Herding in Financial Markets

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Contents

List of Figures

List of Tables

Overview

Throughout the past decades financial markets witnessed prolonged periods of increased volatility and the frequent formation and subsequent burst of bubbles. The dot-com bubble in the beginning of the millennium, the US house pricing bubble of 2006 that culminated in the recent global financial crisis and the 2015 stock market bubble in China are but few examples for apparent inefficiencies if not outright failures of financial markets to correctly reflect asset prices.

Investors acting in sync have been suspected to cause such unwanted market phenomena, compare e.g Wermers (1999). The destabilizing character of investor coordination has been made explicit in the theoretical literature under "herding". The term refers to the behavior of individual investors following the decision of the majority or crowd despite of being endowed with information that advises them to take a different action (see Brunnermeier (2001), p.148).

The claim that such behavior adversely affects financial markets is intuitive. Investors face a decision whether or not to buy a financial asset. As they observe other investors accumulating on one side of the market they loose confidence in their own information regarding the asset's true value and follow the crowd instead. This already leads to amplified stock price movements. If, moreover, the crowd errs in buying or selling the asset, herding on the crowd's action will drive prices away from the asset's true value, which in turn contributes towards the formation of bubbles (or accelerated downturns) and extreme subsequent price reversals.

It is, thus, not surprising that the theoretical herding literature has made great efforts to understand potential drivers of herd behavior. Lead by the seminal work of Bikhchandani et al. (1992) herding theory has identified reputational concerns, momentum trading strategies as well as correlated gathering of information as relevant drivers for investor herding.

At the same time, celebrated empirical studies such as Lakonishok et al. (1992) and Sias (2004) have supplied measures to detect investor herding based on transaction data and provided insights which investor groups and asset types are particularly prone to herding.

It is, however, noted by e.g. Devenow and Welch (1996) and Cipriani and Guarino (2014) that herding theory and the corresponding empirical literature are disconnected. While herd models rarely provide empirically testable hypotheses, empirical works do not rigorously tie their proposed measurement approaches to the theoretical concept of herding.

This thesis contributes towards closing the gap between the theoretical and empirical herding literature.

Papers 1 and 2 of this thesis derive testable hypotheses on two new drivers for investor herding from the model of Park and Sabourian (2011). The hypotheses are confirmed by applying the standard herd measure of Sias (2004) (Sias) to transaction data from the German stock market. Although the Sias measure is the best possible choice for our application, it still does not fully reflect the notion of theoretical herding intensity as implied by the model.

To further bridge this gap, Papers 3 and 4 in this thesis design a new theory-founded herd measure that can be applied to real-world transaction data. Using the measure to analyze German stock market data from the recent financial crisis shows that herding is a rare event but has the potential to destabilize markets.

Paper 5 serves an important integrating function in this thesis as it provides a strong theoretical link between investor herding and the destabilization of financial markets - a fact rarely encountered in the existing financial market herding literature, compare Eyster and Rabin (2010). Paper 5 proposes a framework to study the behavior of investors facing choices under ambiguity as opposed to quantifiable risk. It derives precise conditions under which investor herding moves prices away from fundamentals contributing towards the formation and burst of bubbles.

The results of Paper 5 validate the relevance to study investor herding and, thus, the efforts made in Papers 1 to 4.

A more detailed summary of the main contributions and results of each individual paper of this thesis is provided in the following:

- ***Paper 1:*** *The Impact of Information Risk and Market Stress on Herding in Financial Markets*[1]
  This paper employs numerical simulations of the Park and Sabourian (2011) herd model to derive new theory-based predictions for how information risk and market stress influence aggregate herding intensity. We find that higher information risk increases both buy and sell herding. The model also predicts that in crisis periods buy and not sell herding is more pronounced.

- ***Paper 2:*** *Information Risk, Market Stress and Institutional Herding: Evidence from the German Stock Market*[2]
  This paper empirically tests and confirms the hypotheses regarding the impact of information risk and market stress on herding intensity that are derived in Paper 1. This is done by applying the measure of Sias (2004) to high-frequency, investor-specific transaction data from the German DAX 30 index from to 2006 to 2009. The Sias measure is chosen because it is particularly suited to analyze high-frequency transaction data and because of all prominent herd measures it best reflects the concept of aggregate herding intensity as introduced in Paper 1.

- ***Paper 3:*** *How to Measure Herding in Financial Markets*[3]
  Combining the insights of market microstructure theory with the ideas of Lakonishok et al. (1992) this paper develops a new measure for investor herding. A theoretical analysis of our new measure and the LSV measure

[1] This paper was written in collaboration with my co-author Simon Jurkatis.
[2] This paper was written in collaboration with my co-authors Simon Jurkatis, Dieter Nautz and Stephanie Kremer.
[3] This paper was written in collaboration with my co-author Simon Jurkatis.

reveals testable distributional assumptions underlying both approaches and shows that our measure generalizes the LSV measure. In a comprehensive simulation study we find that our measure differentiates between herd and contrarian behavior as well as independent trading. At the same time the LSV measure fails to reliably detect investor herding.

- ***Paper 4:*** *Herding and Contrarian Behavior on the German Stock Market During the Recent Financial Crisis* [4]
  Gauging transaction data from the German stock market in 2008 with the herd measure developed in Paper 3, we find that investors predominantly exhibit contrarian tendencies or trade independently. Only less proficient traders occasionally engage in herd behavior. When they do, however, they tend to destabilize the German stock market. The data support the assumptions associated with the measurement approach developed in Paper 3, while at the same time strongly rejecting the distributional assumptions underlying the celebrated herd measure of Lakonishok et al. (1992).

- ***Paper 5:*** *Irrational Exuberance and Herding in Financial Markets - How Investors Facing Ambiguity Drive Prices Away From Fundamentals*
  In the context of a two-state, two-trader financial market herd model introduced by Avery and Zemsky (1998) we investigate how informational ambiguity in conjunction with waves of optimism and pessimism affect investor behavior and social learning. Without ambiguity, neither herding nor contrarianism is possible. If on the other hand ambiguity is high and traders become overly exuberant (or desperate) as the asset price surges (or plummets), we establish that investor herding may drive prices away from fundamentals.

[4] This paper was written in collaboration with my co-authors Simon Jurkatis and Puriya Abbassi.

Zusammenfassung

Phasen hoher Unsicherheit sowie das Entstehen und anschließende Platzen von Preisblasen kennzeichneten die Finanzmärkte der vergangenen Jahrzehnte. Beispiele hierfür sind die Dot-com Blase während der Jahrtausendwende, die Preisblase auf dem Wohnungsmarkt der USA von 2006, die in den Folgejahren eine globale Finanzkrise auslöste sowie die 2015 geplatzte Blase auf dem chinesischen Aktienmarkt. Diese Beipiele belegen, dass es auf Finanzmärkten durchaus auch über längere Zeiträume zu Ineffizienzen und Fehlpreisbildungen kommen kann.

Es wird vermutet, dass gleichgerichtetes Handeln von Investoren solches Marktversagen bedingen kann, vergleiche Wermers (1999). Wenn ein solch koordiniertes Investorenverhalten destabilisierend auf Märkte wirkt, spricht die theoretische Literatur von "Herdenverhalten". Der Begriff beschreibt ein Verhaltensmuster, bei dem Investoren blind und wider besseren Wissens der Entscheidung der Mehrheit oder der Masse folgen, z.B. eine Aktie zu kaufen oder verkaufen (siehe Brunnermeier (2001), S. 148). Dass sich solches Herdenverhalten tatsächlich negativ auf das Funktionieren von Finanzmärkten auswirken kann, belegt folgende vereinfachte Argumentation: Investoren sehen sich mit der Entscheidung konfrontiert, z.B. eine Aktie zu kaufen oder zu verkaufen. Sie besitzen Informationen, dass der Kauf der Aktie nicht gewinnversprechend ist. Sie beobachten jedoch, dass viele andere Investoren die Aktie kaufen, was einen steigenden Aktienpreis bedingt. Die Investoren verlieren Vertrauen in ihre eigene Information und folgen wider besseren Wissens und trotz gestiegener Preise der Masse der Anleger und kaufen die Aktie. Dass ein solches Verhalten an sich bereits Preistrends verstärkt und somit zu erhöhten

Aktienkursschwankungen führt, liegt auf der Hand. Falls jedoch obendrein die Masse der Anleger den Wert der Aktie überschätzt hat, trägt Herdenverhalten zur Entstehung von Blasen bei, bei deren Platzen es in kürzester Zeit zu extremen Kurskorrekturen kommt.

Es ist daher kaum verwunderlich, dass die theoretische Literatur viel über die möglichen Treiber von Herdenverhalten diskutiert. Nach der wegweisenden Studie von Bikhchandani et al. (1992) hat die theoretische Herdenliteratur Sorgen um den eigenen Ruf, Momentum Handelsstrategien sowie Analyse identischer Informationen als mögliche Ursachen für Herdenverhalten von Investoren identifiziert. Gleichzeitig entwickelten bekannte empirische Arbeiten wie die von Lakonishok et al. (1992) und Sias (2004) häufig wiederverwendete Maße zur Quantifizierung von Herdenverhalten und lieferten empirische Evidenzen, welche Investorengruppen und welche Aktien besonders von Herdenverhalten betroffen sind.

Devenow und Welch (1996) sowie Cipriani und Guarino (2014) stellen jedoch fest, dass die theoretische und die empirische Forschung zum Thema Herdenverhalten nur lose miteinander verknüpft sind. Die entwickelten theoretischen Modelle liefern beispielsweise nur selten empirisch überprüfbare Hypothesen. Demgegenüber stellen empirische Arbeiten keinen direkten Zusammenhang zwischen den entwickelten Maßen und den entsprechenden theoretischen Konzepten des Herdverhaltens her.

Das Ziel dieser Disseration ist es daher, einen Beitrag zu leisten, die Lücke zwischen theoretischer und empirischer Herdenliteratur zu schließen.

Papiere 1 und 2 leiten Hypothesen hinsichtlich der Auswirkungen von Informationsrisiko und Marktunsicherheit auf Herdenverhalten ab und testen diese empirisch. Zu diesem Zweck wird das Herdenmaß von Sias (2004) auf Transaktionsdaten vom deutschen Aktienmarkt angewendet. Obwohl das Sias Maß die bestmögliche Wahl ist, stellen wir fest, dass es immer noch Diskrepanzen gibt zwischen dem, was Sias misst und dem, was die Theorie als Herdenintensität beschreibt.

Um diese Lücke weiter zu schließen, entwickeln Papiere 3 und 4 ein theoriebasiertes Maß, welches auf echte Transaktionsdaten anwendbar ist. Die Analyse von entsprechenden Daten vom deutschen Aktienmarkt zeigt, dass Herdenverhalten während der globalen Finanzkrise von 2008 ein seltenes Phänomen ist. Wenn es jedoch auftritt, dann wird der Markt dadurch destabilisiert.

Papier 5 bildet einen wichtigen Rahmen für die gesamte Dissertation, da es den Zusammenhang zwischen Herdenverhalten an Finanzmärkten und potentiellem Marktversagen klarer theoretisch fundiert, als dies in der Literatur bisher der Fall ist, vergleiche Eyster und Rabin (2010). Es entwickelt ein Modell, welches die Untersuchung des Verhaltens von Investoren ermöglicht, die mit nicht quantifizierbaren Unsicherheiten (Ambiguität) konfrontiert sind. Es leitet Bedingungen her, unter denen Herdenverhalten von Investoren die Preise tatsächlich langfristig vom wahren Wert einer Anlage entkoppelt und so zu Blasenbildungen führt.

Damit belegt das fünfte Papier die Relevanz des Studiums von Herdenverhalten an Finanzmärkten und hebt damit noch einmal die Wichtigkeit der Analysen der ersten vier Papiere hervor.

Eine detaillierte Aufstellung der Beiträge und Resultate jedes einzelnen Papiers dieser Dissertation wird im Folgenden präsentiert:

- ***Papier 1:*** *The Impact of Information Risk and Market Stress on Herding in Financial Markets*[1]
  Basierend auf numerischen Simualtionen des Modells von Park und Sabourian (2011), leitet dieses Papier Aussagen über die Auswirkungen von Informationsrisko und Unsicherheit im Markt auf die Intensität des Herdenverhaltens her. Höheres Informationsrisiko erhöht sowohl die Intensität von Kauf-Herden als auch von Verkauf-Herden. In Krisenzeiten sagt das Modell interessanterweise vorher, dass Kauf-Herden stärker zunehmen als Verkaufsherden.

[1]Dieses Papier entstand in Zusammenarbeit mit meinem Ko-Autor Simon Jurkatis.

- ***Papier 2:*** *Information Risk, Market Stress and Institutional Herding: Evidence from the German Stock Market*[2]
  Dieses Papier testet und bestätigt die Hypothesen hinsichtlich des Einflusses von Informationsrisiko und Marktunsicherheit auf Herdenverhalten, die im Papier 1 hergeleitet wurden. Zur Durchführung der Tests wird das Herdenmaß von Sias (2004) auf einen hochfrequenten, investorspezifischen Transaktionsdatensatz vom deutschen DAX 30 Index zwischen 2006 und 2009 angewendet. Wir verwenden das Sias Maß, da es besonders geeignet ist, hochfrequente Transaktionsdaten zu analysieren. Darüber hinaus ist es unter denen in der Literatur etablierten Herdenmaßen jenes, welches die Idee der im Papier 1 eingeführten durchschnittlichen Herden-Intensität am besten widergibt.

