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# WAGE AND EMPLOYMENT EFFECTS OF NON-BINDING MINIMUM WAGES

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## **Abstract**

Common wisdom holds that the introduction of a non-binding minimum wage is irrelevant for actual wages and employment. Empirical and experimental research, however, has shown that the introduction of a minimum wage can raise even those wages that were already above the new minimum wage. In this paper, we analyze how these findings can be explained by theoretical wage bargaining models between unions and firms. While the Nash bargaining solution is unaffected by minimum wages below initially bargained wages, we show that such minimum wages can drive up wages – and be harmful to employment – when bargaining follows the Kalai-Smorodinsky solution.

JEL Classification Codes: J38, C78, J52

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## 1. Introduction

The wage and employment effects of a statutory minimum wage are a fiercely debated topic among economists and policymakers. Its proponents argue that raising the wages of the lowest-paid would help fighting poverty, while its critics claim that introducing such rigidities impedes the allocative role of flexible wages, causing more unemployment and possibly even more poverty. Theoretical labor economics cannot provide a clear-cut answer on which view is correct, since the effects of minimum wages depend strongly on the structure of the labor market. While minimum wages are generally expected to be harmful to employment when labor markets are competitive, there is the possibility that introducing a wage floor can actually increase employment if the labor market is characterized by monopsonistic structures and minimum wages are not set at too high levels (see Brown (1999) for an overview). Empirical studies do not give a clear picture either. While some case studies, focusing at specific industries at the regional level, provide evidence for positive employment effects (Card and Krueger 1995), studies that focus on broader groups typically find negative employment effects (Neumark and Wascher 2008).

Despite these controversial views, there appears to be one issue that receives support from proponents and opponents of the minimum wage alike: minimum wages have to be binding to have any effect on wages and employment. According to this commonly held belief, a minimum wage set at a non-binding level, i.e. below what the lowest-paid workers in an economy would receive even without it, cannot be effective in raising actually paid wages and thus will not have any impact on employment. In short: a minimum wage is either binding or de facto non-existent.

This conventional view, however, is at odds with a number of observations obtained from empirical and experimental studies of minimum wages. A large number of empirical studies report that introducing or raising the minimum wage has spillover effects to wages higher up in the wage distribution (see Card and Krueger (1995, Ch. 5) and Neumark and Wascher (2008, Ch. 4) for an overview). Three possible explanations for the existence of spillover effects are provided by the theoretical literature: 1) firms try to replace minimum wage workers with close substitutes higher up in the wage distribution, thereby pushing up their wages, 2) high-wage firms find it harder to recruit employees from low-wage firms and thus have to pay higher wages, and 3) firms adjust wages to maintain the differential between their

lowest-skilled and higher-skilled workers in order to preserve incentives for effort and skill acquisition. Laboratory experiments, however, in which such effects can be eliminated by design, suggest that these three reasons cannot exhaustively explain why spillover effects occur. Falk et al. (2006) report on an experiment in which a rent is distributed between participants acting as “workers” and a “firm”. Even though demand shifts and wage hierarchy considerations cannot play a role in this setting, the experiment shows that introducing a minimum wage still affects the equilibrium wage structure by raising workers’ reservation wages. Falk et al. (2006) argue that the minimum wage affects what people consider to be a “fair” compensation for their work. If increasing the minimum wage also raises the wage demanded by workers paid above the minimum level, even a non-binding minimum wage could have real labor market effects. Fairness concerns might thus also play an important role in explaining the existence of spillover effects. Brandts and Charness (2004) show that the introduction of a minimum wage, even if it is non-binding, affects workers’ effort provision negatively. In their interpretation, wage determination is driven by gift-exchange considerations (Akerlof 1982). Workers perceive a given wage to be less kind if the difference to the wage the firm would have to pay anyway shrinks, which leads them to exert less effort. Under a more general perspective, these findings suggest that preferences over wages are *menu dependent* (Sen 1997). All potentially available, but eventually foregone, choices matter for the determination of wages because workers value their wage in relation to the firm’s options.

In this paper, we will analyze how menu dependence can explain spillover effects in theoretical models of wage bargaining. In particular, we will examine whether a non-binding minimum wage can affect wages and employment if wage bargaining between firms and unions can be described by the Nash and the Kalai-Smorodinsky bargaining solutions (Nash 1950, Kalai and Smorodinsky 1975). For the Nash solution, the “common wisdom” is supported. The introduction of a non-binding minimum wage cannot change the outcome of a Nash bargain because it does not affect any of the relevant parameters of the bargaining solution. A non-binding minimum wage, i.e. a minimum wage that is less than the lowest wage received by any worker in the economy, is irrelevant for the bargaining set and the bargaining parties’ outside options. Only if the minimum wage is binding for at least some workers, it could potentially have spillover effects to other workers by changing the value of their outside option. The fairness concerns that appear to drive the empirical and experimental findings cited above do not play a role in the Nash bargaining solution.

