Introduction

The classical mathematical theory of two-dimensional (plane) elasticity has been thoroghly investigated (see, e.g. [3], [7], [8]-[10], [17], [18], [19], [40], [43], [45], [46], [49], [51], [56]) by using the complex potential method, which is one of the most promising and efficient methods and has become a classic but not lost its significance even today, on the basis of the theory of integral equation and boundary value problem (see, e.g. [1], [2], [16], [21], [25], [30], [37], [39], [47], [48], [54], [57], [58]). The singly periodic problems of plane elasticity theory have also been studied in a number of references (see, e.g. [4], [11], [12], [22], [23], [24], [26], [29], [31], [32], [38], [44], [50]). Doubly-periodic elastic problems are of considerable importance to continuum mechanics and engineering, for example, they are frequently encountered in rock mechanics, concrete mechanics, fracture mechanics and related fields of elasticity theory. However the available information on this topic is far from sufficient. Nevertheless, a few papers have been appeared, e.g. the works on an infinite plate with a doubly-periodic set of equal holes or cracks (much attention to rectilinear cracks) by [14], [15], [28], [41], [53], [59], etc. Plane elastic problems of nonhomogeneous media (the first foundamental problem with no holes or cracks) by [13], (the first and the second foundamental problems with holes or cracks) by [33]-[35], are based on Mushelishivili's complex potential method.

The complete plane strain (CPS) problem is a specific case of the three-dimensional elasticity theory. But the available information on doubly-periodic CPS is rather scarce. In the case of the the first fundamental CPS problem of three-dimensional nonhomogeneous media with no holes or cracks has been investigated by [20]. In the present thesis, we wish to use complex methods to solve some of the doubly-periodic CPS problems of elasticity theory, such as the first and the second fundamental CPS problems of the three-dimensional nonhomogeneous elastic body with a doubly-periodic set of cracks on the x_1, x_2 plane in Chapter 1 and Chapter 2, and the mixed CPS problems of the three-dimensional nonhomogeneous elastic body with holes, which perforated doubly-periodically the x_1, x_2 plane, in Chatper 3. To these problems, at first, we resolve the complete plane strain state, which is a special three-dimensional elastic system, into two linearly independent two-dimensional (plane) elastic systems by the superposition principle of force, one is the generalized plane strain state and another is the longitudinal displacement state. Due to the fact when the stress distributions are doubly-periodic in the elastic body, then the displacements are doubly quasi-periodic (see Lemma 1.1.1), and the complex stress function $\phi(z)$, the expression $z\overline{\phi'(z)}+\overline{\psi(z)}$ and the complex torsion function $F(z)$ are all doubly quasi-periodic (see Lemma 1.1.2). Then, we construct Kolosov functions, say, we must separete the doubly-periodic multi-valued parts of the complex stress functions, on the basis of Lemma 1.2.1. And we establish boundary value problems for analytic functions by using Muskhelishvili's complex potential method. Furthermore, based on a suitable modification of Cauchy-type integrals, which is defined by the replacement of the Cauchy kernel $1/(t-z)$ by the Weierstrass zeta function $\zeta(t-z)$, the general representations for the solutions are constructed, under some general restrictions the boundary value problems are reduced to normal type singular integral equations with Weierstrass zeta kernel, and the existences of the essentially unique solution are proved.

In Chapter 4 we pose three formulations of the modified doubly-periodic second fundamental CPS problems with relative displacements on the basis of a new formulation in the work [42] for plane elasticity problem in the non-periodic case. In the present case, the displacements $u_j + iv_j$, given on the closed boundary contours of the multi-connected elastic region, are relative to certain rigid motions which are different to each other. It is proved that, for the unique existence of the solution, the external resultant principal vectors $X_{1j} + iX_{2j}$ and moments M_j must be given in advance. The solution method is discribed, too.

Chapter 5 is devoted to obtaining closed solutions for several specific cases, such as the doubly-periodic homogeneous and nonhomogeneous cylindrical inlay CPS problem in Section 5.1 and 5.3, respectively. After referring to the transformation introduced in Chapter 1, the boundary value problems are transfered into doubly quasi-periodic boundary value problems. Then, by employing the solutions of doubly quasi-periodic boundary value problem we obtain the general solution in closed form. In the illustrating examples of practical interest, e.g. doubly-periodic homogeneous and nonhomogeneous circular cylindrical inlay problems, the exact solutions are obtained. To the best of the author's knowledge, no exact solution on doubly-periodic elasticity problems has been found in the literature. Moreover when we fix one of the periods, letting the other tends to infinity, as a by-product we get the exact solution of the singly-periodic case. Furthermore, when we let the two periods both tend to infinity, we have immediately the solution of the non-periodic case, which is identical with the classical one. In Section 5.2, the effect of homogeneous cylindrical inlay on cracks in the doubly-periodic CPS problem is investigated, the general solution is obtained in closed form, and approximate analytical expressions of the stress intensity factors, which are identical with the known results, are obtained when there are no any holes or gaskets in the periodic parallelogram and $e_3 = 0$, $F_k = 0$, $T_k = 0$, $k = 1, 2$, with constant load applied at the edges of the crack.