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SIGNALLING RIVALRY AND QUALITY UNCERTAINTY IN A DUOPOLY*

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Abstract

This paper considers a market in which only the incumbent's quality is publicly known. The entrant's quality is observed by the incumbent and some fraction of informed consumers. This leads to price signalling rivalry between the duopolists, because the incumbent gains and the entrant loses when observed prices make the uninformed consumers more pessimistic about the entrant's quality. When the uninformed consumers' beliefs satisfy the 'intuitive criterion' and the 'unprejudiced belief refinement', only a two–sided separating equilibrium can exist and prices are identical to the full information outcome.

Keywords: Quality uncertainty, Signalling, Oligopoly
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1 Introduction

It is well established that in markets with asymmetric information firms may use prices, possibly in conjunction with additional marketing devices, to signal quality information to uninformed market participants. In particular, if only some fraction of consumers is informed about quality, then firms may signal their qualities to the uninformed by setting prices higher than under perfect information. The idea is high-quality firms suffer less from decreased sales to informed consumers due to price increases than low-quality firms. Therefore a high-quality firm can separate itself by setting a high price which is not profitable to imitate for the low-quality firm. Signalling thus leads to distorted pricing and an inefficient reduction in the supply of high-quality goods.

This paper studies an extension of the standard price signalling model to a durable goods duopoly. In this environment the equilibrium outcome is free of distortions and identical to the perfect information equilibrium. We obtain this conclusion for a horizontally and vertically differentiated duopoly market with price-setting competitors engaging in a game of signalling rivalry: An established incumbent, whose quality is known by all market participants, faces an entrant who is either supplying the same quality as the incumbent or a superior quality acquired through some product innovation. Both firms and some fraction of consumers know the entrant's quality. The uninformed consumers use prices set by *both* firms to infer quality information. An important feature of price competition is that the two firms have opposing interests in conveying information, because the incumbent gains and the entrant loses when observed prices make the uninformed consumers more pessimistic about the entrant's quality.

In our model consumers are confronted with two price signals concerning a single uncertain variable, the entrant's quality. For the analysis of equilibrium, we apply two standard refinements for the uninformed consumers' out-of-equilibrium beliefs. First, we use the 'intuitive criterion' of Cho and Kreps (1987) and show that this eliminates all equilibria in which both firms adopt a pooling strategy. This means that at least one firm must use a separating strategy that reveals the entrant's quality to the uninformed consumers. Interestingly, this conclusion can be derived by applying the intuitive criterion to the incumbent's rather than the entrant's pricing. The incumbent facing a low quality entrant can credibly deviate from pooling by setting a price that signals a low quality entrant, whereas under some parameter constellations the high quality entrant may not be able to avoid pooling by appealing to the intuitive criterion.

Second, in situations where one of the firms' pricing is informative we adopt the 'unprejudiced belief criterion' of Bagwell and Ramey (1991) to the pricing strategy of its competitor, because the intuitive criterion is no longer applicable. Under the unprejudiced belief criterion the consumers trust in the price signal of the non–deviating competitor whenever only one of the two firms selects an out-of-equilibrium price. This means that, given the other firm playing an equilibrium separating strategy, a deviating firm cannot influence beliefs by deviating to a non-equilibrium price and therefore always sets its best response price as under perfect information.

The unprejudiced belief criterion therefore excludes all separating equilibria with prices distorted from full-information prices. We show that these prices constitute the unique equilibrium outcome in our model as long as the fraction of informed consumers is not too small. If only rather few consumers are informed, there is no equilibrium satisfying our refinements. The reason is that either the low-type entrant could gain by deviating to the high-type equilibrium price or the incumbent playing against the high-type would deviate to the low-type equilibrium price. Thus the firms' price signals would become contradictory: The entrant would signal that his quality is high and the incumbent that the entrant's quality is low.

Related Literature

The standard prediction of the literature on price signalling is that quality uncertainty leads to distorted pricing for signalling purposes. The earliest contributions to this literature consider a market with a single seller. For example, Milgrom and Roberts (1986) show that a monopolist may use price and advertisement to convince consumers of the quality of a newly introduced product. In their model, which is based on repeat purchases of a non-durable good, prices can be distorted up– or downwards depending on expectations over future sales. Bagwell and Riordan (1991) consider a monopolist who produces a durable good whose quality may be high or low. The existence of informed consumers and cost differences between qualities allow the monopolist to signal high quality through an upward distorted price.[1] Basically, our model extends Bagwell and Riordan (1991) to a horizontally differentiated duopoly in which one of the two firms offers a quality that is known to the competitor but not to all consumers.

One string of the literature extends the analysis of price signalling to oligopolistic markets under the assumption that firms have private information only about their own quality. They are not informed about the other firms and, therefore, have the same prior about their competitors' qualities as the uninformed consumers. Daughety and Reinganum (2007) and Daughety and Reinganum (2008) examine a horizontally and vertically differentiated duopoly and n–firm oligopoly, respectively. Price setting takes into account the ex-ante probabilities of rivals to be high– or low–quality types. Separating equilibria imply upward distorted prices,

[1] Linnemer (2002) shows that in the same setup it would be in some cases more profitable for the high-type firm to combine price and advertising signals.

increasing in the ex-ante probability of firms being high–types. Similarly, Janssen and Roy (2010) show for a homogenous oligopoly that fully revealing mixed strategy equilibria exist in which high–types distort prices upward and low–types randomize prices over an interval, thereby generating sufficient rents to avoid mimicry of the high–types.

Closer related to the information structure in our model is the other string of the literature that assumes the oligopolists to be informed about their rivals' qualities. Hertzendorf and Overgaard (2001a) analyze price setting and advertising in a duopoly where qualities are perfectly negatively correlated and consumers only know that one firm offers high quality and the other low quality. They apply two refinements that lead to a unique separating and a unique pooling equilibrium. In the separating equilibrium, a high degree of vertical differentiation leads to upwards distorted prices and a low degree to downward distorted prices. Yehezkel (2008) introduces some informed consumers into a similar model and examines how pricing and advertising strategies depend on the fraction of informed consumers.

In Fluet and Garella (2002) the ex ante distribution of the firm's qualities is such that either both firms offer low quality or one firm offers low and the other high quality. The authors avoid the use of selection criteria and find multiple separating and pooling equilibria. For small quality differences separation can only be achieved with a combination of upward distorted prices and advertisement. This result is similar to a finding by Hertzendorf and Overgaard (2001b), who show that fully revealing separating equilibria satisfying the unprejudiced belief condition do not exist.

These papers differ from our model in that they consider product differentiation only in the vertical dimension. This implies that the duopolists have a common interest in signalling *different* qualities since they earn zero profits if consumers believe that they both offer the same expected quality. In our model of signalling rivalry such a common interest does not exist because consumer preferences are differentiated horizontally between the firms, and in the vertical dimension all consumers have identical preferences. As a consequence, the incumbent always prefers the consumers to believe that the entrant's quality is identical to his own quality, whereas the entrant gains by convincing the consumers that he offers a superior quality. Another feature that distinguishes our model from the above literature is that the duopolist's are not in a symmetric position. Consumers are uninformed only about the entrant's and not about the incumbent's quality. They interpret the prices of both firms as signals only about the entrant's quality. In our analysis, we do not address expenditures on directly uninformative advertising as an additional signal. Since under our belief refinements only the full–information equilibrium without distortions survives, there is no role for dissipative advertising in equilibrium.