Kalai and Smorodinsky (1975) proposed an alternative bargaining solution (henceforth KS solution) that is often considered to incorporate fairness considerations into the wage bargain (McDonald and Solow 1981). The KS solution states that both bargaining parties agree to a solution that equalizes the relative utility gains, defined as the ratio of the actual gain to the maximum feasible gain. Each party's maximum feasible gain is determined by the payoff it can secure by pushing the other party to the minimum payoff it would just be willing (or allowed) to accept. A reduction in a party's maximum feasible gain will then also diminish its "just" claim to the bargaining set. Contrary to the Nash solution, a non-binding minimum wage can affect the bargaining solution in the KS solution because it reduces the firm's maximum feasible gain. Without a statutory minimum wage, a firm's maximum feasible gain is given by its profit if it paid its workers exactly the value of their outside option. A union's maximum feasible gain in this case would be determined by the highest wage the firm would be willing to pay without closing down or relocating (or the monopoly union solution if this is associated with a lower wage). The KS bargaining outcome requires equal sacrifices of both parties, so that the union will not be able to set its most desired wage, but it will definitively reach a payoff higher than its outside option. If a minimum wage is introduced at a level between the bargained wage and the union's outside option, it will not be binding, i.e. no worker will have received a wage at or below the new minimum level prior to its introduction. Nevertheless, such a non-binding minimum wage will affect the bargaining outcome because it reduces the firm's maximum feasible gain. Prior to the introduction of the minimum wage, the best a firm could achieve was to reduce the union's payoff to its outside option. With a wage floor above the former outside option, however, the firm can at best reduce the wage to its statutory minimum. This results in a smaller maximum feasible gain of the firm, which reduces its claim to the bargaining set and leads to higher wages and lower employment. Hence, the KS bargaining solution can provide a theoretical justification for the empirical observation that even a non-binding minimum wage, set at a level below the wages actually observed in the labor market, can lead to rising wage levels and reduced employment.

We will proceed as follows. Section 2 provides a review of the literature on spillover effects of minimum wages. Section 3 derives the labor market outcome when bargaining follows either the Nash or the Kalai-Smorodinsky solution. Section 4 discusses the labor market effects of a minimum wage introduction. Section 5 concludes.

## 2. Spillover effects: Theory and evidence

The obvious effect of the introduction of a minimum wage is that it raises the wages of workers that were previously paid less than the new minimum (given that they keep their jobs). A less obvious, but nevertheless regularly observed effect of the introduction of, or an increase in, the minimum wage is that it raises the wages of some workers even above the new minimum. Similarly, the wages of some workers who were already paid above the new minimum increase even further. This positive impact of a legal minimum wage on wages higher up in the wage distribution has become known as “spillover” or “ripple” effects.

Economic theory provides three popular explanations for the existence of spillover effects. The first explanation builds on shifts in labor demand induced by the minimum wage. If workers have different productivities because they differ in their levels of human capital, the introduction of a minimum wage will cause firms to substitute away from workers with productivity below the new minimum wage towards workers with higher productivities. This will increase the return to human capital and thus results in higher wages for workers who were already better-paid. Pettengill (1981) develops a model in which the degree of substitutability between different skill groups is greater between workers whose skill levels are relatively close than between workers whose skill levels differ substantially. In this model, spillover effects will be strongest for workers who earn wages above, but close to the new minimum wage, while workers farther up the wage distribution do not benefit.

A second explanation can be derived in equilibrium search models with monopsonistic firm behavior. Manning (2003) shows that, in a modified Burdett-Mortensen (1998) framework, firms that previously paid relatively high wages to attract workers from low-wage firms can only recruit enough new employees if they increase their wages too. This effect is strongest for firms that used to pay wages just above the new minimum, so that spillovers will be concentrated to wage level close to the minimum wage.

A third explanation is provided by efficiency wage models. Grossman (1983) develops a model with skilled and unskilled labor in which the effort exerted by skilled workers depends on their wage relative to that received by unskilled workers. If an increase in the legal minimum wage raises the wages of unskilled workers, then there will be a demand shift effect (as described above) that increases the demand and the wages of skilled workers. In addition, the smaller wage differential between skilled and unskilled work will reduce skilled workers' effort, so that the firm will have to increase the wage received by higher-paid workers as well.

In this type of model, firms react to an increase in the minimum wage by maintaining their internal wage hierarchy.

A number of empirical studies have examined the existence and magnitude of the spillover effect from minimum wages.<sup>1</sup> The first such study was conducted by Grossman (1983). Her results indicate that an increase in the minimum wage increases the wages of occupations just above the new minimum wage, at least in the short run. In the long-run, the effects are less clear, perhaps because the real value of the minimum wage diminished due to inflation. Katz and Krueger (1992) study the effect of the increase in the federal minimum wage from \$3.35 to \$3.80 in 1990 and to \$4.25 in 1991 on fast-food restaurants in Texas. They find that about one-third of surveyed Texan fast-food restaurants reacted to the 1991 minimum wage increase by “maintaining their wage hierarchy”. The wages of workers who earned more than the old minimum wage would be raised as well, so that they would also exceed the new minimum wage. Among firms in which the starting wage was already above the new minimum wage, 60 percent reacted to the higher minimum wage by raising their wages even further. In a similar study for the 1992 minimum wage increase in New Jersey, Card and Krueger (1994) do not observe evidence for spillover effects.

To estimate the magnitude of spillover effects, Manning (2003) assumes that, without the minimum wage, wages would follow a standard log normal distribution and then compares the actual with an estimated latent wage distribution, using data for the U.S. between 1979 and 2000. His findings suggest that spillovers amount to about 11 percent of the minimum wage for wages just above the new minimum, but disappear for wages higher than 50 percent above the minimum wage. In a related study for Great Britain, Dickens and Manning (2004) find only small spillover effects in the short run, and virtually no spillovers if a longer time horizon is considered.

A more direct approach to estimating spillover effects is taken by Neumark et al. (2004), who estimate the impact of changes in the minimum wage on the wages of workers already earning more than the new minimum. To control for contemporaneous general wage growth, they compared workers in states in which the minimum wage was raised to workers at the same position in the wage distribution in states in which the minimum wage stayed constant. The results are indicative of substantial short-run spillovers to higher wage groups. For

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<sup>1</sup> An overview over these studies is provided by Card and Krueger (1995, Ch. 5) and Neumark and Wascher (2008, Ch. 4).

workers with wages close to the new minimum wage (less than 10 percent more than the minimum), the wage elasticity with respect to the minimum wage is 0.8. This elasticity is smaller for higher wage groups, but is still amounts to 0.15 for workers who earn between 1.5 and two times the minimum wage. In the long run, the effects seem to be weaker than in the short run, and might even be negative for higher wage groups. This suggests that wage growth in states in which the minimum wage was raised was weaker than in the control states in the years following the increase in the minimum wage. Apparently, employers were able to reclaim some of the wage increases they had to give to their employees after the minimum wage increase by suspending the nominal wage increases they would have paid otherwise.

