+v)\omega_\kappa}{\alpha^M + \alpha^\Phi} - 2(\alpha^M + \alpha^\Phi)W_\kappa \right] \right] + \\
(1-\psi)\lambda &\left[ P(\omega^* > \omega(\kappa)) \left[ \frac{(1+t)\omega_\kappa}{\alpha^R + \alpha^\Phi} - 2(\alpha^R + \alpha^\Phi)W_\kappa \right] + P(\omega^* \leq \omega(\kappa)) \left[ \frac{(1+v)\omega_\kappa}{\alpha^R + \alpha^\Phi} - 2(\alpha^R + \alpha^\Phi)W_\kappa \right] \right] = 0 \Rightarrow \\
P(\omega^* > \omega(\kappa))(1+t)\omega_\kappa &\left[ \frac{(\alpha^\Phi + \alpha^M)(\alpha^\Phi + \alpha^R) + \alpha^\Phi(2\alpha^\Phi + \alpha^R + \alpha^M)}{\alpha^\Phi(\alpha^\Phi + \alpha^M)(\alpha^\Phi + \alpha^R)} \right] + P(\omega^* \leq \omega(\kappa))(1+v)\omega_\kappa \left[ \frac{(\alpha^\Phi + \alpha^M)(\alpha^\Phi + \alpha^R) + \alpha^\Phi(2\alpha^\Phi + \alpha^R + \alpha^M)}{\alpha^\Phi(\alpha^\Phi + \alpha^M)(\alpha^\Phi + \alpha^R)} \right] \\
-\psi\alpha^\Phi W_\kappa - 2(1-\psi)(2\alpha^\Phi + \alpha^R + \alpha^M)W_\kappa &= 0 \Rightarrow \\
P(\omega^* > \omega(\kappa))(1+t)\omega_\kappa &\left[ \frac{1}{\alpha^\Phi} + \frac{2\alpha^\Phi + \alpha^R + \alpha^M}{(\alpha^\Phi + \alpha^M)(\alpha^\Phi + \alpha^R)} \right] + P(\omega^* \leq \omega(\kappa))(1+v)\omega_\kappa \left[ \frac{1}{\alpha^\Phi} + \frac{2\alpha^\Phi + \alpha^R + \alpha^M}{(\alpha^\Phi + \alpha^M)(\alpha^\Phi + \alpha^R)} \right] \\
-[\psi\alpha^\Phi + 2(1-\psi)(2\alpha^\Phi + \alpha^R + \alpha^M)]W_\kappa &= 0 \Rightarrow \\
\omega_\kappa \left[ \frac{1}{\alpha^\Phi} + \frac{2\alpha^\Phi + \alpha^R + \alpha^M}{(\alpha^\Phi + \alpha^M)(\alpha^\Phi + \alpha^R)} \right] &[P(\omega^* > \omega(\kappa))(1+t) + P(\omega^* \leq \omega(\kappa))(1+v)] = [\psi\alpha^\Phi + 2(1-\psi)(2\alpha^\Phi + \alpha^R + \alpha^M)]W_\kappa \Rightarrow \\
\omega_\kappa = W_\kappa \frac{\psi\alpha^\Phi + 2(1-\psi)(2\alpha^\Phi + \alpha^R + \alpha^M)}{\left[ \frac{1}{\alpha^\Phi} + \frac{2\alpha^\Phi + \alpha^R + \alpha^M}{(\alpha^\Phi + \alpha^M)(\alpha^\Phi + \alpha^R)} \right] [P(\omega^* > \omega(\kappa))(1+t) + P(\omega^* \leq \omega(\kappa))(1+v)]} &\Rightarrow \\
\omega_\kappa = W_\kappa \frac{\alpha^\Phi(4-3\psi) + 2(1-\psi)(\alpha^R + \alpha^M)}{\left[ \frac{1}{\alpha^\Phi} + \frac{2\alpha^\Phi + \alpha^R + \alpha^M}{(\alpha^\Phi + \alpha^M)(\alpha^\Phi + \alpha^R)} \right] [1 + P(\omega^* > \omega(\kappa))t + P(\omega^* \leq \omega(\kappa))v]} &= \\
W_\kappa \frac{[\alpha^\Phi(\alpha^\Phi + \alpha^M)(\alpha^\Phi + \alpha^R)] [\alpha^\Phi(4-3\psi) + 2(1-\psi)(\alpha^R + \alpha^M)]}{[\alpha^\Phi(3\alpha^\Phi + 2(\alpha^R + \alpha^M)) + \alpha^M\alpha^R] [1 + P(\omega^* > \omega(\kappa))t + P(\omega^* \leq \omega(\kappa))v]} &, \text{ which solves for } \kappa^* \Rightarrow \\
\kappa^* = \omega_\kappa^{-1} \left( W_\kappa \frac{[\alpha^\Phi(\alpha^\Phi + \alpha^M)(\alpha^\Phi + \alpha^R)] [\alpha^\Phi(4-3\psi) + 2(1-\psi)(\alpha^R + \alpha^M)]}{[\alpha^\Phi(3\alpha^\Phi + 2(\alpha^R + \alpha^M)) + \alpha^M\alpha^R] [1 + P(\omega^* > \omega(\kappa))t + P(\omega^* \leq \omega(\kappa))v]} \right) &
\end{aligned}$$

It is certain that  $\omega_\kappa$  is monotonically decreasing in the expropriation rate  $\kappa$ , which implies that the extractive capacity of the poor undermines the output they produce. The larger the share of rich and middle in the economy, the higher the expropriation rate to be observed and therefore the lower the level of optimal output. In the Aristotelian economy, class competition is explicit and defines wealth. The decision of the poor to form a government coalition with the rich or the middle is driven by the effect of the two competing classes on the poor's expropriation rate. As the regime analysis in Book V of *Politics* indicates, democracy as the rule of the poor allows the expropriation of the middle and particularly the rich at the expense of the economy (1301a37-1320a1). The same holds for oligarchy as the rule of the rich. However, oligarchies are inclined to be less stable than democracies because the former involve not only redistributive conflicts between the rich and the poor, but also rivalries among the rich themselves (ibid.).