- ***Papier 3:*** *How to Measure Herding in Financial Markets*[3]
  In diesem Papier werden Erkenntnisse der Markt Mikrostruktur Theorie mit Ideen von Lakonishok et al. (1992) kombiniert, um ein neues Herdenmaß zu entwickeln. Die theoretische Analyse unseres Maßes und des LSV Maßes liefert empirisch überprüfbare Verteilungsannahmen, die den jeweiligen Messansätzen zu Grunde liegen. Wir zeigen darüber hinaus, dass unser Maß eine Verallgemeinerung des LSV Maßes darstellt. Weiterhin belegt eine umfassende Simulationsstudie, dass unser Maß verlässlich zwischen Herdenverhalten, Kontrarianismus und unabhängigem Handeln der Investoren unterscheiden kann. Das LSV Maß hingegen nimmt immer positive Werte an unabhängig vom tatsächlichen Verhalten der Investoren.

- ***Papier 4:*** *Herding and Contrarian Behavior on the German Stock Market During the Recent Financial Crisis*[4]
  Die Auswertung von Transaktionsdaten mit dem in Papier 3 entwickelten Herdenmaß zeigt, dass Investoren am deutschen Aktienmarkt in 2008 vornehmlich kontrarianistisch agierten oder unabhängig von einander Han-

[2]Dieses Papier entstand in Zusammenarbeit mit meinen Ko-Autoren Simon Jurkatis, Dieter Nautz und Stephanie Kremer.

[3]Dieses Papier entstand in Zusammenarbeit mit meinem Ko-Autor Simon Jurkatis.

[4]Dieses Papier entstand in Zusammenarbeit mit meinen Ko-Autoren Simon Jurkatis und Puriya Abbassi.

delsentscheidungen getroffen haben. Nicht-institutionelle Anleger jedoch formierten sich zumindest an einigen Tagen in dem Jahr zu einer Herde. Dies ging einher mit einer dramatischen Destabilisierung des deutschen Aktienmarktes. Die Annahmen, die dem in Papier 3 entwickelten Maß zu Grunde liegen, werden von den Daten bestätigt, während die mit dem bekannten Maß von Lakonishok et al. (1992) assoziierten Verteilungsannehmen signifikant abgelehnt werden.

- ***Papier 5:*** *Irrational Exuberance and Herding in Financial Markets - How Investors Facing Ambiguity Drive Prices Away From Fundamentals*
  Wir untersuchen, ob und in welcher Form Informationsambiguität in Verbindung mit Wellen von marktweitem Optimismus und Pessimismus Investorenverhalten beeinflusst. Die Analyse wird im Rahmen eines Zwei-Zustände und Zwei-Investoren Finanzmarktmodels basierend auf Avery und Zemsky (1998) durchgeführt. Ohne Ambiguität ist weder Herdenverhalten noch Kontrarianismus möglich. Wir stellen fest, dass wenn die Informationsambiguität hoch ist und sich die Investoren durch die allgemeine Stimmung am Markt beeinflussen lassen, Überschschwang (Panik) bei steigenden (fallenden) Kursen Herdenverhalten auslösen kann, welches die Preisbewegung langfristig von dem wahren Wert der Aktie entkoppelt.

Paper 1

The Impact of Information Risk and Market Stress on Herding in Financial Markets[1]

1.1 Introduction

Herd behavior by investors can be a significant threat to the functioning of financial markets. The distorting effects of herding range from informational inefficiency to increased stock price volatility, or even bubbles and crashes.

This paper derives two theory-based predictions on how information risk and market stress influence herding intensity. The predictions are tested with high-frequency and investor-specific trading data from the German stock market in Paper 2.

We focus on information risk, defined as the probability of trading with a counterparty who holds private information about an asset (Easley et al. (1996)), since it is easier to assess empirically than true herding. A better understanding of how information risk impacts herding intensity may provide financial regulators with a suitable proxy to ascertain the risk of destabilizing herds.

In light of the recent financial crisis, our second focus is on how herd behavior is affected by market stress, that is, situations in which investors are both pessimistic and uncertain about the stock's value. While herding certainly has the potential to *create* such market stress, it is not obvious whether the reverse relationship holds. If it does, it's existence threatens to create vicious cycles of economic downturns and high volatility regimes.

[1] This paper was written in collaboration with my co-author Simon Jurkatis.

Building on Glosten and Milgrom (1985) and Easley and O'Hara (1987), the literature on information risk deals with estimating the information content of trades, see e.g. Hasbrouck (1991), Easley et al. (1996) and Easley et al. (1997). The effects of information risk on herding intensity, however, are underresearched.[2] While the probability of informed trading is a key parameter in financial market herd models, compare e.g. Avery and Zemsky (1998) and Park and Sabourian (2011), to date these models have not been exploited to discover the impact of information risk on herding intensity. This is surprising, since the effects of information risk on herding intensity are far from obvious. On the one hand, an increase in information risk increases the average information content of an observed trade. As a consequence, traders update their beliefs more quickly and those investors that are susceptible to herding are more easily swayed to follow the crowd. On the other hand, increased information risk amplifies the market maker's adverse selection problem, compare Easley et al. (2002). Given the higher probability of trading at an informational disadvantage, the market maker quotes larger bid-ask spreads which tends to prevent potential herders from trading. Understanding which of these counteracting effects dominates could facilitate the detection of herds.

The impact of market stress on herd behavior has not been analyzed by the theoretical herding literature, either. Typically, herd models focus on the reverse relationship. For example, Park and Sabourian (2011) demonstrate that price paths tend to be more volatile in the presence of herd behavior. Agent based models proposed by, for example, Lee (1998) and Eguíluz and Zimmermann (2000) show that herd behavior contributes to fat tails and excess volatility in asset returns. A notable exception is Avery and Zemsky (1998), who show that herding is possible provided that multiple sources of uncertainty exist. Their model does not imply, however, that more uncertainty actually leads to more herding.

[2] An exception is Zhou and Lai (2009) who provide evidence that herding is positively related to information risk measured by probability of informed trading (PIN), see e.g. Easley et al. (1997). In our empirical application in Paper 2 of this thesis, we choose to approximate information risk differently since PIN and the Sias herd measure are correlated by construction.

The prevalent unidirectional focus of the theoretical literature is particularly puzzling in light of the mixed evidence regarding the impact of market stress on herding intensity. Chiang and Zheng (2010) and Christie and Huang (1995) assume that herding increases during times of market stress, whereas Kremer and Nautz (2013a;b) find that herding in the German stock market slightly decreased during the recent financial crisis, which is similar to the results of Hwang and Salmon (2004) for herding intensity during the Asian and the Russian crisis in the 1990s.

We base our theoretical analysis on the financial market herd model of Park and Sabourian (2011), which can be viewed as a generalization of the seminal work of Avery and Zemsky (1998).[3] One important extension is the broader set of different information structures that allows a differentiated discussion of how information externalities may contribute to herd behavior under various market conditions including scenarios of high and low market stress. Relating investor herding to the shape of the information structure instead of to multi-dimensional uncertainty, Park and Sabourian (2011) identify more explicitly those situations in which the potential for herding is high. Consequently, the Park and Sabourian (2011) framework is more appropriate for finding and explaining high degrees of herding. In fact, experimental evidence suggests that the Avery and Zemsky (1998) framework discovers little or no herd behavior, see Cipriani and Guarino (2009).[4] In contrast, experiments based on the Park and Sabourian (2011) model find that herding in financial markets can be substantial, see Park and Sgroi (2012).

[3] Similar to the bulk of the theoretical literature, both models define herd behavior as a switch in an agent's opinion toward that of the crowd, see Brunnermeier (2001). As herders ignore their private information, herd behavior is informationally inefficient and thus has the potential to distort prices and destabilize markets.

[4] Avery and Zemsky (1998) includes different model setups. The most basic setup extends the traditional herd model of Bikhchandani et al. (1992) by a price mechanism that prevents herd behavior. Prominent experimental tests of the Avery and Zemsky (1998) framework, Drehmann et al. (2005) and Cipriani and Guarino (2005), focus on this setup and confirm the theoretical prediction of no herding. Cipriani and Guarino (2009), on the other hand, focus on one of the more complex setups in which herd behavior is predicted, but again find only little evidence of it.

In Park and Sabourian (2011), herding is triggered by information externalities that an investment decision by one agent imposes on subsequent agents' expectations about the asset value, see Bikhchandani et al. (1992) and Banerjee (1992).[5] Therefore, this model is a natural candidate for investigating the impact of information risk on herding intensity.[6]

The history dependence of trading decisions in financial market herd models drastically impedes the derivation of analytical results on herding intensity. This may explain why these models have not yet been exploited to make empirically testable predictions on the impact of information risk and market stress. Moreover, standard empirical herding measures, including the ones proposed by Lakonishok et al. (1992) and Sias (2004), examine herding intensity on an *aggregate* level. Consequently, empirical testability of our theory-guided hypotheses requires that we analyze herding intensity aggregated over investor groups, time periods, and heterogeneous stocks, compare Paper 2 of this thesis. This further complicates the derivation of analytical results.

We circumvent these problems by simulating the Park and Sabourian (2011) model for more than 13,000 different parameterizations that broadly cover the theoretical parameter space, generating about 2.6 billion trades for analysis. We obtain two testable hypotheses on the model-based measure of aggregate herding intensity. First, an increase in information risk should result in a symmetric increase of buy and sell herding intensity. Second, high market stress should be found to have an asymmetric effect on herding intensity: while buy herding is predicted to surge during crisis periods, the simulation results suggest that sell herding intensity increases only moderately.

[5] Alternative drivers for herd behavior include reputational concerns as well as investigative herding. Reputational herd models modify the agents' objective functions such that their decisions are affected by positive externalities from a good reputation, see e.g. Scharfstein and Stein (1990), Graham (1999) and Dasgupta et al. (2011). Investigative herd models examine conditions under which investors may choose to base their decisions on the same information resulting in correlated trading behavior, see e.g. Froot et al. (1992) and Hirshleifer et al. (1994). For a survey of the early herding literature see Devenow and Welch (1996). For an in-depth discussion of how the herding literature ties into the social learning literature see Vives (1996).

[6] Other financial market herd models such as Lee (1998), Chari and Kehoe (2004), and Cipriani and Guarino (2008), investigate how investor herding is related to transaction costs, endogenous timing of trading decisions, and informational spillovers between different assets, respectively.

The remainder of this paper is structured as follows. In Section 1.2 we review the model of Park and Sabourian (2011). In Section 1.3 we define information risk as well as market stress and provide an initial qualitative assessment of their effect on herding intensity. Section 1.4 formalizes the concept of aggregate herding intensity. It subsequently introduces the simulation setup and derives testable hypotheses regarding the role of information risk and market stress for aggregate herding intensity. Section 1.5 summarizes the results.

1.2 A Model of Investor Herding

This section reviews the herding model of Park and Sabourian (2011) and highlights conceptual additions and modifications that are relevant to our application. Moreover, it formalizes the notion of herding intensity.

1.2.1 The Model Setup

Park and Sabourian (2011) consider a sequential trading model à la Glosten and Milgrom (1985), consisting of a single asset, both informed and noise traders, and a market maker. The model assumes rational expectations and common knowledge of its structure.

The Asset: There is a single risky asset with unknown fundamental value $V \in \{V_1, V_2, V_3\}$, where $V_1 < V_2 < V_3$. Without loss of generality, let $V_1 = 0$, $V_2 = 1$ and $V_3 = 2$. The prior distribution $0 < P(V = V_j) < 1$ for $j = 1, 2, 3$ determines the degree of *public uncertainty* $\text{Var}(V)$ about the asset's true value before trading has started. The asset is traded over T consecutive points in time. In Section 1.4, we choose $T = 100$ for the model simulation.

The Traders: Traders arrive in the market one at a time in a random exogenous order and decide to buy, sell or not to trade one unit of the asset at the quoted bid and ask prices. Traders are either informed traders or noise traders. The fraction of informed traders is denoted by μ. Informed traders base their decision to buy, sell or not to trade on their expectations regarding the asset's true value.

Publicly available information consists of the history of trades $H_t := \{(a_1, p_1), ..., (a_{t-1}, p_{t-1})\}$, where a_i is the action of a trader in period i and p_i the price at which the trader's action is executed, and the risky asset's prior distribution $P(V)$.

In addition to public information, informed traders base their asset valuation on a private signal $S \in \{S_1, S_2, S_3\}$ regarding the true value of the asset. They buy (sell) one unit of the asset if their expected value of the asset $E[V \mid S, H_t]$ is strictly greater (smaller) than the ask (bid) price quoted by the market maker. Otherwise, informed traders choose not to trade. In the empirical herding literature, institutional investors are viewed as a typical example for informed traders. In contrast to informed traders, noise traders trade randomly, that is, they decide to buy, sell or not to trade with equal probability of $1/3$. p_t denotes the price at which the asset is traded in period t.

The Private Signal: The distribution of the private signals S_1, S_2, S_3 is conditional on the true value of the asset. Denote the conditional signal matrix by $P(S = S_i \mid V = V_j) = (p^{ij})_{i,j=1,2,3}$. For each column j, the matrix is leftstochastic, i.e. $\sum_{i=1}^{3} p^{ij} = 1$. For each row i, $\sum_{j=1}^{3} p^{ij}$ is the likelihood that an informed trader receives the signal S_i. An informed trader's behavior is critically dependent on the shape of her private signal. Specifically, Park and Sabourian (2011) define a signal S_i to be

- monotonically decreasing iff $p^{i1} > p^{i2} > p^{i3}$,
- monotonically increasing iff $p^{i1} < p^{i2} < p^{i3}$,
- U-shaped iff $p^{i1} > p^{i2}$ and $p^{i2} < p^{i3}$.

Traders with monotone signals are confident about the asset's true value and rarely change their trading decision. That is, an optimistic trader with an increasing signal will only buy or hold, whereas a pessimistic trader with a decreasing signal will only sell or hold.