While employers' attempts to maintain their internal wage hierarchy, to ensure competitive wages compared to other firms, or to preserve incentives for effort might provide an explanation for spillover effects, such influences can be excluded in experimental studies. Falk et al. (2006) conduct a laboratory experiment indicating that behavioral aspects, in particular menu dependence, play an important role in explaining the effects of minimum wages across the wage distribution. In their experiment, a rent is distributed between participants acting as "workers" and a "firm". In the experiment's first step, workers state their reservation wages, which are not observed by the firm. Then, the firm makes a wage offer, workers with reservation wages below this wage offer are hired by the firm, and the resulting rent is shared between the parties. Falk et al. (2006) show that the introduction of a minimum wage affects the labor market equilibrium by raising workers' reservation wages. Before the minimum wage was introduced, about 91 percent of workers stated a reservation wage below the later minimum. After its introduction, 59 percent of the workers reported that their reservation wage was equal to the new minimum wage, and the other 41 percent said that their reservation wage was even larger than the new minimum.

The results by Falk et al. (2006) suggest that the minimum wage acts as an anchor which workers use to judge the fairness of the actual wage paid. From a bargaining perspective, the fact that the equilibrium wage is increased above the new minimum indicates that reductions in the set of available alternatives can affect bargaining outcomes, even if the alternatives that become unavailable were not chosen initially. In case of the introduction of the minimum wage, such a legal minimum makes a range of low wages unattainable for the firm. While workers may have judged a wage payment fair, or even generous, if there was no minimum wage, the introduction of a minimum wage close to the same wage might cause workers to perceive this wage as unfair and to demand higher wages.



This short review of the theoretical and empirical literature has illustrated that minimum wages do more than simply cut off the lowest part of the wage distribution. Instead, two important stylized facts can be observed:

1. a substantial number of firms raise the wages of workers that used to earn less than the new minimum wage above the minimum level required, and
2. it seems to be common practice that workers already earning wages above the new minimum wage receive wage raises as well.

In the rest of this paper, we will illustrate that such behavior is indeed compatible with the implications of theoretical bargaining models that incorporate notions of fairness.

### 3. Wage bargaining between unions and firms

#### *Firms*

We consider an economy of  $n$  firms which sell their output in competitive goods markets. Output prices are normalized to unity. The profit function of a representative firm can be written as

$$\pi = L^\alpha - wL, \quad (1)$$

where  $w$  denotes the wage,  $L$  is employment, and  $\alpha \in (0,1)$ . The corresponding labor demand function is given by

$$L(w) = \left( \frac{w}{\alpha} \right)^{\frac{1}{\alpha-1}}. \quad (2)$$

#### *Unions and Workers*

For simplicity, we assume that all workers are unionized and that all  $n$  sectors of the economy can be described by a representative union-firm pair. A representative union's objective function is given by the standard utility function of a utilitarian union that weighs all its members equally:

$$V(w) = wL + (N - L)w_0 \quad \text{with } L \leq N, \quad (3)$$

where  $N$  is the total number of union members and  $w_0$  denotes the alternative income. The alternative income is the income workers in each respective sector expect to receive should they lose their job. In this case, they would either find a job somewhere else in the economy or stay unemployed. For expositional convenience, we assume that each sector is sufficiently small so that its impact on aggregate measures is negligible and the alternative income is exogenous from each union's point of view. We can then express the probability of finding, or not finding, a job in the rest of the economy by the employment rate  $(1-u)$  and unemployment rate  $(u)$ , respectively. Moreover, the expected remuneration if finding a job in another sector is given by the average wage in the economy,  $\bar{w}$ . Combining these opportunities, the expected utility of a sector  $i$ -worker from the alternative income can be expressed by

$$w_0 = ub + (1-u)\bar{w}, \quad (4)$$

with  $b$  denoting the wage-equivalent of being unemployed.

#### *The Nash bargaining solution*

We first analyze the bargaining outcome of the generalized Nash solution which has been used to solve most bargaining situations in labor economics. Nash (1950) proposes an axiomatic solution that satisfies four plausible conditions: Pareto efficiency, invariance to equivalent utility representations, symmetry, and independence of irrelevant alternatives. The Nash bargaining solution is the unique solution that satisfies these axioms. It can be formally written as:

$$\max_{v_1, v_2 \in S} \Omega = (v_1 - d_1)(v_2 - d_2), \quad (5)$$

where  $v_i$  is player  $i$ 's utility,  $d_i$  is  $i$ 's respective disagreement point,  $S$  is the utility possibility set, and  $\Omega$  is the value of the Nash product.