From a methodological perspective our analysis is closely related to Bagwell and Ramey (1991) and Schultz (1999). They study limit pricing by two incumbents to affect the entry decision of a third firm. The incumbents' prices signal their information about an industry–wide parameter. The third firm enters the market only if it concludes that the probability of a favorable state is sufficiently high. In the paper by Bagwell and Ramey (1991) the competitors have a common interest, both want to signal an unfavorable state in order to prevent entry. Introducing the unprejudiced belief refinement, the authors find that only non–distorted separating equilibria exist. Further, under additional assumptions the intuitive criterion of Cho and Kreps (1987) eliminates all equilibria with pooling. By applying the same belief refinements to our context, we arrive at similar conclusions for the qualitative features of equilibrium. Schultz (1999) considers a variation of Bagwell and Ramey (1991) where the incumbents have conflicting interests, i.e. one incumbent prefers the entrant to stay out of the market, whereas its competitor profits from entry. Again, separating equilibrium prices are not distorted. But due to signalling rivalry these equilibria only exist if the effect of entry on the incumbent's profits is relatively small. We obtain a related non–existence result in our model when the fraction of informed consumers is rather small.

This paper is organized as follows. In Section 2 we describe the model and, as a reference point, we derive the equilibrium under full information. Section 3 defines the Perfect Bayesian Equilibrium and explains the belief refinements of our analysis. In Section 4 we show that under our refinements only the full information equilibrium prices can survive in a signalling equilibrium and that such an equilibrium exists if the fraction of informed consumers is not too small. Section 5 provides concluding remarks.

2 The Model

We employ the demand structure of the standard Hotelling (1929) duopoly with the modification that the two firms may offer different qualities. One of the firms offers a quality that is publicly known by all market participants. For convenience, we call this firm the incumbent. The other firm, which we call the entrant, produces a quality that is known also by the competing incumbent. Yet, some fraction of potential consumers is not informed about the entrant's quality. In the terminology of Nelson (1970), the entrant's good is an *experience* good so that an uninformed consumer learns its true quality only after purchase. The uninformed consumers use the firms' prices to draw inferences about the entrant's quality. Accordingly, the price setting behavior of *both* firms takes into account that prices are quality signals.

There is a unit mass of consumers whose preference characteristic x is uniformly distributed on the interval $[0,1]$. Each consumer purchases at most one unit of the good from either the incumbent I or the entrant E. Given the incumbent's quality q_I and the entrant's (expected) quality q_E, the valuation of a consumer with characteristic $x \in [0,1]$ is

$$v_I(x) = q_I - tx, \quad v_E(x) = q_E - t(1-x) \tag{1}$$

for the incumbent's and the entrant's good. The parameter t reflects the degree of horizontal product differentiation. The two firms are also vertically differentiated if $q_I \neq q_E$. But the quality differential between the two firms affects the taste of all consumers in the same way, independently of their characteristic x. This aspect distinguishes our model from the price signalling models of Hertzendorf and Overgaard (2001a) and Fluet and Garella (2002) who similarly to Shaked and Sutton (1982) assume that consumers differ in their valuation of quality and that the goods are not horizontally differentiated. In what follows, we assume that the firms' qualities are sufficiently high so that each consumer buys one unit of the good.

All consumers observe the incumbent's price p_I and the entrant's price p_E. The critical consumer type $\tilde{x}$, who is indifferent between purchasing from firm I and firm E, is then determined by $v_I(\tilde{x}) - p_I = v_E(\tilde{x}) - p_E$, and by (1) we have

$$\tilde{x}(p_I, p_E, q_E - q_I) = \frac{p_E - p_I - (q_E - q_I) + t}{2t}. \tag{2}$$

All consumers with $x < \tilde{x}$ optimally buy the incumbent's good, whereas consumers with $x > \tilde{x}$ purchase from the entrant.

There are two possible qualities, q_L and q_H, with $0 < q_L < q_H$. The incumbent's quality is commonly known to be $q_I = q_L$. There is uncertainty, however, about the entrant's quality. Its quality is $q_E = q_H$ with ex ante probability λ and $q_E = q_L$ with probability $1 - \lambda$. One interpretation is that with probability λ the entrant has realized a product innovation which increases the 'standard' quality q_L by the amount $q_H - q_L$. We normalize the unit cost of producing quality q_L to zero and assume that the unit cost of producing quality q_H is $c > 0$.

By (2) only the quality differential between the two firms affects the consumers' demand decisions. Therefore, we can simplify notation by defining

$$\Delta \equiv q_H - q_L. \tag{3}$$

We assume that the high quality entrant has a competitive advantage because $c < \Delta$. Yet, to ensure an interior solution, we take the entrant's product innovation to be *non–drastic* so that

$$0 < \Delta - c < 3t. \tag{4}$$

This will guarantee that the incumbent's market share is positive even when he competes with the high quality entrant.

Both firms observe the realization of q_E before setting prices. In addition some fraction $\gamma \in (0,1)$ of consumers becomes informed about the entrant's true quality before making demand decisions. Each consumer type x is equally likely to be informed. This means that in each subset of the consumers' characteristic set $[0,1]$ the fraction of informed consumers is identically equal to γ.

The uninformed consumers use the observed prices p_I and p_E to draw inferences about the entrant's quality. We denote their posterior belief that the entrant's quality is $q_E = q_H$ by $\mu \in [0,1]$. Thus the uninformed consumers anticipate that the entrant offers the *expected* quality $\mu q_H + (1-\mu) q_L = q_L + \mu\Delta$. Since consumers are risk–neutral with respect to quality, for given prices p_I and p_E their demand decisions depend only on the *expected* quality difference between the two sellers.

In the uninformed consumers' expectation the quality difference $q_E - q_I$ is always equal to $\mu\Delta$, independently of the entrant's true quality. If the entrant's quality is $q_E = q_L$, the informed consumers know that $q_E - q_I = 0$. Therefore, by (2) the incumbent's and the entrant's market shares, D_{IL} and D_{EL}, are given by

$$\begin{aligned} D_{IL}(p_I, p_E, \mu) &= \gamma \tilde{x}(p_I, p_E, 0) + (1-\gamma)\tilde{x}(p_I, p_E, \mu\Delta), \\ D_{EL}(p_I, p_E, \mu) &= 1 - D_{IL}(p_I, p_E, \mu). \end{aligned} \tag{5}$$

If $q_E = q_H$, the informed consumers know that $q_E - q_I = \Delta$. In this case, the two sellers' market shares, D_{IH} and D_{EH}, are equal to

$$\begin{aligned} D_{IH}(p_I, p_E, \mu) &= \gamma \tilde{x}(p_I, p_E, \Delta) + (1-\gamma)\tilde{x}(p_I, p_E, \mu\Delta), \\ D_{EH}(p_I, p_E, \mu) &= 1 - D_{IH}(p_I, p_E, \mu). \end{aligned} \tag{6}$$

If the entrant's quality is $q_E = q_L$, it follows from (2) and (5) that the incumbent's profit $\Pi_{IL} = p_I D_{IL}$ and the entrant's profit $\Pi_{EL} = p_E D_{EL}$ are

$$\Pi_{IL}(p_I, p_E, \mu) = p_I \frac{t - (1-\gamma)\mu\Delta - p_I + p_E}{2t}, \tag{7}$$

$$\Pi_{EL}(p_I, p_E, \mu) = p_E \frac{t + (1-\gamma)\mu\Delta + p_I - p_E}{2t}. \tag{8}$$

If $q_E = q_H$, then by (2) and (6) the duopolists' profits, $\Pi_{IH} = p_I D_{IH}$ and $\Pi_{EH} = (p_E - c) D_{EH}$, are equal to

$$\Pi_{IH}(p_I, p_E, \mu) = p_I \frac{t - [\gamma + (1-\gamma)\mu]\Delta - p_I + p_E}{2t}, \tag{9}$$

$$\Pi_{EH}(p_I, p_E, \mu) = (p_E - c) \frac{t + [\gamma + (1-\gamma)\mu]\Delta + p_I - p_E}{2t}. \tag{10}$$

Note that for all $\gamma \in (0,1)$ it is the case that

$$\frac{\partial \Pi_{EL}}{\partial \mu} > 0, \quad \frac{\partial \Pi_{EH}}{\partial \mu} > 0; \quad \frac{\partial \Pi_{IL}}{\partial \mu} < 0, \quad \frac{\partial \Pi_{IH}}{\partial \mu} < 0. \tag{11}$$

Irrespective of the true quality, the entrant always gains and the incumbent always loses when the uninformed consumers raise their belief that the entrant offers high quality. Since these consumers interpret the firms' prices as quality signals, price competition entails a signalling rivalry: The entrant has an incentive to choose a price that indicates high quality. This is in conflict with the incumbent's interest to convince consumers that the entrant offers low quality.