In contrast, traders with U-shaped signals face a high degree of uncertainty and may decide to buy, sell or hold. U-shaped traders are more easily swayed to change their initial trading decision as they observe trade histories H_t with a strong accumulation of traders on one side of the market. In fact, Park and Sabourian (2011) show that a U-shaped signal is a necessary condition for herding.

Park and Sabourian (2011) also introduce hill-shaped signals which are necessary for contrarian behavior. Since contrarian behavior is self-defeating, it's destabilizing effects are limited and thus of only secondary importance for financial markets. Consequently, we exclude hill-shaped signals from our analysis.

In the following, we assume that S_1 is monotone decreasing, S_2 is U-shaped and S_3 is monotone increasing. The conditional private signal distribution $P(S \mid V)$ determines the degree of information asymmetry between market maker and informed traders. The less noisy the signal, the higher the informational advantage of the informed traders.

The Market Maker: Trading takes place in interaction with a market maker who quotes a bid and an ask price. The market maker only has access to public information and is subject to perfect competition such that he makes zero-expected profit. Accordingly, he sets the ask (bid) price equal to his expected value of the asset given a buy (sell) order and the public information. Formally, he sets $ask_t = E[V|H_t \cup \{a_t = buy\}]$ and $bid_t = E[V|H_t \cup \{a_t = sell\}]$.

1.2.2 Herding Intensity

Park and Sabourian (2011) describe herding as a "history-induced switch of opinion [of a certain informed trader] in the direction of the crowd." Thus, only informed traders can herd. More precisely, a herding trade is defined as follows:

Definition 1.1. ***Herding***
Let b_t (s_t) be the number of buys (sells) observed until period t. An informed trader with signal S ***buy herds*** *in t at history H_t if the following three conditions hold:*

(BH1) $E[V|S] < E[V]$, *i.e. an informed trader with signal S does not buy initially and is more pessimistic regarding the asset's true value than is the market maker.*

(BH2) $E[V|S, H_t] > ask_t$, *i.e. an informed trader with signal S buys in t.*

(BH3) $b_t > s_t$*, i.e. the history of trades contains more buys than sells: the crowd buys.*

Analogously, an informed trader with signal S ***sell herds*** *in period t at history* H_t *if and only if (SH1)* $E[V|S] > E[V]$*, (SH2)* $E[V|S, H_t] < bid_t$*, and (SH3)* $b_t < s_t$ *hold simultaneously.*

Note that (BH1) and (SH1) imply that either buy or sell herding is possible for a given model parameterization. Our definition of herding is less restrictive than the one used in Park and Sabourian (2011), who, for example, define buy herding as an extreme switch from selling initially to buying. In our definition, buy herding also includes switches from *holding* to buying, provided that the trader leans toward selling initially (see (BH1) and (BH2) in Definition 1.1).[7] As a consequence, herd traders always act informationally inefficiently as their trading decisions contradict their private information. From an empirical perspective, including switches from holding to selling or buying is important as these actions may drive amplified stock price movements.

(BH3) and (SH3) also differ slightly from Park and Sabourian (2011) in which, for example, buy herding requires $E[V|H_t] > E[V]$. This condition is based on the idea that prices rise when there are more buys than sells. However, this only holds if the prior distribution of the risky asset $P(V)$ is symmetric around the middle state V_2, i.e. $P(V_1) = P(V_3)$.[8] In fact, for asymmetric $P(V)$, it is possible that even though a history H_t contains more buys than sells, the price of the asset goes down (i.e., $E[V|H_t] < E[V]$). From an empirical perspective, asymmetric prior distributions $P(V)$ should not be ruled out. Therefore, we modify the herding definition to ensure that a herder always *follows the crowd.*

The above definition enables us to decide whether or not a particular trade by a *single* investor at a specific point in time is a herd trade. In contrast, empirical

[7]According to Park and Sabourian (2011), such an extension of the herding definition is theoretically legitimate. They focus on the stricter version to be consistent with earlier theoretical work on herding.

[8]Note that Park and Sabourian (2011) assume symmetry of the risky asset's prior distribution throughout their paper (see Park and Sabourian (2011), p.980).

herding measures are based on a number of trades by different investors observed over a certain time interval, see, e.g., Lakonishok et al. (1992) and Sias (2004). Since we aim to derive theory-based predictions on herd behavior that can be tested empirically, we need to aggregate herding in the model over time as well as over investors. We aggregate over time by considering all relevant trades from $t = 1, \dots, T$. We aggregate over investors by calculating herding intensity for the whole group of informed traders. Therefore, we define *herding intensity (HI)* as the share of herding trades in the total number of informed trades.

Definition 1.2. ***Herding Intensity***
Let b_T^{in} and s_T^{in} be the number of buys and sells of informed traders observed until period T, i.e. during the entire time interval under consideration. Let b_T^h and s_T^h denote the corresponding number of buy and sell herding trades. Then,

$$\begin{aligned} \textit{Buy herding intensity (BHI)} &=: \frac{b_T^h}{b_T^{in} + s_T^{in}} \\ \textit{Sell herding intensity (SHI)} &=: \frac{s_T^h}{b_T^{in} + s_T^{in}} \end{aligned}$$

Standard empirical herding measures including those of Lakonishok et al. (1992) and Sias (2004) are calculated using only buys and sells, see Section 2.2 in Paper 2 of this thesis. To be consistent with empirical herding measures, we exclude holds when calculating the number of informed trades in the definition of theoretical herding intensity.

1.3 Information Risk and Market Stress in the Herd Model

This section shows how the concepts of information risk and market stress are translated into the Park and Sabourian (2011) model. It also provides a qualitative assessment how each concept impacts herding intensity.

1.3.1 Information Risk

In Easley et al. (1996), information risk is the probability that a trade is executed by an informed trader. Hence, information risk coincides with the parameter μ, the fraction of informed traders, in the Park and Sabourian (2011) model.

From a theoretical perspective, the effect of changes in μ on herding intensity is ambiguous. On the one hand, herding may increase with information risk because a higher μ implies that there are more potential herders (U-shaped traders) in the market. Due to the self-enforcing nature of herd behavior a higher μ contributes to longer-lasting herds and, hence, stronger herding intensity. Moreover, a higher fraction of informed traders implies that the average information content of a single trade increases. As a consequence, informed traders update their beliefs more quickly and those traders that are susceptible to herd behavior are more easily swayed to change from buying to selling and vice versa.

On the other hand, a rise in μ may also reduce herding intensity. Since the average information content per trade increases in μ, herds tend to break up more quickly as traders stop herding after observing fewer trades on the opposite side of the market. Higher information risk further amplifies the market maker's adverse selection problem, compare Easley et al. (2002). Given the higher probability of trading at an informational disadvantage, the market maker quotes larger bid-ask spreads in order to avoid losses. The larger spread, in turn, requires potential herders to observe much stronger accumulation of traders on one side of the market before they alter their trading decision.

1.3.2 Market Stress

Times of high market stress and crisis periods are typically understood as situations where investors are confronted with a deteriorating economic outlook and increased uncertainty about stock values, compare e.g. Schwert (2011).

A negative economic outlook in the Park and Sabourian (2011) model is captured by low expectations regarding the asset's true value $E[V]$. A low $E[V]$ not only describes a deteriorated outlook by the public but also a high degree of pessimism among informed traders. First, lower public expectations $E[V]$ result in lower private expectations $E[V|S]$ for all informed traders. Second, there tend to be more decreasing signals (pessimists) among informed traders as well as fewer increasing signals (optimists) for low $E[V]$ than for high $E[V]$.

Uncertainty in the Park and Sabourian (2011) can be sorted into two types: public uncertainty and informed trader uncertainty. *Public uncertainty* is given by the variance of the risky asset $\text{Var}(V)$. *Informed trader uncertainty* (IU) is measured by the probabilities that informed traders receive a U-shaped signal conditional on V_j, $j = 1, 2, 3$: $\text{IU} := \sum_{j=1}^{3} p^{2j}$. The higher IU, the more traders there are in the market with U-shaped signals and, hence, the higher the uncertainty among informed traders.[9] In light of the recent financial crisis, we are particularly interested in comparing herding intensity in times of high market stress with the herding intensity predicted for more optimistic periods.

The overall effect of market stress on herding intensity is not obvious and crucially depends on model parameterization. Particularly, buy and sell herding intensity may react differently to changes in market stress. Consider, for example, an increase in market stress due to a decrease in $E[V]$. More specifically, assume a shift of probability mass from V_3 to lower values.

First, if, for a given model parameterization, buy herding is possible (and hence sell herding is impossible), a marginal reduction in $P(V_3)$ would result in a *decrease* in buy herding intensity, whereas sell herding intensity would remain constant at 0. Similarly, if sell herding is possible for a given model parameterization (and buy herding impossible), a marginal reduction in $P(V_3)$ would result in an *increase* in sell herding intensity while buy herding intensity would remain unaffected. This converse effect on buy and sell herding intensity is due to the fact that a reduction in $P(V_3)$ diminishes the probability of buy-dominated trade histories and increases the probability of sell-dominated histories. Hence, potential sell (buy) herders are more (less) likely to be confronted with a trade history that sways them into herding.

[9]Note that an increase in $\text{Var}(V)$ may reduce the number of U-shaped traders in the market. This effect is not necessarily offset by an increase in IU. One could circumvent this issue by additionally imposing that the total probability that an informed trader receives a U-shaped signal $P(S_2) = \sum_{j=1}^{3} p^{2j} P(V = V_j)$ must also be high in times of market stress. Since this does not affect the results of our simulation, we choose not to complicate the model by adding this characteristic to the uncertainty definition.

Second, if the U-shaped signal is positively biased, i.e., $P(S_2 \mid V_1) < P(S_2 \mid V_3)$, a reduction of $P(V_3)$ diminishes the number of U-shaped traders in the market and, hence, tends to decrease buy as well as sell herding intensity. Finally, for a whole range of model parameterizations, a lower $E[V]$ may even contribute to an increase in buy herding intensity and a decrease in sell herding intensity. Since a lower $E[V]$ implies that more informed traders are initially inclined to sell, the number of potential sell herders declines. Correspondingly, buy herding becomes more likely.

These complex and partly counteracting effects, in conjunction with the history-dependent updating of beliefs, lead to a low analytical tractability of herding intensity in the Park and Sabourian (2011) model, see the Appendix of this paper. This particularly applies to the empirically relevant case where herding intensity is considered as an average over a set of stocks with heterogeneous characteristics.

In the following, therefore, empirically testable predictions about the effects of information risk and market stress on average herding intensity are derived by numerically simulating the model over a broad set of model parameterizations.

1.4 Simulation of the Herd Model for a Heterogeneous Stock Index

1.4.1 Average Herding Intensity

Empirical studies on herd behavior typically derive results for herding intensity as an average for a large set of stocks and over certain time intervals. The stocks under consideration are likely to differ in their characteristics implying that each stock is described by a distinct parameterization for the fraction of informed traders, the prior distribution of the asset, and the distribution of the private signals. In accordance with the empirical literature, we are particularly interested in herding intensity defined as an *average* over a broad range of model parameterizations that reflects the heterogeneity in stock market indices. Specifically, we define average herding intensity as follows:

Definition 1.3. ***Average Herding Intensity***
For a given set of model parameterizations $\mathcal{I}$ and length T of the trading period, average buy herding intensity is defined as

$$\overline{BHI} = \frac{\sum_{i \in \mathcal{I}} w_i BHI_i}{\sum_{i \in \mathcal{I}} w_i},$$

where BHI_i stands for the buy herding intensity obtained for model parameterization i and the weights $w_i = b_{T,i}^{in} + s_{T,i}^{in}$ correspond to the number of informed trades observed for that parameterization.
The definition for average sell herding intensity $\overline{SHI}$ follows analogously.

Weights w_i ensure that average herding intensity is not biased upward by simulation outcomes with a low number of informed trades.[10]

1.4.2 The Simulation Setup

We choose μ, the fraction of informed traders, from

$$\mathcal{M} = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}.$$

Accordingly, we simulate the model for $|\mathcal{M}| = 9$ different levels of information risk. In the German stock market, the share of institutional (i.e. informed) trading for the sample period ranges from 0.2 to 0.7, compare Kremer and Nautz (2013a).

The prior distribution of the risky asset $P(V)$ is chosen from

$$\mathcal{P} = \{P(V) \in \{0.1, 0.2, \dots, 0.9\}^3 : \sum_{i=1}^{3} P(V_i) = 1\}.$$

Since we impose that V takes each value $V_1 = 0, V_2 = 1, V_3 = 2$ with positive probability, $P(V_i)$ cannot be 0.9, which gives us $|\mathcal{P}| = 36$ different prior

[10] Consider, for example, a situation where we observe a herding intensity of 0.5 as 2 out of 4 informed trades are herd trades. Now assume that for another simulation the herding intensity is 0, as 0 out of 16 informed trades are herd trades. In this case, the *unweighted* average of simulated herding intensities would be 0.25, which overestimates herding intensity as only 2 out of 20 trades were herd trades across the whole sample.

distributions.

