

The fade away of an initial bias in longitudinal surveys

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Abstract

We propose a new view of initial nonresponse bias in longitudinal surveys. Under certain conditions, an initial bias may "fade-away" over consecutive waves. This effect is discussed in a Markovian framework. A general contraction theorem for time inhomogeneous Markov chains is presented. The result is that two chains with different starting distributions will eventually converge to equal state distributions. Two conditions are required: transition probabilities must be equal for respondents and nonrespondents, and attrition in later panel waves must not depend on the state of the individuals. The theory is applied to a German survey on social benefit recipience. Minor deviations from assumptions are shown to have only a negligible impact on the strength of the fade-away effect. Results from other European surveys indicate that the fade-away effect is present in them, as well. Extensions are pointed out.

Keywords: panel surveys, panel attrition, nonresponse bias, Markov chains, steady state distribution.

1 Introduction

Longitudinal surveys are plagued by nonresponse not only at their start but also in later phases of the study. For example, in panel surveys the nonresponse at the initial wave may be aggravated by attrition in later panel

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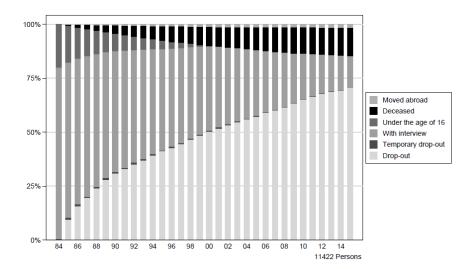


Figure 1: The cumulative attrition of the SOEP after 32 waves. Losses of Subsample A started in 1984 until 2015. The losses of initial nonresponse are not displayed here. Figure 15 taken from Kroh et al (2016).

waves. The attrition can be caused, e.g., by non-cooperation or failure to follow-up residential movers (Watson and Wooden 2009). When such losses are documented in a cumulative fashion, as, e.g., for the German Socio Economic Panel (SOEP) by Kroh et al. (2016), the impression one easily gets is that concomitant with the increasing nonresponse rate there is likely to be increasing nonresponse bias, as well. According to Figure 1 the cumulative losses of attrition amount to more than 50 percent of the sample size of wave 1. These losses have to be added to the losses of the start of the SOEP which amount to 33 percent of the intended sample.

However, such a cumulation of selective effects need not always be the case. To be sure, if nonresponse depends on an unchanging characteristic such as gender, an initial nonresponse bias will not vanish in later panel waves. But, when surveys are launched to study the change in dynamic variables, such as income or poverty, the situation may be different, cf., Atkinson and Marlier (2010) for the European Union Statistics of Income and Living Conditions (EU-SILC). As discussed in Rendtel (2015), there can be considerable exchange between the states "poor" and "non-poor". Therefore, even if there is a substantial over-representation of poor people in the first wave of the panel, it will happen that some of the "poor" become "non-poor" and vice versa. It is not generally understood that this general turn-over has the potential to reduce the initial non-response bias. Rendtel (2013) coined the term "fade-away effect" for this phenomenon.

We present a Markov chain approach to explain the empirical phenomenon. There are two conditions: (A) the state transitions of the respondents and the nonrespondents between panel waves follow the same transition probabilities, and (B) an individual's probability of responding at later waves must not depend on the state he or she is in. The transition probabilities may change over time, and if the turn-over between the states is large enough then even moderate violations of assumption B do not invalidate the result. In the case of time-homogeneity a steady state distribution can be inferred from the transition matrix.

Alternatively, the result applies also to deterministic matrix models of population evolution. This gives us two complementary ways of viewing the time evolution of the panel waves.

The framework is applied to several panel surveys to study its relevance in applications. All panels are sampled from registers, so it is possible to link information on state transitions to data on survey participation. Details for the transitions in and out of the recipience of social benefit payments in the German Panel on Labour Market and Social Security (PASS) are given. We also display results for the European Community Household Panel (ECHP) and EU-SILC for transitions between quintiles of household equivalence income (cf., Rendtel 2015).

The article has five parts. The first part presents a general contraction theorem for the time inhomogeneous case. We then turn to the time-homogeneous case where a steady state distribution exists, and consider Markov chains of higher order that can account for dependencies between consecutive moves. The second part describes the PASS and the link with register information. The third part presents the empirical results for the PASS. In particular, we check the validity of the assumption of equal transition laws, and other modeling assumptions. In part four we present results for the ECHP and EU-SILC, and the effect of selective attrition. Part five points out extensions to the design and analysis of longitudinal surveys.

2 Markov chain model for state transitions

Consider a set of states $S = \{1, ..., I\}$. Let Y_t be the state of an individual at panel wave t = 0, 1, 2... Let R_t be the response indicator, or for $R_t = 1$ we observe the value of Y_t , for $R_t = 0$ we don't. The state transitions of the individual are assumed to be Markovian, or

$$P(Y_t = j | Y_{t-1} = i, Y_{t-2} = s_{t-2}, \dots, Y_0 = s_0) = P(Y_t = j | Y_{t-1} = i)$$

 $\equiv p_{i,j}(t)$

The $I \times I$ matrix of transition probabilities from time t-1 to time t is $P(t) = (p_{i,j}(t))$. In the case of panel surveys this refers to state transitions from one wave to the next. If the transition probabilities are time-homogeneous, we write $P(t) \equiv P = (p_{i,j})$, for all t. Transition probabilities from time 0 to time t are given by $P^{(t)} = P(1)P(2) \dots P(t)$. In the case of time-homogeneous chains we have $P^{(t)} = P^t$.

