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Proxy Wars

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Abstract: Proxy wars are a key pattern of political conflict and interstate competition. Rather than resorting to direct conflicts, which are costly and entail a higher level of uncertainty, governments may opt for proxy wars, which may last longer, but are less costly and render them more immune to exogenous shocks. We start with the modeling of a direct war with two players where a static equilibrium may be neither realizable nor sustainable in the long run. Then, we offer a model of proxy war where the proposed equilibria are realizable, but not always sustainable in the long run. The consolidation level of the double principal-agent relationship predicts the continuation of conflict and thus the emergence of peace.

Keywords: non-cooperative games, principal-agent models, proxy conflicts

JEL Codes: C72, D72, D74, P51

I. Introduction

Proxy wars have been a standard pattern of interstate competition and grand strategies of Great Powers. From the Vietnam War to the Afghanistan involvement, and from the Ukrainian conflict to the Syrian civil war, proxy wars have replaced direct military confrontations and at the same time have facilitated the emergence of protracted and oftentimes "frozen" conflicts. Powell (2013) argues that in a civil war context, the government's decision to monopolize violence and consolidate power depends on the rebel opposition's ability to resist the government's "coercive power", which reduces its payoff to fighting. Furthermore, he indicates that the size of "contingent spoils" is decisive for the method that the government chooses to consolidate power (Powell, 2013). When the spoils are large, then consolidation of power occurs through monopolization of violence; when the spoils are small, then the government buys off the rebels peacefully (ibid.). In another paper, Powell (2012) suggests that the persistence of fighting depends on the speed of power distribution. If the distribution of power occurs slowly or is stable, then fighting does not persist; however, if the distribution of power shifts rapidly, then fighting becomes more attractive (Powell, 2012). It is obvious that proxy wars can decelerate the shift in power distribution even more and in that way prevent the principals from starting a direct war with each other. Proxy wars reduce the total cost of conflict and, therefore, can perpetuate its existence in the long run. At the same time, agents

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deviating from their equilibrium level of conflict involvement may lead one of the two principals to defeat and transition to a form of direct military confrontation.

In this paper, we find that in direct wars (what we call a basic model of conflict – BMC) the realizability of equilibrium depends on whether both players have a starting level of conflict involvement that is strictly positive. Only under this condition is it possible to observe the emergence of an equilibrium. In the case that one player selects a negative starting level, an equilibrium is not realizable and therefore not observable. Furthermore, it needs to be pointed out that even the realizable equilibrium may not be sustainable in the long run if the distance between the conflict involvement levels of the two players becomes extremely high such that the player with the lower level of conflict involvement decides to exit the game.

In our proposed model of proxy conflict (MPC), we argue that proxy conflicts allow the prevalence of negative differentials between the conflict involvement levels of principals and their respective agents. This either leads to the defeat of one principal against the other and therefore the imposition of a disproportional peace in favor of the other principal or it will facilitate a transition to a form of direct confrontation, both depending on the starting point of each principal's conflict involvement level. Furthermore, when both principals and agents deliver positive levels of conflict involvement, the principals are likely to perpetuate the conflict; this equilibrium is both realizable and sustainable in the long run.

Dunning (2011) proposes that armed conflict and electoral politics can be both strategic complements and strategic substitutes. Here, too, the distribution of popular support not only predicts the occurrence and type of armed conflict, but it may also create incentives of investment into institutional mechanisms, which may resolve commitment problems and facilitate peace (Dunning, 2011). The study of coups (Powell and Thyne, 2011) may explain why proxy conflicts are more efficient than direct confrontations: the ability of a principal to install his own agent into power reduces his own level of conflict involvement and brings about similar results in terms of power distribution.

A characteristic feature of the second half of the twentieth century was the replacement of the old traditional forms of conflict resolution with new ones. By the end of the century, conflicts in the form of proxy wars had become predominant, and the term itself had become firmly entrenched in political discourse. At present, a proxy war is defined as a war that is carried out by someone else (through representatives) or a mediated war. It is very important here to distinguish between proxy wars as wars by representatives and wars run by coalitions that are composed by partners of different weight and influence. The history of the Thirty Years' War (1618-1648) can be modeled as a proxy confrontation between France and the Habsburgs, where Denmark and Sweden were agents of France. Similarly, in the Hundred

Years' War in Bretagne, feudal parties were oriented toward France (Maison de Blois) and England (Maison de Montfort).

In our definition of proxy wars, we do not include conflicts in which small states have received significant external support from Great Powers or simply stronger states. Nevertheless, in the broad yet undefined category of proxy wars, a rising number of scholars include historical incidents such as the Spanish Civil War, the Korean War, the Vietnam War, the Afghanistan War, the Civil War in Angola, and the events in Cambodia in the late 1970s. This also holds for modern conflicts such as the Syrian civil war and the Russian-Ukrainian confrontation in Donbass. A strong argument in favor of increasing the relevance of proxy conflicts is the rapid development of this phenomenon due to private military companies. The emergence of cyber wars and hacker attacks has led to these becoming relatively common in recent years and this is also related to proxy conflicts. However, the insufficient formal identification of an agent as such in cyberspace does not allow for direct inferences on that matter.

Mumford (2013) provides interesting insights on conflict identification as proxy war. He analyzes a series of indicators according to which the conflict can be considered a proxy war, and in the absence of which a contrary conclusion can be made. He does not define conflicts where one of the principals interferes indirectly in the conflict (ibid.) as proxy wars. This is why, according to Mumford (2013), the Spanish Civil War and the War in Afghanistan cannot be classified as proxy wars. Eland (2002) provides a brief overview of US policy in Somalia through the lens of proxy wars. The advantages of proxy wars for the US are linked to efficiency gains because of an effective cooptation of local agents, which make up for the absence of a thorough understanding of local realities by the US themselves (ibid.). Sanders (2016) suggests that in norm proxy wars, norm challengers finance networks or localities of agents who contest human rights norms, usually in a transnational context. Biddle (2017) indicates that security force assistance, which is oftentimes provided by US administrations to allies that have weak state capacity, involves a principal-agent game where the higher the aid commitment on the part of the principal, the higher the degree of moral hazard that ensues. The launching of proxy wars requires the emergence of proxy dependencies between principals and agents. When one looks at executive politics and the US presidency, it becomes clear that experience in leadership cannot be substituted by the experience of presidential advisers when it comes to bias aggregation (Saunders, 2017).

The paper has the following structure. In section II, we propose and solve a basic model of conflict (BMC) and a model of proxy conflict (MPC). Section III offers some baseline simulations of our theoretical results. Section IV offers a concise discussion of the findings and concludes.

II. The Model

Preliminary observations & concepts

Proxy war is a complex phenomenon. This determines the variability and ambiguity of the approaches and concepts of mathematical modeling that can be proposed for its description. At first, proxy wars can be modeled as preliminary training contests within which the main aim for the conflict parties is the exchange of information on levels of readiness for confrontation. In this setup, players exchange signals as to how far they are willing to go if the conflict follows the proxy path. A key concept for implementing this approach is dynamic Bayesian games and, in particular, signaling games. The main drawback of this approach is scalability. Indeed, it is very difficult to obtain convincing guarantees regarding the truthfulness of the signals sent. Quite reasonable is the question of what will make participants replicate their strategy in a full-scale conflict. Conversely, the bluff strategy is meaningful for the weaker side that intends to deter the opponent in the preliminary game with the threat of resistance and thus prevent his defeat in a full-scale conflict.

Another concept treats proxy war as confrontation with a binary result. Here, victory becomes a priority for participants. With such an interpretation, the number of victories becomes the crucial criterion. The main idea proposed is to further models that offer and develop mathematical constructions explaining the objective necessity of a transition from direct confrontation to confrontation with the involvement of additional participants (agents). As is well known, in most of the games and competitive situations encountered in practice, the main element of participants' skills is their ability to find a strategy that opponents do not expect. The study of such "outcomes" obviously lies outside game theory *per se*, as it is meaningfully determined by the specifics of concrete games. The point is that, with a meaningful analysis of a particular game and the equilibria existing in it and on the basis of comparing them with the real properties of the modeled situation, we can evaluate the probability of identifying these equilibrium situations in reality.

The improbability of such outcomes can be interpreted as an argument in favor of the hypothesis of "incompleteness" of the game description. It is reasonable to assume that players resort to some other actions that we do not consider in the initial model. We emphasize that we do not have the opportunity to "guess" their specific forms. However, at least there is a signal that such actions should be expected. The following models and methods of research on proxy wars to a large extent lie within the framework of this approach. In the case of political and economic confrontation between individual countries (or country blocs), it is very difficult to formulate an exhaustive list of possible actions. Also, it is not realistic to imagine the possibility of drawing up all admissible configurations of involved allies and agents in the conflict. However, an objective justification of the appropriateness of including additional participants in the conflict itself is an important and constructive result.