The conditional signal distribution $P(S|V) = (p^{ij})_{i,j=1,2,3}$ has to be chosen from the space of leftstochastic 3-by-3 matrices. As before, we discretize this space by imposing a grid ranging from 0.1 to 0.9. All elements of $P(S|V)$ are positive, that is, all signals are noisy in the sense that an informed trader cannot with certainty rule out any of the three possible states for V. Following Park and Sabourian (2011), there are always optimists ($p^{31} < p^{32} < p^{33}$), pessimists ($p^{11} > p^{12} > p^{13}$), and U-shaped traders ($p^{21} > p^{22}, p^{22} < p^{23}$) in the market, see Section 1.2. Finally, informed traders tend to be well-informed, that is, if the bad state $V = V_1$ comes true, most of the informed traders are pessimistic and only few are optimistic ($p^{11} > p^{21} > p^{31}$) and vice versa for $V = V_3$ ($p^{13} < p^{23} < p^{33}$). This implies that the set of simulated signal structures ($\mathcal{C}$) can be summarized as follows:

$$\begin{aligned}\mathcal{C} \;=\; \{ & P(S|V) = (p^{ij})_{i,j=1,2,3} \text{ leftstochastic} : \; p^{ij} \in \{0.1, 0.2, \ldots, 0.9\}, \\ & p^{11} > p^{21} > p^{31}, \; p^{13} < p^{23} < p^{33}, \\ & p^{11} > p^{12} > p^{13}, \; p^{31} < p^{32} < p^{33}, \; p^{21} > p^{22}, \; p^{22} < p^{23}\},\end{aligned}$$

which leads to $|\mathcal{C}| = 41$ different signal structures used in the simulation.

Considering all combinations, one obtains the simulation set $\Omega := \mathcal{M} \times \mathcal{P} \times \mathcal{C}$, where $|\Omega| = 9 \cdot 36 \cdot 41 = 13,284$. Each element $\omega = (\mu, P(V), P(S|V)) \in \Omega$ describes the characteristics of a specific stock.[11] Park and Sabourian (2011) derive upper bounds for μ that have to hold in order for herding to be possible. One can check that these upper bounds are never binding for $\omega \in \Omega$, i.e. in each of the following simulations, either sell or buy herding is possible (see Park and Sabourian (2011), pp. 991-992, 1011-1012). Each stock is traded over $T = 100$ points of time. For each stock, the simulation is repeated $2,000$ times, which produces more than 2.6 billion simulated trades for analysis.

[11] In practice, stock characteristics ω may not be constant over time. For example, the Deutsche Bank share before the financial crisis is likely to have different characteristics than the Deutsche Bank share during the crisis.

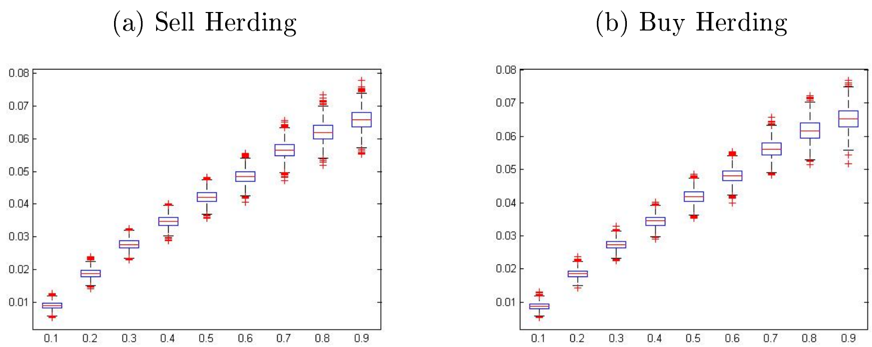


Figure 1.1: Information risk and herding intensity

Notes: $\overline{SHI}$ and $\overline{BHI}$ are plotted against information risk. On the ordinate we plot average herding intensity. Information risk μ is plotted along the horizontal. Average herding intensity is calculated as the weighted cross-sectional average for the simulated $\overline{SHI}$ and $\overline{BHI}$ of stocks contained in $\{\mu\} \times \mathcal{P} \times \mathcal{C}$. The weights correspond to the observed number of informed trades. The boxplots show the variation across 2,000 simulations of average herding intensity for a fixed level of information risk μ.

1.4.3 Simulation Results: Information Risk and Average Herding Intensity

To discover the impact of information risk on average herding intensity, we fix $\mu \in \mathcal{M}$ and calculate average herding intensity as the cross-sectional average over all parameterizations in $\{\mu\} \times \mathcal{P} \times \mathcal{C}$, where $|\{\mu\} \times \mathcal{P} \times \mathcal{C}| = 1 \cdot 36 \cdot 41 = 1,476$.

Figure 1.1 shows the comparative statics for average sell and buy herding intensity with respect to changes in information risk μ. The simulation results clearly indicate that $\overline{SHI}$ and $\overline{BHI}$ symmetrically increase with information risk. The boxplots demonstrate that the simulation results are very stable. Indeed, the variation of average herding intensity for a given level of information risk is relatively low, whereas its increase is rather steep as μ goes up. This particularly applies to the empirically relevant range of $\mu \in [0.2, 0.7]$, compare Kremer and Nautz (2013a) and Paper 2 of this thesis. Only as μ approaches 1, do $\overline{SHI}$ and $\overline{BHI}$ level out and exhibit higher variations.

The model simulation shows that the increasing effects of a rise in information

	$\overline{SHI}$	$\overline{BHI}$
Low market stress	0.0351 (0.0029)	0.0306 (0.0020)
High market stress	0.0382 (0.0023)	0.0635 (0.0038)

Table 1.1: The effects of market stress on average herding intensity

Notes: This table reports the simulated average sell ($\overline{SHI}$) and buy herding intensity ($\overline{BHI}$) for stocks under high market stress and stocks under low market stress. Standard deviations are in parentheses. Welch's t-test reveals that $\overline{SHI}$ as well as $\overline{BHI}$ increase significantly during times of high market stress for usual significance levels. Out of the 13,284 simulated stocks, 1,368 classify as high market stress and 1,008 as low market stress. Average herding intensities are calculated as the weighted cross-sectional averages of the simulated SHI and BHI for stocks in each respective class. The figures in the table are the weighted average and the weighted standard deviation of 2,000 iid simulated outcomes of $\overline{SHI}$ and $\overline{BHI}$ under high and low market stress, respectively. For all calculations, the weights correspond to the observed number of informed trades.

risk on herding intensity dominate the decreasing effects. Only as the share of informed traders surpasses 80%, does the adverse selection problem of the market maker begin to impair market liquidity severely enough that trading among the potential herders breaks down. The ambiguity of their signal prevents them from paying the high premiums now demanded by the market maker via large bid-ask spreads. We summarize the simulation-based insight from Figure 1.1 as follows:

Hypothesis 1.1. ***Information Risk and Herding Intensity***
Average sell and buy herding intensity increase in information risk.

1.4.4 Simulation Results: Market Stress and Average Herding Intensity

For the analysis of the effects of market stress we define two distinct classes of stocks and compare the average herding intensity of each. The first class comprises of all stocks that have high market stress characteristics; the second class includes all stocks that show low market stress characteristics. In line with the definition of market stress developed in Section 1.3.2, a simulated stock $\omega \in \Omega$ is subject to high market stress if it exhibits both, above-average uncertainty and below average $E[V]$. Correspondingly, low market stress stocks

	$\overline{SHI}$	$\overline{BHI}$
Low uncertainty	0.0373 (0.0018)	0.0340 (0.0016)
High uncertainty	0.0557 (0.0022)	0.0555 (0.0022)

Table 1.2: The effects of uncertainty on average herding intensity

Notes: This table reports the simulated $\overline{SHI}$ and $\overline{BHI}$ for stocks with high and low uncertainty respectively. Standard deviations are in parentheses. Welch's t-test reveals that $\overline{SHI}$ as well as $\overline{BHI}$ increase significantly during times of high uncertainty for usual significance levels. Out of the 13,284 simulated stocks, 3,078 exhibit high and, 2,268 low, uncertainty. Average herding intensities are calculated as the weighted cross-sectional averages of the simulated SHI and BHI for stocks in each respective class. The figures in the table are the weighted average and the weighted standard deviation of 2,000 iid simulated outcomes of $\overline{SHI}$ and $\overline{BHI}$ under high and low uncertainty, respectively. For all calculations, the weights correspond to the observed number of informed trades.

are defined by below-average uncertainty and above-average $E[V]$. The averages are the respective medians of the simulated model parameterizations.[12] We compare the cross-sectional average $\overline{SHI}$ and $\overline{BHI}$ over all high market stress stocks with the $\overline{SHI}$ and $\overline{BHI}$ obtained for all low market stress stocks.

The simulation results for the impact of market stress on average sell and buy herding intensity are shown in Table 1.1. As expected, both sell and buy herding are more pronounced during times of high market stress. Interestingly, however, the rise in buy herding intensity is greater than that of sell herding intensity. This puzzling asymmetry can be explained by disentangling the effects of an increase in uncertainty and pessimism.

Table 1.2 shows that $\overline{SHI}$ and $\overline{BHI}$ symmetrically increase with uncertainty. High public uncertainty is associated with lower prior probabilities for the middle state of the risky asset. Since informed traders receiving U-shaped signals discount the probability for the middle state anyway, high public uncertainty amplifies their tendency to form strong beliefs that only the extreme states of the risky asset can be true. As they rule out one of the extreme states

[12] Specifically, we obtain the median degree of pessimism (public uncertainty) by calculating $E[V]$ ($\mathrm{Var}(V)$) for each of the 36 simulated prior distributions $P(V) \in \mathcal{P}$ and then determine their median. Correspondingly, we calculate the median informed uncertainty over the set of simulated signal structures $\mathcal{C}$.

	$\overline{SHI}$	$\overline{BHI}$
High $E[V]$	0.0502 (0.0010)	0.0357 (0.0010)
Low $E[V]$	0.0370 (0.0016)	0.0504 (0.0016)

Table 1.3: The effects of economic outlook on average herding intensity

Notes: This table reports the simulated $\overline{SHI}$ and $\overline{BHI}$ for stocks where traders show high and low degrees of pessimism respectively. Standard deviations are in parentheses. Welch's t-test reveals a highly asymmetric effect for sell and buy herding. Indeed, $\overline{SHI}$ decreases as pessimism increases while $\overline{BHI}$ increases with the degree of pessimism. The results are significant at all usual significance levels. Out of the 13,284 simulated stocks, 5,904 stocks exhibit high and low degrees of pessimism. Average herding intensities are calculated as the weighted cross-sectional averages of the simulated SHI and BHI for stocks in each respective class. The figures in the table are the weighted average and the weighted standard deviation of 2,000 iid simulated outcomes of $\overline{SHI}$ and $\overline{BHI}$ under high and low uncertainty, respectively. For all calculations, the weights correspond to the observed number of informed trades.

based on the observed trading history, they quickly alter their trading decisions toward that of the crowd. This effect is intensified if private uncertainty is also high since such leads to a larger share of U-shaped traders. Since this argument applies equally to sell and buy herding, the increasing effect of uncertainty on herding intensity is symmetric.

In contrast, Table 1.3 reveals that a reduction in $E[V]$ affects $\overline{SHI}$ and $\overline{BHI}$ in opposite ways. While increased pessimism contributes to buy herding, it significantly reduces sell herding. This result is driven by the fact that during times of grim economic outlook, most informed traders sell anyway. Herd behavior, however, requires a trader to *alter* her initial trading decision. For *sell* herding to be possible, for instance, the trader has to be initially inclined to *buy* the asset. Only informed traders receiving U-shaped signals with strong biases toward the high state of the risky asset (i.e., $p^{21} << p^{23}$) may still be inclined to buy initially for low $E[V]$. As $E[V]$ drops, so does the number of simulated signal structures in $\mathcal{C}$ that exhibit a sufficiently strong positive bias of the U-shaped trader for sell herding to be possible. By the same line of reasoning, $\overline{BHI}$ increases with low $E[V]$.

We emphasize that the results in Table 1.3 do not contradict strong accumulations of traders on the sell side during times of deteriorated economic outlook. The Park and Sabourian (2011) model predicts that such a consensus in trade behavior is not driven by a switch in traders' opinion toward that of the crowd but results from a high share of equally pessimistic traders all acting on similar information. Such correlation of trade behavior is called spurious or unintentional herding in the literature, compare e.g. Kremer and Nautz (2013a) and Hirshleifer and Hong Teoh (2003).

The simulation shows that the positive effect of increased uncertainty on sell herding dominates the negative effect of increased pessimism. This leads to an overall slight increase in $\overline{SHI}$ during times of high market stress. In contrast, the complementary effect of uncertainty and pessimism on buy herding results in a surge of $\overline{BHI}$ during times of high market stress. We consolidate these simulation results in the following

Hypothesis 1.2. ***Market Stress and Herding Intensity***
In times of high market stress, the increase in buy herding is more pronounced than that of sell herding.

1.5 Conclusion

Due to data limitations and a lack of testable, model-based predictions on herding-intensity, the theoretical and the empirical herding literature are only loosely connected. This paper takes a first stab at tightening this connection by deriving theory-based predictions regarding the impact of information risk and market stress on aggregate herding intensity. This is done by numerically simulating the financial market herd model of Park and Sabourian (2011).

The model predicts that both buy and sell herding increase symmetrically with information risk. The effects of market stress on herding intensity are more complicated. We show that buy and sell herding — while both increasing with market stress — they do so in an asymmetric fashion. Interestingly, the model-implied hypothesis is that the increase of *buy* herding is more pronounced in times of high market stress than the one of sell herding. This is because the model-based measure of aggregate herding intensity only detects intentional herding as opposed to unintentional one. Traders may very well accumulate on the sell side of a market during downturns. Such coordination of traders, however, tends to be unintentional since they all follow their own private information that advises them to sell and, hence, is not reflected in the aggregate herding intensity. Conversely, the shortage of good news during crisis periods causes investors to be particularly susceptible to signals that the market rebounds. A temporary increase in stock prices due to trader accumulation on the buy-side of the market is such a signal. Consequently, investors are prone to intentionally follow others into buying stocks.

The next step in further tightening the connection between the theoretical and empirical herding literature is to test our model-implied predictions empirically. This is done in Paper 2 of this thesis by employing the empirical herd measure of Sias (2004), which is related in spirit to our notion of aggregate herding intensity. We test the hypotheses from this paper by applying the Sias measure to transaction data from the German stock market.