The optimal output in the Aristotelian economy is defined by  $\tilde{\omega} = \int_{\kappa} \omega_\kappa h(\kappa) d\kappa$  and the indirect utility of the rich is the following:

$$\begin{aligned}
EU^R(y^R; g) &= \psi \left[ \frac{y^R}{\alpha^R + \alpha^M} + H(g) + \kappa r - (\alpha^R + \alpha^M) C(g, \varepsilon) \right] + \\
(1-\psi) &\left[ \lambda \left( \frac{y^R}{\alpha^R + \alpha^\Phi} + H(g) + (1+\kappa)r - (\alpha^R + \alpha^\Phi) C(g, \varepsilon) \right) + \frac{(1-\lambda)y^R}{\alpha^R} \right] \Rightarrow \\
EU^R(y^R; g) &= \psi \left[ \frac{g+r+\theta^R+\tilde{\omega}}{\alpha^R + \alpha^M} + H(g) + \kappa r - (\alpha^R + \alpha^M) C(g, \varepsilon) \right] + \\
(1-\psi) &\left[ \lambda \left( \frac{g+r+\theta^R+\tilde{\omega}}{\alpha^R + \alpha^\Phi} + H(g) + (1+\kappa)r - (\alpha^R + \alpha^\Phi) C(g, \varepsilon) \right) + \frac{(1-\lambda)(g+r+\theta^R+\tilde{\omega})}{\alpha^R} \right] \Rightarrow \\
EU^R(y^R; g) &= \psi \left[ \frac{g+r+\theta^R+\tilde{\omega}}{\tau(\alpha^R + \alpha^M)} + H(g) + \kappa r - (\alpha^R + \alpha^M) C(g, \varepsilon) \right] + \\
(1-\psi) &\left[ \lambda \left( \frac{g+r+\theta^R+\tilde{\omega}}{\tau(\alpha^R + \alpha^\Phi)} + H(g) + (1+\kappa)r - (\alpha^R + \alpha^\Phi) C(g, \varepsilon) \right) + \frac{(1-\lambda)(g+r+\theta^R+\tilde{\omega})}{\tau\alpha^R} \right]
\end{aligned}$$

The first-order condition with respect to  $g$  in the rich class's optimization problem is the following:

$$\begin{aligned} \frac{\partial EU^R}{\partial g} &= \psi \left[ \frac{1}{\tau(\alpha^R + \alpha^M)} + H_g - (\alpha^R + \alpha^M) C_g \right] + \\ (1-\psi) &\left[ \lambda \left( \frac{1}{\tau(\alpha^R + \alpha^\Phi)} + H_g - (\alpha^R + \alpha^\Phi) C_g \right) + \frac{1-\lambda}{\tau\alpha^R} \right] = 0 \Rightarrow \\ \frac{\partial EU^R}{\partial g} &= \frac{\psi}{\tau(\alpha^R + \alpha^M)} + \frac{\lambda(1-\psi)}{\tau(\alpha^R + \alpha^\Phi)} + H_g [\psi + (1-\psi)\lambda] - \\ C_g &\left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right] + \frac{(1-\lambda)(1-\psi)}{\tau\alpha^R} = 0 \Rightarrow \\ \frac{\psi}{\tau(\alpha^R + \alpha^M)} + \frac{\lambda(1-\psi)}{\tau(\alpha^R + \alpha^\Phi)} + \frac{(1-\lambda)(1-\psi)}{\tau\alpha^R} + H_g [\psi + (1-\psi)\lambda] &= C_g \left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right] \Rightarrow \\ \frac{\psi}{\alpha^R + \alpha^M} + (1-\psi) \left( \frac{\lambda}{\alpha^R + \alpha^\Phi} + \frac{1-\lambda}{\alpha^R} \right) + H_g [\psi + (1-\psi)\lambda] &= C_g \left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right] \Rightarrow \\ H_g [\psi + (1-\psi)\lambda] &= C_g \left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right] - \frac{\frac{\psi}{\alpha^R + \alpha^M} + (1-\psi) \left( \frac{\lambda}{\alpha^R + \alpha^\Phi} + \frac{1-\lambda}{\alpha^R} \right)}{\tau} \Rightarrow \\ H_g &= C_g \frac{\left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right]}{\psi + (1-\psi)\lambda} - \frac{\frac{\psi}{\alpha^R + \alpha^M} + (1-\psi) \left( \frac{\lambda}{\alpha^R + \alpha^\Phi} + \frac{1-\lambda}{\alpha^R} \right)}{\tau [\psi + (1-\psi)\lambda]} \Rightarrow \\ H_g &= C_g \frac{\left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right]}{\psi + (1-\psi)\lambda} - \\ \frac{\psi\alpha^R(\alpha^R + \alpha^\Phi) + \lambda(1-\psi)\alpha^R(\alpha^R + \alpha^M) + (1-\lambda)(1-\psi)(\alpha^R + \alpha^M)(\alpha^R + \alpha^\Phi)}{\tau [\psi + (1-\psi)\lambda]} &\Rightarrow \\ H_g &= C_g \frac{\left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right]}{\psi + (1-\psi)\lambda} - \frac{\psi\alpha^R(\alpha^R + \alpha^\Phi) + (1-\psi)(\alpha^R + \alpha^M) \left[ \lambda\alpha^R + (1-\lambda)(\alpha^R + \alpha^\Phi) \right]}{\tau [\psi + (1-\psi)\lambda]}, \end{aligned}$$