Basic Model: War Without Proxies

We consider the following simplified model of bilateral conflict (from now on the BMC – *Basic Model of Conflict)*. There are two players with the following utilities:

$$u_1(x_1, x_2) = \frac{x_1}{x_2 + h} - a_1\left(x_1 + x_2\right)^{1+\alpha} \tag{1}$$

for player 1,

$$u_2(x_1, x_2) = \frac{x_2}{x_1 + h} - a_2\left(x_1 + x_2\right)^{1+\alpha} \tag{2}$$

for player 2,

where $x_1$and x_2 measure the respective levels ("depth") of conflict involvement. Moreover, $\frac{x_i}{x_j + h}$ is the result of the actions of player i against player j, and parameter h denotes the natural level of resistance to aggression – the conflict involvement of player i cannot be infinitely large in the absolute absence of opposition from player j. The parameter a_i shows the comparability of measures of victory and defeat, and the coefficient α captures the non-linear effect of damage on the scale expansion of conflict (in the basic setup we assume that this parameter is the same for both players). It is also important to point out that component $a_i\left(x_i + x_j\right)^{1+\alpha}$ indicates the damage from conflict as the result of joint actions of participants.

The logic of the utility functions of both players allows the possibility that the quantities can take both negative and positive values. Positive x_i is interpreted as the level of aggression (pressure on the opponent) in the conflict. Negative x_i can be interpreted as a measure of concessions to the enemy. If the logic of $\frac{x_i}{x_j + h}$ and $a_i\left(x_i + x_j\right)^{1+\alpha}$ for $x_i, x_j \geq 0$ is completely transparent, then for the case of negative arguments some additional explanations are required. Meaningful interpretations of the damage component, of course, assume the fulfillment of the condition $x_1 + x_2 \geq 0$. If both sides make concessions, then conflict does not arise and, therefore, there is no damage from it. Strictly speaking, it would be more correct to define utility functions as follows:

$$u_i(x_i, x_j) = \begin{cases} \frac{x_i}{x_j + h} - a_i \cdot (x_i + x_j)^{1+\alpha}, & x_1 + x_2 \geq 0, \\ 0, & x_1 + x_2 < 0. \end{cases} \quad (3)$$

Furthermore, the search for the conditional extremum of the function $u_i(x_i, x_j)$ on some admissible area $X_i \times X_j$ leads to rather cumbersome mathematical derivations. Hence, we confine ourselves to the problems of conditional optimization, $u_i(x_i, x_j)$. In this case, solutions that go beyond the framework of the model ($x_i \leq -h$), i.e. the level of concessions to the opponent beyond the *natural* resistance level h, can be interpreted as a conflict exit for this participant. An unambiguous interpretation of such an outcome as a defeat could not be entirely justified. It can mean that this player is inclined to radically change the conditions of conflict and move to a game with different rules and strategies. Figure 1 shows the effect of parameter α on the functional dependency determining the utility of any player *i*.

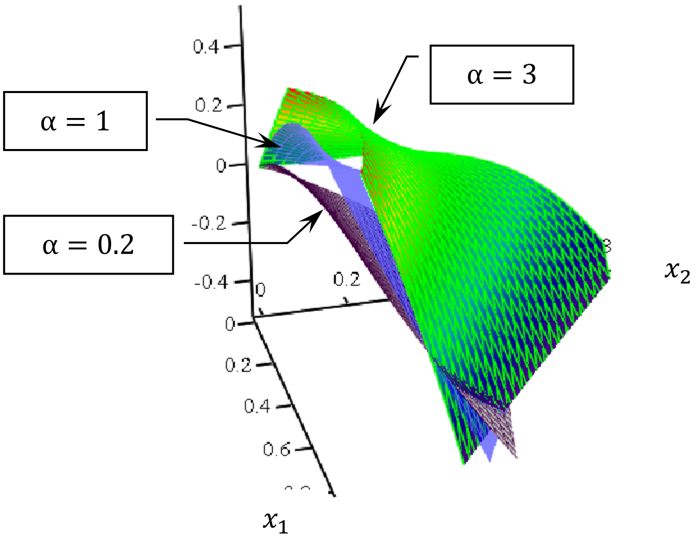


Figure 1: View of surface for function $u_i(x_1, x_2)$ for different α *values* ($a_1 = 1, a_2 = 2, h = 0.7$)

Figure 2 shows the possible mutual disposition of the surfaces of players' utilities (with different coefficients of damage from conflict).

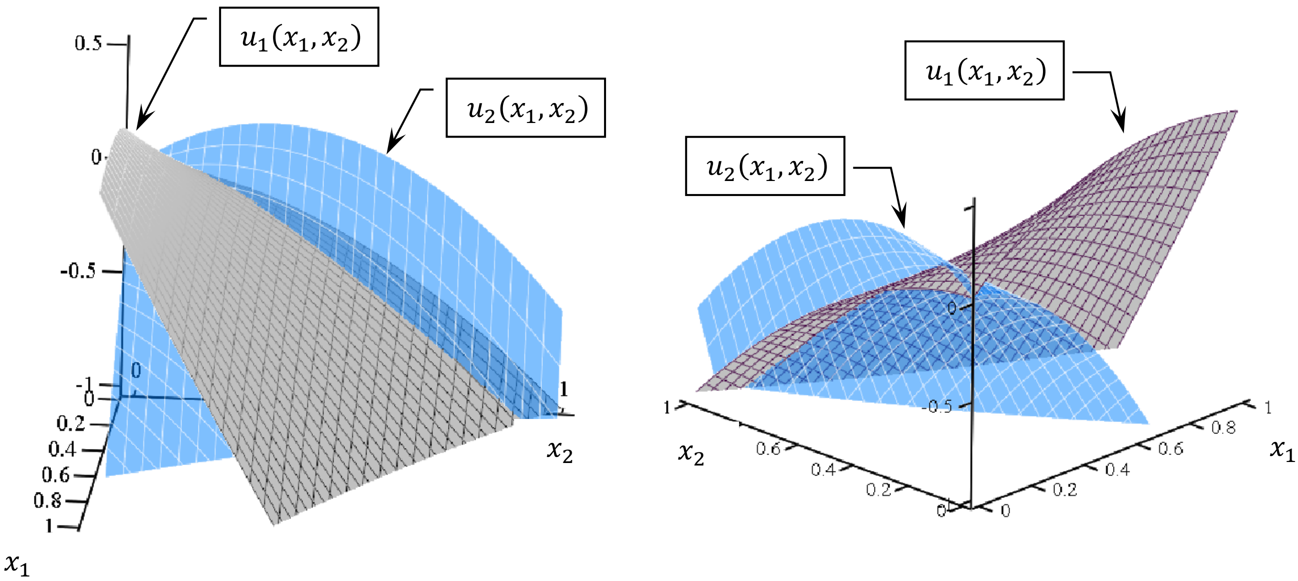


Figure 2: The surfaces for functions $u_1(x_1, x_1)$, $u_2(x_1, x_1)$ $(a_1 = 1, a_2 = 2, h = 0.7, \alpha = 0.8)$*, views from different angles*

We write down the necessary conditions of the stationary point (first-order condition) for the utility $u_i(x_i, x_j)$ (player i against player j) as follows:

$$\frac{\partial u_i(x_i, x_j)}{\partial x_i} = \frac{1}{x_j + h} - a_i \cdot (1 + \alpha) \cdot (x_i + x_j)^{\alpha} = 0 \tag{4}$$

or

$$(1 + \alpha) \cdot (x_i + x_j)^{\alpha} = \frac{1}{a_i \cdot (x_j + h)} = \frac{1}{a_j \cdot (x_i + h)}. \tag{5}$$

As a result, we derive the expression for the best response of player i to a decision of player j:

$$x_i = \left(\frac{1}{a_i \cdot (1 + \alpha) \cdot (x_j + h)}\right)^{\frac{1}{\alpha}} - x_j. \tag{6}$$

It should be emphasized that the transition from (5) to (6) is mathematically correct if and only if the condition $x_1 + x_1 \geq 0$ holds. From the condition $\frac{1}{a_2 \cdot (x_1 + h)} = \frac{1}{a_1 \cdot (x_2 + h)}$ or ($a_2 \cdot (x_1 + h) = a_1 \cdot (x_2 + h)$ in (5) we derive the expression which sets the relationship between the best responses of both players:

$$x_1 = \frac{a_1}{a_2} \cdot x_2 + \frac{a_1 - a_2}{a_2} \cdot h\,,\ x_2 = \frac{a_2}{a_1} \cdot x_1 + \frac{a_2 - a_1}{a_1} \cdot h\,. \tag{7}$$

After the substitution in expression (7) for best response we get:

$$\frac{a_1 + a_2}{a_2} \cdot x_2^* + \frac{a_1 - a_2}{a_2} \cdot h = \left(\frac{1}{a_1 \cdot (1 + \alpha) \cdot (x_2^* + h)}\right)^{\frac{1}{\alpha}} \tag{8}$$

(equation for x_2^*)

$$\frac{a_1 + a_2}{a_1} \cdot x_1^* + \frac{a_2 - a_1}{a_1} \cdot h = \left(\frac{1}{a_2 \cdot (1 + \alpha) \cdot (x_1^* + h)}\right)^{\frac{1}{\alpha}} \tag{9}$$

(equation for x_1^*). The solutions of the equations provide us with the components of the Nash equilibrium point for the BMC.