1.6 Appendix

Financial market herd models including the model of Park and Sabourian (2011) are not designed to provide closed-form solutions for expected herding intensity. In this Appendix, we use two examples to demonstrate why numerical simulations are required for obtaining model-based results regarding the impact of information risk and market stress on herding intensity.

1.6.A The History Dependence of Herding Intensity

Even for a given parameterization model complexity prevents deriving a closed-form analytical formula for herding intensity. The herding definition depends on the market maker's quotes, ask_t and bid_t, as well as the informed traders' expectations regarding the asset's true value $E[V \mid S, H_t]$. These quantities, in turn, depend on the whole history of trades until t. In fact, not only the number of observed buys, sells and holds but also their order affects expectations and quotes at time t.

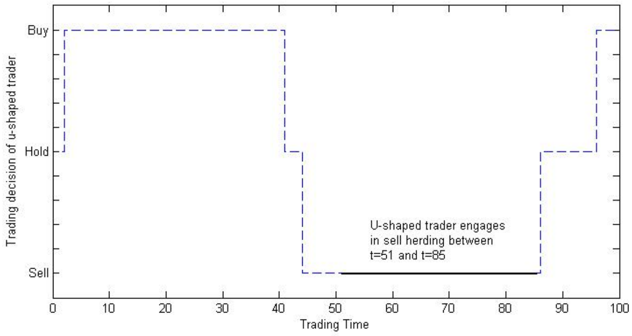
As a consequence, even for a given model parameterization, each history path would need to be analyzed separately to derive results on expected herding intensity.[13]

Let us illustrate this issue with a concrete numerical example. Assume the conditional signal matrix $P(S \mid V)$ to be

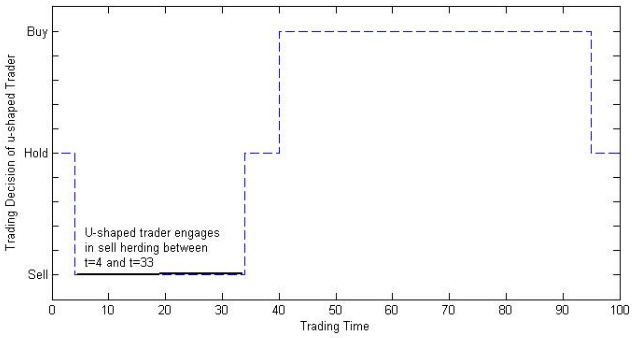
$P(S \mid V)$	$V_1 = 0$	$V_2 = 1$	$V_3 = 2$
S_1	0.6	0.5	0.1
S_2	0.3	0.1	0.4
S_3	0.1	0.4	0.5

The distribution of the risky asset is $P(V) = [0.3\ 0.4\ 0.3]$. Multiplying

[13] Given the sheer number of possible trading histories alone, an analytical derivation of SHI and BHI is not feasible even for relatively small T. For any length T of the history H_T, there are 3^T different history paths.



(a) $H_1^{100} = \{25\ buys,\ 50\ sells,\ 25\ buys\}$



(b) $H_2^{100} = \{25\ sells,\ 50\ buys,\ 25\ sells\}$

Figure 1.2: Trading decisions of U-shaped trader for $\mu = 0.5$

$P(S \mid V) \cdot P(V)$ yields the unconditional probabilities $P(S) = [0.41\ 0.25\ 0.34]$ that a trader receives a signal S given that she is informed. Finally, the share of informed traders is set to be $\mu = 0.5$. Only informed traders receiving the U-shaped signal S_2 can herd. Given that $E[V] = 1 < 1.12 = E[V \mid S_2]$, the U-shaped trader can engage in sell herding only if she is inclined to buy initially.

We discuss two distinct trading histories consisting of 100 trades and the exact same number of buys and sells. The only difference is the order in which the trades are observed. Let $H_1^{100} = \{25\ buys,\ 50\ sells,\ 25\ buys\}$ and $H_2^{100} = \{25\ sells,\ 50\ buys,\ 25\ sells\}$. Figure 1.2 shows how a U-shaped trader would decide to trade at every time $t = 1, ..., 100$ for the respective trading histories.

Note that the number of trades for which S_2 sell herds differs for the two histories. Under H_1^{100}, S_2 potentially sell herds between periods 51 and 85, i.e.

35 times.[14] Under H_2^{100}, S_2 potentially sell herds only 30 times. The share of U-shaped traders among the population of all traders is $\mu P(S_2) = 0.5 \cdot 0.25 = 0.125$. Consequently, we expect to observe a total number of $s_{T,1}^h = 0.125 \cdot 35 = 4.375$ herding sells under H_1^{100}. Correspondingly, under H_2^{100}, we only have $s_{T,2}^h = 0.125 \cdot 30 = 3.75$ expected herd sells.

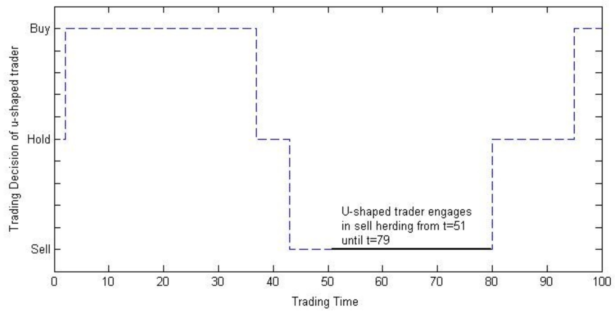
Moreover, since $\mu = 0.5$ and $T = 100$, we expect that both histories contain 50 informed trades. For an arbitrary history, calculation of the expected number of informed trades is much less straight forward since there is the possibility that informed traders hold and we hence have fewer informed trades than 50. Since H_1^{100} and H_2^{100} do not contain any holds, however, this is not an issue here.

According to Definition 1.2, the sell herding intensity is $SHI = s_T^h/(b_T^{in} + s_T^{in})$. Plugging in the expected values for numerator and denominator that we just calculated, we obtain an expected sell herding intensity $SHI_1 = 4.375/50 = 0.0875$ under H_1^{100} and $SHI_2 = 3.75/50 = 0.075$ under H_2^{100}.[15]

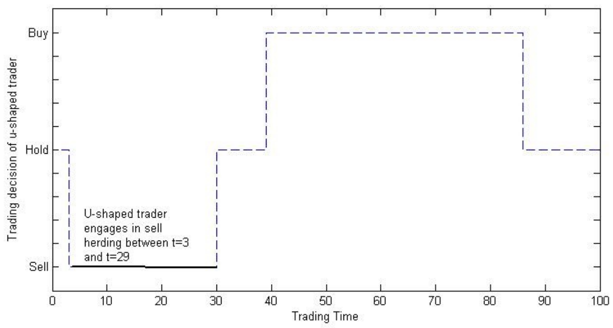
Finally note that the probability of observing these histories $P(H_i^{100})$ is also different for $i = 1, 2$, since the probability of observing a certain trade (i.e., buy or sell) in t depends on the trading decisions of the informed traders at t. This means that in order to calculate an overall expected herding intensity for the model parameterization above, we would need to analyze SHI and $P(H^{100})$ for all 3^{100} possible history paths separately, a task well beyond our current computational capacity. Even if we were able to calculate that number, we still would not have a formula that tells us how SHI would react to changes in certain model parameters such as μ. Indeed, one can illustrate the many counteracting effects of a change in μ that result in quite different outcomes for specific trading histories and thus also prevent the derivation of analytical comparative static results.

[14] Note that S_2 does in fact start herding only in period 51, although she would already have decided to sell in period 44. This is because the complete history does not contain more sells than buys until period 51, which we demand in order to ensure that S_2 actually follows the majority in the market.

[15] Note that since numerator and denominator are clearly correlated, we have that $E[\frac{X}{Y}] \neq \frac{E[X]}{E[Y]}$. A Taylor approximation of order 1, however, yields that the expectation of a ratio can be consistently estimated by the ratio of the expectations. As a consequence, all equations should be understood as approximations. An exact calculation of expected herding intensity would be even more complicated.



(a) $H_1^{100} = \{25\ buys,\ 50\ sells,\ 25\ buys\}$



(b) $H_2^{100} = \{25\ sells,\ 50\ buys,\ 25\ sells\}$

Figure 1.3: Trading decisions of U-shaped trader for $\mu = 0.6$

1.6.B The Impact of a Change in μ on Herding Intensity: An Analytical Approach

Let us now assume that $\mu = 0.6$ and see how SHI changes for H_i^{100}, for $i = 1, 2$. Figure (1.3) shows that the increase in μ causes the number of potential sell herd trades to drop from 35 to 28 and from 30 to 27 for H_1^{100} and H_2^{100} respectively. Given that now $\mu P(S_2) = 0.15$, we expect $SHI_1 = 0.07$ and $SHI_2 = 0.0675$ for the respective histories. In other words, an increase in μ causes a drop in SHI for the above two trading histories.

The effects that drive this result are higher bid-ask-spreads quoted by the market maker in conjunction with a higher average information content of each single trade. Both effects contribute towards a stronger preference of S_2 of holding the asset. In particular, the sell herds are broken much faster than before: While for $\mu = 0.5$, the sell herding U-shaped traders had to observe 9-10 consecutive

$\mu = 0.5$	Number of herd trades	$P(H.)$	$(P(H_1) + P(H_2))/P(H_3)$
H_1^{100}	35	$7.62 \cdot 10^{-38}$	
H_2^{100}	30	$3.75 \cdot 10^{-38}$	$6.72 \cdot 10^{-7}$
H_3^{100}	97	$1.69 \cdot 10^{-31}$	
$\mu = 0.6$			
H_1^{100}	28	$4.15 \cdot 10^{-36}$	
H_2^{100}	27	$1.95 \cdot 10^{-36}$	$7.02 \cdot 10^{-8}$
H_3^{100}	97	$8.69 \cdot 10^{-29}$	

Table 1.4: Probabilities of selected histories

Notes: This table reports the probabilities of three different histories for the previously specified model parameterizations with $\mu = 0.5$ and $\mu = 0.6$ respectively. It also compares the probability ratio of observing histories H_1 or H_2 with observing history H_3 for each scenario. H_1 and H_2 are as before, H_3 is a history consisting of 100 sells.

buys before switching back into holding the asset, the observation of merely 5 consecutive buys already triggers this switch in trading behavior of S_2 when $\mu = 0.6$.

The results in Section 1.4, however, suggest that $\overline{SHI}$ increases with μ. The reason for this is yet another effect of a change in μ. An increase in μ alters the probability with which a certain history is observed. Indeed, an increase in μ shifts probability mass from histories with low or decreasing herding intensity to histories with persistently high herding.

This effect is documented in Table 1.4. Consider the previously introduced histories H_1^{100} and H_2^{100}. Also consider history H_3^{100} consisting of 100 sells. Under H_3, S_2 sell herds from $t = 4$ until $t = 100$ resulting in 97 potential herd sells regardless of μ. Yet, the probabilities for each of the histories changes as μ changes. More specifically, the probability to observe H_1 or H_2 relative to the probability to observe H_3 decreases.

This can be attributed to the self-enforcing nature of herd behavior. Once investors start herding, it is on average more likely that they keep herding than that their herd is broken.

We emphasize that this is not a complete comparative static analysis. For that we would have to consider all 3^{100} different histories. As outlined before, this is beyond current computational capabilities. Also note that the discussed examples are only for a single stock. The calculations further complicate if one aims at calculating average herding intensities for a heterogeneous stock market as we do in Section 1.4.

Paper 2

Information Risk, Market Stress and Institutional Herding: Evidence from the German Stock Market[1]

2.1 Introduction

Paper 1 of this thesis derives two theory-guided predictions on the impact of information risk and market stress on herding intensity by simulating the herd model of Park and Sabourian (2011).[2]

It is the present paper's objective to empirically test the validity of these predictions. This is done by applying the herd measure of Sias (2004) to a set of high-frequency, investor-specific transaction data from the German stock market.

Simulating a herd model allows us to determine for each trade whether herding actually occurred. As a result, the exact intensity of intentional herding in the sense of Hirshleifer and Hong Teoh (2003) can be calculated. In an empirical application, it is much more difficult to decide whether or not a trader herds since researchers have no access to the financial decision makers' private information and preferences. Nevertheless, a rich empirical literature has evolved that has contributed greatly towards overcoming this obstacle.

[1]This paper was written in collaboration with my co-authors Simon Jurkatis, Dieter Nautz and Stephanie Kremer.

[2]For definitions of information risk, market stress and herding intensity and a more detailed discussion of the related theoretical literature, please refer to Paper 1.

In the seminal work of Lakonishok et al. (1992), herding of a group of investors is measured as the tendency to accumulate on one side of the market. Specifically, the authors test whether the share of net buyers in individual stocks significantly deviates from the average share of net buyers across all stocks of the considered stock index. In a more recent study, Sias (2004) investigates whether the accumulation of investors on one side of the market persists over time by measuring the cross-sectional correlation of the share of net buyers over adjacent time periods.

The dynamic nature of the Sias measure does not only make it particularly appropriate for the analysis of high-frequency data but also lends it a conceptual proximity to herding as defined by the theory. Thanks to it's dynamic approach, the Sias measure reflects the theoretical notion of herders' switching behavior more accurately than the static measure of Lakonishok et al. (1992). Moreover, the Sias measure incorporates the intuition of the Park and Sabourian (2011) model that during periods of e.g. buy herding, high shares of net buyers persist over time.

Using intra-day, investor-specific transaction data provided by the German Federal Financial Supervisory Authority (BaFin) further enhances the comparability of our empirical results based on Sias and our theoretical predictions regarding aggregate herding intensity.

In line with herding theory, the use of *intra-day* data is particularly appropriate for measuring herd behavior induced by information externalities. Measuring herding at lower frequencies may bias the results because new information might have reached the market in the meantime, creating a new environment for investor behavior. The use of *investor-specific* data is particularly important as we need to directly identify transactions by each trader in order to determine whether an investor follows the observed actions of other traders.

