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Longitude

This study traces the development of the concept of geographical longitude: from the earliest known Mesopotamian evidence of geographical concepts through its incorporation into mathematical astronomy and to its transmission to ancient Greek philosophy and scholarship. We show that there is a tight connection between the observation of lunar eclipses and the development of a quantitative representation of terrestrial longitudes. It was not until Ptolemy that geographical longitude was systematically quantified through angular differences. It is demonstrated that the ancient scholars failed in their attempts to determine geographical longitude by astronomical means, and that even certain Syriac texts which specify how to determine longitude using a planispheric astrolabe were unable to resolve this scientific challenge.

Ancient astronomy; ancient geography; cardinal directions; ancient cartography; Mesopotamia; longitude; lunar eclipse, history of science; Gaugamela.

1 Introduction

How does one determine the definitive location of a specific place on our planet? Today, this question can be easily answered: one points to a place on a map and uses its geographical coordinates to identify its true location on the Earth's surface. Our modern concept of a geographical coordinate can even be taken to be a 'natural' way of denoting terrestrial locations. We would have no maps at all if geographical locations could not be projected on to diagrams in some form, and, without any kind of coordinates, it would be decidedly more difficult to describe the position of a geographical location, either by means of a textual description or by a map. In this contribution we trace the historical development of the concept of geographical coordinates, placing particular emphasis on the most problematic coordinate – geographical longitude – and analyse the measuring techniques used.

Coordinates were not used in the most common ancient descriptions of places. Rather, sketches or instructions with directions along roads or paths transmitted knowledge about how to find other places. Itineraries recorded parts of routes, sometimes with destinations at the coast or at sea. Cardinal directions or prominent locations, as well as mountains and rivers, characterized the travel paths, and helped to make the routes to distant places identifiable.

The concept of geographical coordinates was developed over a long historical period, starting with early Mesopotamian science during the third millennium BCE. The metaphorical path from distances and directions to coordinates was a long one, with many intermediary steps along the way. Ancient scholars added astronomical knowledge to directions in order to define the cardinal directions. The rise of mathematical astronomy in Babylon from the fifth century BCE until its final form in the early fourth century BCE – documented in Otto Neugebauer's *Astronomical Cuneiform Texts* (*ACT*) – also became paradigmatic of the way questions were modelled in other scientific domains. Yet it took more than four hundred years before geographical coordinates were fully developed – by

Claudius Ptolemy in his masterpiece, the *Geography*, around 160 CE. However, although in this work Ptolemy listed the locations of more than 6000 toponyms of the then-known world to which he assigned geographical longitudes and latitudes, it was not understood how these coordinates had been determined. Determining the sources of these longitudes and latitudes and the methods by which they were incorporated into Ptolemy's *Geography* is the subject of our ongoing studies.[1] In this contribution, we focus on the origin of the concept of geographical longitude. It is much harder to work out geographical longitudes than latitudes using astronomical observations, especially since an almost conceptual tension existed between the requirements of a well-defined geometric geography and the practicalities of quantification, which even the most highly trained ancient geometers could not resolve. We examine the rise of this key geographical concept – and ultimately reveal that the great minds of antiquity were unsuccessful in their attempts to determine longitude by astronomical means.

2 Ancient Mesopotamia

The problem of accurately locating geographical places in ancient Mesopotamia shows up in a wide range of cuneiform sources covering more than two millennia. Royal inscriptions, scholarly compositions, maps, itineraries and administrative documents contain evidence of concepts that were used to record and quantify geographical information, although nothing resembling geographical longitude or latitude has yet been identified. Furthermore, there is no proof that the ancient Mesopotamians assumed that the Earth was spherical, a precondition for the emergence of these concepts. However, Mesopotamian sources do provide qualitative and quantitative information about geographical places in the form of distance measurements and alignments to which the cardinal directions were often added. A selection of these sources is discussed here. At the same time, it should also be noted that these topics cannot be analysed in isolation from developments in Mesopotamian astral science, since a number of important concepts were transferred from geography to astronomy as well as vice versa.

2.1 The cardinal directions

The oldest surviving Mesopotamian map dates from the Old Akkadian period (2400–2300 BCE) and depicts an estate in the city of Nuzi, then named Gašur.[2] On this map, the names of winds ('wind' = *im* in Sumerian) denote the general cardinal directions. The top of the map is labelled *im.kur*, meaning the 'mountain wind' (east), the bottom is marked *im.mar.tu*, meaning the 'wind of Amurru' (west), and the left-hand side of the map is inscribed with *im.mir*, meaning the 'north wind'.[3] The 'mountain wind' denotes the wind blowing from the Zagros mountains, east of Mesopotamia, while the 'wind of Amurru' refers to the Amorites, who then occupied the western lands, that is, the area west of the Euphrates River. The south wind, the mention of which has not been preserved on

1 The studies are part of the project, *Genesis of Ptolemy's 'Geography'*, which was launched at the University of Bern, Switzerland in 2006. See Rinner 2013.

2 Rochberg 2012, 30.

3 The literal meaning of *im.mir* is unknown. Note that what is denoted here as the 'top' of the Nuzi map relates to the image of this map in Rochberg 2012, which follows the orientation of tablets that was in place after the beginning of the second millennium BCE, when cuneiform writing had undergone a 90° anticlockwise rotation. Since the map dates from before this time, this rotation had probably not yet happened. Hence, what is denoted as the top of the map in Rochberg 2012, which is inscribed 'mountain wind', is, in fact, the right side of the tablet and the north wind is at the top, as in a modern map.

this tablet, was usually called *im.u$_{18}$.lu*, the literal meaning of which is unclear. Another frequently used Sumerian name for the north wind was the 'straight wind' (*im.si.sa$_2$*).

From the Old Babylonian period onwards (1800–1600 BCE), these Sumerian terms for winds were also used as logograms in certain Akkadian contexts. For example, in Akkadian the east wind was called *šadû* ('mountain') and the west wind *amurru* ('[land of the] Amorites'), which are both literal translations of the Sumerian terms, although the word for 'wind' has been omitted. The north wind was sometimes equated with the Akkadian word *meḫû* ('violent storm') but more commonly with *ištānu/iltānu*.[4] The Akkadian term for the south wind was *sūtu*, a word of unknown etymology. Another set of logograms that was used to signify the cardinal winds in Akkadian texts involved numerals: IM.1 = 'south wind'; IM.2 = 'north wind'; IM.3 = 'east wind'; and IM.4 = 'west wind'.[5] In this instance, the cardinal directions were not ordered in the clockwise direction of rotation but in a south–north and east–west pairing. The four principle winds were regularly translated as cardinal directions, but it is more probable that they represented 90 degree ranges, with the cardinal directions at their centre. For instance, the 'mountain wind' corresponded to a range of 90 degrees between the north-east and the south-east, and similarly for the other winds.[6]

The cardinal directions were also linked to the ancient concept of the 'four regions of the world'.[7] The Sumerian term for 'the four corners' was *an.ub.da.limmu$_2$.ba*, which had been used since the Old Akkadian period, when the first centralized Mesopotamian state emerged, while the corresponding Akkadian term was *kibrāt arba'i/erbetti*. If taken literally, this term refers to the corners of the Earth, but it more usually signifies the interior regions.[8] In an astrological text known as *The Great Star List*, the four regions are named and connected with the winds as follows: south wind – Elam (actually located to the east); north wind – Akkad; east wind – Subartu and Gutium; west wind – Amurru.[9] Tablets containing lunar eclipse omens from the second and first millennia BCE indicate that this geographical division was also used to identify parts of the Moon. Like the Earth, the Moon was divided into four quadrants, named after four canonical countries – Akkad, Elam, Amurru, and Subartu and Gutium – which allowed astrologers of the period to determine which countries were affected by an eclipse, depending on where the shadow entered or left the lunar disc. Various systems were used to distribute the country names over the lunar quadrants.[10] It is possible that this transfer of geographical concepts to the Moon reflected the belief that the Earth had a similar disc-like shape. It will be shown below that other texts provide considerable evidence for such an assumption.

2.2 Geographical texts from the Third Dynasty of Ur

One of the earliest known texts in which geographical places are located in relation to the cardinal directions is the so-called cadastre of Ur-Nammu (Ur-Nammu founded the Third Dynasty of Ur [Ur III Empire], which lasted from about 2100 to 2000 BCE.). In its

4 This word might derive from *ištēnu* meaning 'one', i.e. the 'first wind' (see Horowitz 1998, 197). However, in the numerical notation for winds (detailed in this paragraph), the south wind was regarded as the first wind.

5 Horowitz 1998, 197.

6 P. V. Neugebauer and Weidner 1931–1932.

7 Glassner 1984; Horowitz 1998, 299.

8 Glassner 1984 argues that Sumer and Akkad were not part of the four regions and that the world, therefore, consisted of five regions, with Mesopotamia at its centre, but this viewpoint has not been generally accepted. There are many examples in cuneiform literature where the term clearly refers to the entire known surface of the Mesopotamian world.

9 Weidner 1959/1960; Koch-Westenholz 1995, 187–205.

10 Rochberg-Halton 1988, 51–55.

original form, this text probably described the boundaries of all the provinces of the Ur III Empire, which was known for its highly centralized and well-developed bureaucracy. Descriptions of four of these provinces have been partly preserved in two later copies from Nippur that date from the Old Babylonian period.[11] The best preserved tablet traces the boundary of three of the provinces along four 'sides,' which were named after the cardinal winds. The boundary of the second province, Apiak, is described as follows:

> From the Tower of Numušda to the Shrine of Numušda, from the Shrine of Numušda to the Tower of the Mountain [...], after you cross the Abgal Canal, from the source of the Ušartum [Canal] you go 9;20 UŠ, it is the *crossing point* [emphasis added][12] of the boundary: its northern side.
>
> From the *crossing point* of the boundary to Me-en-ili: its eastern side.
>
> From Me-en-ili to the bank of the Abgal Canal at the source of the Ilum-bani Canal, after you cross the Abgal Canal, from the source of the Ida'umma Canal to the Imnia Canal: its southern side.
>
> From the Imnia Canal to Nagarbi [...], back to the Numušda Tower: [its western side].
>
> King Ur-Namma has decreed the boundary of the god Meslamtaea of Apiak.

The four sides of Apiak province have been traced in a clockwise direction, setting out from the north-western corner, with each side composed of one or more stretches of land delimited by geographical places. No distance measurements have been given, with one exception: the north-eastern corner is said to be located at a distance of 9;20 (= 9 1/3) UŠ units beyond the preceding location, in a direction that must be roughly eastwards. Since 1 UŠ is about 360 m, 9;20 UŠ comes to approximately 3.2 km. This is perhaps the earliest attestation of the UŠ unit in connection with geographical distances.[13] As we shall see, the UŠ was later used to express time intervals and angular distances in Mesopotamian astronomy and thus it can be viewed as the precursor of the degree, the unit of geographical longitude. Since most locations named in the text remain unidentified, the actual shapes of the provinces are unknown, and they cannot be assumed to form exact rectangles strictly aligned with the cardinal directions. However, the implied concepts that were used to locate places are of considerable interest. The geographical locations, including at least one position defined by a distance measurement, have been arranged in linear sequences and linked to the cardinal directions, which is, as far as we know, the earliest Mesopotamian example of its kind. The other two provincial boundaries have

11 For the published translations of these later copies, see Kraus 1955; Frayne 1997, 50–56; and Robson 2008, 63. The translation reproduced above is based on the published translations, although it has been modified in several places. Kraus suggests that the text was originally part of a monumental inscription that had been made to mark a military conquest and had been displayed at Nippur, in the main temple of the empire, and that the eleven small inscribed stone fragments held by the University of Pennsylvania Museum of Archaeology and Anthropology in Philadelphia, USA, were originally part of this inscription.

12 The published translations do not take the sign RI to denote 'crossing point.' Kraus interprets RI as a deictic element *ri*, which he provisionally translated as 'that beyond [the boundary],' while Frayne translates it as 'this side [of the boundary].' Both translations are problematic, since they make little sense, while the construction '(deictic element) *ri* of x' is awkward from a grammatical point of view. However, RI can be read as *dal*, signifying a 'transversal, dividing line' (Akkadian equivalent: *tallu*), a meaning that is well attested in the third millennium BCE (see the entry for *dal* in the Philadelphia Sumerian Dictionary at http://psd.museum.upenn.edu/epsd/nepsd-frame.html). Hence, *dal* ('crossing point') may denote the point at which the northern side of the boundary touches (or crosses) the eastern side.

13 For the interpretation of this number (9;20 UŠ), see Robson 2008, 63, who amends Frayne 1997, 51. The correct reading and literal meaning of UŠ as a unit of length are unknown.

been described in a similar manner, except that they have been traced in an anticlockwise direction and do not include any measured distances.

The only attestations that exist for the quantitative descriptions of geographical features are for individual fields. About thirty elaborate field plans have survived from the Ur III Empire[14] – perhaps because they were copied to serve as didactic models for land surveyors. On these tablets, the fields have been divided into rectangles, triangles and trapeziums (quadrilateral shapes with parallel sides) in order to make their total area computable. Most of the drawings include cardinal directions in the form of the four canonical winds mentioned earlier. Since it cannot be assumed that the actual fields were, in general, strictly aligned with the cardinal directions, these indications should be interpreted as labels that were used to identify the sides of the fields. All the figures into which a field has been subdivided have been annotated with the lengths of their sides and their areas. From these numbers it is clear that the drawings were not done to scale and that they only represent the spacial relations within the field in a quantitative, topological sense. Unlike in Ur-Nammu's cadastre, perpendicular directions, which have been defined in relation to the orientation of the field and not the cardinal points, have been used in the field plans. However, even here the concept of orthogonality was not applied, since it was not a requirement of Mesopotamian algorithms when computing the areas of rectangles and trapezoidal figures. In other words, these methods were also used to calculate the areas of rectangles without strictly perpendicular sides and trapezoidal figures without parallel sides.[15]

2.3 *The Sargon Geography*

In all the sources discussed so far, the geographical regions were located in relation to the cardinal directions and quantified through their perimeters, which was standard practice in Mesopotamian mathematics and land surveying. A somewhat different approach, however, appears to have been followed in *The Sargon Geography*, a description of the lands conquered by Sargon of Akkad, a legendary king of the Old Akkadian period. Even though this is a literary composition, which was created long after Sargon's reign had ended, and the descriptions are very schematic, the preserved portions of the text are important for its geographical concepts, in particular the references to geographical distances and the empire's total east–west extent. Most of the text is composed of two lists of place names, with sequences of neighbouring lands. Each land is located using two places on its boundary, but no indication of distance is given: "from [place] A to [place] B is Land X, from [place] B to [place] C is Land Y", and so on. Positioned between the two lists is the following section, which extols Sargon's role in delimiting and surveying his empire:[16]

> 120 *bēru* ['miles'] is the length from the Tail of the Euphrates to the border of Meluḫḫa, Magan, that which Sargon, King of the Universe, when he conquered the totality of the land under heaven, defined borders for and measured the *street* of.
>
> 40 *bēru* is the *street* of Marḫaši.
>
> 60 *bēru* is the *street* of Tukriš.
>
> 90 *bēru* is the *street* of Elam.
>
> 180 *bēru* is the *street* of Akkad.