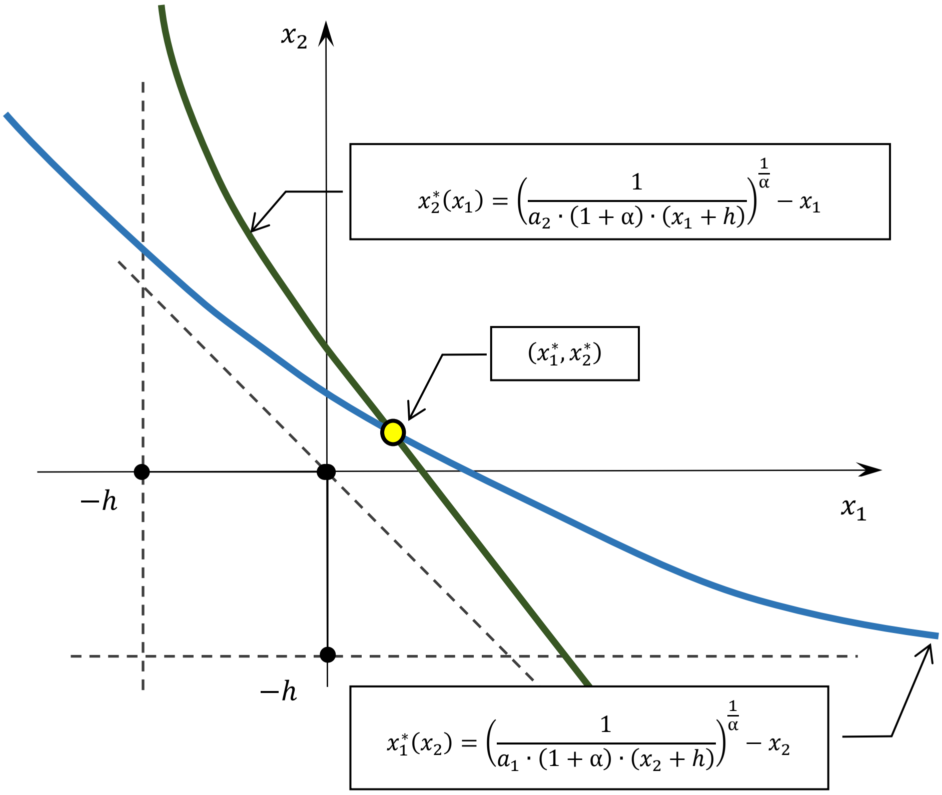


Figure 3: Scenario 1 – Contentious Politics in the BMC, $x_1^*(x_2) > 0, x_2^*(x_1) > 0$

Figures 3 and 4 present two possible scenarios related to the configurations of the best response curves. In the first case, at the Nash equilibrium point, a realistic confrontation in which players are inclined to adhere to certain positive levels of mutual pressure is observed. In the second case, the best response of player 1 turns out to be a negative value. To interpret such an observation as a plausible prediction of a real conflict is rather difficult. From this point of view, the equilibrium represented in Figure 4 can be characterized as unrealizable (in contrast to the realized equilibrium in Figure 3).

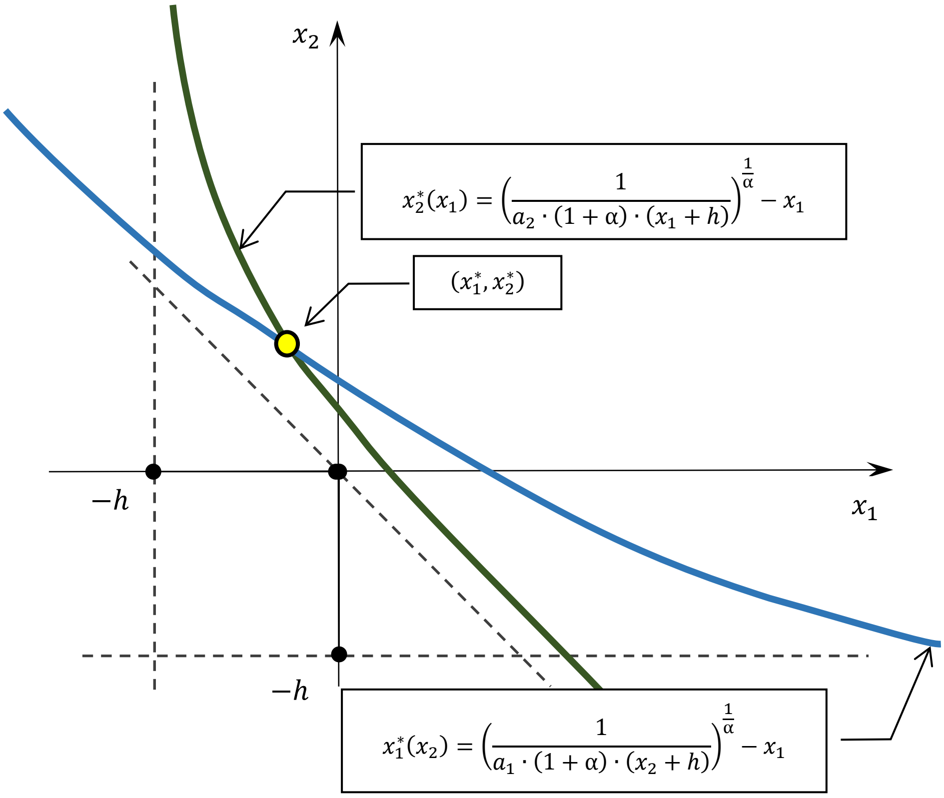


Figure 4: Scenario 2 – Conflict transition in the BMC, $x_1^*(x_2) < 0, x_2^*(x_1) > 0$

For definiteness, we assume that $a_1 < a_2$, i.e. the consequences (damage) from the conflict at the confrontation level $x_1 + x_2$ are more significant for player 2. This assumption, by way of the symmetry of the utility functions of participants, does not in any way reduce the level of generality of the subsequent conclusions. After transforming expression (9), which determines the value x_1^*, we obtain:

$$x_1^* + \frac{\alpha_2 - \alpha_1}{\alpha_1 + \alpha_2} h = \frac{a_1}{(\alpha_1 + \alpha_2)(1+\alpha)^{\frac{1}{\alpha}} \alpha_2^{\frac{1}{\alpha}}} \frac{1}{(x_1^* + h)^{\frac{1}{\alpha}}} . \qquad (10)$$

Graphically, the solution of equation (10) can be represented as a result of the intersection of the line $x_1^* + \frac{\alpha_2 - \alpha_1}{\alpha_1 + \alpha_2} h$ (left-hand side) and hyperbolas $\frac{a_1}{(\alpha_1 + \alpha_2)(1+\alpha)^{\frac{1}{\alpha}} \alpha_2^{\frac{1}{\alpha}}} \frac{1}{(x_1^* + h)^{\frac{1}{\alpha}}}$ (right-hand side). These graphs are shown in figure 5. As can be seen from figure 5, equation (10) has a non-negative root if the point of intersection of the line specified by the left side of the equation with the ordinate axis lies below the intersection point of the hyperbola and the ordinate axis. Otherwise, equation (10) will have a negative root. As a result, from a comparison $\frac{\alpha_2 - \alpha_1}{\alpha_1 + \alpha_2} h \leftrightarrow \frac{a_1}{(\alpha_1 + \alpha_2)(1+\alpha)^{\frac{1}{\alpha}} \alpha_2^{\frac{1}{\alpha}}}$ we obtain the following conditions:

$$x_1^* < 0 \quad \text{if} \quad h^{\left(1+\frac{1}{\alpha}\right)} > \frac{a_1}{(a_1+a_2)\cdot(1+\alpha)^{\frac{1}{\alpha}} \cdot a_2^{\frac{1}{\alpha}}} , \qquad (11)$$

$$x_1^* > 0 \quad \text{if} \quad h^{\left(1+\frac{1}{\alpha}\right)} < \frac{a_1}{(a_1+a_2)\cdot(1+\alpha)^{\frac{1}{\alpha}}\cdot a_2^{\frac{1}{\alpha}}}. \tag{12}$$

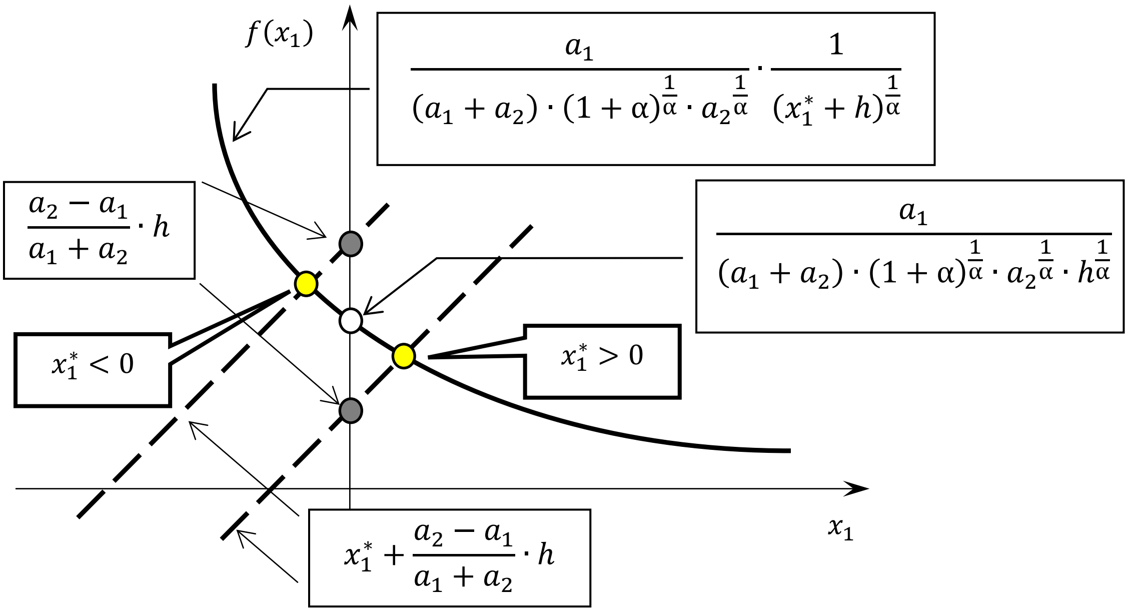


Figure 5: Graphical solution of the equation determining x_1^* as a component of the equilibrium point