Before analyzing how the duopolists' signalling rivalry affects their price competition, we briefly describe the equilibrium under full information. The firms compete by simultaneously setting prices and their pricing strategies are contingent on the entrant's quality. If $q_E = q_L$, we denote the incumbent's and the entrant's price by p_{IL} and p_{EL}, respectively; if $q_E = q_H$ the firms' prices are denoted by p_{IH} and p_{EH}. When all consumers know the entrant's quality, the firms' profits can be calculated from (7)–(10) by setting $\mu \equiv 0$ for $q_E = q_L$ and $\mu \equiv 1$ for $q_E = q_H$.[2] The full information equilibrium prices $\hat{p} = ((\hat{p}_{IL}, \hat{p}_{EL}), (\hat{p}_{IH}, \hat{p}_{EH}))$ are then defined by the conditions for profit maximization so that

$$\begin{aligned} \Pi_{IL}(\hat{p}_{IL}, \hat{p}_{EL}, 0) &\geq \Pi_{EL}(p, \hat{p}_{EL}, 0), \quad \Pi_{EL}(\hat{p}_{IL}, \hat{p}_{EL}, 0) \geq \Pi_{EL}(\hat{p}_{IL}, p, 0), \\ \Pi_{IH}(\hat{p}_{IH}, \hat{p}_{EH}, 1) &\geq \Pi_{EH}(p, \hat{p}_{IH}, 1), \quad \Pi_{EH}(\hat{p}_{IH}, \hat{p}_{EH}, 1) \geq \Pi_{EH}(\hat{p}_{IH}, p, 1). \end{aligned} \tag{12}$$

for all $p \geq 0$. From the corresponding first–order conditions one can easily derive the solution

$$\hat{p}_{IL} = t, \quad \hat{p}_{EL} = t, \quad \hat{p}_{IH} = t - \frac{\Delta - c}{3}, \quad \hat{p}_{EH} = t + \frac{\Delta + 2c}{3}. \tag{13}$$

If $q_E = q_L$, both firms charge the same price and have the same market share $D_{IL} = D_{EL} = 1/2$. If $q_E = q_H$, the incumbent is disadvantaged against the entrant and, even though he sets a lower price, his market share $D_{IH} = (3t - \Delta + c)/(6t)$ is smaller than the entrant's share $D_{EH} = (3t + \Delta - c)/(6t)$.

3 Equilibrium and Belief Restrictions

We envisage the market to operate in three stages. First, both firms and a fraction γ of consumers observe the realization of the entrant's quality. Second, the firms compete simultaneously by setting prices. Finally, in the third stage the uninformed consumers use observed

[2]This is equivalent to setting $\gamma \equiv 1$.

prices to update their beliefs about the entrant's quality, and all consumers decide whether to buy from the incumbent or the entrant.

In what follows we study pricing strategies of the firms and consumer beliefs that constitute a Perfect Bayesian Equilibrium of this game. The firms choose their prices contingent on their information about the entrant's quality, and the uninformed consumers' posterior probability of facing the high quality entrant is a function of the firms' prices. In equilibrium, each firm's price maximizes its profit and the uninformed consumer's posterior belief is consistent with Bayesian updating.[3]

More formally, $(p^*, \mu^*) = ((p^*_{IL}, p^*_{EL}), (p^*_{IH}, p^*_{EH}), \mu^*)$ with $\mu^*\colon \mathbf{R}^2_+ \to [0,1]$ is a *Perfect Bayesian Equilibrium* (PBE) if

(a) for $Q = L, H$

$$p^*_{IQ} = \operatorname{argmax}_p \Pi_{IQ}(p, p^*_{EQ}, \mu^*(p, p^*_{EQ})), \tag{14}$$
$$p^*_{EQ} = \operatorname{argmax}_p \Pi_{EQ}(p^*_{IQ}, p, \mu^*(p^*_{IQ}, p)), \tag{15}$$

and (b)

$$\mu^*(p^*_{IL}, p^*_{EL}) = 1 - \mu^*(p^*_{IH}, p^*_{EH}) = 0, \qquad \text{if} \quad p^*_{IL} \neq p^*_{IH} \quad \text{or} \quad p^*_{EL} \neq p^*_{EH}, \tag{16}$$
$$\mu^*(p^*_{IL}, p^*_{EL}) = \mu^*(p^*_{IH}, p^*_{EH}) = \lambda, \qquad \text{if} \quad p^*_{IL} = p^*_{IH} \quad \text{and} \quad p^*_{EL} = p^*_{EH}. \tag{17}$$

Equilibrium conditions (14) and (15) state that, for each quality $q_E \in \{q_L, q_H\}$, the incumbent and the entrant choose their prices to maximize profits, taking the competitor's price and the uninformed consumers' belief $\mu^*(\cdot)$ as given. Equilibrium conditions (16) and (17) require that on the equilibrium path the buyers' belief is consistent with Bayes' rule. The buyers become fully informed about the entrant's true quality not only in a *two–sided separating equilibrium*, where $p^*_{iL} \neq p^*_{iH}$ for both $i \in \{I, E\}$, but also in a *one–sided separating equilibrium*, where $p^*_{iL} \neq p^*_{iH}$ for some $i \in \{I, E\}$ and $p^*_{jL} = p^*_{jH}$ for $j \neq i$. Prices remain uninformative only if $p^*_{iL} = p^*_{iH}$ for *both* $i \in \{I, E\}$. In such a *pooling equilibrium* the posterior belief is equal to the a priori probability λ.

By (14) and (15), the uninformed consumers' quality expectations affect the duopolists' pricing decisions. But, conditions (16) and (17) impose restrictions on expectations only for prices that are actually chosen in equilibrium. Since out–of–equilibrium beliefs remain arbitrary, there are multiple equilibria, which are a typical feature of signalling games. This is so because the profit of a deviation from the equilibrium price depends on the uninformed

[3] We restrict ourselves to pure strategy equilibria.

consumers' interpretation of this deviation. For example, the incumbent may be deterred from changing its price simply because consumers would interpret this as a signal that the entrant's quality is high. Similarly, the entrant may be kept from changing its price if consumers view this as a signal of low quality. Without restrictions on consumer beliefs multiple equilibria with both upward and downward distorted prices can be found .

To avoid this problem, the literature usually applies refinements that impose restrictions on out–of–equilibrium beliefs. A prominent refinement is the 'intuitive criterion' of Cho and Kreps (1987), which has been used in a variety of price signalling games.[4] Unfortunately, this criterion is not generally applicable in the present context because it is defined for signalling games where each player has private information only about his own and not the other players' characteristics. In our model, however, the duopolists have *common* private information and not only the entrant's but also the incumbent's price may signal the entrant's quality. Therefore, the intuitive criterion cannot be used in our model if both firms' prices are informative. Nonetheless, it remains applicable if one of the firms' equilibrium prices are uninformative, i.e. if $p_{iL}^* = p_{iH}^*$ for some $i \in \{I, E\}$. In this case, the intuitive criterion can be used to refine beliefs for out–of–equilibrium prices of firm $j \neq i$.