14 Rochberg 2012, 24; Liverani 1990.
15 Høyrup 2002, 230–231.
16 This translation is based on that of Horowitz 1998, 71–73, and has been modified in places.

> 120 *bēru* is the *street* of Subartu.
>
> 120 *bēru* is the *street* of Amurru from the Lebanon to Turukki.
>
> 90 *bēru* is the *street* of Lullubi.
>
> 90 *bēru* is the *street* of Anšan.
>
> Anaku and Kaptara, the lands across the Upper Sea, Dilmun and Magan, the lands across the Lower Sea, and the lands from sunrise to sunset, the total of all lands, which Sargon, King of the Universe, conquered three times.

The central and northern parts of Mesopotamia were traditionally known as Akkad and Subartu, respectively. As little is known of many of the other lands, they can only be located approximately.[17] Meluḫḫa and Magan, usually mentioned together, were situated somewhere along the Persian Gulf. The other territories were spread across Mesopotamia's neighbouring regions in the east (Marḫaši, Tukriš, Elam, Anšan) and the west (Amurru, Lullubi), in no particular order. Uniquely in *The Sargon Geography*, the dimensions of a land are expressed in terms of the length of its 'street' (*ribītu*). It is not entirely clear how one should interpret 'street' in this geographical context.[18] Most probably it denoted a linear distance across the centre of a land, analogous to a street crossing a city through its centre. Alternatively, it might have denoted a real or imagined 'street' that traced a section of the perimeter of the land, as in Ur-Nammu's cadastre.[19]

The text gives no indication as to how the 'streets' were orientated. However, the cardinal directions do show up in the text's final passage, in which the total extent of Sargon's empire is defined from east to west. Unlike the cardinal directions in Ur-Nammu's cadastre and in the field plans, east and west are not indicated by winds but by two astronomical phenomena: 'sunrise' (written using the Sumerian logogram ^{d}UTU.E$_3$, of which the Akkadian reading is *ṣīt Šamši*); and 'sunset' (written using the logogram ^{d}UTU.ŠU$_2$.A, of which the Akkadian reading is *ereb Šamši*), respectively. This second parallel tradition of indicating the cardinal directions east and west is attested in different genres (incantations, omens, royal inscriptions, letters, astronomical texts, and so on) from the Old Babylonian period onwards[20] and can be traced back to Sumerian literary texts of the third millennium BCE.[21]

The 'streets' in *The Sargon Geography* are measured in 'miles' (*danna* in Sumerian; *bēru* in Akkadian), which correspond to about 10.8 km, and is a well-attested unit of measurement from the Old Akkadian period onwards. (Note that 1 *bēru* is divided into 30 UŠ, the unit mentioned in Ur-Nammu's cadastre.) These two units constitute another link between the conceptual frameworks underlying the fields of geography and astronomy. From 1200 BCE onwards, the *bēru* and the UŠ also appear in astronomical texts, where they are the

17 Horowitz 1998, 86–88.

18 For *ribītu*, see the *Chicago Assyrian Dictionary*, Vol. R, 317–321. Only in *The Sargon Geography* is this common word for 'street' used in connection with lands. Its first instance in the quoted passage is written logographically as SILA.DAGAL.LA, while the other mentions are written syllabically as RI-*bit* (*status constructus*, 'street of'). Horowitz, taking RI to read *tal*, interprets all these instances as *talbītu* (meaning 'circumference'), a word that is otherwise not attested in cuneiform texts. It is taken to be a *tapristu*-type nominal form of the verb *lamû*, 'to surround', assuming a shift from *talmītu* to *talbītu*. Even though this reading is theoretically possible, the literal meaning of the logogram SILA.DAGAL.LA ('wide street') and nearly all attestations of RI-*bītu* imply that 'street' is the more accurate translation. Furthermore, the entry "120 *bēru* is the RI-*bit* of Amurru from the Lebanon to Turukki" rules out that this denotes a closed, circular path. Hence, we can reject the interpretation that RI-*bit* = *tal-bit* (circumference).

19 The large distances assigned to the 'streets' might speak in favour of the latter interpretation, but this argument is inconclusive, given the schematic, idealized and propagandist nature of the text.

20 See the *Chicago Assyrian Dictionary*, Vol. E, 258; Vol. , 215.

21 Horowitz 1998, 333–334.

most commonly used units of time: 1 *bēru* corresponds to 1/12 of a day, that is, 2 modern hours – the approximate time that it takes to walk a distance of 1 *bēru*. The *bēru* of time is, likewise, divided into 30 UŠ, so that one day amounts to 360 UŠ. Astronomical time intervals were typically measured using the rising or setting of stars as they rotate in the sky. Hence, these units also correspond to the arcs that are traced by the stars and are parallel to the celestial equator. Since 360 UŠ corresponds to a full rotation, these astronomical units of time involve a notion of circularity, unlike their geographical precursors.

2.4 Babylonian 'Map of the World'

Another example of a concept common to geography and astronomy is the circle. In several Akkadian literary texts, the term 'circle (*kippatu*) of the four winds' denotes the edge of the known world, which suggests that the Earth was assumed to have a disc-like shape.[22] The most prominent piece of evidence that this belief was held is the Babylonian 'Map of the World,' also known as the 'mappa mundi,'[23] which is a depiction of the world as known to the Babylonians (see Fig. 1). This map and the descriptions on it do not provide an accurate and complete representation of Mesopotamian geographical knowledge. As Horowitz and Rochberg argue,[24] it should be regarded as an ideological map with mythological connotations. The tablet's colophon (the inscription added by a scribe to the end of a text) states that it was copied from an "old exemplar", but country names such as Assyria, Bit-Yakin, Habban and Urartu indicate that it could not have been created before the ninth century BCE.[25] The map depicts the Earth as a round continent that is completely surrounded by an ocean (*marratu*). Originally, eight triangular areas called *nagû*s, of which only five are preserved owing to the tablet's damaged lower edge, radiated from the outer circle. The *nagû*s contain descriptions of faraway regions, with distance measurements expressed in *bēru*. Interpreting these descriptions poses significant difficulties:[26] at the top of the circular continent, there is an area labelled *šadû* ('mountain'), which some scholars believe represents the cardinal direction 'mountain (wind),' that is, the east. However, the orientation of the river suggests that the top corresponds to the north or north-west.[27] Moreover, neither interpretation can explain the problematic positioning of several of the countries and cities. In short, the mappa mundi is not a world map in the modern sense but an idealized depiction of a disc-shaped Earth, composed in accordance with Babylonian mythological conceptions.

2.5 Coordinates in Babylonian mathematical astronomy

Another Babylonian innovation, which contributed to Greek geographical coordinate systems, was the introduction of the zodiac around 400 BCE. Babylonian astronomers

22 Horowitz 1998, 298.

23 Horowitz 1998, 20.

24 Horowitz 1998; Rochberg 2012.

25 The provenance of the tablet is uncertain, but Sippar and Borsippa are two possibilities (see Horowitz 1998, 26). Although it belongs to a collection in the British Museum that consists mainly of tablets from Sippar, it is possible that the tablet comes from Borsippa, the likely place of origin of the scribe mentioned in the colophon.

26 According to the text, the distances measured were "in between", which perhaps refers to the distances between neigbouring *nagû*s. However, why these distances are of importance remains unexplained. On the reverse side of the tablet, we again find some distance values for the *nagû*s, but they do not correspond to those on the obverse side.

27 Furthermore, on the tablet the word *šadû* is written phonetically and not as the logogram IM.KUR.RA ('mountain wind'), which suggests that the literal meaning of mountain was intended here. For a discussion of the map's orientation, see also Rochberg 2012, 33.

Fig. 1 | Babylonian 'Map of the World' (BM 92687). Photograph courtesy of the Trustees of the British Museum.

introduced this new coordinate system in order to express the positions of celestial bodies. They did this by dividing the apparent path of the Sun, that is, the ecliptic circle, into twelve zodiacal signs, each comprising 30 UŠ. Henceforth, the UŠ was used not only to measure time and geographical distance but also to express the positions of celestial bodies and their distances along and perpendicular to the ecliptic, such that 1 UŠ corresponds to 1 (modern) degree (of an angle).[28] Note that the *bēru* was not incorporated into this coordinate system,[29] which became a central concept of Babylonian mathematical astronomy[30] and zodiacal astrology. This coordinate system was transmitted to Greek astronomers no later than the second century BCE, most probably in the writings of Hipparchus and Hypsicles. In the second century CE, Claudius Ptolemy made it an integral part of his astronomical theories and he was probably the first to construct a related system of geographical coordinates.

28 Subdivisions of the UŠ, as indicated by subsequent sexagesimal digits of a distance or position, were not explicitly named, with the exception of 1/60 of a UŠ, which was sometimes called a 'rod' (*nindanu* in Akkadian).

29 One zodiacal sign is equivalent in length to 1 *bēru*, but no explicit mention of this has been found. The *bēru* does show up as a distance unit in a few star lists and other unusual astronomical texts, but these distances appear to be the result of mathematical or mythological speculations, since they are very large and incompatible with the angular distances that are expressed in UŠ. For a discussion of these texts, see Schaumberger 1952 and Rochberg-Halton 1983.

30 Ossendrijver 2012, 33–34.

To sum up, several concepts related to geographical longitude can be traced back to the coordinate systems, units and definitions of cardinal directions that were developed by Babylonian astronomers. However, other concepts, such as the UŠ unit, unquestionably have their origins in traditional Mesopotamian geography.

3 The concept of longitude in ancient Greek geography

3.1 Aristotle's picture of the world

Readers of Aristotle's *Meteorology* might be surprised to come across a short discussion about the form of the Earth and the size of the inhabited world right in the middle of a section on the theory of winds. Aristotle (384–322 BCE) succinctly describes how only a small part of the spherical Earth is habitable.[31] This brief digression is preceded and followed by a detailed presentation of the causes, number, names and positions of the winds. As in Mesopotamian sources, wind directions are closely linked with spatial orientations as well as with the picture of the world at that time. The key element of Aristotle's work, however – and which leads to major ramifications for the development of Hellenistic geography – is his affirmation that the Earth is spherical. His decisive argument, developed in *De caelo*, is based solely on his astronomical knowledge:

> How else would eclipses of the moon show segments shaped as we see them? As it is, the shapes which the moon itself each month shows are of every kind – straight, gibbous, concave – but in eclipses the outline is always curved; and, since it is the interposition of the earth that makes the eclipse, the form of this line will be caused by the form of the earth's surface, which is therefore spherical.[32]

Observing the sky provides invaluable information about the form of the Earth, the shape of the inhabited regions and the location of specific places in the world. Indeed, believing that the Earth is spherical does imply that places, depending on where they lie on the Earth's surface, are affected by different climates and are subject to different meteorological conditions. Hence Aristotle believes that the *oikoumenē* cannot be round. This fact is also confirmed by empirical estimations:

> For reason proves that the inhabited region is limited in breadth, while the climate admits of its extending all round the earth. For we meet with no excessive heat or cold in the direction of its length but only in that of its breadth; so that there is nothing to prevent our travelling round the earth unless the extent of the sea presents an obstacle anywhere. The observations made on journeys by sea and land bear this out. They make the length far greater than the breadth. If we compute these voyages and journeys the distance from the Pillars of Heracles to India exceeds that from Aethiopia to Maeotis and the northernmost Scythians by a ratio of more than 5 to 3, as far as such matters admit of accurate statement.[33]

According to Aristotle, the *oikoumenē*'s length runs in an east–west direction and its breadth in a north–south direction – a crucial idea for the development of the two concepts of geographical longitude and latitude. Moreover, the Aristotelian picture of the world has another important consequence: depending on the geographical latitude of their location, observers will see differences in the apparent daily motion of the sky. However, if the

31 Arist. *Meteor*. 2.5 (362b). The two habitable zones lie between the polar circles and the tropics. See Harley and Woodward 1987, 144–145.
32 Arist. *De caelo* 2.14 (297b). Transl. J.L. Stocks.
33 Arist. *Meteor*. 2.5 (362b). Transl. E.W. Webster.

observers change their location in an east–west direction, the daily phenomena remain the same. Thus, as it is harder to determine precisely the longitudinal extent of a country or the longitudinal position of a geographical place, ancient geographers needed to develop tools and concepts to compensate for or to solve this difficulty. Aristotle does not develop or improve geographical concepts such as meridian and parallel circles, *klimata*,[34] or computable latitude or longitude, but he does link regular astronomical phenomena (such as lunar eclipses and star observations) to geographical representations.

3.2 Parallels, meridians and Hellenistic maps

Dicaearchus (*fl. c.*326–296 BCE) is commonly credited as being the first scholar to use a meridian and a parallel circle as cartographic descriptive elements, although very little is known about his map of the world.[35] From the few mentions and fragments of his work, however, Agathemerus (*c.* 1st/2nd century CE) attributes to him the creation of a straight line, which runs from the Pillars of Hercules (Gibraltar) through Sardinia, Sicily, the Peloponnese, Caria, Lycia, Pamphylia, Cilicia and the Taurus Mountains until the Imaon Mountains in the east (Central Asia), effectively splitting the *oikoumenē* into two parts: a northern and a southern section.[36] We can, therefore, regard Dicaearchus' line as the archetype of the parallel through Rhodes, which Eratosthenes (*c.* 276–*c.* 194 BCE) would later define. This fundamental line enables Dicaearchus to arrive at an approximation of the whole length of the *oikoumenē*, or at least of some of its sections.[37]

Through his corrections to the 'old map'[38], Eratosthenes develops the concept of parallels and meridians, and combines it with the description of the Earth and its inhabited parts. Strabo (*c.*64/63 BCE–*c.*21 CE) later reports that Eratosthenes used a meridian that went through Meroe, Syene, Alexandria, Rhodes, Byzantium and the Borysthenes,[39] in order to measure the circumference of the Earth and the latitudinal extent of the *oikoumenē*. Hipparchus (*c.*190–post 126 BCE) corrects some of Eratosthenes' distance data, but otherwise preserves the latter's alignment of places on this meridian.[40] It seems, however, that this line did not play the role of a reference axis, as far as the description of the inhabited world and its parts is concerned. Eratosthenes first divides the map of the known world into two zones (or northern and southern divisions), from the parallel through Rhodes. He then divides each zone into smaller sections, which he calls *sphragides*, and likens each *sphragis* to a geometrical figure, so that it can be easily represented on a map.[41] Thus, here it is the *parallel* that plays the role of an axis and divides the *oikoumenē* into two, just

34 The Greek term κλίμα (pl. κλίματα) means 'inclination,' referring to the inclination of the pole above the horizon, by which one can determine the latitude of a place.