2.1 Contraction theorem

Consider two Markov chains with the same transition probabilities P(t) for all t. The first, FULL-sample, represents those who were initially selected to the panel, and the second, RESP-sample, consists of those responding at time t=0 at the start of the panel. The initial distributions of the chains are the (column) vectors $\pi_F(0)$ and $\pi_R(0)$, respectively. The subsequent state distributions satisfy the recursions $\pi_F(t) = P'(t)\pi_F(t-1)$ and $\pi_R(t) = P'(t)\pi_R(t-1)$ for $t=1,2,\ldots$

When all entries of $\pi_R(t)$ are strictly positive, we have the inequalities

$$m_t \equiv \min_i \frac{\pi_{F,i}(t)}{\pi_{R,i}(t)} \le \frac{\pi_{F,j}(t)}{\pi_{R,i}(t)} \le \max_i \frac{\pi_{F,i}(t)}{\pi_{R,i}(t)} \equiv M_t,$$
 (1)

for all $j = 1, \ldots, I$.

The following contraction theorem states that, under regularity conditions, the two distributions $\pi_F(t)$ and $\pi_R(t)$ converge.

Theorem 2.1. Suppose that there is lower bound $0 < p_L \le p_{i,j}(t)$ for all t. Then $\pi_F(t)$ and $\pi_R(t)$ converge uniformly in the sense that

$$\lim_{t \to \infty} (M_t - m_t) = 0. \tag{2}$$

This result is sometimes called *weak ergodicity*. A proof is given in the appendix.

There is an alternative formulation of the contraction theorem that was developed in mathematical demography for large populations (cf., Cohen 1979). Mathematically, the theorem is actually a result that holds for any sequence of non-negative matrices, say, A(t), and state vectors X(t) that evolve recursively as X(t) = A(t)X(t-1), t=1,2,... This matrix model has the property (2), if, say, X(0) > 0 and the elements $a_{ij}(t)$ of matrices A(t) are bounded from below and above, $0 < a \le a_{ij}(t) \le A < +\infty$. This approach involves no assumption of individual level stochasticity, and it provides an approximation to the process even in the presence of absorbing states, like deaths.

A consequence of the theorem is that nonresponse bias in the RESPsample may tend to disappear in later panel waves, and RESP and FULLsamples can become more alike.

However, the RESP-sample is further reduced by panel attrition. The observed sample at t will be denoted by OBS_t . Its state distribution is $\pi_O(t)$.

Condition (B) means that panel attrition must not be selective with respect to the state an individual is in. In terms of the response indicators this means:

$$P(R_t|Y_{t-1}, R_{t-1}) = P(R_t|R_{t-1}) = r_t$$
(3)

Under this assumption the state distribution is unaffected by the attrition, although case counts may go down.

2.2 The speed of the fade away effect

The proofs of the contraction theorem indicate that the convergence is geometric. The rate of convergence depends on the bounds that hold for the elements of the transition matrices. For time-homogeneous chains the convergence is also geometric, but the situation is much simpler in other ways, as a limiting distribution exists.

Consider an irreducible, aperiodic time-homogeneous chain with $I \times I$ transition matrix P. The largest eigenvalue of P is 1, it is simple, and the corresponding eigenvector π^* can be chosen strictly positive, $\pi^* = P'\pi^*$. These results can be proven directly (e.g., Cinlar 1975) or they follow from the so-called Perron-Frobenius theorems (e.g., Gantmacher 1959).

The transition matrix from time 0 to time t is $P^t = (p_{ij}^{(t)})$ in terms of the (i,j) element. The existence of the steady state distribution of P can be complemented by the following

Theorem 2.2. Suppose λ_2 is, in absolute value, the second largest eigenvalue of P. Then

$$|p_{ij}^{(t)} - \pi_j^*| = O(|\lambda_2|^t) \text{ for all } i, j \in S.$$
 (4)

For a proof, see Seneta (1980, Theorem 4.2).

If P is strictly positive, the speed of convergence is directly related to the minimum entry of P (cf., Behrends (2000, p.83 ff)). Thus, processes with low transition probabilities tend to need long time-intervals to reach the steady state.