Consider the incumbent in a situation where the entrant charges $p_{EL}^* = p_{EH}^*$ and the incumbent knows that the entrant's quality is low. Suppose the incumbent wishes to deviate to some price p_I if the uninformed consumers interpret p_I as a signal that indicates a low quality entrant. Then the idea of the intuitive criterion is that p_I should indeed convince the consumers that the entrant offers low quality if the following is true: If the incumbent knew that the entrant's quality is high, he would not gain from deviating to p_I even if the consumers would respond favorably for the incumbent by believing that p_I indicates a low quality entrant.

An analogous argument applies to the high quality entrant in a situation where the incumbent's pricing $p_{IL}^* = p_{IH}^*$ reveals no information. In this case, the intuitive criterion requires the uninformed consumers to believe that a price p_E signals high quality if for this belief deviating to p_E is profitable only for the high quality entrant and not for the low quality entrant.

More formally, the PBE (p^*, μ^*) satisfies the *intuitive criterion* if the following two conditions (a) and (b) are satisfied:

(a) If $p_{EL}^* = p_{EH}^* = p_E^*$, then $\mu^*(p_I, p_E^*) = 0$ for all p_I such that

$$\Pi_{IH}(p_I, p_E^*, 0) \leq \Pi_{IH}(p_{IH}^*, p_E^*, \mu^*(p_{IH}^*, p_E^*)) \tag{18}$$

[4] See, for example, Bagwell and Riordan (1991), Bagwell and Ramey (1991), Bester (1993), Bester and Ritzberger (2001).

and

$$\Pi_{IL}(p_I, p_E^*, 0) > \Pi_{IL}(p_{IL}^*, p_E^*, \mu^*(p_{IL}^*, p_E^*)). \tag{19}$$

(b) If $p_{IL}^* = p_{IH}^* = p_I^*$, then $\mu^*(p_I^*, p_E) = 1$ for all p_E such that

$$\Pi_{EL}(p_I^*, p_E, 1) \leq \Pi_{EL}(p_I^*, p_{EL}^*, \mu^*(p_I^*, p_{EL}^*)) \tag{20}$$

and

$$\Pi_{EH}(p_I^*, p_E, 1) > \Pi_{EH}(p_I^*, p_{EH}^*, \mu^*(p_I^*, p_{EH}^*)). \tag{21}$$

As our analysis will show, the intuitive criterion eliminates all PBE in which both duopolists use a pooling strategy. Thus, only separating equilibria remain in which the entrant's quality is revealed to the uninformed buyers. As we have explained above, for this type of equilibrium the intuitive criterion is not generally applicable because, if one of the firms unilaterally deviates from its equilibrium pricing strategy, the buyers may still be able to infer the entrant's quality from the other firm's price.

As a refinement for situations where firm $i \in \{I, E\}$ defects from the equilibrium and firm $j \neq i$ uses a separating strategy $p_{jL}^* \neq p_{jH}^*$, we employ the 'unprejudiced belief criterion' introduced by Bagwell and Ramey (1991). The basic idea of this criterion is that upon observing an out–of–equilibrium price pair (p_I, p_E) the uninformed consumers rationalize their observation with the fewest number of deviations from the equilibrium strategies. Therefore, if a price pair occurs where one of the prices is out–of–equilibrium while the other price belongs to the separating pricing strategy of the competitor, the consumers believe that the entrant's quality is signaled by the competitor.

Actually, since there are only two types of the entrant, in our context it is sufficient to consider a simplified version of the unprejudiced belief criterion: If only the entrant chooses an out–of–equilibrium price p_E and the incumbent's equilibrium price p_{IH}^* indicates a high quality entrant, then the uninformed consumers should conclude that the entrant offers high quality; there are no belief restrictions if the incumbent's price p_{IL}^* signals low quality. Indeed, a high quality signal of the incumbent looks rather convincing since it is against his interest to admit that his competitor offers a superior good. An analogous reasoning applies when the uninformed consumers conjecture that the price p_I constitutes a unilateral deviation by the incumbent. In this situation, they should infer from the entrant's price p_{EL}^* that his quality is low; there are no belief restrictions if the entrant's price is p_{EH}^*. Again, this seems plausible because expecting high quality makes little sense if the entrant acknowledges that his quality is low.

More formally, the PBE (p^*, μ^*) satisfies the *unprejudiced belief criterion* if the following two conditions (a) and (b) are satisfied:

(a) If $p^*_{IL} \neq p^*_{IH}$, then $\mu^*(p^*_{IH}, p_E) = 1$ for all $p_E \neq p^*_{EL}$.

(b) If $p^*_{EL} \neq p^*_{EH}$, then $\mu^*(p_I, p^*_{EL}) = 0$ for all $p_I \neq p^*_{IH}$.

Notice that in a two–sided separating equilibrium the criterion does not impose belief restrictions on the out–of–equilibrium price constellations (p^*_{IH}, p^*_{EL}) and (p^*_{IL}, p^*_{EH}), under which the signals of the incumbent and the entrant appear contradictory. For these constellations it is not clear whether the incumbent or the entrant has deviated from his equilibrium strategy.

In what follows, we call a PBE (p^*, μ^*) that satisfies the intuitive and the unprejudiced belief criterion a *signalling equilibrium*. In the following section, we investigate the existence and properties of such an equilibrium.

4 Signalling Equilibria

Pooling Equilibria

We first consider pooling equilibria, in which the pricing strategies of both firms reveal no information about the entrant's quality. Let $p^*_I = p^*_{IL} = p^*_{IH}$ denote the incumbent's and $p^*_E = p^*_{EL} = p^*_{EH}$ the entrant's price in a pooling equilibrium. The uninformed consumers' belief then satisfies $\mu^*(p^*_I, p^*_E) = \lambda$.

We will show that the existence of pooling equilibria is not consistent with the intuitive criterion. This is so because after observing that the entrant offers low quality, the incumbent can gain by credibly signalling the entrant's true quality through some price $p > p^*_I$. Indeed, if $q_E = q_L$ the incumbent's gain from deviating to a price p that signals a low quality entrant is

$$\varphi_{IL}(p) \equiv \Pi_{IL}(p, p^*_E, 0) - \Pi_{IL}(p^*_I, p^*_E, \lambda). \tag{22}$$

If $q_E = q_H$, the incumbent's gain from deceiving the uninformed consumers by choosing p is

$$\varphi_{IH}(p) \equiv \Pi_{IH}(p, p^*_E, 0) - \Pi_{IH}(p^*_I, p^*_E, \lambda). \tag{23}$$

The following lemma shows that the incumbent's gain from signalling a low quality of the entrant by some price $p > p^*_I$ is higher when the entrant's true quality is low than when it is high. In fact, for some critical $p' > p^*_I$ the incumbent benefits from deviating to p' and inducing the belief $\mu(p', p^*_E) = 0$ only if he is not cheating.

Lemma 1 *(a) $\varphi_{IL}(p) - \varphi_{IH}(p)$ is strictly increasing in p, and $\varphi_{IL}(p_I^*) = \varphi_{IH}(p_I^*) > 0$. (b) There exists a unique $p' > p_I^*$ such that $\varphi_{IH}(p') = 0$.*

Proof: (a) By (7) and (9) we have

$$\varphi_{IL}(p) - \varphi_{IH}(p) = \frac{(p - p_I^*)\gamma\Delta}{2t}, \tag{24}$$

and

$$\varphi_{IL}(p_I^*) = \varphi_{IH}(p_I^*) = \frac{(1-\gamma)\lambda\Delta p_I^*}{2t} > 0. \tag{25}$$

Since $\varphi'_{IL}(p) - \varphi'_{IH}(p) = \gamma\Delta/(2t) > 0$, this proves part (a).