We emphasize that for the correctness of conditions (11)-(12), the assumption $a_2 - a_1 > 0$ ($a_2 > a_1$) is fundamental. If the condition of nonnegativity is controlled for the second component of the equilibrium point (x_2^*), the exercise is reduced to a comparison:

$$\frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} h \leftrightarrow \frac{a_2}{(\alpha_1+\alpha_2)(1+\alpha)^{\frac{1}{\alpha}}\alpha_1^{\frac{1}{\alpha}}}.$$

It is obvious that the left-hand side is always negative, and the right-hand side is always positive, which means that always x_2^* holds for $a_2 > a_1$. As an example, the calculation results for specific values of the parameters a_i, h and α are placed in Table 1.

Table 1. The equilibrium points and values of the players' utilities in the BMC for different values of the parameters h, α.

h	0.5			0.7		
α	1	1.2	1.4	0.5	0.8	1
a_1	1	1	1	1	1	1
a_2	2	2	2	2	2	2
x_1^*	-0.000	0.005	0.009	-0.122	-0.105	-0.095
x_2^*	0.500	0.510	0.519	0.455	0.490	0.509
$u_1(x_1^*, x_2^*)$	-0.250	-0.227	-0.207	-0.298	-0.268	-0.250
$u_2(x_1^*, x_2^*)$	0.500	0.546	0.586	0.404	0.464	0.500

Table 1 shows that an increase in the parameters h and α leads to an increase in conflict participation levels in equilibrium. This result is not surprising, as both resistance to aggression and the non-linear magnitude of damage have a monotonically direct relationship with conflict expansion. Furthermore, it may also be the case that $a_1 = a_2 = a$. Hence, expressions (8)–(9) can be simplified as follows:

$$x_i^*(x_i^* + h)^{\frac{1}{\alpha}} = \frac{1}{2\left((1+\alpha)a\right)^{\frac{1}{\alpha}}} \tag{13}$$

The value of expression (13) for small h can be approximately estimated as $x_i^* = x_i^* \approx \left(2^{\alpha} \cdot \left((1+\alpha) \cdot a\right)\right)^{-\frac{1}{1+\alpha}}$. For example, when $a \approx 1$, $\alpha \approx 1$, then $x_i^* = x_i^* \approx 0.5$. The problem of stability of equilibrium in the BMC given by (9)-(10) requires special attention.

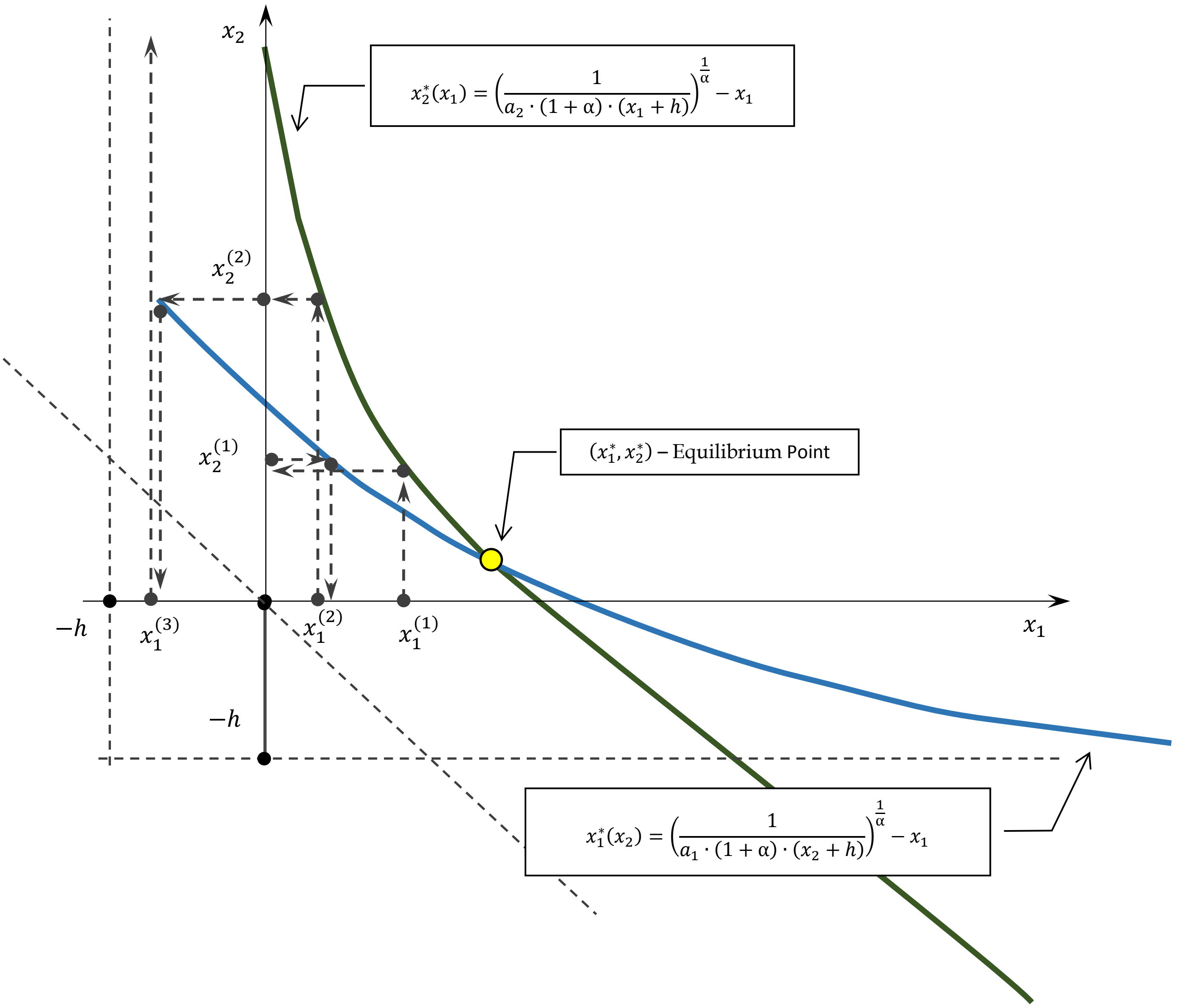


Figure 6: A sequence of reciprocal best responses, starting point $x_1^{(1)} < x_1^*$

In Figure 6, there is the process of mutual reaction of conflict participants (the path of successive responses). This trajectory begins with the choice of participation level for the first player $x_1^{(1)}$. For this action, the second player chooses $x_2^{(1)} = x_2\left(x_1^{(1)}\right)$ in accordance with his best-response function. This is followed by the reaction of the first player $x_1^{(2)} = x_1\left(x_2^{(1)}\right)$. However, the sequence of reciprocal responses does not converge to equilibrium (x_1^*, x_2^*). Thus, we may conclude its *instability*. The sequence of best responses in Figure 6 is characterized by an aggression containment by player 1:

$$x_1^{(1)} > x_1^{(2)} > 0 > x_1^{(3)}$$

and conversely, by an increase in the level of conflict involvement (and the cost of maintaining it) by player 2. According to Figure 6, the best response of player 2 to player 1's concessions

$\left(x_1^{(3)} < 0\right)$ must be at some "beyond" level, and then the "exit" of this player from the conflict follows. Another important property of the BMC is the dependence of the outcome on the choice of the "starting point" and its location relative to Nash equilibrium point. In the case just analyzed, it was $x_1^{(1)} < x_1^*$.