In the application we will meet a situation where the distribution of the gross sample $\pi_F(0) = (\pi_{F,1}(0), \dots, \pi_{F,I}(0))'$ and the net sample of the first

wave $\pi_R(0) = (\pi_{R,1}(0), \dots, \pi_{R,I}(0))'$ may both be far away from the steady state distribution. Yet the differences $D_j(t) = \pi_{F,j}(t) - \pi_{R,j}(t)$ between the two distributions converge to 0 in a geometric fashion. This is due to the above theorem and the triangle inequality, whereby for all $j = 1, \dots, I$ we have:

$$|D_j(t+1)| \le |\sum_i \pi_{F,i}(p_{ij}^{(t)} - \pi_j^*)| + |\sum_i \pi_{R,i}(p_{ij}^{(t)} - \pi_j^*)| = O(|\lambda_2|^t) \quad (5)$$

2.3 Extensions to longitudinal profiles

The contraction theorem establishes the convergence of the cross-sectional distributions on the state space. In panel survey, however, there is more interest in longitudinal profiles over the state space. In order to apply the contraction theorem for longitudinal profiles one has to extent the state space to profiles. In the case of a profile over two waves the state space consists of pairs $(i, j) \in S \times S$ where the first entry is the observation at time t-1 and the second entry is the observation at time t. In this case, it is logically impossible to reach all states in one step. For example, we cannot reach the state (1,1) from (2,2) in one step. However, we may step from (2,2) to (2,1) and from (2,1) to (1,1), so a transition in two steps is feasible.

With this extended state space the contraction results can be still be applied. Note, that the requirement of equal transition probabilities covers now a period of three waves while for the cross-sectional case only two waves are covered. If the contraction theorem holds for the distribution on longitudinal profiles of length two, then it holds also for the conditional distribution on the state at time t, given the state at time (t-1). Such conditional probabilities are of interest in the analysis of individual stability.

3 PASS data base

3.1 Causes of nonresponse

The German Panel Study on Labor Market and Social Security started in 2006, with some 19,000 interviewed persons in more than 12,500 households (Promberger 2007 and Trappmann et al. 2010, 2013). The primary purpose was to create a longitudinal database for research on the so-called Hartz-Reforms of social security that came into effect in January 2005. The most important part of the reform was the introduction of a new means-tested

Unemployment Benefit II (UBII). PASS was set up as a household survey, since UBII is administered at the household level.

For our analysis we use the so-called recipient sample of the PASS. It was drawn in July 2006 from the register of welfare recipients of the Federal Employment Agency and contains households with at least one individual drawing UBII-benefits (Rudolph and Trappmann 2007). The survey population has many target persons with low education or with migration background, so a series of measures to reduce non-response was implemented, such as questionnaires in Turkish and Russian, refusal avoidance training for interviewers, monetary incentives for respondents, intense panel maintenance and tracking activities etc. (cf. Trappmann et al. 2011).

Surveys on low income populations have nonresponse problems (Verploeg et al. 2001 and Hernandez 1999). Compared to the U.S., Canada, and northern European countries, response rates for face-to-face surveys are lower in Germany. E.g., for the European Social Survey response rates for Germany were as low as 30% and never higher than 56% (http://ess.nsd.uib.no). Nevertheless, in the PASS initial nonresponse and attrition was high as compared to government surveys in other countries. Table 1 displays the development of the gross and net sample sizes of the recipient sample as well as the attrition rate for each panel wave. Note, that the gross sample in wave t + 1 can in some waves be larger than the net sample in wave t, because cases who participated in wave t - 1, but not in wave t, are approached in wave t + 1.

Table 1: Sample size of gross and net sample and attrition rate in the recipient sample of the PASS

110 1 1100			
Wave	gross sample	net sample	attrition rate
wave 1	23,773	6,798	71.40 %
wave 2	6,444	3,468	46.20 %
wave 3	5,737	3,665	36.12 %
wave 4	3,760	2,697	28.27~%
wave 5	3,199	$2,\!257$	29.44~%

3.2 Link with administrative data

For the recipient sample, we have sampling frame information and data from the Integrated Employment Biographies (http://fdz.iab.de/en.aspx; IEB Version 10.00). These files contain the exact dates of all episodes of UBII receipt that derive from the notification process of the social security system (Jacobebbinghaus and Seth 2007 and Biewen et al. 2007). Together with linked

Table 2: Comparison of transition probabilities for respondents and nonrespondents in the recipient sample of the PASS

_	1 1									
			Respondents							
	Start	Transit	$\sin 1/2$	Transit	tion $2/3$	Transit	$\sin 3/4$	Transition 4/5		
		UI	3II	U.	BII	U]	BII	UBII		
	UBII	yes	no	yes	no	yes	no	yes	no	
	yes	0.82	0.18	0.82	0.18	0.84	0.16	0.84	0.16	
	no	0.20	0.80	0.14	0.86	0.13	0.87	0.09	0.91	
					Nonresp	ondents				
	Start	Transit	ion 1/2	Transition 2/3		Transition 3/4		Transition 4/5		
		UI	3II	UBII		UBII		UBII		
	UBII	yes	no	yes	no	yes	no	yes	no	
	yes	0.82	0.18	0.83	0.17	0.85	0.15	0.83	0.17	
	no	0.19	0.81	0.15	0.85	0.14	0.86	0.10	0.90	

fieldwork protocols, we are able to determine the UBII and participation status for any sampled unit of the wave 1 recipient sample.

Wave 1 recipient sample has 23,773 cases. We focus on the initial sample of households, and ignore any subsequent split-offs caused by individuals who move out of a PASS households to found or enter a new or non-sampled household.

The UBII status for each unit was determined in each wave, as of May 1. This generally marked the mid-point of a fieldwork period that typically ran from February to September.

