

Applications of Doubly Quasi-Periodic Boundary Value Problems in Elasticity Theory

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ABSTRACT

In the present thesis, we investigate the first and the second fundamental complete plane strain (CPS) problems of the three-dimensional nonhomogeneous elastic body with a doubly-periodic set of cracks and the mixed CPS problems of the three-dimensional nonhomogeneous elastic body with a doubly-periodic set of holes. At first, we resolve the complete plane strain state into two linearly independent plane elastic systems by the superposition principle of forces. Then, on the basis of that when the stress distributions are doubly-periodic in the elastic body, then the displacements, the complex stress function $\phi(z)$, the expression $\overline{z\phi'(z)} + \overline{\psi(z)}$ and the complex torsion function $F(z)$ are all doubly quasi-periodic, we construct Kolosov functions, and establish boundary value problems by using the complex potential method, furthermore, based on a suitable modification of Cauchy-type integrals, which is defined by the replacement of the Cauchy kernel $1/(t-z)$ by the Weierstrass zeta function $\zeta(t-z)$, the general representations for the solutions are constructed, under some general restrictions the boundary value problems are reduced to the normal type singular integral equations with Weierstrass zeta kernel, and the existences of the essentially unique solution are proved. In addition, we pose three formulations of the modified doubly-periodic second fundamental CPS problem with relative displacements. It is proved that, for the unique existence of solution, the external resultant principal vectors and moments must be given in advance. At last, the general solutions are obtained in closed form for several specific cases. For some illustrating examples of practical interest, the exact solutions are obtained. moreover when we fix one of its periods, while the other tends to infinity, we get the exact solutions of the singly-periodic case, furthermore, when we let the two periods both tend to infinity, we have immediately the solutions of non-periodic case, which are identical with the classical ones.

Contents

Introduction	1
Chapter 1. First Fundamental CPS Problem	
of a Nonhomogeneous Body with a Doubly-Periodic Set of Cracks	4
1.1 Preliminaries, Definition and Lemmas	4
1.2 Kolosov Functions	15
1.3 Formulation of the First Fundamental CPS Problem	17
1.4 Solution, Reduction to Integral Equations	20
1.5 Unique Solvability of the First Fundamental Problem	26
Chapter 2. Second Fundamental CPS Problem	
of a Nonhomogeneous Body with a Doubly-Periodic Set of Cracks	32
2.1 Formulation, Solution of the Second Fundamental CPS Problem ..	32
2.2 Unique Solvability of the Second Fundamental Problem	38
Chapter 3. Mixed CPS Problem of a Nonhomogeneous	
Body with a Doubly-Periodic Set of Holes	41
3.1 Preliminaries and Kolosov Functions	41
3.2 Formulation of the Mixed CPS Problem	44
3.3 Solution of the Mixed CPS Problem	46
3.4 Unique Solvability of the Mixed CPS Problem	51
Chapter 4. Modified Doubly-Periodic Second Fundamental	
CPS Problem with Relative Displacements	59
4.1 Three Formulations of MDPP	59
4.2 Solution of MDPP	64
Chapter 5. Closed Solution of Several Specific Cases	72
5.1 Doubly-Periodic Homogeneous Cylindrical Inlay CPS Problem	72

5.2 Effect of Homogeneous Cylindrical Inlay on Cracks in the Doubly-Periodic CPS Problem	81
5.3 CPS Problem of a Nonhomogeneous Body with a Doubly-Periodic Set of Cylindrical Inlay	90
Bibliography	95

List of Figures

Fig. 1.1 A nonhomogeneous body with a doubly-periodic set of cracks ..	6
Fig. 3.1 A nonhomogeneous body with a doubly-periodic set of holes ..	43
Fig. 4.1 Model of an elastic body with a doubly-periodic set of holes ..	61
Fig. 5.1 Model of the doubly-periodic cylindrical inlay CPS problem...	74
Fig. 5.2 Doubly-periodic circular cylindrical inlay CPS problem	78
Fig. 5.3 Displacement u for $-1 < x < 2, 0.75 < y < 2$	81
Fig. 5.4 Displacement v for $-1 < x < 2, 0.75 < y < 2$	82
Fig. 5.5 Displacement u for $-1 < x < 1, y = 0$	83
Fig. 5.6 Displacement v for $-1 < x < 1, y = 0$	84
Fig. 5.7 Model of the doubly-periodic inlay and crack CPS problem ...	85

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