which solves for  $g^{**} \Rightarrow$

$$g^{**} = H_g^{-1} \left( C_g \frac{\left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right]}{\psi + (1-\psi)\lambda} - \frac{\psi\alpha^R(\alpha^R + \alpha^\Phi) + (1-\psi)(\alpha^R + \alpha^M) \left[ \lambda\alpha^R + (1-\lambda)(\alpha^R + \alpha^\Phi) \right]}{\tau [\psi + (1-\psi)\lambda]} \right)$$

The marginal utility from the provision of the public good is negative for the rich, resulting in  $H_g < 0$ . Nevertheless, there are significant differences when compared to the level of public goods provision in the Platonic economy. First, in the Aristotelian economy  $g^{**}$  is monotonically increasing with the tax

rate  $\tau$ , resulting in a positive correlation between taxation and redistribution. In *Politics*, justice implies that the accumulated wealth of the rich is compensated through public goods provision in favor of the middle and the poor, who form the majority in society. The concept of justice here is linked to the provision of public goods, but also to the individual commitment to the common good as a function of rents extraction. Therefore, we also take the first-order condition with respect to  $\theta^R$ :

$$EU^R(y^R; g, \theta^R) = \psi \left[ \frac{g + r + \theta^R + \tilde{\omega}}{\tau(\alpha^R + \alpha^M)} + H(g) + \kappa r - (\alpha^R + \alpha^M)C(g, \varepsilon) \right] +$$

$$(1 - \psi) \left[ \lambda \left( \frac{g + r + \theta^R + \tilde{\omega}}{\tau(\alpha^R + \alpha^\Phi)} + H(g) + (1 + \kappa)r - (\alpha^R + \alpha^\Phi)C(g, \varepsilon) \right) + \frac{(1 - \lambda)(g + r + \theta^R + \tilde{\omega})}{\tau\alpha^R} \right] \Rightarrow$$

$$\begin{aligned}
EU^R = & \psi \left[ \frac{g + r + \frac{\psi}{\alpha^R} \iint_{\xi^R, \xi^M} \Gamma(r, \varepsilon) \chi(\xi^R) h(\xi^M) d\xi^R d\xi^M + \frac{(1-\psi)\lambda}{\alpha^R} \iint_{\xi^R, \xi^\Phi} \Gamma(r, \varepsilon) \chi(\xi^R) q(\xi^\Phi) d\xi^R d\xi^\Phi + \tilde{\omega}}{\tau(\alpha^R + \alpha^M)} + H(g) + \kappa r - (\alpha^R + \alpha^M) C(g, \varepsilon) \right] + \\
(1-\psi) & \left[ \lambda \left( \frac{g + r + \frac{\psi}{\alpha^R} \iint_{\xi^R, \xi^M} \Gamma(r, \varepsilon) \chi(\xi^R) h(\xi^M) d\xi^R d\xi^M + \frac{(1-\psi)\lambda}{\alpha^R} \iint_{\xi^R, \xi^\Phi} \Gamma(r, \varepsilon) \chi(\xi^R) q(\xi^\Phi) d\xi^R d\xi^\Phi + \tilde{\omega}}{\tau(\alpha^R + \alpha^\Phi)} + H(g) + (1+\kappa)r - (\alpha^R + \alpha^\Phi) C(g, \varepsilon) \right) \right. \\
& \left. + \frac{(1-\lambda)(g + r + \frac{\psi}{\alpha^R} \iint_{\xi^R, \xi^M} \Gamma(r, \varepsilon) \chi(\xi^R) h(\xi^M) d\xi^R d\xi^M + \frac{(1-\psi)\lambda}{\alpha^R} \iint_{\xi^R, \xi^\Phi} \Gamma(r, \varepsilon) \chi(\xi^R) q(\xi^\Phi) d\xi^R d\xi^\Phi + \tilde{\omega})}{\tau\alpha^R} \right]
\end{aligned}$$