Empirical studies using transaction data typically have to rely on *either* investor-specific but low-frequency data (e.g. Lakonishok et al. (1992), Sias (2004), Wermers (1999)), *or* on high-frequency but anonymous transaction data (compare, e.g., Barber et al. (2009b)). Kremer and Nautz (2013a) regress daily herding measures on size, volatility, and other stock characteristics to analyze the causes of herding.

To the best of our knowledge, this paper is the first to analyze *intra-day* herding intensity using investor-specific data. It confirms both theoretical predictions on herding intensity derived in Paper 1 of this thesis.

The remainder of this paper proceeds as follows. Section 2.2 presents the herd measure proposed by Sias (2004). Section 2.3 discusses the employed data in further detail. The empirical results on the impact of information risk on herding intensity are provided in Section 2.4, while Section 2.5 confirms the impact of Market Stress on investor herding. Finally, Section 2.6 contains concluding remarks.

2.2 Empirical Herding Measure

The dynamic herding measure proposed by Sias (2004) is designed to explore whether (institutional) investors follow each others' trades by examining the correlation between the traders' buying tendency over time. The Sias herding measure, therefore, is particularly appropriate for high-frequency data. Similar to the static herding measure proposed by Lakonishok et al. (1992), the starting point of the Sias measure is the number of buyers as a fraction of all traders. Specifically, consider a number of N_{it} institutions trading in stock i at time t. Out of these N_{it} institutions, a number of b_{it} institutions are net buyers of stock i at time t. The buyer ratio br_{it} is then defined as $br_{it} = \frac{b_{it}}{N_{it}}$. According to Sias (2004), the ratio is standardized to have zero mean and unit variance:

$$\Delta_{it} = \frac{br_{it} - \bar{br}_t}{\sigma(br_{it})}, \tag{2.1}$$

where $\bar{br}_t := \sum_{i=1}^{I}$ with I denoting the number of stocks in the cross-section and $\sigma(br_{it})$ is the cross-sectional standard deviation of buyer ratios across I stocks at time t. The Sias herding measure is based on the correlation between the standardized buyer ratios in consecutive periods:

$$\Delta_{it} = \beta_t \Delta_{i,t-1} + \epsilon_{it}. \tag{2.2}$$

The cross-sectional regression is estimated for each time t. In the second step, the Sias measure for herding intensity is calculated as the time-series average of the estimated coefficients: $Sias = \frac{\sum_{t=2}^{T} \beta_t}{T-1}$.

The Sias methodology further differentiates between investors who follow the trades of others (i.e., *true herding* according to Sias (2004)) and those who follow their own trades. For this purpose, the correlation is decomposed into two components:

$$\begin{aligned}
\beta_t &= \rho(\Delta_{it}, \Delta_{i,t-1}) \\
&= \left[\frac{1}{(I-1)\sigma(br_{it})\sigma(br_{i,t-1})}\right] \sum_{i=1}^{I} \left[\sum_{n=1}^{N_{it}} \frac{(D_{nit} - \bar{br}_t)(D_{ni,t-1} - \bar{br}_{t-1})}{N_{it} N_{i,t-1}}\right] \\
&+ \left[\frac{1}{(I-1)\sigma(br_{it})\sigma(br_{i,t-1})}\right] \sum_{i=1}^{I} \left[\sum_{n=1}^{N_{it}} \sum_{\substack{m=1, \\ m \neq n}}^{N_{i,t-1}} \frac{(D_{nit} - \bar{br}_t)(D_{mi,t-1} - \bar{br}_{t-1})}{N_{it} N_{i,t-1}}\right],
\end{aligned} \tag{2.3}$$

where I is the number of stocks traded. D_{nit} is a dummy variable equal to 1 if institution n is a buyer in i at time t and 0 otherwise. $D_{mi,t-1}$ is a dummy variable equal to 1 if trader m (who is different from trader n) is a buyer at time $t-1$. Therefore, the first part of the measure represents the component of the cross-sectional inter-temporal correlation that results from institutions following their own strategies when buying or selling the same stocks over adjacent time intervals. The second part indicates the portion of correlation resulting from institutions following the trades of others over adjacent time intervals. A positive correlation that results from institutions following other institutions, that is, the latter part of the decomposed correlation, can be regarded as evidence of herd behavior. In the subsequent empirical analysis, we therefore focus on the latter term of Equation (2.3), which we denote by $\overline{Sias}$. According to Choi and Sias (2009), Equation (2.3) can be further decomposed to distinguish between the correlations associated with "buy herding" ($br_{i,t-1} > 0.5$) and "sell herding" ($br_{i,t-1} < 0.5$).

2.3 Data

The data are from the German Federal Financial Supervisory Authority (BaFin).[3] Under Section 9 of the German Securities Trading Act, all credit institutions and financial services institutions are required to report to BaFin any transaction in securities or derivatives that trade on an organized market. These records make it possible to identify all relevant trade characteristics, including the trader (the institution), the particular stock, time, number of traded shares, price, and the volume of the transaction. Moreover, the records specify on whose behalf the trade was executed, that is, whether the institution traded on it's own account or on behalf of a client that is not a financial institution.

Only institutions that fall under Section 9 of the German Securities Trading Act are allowed to submit trade orders to German trading platforms. Therefore, the data are a comprehensive repository of *all* trades executed on German stock exchanges during the sample period. Since this study is concerned with institutional trades, particularly those of financial institutions, we restrict our attention to the trading of own accounts, that is, those cases where a bank or financial services institution is clearly the originator of the trade. We exclude institutions trading exclusively for the purpose of market making. We also exclude institutions that are formally mandated as designated sponsors, i.e., liquidity providers, for a specific stock. For each stock, there are usually about two institutions formally mandated as market maker. The institutions are not completely dropped from the sample (unless they have already been excluded due to engaging in purely market maker business), but only for those stocks for which they act as designated sponsors.[4] We are particularly interested in the herding behavior of institutional investors because they are more likely to be informed compared to, for example, retail investors. Moreover, institutional investors are the predominant class in the stock market, with the power to move the market and impact prices, particularly if they herd.

[3]Due to the sensitivity of the data, BaFin does not allow to share the data with third parties. To access the data for replication purposes, please contact Stephanie Kremer (stephanie.kremer@fu-berlin.de).

[4]The designated sponsors for each stock are published at http://www.deutsche-boerse.com. For more information about the data, see Kremer and Nautz (2013a;b).

The analysis focuses on shares listed on the DAX 30 (the index of the 30 largest and most liquid stocks), where stocks are selected according to the index compositions at the end of the observation period on March 31, 2009. Following the empirical literature, we require that at least five institutions were active in the market at each trading interval. Using data from July 2006 to March 2009 (698 trading days), we are able to investigate whether trading behavior has changed during the financial crisis. Over the sample period, there are 1,120 institutions engaging in proprietary transactions. Among those 1,120 traders, 1,044 trade the DAX 30 stocks.

2.4 Information Risk and Herding Intensity in the German Stock Market

According to Hypothesis 1.1 from Paper 1, average herding intensity increases with information risk. Information risk, i.e. the probability of informed trading increases with the number of informed traders active in a market and their share of the trading volume. Based on this intuition, we use two empirical proxies for the level of information risk: (i) the number of active institutional traders and (ii) the share of the institutional trading volume.

We deliberately do not follow Zhou and Lai (2009) in using Easley et al. (1997)'s PIN measure to proxy information risk as the PIN is positively related to the Sias herd measure by construction.[5]

According to Foster and Viswanathan (1993) and Tannous et al. (2013), the fraction of informed traders and, thus, information risk cannot be expected to be constant over a trading day. To account for intra-day trading patterns in the German stock market, we divide each trading day into 17 half-hour intervals. A trading day is defined as the opening hours of the trading platform XETRA (9 a.m. to 5:30 p.m.), on which the bulk of trades occur. The use of half-hour intervals ensures that the number of active institutions is sufficiently high for

[5] The idea underlying the PIN is that there are distinct trading patterns on days when information events occur. Days with information events (i.e. high information risk) are characterized by a strong accumulation of (informed) traders on one side of the market. The Sias measure also identifies herding as a (persistent) accumulation of traders on one side of the market.

	Information risk		Herding intensity	
Time	*Traders*	*Trading Volume*	*Sias*	$\overline{Sias}$
09 : 00 – 09 : 30	25.33	6.73	–	–
09 : 30 – 10 : 00	21.05	5.34	25.92 (0.23)	9.92 (0.26)
10 : 00 – 10 : 30	15.75	2.57	28.59 (0.22)	7.54 (0.24)
10 : 30 – 11 : 00	22.88	6.73	30.43 (0.29)	7.85 (0.23)
11 : 00 – 11 : 30	19.58	4.51	34.30 (0.31)	9.98 (0.22)
11 : 30 – 12 : 00	18.72	4.15	33.98 (0.29)	8.24 (0.23)
12 : 00 – 12 : 30	17.96	3.77	33.91 (0.30)	7.83 (0.24)
12 : 30 – 01 : 00	17.08	3.39	33.81 (0.25)	6.96 (0.21)
01 : 00 – 01 : 30	17.36	4.31	33.28 (0.24)	7.84 (0.21)
01 : 30 – 02 : 00	16.57	3.28	34.00 (0.28)	8.56 (0.21)
02 : 00 – 02 : 30	17.85	3.96	34.74 (0.25)	8.60 (0.26)
02 : 30 – 03 : 00	18.90	4.63	33.38 (0.24)	8.29 (0.26)
03 : 00 – 03 : 30	18.32	4.42	34.21 (0.26)	9.31 (0.26)
03 : 30 – 04 : 00	20.42	6.43	34.19 (0.28)	10.60 (0.26)
04 : 00 – 04 : 30	20.70	6.98	35.65 (0.28)	12.86 (0.26)
04 : 30 – 05 : 00	20.74	7.64	34.62 (0.27)	11.90 (0.26)
05 : 00 – 05 : 30	22.50	10.13	32.94 (0.28)	12.53 (0.26)

Table 2.1: Information risk and herding intensity within a trading day

Notes: The table shows how information risk and herding intensity evolves over the trading day. *Traders* denotes the average number of active institutional traders; *Trading Volume* refers to the average percentage share of the daily trading volume of institutional investors. For instance, on average, 6.73% of the daily institutional trading volume occured between 9 a.m. and 9:30 a.m. The columns do not add to 1 because we focus on the predominant German platform XETRA®, where trading takes place from 9 a.m. till 5.30 p.m. CET, while the opening period for the German stock exchange at the floor ends at 8 p.m. *Sias* and $\overline{Sias}$ represent the overall and the adjusted Sias herding measure (in percent), where the latter only considers institutions that follow the trades of others, see Equation (2.3). Standard errors are in parentheses.

calculating intra-day herding measures.[6] The first two columns of Table 2.1 show how both empirical proxies for information risk are distributed within a day. For both measures of trading activity, institutional traders are more active during the opening and closing intervals.

To investigate the intra-day pattern of herding intensity, we calculate the Sias herding measure for each half-hour interval separately. The results of this exercise are also shown in Table 2.1. The third column shows for each interval the overall Sias measure ($Sias$), which is based on the average correlation of buy ratios between two intervals (see Equation (2.2)). Following Sias (2004), this correlation may overstate the true herding intensity because it does not account for correlation resulting from traders who follow themselves. It is a distinguishing feature of our investor-specific data that they allow addressing that problem even on an intra-day basis. In particular, Column 4 reports the correlation due to investors following the trades of others ($\overline{Sias}$) (see Equation (2.3)).

Table 2.1 offers several insights into the intra-day pattern of institutional herding. First, both Sias measures provide strong evidence for the presence of herding for each half-hour interval of the trading day. Second, intra-day herding measures are significantly larger than those obtained with low-frequency data, compare Kremer and Nautz (2013a;b). Third, the sizable differences between $Sias$ and $\overline{Sias}$ highlight the importance of using investor-specific data.

How is the observed intra-day variation of information risk related to the intra-day herding intensity of institutional investors? In line with the intuition of Park and Sabourian (2011), the Sias herding measure depends on the trading behavior in two subsequent time periods. On the one hand, high information risk in $t-1$ leads institutional investors to believe that there is a high degree of information contained in previously observed trades. On the other hand, high information risk in t ensures that there is a high number of potential herders active in the market. Both effects contribute positively to herding intensity in period t. Therefore, for each time interval herding intensity is compared with the average information risk of the corresponding time intervals. Figure 2.1 reveals a strong intra-day co-movement between both proxies of information risk and $\overline{Sias}$. In fact,

[6]For the sake of robustness, we also divide the trading day into nine one-hour intervals, but our main results do not depend on this choice.