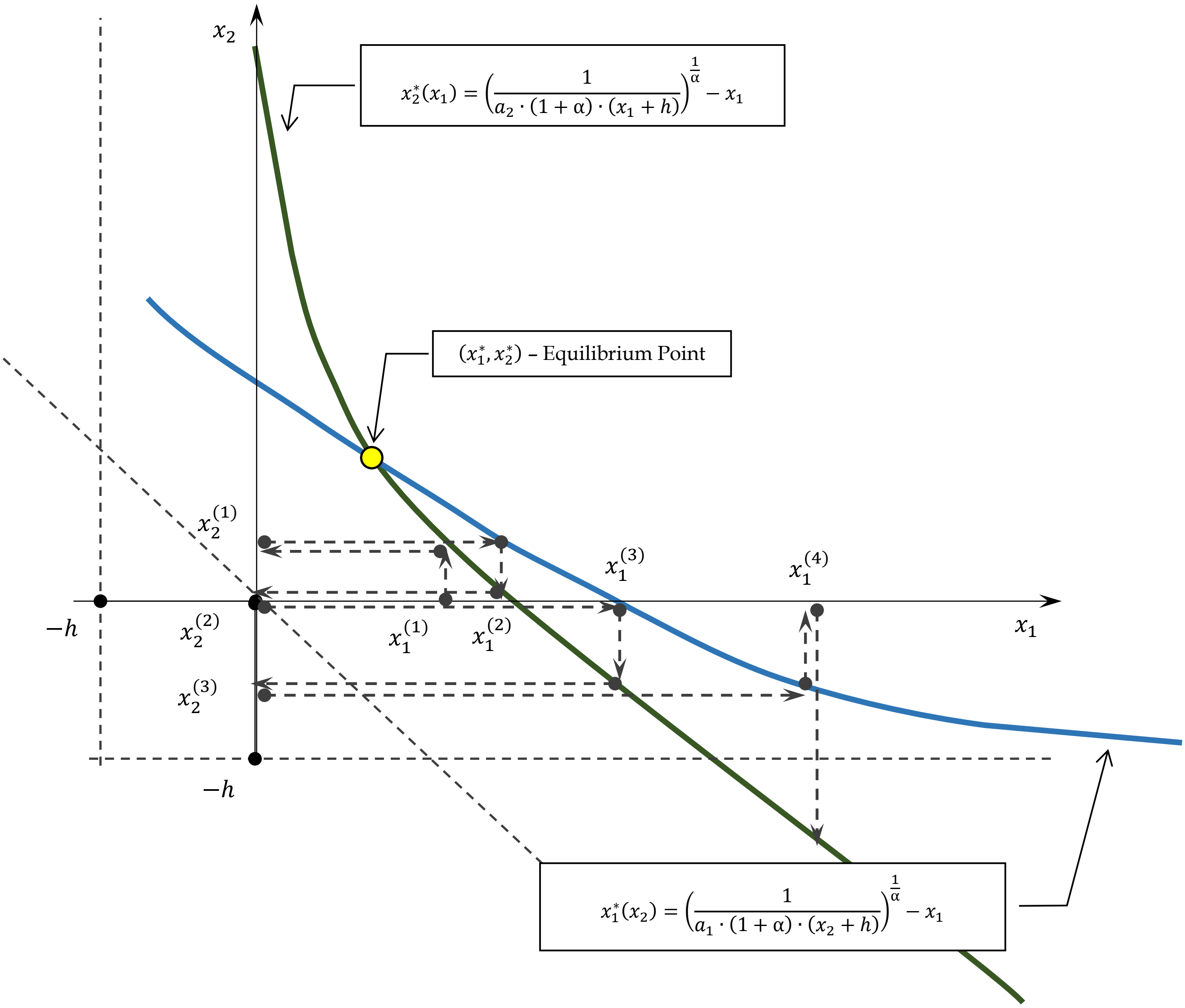


Figure 7: A sequence of reciprocal best responses, starting point $x_1^{(1)} > x_1^*$

In Figure 7, there is a sequence of mutual best answers starting with $x_1^{(1)} > x_1^*$. In this case, the opposite is observed: there is a buildup of pressure by player 1 and a decrease in resistance by player 2. Consequently, player 2 is eliminated from the game. Similar reasoning can be introduced for variants of best response trajectories that start after the second player's action. Even here identical conclusions could be obtained. From a mathematical standpoint, the unsustainability of Nash equilibrium and the difference among the variants of best response trajectories are determined by the ratio of the slopes of the best-response curves $\partial x_2(x_1)/x_1$ and $\partial x_1(x_2)/x_2$. However, the instability of equilibrium in the BMC should not be negatively interpreted. "Setting" the players' actions at the Nash equilibrium point means fixing the conflict for an extended period (in the context of comparative statics) while maintaining confrontation at equilibrium levels.

One of the most important characteristics of the BMC is an indicator reflecting the relative, "strength" of the player's response to his opponent's actions in the previous period $t-1$:

$$\frac{x_i^t(x_j^{t-1})}{x_j^{t-1}} = \frac{\left(a_i(1+\alpha)(x_j^{t-1}+h)\right)^{\frac{1}{\alpha}} - x_j^{t-1}}{x_j^{t-1}} \tag{14}$$

which is the *response multiplier*. From (14) we obtain the condition for reducing the reaction capacity $x_i^t(x_j^{t-1}) < x_j^{t-1}$:

$$x_j^{t-1}(x_j^{t-1}+h)^{\frac{1}{\alpha}} > \frac{1}{2\left(a_i(1+\alpha)\right)^{\frac{1}{\alpha}}} \ . \tag{15}$$

Obviously, the opposite holds for increasing the reaction capacity of any of the two players to his opponent in the previous period $t-1$.

Model of Proxy Conflict (MPC)

We now assume that each participant of the previous model (BMC) can become a principal and therefore involve his own agent into the conflict. A schematic diagram of the model is shown in Figure 8. The key assumption in Figure 8 is that only a univocal relationship between the principal and the agent (i to i) is allowed. For a significant number of proxy war situations this univocal hypothesis is realistic.

We introduce y_i as the level of expenditures for principal $i \in \{1,2\}$ so that the agent $i \in \{1,2\}$ is also involved in the conflict. The principals take decisions on the levels of participation in the conflict (x_i), but they can delegate some of this level ("resource" for maintaining the conflict) to their agents. Thus, direct confrontation of agents occurs at $x_i - y_i \geq -h$. The negativity of these values explains why the principal would give the agent a higher level of participation in the conflict than in the BMC. We underscore that the variable y_i reflects exactly the principal's decision about the involvement of his agent. To denote the agent's solution (the actual participation of the agent in the conflict) we use the variable y_i^a. The difference in notation allows one to consider the potential inconsistency between the decisions of the principal and the agent.

The principal's utility in the MPC is as follows:

$$u_i^p(x_i, x_j, y_i, y_j) = \frac{x_i - y_i}{x_j - y_j + h} + c_i \cdot \frac{y_i}{y_j + h} - a_i \cdot (x_i + x_j - y_i - y_j)^{1+\alpha} - b_i \cdot (y_i + y_j)^{1+\beta}. \quad (16)$$

Comments and explanations on the parameters of the formula (16) are given in Table (2).

Table 2: MPC Parameters

Variable	Denotation
$b_i \in (0,1)$	transformation coefficient ("price") of expenditures ("investing") in agent
$c_i \in (0,1)$	coefficient ensuring comparability of measures of success (victory) in the BMC and the MPC
$\beta > 0$	exponent capturing the non-linear effect of damage on the scale expansion of conflict for agent i (equivalent of coefficient α)
$b_i < a_i$	the negative effect of conflict on the principals' payoffs is lower in proxy conflict
$\beta < \alpha$	the rate of increase of "cumulative" damage for principals is lower in proxy conflict

The variables a_i and α have the same content as in the previous model (BMC). It is impossible to draw unequivocal conclusions on parameter c_i, which characterizes the relationship between the effects of direct and indirect participation in the conflict. Depending on the specificity of the simulated situation, they can be higher than, lower than or equal to 1. We define the utility function of agents in the standard form:

$$u_i^a(y_i^a, y_j^a) = \frac{y_i^a}{y_j^a + h} - d_i \cdot (y_i^a + y_j^a)^{1+\gamma}. \quad (17)$$

The coefficient d_i in (17) is identical to the coefficients a_i and b_i in (16), and the coefficient γ identical to the coefficients α, β. In the general case, it is difficult to link γ to unidirectional relations with α or β. Of undoubted interest is the case $\gamma = \alpha$, i.e. when the "cumulative" damage properties for the agent are identical to those for his principal.