Given that  $\theta^R$  is a function of  $r$ , we implement the implicit function theorem, yielding:

$$\begin{aligned}
\frac{\partial EU^R}{\partial \theta^R} &= \psi \left[ \frac{1}{\tau(\alpha^R + \alpha^M)} - (\alpha^R + \alpha^M) C_{\theta^R} \right] + (1-\psi) \left[ \lambda \left( \frac{1}{\tau(\alpha^R + \alpha^\Phi)} - (\alpha^R + \alpha^\Phi) C_{\theta^R} \right) + \frac{1-\lambda}{\tau\alpha^R} \right] = 0 \Rightarrow \\
\frac{\psi}{\tau(\alpha^R + \alpha^M)} + (1-\psi) \frac{\lambda}{\tau(\alpha^R + \alpha^\Phi)} + (1-\psi) \frac{1-\lambda}{\tau\alpha^R} &= C_{\theta^R} \left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right] \Rightarrow \\
C_{\theta^R} &= \frac{\frac{\psi}{\tau(\alpha^R + \alpha^M)} + (1-\psi) \frac{\lambda}{\tau(\alpha^R + \alpha^\Phi)} + (1-\psi) \frac{1-\lambda}{\tau\alpha^R}}{\psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi)} = \frac{\frac{\psi}{\alpha^R + \alpha^M} + \frac{\lambda(1-\psi)}{\alpha^R + \alpha^\Phi} + \frac{(1-\lambda)(1-\psi)}{\alpha^R}}{\tau \left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right]} \Rightarrow \\
C_{\theta^R} &= \frac{\psi\alpha^R(\alpha^R + \alpha^\Phi) + \lambda(1-\psi)\alpha^R(\alpha^R + \alpha^M) + (1-\lambda)(1-\psi)(\alpha^R + \alpha^M)(\alpha^R + \alpha^\Phi)}{\tau \left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right]} \Rightarrow
\end{aligned}$$

$$\begin{aligned}
\theta^{R*} &= C_{\alpha^R}^{-1} \left( \frac{\psi \alpha^R (\alpha^R + \alpha^\Phi) + \lambda (1-\psi) \alpha^R (\alpha^R + \alpha^M) + (1-\lambda)(1-\psi) (\alpha^R + \alpha^M) (\alpha^R + \alpha^\Phi)}{\tau [\psi (\alpha^R + \alpha^M) + \lambda (1-\psi) (\alpha^R + \alpha^\Phi)]} \right) \\
\frac{\partial EU^R}{\partial r} &= \psi \left[ \frac{1 + \frac{\partial}{\partial r} \left[ \frac{\psi}{\alpha^R} \iint_{\xi^R, \xi^M} \Gamma(r, \varepsilon) \chi(\xi^R) h(\xi^M) d\xi^R d\xi^M + \frac{(1-\psi)\lambda}{\alpha^R} \iint_{\xi^R, \xi^\Phi} \Gamma(r, \varepsilon) \chi(\xi^R) q(\xi^\Phi) d\xi^R d\xi^\Phi \right]}{\tau (\alpha^R + \alpha^M)} + \kappa - (\alpha^R + \alpha^M) C_r \right] + \\
(1-\psi) &\left[ \lambda \left( \frac{1 + \frac{\partial}{\partial r} \left[ \frac{\psi}{\alpha^R} \iint_{\xi^R, \xi^M} \Gamma(r, \varepsilon) \chi(\xi^R) h(\xi^M) d\xi^R d\xi^M + \frac{(1-\psi)\lambda}{\alpha^R} \iint_{\xi^R, \xi^\Phi} \Gamma(r, \varepsilon) \chi(\xi^R) q(\xi^\Phi) d\xi^R d\xi^\Phi \right]}{\tau (\alpha^R + \alpha^\Phi)} + 1 + \kappa - (\alpha^R + \alpha^\Phi) C_r \right) \right. \\
&\left. + \frac{(1-\lambda) \left( 1 + \frac{\partial}{\partial r} \left[ \frac{\psi}{\alpha^R} \iint_{\xi^R, \xi^M} \Gamma(r, \varepsilon) \chi(\xi^R) h(\xi^M) d\xi^R d\xi^M + \frac{(1-\psi)\lambda}{\alpha^R} \iint_{\xi^R, \xi^\Phi} \Gamma(r, \varepsilon) \chi(\xi^R) q(\xi^\Phi) d\xi^R d\xi^\Phi + \tilde{\omega} \right]}{\tau \alpha^R} \right)}{\right. \\
\frac{\partial EU^R}{\partial r} &= \psi \left[ \frac{1 + \Gamma_r X(\xi^R) \left[ \frac{\psi}{\alpha^R} H(\xi^M) + \frac{(1-\psi)\lambda}{\alpha^R} Q(\xi^\Phi) \right]}{\tau (\alpha^R + \alpha^M)} + \kappa - (\alpha^R + \alpha^M) C_r \right] + \\
(1-\psi) &\left[ \lambda \left( \frac{1 + \Gamma_r X(\xi^R) \left[ \frac{\psi}{\alpha^R} H(\xi^M) + \frac{(1-\psi)\lambda}{\alpha^R} Q(\xi^\Phi) \right]}{\tau (\alpha^R + \alpha^\Phi)} + 1 + \kappa - (\alpha^R + \alpha^\Phi) C_r \right) \Rightarrow \right. \\
\left[ \psi (\alpha^R + \alpha^M) + (1-\psi) \lambda (\alpha^R + \alpha^\Phi) \right] C_r &= \psi \left[ \frac{1 + \Gamma_r X(\xi^R) \left[ \frac{\psi}{\alpha^R} H(\xi^M) + \frac{(1-\psi)\lambda}{\alpha^R} Q(\xi^\Phi) \right]}{\tau (\alpha^R + \alpha^M)} + \kappa \right] + \\
(1-\psi) \lambda &\left[ \frac{1 + \Gamma_r X(\xi^R) \left[ \frac{\psi}{\alpha^R} H(\xi^M) + \frac{(1-\psi)\lambda}{\alpha^R} Q(\xi^\Phi) \right]}{\tau (\alpha^R + \alpha^\Phi)} + 1 + \kappa \right] \Rightarrow
\end{aligned}$$



$$\left[ \psi(\alpha^R + \alpha^M) + (1-\psi)\lambda(\alpha^R + \alpha^\Phi) \right] C_r = \frac{1 + \Gamma_r X(\xi^R) \left[ \frac{\psi}{\alpha^R} H(\xi^M) + \frac{(1-\psi)\lambda}{\alpha^R} Q(\xi^\Phi) \right]}{\tau} \left[ \frac{\psi}{\alpha^R + \alpha^M} + \frac{\lambda(1-\psi)}{\alpha^R + \alpha^\Phi} \right] + \kappa[\psi + (1-\psi)\lambda] + (1-\psi)\lambda$$