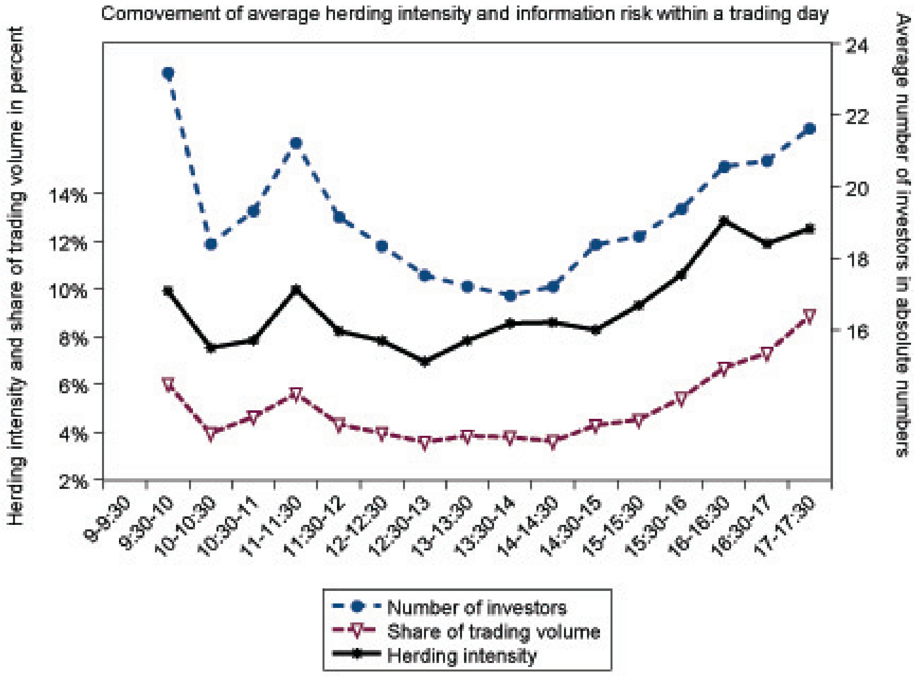


Figure 2.1: Information risk and average herding intensity within a trading day

we find overwhelming evidence in favor of Hypothesis 1.1: the rank-correlation coefficient between the average trading volume and the corresponding Sias measure is 0.80, which is both economically and statistically highly significant. Very similar results are obtained for the number of active institutional traders, where the correlation coefficient equals 0.67.[7]

Note that the peaks in $\overline{Sias}$ at market opening and following the opening of the U.S. market at 3:30 p.m. – 4 p.m. correspond with high activity by informed traders, suggesting that at market openings there is a lot of information contained in observed trades on which subsequent traders herd. This confirms the experi-

[7]These results can be confirmed using standard correlation coefficients, which are also large and significant at all conventional levels for both empirical proxies of information risk. Note that a rank-correlation coefficient might be more appropriate than the standard correlation coefficient, since it accounts for the potentially non-linear relation between information risk and herding intensity suggested by the numerical simulation of the herd model (see Figure 1.1 in Paper 1 of this thesis).

Buy Herding	*Sias*	$\overline{Sias}$
Pre-crisis period	14.37 (0.37)	4.10 (0.10)
Crisis period	13.87 (0.35)	5.09 (0.11)
Sell Herding		
Pre-crisis period	18.87 (0.23)	5.41 (0.09)
Crisis period	15.65 (0.25)	5.74 (0.08)

Table 2.2: Herding intensity - before and during the financial crisis

Notes: This table reports adjusted ($\overline{Sias}$) and unadjusted ($Sias$) herding measures based on half-hour intervals estimated separately for the pre-crisis and the crisis period. The Sias measures are further decomposed into buy and sell herding components (see Section 2.2). Standard errors are in parentheses.

mental findings of Park and Sgroi (2012), who observe that traders with relatively strong signals trade first, while potential herders delay.

2.5 Herding Intensity in the German Stock Market Before and During the Financial Crisis

Hypothesis 1.2 of Paper 1 tells us that both sell and buy herding should increase in times of high market stress when uncertainty increases and markets become more pessimistic about the value of the asset. However, the increase in sell herding is predicted to be smaller than the one in buy herding. In our application, a natural candidate to test this hypothesis is the outbreak of the financial crisis. To investigate the effect of the crisis on herding intensity, we calculate sell and buy herding measures for the crisis and the pre-crisis period separately. The pre-crisis period ends on August 9, 2007 as this is widely considered to be the starting date of the financial crisis in Europe, see, e.g., European Central Bank (2007) and Abbassi and Linzert (2012).

Herding measures obtained before and during the crisis are displayed in Table 2.2. The results confirm the predictions of the simulated model of Paper 1. The statistically significant yet small increase in sell herding ($5.74 > 5.41$) is well in line with Hypothesis 1.2 as is the more pronounced surge in buy herding $(5.09 > 4.10)$.

Apparently, in times of deteriorated economic outlook when traders are exposed to recurring bad news, a small but unexpected accumulation on the buy side is quickly interpreted as good news about an asset's value and induces investors to follow the crowd (as small as it may be) into the alleged investment opportunity. Such behavior in light of Hypothesis 1.2 is by no means purely based on investor sentiment or irrationality, but may be perfectly rational. In line with our theoretical results, the increase in sell herding during the crisis period indicates that the high uncertainty effect dominates the low expectation effect discussed in Section 1.4 of Paper 1. The increase, however, may also be explained by reasons outside the model. If asset prices start to fall, selling may become necessary in order for institutional traders to meet regulatory requirements. The resulting accumulation of institutional traders on the sell side of the market may upward bias the sell herding intensity detected by the empirical herding measure. Yet, the small increase in sell herding intensity in the German stock market during the crisis period indicates that these diluting effects of unintentional herding are not of particular relevance for our sample.

2.6 Concluding Remarks

This paper further strengthens the link between the theoretical and the empirical herding literature.

Having derived two theory-based predictions regarding the impact of information risk and market stress on herding intensity in Paper 1, this paper focuses on testing these predictions empirically using a comprehensive data set from the German stock market. As predicted, we find that both buy and sell herding increase symmetrically with information risk. Our empirical results further show that the herd model can explain why buy and sell herding in the German stock market evolve asymmetrically in response to increased market stress induced by the financial crisis.

We should stress, however, that despite our careful choice of the empirical herd measure and it's application to a microstructure theory compatible data set, the fact remains that the measure proposed by Sias (2004) and the theoretical concept of herding of e.g. Park and Sabourian (2011) remain only loosely connected. Indeed, the general consensus that measures such as the ones proposed by Lakonishok et al. (1992) and Sias (2004) are valid tests for (persistent) investor coordination in general and herding in particular, compare Bikhchandani and Sharma (2001), has not yet been proven rigorously.

This poses the question whether we can use herding and market microstructure theory to qualify potential weaknesses of established herd measures if there are any. This could be done by applying empirical herd measures to simulated trade data from a herd model. Insights generated from such an analysis could be used to modify the existing measures to obtain a new theory-founded measurement approach that accurately tests for the presence of informationally inefficient and, thus, also potentially price-distorting herd behavior. Papers 3 and 4 of this thesis address these tasks in great detail.

Paper 3

How to Measure Herding in Financial Markets[1]

3.1 Introduction

Investor herding describes the behavior of individual investors that follow the decision of the majority although they hold private information that advises them to act differently, compare Brunnermeier (2001). There is strong consensus in the literature that herding has the potential to cause informational inefficiencies, distort prices and ultimately destabilize financial markets altogether, see e.g. Bikhchandani and Sharma (2001).

Consequently, empirical studies have been putting great efforts into detecting destabilizing herd behavior by assessing whether groups of investors coordinate and by gauging the effect of their coordination on asset prices, see e.g. Brown et al. (2014), Dorn et al. (2008), Wermers (1999) or Grinblatt et al. (1995). This literature strand is strongly influenced by the seminal work of Lakonishok et al. (1992). Their well-known LSV measure has long become a benchmark to test for the presence of investor coordination, see e.g. Kremer and Nautz (2013b) and Barber et al. (2009b) in addition to the already mentioned studies.

This paper shows, however, that the LSV measure generally does not provide the right means test for investor coordination and, consequently, provides a measure that does.

[1] This paper was written in collaboration with my co-author Simon Jurkatis.

To that end we adjust the LSV measure in accord with implications from a market microstructure framework. We thereby obtain a new measure for investor coordination and show it to be a generalization of the LSV measure. We use our model framework to simulate trade data that allows us to further quantify the differences between the two approaches. In particular, we show that our measure accurately distinguishes between different types of investor coordination, i.e. herding and contrarianism, as well as independent trading.[2] The simulation also reveals that the LSV measure generally fails to correctly test for investor coordination if the trade data does not fulfill the rather restrictive assumptions associated with the LSV approach.

The LSV measure uses transaction data for a specific group of investors. It assesses the deviation of the investors' observed buy propensity in each stock from their average buy propensity across all stocks to determine if and to what extent investors coordinate.

In line with our model framework, such a comparison of buy propensities under actual and independent trading is a reasonable approach to detect investor coordination. Yet, we also discover that the LSV approach has two crucial weaknesses that can be remedied with proper adjustments.

First, the LSV approach assumes that under the null hypothesis of independent trading the chance to observe a buy is equal to the average buy propensity *for all stocks*, compare Wermers (1999). The assumption that investors exhibit exactly the same proclivity to buy each stock of a potentially large cross-section is likely to be too rigid to ever hold for actual trade data. We, therefore, propose to take into account that buy propensities under independent trading are stock-specific.

Second, Lakonishok et al. (1992) estimate the unique average buy propensity under independent trading using all trades, that is, trades that may have been carried out in a dependent fashion. This almost inevitably results in a bias of the LSV if the null of independent trading is rejected. To avoid such a bias, the

[2] Contrarianism can be seen as the counter-part of herding. Instead of following the crowd, contrarians act against it although they have information that tells them to trade in the same direction as the majority of the traders, compare e.g. Park and Sabourian (2011).

estimation of the buy ratios under independent trading should focus on those trades that are in fact carried out independently. We argue that these trades can be identified among the early trades after the start of trading, see Avery and Zemsky (1998).

We show that with the proposed modifications of the LSV approach, the distribution of the buy ratios under independent trading can still be estimated accurately even in small cross-sections if the independent buy ratios are iid distributed.[3]

We, thus, obtain a new operational measure for investor coordination. Similar to the LSV measure, it compares the observed buy ratios with the estimated independent benchmark to detect investor coordination. The estimation of the independent benchmark, however, is quite different for our approach and more in line with herding theory than the LSV approach. For instance, as we obtain buy ratios under independent trading from the early trades and compare it to the subsequent trading behavior we capture the notion of switching behavior that underlies herding and contrarianism, compare Park and Sabourian (2011) and Avery and Zemsky (1998).

Other modifications of the LSV measure have been proposed in the literature to boost its performance. Frey et al. (2014) modify the LSV measure by taking the squared instead of the absolute difference between the observed buy propensities and the the average one. Wylie (2005) corrects the LSV measure to account for possible biases that can arise from short-selling constraints and varying liquidity requirements. Yet, since both maintain the assumption of a constant buy propensity under the null and estimate it based on all trades our arguments apply to their approaches as well.[4]

[3]In Paper 4 of this thesis, we uncover strong evidence in favor of this assumption.

[4]Statistically put, the assumption of equal buy propensities under independent trading stems from the fact that the LSV measure tests whether the observed number of buys are more dispersed than suggested by a Binomial distribution. Consequently, our arguments generally apply to any test of Binomial dispersion (e.g. Cochran (1954), Tarone (1979)) that is applied for the purpose of finding deviations from independent trading.

Another measure related to the LSV measure is the one proposed by Sias (2004). Like Lakonishok et al. (1992), Sias (2004) uses the buy propensity of investors as the underlying statistic. Yet, the Sias measure assesses whether buy propensities are persistently high or low over time by measuring the correlation of buy propensities between adjacent time periods. Though we will not compare our approach to the one of Sias (2004) directly, our arguments are valid for his measure as well. By assessing the serial correlation of buy propensities, the cross-sectional averages of the buy propensities constitute a part of the Sias measure and, therefore, our arguments in favor of an approach that accounts for the idiosyncrasy of these propensities apply here as well.

The disconnect of empirical measures on coordinated trading with the theoretical literature has also been noted by Devenow and Welch (1996) and Cipriani and Guarino (2014). To provide a rigorous test of a theoretical herding model, the latter conduct a structural model estimation. Though we attempt to bring the empirical literature closer to the theoretical idea of herding and contrariansm, we do not go as far as estimating a specific model of herding.

Our microstructure framework is statistical in nature. Instead of explicitly modeling drivers for investor behavior, we treat herding, contrarianism and independent trading as well as single trade decisions as probabilistic events.[5] Our model is not designed to explain why investors coordinate but to produce simulated trade data that allows us to understand whether different measures of coordinated trading can accurately detect investor coordination if it is present.

This paper is structured as follows. In Section 3.2 we introduce a model of investor coordination. Section 3.3 provides a detailed discussion of the assumptions associated with the LSV approach and how we aim to modify them

[5]For a better understanding of potential drivers for investor coordination, we refer the reader to the rich theoretical herding literature. The seminal works of Bikhchandani et al. (1992) and Banerjee (1992) demonstrate that herding is triggered by information externalities that a decision by one agent imposes on the decisions of the subsequent agents. Reputational concerns of financial decision makers are identified as another important driver for herd behavior, see Scharfstein and Stein (1990), Graham (1999) and Dasgupta et al. (2011). So-called investigative herding, that is, agents basing their decision on the same information, has been discussed by Froot et al. (1992), Hirshleifer et al. (1994). Paper 1 shows that information risk and market stress are also relevant drivers.

to derive our new measure for herding. It also contains theoretical results that our measure is accurately estimable. In section 3.4 a comparative analysis of the LSV and our measure including a comprehensive simulation study is provided. Finally, Section 3.5 summarizes the results.