(b) For all $p \geq p_E^* + t - \gamma\Delta$, $\varphi_{IH}(p) < 0$ because $D_{IH}(p, p_E^*, 0) = \Pi_{IH}(p, p_E^*, 0) = 0$. Since $\varphi_{IH}(p_I^*) > 0$, the intermediate value theorem therefore implies that there exist a $p' > p_I^*$ such that $\varphi_{IH}(p') = 0$. Moreover, p' is unique because $\varphi''_{IH}(p) = -1/t < 0$. Q.E.D.

When the uninformed consumers' belief decreases from λ to zero, then at the price p_I^* the incumbent's demand increases by an amount which is independent of the entrant's true quality. This is so because the informed consumers' purchasing decisions are not affected and only some fraction of uninformed consumers switches to the incumbent. But if the incumbent raises its price above p_I^* he loses more informed consumers if $q_E = q_H$ than if $q_E = q_L$. Therefore, signalling a low quality entrant by a price p' that satisfies part (b) of Lemma 1 is attractive for the incumbent only if this signal is truthful. By the reasoning of the intuitive criterion, this makes it profitable for the incumbent to deviate from his pooling strategy.

Proposition 1 *There exists no signalling equilibrium (p^*, μ^*) such that $p_{IL}^* = p_{IH}^*$ and $p_{EL}^* = p_{EH}^*$.*

Proof: By Lemma 1 there exists a unique price $p' > p_I^*$ such that $\varphi_{IL}(p') > \varphi_{IH}(p') = 0$, i.e.

$$\Pi_{IH}(p', p_E^*, 0) = \Pi_{IH}(p_I^*, p_E^*, \lambda) \tag{26}$$

$$\Pi_{IL}(p', p_E^*, 0) > \Pi_{IL}(p_I^*, p_E^*, \lambda) \tag{27}$$

Thus p' satisfies conditions (18) and (19) of the intuitive criterion. This implies that $\mu^*(p', p_E^*) = 0$. Therefore, we have

$$\Pi_{IL}(p', p_E^*, \mu^*(p', p_E^*)) = \Pi_{IL}(p', p_E^*, 0) > \Pi_{IL}(p_I^*, p_E^*, \lambda) = \Pi_{IL}(p_I^*, p_E^*, \mu^*(p_I^*, p_E^*)). \tag{28}$$

Because the price strategies violate equilibrium condition (14) for $Q = L$, there cannot exist a signalling equilibrium with $p_I^* = p_{IL}^* = p_{IH}^*$ and $p_E^* = p_{EL}^* = p_{EH}^*$. Q.E.D.

Interestingly, the conclusion that the intuitive criterion eliminates all pooling equilibria relies on the ability of the incumbent to credibly signal a low quality entrant rather than on the entrant's ability to provide a credible price signal of high quality. Indeed, one cannot use an analogous argument as in Lemma 1 to show that the high quality entrant always gains more than the low quality entrant from a price $p > p_E^*$ that the uninformed consumers interpret as a high quality signal. The reason is that the entrant's unit cost depends on his quality. If consumers become more optimistic and raise μ, then at a given price p_E^* the low and the high quality entrant's demand increases by the same amount. Yet, the low quality entrant's profit increases more than the high quality entrant's profit because the latter has a higher production cost and therefore a smaller profit margin. For some parameter constellations, this may prevent the high quality entrant to gain by deviating from a pooling strategy and appealing to the intuitive criterion.[5]

One–Sided Separating Equilibria

We now turn to the analysis of one–sided separating equilibria, in which one firm chooses a pooling and the other a separating pricing strategy. We will show that such equilibria typically do not exist, except for special parameter constellations. First, consider the case where the incumbent's price $p_I^* = p_{IL}^* = p_{IH}^*$ is independent of the entrant's quality, whereas the entrant chooses quality contingent prices p_{EL}^* and p_{EH}^* with $p_{EL}^* \neq p_{EH}^*$. Because in equilibrium the uninformed consumers infer the entrant's quality from his price, their beliefs satisfy $\mu^*(p_I^*, p_{EL}^*) = 0$ and $\mu^*(p_I^*, p_{EH}^*) = 1$.

The following lemma establishes necessary conditions for this type of equilibrium.

Lemma 2 *Suppose that the prices p, with $p_I = p_{IL} = p_{IH}, p_{EL} \neq p_{EH}$, can be supported as a signalling equilibrium (p, μ) by some belief μ. Then p must satisfy*

$$p_{EL} = \text{argmax}_p\, \Pi_{EL}(p_I, p, 0), \tag{29}$$

$$p_I = \text{argmax}_p\, \Pi_{IH}(p, p_{EH}, 1) = \text{argmax}_p\, \Pi_{IL}(p, p_{EL}, 0), \tag{30}$$

$$p_{EH} \textit{ maximizes } \quad \Pi_{EH}(p_I, p, 1) \quad \textit{subject to} \quad \Pi_{EL}(p_I, p, 1) \leq \Pi_{EL}(p_I, p_{EL}, 0). \tag{31}$$

Proof: Since $p_{EL} \neq p_{EH}$ implies $\mu(p_I, p_{EL}) = 0$ and $\partial \Pi_{EL}/\partial \mu > 0$, it follows from equilibrium condition (15) that for all $p \geq 0$

$$\Pi_{EL}(p_I, p_{EL}, 0) \geq \Pi_{EL}(p_I, p, \mu(p_I, p)) \geq \Pi_{EL}(p_I, p, 0). \tag{32}$$

[5] This is related to the observation of Bagwell and Riordan (1991) that in a monopoly model pooling equilibria satisfying the intuitive criterion may exist for some range of parameter values.

This proves that (29) must hold. Analogously, $\mu(p_I, p_{EH}) = 1$ and $\partial \Pi_{IH}/\partial \mu < 0$ imply by (14) that for all $p \geq 0$

$$\Pi_{IH}(p_I, p_{EH}, 1) \geq \Pi_{IH}(p, p_{EH}, \mu(p, p_{EH})) \geq \Pi_{IH}(p, p_{EH}, 1). \tag{33}$$

This proves that p_I must satisfy the first condition in (30).

Suppose that p_I does not satisfy the second condition in (30). Since part (b) of the unprejudiced belief criterion implies $\mu(p, p_{EL}) = 0$ for all $p \neq p_I$, then there exist some p such that

$$\Pi_{IL}(p_I, p_{EL}, \mu(p_I, p_{EL})) = \Pi_{IL}(p_I, p_{EL}, 0) < \Pi_{IL}(p, p_{EL}, 0) = \Pi_{IL}(p, p_{EL}, \mu(p, p_{EL})). \tag{34}$$

This is a contradiction to the condition that in equilibrium p_I has to satisfy (14) for $Q = L$.

Note that p_{EH} must satisfy the constraint in (31) because equilibrium condition (15) implies that

$$\Pi_{EL}(p_I, p_{EL}, 0) = \Pi_{EL}(p_I, p_{EL}, \mu(p_I, p_{EL})) \geq \Pi_{EL}(p_I, p_{EH}, \mu(p_I, p_{EH})) = \Pi_{EL}(p_I, p_{EH}, 1). \tag{35}$$

Suppose that p_{EH} does not solve the maximization problem in (31). Then there exists some p that satisfies the constraint in (31) and $\Pi_{EH}(p_I, p, 1) > \Pi_{EH}(p_I, p_{EH}, 1)$. Because part (b) of the intuitive criterion then implies $\mu(p_I, p) = 1$, this yields

$$\Pi_{EH}(p_I, p, \mu(p_I, p)) = \Pi_{EH}(p_I, p, 1) > \Pi_{EH}(p_I, p_{EH}, 1) = \Pi_{EH}(p_I, p_{EH}, \mu(p_I, p_{EH})), \tag{36}$$

a contradiction to equilibrium condition (15) for $Q = H$. Q.E.D.