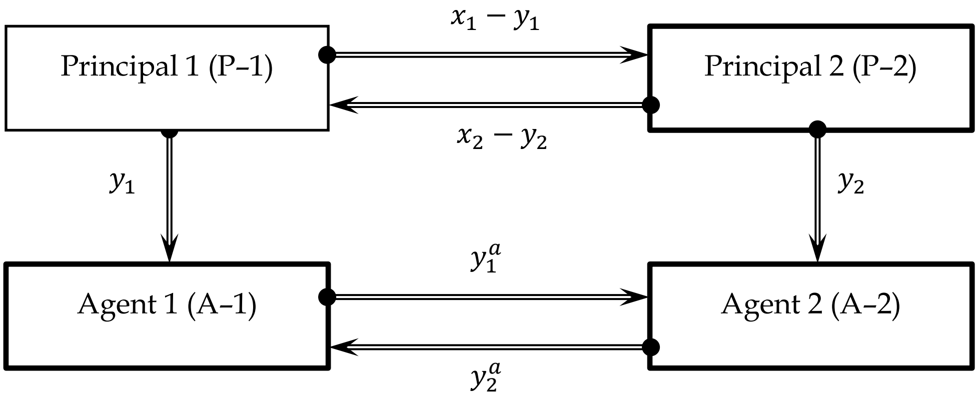


Figure 8: A scheme of interaction between participants in the MPC

The condition for a stationary point (first-order condition) for the function $u_i^p(x_i, x_j)$ (player i against player j) is the following:

$$\frac{\partial u_i^p(x_i, x_j, y_i, y_j)}{\partial x_i} = \frac{1}{x_j - y_j + h} - a_i \cdot (1 + \alpha) \cdot (x_i + x_j - y_i - y_j)^{\alpha} = 0 \quad (18)$$

or

$$(1+\alpha)\cdot\left((x_i-y_i)+(x_j-y_j)\right)^{\alpha}=\frac{1}{a_i\cdot\left((x_j-y_j)+h\right)}=\frac{1}{a_j\cdot((x_i-y_i)+h)}, \tag{19}$$

$$\frac{\partial u_i^p(x_i,x_j,y_i,y_j)}{\partial y_i}=-\frac{1}{x_j-y_j+h}+\frac{c_i}{y_j+h}+a_i\cdot(1+\alpha)\cdot\left(x_i+x_j-y_i-y_j\right)^{\alpha}-$$

$$-b_i\cdot(1+\beta)\cdot\left(y_i+y_j\right)^{\beta}=0. \tag{20}$$

By considering (18), we derive the following condition:

$$\frac{c_i}{y_j+h}=b_i\cdot(1+\beta)\cdot\left(y_i+y_j\right)^{\beta}. \tag{21}$$

Similarly, with (6) from (19):

$$(x_i-y_i)=\left(\frac{1}{a_i\cdot(1+\alpha)\cdot\left((x_j-y_j)+h\right)}\right)^{\frac{1}{\alpha}}-\left(x_j-y_j\right). \tag{22}$$

and from (21):

$$y_i=\left(\frac{c_i}{b_i\cdot(1+\beta)\cdot(y_j+h)}\right)^{\frac{1}{\beta}}-y_j. \tag{23}$$

As we can see from the best-response functions for agents (21) and principals (19), an important and significant advantage of the MPC is the possibility of expressing the optimality condition via the differences (x_i-y_i) in a form similar to the optimality conditions for the BMC. This provides the option of comparing the conclusions obtained in both models, and, consequently, measuring the influence exerted by the agents (the proxy element of the conflict). Similarly to (8)–(9), we can write the equation which solves the equilibrium values of the direct participation levels of the principal proxy conflict:

$$\frac{a_1+a_2}{a_1}\cdot(x_1^*-y_1^*)+\frac{a_2-a_1}{a_1}\cdot h=\left(\frac{1}{a_2\cdot(1+\alpha)\cdot\left((x_1^*-y_1^*)+h\right)}\right)^{\frac{1}{\alpha}} \tag{24}$$

(equation for $(x_1^*-y_1^*)$),

$$\frac{a_1+a_2}{a_2}\cdot(x_2^*-y_2^*)+\frac{a_1-a_2}{a_2}\cdot h=\left(\frac{1}{a_1\cdot(1+\alpha)\cdot\left((x_2^*-y_2^*)+h\right)}\right)^{\frac{1}{\alpha}} \tag{25}$$

(equation for $(x_2^*-y_2^*)$).

Similarly, equations are obtained to identify the equilibrium levels of attracting agents to the conflict (in terms of the utility functions of the principals):

$$\frac{b_1\cdot c_2+b_2\cdot c_1}{b_1\cdot c_2}\cdot y_1^*+\frac{b_2\cdot c_1-b_1\cdot c_2}{b_1\cdot c_2}\cdot h=\left(\frac{c_2}{b_2\cdot(1+\beta)\cdot(y_1^*+h)}\right)^{\frac{1}{\beta}} \tag{26}$$

(equation for y_1^*),

$$\frac{b_1\cdot c_2+b_2\cdot c_1}{b_2\cdot c_1}\cdot y_2^*+\frac{b_1\cdot c_2-b_2\cdot c_1}{b_2\cdot c_1}\cdot h=\left(\frac{c_1}{b_1\cdot(1+\beta)\cdot(y_2^*+h)}\right)^{\frac{1}{\beta}} \tag{27}$$

(equation for y_2^*).

Finding equilibrium solutions without taking into account the intrinsic utility of agents implies a consistent solution of equations (26) then (24), and (27) then (25). The main advantage of the MPC is that it allows us to reasonably analyze a conflict situation when there are negative components defining the equilibrium point. Indeed, the difference $x_i^* - y_i^*$ can also be negative in the case when $x_i^* \geq 0$, $y_i^* \geq 0$, $x_i^* < y_i^*$. This suggests the emergence of an equilibrium in which the agent is delegated a greater share of responsibility for maintaining the conflict than the principal himself. Similar situations correspond to stable (protracted) proxy wars. It may be the case that without agent participation equilibrium assumes asymmetric concessions of one side (x_i^*) under the pressure of the other (x_j^*). This property of the MPC can be considered as a reliable and plausible explanation of the determinants of proxy wars.

An example of a constructive implementation of a problematic (asymmetric in pressure and concessions) equilibrium is given in Table 3. To preserve the continuity of examples, the situation that arose in the BMC was chosen (for $a_1 = 1$, $a_2 = 2$, $h = 0.7$, $\alpha = 0.8$, see Table 1). In this case, at the equilibrium point, player 2 must maintain a positive level of participation in the conflict (0.490), but player 1, in contrast, makes concessions (–0.105). In the case of a conflict transformation from direct confrontation to a proxy war (with the same values of h and α) an equilibrium arises in the MPC, in which principal 1 together with his agent can maintain a positive level of confrontation. At the chosen values of parameters, the utility of principal 1 in the MPC is less than the value of player 1's utility in the BMC $(-0.455 < -0.268)$. This observation is in line with the objective properties of conflicts. Involvement of additional participants (agents) in the conflict helps maintain a "moderate" level of confrontation (due level of conflict participation), but this undoubtedly requires additional costs.

In accordance with the equilibrium presented in Table 3, when there is a proxy conflict, the agent's involvement becomes useful not only for player 1 but also for player 2, despite his positive level of participation in the BMC equilibrium. The utility of player 2 in the case of a proxy conflict increases compared to his utility in the baseline model $(0.477 > 0.464)$.

Table 3 Constructive implementation of an equilibrium: negative best response by player 1

i	h	α	β	a_i	b_i	c_i	$x_i^* - y_i^*$	x_i^*	y_i^*	$u_i^*(\circ)$
1	0.7	0.8	0.5	1	0.8	1	–0.105	0.074	0.178	–0.455
2				2	0.8	0.8	0.490	0.888	0.398	0.477

At the same time, the MPC properties do not exclude the possibility of equilibrium existence when x_i^* and y_i^* are negative. Proxy conflict can be considered as a mechanism for implementing equilibrium in situations with "bad" asymmetries in terms of balance of power and one-sided concessions.[3] The conditions determining the sign of equilibrium levels of direct participation for principals are completely identical to condition (11). As before, without loss of generality we can assume that $a_1 < a_2$. Hence, the conditions for y_i^* are the following:

$$y_1^* > 0 \quad \text{if} \quad \left[\begin{array}{l} \left(h^{\left(1+\frac{1}{\beta}\right)} < \frac{b_1 \cdot c_2}{(b_2 \cdot c_1 - b_1 \cdot c_2)\cdot(1+\beta)^{\frac{1}{\beta}} \cdot b_2^{\frac{1}{\beta}}}\right) \wedge \ (b_2 \cdot c_1 > b_1 \cdot c_2)\,, \\ b_2 \cdot c_1 \leq b_1 \cdot c_2 \end{array}\right. \tag{28}$$

$$y_2^* > 0 \quad \text{if} \quad \left[\begin{array}{l} \left(h^{\left(1+\frac{1}{\beta}\right)} < \frac{b_2 \cdot c_1}{(b_1 \cdot c_2 - b_2 \cdot c_1)\cdot(1+\beta)^{\frac{1}{\beta}} \cdot b_1^{\frac{1}{\beta}}}\right) \wedge \ (b_1 \cdot c_2 > b_2 \cdot c_1)\,, \\ b_1 \cdot c_2 \leq b_2 \cdot c_1 \end{array}\right. \tag{29}$$

It is necessary to indicate that equilibrium deviations of players within the framework of proxy conflicts (as well as in the BMC) can lead to different scenarios for their development.

III. Simulations

Of fundamental importance is the problem of equilibrium consistency from the perspective of principals and their preferred levels of agent involvement in the conflict (y_i^*) as well as the values of $\hat{y}_i^a$, which solve the problem of utility maximization of agents determined by functions (17). Computational simulation experiments in which potential successive path trajectories can be identified are of major interest for understanding the nature of the proposed proxy conflicts. To begin with, we analyze situations in which agents in the first period deviate from equilibrium defined by equations (24)–(27), i.e. the "principal equilibrium", and select their optimal levels of participation in the conflict to be $\hat{y}_i^a$. Over the next periods, we assume that the principals have enough "influence" on their agents to force them to act in accordance with the levels of participation y_i^* dictated by them. Based on the properties of the MPC, the mutual arrangement of the points $\left(y_1^*, y_2^*\right)$ and $((\hat{y}_1^a, \hat{y}_2^a)$ is of central importance for the simulation experiment outcomes (see Figure 9).