4 Empirical results for PASS

4.1 Assumptions A and B

Consider first assumption (A). Table 2 displays the estimated transition matrices for respondents and nonrespondents. The null-hypothesis of equal transition probabilities cannot be rejected for any wave. A likelihood ratio test on the equality of the transition matrices delivered LRT=0.493 (2 DF, p-value=0.782) for wave 1/2, LRT=0.326 (2 DF, p-value=0.849) for wave 2/3, LRT=0.052 (2 DF, p-value=0.974) for wave 3/4 and LRT=1.246 (2 DF, p-value=0.563) for wave 4/5. As all empirical transition matrices are strictly positive the prerequisites of the contraction theorem are fulfilled.

Consider assumption (B). Table 3 gives some indication that UBII recipients tend to have a lower attrition probability. However, the differences in participation rates are small and there is no stable trend over waves visi-

Table 3: Attrition rate in wave t conditional on UBII-participation in wave t-1

Transition	UBII	Attrition
wave	at wave $t-1$	Rate
1/2	yes	46.0%
	no	47.5%
2/3	yes	34.2%
	no	41.0%
3/4	yes	28.2%
	no	28.4%
4/5	yes	28.6%
	no	30.9%

ble. In particular, in transitions from wave 2 to wave 3, where the difference is most notable, about 40 percent of non-response was due to temporary nonrespondents of wave 2.

We will see below that despite the differences in response rates, the contraction property still holds reasonably well.

4.2 The fade away effect for the recipient sample

Table 4: Comparison of the percentage of UBII-persons in the FULL, RESP and OBS sample

sample	wave 1	wave 2	wave 3	wave 4	wave 5
FULL	79.0	68.9	61.6	57.6	52.3
RESP	81.3	70.6	61.9	57.2	51.9
OBS	81.3	73.2	63.6	60.2	54.8

Table 4 compares the percentage of UBII recipients for the FULL, RESP and OBS samples. Despite an initial nonresponse rate of 71.5 %, there is a bias of 2.3 percentage points only. Comparing the percentages in the line FULL with those of the line RESP we see that the over-representation of persons with UBII-payments has fallen from 2.3 percentage points at the start of the panel to 0.4 percentage points in wave 5, and the difference between the FULL with the OBS-samples remains stable at about 2.5 percentage points. In this case, the level of initial nonresponse bias is too low to demonstrate the potential of the contraction theorem.

To address this, we have selected an artificial starting distribution $\tilde{\pi}_R$ with a percentage of UBII recipients of 95 %. This results in a bias of 16

Table 5: Results of an artificial nonresponse bias experiment

	OBS Sample	FULL Sample	$Bias_t$	$Bias_t/Bias_{t-1}$
Wave 1	95.0	79.0	16.0	
Wave 2	79.0	68.8	10.2	0.64
Wave 3	68.9	61.8	7.1	0.70
Wave 4	62.0	57.9	4.1	0.58
Wave 5	56.0	52.8	3.2	0.79

percentage points. This starting distribution is multiplied by the empirical transition laws of Table 2, and the attrition rates of Table 3 are applied. The result of this scenario is displayed in Table 5. The contraction reduces the original bias of 16 percentage points by a factor 0.20 to 3.2 percentage points in wave 5. So despite small violations of assumption (B) the turn-over on the state space according to the Markov chain is strong enough to keep the contraction at a reasonable rate.

4.3 The speed of convergence and the steady state distribution

The transition probabilities of Table 2 of the recipient sample show a clear trend that the risk to fall back in the UBII state continuously decreases from 0.20 to 0.09. A formal likelihood ratio test for time-homogeneity for the FULL sample rejects the null-hypothesis of equal transition matrices over time with LRT = 285.7 (3 DF, p-value < 0.001). However, in order to get some ideas about the speed of the convergence process we computed the pooled transition matrix. This resulted in the values given in Table 6.

Table 6: Pooled transition matrices for respondents and nonrespondents

	Respondents		Nonrespondents		
Start	UBII		UBII		
UBII	yes	no	yes	no	
yes	0.83	0.17	0.84	0.16	
no	0.14	0.86	0.14	0.86	

Both matrices are virtually identical. They yield a second eigenvalue of $\lambda_2 = 0.69$. Note that this is about the same size as the average contraction factor in the last column of Table 5. So even under time-inhomogeneity and moderate violations of assumption (B) the second eigenvalue of the pooled transition matrix may be a good approximation for the speed of the fade

away effect.

4.4 Longitudinal profiles

If one is interested in histories of UBII, the Markov chain model can be used with an extended state space. In the case of profiles over two waves the state space of UBII status is given by the pairs (yes, yes), (yes, no), (no, yes), and (no, no). For example, a person is said to be in state (yes, yes) in waves 1 and 2, if the person receives the benefit at both times. The transition matrix 1/2 to 2/3 describes the transition from the profiles of wave 1/2 to the profiles of wave 2/3.

Table 7 shows the transition probabilities over the panel waves. As in the previous case the transition matrices for respondents and nonrespondents are virtually equal. For the sake of brevity the results are omitted here. Note, that the manifest time-inhomogeneity of the Markov first order transitions is greatly relaxed here. A formal likelihood ratio test rejects the hypothesis of equal transitions with LRT = 38.8 (18 DF, p-value = 0.003). However, there is no evidence of an apparent trend in the transition risks over time.