Hence, according to the implicit function theorem:

$$\frac{\partial \theta^R}{\partial r} = - \frac{\frac{\psi}{\tau(\alpha^R + \alpha^M)} + (1-\psi)\frac{\lambda}{\tau(\alpha^R + \alpha^\Phi)} + (1-\psi)\frac{1-\lambda}{\tau\alpha^R} - C_{\theta^R} \left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right]}{1 + \Gamma_r X(\xi^R) \left[ \frac{\psi}{\alpha^R} H(\xi^M) + \frac{(1-\psi)\lambda}{\alpha^R} Q(\xi^\Phi) \right]} \left[ \frac{\psi}{\alpha^R + \alpha^M} + \frac{\lambda(1-\psi)}{\alpha^R + \alpha^\Phi} \right] + \kappa[\psi + (1-\psi)\lambda] + (1-\psi)\lambda \Rightarrow$$

$$\frac{\partial \theta^R}{\partial r} = - \frac{\frac{\psi}{\alpha^R + \alpha^M} + \frac{\lambda(1-\psi)}{\alpha^R + \alpha^\Phi} + \frac{(1-\lambda)(1-\psi)}{\alpha^R} - \tau C_{\theta^R} \left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right]}{\left[ 1 + \frac{\Gamma_r X(\xi^R)}{\alpha^R} \left[ \psi H(\xi^M) + (1-\psi)\lambda Q(\xi^\Phi) \right] \right]} \left[ \frac{\psi}{\alpha^R + \alpha^M} + \frac{\lambda(1-\psi)}{\alpha^R + \alpha^\Phi} \right] + \tau \left[ \kappa[\psi + (1-\psi)\lambda] + (1-\psi)\lambda \right]$$

### **Proposition 2 (The Aristotelian Equilibrium)**

There is a unique subgame perfect equilibrium of the Aristotelian economy game that has the following form:

1. If  $\kappa < \alpha^R$  or  $\kappa \leq \alpha^R + \alpha^M$ , then the poor do not deliver  $\tilde{\omega}$  and polity collapses in favor of democracy.
2. If  $|\kappa - \nu| \leq |\iota - \kappa|$  and  $\kappa < \alpha^R$  or  $\kappa \leq \alpha^R + \alpha^M$ , then the poor do not deliver  $\tilde{\omega}$  and tyranny occurs.
3. If  $|\kappa - \nu| > |\iota - \kappa|$  and  $\kappa < \alpha^R$  or  $\kappa \leq \alpha^R + \alpha^M$ , then the poor do not deliver  $\tilde{\omega}$  and oligarchy occurs.
4. If  $\kappa \geq \alpha^R$  or  $\kappa > \alpha^R + \alpha^M$ , then the poor deliver  $\tilde{\omega}$  and the following solutions are observed:
  - a. If  $\theta^R \leq r$  and  $\bar{\xi}^M > \frac{1}{2}$ , then the rich underprovide the public good  $g$  and kingship occurs.
  - b. If  $\theta^R \leq r$  and  $\bar{\xi}^M \leq \frac{1}{2}$ , then the rich underprovide the public good  $g$  and aristocracy occurs.
  - c. If  $\theta^R > r$ , then the rich provide the public good  $g$  at least at  $g^*$  and polity is preserved.

The main drivers of the Aristotelian economy are the expropriation capacity of the poor  $\kappa$  and the rent extraction of the rich  $r$ . When the expropriation capacity of the poor is strictly dominated by the share of the rich in society or weakly dominated by the joint share of the rich and the poor, then the poor have no incentive to be productive and do not deliver the optimal output that maximizes cross-class collective welfare and the political incumbency of the rich. Democracy emerges as a coalition of the poor and the middle against the rich, and allows the poor to produce the output level that maximizes the utility of the poor and the middle only. Tyranny emerges when the rich implement a repressive

form of monitoring over production processes, such that the poor have a much higher incentive to expropriate and at the same time underfulfill rather than overfulfill and keep expropriation at modest levels. The difference between tyranny and oligarchy is that in an oligarchy monitoring is less repressive and the productive poor are rewarded relatively more than the less productive ones are penalized.