Condition (29) simply states that the low quality entrant's price reaction against p_I is not distorted by signalling considerations. Indeed, some price p not satisfying (29) can maximize the low quality seller's profit only if $\mu(p_I, p) > 0$. But this is inconsistent with an equilibrium where prices reveal the true quality. The same argument underlies the first condition in (30) for the incumbent's price when competing against the high quality entrant. The incumbent's price reaction against p_{EH} cannot be distorted because the consumers' belief that the entrant has high quality is already the worst possible belief from the incumbent's perspective.

The second condition for p_I in (30) is implied by part (b) of the unprejudiced belief criterion. This criterion restricts the consumers' belief to $\mu(p, p_{EL}) = 0$ for all $p \neq p_I$. Further, Bayes' rule in (16) requires that $\mu(p_I, p_{EL}) = 0$. Thus, the incumbent's pricing has no impact on consumer beliefs when facing the low quality entrant, and so in this situation there are also no signalling distortions.

Finally, the constraint in condition (31) has to be satisfied because otherwise the low quality entrant would gain by imitating the high quality entrant's price. Further, the intuitive criterion implies that consumers infer high quality whenever the entrant gains by deviating to some price satisfying this constraint. Accordingly, the high quality entrant's price p_{EH} must solve the constrained maximization problem in (31).

Lemma 2 allows us to show that a one–sided separating equilibrium with $p^*_{EL} \neq p^*_{EH}$ exists at most for a single value of the parameter γ. Since there is no reason for why the fraction of informed consumers should be identical to this value, an equilibrium of this type effectively fails to exist.

Proposition 2 *For all $\gamma \neq t/(t+\Delta)$ there exists no signalling equilibrium (p^*, μ^*) such that $p^*_{IL} = p^*_{IH}$ and $p^*_{EL} \neq p^*_{EH}$.*

Proof: The first–order conditions for (29) and (30) in Lemma 2 are

$$\frac{\partial \Pi_{EL}(p_I, p_{EL}, 0)}{\partial p_{EL}} = \frac{t + p_I - 2p_{EL}}{2t} = 0, \tag{37}$$

$$\frac{\partial \Pi_{IH}(p_I, p_{EH}, 1)}{\partial p_I} = \frac{t - \Delta + p_{EH} - 2p_I}{2t} = 0, \quad \frac{\partial \Pi_{IH}(p_I, p_{EL}, 0)}{\partial p_I} = \frac{t + p_{EL} - 2p_I}{2t} = 0.$$

The solution of these equations is

$$p^*_I = t, p^*_{EL} = t,\ p^*_{EH} = t + \Delta. \tag{38}$$

If the constraint in (31) is not binding, we obtain from the first–order condition

$$\frac{\partial \Pi_{EH}(p^*_I, p_{EH}, 1)}{\partial p_{EH}} = \frac{2t + \Delta + c - 2p_{EH}}{2t} \tag{39}$$

that $p^*_{EH} = (2t + \Delta + c)/2$. This, however, is inconsistent with the last equation in (38) as $\Delta > c$. If the constraint in (31) is binding, then $\Pi_{EL}(p^*_I, p^*_{EH}, 1) = \Pi_{EL}(p^*_I, p^*_{EL}, 0)$. By (38) this equality is equivalent to

$$\frac{(\Delta + t)(t - \gamma\Delta)}{2t} = \frac{t}{2}. \tag{40}$$

From this equation it follows that the conditions of Lemma 2 are satisfied only if $\gamma = t/(t+\Delta)$. Q.E.D.

The nonexistence result stated in Proposition 2 is a straightforward implication of Lemma 2. The lemma shows that prices in a one–sided separating equilibrium have to satisfy four conditions. Yet, such an equilibrium determines only three prices. This means that not all conditions can hold simultaneously, unless the exogenous parameters accidentally make one of the conditions redundant. The following lemma shows that a similar observation applies to the other type of one–sided separating equilibria, in which the entrant adopts a pooling strategy $p_E^* = p_{EL}^* = p_{EH}^*$ and only the incumbent's prices p_{IL}^* and p_{EH}^* reveal the entrant's quality so that $\mu^*(p_E^*, p_{IL}^*) = 0$ and $\mu^*(p_E^*, p_{IH}^*) = 1$.

Lemma 3 *Suppose that the prices p, with $p_{IL} \neq p_{IH}, p_E = p_{EL} = p_{EH}$, can be supported as a signalling equilibrium (p, μ) by some belief μ. Then p must satisfy*

$$p_{IH} = \text{argmax}_p\, \Pi_{IH}(p, p_E, 1), \tag{41}$$

$$p_E = \text{argmax}_p\, \Pi_{EL}(p_{IL}, p, 0) = \text{argmax}_p\, \Pi_{EH}(p_{IH}, p, 1), \tag{42}$$

$$p_{IL} \textit{ maximizes } \quad \Pi_{IL}(p, p_E, 0) \quad \textit{subject to} \quad \Pi_{IH}(p, p_E, 0) \leq \Pi_{IH}(p_{IH}, p_E, 1). \tag{43}$$

We omit a proof of this lemma because it is analogous to the proof of Lemma 2. By our next proposition, also the implications the two lemmas are similar. In fact, Lemma 3 shows that a one–sided separating equilibrium with $p_{IL}^* \neq p_{IH}^*$ may exist merely under a single parameter constellation.

Proposition 3 *For all $\gamma \neq (3t\Delta - 2c\Delta - \Delta^2 - 3c^2)/(3t\Delta + 4c\Delta + 2\Delta^2)$ there exists no signalling equilibrium (p^*, μ^*) such that $p_{IL}^* \neq p_{IH}^*$ and $p_{EL}^* = p_{EH}^*$.*

Proof: From the first–order conditions for (41) and (42) in Lemma 3,

$$\frac{\partial \Pi_{IH}(p_{IH}, p_E, 1)}{\partial p_{IH}} = \frac{t - \Delta + p_E - 2p_{IH}}{2t} = 0, \tag{44}$$

$$\frac{\partial \Pi_{EL}(p_{IL}, p_E, 0)}{\partial p_E} = \frac{t - 2p_E + p_{IL}}{2t} = 0, \quad \frac{\partial \Pi_{EH}(p_{IH}, p_E, 1)}{\partial p_E} = \frac{t + \Delta + c - 2p_E + p_{IH}}{2t} = 0,$$

we obtain the solution

$$p_{IL}^* = \frac{3t + 2\Delta + 4c}{3},\ p_{IH}^* = \frac{3t - \Delta + c}{3},\ p_E^* = \frac{3t + \Delta + 2c}{3}. \tag{45}$$

If the constraint in (43) is not binding, we obtain from the first–order condition

$$\frac{\partial \Pi_{IL}(p_{IL}, p_E^*, 0)}{\partial p_{IL}} = \frac{\Delta + 2c + 6t - 6p_{IL}}{6t} \tag{46}$$

that $p_{IL}^* = (6t + \Delta + 2c)/6$. This, however, is inconsistent with the first equation in (45). If the constraint in (43) is binding, then $\Pi_{IH}(p_{IL}^*, p_E^*, 0) = \Pi_{IH}(p_{IH}^*, p_E^*, 1)$. By (45) this is equivalent to

$$\frac{(2\Delta + 4c + 3t)(3t - 3\gamma\Delta - \Delta - 2c)}{18t} = \frac{(3t - \Delta + c)^2}{18t}. \tag{47}$$

Solving this equation for γ yields $\gamma = (3t\Delta - 2c\Delta - \Delta^2 - 3c^2)/(3t\Delta + 4c\Delta + 2\Delta^2)$. Thus, if γ does not satisfy this condition, also the conditions of Lemma 3 cannot hold. Q.E.D.