Therefore we may use the steady state distribution of the pooled transition matrix to measure the speed of convergence of the fade away effect. Note that P^2 is strictly positive as all states can be reached within two transitions. Therefore P is ergodic and a steady state distribution exists. The second eigenvalue is $\lambda_2 = 0.78$ and thus the speed of convergence is somewhat slower as in the case of the Markov first order case with $\lambda_2 = 0.69$. For three transitions we can expect therefore a bias reduction of $(\lambda_2)^3 = 0.47$.

Table 8 compares the distribution on the state space for the FULL and the RESP sample. There is an over-representation of the state (yes,yes) by 1.9 percentage points which reduces to 0.2 percentage points in wave 4/5. This is a decline by a factor 0.1! On the other side the state (no, no) is under-represented by 2.0 percentage points. Here the bias reduces to 0.6 percentage points with a decline factor of 0.3. Note that all distributions are still far away from the steady state distribution displayed in the last column of Table 8.

5 Empirical results from the ECHP and EU-SILC

An analysis similar to the one here given for PASS has been performed for the household equivalence income in the Finnish subsamples of the ECHP and EU-SILC (cf., Rendtel 2015). The income of the FULL sample was cut

Table 7: Transition matrices between UBII profiles

	Transition $1/2$ to $2/3$						
Start		UBII					
UBII	yes,yes	yes,no	no,yes	$_{ m no,no}$			
yes,yes	0.83	0.17	0.00	0.00			
yes,no	0.00	0.00	0.19	0.81			
no,yes	0.70	0.30	0.00	0.00			
no,no	0.00	0.00	0.11	0.89			
	Tra	ansition ?	2/3 to 3/	$^{\prime}4$			
Start		UB	$_{ m II}$				
UBII	yes,yes	yes,no	no,yes	no,no			
yes,yes	0.86	0.14	0.00	0.00			
yes,no	0.00	0.00	0.21	0.79			
no,yes	0.74	0.26	0.00	0.00			
no,no	0.00	0.00	0.10	0.90			
	Tra	ansition 3	3/4 to 4/	['] 5			
Start		UB	$_{ m II}$				
UBII	yes,yes	yes,no	no,yes	no,no			
yes,yes	0.85	0.15	0.00	0.00			
no,yes	0.00	0.00	0.20	0.80			
yes,no	0.69	0.31	0.00	0.00			
no,no	0.00	0.00	0.07	0.93			

Table 8: Display of the fade away effect for profiles over two waves. Comparison with the steady state distribution π^* of the transition matrix

	Wave 1/2		Wave		
UBII	FULL	RESP	FULL	RESP	π^*
yes,yes	64.9	66.8	48.1	47.9	31.0
yes,no	14.1	14.5	9.5	9.3	6.7
no,yes	4.1	3.8	4.2	4.0	6.7
no,no	16.9	14.9	38.2	38.8	55.6

Table 9: Comparison of the initial bias for income quintiles in the ECHP and SILC

	FULL	ECHP	SILC			
Quintile	Sample	RESP Sample	RESP Sample			
1	20.0	21.8	19.3			
2	20.0	20.7	20.1			
3	20.0	21.8	20.0			
4	20.0	20.1	20.5			
5	20.0	15.6	20.1			
Results from Junes (2012) and Rendtel (2015)						

into quintiles. Thus at the start of the panel the distribution of the FULL sample was 0.2 for each quintile. The upper and lower limits of the first-wave brackets were then inflated to avoid temporal trends in the distribution on the quintiles.

Table 9 compares the distribution on the income brackets for the FULL sample with the RESP sample. The starting year was 1996 for the ECHP and 2006 for EU-SILC. While we have a virulent under-representation of high incomes in the ECHP, there is virtually no bias in the SILC survey. The reason for this seems to be the organisation of the field work: while the ECHP questionnaire was run as a separate survey meaning some extra respondent burden, the SILC questionnaire was completely integrated into the general Finnish income survey.

Table 10 displays the transition matrix between income quintiles for EU-SILC for respondents and nonrespondents. For each of the two groups the transitions are pooled over the panel waves. A likelihood ratio test on differences of the transition matrices between the two groups resulted in 2*(-12189.03 + 12197.07)=16.06 with 5*4=20 degrees of freedom. This results in a p-value of 0.72. Hence the null-hypothesis of equal transition matrices cannot be rejected here. Similar results were obtained for the ECHP.

Table 11 displays the nonresponse rate of wave 1 (2005) of EU-SILC and the attrition rates in waves 2, 3 and 4 (2008). Here the nonresponse rate declines sharply after wave 1, which is typical for panel surveys. However, a look to the case numbers indicates a cumulation of losses which amount at wave 4 to 100-61=49 percent of the gross-sample size at the start of the panel.