[3] However, this does not mean that any proxy conflict with any parameters of agents involved can be analytically powerful.

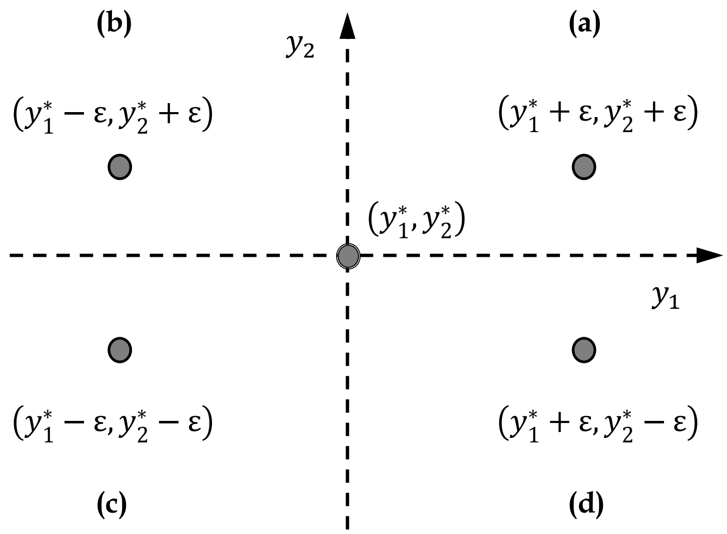


Figure 9: Mutual arrangement of the points (y_1^*, y_2^*) *and* $((\hat{y}_1^a, \hat{y}_2^a)$*,* $\varepsilon > 0$

Table *3* contains the values of the mutual best responses of the principals and agents under the assumption that both agents deviate from the equilibrium values (y_1^*, y_2^*) in the direction of decreasing (reducing the tension of the conflict)

$$\hat{y}_i^a = y_i^* + \varepsilon_i ,$$

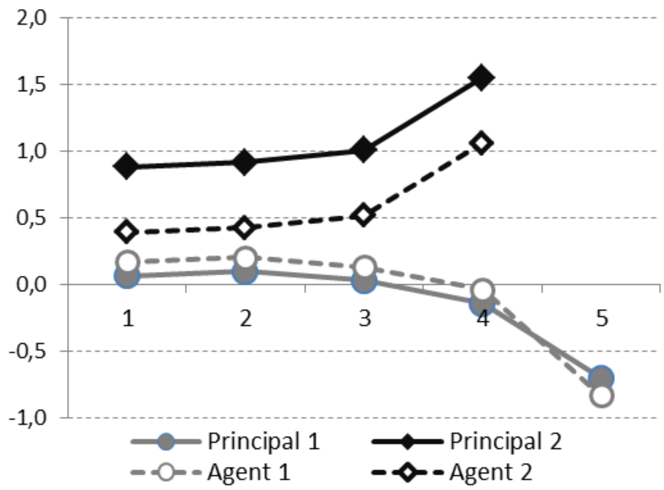
where $\varepsilon_1 = \varepsilon_2 = 0.1$ (variant (c), in accordance with the classification of Figure 9). The values of the equilibrium point $\{(x_i^*, y_i^*)\}_{i=1,2}$ are calculated from the data in Table 3.

Table 4: Trajectory of mutual best responses by MPC participations: agents already reject their conflict involvement levels at the beginning

t	Principal 1				Principal 2			
	$x_1^* - y_1^*$	x_1^*	y_1^*	$u_1^*(\circ)$	$x_2^* - y_2^*$	x_2^*	y_2^*	$u_2^*(\circ)$
Equilibrium	-0,105	0,074	0,178	-0,455	0,490	0,888	0,398	0,477
0	-0,105	0,064	0,168	-0,445	0,490	0,878	0,388	0,490
1	-0,105	0,094	0,199	-0,481	0,490	0,912	0,421	0,448
2	-0,105	0,026	0,131	-0,573	0,490	1,003	0,512	0,544
3	-0,105	-0,144	-0,039	-1,113	0,490	1,548	1,058	0,923
4	-0,105	-0,700	-0,833	–	0,490	-0,700	-0,700	–

The mutual best responses of the conflict participants are computed based on the assumption that at the start point the agents deviate from equilibrium, while the principal acts in accordance with the logic $x_i^0 = (x_i^* - y_i^*) + \hat{y}_i^a$ and then follows a series of consecutive mutual best responses to the opponent's actions in the previous period. As the data in Table 4 indicates, the series of mutual best responses is characterized by successive weakening of side 1 (both principal and agent) and their “exit from the game” at the fourth period. Figure 10 presents the dynamics of the consecutive mutual best responses of the principals and agents.

Figure 10: Dynamics of successive mutual best answers of participants (agents deviate at the beginning of the first period such that $\varepsilon = 0.1$*)*



The mutual relationship of the participants' best responses are depicted in Figure 9. Each point corresponds to the selection of each conflict party (the principal plus his agent). On the abscissa axis, the selections of conflict involvement levels by the principals are laid out, while along the ordinates are those of the agents.

Figure 11: Trajectory of consecutive mutual best responses of participants in a proxy conflict $(x_1^*, y_1^*), (x_2^*, y_2^*)$

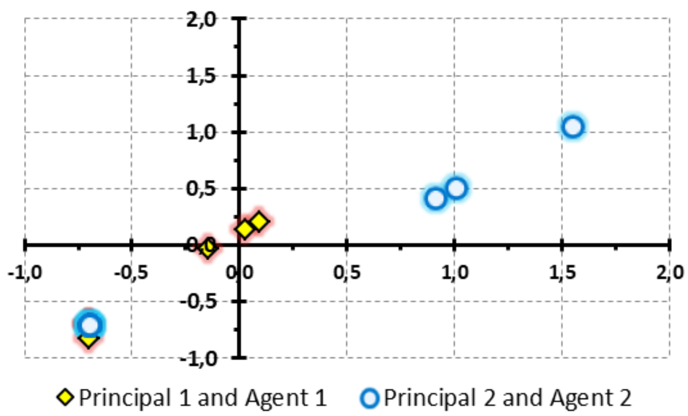


Figure 12: Trajectory of the utilities of principals in a proxy conflict given their mutual consecutive best responses $u_1(x_1^*, y_1^*, x_2^*, y_2^*), u_2(x_1^*, y_1^*, x_2^*, y_2^*)$

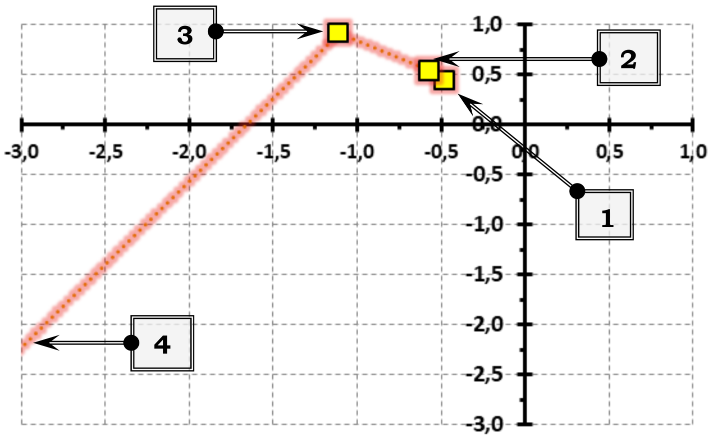


Figure 12 depicts the payoff dynamics of the principals in the process of conflict. The abscissa axis corresponds to the utility of principal 1, the ordinate axis to the utility of principal 2. The graphs of mutual best response trajectories determined by the conditions (26)-(27) for the MPC are presented in the Appendix (see below).[4] Simulation procedures allow us to obtain interesting and informative results about the development of proxy conflicts. We can assess the consequences for principals when agents independently select their level of involvement in the conflict. The significance of the model proposed in this paper is that it shows the cardinal importance for the principal of the choice of such an agent who has an objective self-interest in participating in the conflict *exactly* at the level which corresponds to his own equilibrium condition (26)-(27). Otherwise, a mutual best-response trajectory arises that does not converge to equilibrium and leads to the retirement of one of the parties from the conflict.

Another important feature of this model is its ability to question the effectiveness of contemporary peace-making processes, by suggesting that a simultaneous reduction in the levels of confrontation will only have short-term effects on conflict resolution. As discussed above, a trajectory that starts with a simultaneous decrease in the deviation of agents from equilibrium levels leads to a gradual surrender of positions by principal 1 and, ultimately, his exit from the conflict. This outcome can be interpreted either as defeat for principal 1 or as a costly transition to another form of direct or indirect confrontation. *Therefore, proxy conflicts are much more likely to persist than direct conflicts.* Extensions of our model may include Bayesian games. Informational asymmetry may be related to the incompleteness of the principal's knowledge about the true levels of the agent's conflict involvement.

[4] They correspond to other deviation possibilities of agents from the equilibrium values of y_i^*.