Our results so far show that in a signalling equilibrium it cannot happen that one or both of the duopolists adopt a pooling strategy. In Proposition 1, the intuitive criterion rules out two–sided pooling. Propositions 2 and 3 eliminate one–sided pooling by combining the intuitive and the unprejudiced belief criterion. This leaves a two–sided separating equilibrium as the remaining candidate for a signalling equilibrium.

Two–Sided Separating Equilibria

In a two–sided separating equilibrium the uninformed consumers' equilibrium belief is $\mu^*(p^*_{IL}, p^*_{EL}) = 0$ and $\mu^*(p^*_{IH}, p^*_{EH}) = 1$ as $p^*_{IL} \neq p^*_{IH}$ and $p^*_{EL} \neq p^*_{EH}$. Since each firm's price is informative, the intuitive criterion is no longer applicable. Therefore, only the unprejudiced belief criterion plays a role in the following lemma which provides necessary and sufficient conditions for a two–sided separating equilibrium.

Lemma 4 *The prices p, with $p_{IL} \neq p_{IH}, p_{EL} \neq p_{EH}$, can be supported as a signalling equilibrium (p, μ) by some belief μ if and only if*

(a) p is identical to the perfect information equilibrium $\hat{p}$ in (13), and

(b) there exists some $\bar{\mu} \in [0, 1]$ such that

$$\Pi_{IH}(p_{IH}, p_{EH}, 1) \geq \Pi_{IH}(p_{IL}, p_{EH}, \bar{\mu}), \quad \Pi_{EL}(p_{IL}, p_{EL}, 0) \geq \Pi_{EL}(p_{IL}, p_{EH}, \bar{\mu}). \tag{48}$$

Proof: We first show that (a) and (b) must hold in a signalling equilibrium (p, μ). By (14)

$$\Pi_{IH}(p_{IH}, p_{EH}, 1) \geq \Pi_{IH}(p, p_{EH}, \mu^*(p, p_{EH})) \geq \Pi_{IH}(p, p_{EH}, 1) \tag{49}$$

for all $p \geq 0$, where the second inequality follows from $\partial \Pi_{IH}/\partial \mu < 0$. Similarly, (14) and part (b) of the unprejudiced belief criterion imply

$$\Pi_{IL}(p_{IL}, p_{EL}, 0) \geq \Pi_{IL}(p, p_{EL}, \mu^*(p, p_{EL})) = \Pi_{IL}(p, p_{EL}, 0) \tag{50}$$

for all $p \neq p_{IH}$. By continuity of $\Pi_{IL}(\cdot, p_{EL}, 0)$, therefore also

$$\Pi_{IL}(p_{IL}, p_{EL}, 0) \geq \Pi_{IL}(p_{IH}, p_{EL}, 0). \tag{51}$$

By an analogous argument it follows from (14), $\partial \Pi_{EL}/\partial \mu > 0$, and part (a) of the unprejudiced belief criterion that

$$\Pi_{EL}(p_{IL}, p_{EL}, 0) \geq \Pi_{EL}(p_{IL}, p, 0), \quad \Pi_{EH}(p_{IH}, p_{EH}, 1) \geq \Pi_{EH}(p_{IH}, p, 1) \tag{52}$$

for all $p \geq 0$. By (49)–(52), p satisfies the conditions that define $\hat{p}$ in (12). This proves that (p, μ) must satisfy claim (a) that $p = \hat{p}$. Note that by (14) and (15)

$$\begin{aligned} \Pi_{IH}(p_{IH}, p_{EH}, 1) &\geq \Pi_{IH}(p_{IL}, p_{EH}, \mu(p_{IL}, p_{EH})), \\ \Pi_{EL}(p_{IL}, p_{EL}, 0) &\geq \Pi_{EL}(p_{IL}, p_{EH}, \mu(p_{IL}, p_{EH})). \end{aligned} \tag{53}$$

This proves that statement (b) holds for $\bar{\mu} \equiv \mu(p_{IL}, p_{EH})$.

Next we show that $(\hat{p}, \mu)$ is a signalling equilibrium for some μ only if (b) holds. Note that the intuitive criterion does not apply to $\hat{p}$ because $\hat{p}_{IL} \neq \hat{p}_{IH}$ and $\hat{p}_{EL} \neq \hat{p}_{EH}$. In line with the unprejudiced belief criterion, define

$$\mu(\hat{p}_{IH}, p) \equiv 1 \text{ for all } p \neq \hat{p}_{EL}, \quad \mu(p, \hat{p}_{EL}) \equiv 0 \text{ for all } p \neq \hat{p}_{IH}, \quad \mu(\hat{p}_{IH}, \hat{p}_{EL}) \equiv \lambda. \tag{54}$$

Further, if (48) in part (b) of the lemma holds for $p = \hat{p}$ we can set

$$\mu(\hat{p}_{IL}, p) \equiv 0 \text{ for all } p \neq \hat{p}_{EH}, \quad \mu(p, \hat{p}_{EH}) \equiv 1 \text{ for all } p \neq \hat{p}_{IL}, \quad \mu(\hat{p}_{IL}, \hat{p}_{EH}) \equiv \bar{\mu}. \tag{55}$$

The beliefs for all other price pairs (p_I, p_E) play no role in the definition of a PBE and so they are arbitrary. Since $\mu(\hat{p}_{IL}, \hat{p}_{EL}) = 0$ and $\mu(\hat{p}_{IH}, \hat{p}_{EH}) = 1$ by (54) and (55), these beliefs satisfy Bayes rule (16) in part (b) of the definition of a PBE. Further since $\hat{p}$ satisfies (12) and (53) holds for $p = \hat{p}$, it is easily verified that $(\hat{p}, \mu)$ satisfies also the conditions (14) and (15) for profit maximization in part (a) of the definition of a PBE. This proves that $\hat{p}$ and the beliefs μ in (54) and (55) constitute a signalling equilibrium if (48) in part (b) of the lemma holds for $p = \hat{p}$. If the latter condition does not hold, then there is no belief $\mu(p_{IL}, p_{EH})$ that satisfies both conditions in (53) for $p = \hat{p}$. In this case, there exists no PBE (p, μ) with $p = \hat{p}$ because at least one of the conditions (14) and (15) for profit maximization is violated. Q.E.D.

By statement (a) of Lemma 4, in a two–sided separating equilibrium the firms' prices are identical to the outcome of price competition under full information of all market participants about the entrant's quality. Thus, even though prices act as signals, they are not distorted by incentive restrictions. This observation is a well–known implication of the unprejudiced beliefs refinement (see Bagwell and Ramey (1991)).[6] The idea is simply that the high quality entrant can ignore signalling effects when already the incumbent's price convinces the uninformed consumers of high quality. Similarly, the incumbent does not have to resort to distorted pricing to indicate a low quality entrant, because the entrant himself already reveals his quality through his price setting strategy. In a two–sided separating equilibrium,

[6]Yehezkel (2006) proposes a generalization of the unprejudiced belief criterion that eliminates all possible separating equilibria but the full information outcome.

therefore, the firms' prices are determined as mutually undistorted best responses against the competitor and are thus identical to the full information equilibrium.

While prices are not distorted by signalling effects, statement (b) of Lemma 4 shows that they have to satisfy an incentive compatibility restriction, which is related to the signalling rivalry between the duopolists. The uninformed consumers will be perplexed when they observe the out–of-equilibrium price pair $(\hat{p}_{IL}, \hat{p}_{EH})$. These prices are contradictory because the incumbent's price signals a low quality entrant and the entrant's price a high quality. Also, it is not clear which firm has deviated from its equilibrium strategy. The prices $(\hat{p}_{IL}, \hat{p}_{EH})$ could originate from the equilibrium pair $(\hat{p}_{IH}, \hat{p}_{EH})$ because the incumbent has deviated to $\hat{p}_{IL}$; or they could originate from the equilibrium pair $(\hat{p}_{IL}, \hat{p}_{EL})$ because the entrant has deviated to $\hat{p}_{EH}$. Condition (48) states that there must be some belief $\bar{\mu} = \mu(\hat{p}_{IL}, \hat{p}_{EH})$ that deters both kinds of deviations. On the one hand, by the first inequality in (48), $\bar{\mu}$ must be high enough so as to make it unattractive for the incumbent to deviate from $\hat{p}_{IH}$ to $\hat{p}_{IL}$. On the other hand, the second inequality in (48) requires that $\bar{\mu}$ is small enough so that the entrant cannot gain by deviating from $\hat{p}_{EL}$ to $\hat{p}_{EH}$.