Table 12 compares the distribution of the quintiles for the FULL, RESP and OBS samples for the ECHP (5 waves) and EU SILC (4 waves). While for the ECHP we see only minor discrepancies between the FULL and the OBS sample, the findings for SILC might indicate an attrition effect with an

Table 10: Transition rates in percent between income states. Upper panel: transitions for wave 1 respondents, lower panel: transitions for wave 1 non-respondents

	Respondents					
Quintile	1	2	3	4	5	
1	76.5	16.2	4.4	2.1	0.7	
2	15.7	57.6	19.1	5.7	1.8	
3	4.6	17.2	51.4	22.9	3.9	
4	3.0	5.9	16.1	58.9	16.1	
5	2.8	1.2	3.3	14.0	78.6	
		Non-	respon	dents		
Quintile	1	2	3	4	5	
1	73.9	17.9	5.0	2.1	1.0	
2	16.8	58.4	17.1	5.8	1.7	
3	4.2	16.7	55.9	18.5	4.6	
4	1.2	5.5	15.7	63.9	13.7	
5	3.7	2.0	3.9	10.1	79.4	

over-representation of the above median incomes and under-representation of low incomes (cf., Junes 2012).

A direct check of assumption (B) for EU-SILC is given in Table 13. In waves 2 and 3 persons with low income states have a significantly lower response probability than persons in the upmost income state. However, this tendency has completely disappeared until wave 4. Thus, while (B) is violated, the selective effect is not persistent.

As there is almost no initial nonresponse bias in the case of EU-SILC we did run some simulation experiments with different starting distributions for the RESP sample. We used six scenarios which are displayed in Table 14. For the transitions we used a pooled version of respondents and non-respondents.

Scenario 1 is the situation of the Finnish ECHP at the start with an under-representation of the persons in the upmost quintile. In Scenario 2 the situation is even more skew with an additional moderate over-representation of the lowest quintile. Scenario 3 is even more extreme with a substantial over-representation of the lowest quintile and a substantial under-representation of the upmost quintile. Scenario 4 is in the opposite direction. Here the poor people are under-represented while the rich persons are over-represented. In Scenario 5 we have an over-representation of the mid-quintile positions. And finally Scenario 6 displays a situation where the extreme categories are over-represented.

Table 11: Response and attrition rates in the Finnish subsample of EU-SILC

	Number of inter-	•	Response rate	Nonres- pondents	Attrition-
	viewees	dents	$(Basis\ 2005)$		rate
2005	2 353	1 769	75%	584	25%
2006	1 769	1634	69%	135	8 %
2007	1634	1522	65%	112	7%
2008	1 522	1448	61%	74	5%

Table 12: Comparison of the distribution on income states for the three samples FULL, RESP and OBS

TOLE, REST and OBS							
	ECHP			EU-SILC			
		Sample			Sample		
Quintile	FULL	RESP	OBS	FULL	RESP	OBS	
	14616	7809	5192	2353	1769	1448	
1	23.9	22.2	22.4	20.4	20.5	18.9	
2	16.9	16.6	17.4	19.8	19.3	18.7	
3	18.3	17.9	17.6	18.7	18.2	18.1	
4	20.6	21.4	21.8	21.1	21.7	22.2	
5	20.4	22.0	20.9	20.1	20.4	22.1	
Results fro	m Junes (2012) and	Rendtel	(2015)			

If we measure the initial nonresponse bias by the Euclidian distance to the distribution of the FULL-Sample, which is $(0\ 2,0\ 2,0\ 2,0\ 2,0\ 2)$, we get the values displayed in Table 15

We also want to check the impact of violations of assumption (B). For this purpose we combined the six nonresponse scenarios with six attrition scenarios which are displayed in Table 16. Each row represents a different attrition scenario. The first five columns display the response probabilities with respect to the previous income position. The last column under symbol $|\cdot|$ measures the maximum difference between the response probabilities, which is a measure of selective attrition.

Attrition scenario **A** reflects a linear trend in probability to respond. The maximum difference in the response rates is 5 percentage points which is regarded as a mild violation of assumption (B). Scenario **B** increases this difference to 11 percentage points and generates a clear differential attrition between low and high income people. Scenario **C** is even more dramatic in the same direction. Scenario **D** reverses Scenario **B**. Now the rich ones are

Table 13: Comparison of the impact of the income quintile position in previous panel wave on the response probability

Quintile in	Response probability				
previous wave	wave 2	wave 3	wave 4		
1	0.918	0.868	0.951		
2	0.901	0.916	0.947		
3	0.901	0.954	0.948		
4	0.934	0.953	0.956		
5	0.964	0.965	0.954		

Table 14: Starting distributions of the RESP-sample in 6 different simulation scenarios

		Scenario						
Quintile	1	2	3	4	5	6		
1					0.150			
2					0.225			
3	0.218	0.225	0.190	0.215	0.240	0.100		
4					0.225			
5	0.156	0.130	0.090	0.260	0.160	0.290		

Table 15: Initial nonresponse bias of the 6 simulation scenarios

Scenario	1	2	3	4	5	6
	0.0513	0.0828	0.1778	0.0995	0.0834	0.1794

not so willing to cooperate. A two sided approach is displayed in scenario \mathbf{E} . Here the extreme categories have a lower tendency to stay in the panel. Finally, scenario \mathbf{F} reflects a situation where the extreme categories are more cooperative than the middle income groups.