35 Berger 1903, 367–384; Harley and Woodward 1987, 152–153; Rathmann 2013, 103–104. M. Rathmann also believes that Dicaearchus was responsible for developing the idea of degrees of latitude and longitude.

36 This imaginary line is the famous *diaphragma*, a term that is frequently used in modern scholarly publications. There is, however, no evidence that Dicaearchus himself used this term. Agathemerus writes of a simple 'right line' (Agathem. *Hypotyp.* 1.5), while Strabo, reporting on the work of Eratosthenes, uses only 'line.' There are few mentions in antique texts of the term *diaphragma* in a geographical context.

37 Dicaearchus estimates, for example, that the distance between the Peloponnese and the Pillars of Hercules measures 10 000 stadia (Str. *Geogr.* 2.4.2), which is the first evaluation of the longitudinal extent of this part of the Mediterranean Sea.

38 Str. *Geogr.* 2.1.2.

39 Str. *Geogr.* 1.4.1–2. The Borysthenes is the present-day Dnieper River. H. Berger and many modern scholars have tried to attribute the introduction of this main meridian to Dicaearchus, as it is visible on many reconstructions of the 'Dicearchus map.' See Harley and Woodward 1987, 153. However, despite its plausibility, there is no evidence for this attribution.

40 Str. *Geogr.* 1.4.1–2.

41 Str. *Geogr.* 2.1.22. See Roller 2010, 26.

as in Dicaearchus' map, thereby providing a useful framework for organizing countries. The role of the *meridian* through Rhodes is less clear, though, and none of the surviving fragments of Eratosthenes' *Geography* contains enough evidence to support the idea that the meridian was combined with the parallel in order to arrange the *sphragides*.[42]

Even in the much later geographical work of Pliny the Elder (23–79 CE), in which he describes twelve parallel circles, it is clear that a grid of meridians to locate places on the Earth's surface has still not been developed. In his list, Pliny provides a description of the countries and cities situated under each parallel, from east to west, but he does not mention any meridians.[43] Consequently, the cities linked to one parallel are spatially disconnected from the ones of the other circles, so that it is impossible to determine if, for example, Syracuse (under the 3rd parallel) is to the west or to the east of Massilia (under the 6th).

The experiences of navigators played a distinct role in this cartographic concept of places aligned on the same meridian.[44] One of the most valuable sources, according to Eratosthenes, was Timosthenes of Rhodes (*fl.* *c.*270 BCE), a navigator and author of a work on harbours,[45] who developed a system of spatial orientation based on wind directions and completed the Aristotelian compass rose.[46] According to Strabo, Timosthenes also referred to some places lying in a north–south direction.[47] Therefore, we can reasonably assume that some of the north–south alignments of places used by Eratosthenes were inspired by the work of Timosthenes. The idea of a set of places aligned along a common meridian also prefigures Ptolemy's systematization of meridians for his world map. It is, therefore, not surprising to see Timosthenes among the very few sources explicitly quoted by Ptolemy (*c.*90–*c.*168 CE).[48]

3.3 A geographical convention

The Greek word commonly used – by Ptolemy among others – to designate the longitude of a place is τὸ μῆκος, which originally referred to the *length* of an object, that is, an object's largest dimension compared with its smaller one.[49] Therefore, when the term μῆκος is used in a geographical text, its meaning relates to the field of geometry.[50] When commenting on the *sphragides* of Eratosthenes, Strabo points out that defining the length or breadth of countries that have a complicated geometrical form can be problematic:

> The fourth *sphragis* would be the one composed of Arabia Felix, the Arabian Gulf, all Egypt, and Ethiopia. Of this section, the length [μῆκος] will be the space bounded by two meridian lines, of which lines the one is drawn through the most western point on the section and the other through the most eastern point. Its breadth [πλάτος] will be the space between two parallel lines, of which the one is drawn through the most northern point, and the other through the most southern

42 Although Strabo refers several times to the north–south division of the Eratosthenian *oikoumenē* (Str. *Geogr.* 2.1.1; 2.1.21; 2.1.31), he never alludes to a western or an eastern part. Moreover, even if Eratosthenes had used meridian alignments of places to determine, for example, the edges of some of his *sphragides* (e.g. Str. *Geogr.* 2.1.39–40), no traces of any systematization of a meridian grid have survived.

43 Pl. *Nat. Hist.* 6.211–220.

44 Ptol. *Geogr.* 1.4.2. Strabo gives a similar explanation for the Rhodes–Alexandria alignment (Str. *Geogr.* 2.5.24).

45 Str. *Geogr.* 2.1.40.

46 Harley and Woodward 1987, 153; Arnaud 2005, 53–59.

47 Such as Massilia (Marseille) and Metagonion (Ras el Ma) in modern-day Morocco (Str. *Geogr.* 17.3.6).

48 Ptol. *Geogr.* 1.15.2; 4.

49 Euclid *Elem.* 1 Defs. 2 & 5; Arist. *Phys.* 4.1 (209a).

50 For example, Strabo gives the dimensions of the Iberian peninsula in the following way: "The length (μῆκος) of [Iberia] is about 6,000 stadia, its breadth (πλάτος) 5,000 stadia."(Str. *Geogr.* 2.5.27)

> point; for in the case of irregular figures whose length and breadth it is impossible to determine by sides, we must in this way determine their size.[51]

Incidentally, it is Strabo who introduces the idea of a *longitudinal extent* as opposed to a simple *length*. In the context of an oriented space (thanks to the cardinal directions), the word μῆκος designates not the largest dimension of a country but its extent from east to west, whatever the measurements of its dimensions. In order to use this term in that particular way, it is necessary to have an idea not only of the overall shape of a given country but also of its location on the Earth's surface. It is reasonable to assume that Eratosthenes already formalized this fact in his own work – even if Strabo's text is unclear on this point. Strabo places this cartographic convention, which was systematically decided upon and is clearly based on the Hellenistic picture of the *oikoumenē*, into the spotlight:

> And, in general, we must assume that *length* and *breadth* are not employed in the same sense of a whole as of a part. On the contrary, in case of a whole the greater distance is called *length*, and the lesser distance *breadth*, but, in case of a part, we call *length* any section of a part that is parallel to the length of the whole – no matter which of the two dimensions is the greater, and no matter if the distance taken in the breadth be greater than the distance taken in the length.
>
> Therefore, since the *oikoumenē* stretches lengthwise from east to west and breadthwise from north to south, and since its length is drawn on a line parallel to the equator and its breadth on a meridian line, we must also, in case of the parts, take as *lengths* all the sections that are parallel to the length of the inhabited world, and as *breadths* all the sections that are parallel to its breadth.[52]

By the time of Ptolemy, the assimilation of the term μῆκος with the concept of the east–west direction has become well and truly established.[53]

3.4 From longitudinal intervals to longitudes

With the development of a framework made up of parallels and meridians, the term μῆκος acquires over time the modern meaning of the *longitude* of a place. A particularly decisive innovation occurs when a description of places along parallels and the distance data between these places are combined. While Dicaearchus provides distances measured between places that lie on his main parallel,[54] Eratosthenes, paradoxically, does not link the parallel through Rhodes to his own distances.[55]

Artemidorus (*fl.* c. 100 BCE) remains the major source on this topic in antiquity, being quoted later by Agathemerus, Pliny the Elder (who gives some of his distances), Strabo and Marcian of Heraclea (5th to 6th century CE). Artemidorus' line is not explicitly presented

51 Str. *Geogr*. 2.1.32. Transl. H.L. Jones modified.

52 Str. *Geogr*. 2.1.32. Transl. H.L. Jones modified.

53 Ptol. *Alm*. 2.1; Ptol. *Geogr*. 1.6.3–4.

54 For instance, 3,000 stadia from the Peloponnese to Sicily and 7,000 stadia to the Pillars of Hercules (Str. *Geogr*. 2.4.2).

55 Eratosthenes' parallel of Rhodes goes through the Pillars of Hercules, the Straits of Sicily, south of the Peloponnese, through Rhodes of course, the Gulf of Issus and the Tauros Mountains (Str. *Geogr*. 2.1.1). However, Eratosthenes does not use this parallel to record distance data; rather, he gives some intermediate distances between the Indus River and the Caspian Gates, from there to the Euphrates River, to the Nile, to the Canopic mouth of the Nile, to Carthage and from there to the Pillars of Hercules (Str. *Geogr*. 1.4.5). This succession of places is not explicitly presented as a parallel circle – and this was certainly not Eratosthenes' intention. In fact, Strabo reproaches him for using broken lines to measure the inhabited world, and thereby introducing significant inaccuracies (Str. *Geogr*. 2.1.37).

by Pliny as a parallel circle,[56] but it does match the Hellenistic parallel of Rhodes: it goes through the Ganges, the Gulf of Issus, Cyprus, Rhodes, Astypalaia, Lilybaion (Marsala), Caralis (Cagliari), Gades (Cadiz) and the Sacred Cape (Cape St. Vincent in modern-day Portugal). The related distances – 8568 miles from the Ganges to the Gulf of Issus, then 2103 miles to Caralis and 1250 miles to Gades – were, for the most part, used to measure the length of the whole *oikoumenē*.

Marinos of Tyre (*fl. c.* 100 CE) improves the concept of meridians and in his work provides some tables of hour-intervals, on the model of the *klimata*, which should be understood as longitudinal sectors into which places are classified.[57] Each zone, which Marinos numbers, covers a longitudinal interval of 15°,[58] allowing Marinos to locate approximately some places at a longitudinal interval, no matter which parallel circle they occupy. Tables of both *klimata* and hours-intervals are, indeed, kept separate in Marinos' work, and it is clear that he did not use hours as a unit for the absolute longitude of places.

Ptolemy, however, found Marinos' tables unsatisfying,[59] and the former must be credited for coming up with the idea of taking distances measured along a parallel to determine the longitude of a particular place on Earth. For example, Ptolemy uses the measures established by Marinos of Tyre on the parallel through Rhodes and converts them into degrees, first, like his predecessors, in order to discuss the length of the *oikoumenē*,[60] and then to get individual longitudes for each of the mentioned places. In Ptolemy's work, the term μῆκος designates not only the length or the longitudinal extent of a country but also the absolute *longitude* of any place, as in our modern understanding.[61] In order to make this possible, Ptolemy has to fix a prime meridian. He draws the first meridian through the Fortunate Isles (entered in the catalogue of localities of his *Geography*), situated at the extreme western point of the *oikoumenē*, and gives it exactly the same function as today's Greenwich Meridian:

> [By 'locations' I mean] the number of degrees (of such as the great circle is 360) in longitude [κατὰ τὸ μῆκος] along the equator between the meridian drawn through the place and the meridian that marks off the western limit [of the *oikoumenē*], and the number of degrees in latitude [κατὰ τό πλάτος] between the parallel drawn through the place and the equator [measured] along the meridian.[62]

This specific meaning of the term μῆκος, as well as the idea of a catalogue of localities with both longitudes and latitudes, is certainly inspired by Ptolemy's astronomical work and his catalogue of stars.[63] The influence of astronomy is also clear from the geographical vocabulary he uses. When Ptolemy needs to give an absolute longitude, a particular place always lies *under* a meridian, and he takes the celestial sphere as the reference point of the location:[64]

56 Pl. *Nat. Hist.*. 2.242–243.
57 Ptol. *Geogr*. 1.15.5.
58 Ptol. *Geogr*. 1.14.9.
59 Ptol. *Geogr*. 1.18.4–6.
60 Ptol. *Geogr*. 1.12.11.
61 Ptolemy was, of course, aware of the polysemy of μῆκος in the field of mathematical cartography. When he describes the shape of Ireland (*Geogr*. 1.11.8), he needs, e.g., to specify "its length [μῆκος] *from east to west*", because the largest dimension of this island (its *geometrical* length) is in its north–south direction.
62 Ptol. *Geogr*. 1.19.2. Transl. J.L. Berggren and A. Jones modified.
63 Ptol. *Alm*. 7.5–8.1. See Graßhoff 1990, 6–16.
64 Ptol. *Geogr*. 1.1.8: "so that it will be possible to specify under [ὑπὸ] which parallels of the celestial sphere [τῆς οὐρανίου σφαίρας] each of the localities in this known part lies." Transl. J.L. Berggren and A. Jones modified.

> Hipparchus has transmitted to us …lists of the localities that are situated under the same parallel [ὑπὸ τοὺς αὐτοὺς παραλλήλους]; and a few of those who came after him have transmitted some of the localities that are *oppositely situated* …meaning those that are under a single meridian [ὑπὸ τοὺς αὐτοὺς μεσημβρινούς].[65]

However, in two cases meridians and parallels are taken to be the projection of celestial lines on the Earth's surface. On the one hand, when Ptolemy and, with some exceptions, Strabo measure a location's longitudinal distance, it is always *on to* a parallel circle:

> One should follow the number of stades from place to place, set down by Marinos, …as measured onto the parallel [ἐπὶ τοῦ παραλλήλου] through Rhodes.[66]

> It would be adequate …to divide the parallel through Rhodes, on which [ἐφ' οὗ] most of the investigations of the longitudinal distances have been made, in proportion to the meridian, as Marinos does.[67]

On the other hand, when a parallel is used to list places from east to west, it always goes *through* the cities and countries. Strabo writes:

> The Sacred Cape …lies approximately on the line that passes through Gades [διὰ Γαδρείρων], the Pillars, the Strait of Sicily and Rhodes.[68]

And Ptolemy:

> In the description of the parallels, Marinos puts the parallel through Byzantium [διὰ Βυζαντίου] through Satala [διὰ Σατάλων] and not through Trapezous [διὰ Τραπεζοῦντος].[69]

Theon of Alexandria later continues this practice of alternating between the prepositions 'under' (ὑπό) and 'on to' (ἐπί), depending on the context.[70] To sum up, Ptolemy establishes in the *Geography* a geographical and cartographic tradition to express the longitudes of places. In order to make it possible for him to find the absolute locations of places anywhere on the Earth's surface, Ptolemy perfects the Hellenistic concept of parallels and meridians by giving the word μῆκος a new meaning as well as creating a set of descriptive conventions.

4 The development of methods in Greek geography

Many descriptions of the spatial relationships between distant locations within different conceptual frameworks can be found in ancient Greek texts. However, there exists no detailed information about the methods that were used to determine these relationships. What we do have, though, are some argumentative passages which contain deductions of such values as well as data sets that are the result of ancient geographical practices. In addition, there exist a number of brief statements about the general principles that are said to have been used within the process of determining these values.

Particularly with regard to the argumentative textual passages, the only sources that can be fruitfully analysed are the *Geōgraphiká* of Strabo (*c.*64/63 BCE–*c.*21 CE), in which the author summarizes and discusses some of the arguments of Eratosthenes (*c.*276–*c.*194

65 Ptol. *Geogr.* 1.4.2. Transl. J.L. Berggren and A. Jones modified.
66 Ptol. *Geogr.* 1.11.2. Transl. J.L. Berggren and A. Jones modified.
67 Ptol. *Geogr.* 1.21.2. Transl. J.L. Berggren and A. Jones modified.
68 Str. *Geogr.* 2.5.14. Transl. H.L. Jones.
69 Ptol. *Geogr.* 1.15.9. Transl. J.L. Berggren and A. Jones.
70 Theon of Alex. *Great Commentary on Ptolemy's Handy Tables* 1.1.