Gaertner (1994) underscores the significance of distributive justice theories and particularly utilitarianism. Cohen (1997) reminds us that societies entail deterrence and prevention functions that contribute to a novel concept of rationality, where agents make distributive decisions based on these *ex-ante* coercive structures. The proposed model of Aristotelian *Politics* treats the expropriation capacity of the poor and the rent-seeking activities of the rich as morally equivalent. There is no moral advantage for the rich in the Aristotelian economy, as there is for the guardians in the Platonic economy. Furthermore, the upper bound of the joint level of skills for auxiliaries and producers  $\bar{\sigma}^L$  distinguishes itself from the upper bound of the competence level of the middle class  $\bar{\xi}^M$ . When the expropriation rate of the poor dominates the electoral magnitude of the rich and/or the middle in society, then optimal output will be produced. Three political regimes are then possible: kingship, aristocracy and polity. Kingship occurs when the rent-seeking activities of the rich weakly dominate their individual commitment to the common good and their level of skills is defined by a closed set whose lower bound is above the average level of skills in society. Hence, it becomes obvious why kingship is a regime that requires a coalition between the rich and the middle: the rich are competent enough to rule society through a representative. Nevertheless, they are inclined to coalesce with the middle rather than the poor because their high level of rent extraction is more likely to be compensated by the higher skill level of the middle class in relation to the provision of public goods.

In contrast, when rent-seeking dominates the common good mission of the rich, but their level of skills takes a value from a closed set whose lower bound is below the average level of skills in society, then aristocracy emerges. The difference between kingship and aristocracy lies in the comparison of the lower bound of the competence level of the rich in society with the average level of skills for any citizen, which we assume to be  $\frac{1}{2}$ . When the rich class is extractive and reveals high levels of skills, then it is possible for one enlightened representative of the rich class to function as a king. When the rich class is extractive but it operates at low skill levels, then a collective solution such as aristocracy is Pareto-improving with respect to kingship such that the distance between the lower

bound of skill levels for the rich and the upper bound of skill levels for the poor  $\left| \bar{\xi}^M - \bar{\xi}^\Phi \right|$  is minimized.

This is why the provision level of public goods is higher under kingship than under aristocracy: lower skills of the rich under aristocracy and coalition with the poor produce an overall more extractive ruling coalition. As Moreno-Ternero and Roemer (2012) point out, there may be a common justification both for resource and welfare egalitarianism. Similarly, in the Aristotelian economy, the achievement of polity becomes possible when the notions of no-domination, solidarity and composition also apply to the rich. This is why polity is the Pareto optimal regime driven by the coalition between the poor and the middle.

### *Comparative Statics*

As in the Platonic economy, I identify the relationship between the distributive commitment of the rich class to the provision of the public good and its degree of rent-seeking, the relationship between public goods provision and the share of the rich class in the Aristotelian economy, and the relationship between the expropriation rate of the poor and the joint share of rich and middle in society. I find that

$$\frac{\partial \theta^R}{\partial r} = - \frac{\frac{\psi}{\alpha^R + \alpha^M} + \frac{\lambda(1-\psi)}{\alpha^R + \alpha^\Phi} + \frac{(1-\lambda)(1-\psi)}{\alpha^R} - \tau C_{\theta^R} \left[ \psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi) \right]}{\left[ 1 + \frac{\Gamma_r X(\xi^R)}{\alpha^R} \left[ \psi H(\xi^M) + (1-\psi)\lambda Q(\xi^\Phi) \right] \right] \left[ \frac{\psi}{\alpha^R + \alpha^M} + \frac{\lambda(1-\psi)}{\alpha^R + \alpha^\Phi} \right] + \tau \left[ \kappa \left[ \psi + (1-\psi)\lambda \right] + (1-\psi)\lambda \right]} < 0$$

which means that the distributive commitment of the rich is monotonically decreasing with their rent extraction rate. Therefore, as per Aristotle, political rents have a negative effect on the common good. The higher the rate of rent extraction, the lower the distributive commitment of the rich class to the common good. Hence, we derive  $r^*$ , which solves  $\frac{\partial \theta^R}{\partial r}$  and is defined by  $(\underline{r}, \bar{r}]$  such that  $r \in (\underline{r}, \bar{r}]$ .

Polity occurs at low levels of rent-seeking such that  $r \leq \underline{r}$ , whereas kingship occurs at intermediate levels of rent-seeking such that  $\underline{r} < r \leq \bar{r}$ . Similarly, at high levels of rent-seeking aristocracy occurs such that  $r > \bar{r}$ . Rent extraction by the rich undermines their commitment to the common good and leads to less representative forms of government. The rich are then more willing to seek coalitions that lead to kingship or aristocracy, depending on their own level of skills and the compensatory power of the skills of the middle or the poor toward the common good, and in compensation of their own rent-seeking activities.

### **Lemma 3**

In the Aristotelian economy game, there is a positive monotonic relationship between the distributive commitment to the common good by the rich and their rate of rent extraction. When  $r \leq \underline{r}$ , politeia is observed, whereas for intermediate values of  $r$  such that  $\underline{r} < r \leq \bar{r}$  kingship occurs. Aristocracy emerges at high levels of rent extraction such that  $r > \bar{r}$ .