Applying this solution to our bargaining problem requires the specification of the relevant parameters in (5). First, both parties' utilities are given by union's utility function (3) and firm's profit (1). Second, the parties' disagreement payoffs are determined by their outside options. We assume that the firm's outside option is given by zero profits.<sup>2</sup> If the firm and the

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<sup>2</sup> This assumption is uncritical and commonly used in the literature. We could instead assume any other value as long as it is constant and not too large without changing our results qualitatively.

union do not come to an agreement, union members will have to look for a new job or become unemployed. Their expected income in this case is  $w_0$ , given by (4). Therefore, union's outside option is given by

$$V_0 = w_0 N. \quad (6)$$

The general Nash bargaining solution can then be written as

$$\max_w \Omega = (V(w) - V_0) \pi(w). \quad (7)$$

Maximizing (7) with respect to  $w$  yields the bargained wage as a markup on the alternative income:

$$\Omega_w = 0 \Leftrightarrow w^{Nash} = \frac{1}{2} \frac{\alpha + 1}{\alpha} w_0. \quad (8)$$

Figure 1 illustrates this result in  $\pi$ - $V$ -space. The utility possibility frontier (UPF), depicting all the combinations of the firm's profit  $\pi(w)$  and the union's utility  $V(w)$  that correspond to various wage levels, is hill-shaped. At very high wages, profits and employment levels are close to zero, such that the union's utility is close to its outside option  $V_0$ .<sup>3</sup> If wages fall, both the firm's profits as well as the union's utility increase. In this range, the union benefits from lower wages and higher employment because the wages earned by the additional employees strictly outweigh the wage reduction of those employees that would also have been employed at higher wages. At point  $A$ , the two effects exactly balance and the union utility function reaches a maximum. Hence, point  $A$  indicates the monopoly union outcome. Maximization of (3), taking the firm's labor demand function (2) into account, shows that the monopoly union's wage is  $w^{mon} = (1/\alpha) w_0$ . To the right of  $A$ , further wage reductions increase profits but reduce the utility of the union. We restrict our attention to the case where employment does not exceed the number of union members. Hence, the UPF ends to the right at a point where all union members are employed, i. e.  $L(w) = N$ .<sup>4</sup>

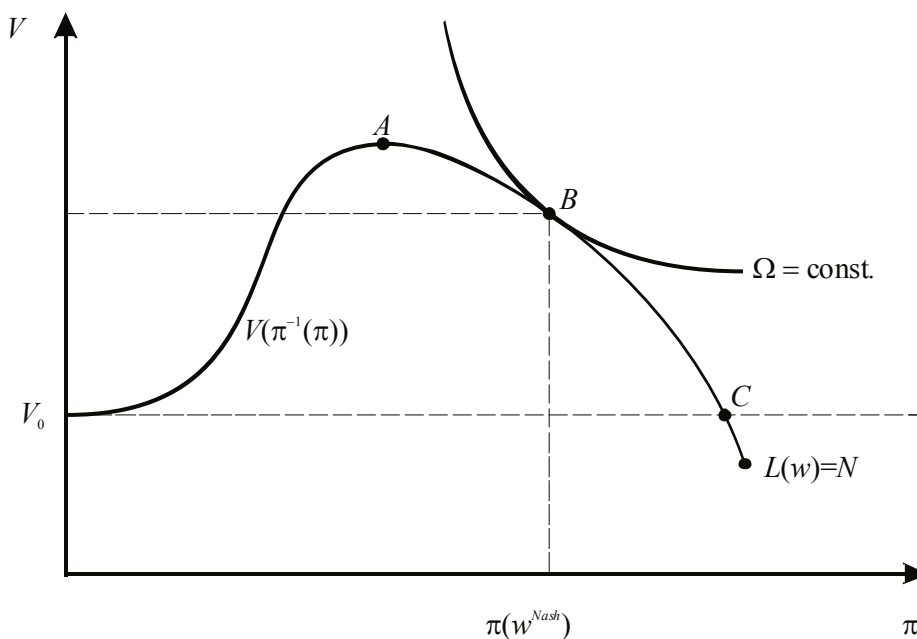
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<sup>3</sup> Since profits are strictly positive for any finite wage level, the UPF does not intersect the ordinate.

<sup>4</sup> Restricting our attention to  $L(w) \leq N$  can be justified by assuming that  $V_0$  exceeds the union's utility at  $L=N$ , in which case the union would never set, or agree to, a wage low enough to increase employment beyond  $N$ . This assumption does not reduce the generality of the model, as it could be extended to cover also the case where  $L(w) > N$ . The UPF would extend to the right, but would be kinked at  $L=N$  (Oswald 1985). Our qualitative findings would remain unchanged.

The Nash bargaining solution can also be represented graphically (Figure 1). The different values of the Nash product  $\Omega$  can be illustrated by a set of iso-Nash-curves, each representing all combinations of  $\pi$  and  $V$  that yield the same value of  $\Omega$ . The maximization problem in (7) then implies that the Nash bargaining solution will be found at the point where the highest iso-Nash-curve is tangent to the utility possibility frontier (point  $B$ ). The bargaining solution will necessarily be somewhere between the monopoly union solution (point  $A$ ) and the point where the union would only be able to obtain its disagreement payoff (point  $C$ ).

Figure 1: The Nash bargaining solution



*The Kalai-Smorodinsky bargaining solution*

The plausibility of some of the axioms necessary to derive the Nash bargaining solution has been questioned. In particular, the axiom of independence of irrelevant alternatives has received strong criticism (Luce and Raiffa 1957). This axiom requires that, if  $X$  is the Nash bargaining solution for a bargaining set  $S_1$ , then for any subset  $S_2$  of  $S_1$  containing  $X$ ,  $X$  continues to be the Nash bargaining solution.

Kalai and Smorodinsky (1975) replace this axiom with the property of individual monotonicity. This axiom implies that the players must not suffer from an enlargement of the bargaining set that leaves the maximum utilities attainable by both players unchanged. Kalai and Smorodinsky (1975) prove that there is only one bargaining rule satisfying this new set of axioms. The KS solution consists of equalizing the parties' relative sacrifice of their maximum

payoff in excess of their conflict utilities. Although earlier works on wage bargaining (e.g. McDonald and Solow 1981) considered both the Nash and the KS solutions, subsequent work on wage bargaining has largely ignored the latter one. This negligence is surprising. First, although both solutions were originally derived axiomatically, they also have sound game-theoretic foundations (see Binmore et al. (1986) for the Nash solution and Moulin (1984) for the KS solution). Second, economic as well as psychological experiments provide evidence for the view that people compare relative payoffs and bargain by mutually making relative concessions (Nydegger and Owen 1974, Roth and Malouf 1979). The Nash approach cannot capture this stylized fact because of the independence of irrelevant alternatives axiom. Replacing this axiom by the monotonicity axiom, as it has been done by Kalai and Smorodinsky (1975), allows individuals to compare relative payoffs and is thus more in line with the experimental evidence.