IV. Discussion

Proxy wars reveal the contradictory nature of retaliatory strike and of symmetric peace-making processes. In the case of retaliatory strike, for principal payoffs structurally identical to functional dependencies, the more reactive sequences of mutual best responses inevitably lead conflict participants away from the equilibrium point. This observation suggests, on the one hand, an unstable evolutionary path and, on the other hand, a possible instrument for conflict resolution. At the same time, the simultaneous reduction of conflict involvement for agents may lead one of the two parties to exit from the conflict, which may be desirable in the short run, but it is uncertain whether it will produce a long-lasting peace.

There are several extensions to be proposed based on our *Model of Proxy Conflict* (MPC). First, the interaction scheme between principals and agents may differ, including asymmetric derivations with two principals and three agents. Moreover, the same participant under different conditions may be an agent of different principals (the problem of the unprincipled ally or so-called "Myrmidon problem"). It can also be the case that there is a high degree of the principal's dependence on the agent if the latter uses feedback strategies (the so-called North Korea problem). Finally, cooperative game-theoretic models underscore the significance of coalition configurations between agents and principals. It is also possible to consider the introduction of constraints on the participants' resources, both principals' and agents'. In our paper, the absence of resource constraints is partially compensated for by the effect of the "damage" component in the participants' respective payoffs. It is, therefore, necessary to suggest that from the point of view of mathematical and computational methods the models proposed in this paper are very close to the methodology of penalty functions that are widely used in solving optimization problems in the fields of engineering and natural sciences.

What needs to be pointed out here is the importance of randomization processes for the predictive power of proxy war modeling. Randomization primarily involves the choice of "starting" points in the simulation of consecutive mutual best responses. As has already been mentioned above, the evolution of best-response trajectories within the framework of the BMC and the MPC depends on the mutual location of the starting point of each trajectory and the equilibrium point. This is why it can lead to radically different outcomes. Nevertheless, in the case of asymmetric information (the players do not have knowledge about the parameters of the utility function of each other), the choice of a starting point of conflict involvement becomes more complex. We are convinced that the inclusion of incomplete information and stochastic processes can significantly increase their validity in the long run. What we underscore in our theory of proxy wars is that they can facilitate efficient victories for principals compared to direct confrontation. Nevertheless, high levels of deviations by the appointed agents may speed up the principal's exit from the conflict either in the form of defeat or transition to a costly direct confrontation. Hence, agents are not blind executors of the principals' will, but they are drawn by them to participate in the conflict, while preserving their own system of incentives and interests. The success of the principals' actions depends to a large extent on their ability to harmonize their own interests with the respective goals of their selected agents.

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Appendix

Table 5: Trajectory of mutual best responses of proxy conflict participants – scenario 1 (agents deviate at the starting point)

t	Principal 1				Principal 2			
	$x_1^* - y_1^*$	x_1^*	y_1^*	$u_1^*(\circ)$	$x_2^* - y_2^*$	x_2^*	y_2^*	$u_2^*(\circ)$
Equilibrium	-0,105	0,074	0,178	-0,455	0,490	0,888	0,398	0,477
0	-0,105	0,084	0,188	-0,466	0,490	0,898	0,408	0,463
1	-0,105	0,053	0,158	-0,432	0,490	0,865	0,375	0,502
2	-0,105	0,121	0,226	-0,339	0,490	0,783	0,292	0,418
3	-0,105	0,308	0,413	0,199	0,490	0,436	-0,054	0,253
4	-0,105	1,614	1,719	-2,108	0,490	-0,700	-1,643	-0,096

Figure 13: Dynamics of successive mutual best responses of proxy conflict participants – scenario 1 (agents deviate at the starting point $\varepsilon = 0.1$)

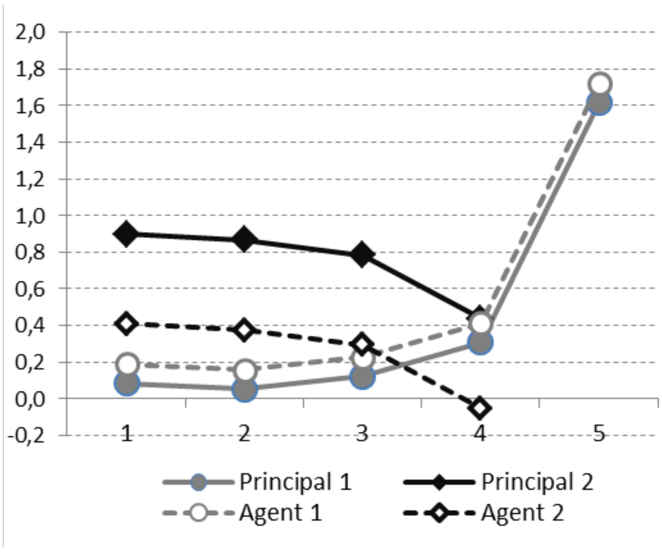


Table 6: Trajectory of mutual best responses of proxy conflict participants – scenario 2 (agents deviate at the starting point)

t	Principal 1				Principal 2			
	$x_1^* - y_1^*$	x_1^*	y_1^*	$u_1^*(\circ)$	$x_2^* - y_2^*$	x_2^*	y_2^*	$u_2^*(\circ)$
Equilibrium	-0,105	0,074	0,178	-0,455	0,490	0,888	0,398	0,477
0	-0,105	0,064	0,168	-0,466	0,490	0,898	0,408	0,490
1	-0,105	0,053	0,158	-0,480	0,490	0,912	0,421	0,504
2	-0,105	0,026	0,131	-0,573	0,490	1,003	0,512	0,544
3	-0,105	-0,144	-0,039	-1,113	0,490	1,548	1,058	0,923
4	-0,105	-0,700	-0,833	–	0,490	-0,700	-0,700	–

Figure 14: Dynamics of successive mutual best responses of proxy conflict participants – scenario 2 (agents deviate at the starting point $\varepsilon = 0.1$)

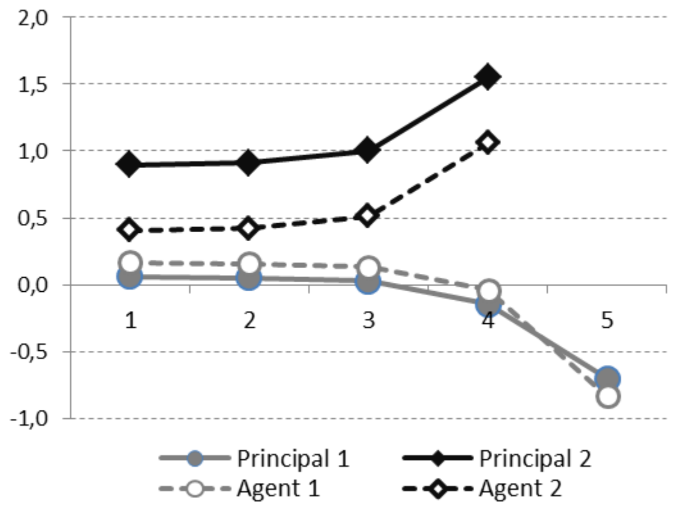
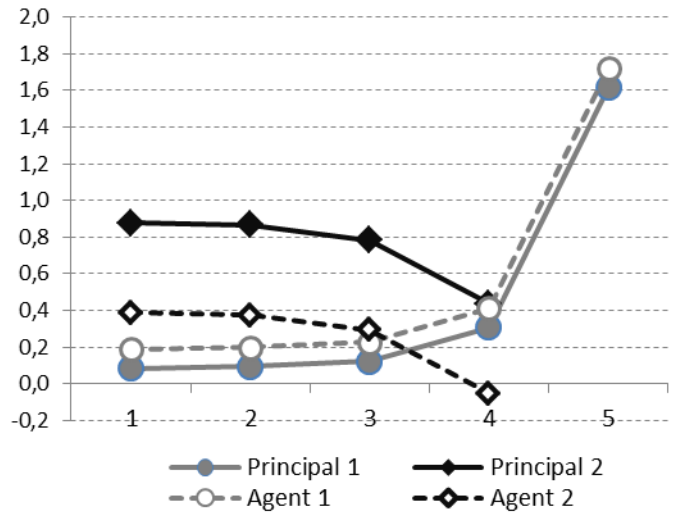


Table 7: Trajectory of mutual best responses of proxy conflict participants – scenario 3 (agents deviate at the starting point)

t	Principal 1				Principal 2			
	$x_1^* - y_1^*$	x_1^*	y_1^*	$u_1^*(\circ)$	$x_2^* - y_2^*$	x_2^*	y_2^*	$u_2^*(\circ)$
Equilibrium	-0,105	0,074	0,178	-0,455	0,490	0,888	0,398	0,477
0	-0,105	0,084	0,188	-0,445	0,490	0,878	0,388	0,463
1	-0,105	0,094	0,199	-0,431	0,490	0,865	0,375	0,450
2	-0,105	0,121	0,226	-0,339	0,490	0,783	0,292	0,418
3	-0,105	0,308	0,413	0,199	0,490	0,436	-0,054	0,253
4	-0,105	1,614	1,719	-2,108	0,490	-0,700	-1,643	-0,096

Figure 15: Dynamics of successive mutual best responses of proxy conflict participants – scenario 3 (agents deviate at the starting point $\varepsilon = 0.1$)



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