The relationship between the provision of public goods and the share of rich in the population of the city-state is defined by the following expression:

$$g^{**} = H_g^{-1} \left( C_g \frac{[\psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi)]}{\psi + (1-\psi)\lambda} - \frac{\psi\alpha^R(\alpha^R + \alpha^\Phi) + (1-\psi)(\alpha^R + \alpha^M)[\lambda\alpha^R + (1-\lambda)(\alpha^R + \alpha^\Phi)]}{\tau[\psi + (1-\psi)\lambda]} \right)$$

such that:

$$\frac{\partial g}{\partial \alpha^R} = H_{g\alpha^R}^{-1} \left( C_g \frac{[\psi(\alpha^R + \alpha^M) + \lambda(1-\psi)(\alpha^R + \alpha^\Phi)]}{\psi + (1-\psi)\lambda} - \frac{\psi\alpha^R(\alpha^R + \alpha^\Phi) + (1-\psi)(\alpha^R + \alpha^M)[\lambda\alpha^R + (1-\lambda)(\alpha^R + \alpha^\Phi)]}{\tau[\psi + (1-\psi)\lambda]} \right)^*$$

$$\left[ C_g - \frac{2\alpha^R + \alpha^\Phi[\psi + (1-\psi)(1-\lambda)] + \alpha^M(1-\lambda)(1-\psi)}{\tau[\psi + (1-\psi)\lambda]} \right] < 0.$$

The higher the share of the rich in society, the lower the provision of public goods. Because polity emerges only as a coalition between the poor and the middle, the provision level of public goods is maximized only if the rich are unable to form a ruling coalition and preserve their incumbency advantage. However, democracy is more likely to emerge than polity when the poor maintain a low rate of expropriation. A coalition between the poor and the middle facilitates the provision of the public good under democracy, but at lower levels than in a polity. When the share of rich in society is very low and it strictly dominates the expropriation capacity of the poor, tyranny emerges as a political regime that also underprovides public goods. The same observation holds when the joint share of rich and middle weakly dominate the expropriation rate of the poor. In contrast to the case with democracy, under tyranny and oligarchy the participation of the rich in the government of the city-state leads to higher levels of underprovision of the public good. Under oligarchy, in particular, the level of public goods provision reaches its lower bound because the high share of the rich in society leads to higher levels of aggregate extraction of rents and makes the poor more indifferent toward the provision of the public good.

#### **Lemma 4**

In the Aristotelian economy game, there is a negative monotonic relationship between the provision of the public good and the share of the rich. When  $\kappa < \alpha^R$  or  $\kappa \leq \alpha^R + \alpha^M$ , tyranny is observed for low values of  $\alpha^R$  such

that  $\alpha^R \leq \underline{\alpha}^R$ , whereas for high values of  $\alpha^R$  such that  $\alpha^R > \bar{\alpha}^R$  oligarchy occurs. Democracy occurs for intermediate values of  $\alpha^R$  such that  $\underline{\alpha}^R < \alpha^R \leq \bar{\alpha}^R$ .

Furthermore,  $\frac{\partial \kappa}{\partial \alpha^R} < 0$  and  $\frac{\partial \kappa}{\partial \alpha^M} < 0$ , which suggests that the expropriation rate by the poor decreases with the share of the rich and the middle. Hence, Aristotle treats socio-economic classes as agents that mutually restrain each other's extractive activities. The realization of polity does not constitute an equilibrium in relation to a continuous individual commitment to the common good by the rich or an implementation of the production plan by the poor. On the contrary, the Aristotelian logic underscores the significance of mutually restraining the accumulation of political rents by the rich and expropriation activities by the poor.

#### **Corollary 4A**

*In the Aristotelian economy game, there is a negative monotonic relationship between the expropriation rate by the poor and the joint share of the rich and middle. When  $\alpha^R + \alpha^M > \underline{\alpha}^R + \underline{\alpha}^M$ , then  $\tilde{\omega}$  is not delivered and tyranny or democracy or oligarchy emerges. When  $\alpha^R + \alpha^M \leq \underline{\alpha}^R + \underline{\alpha}^M$ , then  $\tilde{\omega}$  is delivered and politeia or kingship or aristocracy emerges.*

As in the Platonic economy, I also define  $\underline{\alpha}^R$  and  $\underline{\alpha}^M$  in the Aristotelian economy as the respective lower bounds of the rich and middle class population shares that facilitate the implementation of the production schedule by the poor. When  $\tilde{\omega}$  is not delivered by the poor, the level of public goods provision collapses and we observe the emergence of three regime types, which Aristotle considers to be deviations from the correct forms of regimes: democracy, tyranny and oligarchy. An overwhelmingly large size of the rich and middle undermines the delivery of the optimal output  $\tilde{\omega}$  by the poor and allows an excessive gross rent extraction by the rich. This implies an underprovision of the public good, where democracy becomes an equilibrium solution that punishes the rent-seeking of the rich without introducing punitive sanctions for underfulfillment against the poor. Tyranny as a political-economic outcome in the Aristotelian economy suggests that the implied coalition between the rich and the poor does not advance social welfare. In reality, it subdues governmental policy to rent accumulation by the rich. The underprovision of the public good becomes even more acute under oligarchy. The coalition between the rich and the middle increases the predatory activities of the rich against the poor even more, while being more dependent on the output produced by the poor as a compensatory mechanism for their own extractive history.