Taking these facts into account, we analyze the labor market outcome under the assumption that the bargaining process follows the KS solution. This solution implies that both parties make equal proportional concessions from their respective favored points. Formally, the KS solution is given by

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} + \bar{\lambda} \begin{pmatrix} v_1^* - d_1 \\ v_2^* - d_2 \end{pmatrix} \text{ with } \bar{\lambda} = \sup \{ \lambda \in \mathbb{R} : d + \lambda(v^* - d) \in S \}, \quad (9)$$

where  $v^*$  denotes the so-called ‘‘utopia point’’. The utopia point is the vector of the largest utility each player could hope to obtain, given that the other player receives at least his disagreement payoff. The utopia payoff  $v_i^*$  is then given by

$$v_i^* = \sup \{ v_i : (v_i, v_j) \in S \text{ and } v_i \geq d_i, v_j \geq d_j \}. \quad (10)$$

Since the utopia point will typically be outside the bargaining set  $S$ , the KS bargaining solution (9) gives both parties the largest feasible share  $\lambda$  of their respective maximum rent  $v_i^* - d_i$ , in excess of their disagreement payoffs. The solution can then also be described by the so-called KS curve:

$$\frac{v_1 - d_1}{v_1^* - d_1} = \frac{v_2 - d_2}{v_2^* - d_2}. \quad (11)$$

In order to describe the labor market outcome, we apply the formal concept of the KS solution to our bargaining model. Therefore, we specify the KS curve as follows. The utility

from reaching a bargain for both parties is again described by union's utility (3) and firm's profit function (1). As in the Nash solution discussed above, the respective utilities in the case of disagreement are given by  $d_1 = V_0 = w_0 N$  and  $d_2 = \pi_0 = 0$ .

In a next step, we calculate both parties' utopia payoffs. For the union, its maximum feasible gain is determined by the highest wage the firm would be willing to pay without closing down. This wage can be derived by maximizing the union's utility function subject to the condition that firm's profit is at least zero:

$$V^* = \max_w V(w) = wL + (N - L)w_0 \quad \text{s.t.} \quad \pi = L^\alpha - wL \geq 0. \quad (12)$$

It is easy to see that the the union reaches its highest feasible utility if the wage equals the monopoly union wage,  $w^{mon} = w_0/\alpha$ . The utopia point of the firm  $\pi^*$  results from maximizing the profit function subject to the constraint that the union reaches at least its conflict utility:

$$\pi^* = \max_w \pi = L^\alpha - wL \quad \text{s.t.} \quad V \geq V_0. \quad (13)$$

The firm achieves its highest profit by pushing the union down to the utility associated with its outside option. Hence, the solution to (13) is given by  $w_0$ .

Substituting the values for the firm's and union's objective functions, their utopia payoffs and their disagreement utilities into (11), we obtain an implicit determination of the bargained wage:

$$\frac{L(w)[w - w_0]}{L(w^{mon})[w^{mon} - w_0]} = \frac{L(w)^\alpha - wL(w)}{L(w_0)^\alpha - w_0L(w_0)}. \quad (14)$$

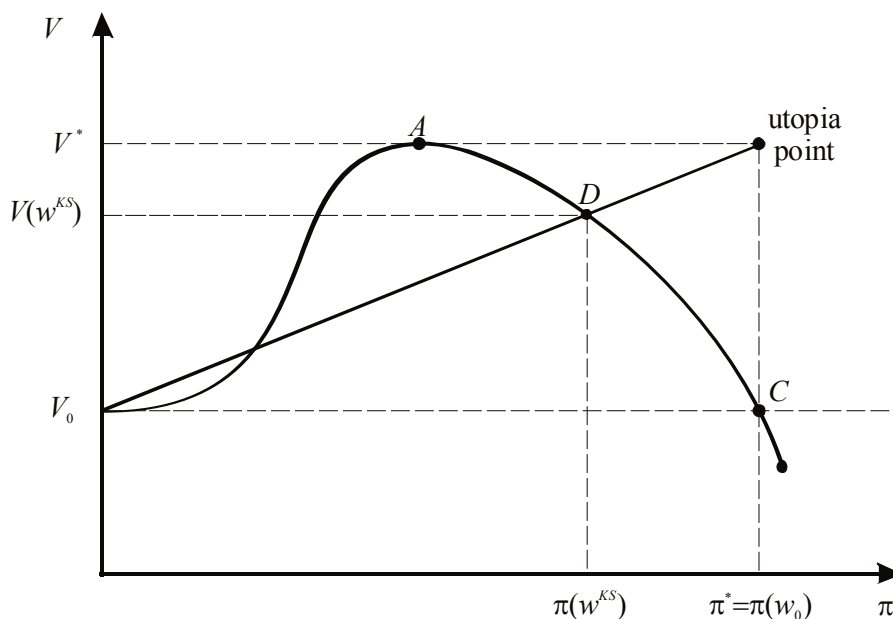
Taking the firm's employment decision (2) and the monopoly union wage into account, we can solve (14) for the KS wage:

$$w^{KS} = \frac{w_0}{1 - (1 - \alpha)\alpha^{\alpha/(1-\alpha)}}. \quad (15)$$

Figure 2 illustrates this general result. The utopia point is given by the vector  $(\pi^*, V^*)$ .  $\pi^*$  is the profit level at which the decreasing part of the UPF corresponds to the union's outside option  $V_0$ . Likewise,  $V^*$  is the largest union utility level at which the firm makes at least zero profits. This corresponds to point  $A$ , which illustrates that a party's utopia payoff does not necessarily entail reducing the other party's utility to its disagreement point. The KS curve

connects the disagreement point  $(0, V_0)$  with the utopia point. According to (9), the KS solution is given by the highest point on the KS curve that is still part of the bargaining set. Hence, the KS bargaining solution is given by point  $D$ , where the KS curve intersects the utility possibility frontier.