BCE) and Hipparchus (*c.* 190–post 126 BCE),[71] and the *Geography*[72] of Ptolemy (*c.* 90–*c.* 168 CE). There is also a dearth of sources about the methods that concurred with certain conceptual developments, of which we are aware through what is known about the geographical work of Artemidorus (*fl. c.* 100 BCE) and Marinos of Tyre (*fl. c.* 100 CE).

4.1 Path distances, the Earth's circumference and longitudinal relationships

Questions pertaining to longitudinal relationships are an intrinsic part of Eratosthenes' approach to geography in which he describes the *oikoumenē* by means of sections and *sphragides*, particularly when determining their metrical properties and orientations. For example, Strabo's description of Eratosthenes' third section of the *oikoumenē* starts by specifying its eastern, northern, western and southern sides. Strabo construes some of these sides in terms of natural linear features, such as the Euphrates River, which is used as its western side, but, as in the case of the southern side, they do not necessarily correspond to any natural outline of the landscape:

> [T]he Persian Gulf breaks into the southern side, as Eratosthenes himself says, and therefore he has been forced to take the line beginning at Babylon as though it were a straight line running through Susa and Persepolis to the frontiers of Carmania and Persis, on which he was able to find a measured highway, which was slightly more than nine thousand stadia long, all told. This side Eratosthenes calls 'southern,' but he does not call it parallel to the northern side.[73]

All in all, the remaining fragments show that, at least in the words of Strabo, Eratosthenes' system of describing the world is much more abstract than a figurative description of the shapes of parts of countries. In addition, even though terms such as the 'southern side' or 'northern side' (which could be read to mean sides that are parallel to each other and that run from east to west) are employed and countries are described using geometrical forms, Strabo states on more than one occasion that they should be taken to be solely general descriptions. So, if in this context some distances are defined in the east–west direction, these orientations are more or less the result of chance.

If Strabo is right about how he says Eratosthenes arrived at the dimensions of these geometrical figures, the text shows at least one way of determining them: in the case of the third section, they come from taking measurements of the distances along the lines that were used as the sides of this figure. Furthermore, the discussion of the western side (which is determined by the Euphrates), in particular, suggests that no additional methods were used to cancel out the elongations that arose from using the lengths of non-straight pathways rather than the direct distances between places. Owing to the lack of sources, it is impossible to establish whether the same applies to Eratosthenes' determination of the longitudinal extent of the whole *oikoumenē*.

In sum, this means that distances that run in an east–west direction neither describe the longitudinal distance between two places nor the longitudinal extent of an area, and nothing is known about how Eratosthenes determined such values, if he ever did so. Additionally, his representation of the world using the concept of *sphragides*, as presented by Strabo, has much in common with the description of the provinces found in the so-called cadastre of Ur-Nammu.[74]

71 Especially Str. *Geogr.* 2.1.

72 Especially Ptol. *Geogr.* 1.

73 Str. *Geogr.* 2.1.23. Transl. H.L. Jones.

74 See above, 2.2 Geographical texts from the Third Dynasty of Ur.

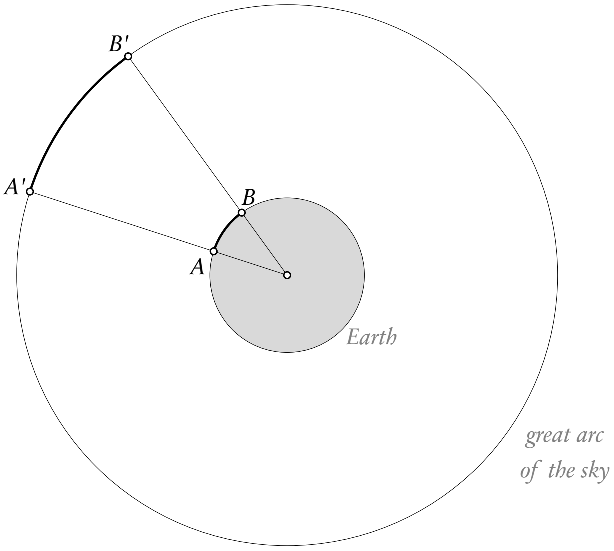

Fig. 2 | Illustration of the methods attributed to Eratosthenes to determine the Earth's circumference.

However, according to several ancient Greek and Roman authors, Erastosthenes used the knowledge he had about equal longitudinal positions of places far distant from the main parallel to determine the circumference of the Earth using a procedure that is attributed to him. Although there are almost as many different versions of what Eratosthenes did as there are reports about it,[75] and despite the different intentions of those descriptions, all share a common principle: owing to the sphericity of the Earth and its position at the centre of the universe, the ratio of an arc on the celestial sphere to a full great circle equals the ratio of the direct distance between two places A and B lying on the surface of the Earth beneath the two end points A' and B' of the aforementioned arc to the whole circumference of the Earth (Fig. 2). Even though the method, as described by Ptolemy more than 300 years later in his *Geography*,[76] works for direct distances in any direction, the sources agree that Eratosthenes evaluated a situation in which he is said to have believed that a pair of localities (Alexandria and Syene, according to Cleomedes, and Syene and Meroe, according to Martianus Capella) lay along the same meridian. In the case of this special spatial relationship between the localities, one has to know the length of a portion of a celestian meridian circle. Unlike the case of arcs oriented in arbitrary directions in the sky, the length of this arc can be derived either from solely knowing the geographical latitudes of the two places or from other, equivalent information. The sources name a large number of astronomical phenomena, methods and instruments that can be useful in this respect – even though it is by no means clear what Eratosthenes in actual fact did. In contrast to the descriptions of methods for determining latitudinal differences, there is not a single indication in any of the reports – even in the more didactic of the ancient texts – that explains how Eratosthenes determined which places were located on the same meridian.

It is not evident to what extent the views of Hipparchus differ from Eratosthenes with regard to concepts and terminology. At the very least, Hipparchus' treatise, *Against the*

75 Extensive descriptions of his method can be found in Cleomedes *On the Circular Motions of the Celestial Bodies* 1.7. Other sources are: Vitruvius *De Architectura* 1.6.9; Pliny *Nat. Hist.* 2.247; Censorinus *De Die Natali* 13.2; and Martianus Capella 6.596–598.

76 Ptol. *Geogr.* 1.3.

Geography of Eratosthenes, as handed down through Strabo, reveals a completely different way of thinking about spatial relationships and the dimensions of the world.

According to Strabo, Hipparchus, within his critique of Eratosthenes' description of the world, derived some distances from the information handed down by Eratosthenes. For example, he argues:

> [S]ince the northern side of the Third Section is about ten thousand stadia, and since the line parallel thereto, straight from Babylon to the eastern side, was reckoned by Eratosthenes at slightly more than nine thousand stadia, it is clear that Babylon is not much more than a thousand stadia farther east than the passage at Thapsacus.[77]

Such derived data provided the basis for further calculations:

> [I]f we conceive a straight line drawn from Thapsacus towards the south and a line perpendicular to it from Babylon, we will have a right-angled triangle, composed of the side that extends from Thapsacus to Babylon, of the perpendicular drawn from Babylon to the meridian line through Thapsacus, and of the meridian itself through Thapsacus. Of this triangle he makes the line from Thapsacus to Babylon the hypotenuse, which he says is four thousand eight hundred stadia; and the perpendicular from Babylon to the meridian line through Thapsacus, slightly more than a thousand stadia – the amount by which the line to Thapsacus exceeded the line up to Babylon; and then from these sums he figures the other of the two lines which form the right angle to be many times longer than the said perpendicular.[78]

On the basis of these considerations, Hipparchus deduced the latitudinal difference between Babylon and Thapsacus.

On the whole, Hipparchus used geometrical calculations, based on some given data, to deduce the properties of the spatial relationship between places. However, it is not clear from the extant parts of his text whether he made any adjustments to allow for the sphericity of the Earth. By introducing purely abstract geometrical figures, whose sides lie in an east–west or a north–south direction, Hipparchus, in the surviving fragments of his work, for the very first time systematically considered the longitudinal and latitudinal aspects of the position of a locality separately and gave a quantitative description of the positions of places relative to abstract lines, that is, lines that do not correspond to naturally occurring linear features such as rivers, shorelines and mountains.

However, all these calculations and considerations make up Hipparchus' criticism of Eratosthenes' description of the world. According to Strabo, Hipparchus' main argument consisted of drawing conclusions from Eratosthenes' data and showing that this leads to contradictions. Whereas it is clear that Hipparchus believed that this was an appropriate way of justifying his criticisms, there is no indication that such calculations were really meant to determine the spatial relationships of places on the Earth's surface. This, too, is attested by Strabo, who rebuked Hipparchus several times for failing to provide any corrections to his criticisms of Erastosthenes' work.[79]

Ptolemy's criticism of how Marinos of Tyre determined the longitudinal extent of the world should be read in a similar way: Ptolemy estimated the longitudinal differences between pairs of places along the coastline of India by calculating the east–west aspect of the distances of sea journeys in a multi-step procedure that included reducing distances to account for elongations arising from the shapes of coastlines and from non-straight

77 Str. *Geogr.* 2.1.27. Transl. H.L Jones.
78 Str. *Geogr.* 2.1.29. Transl. H.L. Jones.
79 For example, Str. *Geogr.* 2.1.38; 40; 41.

sea journeys, thereby arriving at direct distances. When carrying out the calculation, he also considered the effects of the Earth's spherical shape, until he finally devised a way of representing the result by incorporating the distances between the meridian lines of the localities into a great circle.[80] However, even though Ptolemy set out the calculation with explanations of all its intermediate steps very clearly, this is not the method he used to derive his own longitudinal positions in his *Geography*. As he himself explained, such calculations only served to justify his criticism of Marinos' value of the longitudinal extent of the *oikoumenē*.

Fig. 3 | Reconstruction of the method used by Ptolemy for determining the positions of places on the west coast of Asia Minor in his *Geography*.

80 Ptol. *Geogr.* 1.13.

What Ptolemy did in fact do to obtain the positions of places was make a geometrical construction on a plane map (Fig. 3). For example, on the western coast of Asia Minor, the positions of three locations that served as the starting points for finding the locations of other places were constructed successively, beginning at the island of Rhodes. The position of the next location, the island of Chios, was derived from its latitude and its distance from Rhodes. Whereas the latitudinal aspect of its position was given by the parallel of the place, which corresponds to a circle on the map, the location itself was determined by drawing a circle around Rhodes with the given distance between the two islands as the radius. This circle took the place of the actual, slightly egg-shaped curve on which localities with that distance from Rhodes lie on a map with Ptolemy's map projection. However, if errors are made in the latitude or in the direct distances, as in the case of the position of Byzantium, this method can lead to the introduction of large inaccuracies in the longitudinal relationships.[81]

As we have seen, the methods referred to in the argumentation for and against single statements on spatial relationships and found as explanations of the ancient transmitted geographical data made extensive use of distances, which were given in *stadia* ('stades'), of latitudinal data and of the size of the Earth's circumference. Methods that are known from astronomy also become part of the geographical toolbox, particularly where the two latter types of data are concerned. This emergence of astronomical methods in geography can be traced back to the time of Eratosthenes or earlier. The size of the Earth is crucial when determining the relative positions of localities or countries in the procedures identified above. Differing values for the circumference of the Earth, as found in the texts of ancient geographers, not only mean that a distance measured between two places corresponds to different portions of a great circle of the Earth; owing to the spherical shape of the Earth, variations in the resulting figures are generated from the combinations of distances and directions on that sphere, too. As in the case of Ptolemy's construction of the coordinates of the *Geography*, this would also affect the shapes of coastlines, countries and the relative positions of localities. So, the varying values for the Earth's circumference that were held by the ancient Greek geographers is a problematic factor of these methods, and recognised as such by the ancient scholars.[82]

4.2 Determining longitudinal distances without the aid of surveys

All the methods for determining the longitudinal relationships mentioned in the sources that explicate general principles are related to the field of astronomy. In the three extant cases detailed below, knowledge about longitudinal relationships was derived solely from observing eclipses:

1. According to Strabo, Hipparchus – in his treatise *Against the Geography of Eratosthenes* – explains that a knowledge of astronomy is essential for dealing appropriately with geographical topics. One of his reasons is that "we cannot decide accurately whether places are situated to a greater or less degree towards the east or west except by comparison of [the times of] eclipses of the sun and moon".[83]

2. In his *Almagest*, Ptolemy explains why the known parts of the world cover a quarter of the Earth, stating that this area is bounded by the equator in the south and a great circle through the poles:

81 Rinner 2013, esp. 201–231.
82 For example, see Ptol. *Geogr.* 1.2–4.
83 Str. *Geogr.* 1.1.12. Transl. D.R. Dicks.

> In the case of longitude (that is, in the east–west direction) the main proof is that observations of the same eclipse (especially a lunar eclipse) by those at the extreme western and extreme eastern regions of our part of the inhabited world (which occur at the same [absolute] time), never are earlier or later by more than twelve equinoctial hours [in local time]; and the quarter [of the Earth] contains a twelve-hour interval in longitude, since it is bounded by one of the two halves of the equator.[84]

Besides the reasoning, the whole statement about the size and position of the *oikoumenē* can be traced back to Hipparchus.

3. In his *Geography*, Ptolemy informs his readers about the general quality of the available geographical data, which was based on astronomical observations:

> Most intervals, however, and especially those to the east or west, have been reported in a cruder manner, not because those who undertook the researches were careless, but perhaps because it was not yet understood how useful the more mathematical mode of investigation is, and because no one bothered to record more lunar eclipses that were observed simultaneously at different localities (such as the one that was seen at Arbēla at the fifth hour and at Carthage at the second hour), from which it would have been clear how many equinoctial time units separated the localities to the east or west.[85]

Eclipses play a central role in all three of the above sources. Explanations of lunar and solar eclipses as the effects of two special constellations of the Sun, Moon and Earth appear in extant Greek texts that can be traced back to long before Ptolemy and Hipparchus. Solar eclipses occur when the Sun, Moon and Earth are aligned, with the Moon lying between the Sun and and the Earth (we say that the Moon is in conjunction with the Sun). In lunar eclipses, the three celestial bodies are again aligned, this time with the Earth situated between the Sun and the Moon (we say that the Moon is in opposition to the Sun). Geometrical representations of this model can also be found in ancient Greek sources: for example, in *On the sizes and distances of the sun and moon*[86] from the third century BCE, Aristarchus uses the geometry of both phenomena to determine the dimensions of the celestial bodies (Fig. 4).