These response probabilities are applied for the three transitions to waves 2, 3 and 4. Note, that this creates a steady selective drift of attrition that has not been observed empirically. These attrition pattern are combined with the six initial bias scenarios. We compare at each wave t=1,2,3,4 the simulated distribution on the state space for the FULL-, the RESP- and the OBS-sample. Denote the Euclidian distance between the distributions in FULL and the RESP sample in wave t by B_t^{FR} . Similarly, B_t^{FO} denotes the distance between the FULL and the OBS sample in wave t. These distances are used here as measures for the absolute bias of the nonresponse.

In order to assess the attrition effect on the speed of the fade-away effect

Table 16: 6 attrition scenarios with differential response probabilities in each panel wave. $|\cdot| = \text{maximum difference}$ in response probability

					·	
	Rea	Response probability at quintile				
Attrition scenario	1	2	3	4	5	•
A	0.9120	0.9242	0.9364	0.9485	0.9607	0.05
В	0.8570	0.8935	0.9164	0.9475	0.9718	0.11
\mathbf{C}	0.8000	0.8532	0.9365	0.9607	0.9807	0.18
D	0.9720	0.9443	0.9274	0.9185	0.8790	0.10
${f E}$	0.9020	0.9242	0.9663	0.9424	0.9020	0.06
${f F}$	0.9720	0.9242	0.8564	0.9085	0.9420	0.12

we compute the relative bias, i.e. B_4^{FR}/B_1^{FR} or B_4^{FO}/B_1^{FO} . Table 17 compares the relative bias for the six attrition scenarios with the RESP-sample. Here small values indicate a high fade-away effect. If the relative bias is 15 % above the corresponding RESP-value we mark the combination of the start and the attrition scenario with dark grey. In case of a relative bias less than 15 % the corresponding RESP-value we use a light grey mark. Table 17 reveals 17 (out of 36) fields which are uncoloured. 12 dark grey fields have to be balanced against 7 light grey fields. For the attrition Scenarios A and E with a maximum differential of 6 percentage points the fade-away effect is almost preserved. In all cases there is a substantial reduction of the initial response bias. If we include scenarios A,B and D to F the maximum attrition differential is 12 percentage points. Yet 28 out of 30 scenarios display a reasonable fade-away effect.

Table 17: Relative bias for RESP und OBS after four waves

		Scenario attrition					
Scenario at start	\mathbf{A}	\mathbf{B}	\mathbf{C}	\mathbf{D}	${f E}$	${f F}$	\mathbf{RESP}
1	0.20	0.38	0.90	1.00	0.59	0.75	0.47
2	0.28	0.20	0.47	0.77	0.49	0.62	0.45
3	0.46	0.34	0.21	0.69	0.51	0.66	0.55
4	0.71	0.92	1.14	0.25	0.55	0.53	0.54
5	0.34	0.55	0.83	0.40	0.46	0.09	0.28
6	0.30	0.38	0.49	0.36	0.20	0.40	0.28

6 Discussion

It is clear that the fade away effect depends on a fast turnover of the variables of interest. For stable characteristics like gender, ethnicity, family status or the level of education one cannot expect a decline of an initial nonresponse bias. On the other hand for many socio-economic indicators like income position or receipt of social benefits the turnover can be strong enough to reduce a potential initial bias and also a mild attrition bias within a few panel waves.

An inspection of the proof of the contraction theorem reveals that similar arguments will hold also for positive densities (cf., Le Bras 1977, Cohen 1979). Thus, the fade away phenomenon exists also for infinite state spaces. Extensions to regression in a time-series setting can similarly be developed and will be reported elsewhere.

Our findings do not mean that one should merely trust in the fade away and do nothing about nonresponse. For variables that are stable over time, calibration information can be useful (e.g., Särndal 2007). Assumption (B) can be relaxed by using weights to compensate for attrition. For correcting the initial nonresponse, one can take information from the steady state distribution, or use a convex combination of a design-based estimator and the model-based steady state distribution (cf., Rao 2003).

Panel surveys are often augmented by refreshment samples, in part to counteract attrition effects and to stabilize case numbers. Our results show that this seemingly innocuous practice has the potential to incur fresh non-response biases! The same can happen, when refreshment samples are used to fight panel fatigue in rotation panels.

On a more positive note, our results suggest that when commercial internet panels recruit their initial sample by advertising, the self-selection effects that may initially be very large, can for some variables reduce over time, at least within strata defined by variables such as gender, age, or family status which can be controlled by stratification.

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Appendix

Proof of the Contraction Theorem

Let $\pi_{\mathbf{F}}(\mathbf{t})$ be the distribution on the state space at wave t for the first Markov chain which started as the FULL-sample. It's components are denoted by $\pi_{F,i}(t)$. Similarly $\pi_{\mathbf{R}}(\mathbf{t})$ denotes the distribution on the state space for the second Markov chain which started as the RESP-sample.

We assume that P(t) > 0, $\pi_{\mathbf{F}}(\mathbf{0}) > 0$, and $\pi_{\mathbf{R}}(\mathbf{0}) > 0$, so for all t the minima m_t and the maxima M_t of the ratios $\pi_{F,i}(t)/\pi_{R,i}(t)$ are well defined. Moreover, we assume that the elements of the transition matrices P(t) are bounded from below by $p_{ij}(t) > p_L > 0$, and as probabilities they satisfy $p_{ij}(t) \leq 1$. All these assumptions can be relaxed but those mathematical details are not central for our applications, and we omit them.