Figure 2: KS bargaining solution



#### 4. Labor market effects of minimum wages

We now turn to the analysis of the labor market effects of minimum wages. We analyze how the introduction of minimum wages affects the bargained wage and the resulting employment level in both the Nash and the KS bargaining solution. We will distinguish between the introduction of (a) a minimum wage at the level of a single sector that does not apply to other sectors of the economy and (b) a national minimum wage that applies uniformly to all sectors in the economy. Our main focus will be on the effects of non-binding minimum wages, i.e. minimum wages that are below the level already agreed upon between a union-firm-pair (if introduced at the sectoral level) or between all union-firm-pairs (if the minimum wage covers the whole economy).

##### *Nash bargaining*

We first analyze the case of a sectoral minimum wage that applies to a single sector only. If such a sectoral minimum wage  $w^{min}$  is introduced, we have to distinguish between the case

in which it is ex ante binding and the case where it is not. The minimum wage is ex ante not binding if it does not exceed the previously bargained wage in this sector, i. e.  $w^{min} \leq w^{Nash}$ . Since the minimum wage is set only in this sector and does not apply to other sectors, it is irrelevant for both parties' outside options. The bargaining set, however, becomes smaller since the minimum wage excludes a range of low wages (a segment to the very right of the utility possibility frontier in Figure 1). However, the bargaining outcome without the minimum wage remains part of the bargaining set since the minimum wage is not binding. Due to the axiom of independence of irrelevant alternatives, the reduction of the bargaining set must not have an impact on the bargaining outcome. Hence, a non-binding minimum wage at the sectoral level does not affect wages or employment. This illustrates that the Nash solution does not exhibit menu dependence.

If  $w^{min} > w^{Nash}$  in the respective sector, the minimum wage is ex ante binding. The minimum wage cuts off a segment of the UPF, including the initial bargaining outcome. The minimum wage then becomes a binding constraint on the bargaining outcome, thus lifting the bargained wage to the new minimum wage level. An ex ante binding minimum wage will then also be ex post binding. Summarizing these two cases, the new bargained wage  $\tilde{w}^{Nash}$  can be expressed as

$$\tilde{w}^{Nash} = \begin{cases} w^{Nash} & \text{if } w^{min} \leq w^{Nash} \\ w^{min} & \text{if } w^{min} > w^{Nash} \end{cases} \quad (16)$$

Let us now consider the case of a national minimum. If the minimum wage is non-binding in all sectors of the economy, the bargained wages will not change in any sector. Since there are no wage changes, there is also no change in any union's outside option and, therefore, no labor market effects in the economy.

If the national minimum wage is binding in at least one sector, however, the Nash wage in this sector rises to the minimum wage level. This wage rise in turn affects the outside option of all other unions in the economy. The change in the outside option resulting from the introduction of a minimum wage can be calculated by differentiating (4) with respect to the wage in sector  $i$ ,  $w_i$ , and taking into account the endogeneity of the unemployment rate and the average wage in equilibrium:

$$\frac{dw_0}{dw_i} = (1-u) \frac{\partial \bar{w}}{\partial w_i} - \frac{\partial u}{\partial w_i} (\bar{w} - b) \geq 0. \quad (16)$$



The ambiguous sign follows from the endogeneity of both the unemployment rate and the average wage at the macroeconomic level. On the one hand, a higher wage in another sector increases the outside option as the average wage rises. On the other hand, the wage rise causes less employment in that sector, resulting in higher unemployment, a smaller chance to find a job, and thus in a lower outside option. Although the aggregate outside option effect is ambiguous, the sign of the change in the bargained wage in any sector  $j \neq i$  in which the minimum wage is not binding is the same as the sign of the change in the outside option due to a binding minimum wage in sector  $i$ . This effect can be shown by differentiating the Nash wage (8) with respect to  $w_i$ :

$$\text{sgn}\left(\frac{dw_j^{Nash}}{dw_i}\right) = \text{sgn}\left(\frac{dw_0}{dw_i}\right). \quad (17)$$

Summing up, the minimum wage can only have an effect on wages and employment if it is binding in at least one sector. A minimum wage that is not binding anywhere will be ineffective.

**Proposition 1:** *If bargaining follows the Nash solution, a sectoral minimum wage that is ex ante not binding is ineffective. A national minimum wage can only have an effect on wages and employment if it is ex ante binding in at least one sector.*

#### *Kalai-Smorodinsky bargaining*

As discussed above, the KS solution implies that both bargaining parties agree on a wage that equalizes their relative utility gains. These relative utility gains are given by the ratio of the actual gain to the maximum feasible gain, where the maximum is defined by the payoff each party can secure by pushing the other party to the minimum it would just be willing (or allowed) to accept. If a sectoral minimum wage is introduced below the level of a union's alternative income  $w_0$ , this legal minimum would have no effect on the disagreement and utopia points. Since the cut-off segment of the UPF is to the right of the firm's utopia payoff, the minimum wage would also leave the relevant segment of the UPF unchanged. Hence, non-binding minimum wages are ineffective also in the case of Kalai-Smorodinsky bargaining if they do not exceed the union's alternative wage.