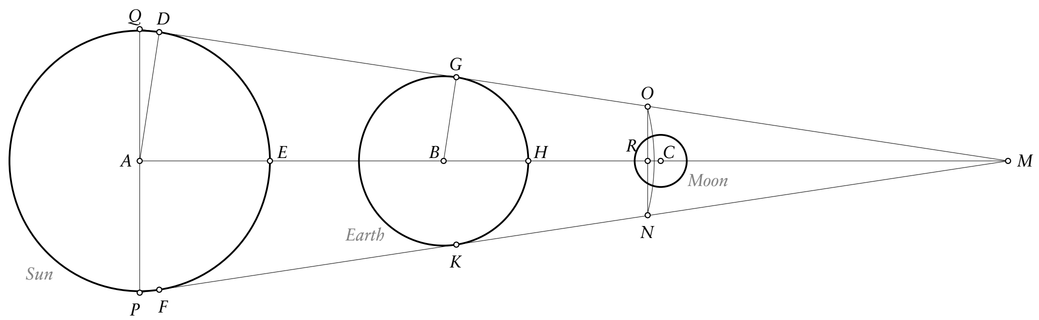

Fig. 4 | Diagram based on the geometrical explanation of lunar eclipses as used in Aristarchus' *On the sizes and distances of the sun and moon*, proposition 15.

84 Ptol. *Alm.* 2.1. Transl. G.J. Toomer modified.
85 Ptol. *Geogr.* 1.4.2. Transl. J.L. Berggren and A. Jones.
86 Heath 1913.

The general principle of Ptolemy's method of determining the longitudinal difference between two places as described in the *Geography* can be explained as follows: if it is possible to observe a lunar eclipse from two different places, the same event – the shadow of the Earth falling on the Moon – will be visible simultaneously from both places. This means that the lunar eclipse is an indicator of one and the same moment for all localities on the moon-facing side of the Earth. Through observing the local time,[87] as in the example of Arbēla and Carthage, at a defined moment of an eclipse, it is possible to calculate the time differences in equinoctial hours between two places, which is equivalent to a longitudinal difference.

Besides all their similarities, there are some fundamental differences between the three texts on determining longitudinal relationships:

- Whereas texts 1 and 3 refer at least to the existence of methods to dermine the spatial relationships between places, text 2 contains a statement about lunar and solar eclipses and its consequences for determining the longitudinal extent of the world.

- The results of these methods differ: in the method described in 3, one reaches an absolute value of the difference in equinoctial hours between two places, while 1 leads to a statement about which place lies more westerly or easterly, and it is not clear if the result includes a quantitative value of the difference.

- Ptolemy emphasizes that both the procedure in text 3 and the statement about the longitudinal extent of the *oikoumenē* in text 2 are based on comparisons of observations of the same eclipses. Strabo provides no such information for Hipparchus' procedure in text 1. Furthermore, it is not made at all clear what exactly should be observed, particularly in texts 1 and 2: the statement on the time difference also does not imply that one has to record the local times of eclipses.

- The procedure in text 3 is restricted to the usage of lunar eclipses, whereas in text 1 Strabo explicitly mentions solar eclipses. The addition of "especially a lunar eclipse" in text 2 also implies that there was at least one other type of eclipse that could be used.

Although texts 1 and 2 are unclear and differ from Ptolemy's statement in the *Geography*, they may still – at least with regard to lunar eclipses – relate to the same principle.

Even though the geometrical model of lunar eclipses is probably in some way part of the reasoning of his method, Ptolemy does not propose a geometrical procedure in the *Geography*: his text clearly suggests that one should carry out a calculation. However, what is more important than considering a specific geometrical situation is the fact that a lunar eclipse is simultaneously visible from different places on the Earth. This, too, had been known from at least the 1st century BCE, when Geminus wrote in his *Introduction to the Phenomena*:

> The eclipses of the Moon are, however, equal for all [observers]. For the coverings that occur in the eclipses of the Sun are different on account of the locations [of the observers], for which reason the magnitudes of the eclipses are different. But the Moon's falling into the shadow is equal for all during the same eclipse.[88]

87 The passage does not make clear exactly what should be observed or what kind of data needed to be documented; the recording of local times fits Ptolemy's specific example, but there are equivalent data that could have been recorded, such as the culminations of stars (see section 6).

88 Gem. *Phaen.* 11.5. Transl. J. Evans and J.L. Berggren.

Geminus contrasts this property of lunar eclipses with the characteristics of solar eclipses. He describes the spatial relationship of the celestial bodies and the variations in the phenomena, which he explains is a result of the different perspectives of the Sun and the Moon for observers at different locations:

> The eclipses of the Sun occur because of covering by the Moon. Since the Sun moves higher, and the Moon lower, when the Sun and the Moon are at the same degree, the Moon, having run in under the Sun, blocks the rays leading from the Sun toward us. Therefore, one must not speak of them as eclipses in the proper sense, but rather as coverings. For not one part of the Sun will ever be eclipsed [i.e., 'fail']: it [simply] becomes invisible to us through the covering of the Moon.
>
> For this reason the eclipses are not equal for all [observers]; rather, there are great variations in the magnitudes of the eclipses, in accordance with the differences of the *klimata*. For at the same time the whole Sun is eclipsed for some, half [is eclipsed] for others, less than half for others, while for still others no part of the Sun is observed to have been eclipsed at all. For those dwelling vertically beneath the covering, the whole Sun is invisible; for those dwelling partly outside the covering, a certain part of the Sun is seen to have been eclipsed; and for those dwelling wholly outside the covering not one part of the Sun is observed to have been eclipsed.[89]

Since Ptolemy's description of eclipses in his *Almagest* is very similar, we can be confident that he was familiar with these characteristics.

As a consequence, solar eclipses cannot be used as indicators of exactly the same moment for different places on the Earth. So, replacing observations of lunar eclipses with observations of solar eclipses in the method that is explained in the *Geography* will not lead to the correct results. Provided that it was the simultaneous occurrence of lunar eclipses that led to Ptolemy's method, we have to assume that he was aware of this consequence. This tallies with the text: his method explicitly uses lunar eclipses.

It is not easy to explain why Strabo, citing Hipparchus in text 1, and Ptolemy in text 2 both refer to solar eclipses. Of course, they may have just made a mistake – Hipparchus, or Strabo, might simply have been wrong about solar eclipses, and perhaps Ptolemy's addition of "especially a lunar eclipse" in the *Almagest* was a gentle way of pointing out Hipparchus' error. It is also possible that the errors occurred later during the process of transmission of the various texts. However, it is striking that the reference to solar eclipses as a useful data source coincides with the determinations of more general statements about the east–west relationship between two places and not with the determinations of their absolute distance. But whether Ptolemy and Hipparchus had considered any particular method of doing so using solar eclipses is simply not known.

4.3 Problem solved?

Hipparchus and Ptolemy agreed that the best way of determining longitudinal differences, both absolutely and in comparison with other methods, was to evaluate observations of eclipses, a method they described as being exact (ἀκριβής)[90] and based on unquestionable (ἀδίστακτος)[91] sources. According to Strabo, Hipparchus even demonstrated that it was not possible to determine these values using any other procedures.[92] Ptolemy

89 Gem. *Phaen.* 5.1–5. Transl. J. Evans and J.L. Berggren.

90 Str. *Geogr.* 1.1.12.

91 Ptol. *Geogr.* 1.4.2.

92 Str. *Geogr.* 1.1.12.

clearly favoured this method, as it was based solely on the observations of astronomical phenomena and thus did not depend on measurements carried out on the Earth's surface and on its circumference, which he saw as problematic.[93]

When it comes to summing up the development of ancient Greek methods to determine the longitudinal relationships of places on a spherical Earth – together with all the concessions that have to be made owing to the lack of sources and their state of transmission – one sees, first of all, the emergence of different ways of evaluating measured distances, which developed closely alongside the changes in concepts and terminology, albeit not in exactly the same way. In addition, and by Hipparchus at the latest, a second approach to this issue – that of evaluating eclipses – had become evident in the sources. According to the self-attestations in the extant sources, the problem of how to determine longitudinal differences, particularly using the evaluation of eclipses, was considered to be solved, at least in theory.

However, a thorough evaluation of the coordinate values of the *Geography* has shown that, conversely, the main methods used to determine these values in truth evaluated data pertaining to the dimensions of the Earth.[94] Moreover, there is no evidence that the eclipse-based method was ever used to generate coordinate values. This tallies with what Ptolemy told his readers in the *Geography* that "if the people who visited the individual countries had happened to make use of some such observations, it would have been possible to make the map of the *oikoumenē* with absolutely no error".[95] So, if we believe what Ptolemy told us in the *Geography*, we find a story of a method that should, in principle, have succeeded but in fact failed because of the lack of suitable observational data.

Contrary to this, one could take Ptolemy's reference to the evaluation of eclipses in the *Almagest* as the starting point of a completely different story: that an analysis of the observed eclipses was the only way to justify the reported longitudinal extent of the known part of the world.[96] If one takes this to be true, then there would have been enough observational data to carry out the calculations: there should have been either a pair of observations of the same lunar eclipse from the western and eastern ends of the *oikoumenē*[97] or observations of more than one eclipse from a sequence of pairs of places. This stands in stark contrast to what Ptolemy related in his *Geography*.

In order to add further evidence to the success story, as it has been described above, one could also refer to a procedure that is mentioned in *Dioptra*, an earlier first-century work of Heron of Alexandria.[98] In this text, Heron presents a geometrical construction procedure for determining the portion of a great arc that lies between two places, starting with their latitudes and an observation of the local times of the same lunar eclipse from both localities.[99] As concerns the longitudinal aspect, the underlying principle is the same as the general principle for determining the longitudinal difference, and so through this he extends an already known procedure to a more general case. By relating the portion of the great arc to the Earth's circumference, the direct distance between the two places is only determined in the last step of the procedure. Thus, by using the advantages of astronomical and mathematical methods within a geographical approach, Heron's procedure can be seen to be in line with Ptolemy's assessment of which methods should be used in geography.

93 Ptol. *Geogr.* 1.2–4.
94 Rinner 2013, 201–324.
95 Ptol. *Geogr.* 1.4.1.
96 Ptol. *Alm.* 2.1.
97 It is only possible to observe the same lunar eclipse from places with a longitudinal difference of about 180°.
98 Heron, *Dioptra* 35. Edition in Schöne 1903. For an English translation, see Sidoli 2005.
99 Sidoli 2005 and Sidoli 2011.

Nevertheless, a lack of suitable observational data may also be present in Heron's explication of his extended procedure: the observational data of the same lunar eclipse seen from Alexandria and Rome that was used as the example in the construction procedure suggests that it was, at least in part, fabricated.[100]

All in all, the extant self-attestations of the ancient scholars on the usage and usability of eclipses to determine the longitudinal relationships between two places as they are preserved in the context of geography have led us to a totally inconsistent picture of the way they worked. Thus, we still have some way to go before we are able to understand fully the connection of observational data of eclipses, particularly of lunar eclipses and longitude in Greek scientific texts.

5 On the feasibility of using eclipses for measuring geographical longitude

5.1 Lunar theory and observations

Ptolemy developed his lunar theory in the fourth book of the *Almagest*, his aim being to improve on all the astronomical knowledge that had so far been acquired. Knowledge about the motion of the Moon was the most complex challenge facing astronomers of antiquity, for without it calendars, lunar phases and eclipses could not be calculated. Despite the fact that it was known that the Moon's position changes from one to the next Full Moon and that the mean length of the synodic month is 29.530594 days, ancient scholars were still unable to calculate accurately enough even the beginning of the month as a pattern of 29 or 30 days using these precise mean motions. The Babylonians, who discovered the periods of lunar motion, solved this challenge by introducing, around 400 BCE, a set of remarkably accurate mathematical procedures to determine these periods.[101] Greek astronomers could not possibly compete with the accuracy of Babylonian models by developing independent theories; they could only build on the achievements of their predecessors. However, to do so, Greek scholars needed to be able to refer to the Babylonian observational records, which only became possible shortly before the time of Hipparchus (*c.* 190–post 126 BCE), although Babylonian data on the principal lunar period relations were certainly known in Greece when Euclid was active (*fl. c.* 300 BCE), and possibly slightly earlier.

References in the *Almagest*, particularly in Books 3 and 4, reveal that, around 130 BCE, Hipparchus had access to the complete observational records of the Babylonians and was aware of their work on the primary lunar period relations. With this information he was able to transform the highly precise numerical periods of the Babylonians, together with their huge empirical basis, into geometrical models of celestial motion, from which he successfully constructed a solar theory. Hipparchus also drafted a first model for a lunar theory, although he failed to transfer the method to planetary models. Hipparchus himself acknowledged that he was unable to fully develop his lunar theory;[102] he reported that there were discrepancies between his models and the Babylonian observations, and concluded that they were incomplete.

It was then left to Ptolemy (*c.* 90–*c.* 168 CE), as he wrote in the introductory passages of Book 4 of the *Almagest*, to take over, three centuries later, from where Hipparchus had left off. The titles of Hipparchus' major works, like the majority of his theoretical writings, are mostly only known from references in the *Almagest*, and it is clear that Ptolemy's main and

100 See Sidoli 2011 for a discussion on the problem of the eclipse in Heron, *Dioptra* 35.
101 O. Neugebauer 1975.
102 O. Neugebauer 1975, 175.

extensive source of data and theories for this work of his came from Hipparchus. Where possible, Ptolemy adopted Hipparchus' results,[103] simply copying, for example, the solar theory of his esteemed predecessor. However, he did make significant modifications to Hipparchus' lunar models.

To construct his epicyclic models for lunar theory, Hipparchus took over the numerical parameters of the motions from the Babylonians, checking their initial parameters against their observational records. Ptolemy also utilized the Babylonian periodicities of the mean lunar phenomena and used them, unchanged, in his basic geometrical models.[104] These periodicities are cited at the beginning of Book 4 of the *Almagest*. Then, however, Ptolemy adopted a twofold strategy to solve the remaining discrepancies with the observational records: he reworked Hipparchus' evaluations of Babylonian data and introduced additional refinements to the models and the parameters. To do so, Ptolemy must have analysed the Babylonian observations as well as later Greek data, most importantly those collected at Alexandria.

Hence, Ptolemy's methodology involved an iterative process of model development by which he tried to identify, step by step, the critical discrepancies that existed between preceding theories and a number of selected historical observations, in order to improve on the theories. The discrepancies did not falsify the earlier theories. What is clear is that Ptolemy did not introduce a lunar theory based on his own, independently acquired empirical evidence. Rather, he extended the Babylonian theory of periodicities, which had been transferred by Hipparchus, and introduced necessary corrections to the second anomaly of the Moon's motion. In order to carry out these corrections, Ptolemy must have had access to the entire set of available empirical records.