Step 1. The ratios of the vector elements contract over time, i.e. for all t we have:

$$m_t \le \pi_{F,i}(t+1)/\pi_{R,i}(t+1) \le M_t, \quad i = 1, \dots, I.$$
 (6)

Because of $\pi_{F,j}(t+1) = \sum_{i} p_{i,j}(t) \pi_{F,i}(t)$ we have

$$\frac{\pi_{F,j}(t+1)}{\pi_{R,j}(t+1)} = \sum_{i} \frac{p_{i,j}(t)\pi_{R,i}(t)}{\sum_{h} p_{h,j}(t)\pi_{R,h}(t)} \quad \frac{\pi_{F,i}(t)}{\pi_{R,i}(t)}$$

Thus, for all j, the ratios $\pi_{F,j}(t+1)/\pi_{R,j}(t+1)$ are convex combinations of the ratios $\pi_{F,i}(t)/\pi_{R,i}(t)$ with weights

$$w_{i,j}(t) = \frac{p_{i,j}(t)\pi_{R,i}(t)}{\sum_{h} p_{h,j}(t)\pi_{R,h}(t)}.$$

Therefore it follows that

$$\frac{\pi_{F,j}(t+1)}{\pi_{R,j}(t+1)} = \sum_{i} w_{i,j} \frac{\pi_{F,i}(t)}{\pi_{R,i}(t)}$$

$$\leq \sum_{i} w_{i,j} \max_{h} \frac{\pi_{F,h}(t)}{\pi_{R,h}(t)}$$

$$= \max_{h} \frac{\pi_{F,h}(t)}{\pi_{R,h}(t)} \sum_{i} w_{i,j}$$

$$= M_{t}$$

A similar argument holds for the minima. This proves Equation 6. As a consequence we have $m_t \uparrow$ and $M_t \downarrow$, and the task is to show that the limits are the same.

Step 2. As an intermediate step, we show that the weights are bounded from below,

$$w_{ij}(t) \ge (p_L)^2 / I \quad i, j = 1, \dots, I.$$
 (7)

We first show that the numerator of the weights is bounded from below:

$$p_{ij}(t)\pi_{R,j}(t) \geq p_L \pi_{R,j}(t)$$

$$= p_L \sum_h p_{hj}(t-1)\pi_{R,h}(t-1)$$

$$\geq (p_L)^2 \sum_h \pi_{R,h}(t-1)$$

$$= (p_L)^2$$

The summands of the denominator can be bounded from above by 1. Hence the sum is smaller than $I \times 1$. From both inequalities equation 7 is proven.

Step 3. To bound the difference $M_t - m_t$, define first adjusted weights $w_{ij}^*(t) = w_{ij}(t) - (p_L)^2/I \ge 0$, according to step 2. Their sum over i is less than 1. We define adjusted ratios for time t+1 which use these adjusted weights. The minima and maxima with respect of the weighted ratios are defined as:

$$m_{t+1}^* = \min_{j} \{ \sum_{i=1}^{I} \frac{\pi_{F,i}(t)}{\pi_{R,i}(t)} w_{ij}^*(t) \}$$

and

$$M_{t+1}^* = \max_{j} \{ \sum_{i=1}^{I} \frac{\pi_{F,i}(t)}{\pi_{R,i}(t)} w_{ij}^*(t) \}.$$

Nevertheless, $M_t - m_t = M_t^* - m_t^*$ as the same constant is subtracted from both M_t and m_t . Now we obtain,

$$m_{t+1}^{*} = \min_{j} \left\{ \sum_{i=1}^{I} \frac{\pi_{F,i}(t)}{\pi_{R,i}(t)} w_{ij}^{*}(t) \right\}$$

$$= \min_{j} \left\{ \sum_{i=1}^{I} \frac{\pi_{F,i}(t)}{\pi_{R,i}(t)} (w_{ij}(t) - (p_{L})^{2}/I) \right\}$$

$$\geq \sum_{i=1}^{I} \min_{j} \left\{ \frac{\pi_{F,j}(t)}{\pi_{R,j}(t)} \right\} (w_{ij}(t) - (p_{L})^{2}/I)$$

$$= m_{t} (1 - (p_{L})^{2})$$

Similarly it is shown that

$$M_{t+1}^* \leq M_t(1 - (p_L)^2)$$

Therefore we obtain the inequalities

$$0 < M_{t+1} - m_{t+1} \le (M_t - m_t)(1 - (p_l)^2)$$

$$\vdots$$

$$\le (M_0 - m_0)(1 - (p_l)^2)^t$$

As $(1-(p_l)^2)^t$ converges to 0 as $t\to +\infty$ we have completed the proof.

More generally a contraction theorem may be proven, if the matrices P(t) have strictly positive elements in the same positions, there is some $t_0 \geq 1$ such that the product $P(1) \cdots P(t_0)$ is strictly positive, and the positive elements of matrices P(t) are uniformly bounded away from zero.

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