If the sectoral minimum wage is set at a non-binding level but above the alternative wage, this would reduce the firm's utopia payoff because the best the firm can expect to achieve

now would be to push the wage down to its legal minimum. The firm's utopia point, initially given by (13), changes to:

$$\pi^* = \arg \max_w \pi = L^\alpha - wL \quad \text{s.t.} \quad w \geq w^{\min} > w_0. \quad (18)$$

The corresponding KS curve is given by

$$\frac{L(w)[w - w_0]}{L(w^{\min})[w^{\min} - w_0]} = \frac{L(w)^\alpha - wL(w)}{L(w^{\min})^\alpha - w^{\min}L(w^{\min})}. \quad (19)$$

Comparison with (14) shows that the non-binding minimum wage replaces the alternative income in the denominator of the firm's relative gain (right-hand side of (20)). The bargained wage resulting from the introduction of a non-binding sectoral minimum wage can then be obtained by solving (20):

$$\tilde{w}^{KS} = \begin{cases} w^{\min} & \text{if } w^{\min} \leq w^{\min} \\ \frac{w_0}{1 - (1 - \alpha)(\alpha w^{\min}/w_0)^{\alpha/(1-\alpha)}} & \text{if } w_0 < w^{\min} < w^{\min} \\ w^{KS} & \text{if } w^{\min} \leq w_0 \end{cases}. \quad (20)$$

If the minimum wage exceeds the alternative wage,  $w^{\min} > w_0$ , the bargained wage depends not only on  $w_0$  but also on  $w^{\min}$ . This gives our first result for the effect of minimum wages in the KS solution.

**Proposition 2:** *If bargaining follows the KS solution, a sectoral minimum wage between the union's alternative wage and the originally bargained wage raises the bargained wage to a level above the initially bargained wage.*

This result,  $\tilde{w}^{KS} > w^{KS}$  if  $w^{KS} > w^{\min} > w_0$ , follows directly from the assumption  $w^{\min} < w^{KS}$  and from comparing (15) with (21). Since the new bargained wage contains an additional markup  $(w^{\min}/w_0)^{\alpha/(1-\alpha)} > 1$ , this new wage exceeds the former bargained wage, i. e.  $\tilde{w}^{KS} > w^{KS}$ . Therefore, the minimum wage is not binding but nevertheless effective for wages and employment. This shows that the Kalai-Smorodinsky solution is menu dependent. A change in the choices available to a firm-union-pair that does not affect their ability to choose their original agreement causes a shift in the bargaining outcome.

Let us now turn to the case in which the minimum wage is ex ante binding, i.e. it is set above the wage that workers would receive if there was no minimum wage. We summarize our second result in

**Proposition 3:** *If bargaining follows the KS solution, a sectoral minimum wage between the originally bargained wage and the monopoly union wage raises the bargained wage to a level above the minimum wage. If the sectoral minimum wage is set above the monopoly union wage, the bargained wage equals the minimum wage.*

The first part of this result follows for the case  $w^{mon} > w^{min} > w^{KS}$ . As we want to prove that  $\tilde{w}^{KS} > w^{min}$  holds, (21) implies that the following inequality must be fulfilled:

$$\frac{w_0}{1 - (1 - \alpha) \left( \alpha \frac{w^{min}}{w_0} \right)^{\alpha/(1-\alpha)}} > w^{min}. \quad (21)$$

Rearranging yields

$$\left( \frac{w_0/w^{min}}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} > \frac{1 - w_0/w^{min}}{1 - \alpha}. \quad (22)$$

If the minimum wage is lower than the monopoly union wage, i.e.  $w^{min} < w_0/\alpha$ , then the left-hand side in (23) is always greater than 1, while the right-hand side is smaller than 1. Hence, inequality (22) holds, which yields  $\tilde{w}^{KS} > w^{min} > w^{KS}$ . This result implies that a sectoral minimum wage that is ex ante binding can nevertheless be ex post non-binding.

If  $w^{min} > w^{mon}$ , the corresponding KS curve becomes

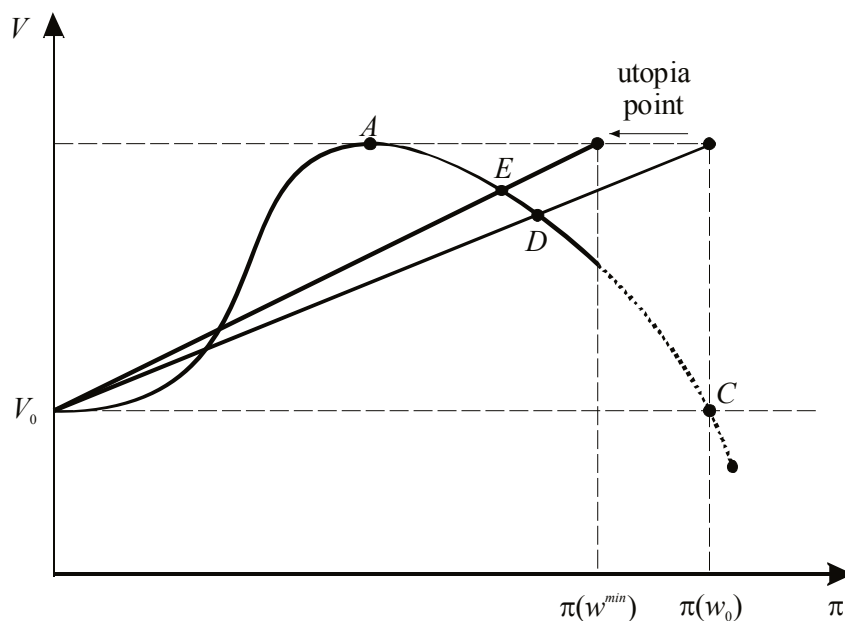
$$\frac{L(w)[w - w_0]}{L(w^{min})[w^{min} - w_0]} = \frac{L(w)^\alpha - wL(w)}{L(w^{min})^\alpha - w^{min}L(w^{min})}, \quad (23)$$

which immediately implies that the bargained wage corresponds to  $w^{min}$ . In this case, the minimum wage will be binding ex ante as well as ex post.