It has often been observed that Ptolemy could not have carried out the corrections to his lunar theory on the basis of his own observations. Ptolemy knew only too well that he could not use just any observations of the Moon's position to prove his theory. For the discrepancies to be explained clearly by the corrections, the geometrical positions of the Moon relative to the Earth and the Sun needed to be set in a particular way. And only lunar eclipses were appropriate to such a procedure, as Ptolemy states explicitly in the *Almagest*:

> Rather, to establish our general notions [on this topic], we should rely especially on those demonstrations which depend on observations which not only cover a long period, but are actually made at lunar eclipses. For these are the only observations which allow one to determine the lunar position precisely: all others, whether they are taken from passages [of the moon] near fixed stars, or from [sightings with] instruments, or from solar eclipses, can contain a considerable error due to lunar parallax. […]
>
> This is the reason why in the case of solar eclipses, which are caused by the moon passing below and blocking [the sun] (for when the moon falls into the cone from the observer's eye to the sun it produces the obscuration which lasts until it has passed out [of the cone] again), the same eclipse does not appear identical, either in size or in duration, in all places.[105]

Characteristically, the other observation data cited by Ptolemy are exactly of the kind that can be found in Babylonian astronomical data. Thus, Ptolemy did not even use those ob-

103 See O. Neugebauer 1975 and Toomer 1984.

104 Although Ptolemy attributed a small correction of 1/12 of a second to the mean synodic month to Hipparchus, he notably based his lunar theory on the Babylonians' standard value of 29;31,50,8,20d. Cf. O. Neugebauer 1975, 78.

105 Toomer 1984, 173–174.

servations that could be made by the observation tools described in the *Almagest*. Ptolemy used only what he believed were the most reliable and accurate data: the observational reports so carefully compiled by the Babylonians.

> So, for the above reasons, we declare that we must not use, for this purpose, observations of the moon into which the observer's position enters, but only lunar eclipse observations, since [only] in these does the observer's position have no effect on the determination of the moon's position.[106]

5.2 Hipparchus and the Babylonians

In order to make a thorough comparison between the observational data and the geometrical models, Hipparchus had tried to reduce the former. However, as Ptolemy later reported, Hipparchus had been unable to carry out this correction, and, therefore, Ptolemy had no option but to recalculate the eclipses that Hipparchus had selected and correct them where necessary:

> However, Hipparchus already proved, by calculations from observations made by the Chaldaens and in his time, that the above relationships were not accurate.
>
> That is why, as we can see, Hipparchus too used his customary extreme care in the selection of the intervals adduced for his investigation of this question: he used [two intervals], in one of which the moon started from its greatest speed and did not end at its least speed, and in the other of which it started from its least speed and did not end at its greatest speed.[107]
>
> We shall now demonstrate the lunar anomaly in question, by means of the epicyclic hypothesis, for the reason mentioned. [For this purpose] we shall use first, among the most ancient eclipses available to us, three [which we have selected] as being recorded in an unambiguous fashion, and, secondly, [we shall repeat the procedure] using, among contemporary eclipses, three which we ourselves have observed very accurately.[108]

Ptolemy went on to give an account of the oldest observed lunar eclipse of antiquity, which is reproduced below (dates in square brackets are Toomer's additions), and then followed it with an analysis of the observation.

> First, the three ancient eclipses which are selected from those observed in Babylon.
>
> The first is recorded as occurring in the first year of Mardokempad, Thoth [I] 29/30 in the Egyptian calendar [-720 Mar. 19/20]. The eclipse began, it says, well over an hour after moonrise, and was total.[109]

Ptolemy does not quote the source verbatim: some information was converted to the Egyptian calendar, and the time unit of 'an hour' does not correspond to the unit used to record Babylonian eclipse reports.[110] The unit of time used by the Babylonians was the 'UŠ' or 'time degree', 360 degrees of which make up a full rotation of the night sky, with 1 UŠ thus corresponding to four equinoctial minutes. It is generally assumed that water clocks were used to take these measurements. In the early reports of this eclipse, the numer-

106 Toomer 1984, 174.
107 Toomer 1984, 178.
108 Toomer 1984, 190.
109 Toomer 1984, 191.
110 Steele 2000 and Steele 2005, 5.11–5.12

ical resolution of the time is 5 UŠ or 20 equinoctial minutes, which is problematic when one considers that the Babylonians had a sophisticated way of recording time intervals (for instance, a set of six time intervals between the rising and setting of the Sun and Moon). I have another difficulty with Ptolemy's reporting of this early observational record of an eclipse. Recalculations of the data show that the eclipse did not begin until over an hour after *moonrise*, as reported by Ptolemy, whereas Babylonian astronomers typically recorded the time that had passed after *sunset*, as just mentioned.[111]

Then, in three critical steps, Ptolemy converted the reported local time at Babylon to standard time at Alexandria, which henceforth became the reference time on which Ptolemy based all his astronomical theories:

> Now since the sun was near the end of Pisces, and [therefore] the night was about 12 equinoctial hours long, the beginning of the eclipse occurred, clearly, 4 1/2 equinoctial hours before midnight, and mid-eclipse (since it was total) 2 1/2 hours before midnight.
>
> Now we take as the standard meridian for all time determinations the meridian through Alexandria, which is about 5/6 of an equinoctial hour in advance [i.e. to the west] of the meridian through Babylon. So at Alexandria the middle of the eclipse in question was 3 1/3 equinoctial hours before midnight, at which time the true position of the sun, according to the [tables] calculated above, was approximately Pisces 24 1/2 degrees.[112]

Thus, in order to be able to convert the reported local time in UŠ to equinoctial hours, Ptolemy needed to obtain more precise information on the duration of the day of the eclipse, such as the length of that day's night. He deduced this information from the data relating to the position of the Sun, the seasonal position of which determines the duration of the day. Tables to carry out this conversion were available to both the Babylonians and to Ptolemy. The Babylonian data on the duration of the eclipse are not cited in this passage, and Ptolemy assumed a length of five hours, even though the total phases of lunar eclipses are always much shorter in duration. Ptolemy determined that the middle of the eclipse lasted for half of that time.

The moment of mid-eclipse is, for the eclipses that Hipparchus selected, significant, as it is only at the middle point of a lunar eclipse that the Moon is positioned exactly opposite the Sun. Since it was known how to calculate the position of the Sun, the position of the Moon – and consequently the position of the Moon in mid-eclipse, irrespective of the peculiarities of the lunar models – could also be accurately calculated. However, in order for Ptolemy to compare the data of the Babylonian astronomers with his own observations from Alexandria, he had to make allowances for an effect known since the time of Hipparchus: as Alexandria is located west of Babylon, Ptolemy had to take into account the time difference between Babylon and Alexandria when converting the eclipse observations of the Babylonians. These calculations are not, of course, to be found in the Babylonian texts, where it is implicitly assumed that all the observations were made at locations on similar geographical longitudes. As these differences could not be ignored, Hipparchus and Ptolemy were obliged to account for longitudinal differences in the process of constructing their lunar theories. Thus, it is at this point that a quantification of longitudinal differences becomes apparent in the sources.

111 Steele, Stephenson, and Morrison 1997, 344–345

112 Toomer 1984, 191.

5.3 Determining geographical longitude using lunar eclipses

In the *Geography*, Ptolemy writes that he considers one method of measuring geographical longitude to be particularly applicable: the simultaneous observation of the same lunar eclipse from different places on Earth. While the part of the Moon being covered by the shadow of the Earth takes place at the same instant and appears the same to all observers over half of the surface of the Earth, observers at different geographical longitudes will see the same lunar eclipse at different local times. As the Earth rotates once every twenty-four hours, it turns at an angle of 15° per hour. Only a simple multiplication is, therefore, required to convert the observed time differences in equinoctial hours to degrees of longitude – a calculation that Ptolemy's predecessor Hipparchus had proposed as the basis for obtaining longitudinal differences.

However, even though the underlying mathematical operations are simple and the phenomenon of a lunar eclipse can be observed without the need for complex instruments, measuring local time was the challenge faced by the scientists of Antiquity. In the *Geography*, Ptolemy complains about the lack of available data:

> Most intervals, however, and especially those to the east or west, have been reported in a cruder manner, not because those who undertook the researches were careless, but perhaps because it was not yet understood how useful the more mathematical mode of investigation is, and because no one bothered to record lunar eclipses that were observed simultaneously at different localities (such as the one that was seen at Arbēla at the fifth hour and at Carthage at the second hour), from which it would have been clear how many equinoctial time units separated the localities to the east or west.[113]

In this citation, Ptolemy is not complaining about the absence of reports on the occurrences of lunar eclipses; there existed a large number of such accounts in Antiquity. What he is criticising is the fact that these reports did not include the precise timings of the phases of lunar eclipses as observed from different locations.

In order to demonstrate the geometrical features of his lunar theory, Ptolemy describes eighteen lunar eclipses in the *Almagest*. The oldest lunar eclipse he mentions dates from 720 BCE; he also refers to three lunar eclipses from the fourth century BCE, but not the eclipse that took place at Arbēla in 331 BCE, even though this eclipse had been observed by Babylonian astronomers, and it is highly probable that their records were included in the list that was transmitted to Ptolemy. There are two factors that could explain this omission. Firstly, it is possible that this particular eclipse might not have been considered as suitable as the other eclipses for determining the geometry of Ptolemy's lunar model. Secondly, and more importantly, the times of the different stages of this eclipse would have had to be corrected by the geographical longitudes of the locations, the values of which Ptolemy analysed only later in his life, that is, when writing his *Geography*, where it served a different purpose and featured prominently in his exposition of the geometrical differences in geographical longitude.

The eclipse at Arbēla that Ptolemy analyses in the *Geography* took place on 20 September 331 BCE, eleven days before Alexander the Great's victory over the Persians at Gaugamela. Many historical sources refer to this eclipse: since the ancient Greeks as well as the Persians regarded lunar eclipses as being divine signs, this lunar eclipse, occurring so shortly before the Battle of Gaugamela, was regarded as being particularly significant. Its astronomical data are shown in Figure 5.

113 L. Berggren and Jones 2000, 1.4.2.

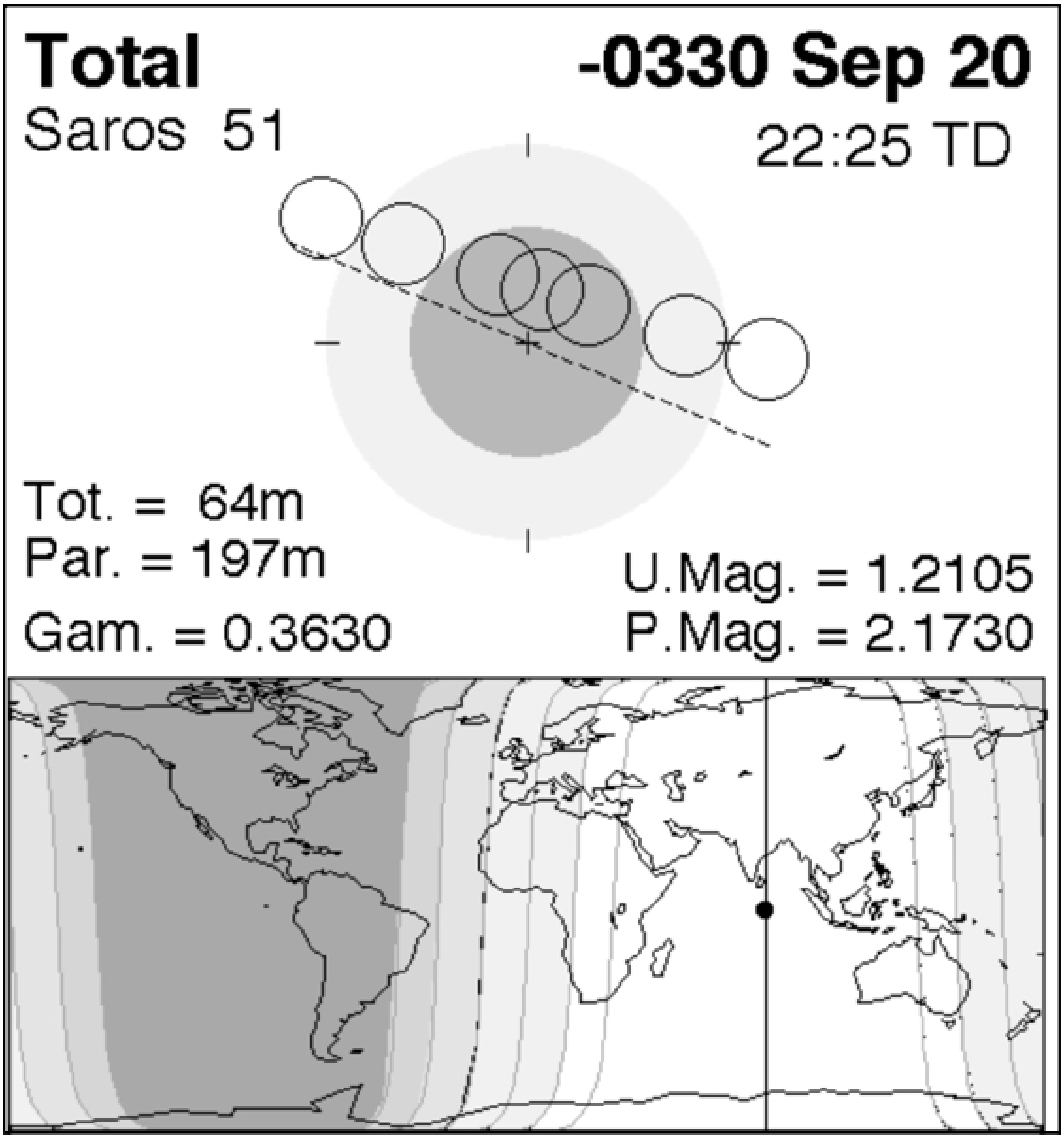

Fig. 5 | Circumstances of the lunar eclipse of 20 September -330.

Figure 6 explains the information shown in Figure 5. The meridian at the time of the middle of the eclipse passed through the Bay of Bengal. The black spot on the world map indicates the point at which the Moon crossed the zenith at mid-eclipse.

Using the Besselian elements provided by Jean Meeus, Fred Espenak has recalculated the astronomical characteristics of the lunar eclipse at Arbēla, which have been published in a five millennium catalogue of eclipses by NASA.[114] This work has greatly facilitated the calculating of the eclipse times for the locations referred to by Ptolemy.

The eclipse of Arbēla lasted for 197 minutes, from the moment when the Moon entered the umbra to its exit. Of these 197 minutes, the Moon was in total eclipse for 64 minutes. The upper half of Figure 5 illustrates schematically the movement of the Moon through the Earth's umbra. The Moon moves from right to left through the Earth's umbra during the course of one night, while the sky moves from east to west. The darkly shaded parts of the diagram represent the umbra. In these areas, no direct sunlight reaches the

114 http://eclipse.gsfc.nasa.gov/LEcat5/LEcatalog.html.