Whether condition (b) of Lemma 4 holds or not, depends on how large the fraction γ of informed consumers is. To state our next result, we define the critical parameter

$$\bar{\gamma} \equiv \frac{27\Delta t^2 + (\Delta - c)(3t\Delta + 15ct + 2c^2) - \Delta(\Delta^2 - c^2)}{27\Delta t^2 + 9\Delta^2 t + 18\Delta tc}. \tag{56}$$

Note that, since

$$\frac{\partial \bar{\gamma}}{\partial t} > 0, \qquad \lim_{t \to (\Delta - c)/3} \bar{\gamma} = \frac{\Delta^2 - c^2}{2\Delta^2 + c\Delta} > 0, \qquad \lim_{t \to \infty} \bar{\gamma} = 1, \tag{57}$$

our assumption (4) implies that $\bar{\gamma} \in (0, 1)$.

Proposition 4 *(a) Let $\gamma \geq \bar{\gamma}$. Then there exists a signalling equilibrium (p^*, μ^*) with $p^*_{IL} \neq p^*_{IH}$ and $p^*_{EL} \neq p^*_{EH}$. The prices p^* in this equilibrium are identical to the perfect information equilibrium $\hat{p}$. (b) If $\gamma < \bar{\gamma}$, there exists no signalling equilibrium (p^*, μ^*) such that $p^*_{IL} \neq p^*_{IH}$ and $p^*_{EL} \neq p^*_{EH}$.*

Proof: By Lemma 4 it is sufficient to show that for $p = \hat{p}$ (48) has a solution $\bar{\mu} \in [0, 1]$ if and only if $\gamma \geq \bar{\gamma}$. Using $\hat{p}$ in (13), the first inequality in (48) is equivalent to

$$\frac{(3t - \Delta + c)^2}{18t} \geq \frac{3t + 2c + \Delta(1 - 3\gamma) - 3\bar{\mu}\Delta(1 - \gamma)}{6}. \tag{58}$$

Solving this inequality for $\bar{\mu}$ yields

$$\bar{\mu} \geq \bar{\mu}_I(\gamma) \equiv \frac{9t\Delta(1 - \gamma) - (\Delta - c)^2}{9t\Delta(1 - \gamma)}. \tag{59}$$

$\bar{\gamma}\|_{\Delta=10}$	$t=2$	$t=4$	$t=6$
$c=5$	0.35	0.51	0.6
$c=7$	0.31	0.44	0.53
$c=9$	0.23	0.34	0.43

Table 1: *Numerical values for $\bar{\gamma}$.*

By (13) the second inequality in (48) is equivalent to

$$\frac{t}{2} \geq \frac{(3t+2c+\Delta)(3t-2c-\Delta+3\bar{\mu}\Delta(1-\gamma))}{18t}. \tag{60}$$

Solving this inequality for $\bar{\mu}$ yields

$$\bar{\mu} \leq \bar{\mu}_E(\gamma) \equiv \frac{(\Delta+2c)^2}{3\Delta(1-\gamma)(3t+2c+\Delta)}. \tag{61}$$

The inequalities (59) and (61) admit a solution $\bar{\mu}$ if and only if $\bar{\mu}_I(\gamma) \leq \bar{\mu}_E(\gamma)$. It is easily verified that $\bar{\gamma}$, as defined in (56), satisfies $\bar{\mu}_I(\bar{\gamma}) = \bar{\mu}_E(\bar{\gamma})$. Note that $\bar{\mu}_I(0) < 1, \bar{\mu}_E(0) > 0$, $\bar{\mu}_I'(\gamma) < 0$ and $\bar{\mu}_E'(\gamma) > 0$. Since $\bar{\gamma} \in (0,1)$, this implies that there exists a $\bar{\mu} \in [\bar{\mu}_I(\gamma), \bar{\mu}_E(\gamma)] \cap [0,1]$ if and only if $\gamma \geq \bar{\gamma}$. Q.E.D.

In a two–sided separating equilibrium prices are not distorted by signalling. The incumbent or the entrant can gain by a unilateral deviation only because this changes the uninformed consumers' beliefs. Therefore, a deviation is not profitable as long as not too many consumers are uninformed. This explains why $(\hat{p}, \mu^*)$ can constitute a signalling equilibrium for $\gamma \geq \bar{\gamma}$. If $\gamma < \bar{\gamma}$, then the firms' signalling rivalry is too intense to prevent profitable deviations: Either the incumbent will defect from the equilibrium if $q_E = q_H$, or the entrant will defect if $q_E = q_L$. As observed by Schultz (1999) in a different context, conflicting interests may thus rule out the existence of a two–sided separating equilibrium for some parameter constellations.

In Table 1 some numerical values illustrate how $\bar{\gamma}$ depends on c and t if $\Delta = 10$. Prices can be used as credible signals because of their effect on demand. Since the price sensitivity of demand is negatively related to the product differentiation parameter t, this implies that $\bar{\gamma}$ is increasing in t. An increase in the cost c of high quality raises the price differences $|\hat{p}_{IH} - \hat{p}_{IL}|$ and $|\hat{p}_{EL} - \hat{p}_{EH}|$. Therefore, a deviation of the incumbent from $\hat{p}_{IH}$ to $\hat{p}_{IL}$ or of the entrant from $\hat{p}_{EL}$ to $\hat{p}_{EH}$ is less profitable for high values of c. Consequently, if c is increased, a smaller fraction $\bar{\gamma}$ of informed consumers suffices for existence of a signalling equilibrium.

5 Conclusion

Our analysis shows that a firm may not have to resort to distorted pricing to signal its quality to the uninformed consumers. If its quality is known to a competitor, then the prices of both firms become quality signals and signalling competition may lead to non–distorted pricing in equilibrium. Indeed, in our model only the full information equilibrium can survive under two belief refinements that have frequently been used in the literature.

This finding has obvious implications for other strategic choices. For example, consider the market entry decision of a firm whose quality is not publicly observable. In this situation our analysis indicates that entry decisions are not distorted when at least one of the incumbent firms learns the new firm's quality after it has entered the market. A similar conclusion obtains for *R&D* investments in product innovation when some consumers cannot observe whether the investment has been successful or not. As long as competing firms become informed about the outcome, our results suggest that the incentives for product innovation are not distorted by the presence of uninformed consumers.

Our analysis also reveals that the two refinements, which we adopt to restrict out–of–equilibrium beliefs, can become incompatible with existence of an equilibrium. When the fraction of informed consumers is too small in our model, there is no equilibrium satisfying both the intuitive criterion and the unprejudiced belief refinement. One way out of this problem would be to weaken these refinements. But it is not obvious how one should proceed along these lines because both refinements look rather appealing and convincing in the context of our model. Another approach would be modifying our model by assuming that the incumbent is not perfectly informed about the entrant's quality but that he receives noisy information. This would eliminate the problem of specifying beliefs for 'contradictory' price signals. With noisy information such signals would no longer be an out–of–equilibrium event in a two–sided separating equilibrium because it happens with positive probability that the incumbent receives information that the entrant's quality is low even though its quality is actually high. It may be interesting for future research to investigate whether with noisy information a signalling equilibrium always exists and whether it approaches the full information equilibrium as the noise becomes negligible.

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