Figure 3 illustrates these results. The introduction of a minimum wage above the alternative wage cuts off a segment of the UPF (dotted) and shifts the firm's utopia payoff to the left. Even at its best conceivable outcome, its profit will be lower than without the minimum wage. The disagreement point stays unchanged because a sectoral minimum wage does not affect the payoffs the firm and the union could obtain outside their own sector. This

causes a counter-clockwise rotation of the KS curve, which leads to an increase in the bargained wage. The bargaining outcome shifts from  $D$  to  $E$ . The logic of Figure 3 also suggests that any minimum wage below the monopoly union wage  $w^{mon}$  (point  $A$ ) will lead to a bargained wage above the new minimum. If  $w^{min} > w^{mon}$ , the utopia point will coincide with the UPF because both parties' most preferred, and legal, wage is the minimum wage. Only in this case can a sectoral minimum wage be ex post binding in the sense that the new bargained wage is equal to the minimum wage.

Figure 3: Change in the utopia point in the KS bargaining solution.



Finally, we want to briefly comment on the labor market effects of a national minimum wage. If  $w^{min} \leq w_0 \forall i$ , i. e. the national minimum is below the respective outside option in each sector, then the minimum wage will be ineffective in any sector and thus also at the national level. If  $\exists i : w^{min} > w_0$ , however, the minimum wage changes the utopia point in this sector  $i$ . Consequently, the wage in this sector rises. Although the sign of this change on the outside option in some other sector  $j$  is ambiguous, the effect on the KS wage can be shown by differentiating (21) with respect to  $w_0$ :

$$\text{sgn} \left( \frac{dw_j^{KS}}{dw_i} \right) = \text{sgn} \left( \frac{dw_0}{dw_i} \right). \quad (24)$$

This implies the following result:

**Proposition 4:** *If bargaining follows the KS solution, a national minimum wage affects wages and employment if it exceeds the union's outside option in at least one sector.*

While Proposition 2 shows that a minimum wage does not have to be binding to have an effect on the sector in which it is introduced, Proposition 4 demonstrates that wage and employment effects can arise in some sector even if the minimum wage is less than the union's outside option as long as it exceeds that alternative wage in some other sector.

## 5. Conclusion

The labor market effects of statutory minimum wages are a hotly debated topic in economics and politics. Despite the often controversial debate between proponents and opponents of minimum wages, there is one issue that receives general support from both sides alike: minimum wages set at a non-binding level cannot be effective in raising wages. Unless the minimum wage is set above the wage level of at least some workers, it will not have any effect on wages and employment.

We briefly discussed a number of empirical observations that cast doubt on the universal validity of this conventional view. Empirical evidence shows that minimum wages do not only affect wages previously below the new minimum, but that they also have spillover effects on other wages higher up in the wage distribution (Card and Krueger 1995, Neumark and Wascher 2008). Experimental findings by Falk et al. (2006) suggest that the introduction of a minimum wage affects the wage level that people are willing to accept even if the minimum wage is too low to affect them directly. A potential explanation for this observed behavior is that minimum wages act as a focal point used by people to determine what remuneration they consider to be a "fair" compensation for their work. If increasing the minimum wage raises the wage that workers demand in the labor market, even a non-binding minimum wage has labor market effects. Therefore, fairness concerns help to understand why firms react to a higher minimum wage by raising wages more than necessary to comply with the new minimum. Two important stylized facts emerge from the empirical and experimental evidence on minimum wages. First, firms raise the wages of workers that used to earn less than the new minimum wage above the minimum level required. Second, workers already earning wages above the new minimum wage receive additional wage raises as well.

In this paper, we have analyzed how these observations can be explained by theoretical wage bargaining models. The most common bargaining model in labor economics is the Nash

bargaining model. We showed that the introduction of a non-binding minimum wage cannot change the outcome of a Nash bargain because it does not affect any of the relevant parameters of the bargaining solution. In particular, non-binding minimum wages eliminate a segment of the bargaining set in which no bargaining agreement was found to begin with. Due to the assumption of independence of irrelevant alternatives, a non-binding minimum wage is irrelevant for the bargaining outcome.

Contrary to the Nash solution, the Kalai-Smorodinsky bargaining solution is able to explain the empirically observed effects of non-binding minimum wages. This bargaining solution allows for menu dependence because each side's bargaining outcome does not only depend on its outside option, but also on how much it could have at best obtained from the bargain. The smaller is a party's maximum attainable gain, the smaller will also be its share in the bargaining outcome. In the case of wage bargaining, a minimum wage reduces the maximum attainable gain for the firm. Prior to the introduction of the minimum wage, the best a firm could achieve was to reduce the union's payoff to its outside option. With a wage floor above the former outside option, the firm now can at best reduce the wage to its statutory minimum. This reduces its claim to the bargaining set and leads to higher wages.

On a more general level, our findings indicate that the choice of the specific bargaining solution used in theoretical models of the labor market is not innocuous. Uncritically applying the Nash solution can obstruct the view on important mechanisms through which labor market policies can affect wages and employment. As we demonstrated in this paper, minimum wages are one application where the choice of the bargaining solution strongly affects the implications derived from theoretical models. There are certainly many other policies for which this is the case, too.<sup>5</sup> Identifying those policies, and deriving a more differentiated picture of their labor market effects for various bargaining solutions, provides ample room for further research.

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<sup>5</sup> For example, Gerber and Upmann (2006) show that changes in the reservation wage can have qualitatively different effects in Nash and Kalai-Smorodinsky wage bargaining models when the parties negotiate over wages and employment.

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