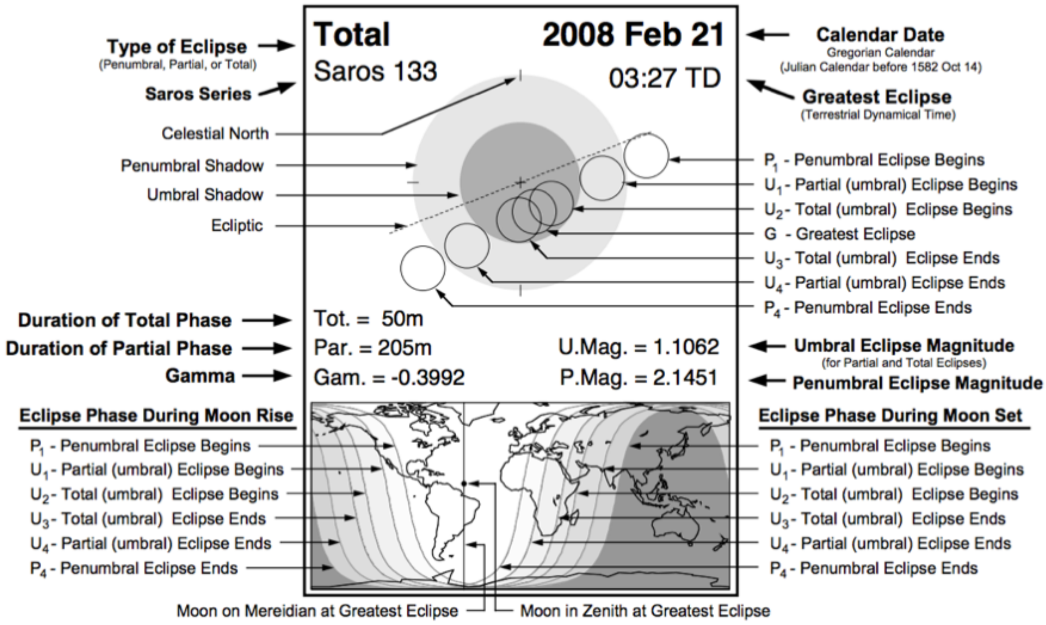

Fig. 6 | Key to the eclipse diagram of Figure 5, according to Espenak/Meeus.

surface of the Moon. A lunar eclipse occurs at the same instant to every observer on the night side of the Earth; only the observed local times at which an eclipse occurs differ.

In the lower, left part of the diagram in Figure 5, the vertical lines show those geographical borders within which the different phases of the lunar eclipse became visible during moonrise. The line called "U1" (see Figure 6) indicates the entry of the Moon into the Earth's umbra at moonrise, and in Figure 5 it passes through the western coast of northern Africa. Thus, in Carthage – unlike in Arbēla – the lunar eclipse was only visible on the eastern horizon after the Moon had risen.

In the aforementioned quotation from the *Geography*, Ptolemy refers only to the Arbēla eclipse as an example of an observation of the same lunar eclipse from two different locations. Ptolemy reports that the eclipse occurred at the fifth hour of night, while in Carthage, the same eclipse was said to have been observed at the second hour of the night after sunset. As the eclipse took place near the equinoxes, Ptolemy could equate temporal hours with equinoctial hours. A difference of three equinoctial hours yields a difference in length of 3 x 15°, which corresponds to a difference of 45° in geographical length. In reality, the time and longitudinal differences between Arbēla and Cartage were significantly smaller: the value given in the *Geography* for the difference in geographical longitude is 45° 10',[115] while, rather than three equinoctial hours, the observed time difference comes to only two and a quarter equinoctial hours.

Different viewpoints concerning the reasons for these discrepancies can be found in the literature.[116] In contrast to Hipparchus, Ptolemy introduced a methodological novelty in the way he calculated quantitative data: he stopped using intervals to represent quantitative data and calculated definite values, which fell within an interval that could be theoretically corroborated.[117]

Pliny the Elder's observations of the same eclipse differ from Ptolemy's:

115 A geographical longitude of 34° 50' is recorded for Carthage and 80° for Arbēla.
116 Cf. L. Berggren and Jones 2000 and O. Neugebauer 1975.
117 Graßhoff 1990, Graßhoff 2014.

> Consequently inhabitants of the East do not perceive evening eclipses of the sun and moon, nor do those dwelling in the West see morning eclipses, while the latter see eclipses at midday later than we do. The victory of Alexander the Great is said to have caused an eclipse of the moon at Arbēla at 8 p.m. while the same eclipse in Sicily was when the moon was just rising.[118]

The two times that Pliny mentions closely match the actual start of the partial lunar eclipse at both locations. The times of the Arbēla lunar eclipse, which can be found on the NASA website, in Universal Time (UT) for the time zone + 3h are, the same eclipse for Carthage in UT + 1h:
According to UT, in Arbēla the partial eclipse started at 19:46, the total eclipse at 20:52 and the middle of the eclipse took place at 21:24, while the total eclipse ended at 21:56 and the partial eclipse ended at 23:03 (at the beginning of the fifth hour). In Carthage (in the time zone UT + 1 hour), the partial eclipse started at 17:46, with the moon just below the horizon. The total eclipse began at 18:52 and the mid-eclipse took place at 19:24. The total eclipse ended at 19:56 and the partial eclipse at 21:03. The time differences reflect the adjustments of the universal time for the appropriate time zones. The differences in local time (the angular position of the celestial sphere) for both places reflects the difference in geographical longitude.

At first sight, the reports of Pliny and Ptolemy seem very different. Pliny refers to Sicily, while Ptolemy cites Carthage. According to Pliny, the eclipse of Arbēla took place at the second hour, while Ptolemy records that it took place at the fifth hour. However, the apparent contradiction can be resolved if one takes into account the data reduction procedures that Ptolemy probably used.

Ptolemy needed to develop a methodological procedure for generating appropriate data from witness reports, which should not be confused with data that had been methodically obtained. A suitable way of measuring time outside astronomical procedures was not practiced in Antiquity and would only be developed much later. Ptolemy made some headway in this area, which marks an important step in the development of quantitative observations. It is quite plausible that Ptolemy used the following method to compute the eclipse data:

1. For his data analysis, Ptolemy had to rely on historical reports of lunar eclipses.
    (a) Apart from the difficulty of decoding the local calendar, these historical data refer either to the entire day or, in exceptional cases, to a watch, which comprised a third of a night.
    (b) Besides Pliny, there are no non-astronomical reports of eclipses that attest to the use of hours as a way of measuring time.

Beginning of eclipse (in Universal Time)						
	begin partial	alt	begin total	alt	mid-eclipse	alt
Arbēla	19:46	+19	20:52	+30	21:24	+36
Carthage	17:46	+11	18:52	-08	19:24	+05
End of eclipse						
	end total	alt	end partial	alt		
Arbēla	21:56	+40	23:03	+48		
Carthage	19:56	+18	21:03	+29		

Tab. 1 | Lunar eclipse data in UT for Arbēla (+ 3h) and Carthage (+ 1h).

118 Pliny, Nat. Hist. II, 180.

(c) A report might include mentions of Arbēla at the beginning of the eclipse during the first watch, while, according to another source, the moon started to rise just after the eclipse began in Sicily.

2. The next stage of the analysis might have involved Ptolemy standardising the existing, non-scientific information. In a comparison of the eclipse times of both locations, the same phase of the eclipse – either the beginning, the middle or the end of a lunar eclipse – needed to be observed. As the moon had already risen in Sicily after the eclipse had begun, only the end of the eclipse remained a possible reference point. Determining the middle of an eclipse is a task that only an astronomer, not a casual observer, can carry out, since a mid-eclipse cannot be observed directly.
   (a) Thus, the duration of the partial eclipse had to be added to the time of the beginning of the eclipse. Ptolemy knew this value from the calculations that formed the basis of his lunar theory. This would explain how the second hour became the fifth hour at Arbēla, while the eclipse seen from Sicily ended at the second hour.
   (b) Since Sicily and Carthage were considered to be located roughly on the same meridian, Carthage, which lies on approximately the same geographical latitude as Arbēla, was the better location for the second observation.

3. The calculated time difference of the simultaneous event at two locations yields the longitudinal difference of the places.

Such procedures for evaluating raw data were self-evident to Ptolemy: non-scientific reports on natural phenomena could not have the same systematic quality of data that had been corroborated by a mathematician. Ptolemy would have chosen a suitable data format (for example, for measuring the time precisely) and appropriate instruments, and he would have avoided systematic errors and taken into account homogeneous measuring conditions. However, Ptolemy had no documented quality data at his disposal. And, as he was unable to develop an alternative procedure for determining geographical longitude astronomically other than by lunar eclipse observations, Thus, Ptolemy had to leave the measuring of geographical longitude by astronomical means to future astronomers.

6 Syriac reception

Attention has recently been focused on Syriac scholars who took an interest in the field of geography,[119] and, since some of them were active in late antiquity and had strong links with Alexandria,[120] their work deserves to be included in studies of this branch of knowledge over this time period. In our inquiry about longitude, we will examine, in particular, the writings of Severus Sebokht (*fl.* mid-seventh century CE), who wrote original scientific texts in the Syriac language and who also translated Greek Alexandrian astronomical texts, which would otherwise have been lost, into Syriac.

We find discussions about the connections between geographical longitude and astronomical observations in Chapters 14 and 15 of Severus' *Treatise on the Constellations* and in the second part of the *Treatise on the Astrolabe*.[121] Severus Sebokht, who was a

119 For a panorama of Syriac sources (fifth to thirteenth centuries CE), see Defaux 2014. In particular, the author draws attention to the repartition of the Earth into seven climate zones, which is a notable characteristic of these sources.

120 See Villey 2014.

121 An edition of the *Treatise on the Astrolabe* in Syriac and a French translation were published by François Nau in 1899 (see Sev. Seb. *Treat. Astro.*); a French translation of the *Treatise on the Constellations* was published by Nau in 1931–1932 and was reprinted in Nau 2014, 183–290 (see Sev. Seb. *Treat. Const.*).

miaphysite bishop and abbot of the monastery of Qenneshre (a seat of Greek and Syriac learning in north Syria), wrote the *Treatise on the Constellations* in Syriac in 660 CE. The *Treatise on the Astrolabe* pre-dates it – the *Treatise on the Constellations* refers twice to the other work – and is composed of two parts, of which the second section is believed to be a translation of a lost, late antique Greek treatise.[122] In both treatises, the late antique authors (Severus and a Greek Alexandrian author of the beginning of the sixth century) assume that a planispheric astrolabe can locate a place in longitude. Since the Syriac translation of the second part (also called *Skolyon on the Astrolabe* by Severus) of the *Treatise on the Astrolabe* is the earliest extant evidence of the use of the planispheric astrolabe for geographical purposes, its documentation, which exists only in Syriac, is extremely valuable.[123] Below, we give the English translations of the more interesting passages and provide a few comments.

6.1 *Skolyon on the Astrolabe*

In *Qānonē* (or 'exercises') 14 and 15 of the *Skolyon on the Astrolabe*, that is to say, the part Severus translated, the author – whom Severus calls "the *philosophos*" – explains how to use the astrolabe to calculate the longitude of two cities. A new critical edition of the whole treatise in Syriac, with a translation in English, is in preparation, and on this basis we reproduce the following translated passages: [124]

> *Qānonā* 14: How shall we know the longitude of a first city in relation to another, which is the more easterly and which is the more westerly? It is possible to know this, thanks to an eclipse of the Moon or an eclipse of the Sun. For example: with the astrolabe, we take the middle of the sky [the meridian] of both cities at the moment of the beginning of the eclipse, or at the end, or at any moment of the eclipse. And we also send the astrolabe[125] to the wanted [other] city to know their relative position, east or west. It has been written above how we take the degree of the meridian of both cities;[126] afterwards we compare the degree of the diopter's degrees indicator of the two cities with one another. Where the number – given by the diopter's degrees indicator – is greater, we say that it [the city] is more easterly than where the number – given by the indicator of the degrees indicator – is smaller.

122 Thanks to the discovery and examination of a textual supplement in a new Syriac manuscript, I was able to demonstrate (Villey 2015) that the Greek text was most probably written by Ammonius of Alexandria in 523 CE. On the basis of a later Arabic document, both Otto Neugebauer and Alain Ph. Segonds previously put forward the hypothesis that the author of the second part was Theon of Alexandria (see O. Neugebauer 1949, 245; Segonds 1981, 29–32).

123 John Philoponus tells us nothing about the geographical application of planispheric astrolabes in his mid-sixth-century treatise on the instrument (see Segonds 1981, 37). Ptolemy wrote a treatise on the planispheric astrolabe (also called a *planispherium*), which is only preserved in Arabic, but it deals solely with the construction, not the use, of the instrument (see Sidoli and J. L. Berggren 2007). The planispheric astrolabe needs to be distinguished from the armillary sphere (also known as a spherical astrolabe), which Ptolemy used in Book 5 of his *Almagest* and which was described in detail by Pappus of Alexandria, Theon of Alexandria and Proclus of Athens (for a complete study of these sources, see Rome 1927). The Greek writings of Pappus and Theon on the armillary sphere were published in Rome 1931; Proclus' treatise is preserved in a Latin translation of an Arabic version of the original (see Proclus *Hypotyposis* 6, 198–202).

124 See also Nau 1899, 58–60 (text) and 292–293 (French translation). I would advise readers not to refer to the English translation in Gunther 1932 because of its inaccuracies.

125 As F. Nau suggested, we should probably take "we also send the astrolabe" to mean "we also send the results obtained using the astrolabe".

126 The reader is clearly invited to reread the method described in the first exercise (*Qānonā*) of the *Skolyon* (see Sev. Seb. *Treat. Astro.* 2.1, transl. Nau 1899, 87–90).

Qānonā 15: How shall we know the difference of midday between two cities? We find again the difference of midday between two cities in this way: we subtract the smaller <indications of> time of the diopter's degrees indicator from the greater <indications of> time; we divide the remainder by 15, and the number of <indications of> times that is found corresponds to the number of equinoctial hours. Thanks to this number, we say that we have the hours, but also the distance between these cities in respect of their position, west and east. It is always midday first in the city which is more easterly: when at Carthage it is the third hour, it is the sixth at Arbela, because the difference between the meridian of Carthage and the one of Arbela is three hours. Arbela is three hours more easterly than Carthage. For example: the longitude of Arbela is 80 degrees; the longitude of Carthage is 35 degrees. When the smaller <number of> degrees is subtracted from the greater <number>, 45 degrees remain; when divided by 15, they make three equinoctial hours.

In these extracts, the author is clearly using the antique world's well-known method of determining the longitude of a location: the first step consists of observing an astronomical phenomenon;[127] the second of measuring the local time, with the planispheric astrolabe used in both operations. In relation to the problem of determining geographical longitudes, it is significant that in the extracts quoted above the author does not lay claim to finding the absolute longitudinal coordinates of a place; thanks to the planispheric astrolabe, he can establish which of the two cities is the most easterly as well as give the distance between the cities in degrees longitude.