#### IV. The Classical Origins of Plan & Market

In this paper, I argue that the distinction between Platonic and Aristotelian economies constitutes the archetypical comparison that has underpinned the study of economic systems, the comparison between plan and market. While Plato's *Republic* and Aristotle's *Politics* propose five and six regime types, respectively, as possible equilibrium solutions to the distribution of resources and political power, there are significant differences even for regime types that share the same characterization: democracy, oligarchy and tyranny. The Platonic *politeia* is also significantly distinctive from the Aristotelian polity. As Faravelli (2007) indicates, justice conceptualizations are context-dependent. Tyranny under Plato is a planning pathology that emerges when guardians penalize producers that deviate from production targets more than they reward producers that meet them. Tyranny under Aristotle occurs when the rich are better off by forming a coalition government with poor and underprovide public goods under conditions of limited production. Cross-class competition not only reveals the close affinity between Aristotle and Marx (Schwartz, 1979), but it also proposes that government coalitions may reduce the efficiency losses generated by the underfulfillment of production targets. This tradeoff between the political effect of representative governance and the efficiency loss related to a vertical production process is crucial for our understanding of Platonic and Aristotelian economies. While the performance of a Platonic economic system depends on the enforcement capacity of the guardian and his degree of individual commitment to the common good, the performance of an Aristotelian economic system depends on the relative competence advantage of the rich and the fulfillment of the production plan by the poor. Extractive poor can be much more detrimental for the emergence of polity per Aristotle than for the emergence of *politeia* per Plato. The guardian-centric nature of the Platonic economy makes oligarchy a relatively more efficient regime than its Aristotelian equivalent. The same observation holds for tyranny. Commitment to the common good is a much more powerful constraint than the interests of the middle class in singular regimes. The reverse observation holds for democracy: the Platonic version of democracy is more extractive than its Aristotelian equivalent.

Nussbaum (2000) argues that the roots of political liberalism and plurality are Aristotelian. Van Johnson (1939) recognizes yet another intellectual line linking Aristotle with Marx: the concepts of value as use in Aristotle and value as labor in Marx. The existence of hierarchy both in Plato and Aristotle resonates with the logic of *oikos* that Marx observed in Aristotelian forms of government

(Booth, 1991). Nevertheless, the optimal regime – polity – is achieved differently in these two types of economic systems. In the *Republic*, *politeia* is the equilibrium regime where the guardians are self-invested in the maximization of social welfare and place the interests of the community before their own. In *Politics*, polity is the equilibrium regime that consolidates the dominance of the middle class while preserving the expropriation capacity of the poor such that the production process is not interrupted. Hence, the concept of just society in the Platonic equilibrium is related to the enlightened nature of society's leadership, whereas in the Aristotelian equilibrium it is linked to the majoritarian imposition of the middle class in coalition with the less extractive of the two remaining classes, the poor. Just society in the *Republic* is a phenomenon *ad personam*; in *Politics*, it is *ad rem*. I argue that this is yet another powerful distinction between plan and market and, in particular, the way they reflect on political-economic outcomes.

Marx's expropriation theory does not imply any equality of resources *ex-ante* (Roemer, 1985b). Neither the Platonic nor the Aristotelian economy does this. Guardians are the persistent winners of a Platonic economy with differential rates of success, depending on the type of regime that emerges. The frequent change in government coalitions points to a more complex institutional environment. In this sense, Aristotle is an advocate of what Roemer calls the egalitarian principle (Roemer, 1993): the core of human nature and the resources related to it should be common to all people. Moreover, the transition from modeling economic environments to the modeling of ethical environments is central here (Roemer, 1986). What makes the Aristotelian equilibrium more interesting and diverse is its reliance on political majorities rather than on hierarchical structures that are fixed *ex-ante*. While the incumbency advantage of the rich is assumed in society for purposes of comparability with its Platonic counterpart, it may also be the case that the starting point of production may involve the incumbency of the middle class. I have not included that in my model because I also intended to produce a model that would be historically contingent upon the historical realities developed in the 5<sup>th</sup> and 4<sup>th</sup> century BC in Athens. The rich are winners at differential levels under tyranny, kingship, aristocracy and oligarchy, while the middle under polity and the poor under democracy. Despite his straightforward support of the common good, Aristotle is also convinced that injustice undermines the realization of the common good by prioritizing individual profit over collective welfare (Smith, 1999). The prevalence of a replacement mechanism for the ruling coalition and the achievement of a just society through the elimination of rent-seeking activities of the rich indicate the market-based orientation of the Aristotelian economic system, whereas the existence of a fixed hierarchy and the achievement of a just

society through the distributive commitment of the guardian to the common good and the high ex-ante quality of monitoring institutions reveal the plan-based orientation of the Platonic economy.

## V. Conclusions

In this paper, I model the emergence of political regimes and their relationship to justice when it comes to the distribution of resources and the maximization of collective welfare. On the basis of the regime outcomes proposed in Plato's *Republic* and Aristotle's *Politics*, I argue that the Platonic economic system constitutes an archetypical form of a centrally planned economy, whereas the Aristotelian economic system represents an archetypical form of a market economy. What drives plan fulfillment in the Platonic economy is the ex-ante quality level of monitoring institutions and the distributive commitment of the guardians to the common good. What drives production processes in the Aristotelian economy is the rent-seeking of the rich class and the expropriation rate of the poor class. The frequent replacement of the ruling coalition in the Aristotelian economy leads to higher levels of overall efficiency since it identifies tradeoffs between the political effect of representative governance and vertical production processes.

This paper suggests that the distinction between plan and market is not unknown, or at least it should not be considered unrelated, to classical political philosophy and its two main Greek thinkers, Plato and Aristotle. While Aristotle became the intellectual force behind the Renaissance and the reconceptualization of the West, Plato defined the development of Byzantine and Near Eastern political thought and later on became very popular in the Soviet Union, which never integrated Aristotle into the core of its philosophical and political-economic thinking. Developing an analytical framework to delineate the distributive origins of the Cold War within the context of classical philosophy is only the first step in what may become an extensive research program.

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