However, since the coordinates given in the text for the longitude of Arbēla and Carthage have clearly been taken from the work of Ptolemy[128] and since no attempt seems to have been made to improve these figures or to apply another example of an eclipse, we can legitimately question whether the original Greek author of this text ever took these measurements himself and whether he compared the longitude of the two cities with an astrolabe. The author even erroneously suggests that a solar eclipse can be used to determine the longitude of a place.[129]

6.2 *Treatise on the Constellations*

One hundred and fifty years after the Greek *philosophos* had composed a text on the astrolabe, Severus Sebokht translated it and used the information within it in another tract of his composition, the *Treatise on the Constellations*. Chapters 17 and 18 of this Syriac treatise deal only with the discipline of geography, while in Chapters 12 and 15 Severus makes some interesting remarks on the notion of linking geographical longitude to astronomical longitudes.

In Chapter 12 Severus lists the ten most important circles of the celestial sphere (the Arctic, the two tropics, the equator, the Antarctic, the zodiac, the ecliptic, ἀξώνιος, the meridian and the horizon) and in so doing explains his understanding of the concept of the *meridian*.[130]

127 In reality this involves measuring the sidereal time.

128 Ptolemy used this example of a lunar eclipse observed simultaneously at Arbēla and Carthage in the *Geography*. See Chapter 6, 34–36, for a discussion of this observation.

129 Strabo (*Geogr*. 1.1.12) attributes this fallacy to Hipparchus.

130 "Le neuvième [cercle] est celui qui est nommé μεσημβρινός (méridien), qui est tracé directement par les pôles, du nord au sud, par les deux moitiés de sphère, à savoir du haut en même temps et du bas et qui coupe en même temps tous les cercles dont nous avons parlé, c'est-à-dire qu'il est coupé par eux, mais il ne tourne pas avec l'ensemble des cercles de la sphère, mais il reste immobile par le κέντρον du milieu du ciel, à savoir celui du dessus et celui du dessous de la terre (zénith et nadir), et il partage exactement

In Chapter 15 he explains that geographical longitude is measured by using the 'Fortunate Isles'[131] as the basis for a prime meridian, which corresponds to the prime meridian used by Ptolemy in the *Handy Tables* and the *Geography*. Severus then refers specifically to Ptolemy's *Handy Tables* and to the use of the astrolabe, writing that the meridian circle changes according to the longitudinal location of a city.[132] After that, he assumes that the difference in longitude between two cities can be observed and calculated by making an astronomical observation:

> C'est comme aussi par l'observation, par exemple des éclipses de soleil et de lune. Car lorsqu'il y a une éclipse, par exemple pour la ville de Ctésiphon, et aussi pour Alexandrie, le même jour, on ne la trouve pas à la même heure, à savoir à l'heure de Ctésiphon et à celle d'Alexandrie. Cette éclipse a été vue à Ctésiphon avant Alexandrie de une heure $\frac{1}{5}$ et $\frac{1}{10}$. Comme la longitude de Ctésiphon est de 80° et celle d'Alexandrie de $60°\frac{1}{2}$; si nous retranchons $60\frac{1}{2}$ de 80 il reste $19\frac{1}{2}$ ce qui fait une heure $\frac{1}{5}$ et $\frac{1}{10}$ puisque 15° font une heure. De ce que le soleil se lève à Ctésiphon, avant de se lever à Alexandrie, il est évident qu'il y fera aussi plus tôt le milieu du jour et qu'il s'y couchera plus tôt. Voici la différence que nous disons exister entre l'horizon et le méridien de Ctésiphon par rapport à Alexandrie: l'horizon de l'une ou de l'autre ville aura lieu quand le soleil se lève ou se couche sur elle, et le méridien, c'est-à-dire (le cercle) de la moitié du jour aura lieu, quand le soleil sera vu (en chaque endroit) au milieu du ciel.[133]

Although Severus mentions a prime meridian placed at the Fortunate Isles, the obvious reference given in the framework of this astronomical observation is the meridian at Alexandria. But since the figure given for Ctesiphon is added to the meridian of Alexandria not as a prime meridian but as a meridian distance of $60° \frac{1}{2}$ from the prime meridian, his approach is mathematically correct.

Once again, as in the originally Greek text of the first half of the sixth century, this Syriac document demonstrates that an author writing in the second half of the seventh century could still believe that it was possible to measure longitudinal geographical coordinates from the observations made during a solar eclipse.

Another particularly interesting aspect of this seventh-century passage is that Severus Sebokht seems to have used an original example of a simultaneously observed lunar eclipse. This example was not even used in Ptolemy's work, where the only recorded instance of a simultaneously observed lunar eclipse is that of the eclipse viewed at

en chaque lieu autre la sixième et la septième heure, c'est de là qu'il est nommé μεσημβρινός, c'est-à-dire milieu de midi." From Sev. Seb. *Treat. Const.* 12.11, transl. Nau 1929–1930, 1931–1932, 397 (and reprinted in Nau 2014, 253).

131 "La latitude, constamment, dans tout climat et toute ville, est comptée du cercle de l'équateur vers le nord ; mais la longitude est comptée de l'Occident à l'Orient, c'est-à-dire depuis les îles qui sont dans l'Océan, que les poètes ont nommées îles des Bienheureux – Bardesane le Syrien et ceux qui se sont attachés à lui (les nomment) îles de la Béatitude." From Sev. Seb. *Treat. Const.* 15.7, transl. Nau 1929–1930, 1931–1932, 87–88 (and reprinted in Nau 2014, 269–270).

132 "Nous rappelons encore, ô ami de la science, que ces deux cercles de l'horizon et du méridien ne sont pas fixés sur la sphère du ciel, comme les cinq dont on vient de parler ; ils sont conçus par l'esprit, de manière différente et accidentelle, non seulement selon les différences des sept climats, comme nous l'avons déjà démontré, mais encore selon les différences des villes qui sont dans un même climat, le méridien suivant la longitude seulement, mais l'horizon aussi selon la latitude, comme il a déjà été montré dans ce qui a été dit. Il leur arrive d'être fréquemment changés, parce que le soleil ou les parties de la sphère ne se lèvent pas en même temps pour toutes les villes d'un même climat, pour celles de l'Orient et pour celles de l'Occident, comme le *Canon* Πρόχειρος de Ptolémée et aussi l'astrolabe le montrent." From Sev. Seb. *Treat. Const.* 15.8, transl. Nau 1929–1930, 1931–1932, 88 (and reprinted in Nau 2014, 270).

133 From Sev. Seb. *Treat. Const.* 15.8, transl. Nau 1929–1930, 1931–1932, 88 (and reprinted in Nau 2014, 270).

Carthage and at Arbēla that occurred on 20 September in 330 BCE,[134] which was cited in the *Geography* and reappears in the *Skolyon on the Astrolabe*. In the *Almagest* Ptolemy does compare the results pertaining to the cities of Alexandria and Babylon several times – admittedly in his references to the instances of lunar eclipses used by Hipparchus – but he does not give an account of a simultaneous observation of an astronomical event in both cities.[135] Does this imply that Severus made this observation himself or that he recorded an observation made by some astronomers after the sixth-century *Skolyon on the Astrolabe*? The Syriac text is clear on this point: Severus states unambiguously that one lunar eclipse was observed simultaneously at Alexandria and at Ctesiphon. Unfortunately, he does not give a date, nor does he identify the observers. Nevertheless, we should still pay heed to the results cited by the Syriac author, that is, the longitudinal coordinates that resulted from observing the lunar eclipse. Indeed, the figures given for Ctesiphon are exactly the same as those given by Ptolemy in the *Handy Tables*[136] and also in his *Geography*[137]. If Severus or some of his contemporaries had really made these astronomical observations themselves, they would certainly not have obtained the same result, since it is incorrect.[138] Consequently, the longitude that Severus gives for the two cities of Alexandria and Ctesiphon must have originated from Ptolemy's *Handy Tables*. This does not come as much of a surprise, given that Severus cites the *Handy Tables* in his *Treatise on the Constellations* before giving an explanation of that eclipse; moreover, it is well known that both the *Handy Tables* and the *Geography* were studied at the monastery of Qenneshre (where Severus tutored other residents) and that they were widely used in Syriac treatises and biblical commentaries.[139]

To conclude: we have two late antique documents preserved in the Syriac language that attest to an interest in determining geographical longitude through observing astronomical phenomena (lunar and solar eclipses). However, neither of the two authors whose works we have examined used their theoretical knowledge to measure the longitude of cities themselves; and in both cases the examples provided to explain their method were taken from Ptolemy's *Geography*.

7 Conclusion

The history of science has tended to regard the development of spherical geographical coordinates as a transfer of astronomical coordinate systems onto the terrestrial globe, with the purpose of acquiring greater geographical precision by improving mapmaking. In such a coordinate system, the coordinates provided by the intersections of the lines of latitude and longitude denote the locations. Whereas it was easy to observe latitude, for example by noting differences in shadow lengths or in the duration of the longest

134 This was pointed out in Toomer 1984, 75, note 3.

135 In *Alm.* 4.6, Ptolemy gives three examples "among the most ancient eclipses available to us" (documented in Babylon in 720 BCE and 719 BCE; see H301–304, transl. Toomer 1984, 190–192) and then three other examples "among contemporary eclipses", which he carried out himself in Alexandria in 133, 134 and 136 CE (see H314–315, transl. Toomer 1984, 198).

136 The longitude of Ctesiphon (80°) is given in Ptolemy's *Table of the important cities* 19. 1, Stückelberger and Mittenhuber 2009, 201. For the longitude of Alexandria (60° 30'), see Ptolemy's *Table of the important cities* 13.4, Stückelberger and Mittenhuber 2009, 179.

137 Ctesiphon is given a longitude of 80° (Ptol. *Geogr.* 6.1.3, transl. Stückelberger and Graßhoff 2006, t. 2, 597) and Alexandria 60° 30' (Ptol. *Geogr.* 4.5.9, transl. Stückelberger and Graßhoff 2006, t. 1, 423).

138 The actual difference in longitude between Alexandria and Ctesiphon is 14° 39' (that is, there is a time difference of $58\frac{1}{2}$ minutes). Ptolemy writes, in the *Handy Tables* and the *Geography*, that the difference in longitude is 19° 30'.

139 See Villey 2014 for an analysis of the use of the *Handy Tables* in Syriac astronomical treatises. See the convincing demonstration made in Defaux 2014 about how Jacob of Edessa (*c.* 640–708 CE), a student of Severus in Qenneshre, reworked sections of the *Geography* into his *Hexaemeron*.

day, more sophistication was required to notice the effects of geographical longitude. This could almost only be measured by comparing the differences in local time at the different locations from which a particular lunar eclipse had been observed. With the completion of Ptolemy's *Geography* around 160 CE, the development of this concept reached its apogee. Our study makes it clear that the development of this concept is considerably more complex than commonly assumed.

In ancient Mesopotamia, geographical regions were located in relation to cardinal directions, which were originally indicated by wind directions, later also by the astronomically defined directions of sunrise and sunset. The same can be observed in early Greek sources. Over the years, some changes of conception, terminology and representation of longitude occurred. In general, the initially close connection between descriptions of spatial relationships and natural and manmade features, such as mountains, rivers, streets or countries became looser. The extant sources suggest that Hipparchus was the first to make wide use of this more abstract description of geographical space. The later Ptolemaic conception of longitude and its quantification, measured along the Earth's equator from the western end of the oikoumenē to the meridian on which a place is located, makes it clear that this concept is based on at least two different developments: First, the inclusion of the sphericity of the Earth (commonly accepted at least since the times of Aristotle), and second, measuring arcs of a circle by parts corresponding to 1/360 of its circumference. The latter can be traced back to Mesopotamia, where the ancient unit of geographical distance UŠ had been transformed into a unit of astronomical time and of angular distance along the ecliptic. It entered Greek science no later than the time of Hipparchus in the context of astronomy. Despite its wide use in astronomical and mathematical texts, it is Ptolemy´s *Geography* where we can see, for the first time in a geographical text, a representation of longitudinal relationships that quantifies them independently of the actual circumference of the Earth – be it by giving distances in degree or in hours of differences in local time when reporting Marinos´s longitudinal extension of the oikoumenē.

No significant changes to the concept of longitude and its quantification can be found in the Syriac reception of the Greek texts in the seventh century CE. The procedures given in Syriac texts for measuring longitudinal distance between two places exploit the same property of observations of lunar eclipses that is described in earlier Greek texts. Again, Hipparchus is the earliest scholar known to have suggested that lunar eclipses, being widely visible indicators for the same moment of time, can be used for deducting new knowledge about the longitudinal relationship between different places by comparing their local times. This method is based on an understanding of lunar eclipses that is rooted in the Greek geometrical models of the Heavens with rotating spheres.

But had such methods indeed been as successful, or at least useful, as Hipparchus, Ptolemy and Severus Sebokht assert to their readers? Our research shows that there is no evidence that methods for observing and computing longitudinal differences by means of a plane astrolabe were ever used in practice, nor that any other method based on the same principle, as they can be found in Greek texts, was ever applied. Moreover, the examples given for such calculations all relate to the same pair of observations of a lunar eclipse, and the data that are quoted in the sources may not have been actually observed, but made up in order to provide a clear example. Our studies have also revealed that simultaneous observations of lunar eclipses were, in fact, not carried out in antiquity: for example, ancient astronomers failed to combine the data of the renowned lunar eclipse that occurred shortly before the Battle of Gaugamela in 331 BCE with accounts of the same eclipse from Carthage, while even Ptolemy, whose archives contained a large set of Babylonian observations, was unable to produce a single example of the simultaneous observation of a lunar eclipse. What Ptolemy did do was to make exemplary evaluations and hypothetical assumptions about astronomical data, even though they had not, in truth, been observed. Instead, the terrestrial measurements that Ptolemy actually used himself were not only widespread but also much more precise than their astronomical counterparts. All in all, we note a large discrepancy between the actual scientific practice and the approaches reported by various authors, which nevertheless initiated a long tradition in the following reception. Even the erroneous suggestion to use observational data of solar eclipses for measuring longitudes took its course through different times and texts in different cultures.

However, our study does show that there is nevertheless a tight connection between the observation of lunar eclipses and the development of a quantitative representation of terrestrial longitudes that is independent from the circumference of the Earth. This becomes apparent when astronomical theory needed to account for the irregularities in the Moon´s motion. Although lunar theory was first developed by Babylonian astronomers in the early fourth century BCE, it could not be adopted by scholars from other regions, since the Babylonian theory was entirely local, that is, the motions of the Moon were exclusively described from the viewpoint of the Babylonian observer. The dissemination of astronomical knowledge over the eastern Mediterranean, however, meant that lunar theory had to account for the quantitative differences of positional observations made at different locations by using the concept of geographical latitude and longitude. No comprehensive lunar theory could neglect these central topocentric parameters, and it was exactly this context in which the concept of quantified longitude was rooted.

The close interaction between geography and astronomy and, especially, the enormous influence of astronomy are important factors in the development of mathematical geography, not only with regard to the development of the concept of longitude and the methods of its determination. Again, it is in the context of astronomy (and the dependency of celestial phenomena on longitude) and for astronomical reasons that for the very first time the need for a geographical treatise with coordinates as in the later *Geography* is named.

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