3.2 A Model of Coordinated Investor Behavior

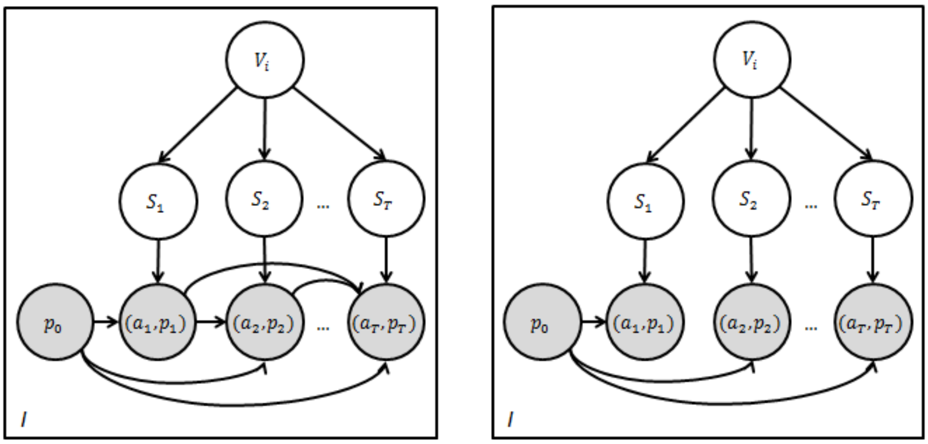
In this section, we revisit a sequential trading framework in the spirit of Glosten and Milgrom (1985). We use this model to illustrate that potentially price-distorting investor coordination such as herding requires financial decision makers to deviate from independent trading. In line with the seminal work of Lakonishok et al. (1992), we propose to use investor buy ratios under actual and independent trading to detect such deviations. We show that the corresponding statistic accurately reflects investor coordination in our model. We conclude this section by discussing how buy ratios can be related to herding and contrarianism, respectively.

3.2.1 Dependent and Independent Trading

Figure 3.1 illustrates the principal setup of Glosten and Milgrom (1985)'s sequential trading model. Consider some stock $i \in \mathcal{I} \subset \mathbb{N}^+$ that is traded for a day.[6] During the course of the day we sequentially observe transactions a_t at price p_t for $t = 1, 2, \ldots, T$.

Each transaction is influenced by two, possibly competing, types of information. First, traders gather information about the stock's value and form a private opinion on how much the asset is worth. Comparing their assessment with the price, they decide what their action should be. Second, as the trading process evolves, investors observe the transactions of others and the corresponding price movements. They include the information contained in these observations into their decision rule. In line with the literature we refer to the first information

[6] In Paper 4 we apply our method at the frequency of days. The method, however, is not restricted to that frequency and the word "day" may be replaced with any other frequency in mind, e.g. half-hour interval, week, month, etc., in what follows.



(a) Actual trading (b) Independent trading

Figure 3.1: Microstructure trading model

Notes: This figure depicts a sequential trading process as a directed graphical model under actual and independent trading. Nodes represent random variables. Links between the nodes represent probabilistic relationships between those variables. The direction of a link indicated by an arrow points from the parent node to the child node. The distribution of the variable from a child node is conditioned on the parent variable. White nodes represent latent variables, while grey shaded nodes represent observable variables. The square indicates that there are I trading processes.

type as private information and to the second one as public information, compare Park and Sabourian (2011).

Private information is depicted in Figure 3.1 by the latent random variables S_1 to S_T, whose distribution is conditioned on the latent, random variable V_i. In accordance with market microstructure theory, we assume that V_i corresponds to the unknown, fundamental value of the stock and S_t to the fundamentally linked private signal of trader t. Public information is depicted by the directed links from any shaded node (a_{t-j}, p_{t-j}) to the node (a_t, p_t) with $t = 1, \ldots, T$ and $j = 1, \ldots, t-1$, see Figure 3.1 (a). Public information also includes the opening price p_0, which can be seen as a consolidated measure of the market's prior belief regarding the stock's value.

If traders are not influenced by the actions of others and base their trade decision solely on their private information and p_0, we say they trade independently, compare Figure 3.1 (b). In that case the links between the

(a_t, p_t) are switched off. Under independent trading the stock price moves towards the asset's fundamental. The view that independent trading should have this desirable effect is based on the assumption that private information is linked to the asset's fundamental value, compare e.g. Avery and Zemsky (1998) and Park and Sabourian (2011). If investors in these models follow their private information signals, the market learns fairly quickly about the asset's true value. This results in prices that accurately reflect the asset's fundamental and that exhibit low degrees of volatility.

Figure 3.1 (a) illustrates the case of actual trading, i.e. when investor t's trade decision is influenced by the actions of her predecessors. In this case trader t may decide to act against her private information signal S_t. She may decide to sell the stock although her private information advises her to buy it and vice versa. We refer to this as dependent trading. Dependent trade behavior bears the potential of informational inefficiencies, as the fundamentally linked private information does not reach the market any more. If many investors act against their privately held information prices may move away from fundamentals.

As a consequence, we want to provide a statistic that allows us to test whether actual trading depicted in Figure 3.1 (a) deviates from the unobserved independent trading in Figure 3.1 (b). If there is dependent trading, the statistic should measure the extent to which traders coordinate.[7]

We combine the concepts of dependent and independent trading with the ideas of the literature on coordinated trading based on transaction data. This strand of the literature was shaped by the seminal work of Lakonishok et al. (1992). Building on their ideas, we define $h := |br - \widetilde{br}|$ as a measure for deviation from independent trading. br and $\widetilde{br}$ are the number of buys over the number of trades (buy ratios) under observed and the hypothesized independent trading, respectively. The measure h captures, to what degree observed trading decisions deviate from what they should have been under independent trading.

[7]Note that models such as Avery and Zemsky (1998) and Park and Sabourian (2011) by no means imply that herding or contrarianism always lead to price distortions. In that sense testing for deviations from independent trading can only be a test on a necessary condition for price-distorting investor behavior.

As we make explicit how we model independent and dependent trade data as well as buy ratios in the next subsections, we shall highlight the intuitive appeal of h in our model context.

3.2.2 Modeling Dependent and Independent Trade Data

In line with market microstructure theory we focus on the initiator of a trade when modeling trade data. Normally a trade is executed by at least two counterparties: the bespoken initiator or active trader and one or more passive traders. As long as there is enough liquidity in the market, however, passive trades are not likely to cause systematic price distortions. In the works of e.g. Park and Sabourian (2011) or Avery and Zemsky (1998), the passive trade side is represented by a market maker who guarantees liquidity as long as there is noise trading. Yet, this market maker does not actively influence stock prices or trading outcomes. The mechanism is similar in order driven markets, where the passive side of a trade typically consists of limit orders in an order book. As long as the order book is deep enough, i.e. the stock is traded liquidly, there is no reason to assume that limit orders have systematic price effects. Dorn et al. (2008) and Barber et al. (2009b) even argue that considering limit orders tends to bias the assessment of investor coordination. Hence, we exclude the passive trade side from our model.

We assume that each stock $i \in \mathcal{I}$ is traded actively T_i times during the course of a day. Each active trade can either be a buy or a sell. Hence, we think of a trade as a bernoulli distributed random variable X^i. We can view it's success probability π_i as the traders' propensity to buy that particular stock.

Theory implies that π_i is related to the asset's fundamental V_i and the corresponding distribution of private information S as well as many other parameters depending on the investor's objective function, compare e.g. Froot et al. (1992), Graham (1999) and Dasgupta et al. (2011). In this paper, however, we do not model this relationship explicitly, since our focus is to *detect* investor coordination rather than to explain where it stems from.

If a trade is carried out independently, we have $\pi_i = \widetilde{br}_i$. Now assume that we have τ trades executed under independent trading. The number of buys among those τ trades, $B_i^\tau := \sum_{t=1}^{\tau} \mathbb{1}_{\{X_t^i=\text{buy}\}}$, then follows a Binomial distribution of length τ with success probability $\widetilde{br}_i$.

During actual trading the investors' buy propensity may deviate from the one under independent trading. Hence, under actual trading, we set $\pi_i = \widetilde{br}_i + \varepsilon_i$, where $\varepsilon_i \in [-\widetilde{br}_i; 1 - \widetilde{br}_i]$. If trades are executed in a dependent fashion, then $\varepsilon_i \neq 0$. If trades are carried out independently, then $\varepsilon_i = 0$. We assume that given there are $T_i - \tau$ potentially dependent trades, then the number of buys under dependent trading $B_i^{T_i-\tau} \sim \text{Bino}(T - \tau_i, \widetilde{br}_i + \varepsilon_i)$.[8]

We now turn to the question whether the measure h from the previous section accurately reflects investor coordination in this model context. If $T_i - \tau$ is sufficiently large, then the observed buy ratio is

$$br_i = \frac{B_i^{T_i-\tau}}{T_i - \tau} \approx \pi_i$$

by the law of large numbers. Thus, for our previously defined measure of investor coordination, we have

$$h = |br_i - \widetilde{br}_i| \approx |\pi_i - \widetilde{br}_i| = |\varepsilon_i|.$$

Hence, h is a reasonable approximation of the extent to which investors systematically deviate from independent trading in our model.[9]

[8] This is a simplifying assumption since the buy probability of a particular trade can change multiple times during a trading day. Incorporating this feature into the model, however, complicates it without providing additional insights.

[9] Since br_i may be different from $\widetilde{br}_i$ due to random fluctuations, h will generally be positive, even if $\varepsilon_i = 0$. We need to account for this fact as we operationalize h in Section 3.3.

3.2.3 Using Investor Buy Ratios to Distinguish Between Herding and Contrarianism

The question remains whether the observed buy ratio br_i and the buy ratio under independent trading $\widetilde{br}_i$ tell us something more about the nature of the investor coordination.

The theoretical herding literature classifies two forms of dependent investor behavior. First, *herding* is defined as a trader's decision to act against her private information and to *follow the crowd* instead. Second, if a trader decides to ignore her private information and acts *against the crowd*, this is called *contrarianism.*[10]

How does such behavior affect br_i relative to $\widetilde{br}_i$? Since herders are crowd-followers, we expect the observed buy ratio to be more extreme than the buy ratio under independent trading, i.e. $|br_i - 0.5| > |\widetilde{br}_i - 0.5|$. As a numerical example, consider $\widetilde{br}_i = 0.6$. That is, under independent trading 40 out of 100 traders sell the stock and 60 out of 100 traders buy the stock. If on average 5 out of 40 sellers decide to follow the majority of the traders into buying the stock, then $br_i \approx 0.65$ and hence $|br_i - 0.5| = 0.15 > 0.1 = |\widetilde{br}_i - 0.5|$.

Conversely, as contrarians are leaning against the crowd, we expect the observed buy ratio to be less extreme than the one under independent trading, i.e. $|br_i - 0.5| < |\widetilde{br}_i - 0.5|$. Consider, for instance, traders who - based on their private information S - value the asset at some fixed price p^* throughout the whole trading day. That is, these traders completely discount the possibility that recent price movements or trade decisions of other investors have informational value. Assume that the opening price $p_0 < p^*$. Hence, under independent trading these traders buy the asset. Yet, as soon as a buy side majority of traders drives the price above p^*, contrarians divest the asset, deviating from their independent trading decision and drawing br_i towards 0.5.

Now that we have developed an understanding of independent and dependent trading and how the buy ratio statistic is related to herding and contrarianism, i.e. different forms of dependent trading, we address the question of how to operationalize our measure for coordinated trading $h := |br - \widetilde{br}|$. Key challenge

[10] Compare e.g. Park and Sabourian (2011) for formal definitions.

of this endeavor is the retrieval of the independent buy-ratio, $\widetilde{br}$, as private opinions are not observable.

3.3 When Investors Coordinate - Detecting Deviations From Independent Trading

A natural first step is to consider the prominent LSV measure for coordinated trading introduced by Lakonishok et al. (1992) as a candidate for h.

3.3.1 The LSV Measure

The LSV measure has already been widely used as a statistic for coordinated trading and herding among investors, compare e.g. Wermers (1999), Dorn et al. (2008) and Kremer and Nautz (2013a) to name but a few. The LSV measure can be calculated for any desired time horizon — in our case a day. For each stock i in the considered cross-section $\mathcal{I}$, the daily stock-specific LSV measure is given by

$$LSV_i = |br_i - p| - AF_i^{LSV}, \tag{3.1}$$

where $br_i = \#\,buys_i/\#\,trades_i = B_i/T_i$ is the observed buy ratio in stock i and p is the expected proportion of traders buying, where p is estimated by $\hat{p} = \sum_i B_i / \sum_i T_i$. AF_i^{LSV} is an adjustment factor to account for random deviations of br_i from p and is given by

$$\begin{aligned} AF_i^{LSV} &:= \mathbb{E}_{\zeta_k} \left|\frac{k}{T_i} - p\right| \\ &= \sum_{k=0}^{T_i} \zeta(k|T_i, p) \left|\frac{k}{T_i} - p\right|, \end{aligned} \tag{3.2}$$

where $\zeta(k|\cdot)$ is the Binomial distribution.[11] The design of AF_i essentially entails the view that the number of buys under independent trading, B^τ, is binomially distributed with success probability (buy propensity) p, compare Wermers (1999).

[11] A closed form solution for the expectation is given by $\frac{1}{T_i}\mathbb{E}_{\zeta_k}|k - pT_i| = \frac{1}{T_i} 2(1-p)^{T_i - \lfloor T_i p \rfloor} p^{\lfloor T_i p \rfloor + 1} (\lfloor T_i p \rfloor + 1) \binom{T_i}{\lfloor T_i p \rfloor + 1}$ (see Diaconis and Zabell 1991).

At first glance, this seems to be very much line with the distribution of the buys under independent trading in our model, see Section 3.2.2. A closer look, however, reveals that the LSV implied distributional assumption under independent trading is far more restrictive than in our model framework. While our model allows for stock specific buy-ratios under independent trading $\widetilde{br}_i$, the LSV approach implicitly assumes that p is the same across stocks.

This appears to be somewhat rigid to be applicable to real-world trading contexts. Our model as well as practical intuition stipulate that information signals S_t and, thus, trade decisions under independent trading are subject to stock-specific variables V_i. Idiosyncratic determinants of say, an automotive stock and the stock of a financial institution may be very different on any given trading day.[12] This implies, that we should expect idiosyncratic buy ratios under independent trading $\widetilde{br}_i$ for different stocks. Approximating the $\widetilde{br}_i$ through a single p will, hence, prove inaccurate. More precisely, the LSV measure will tend to overestimate investor coordination as it registers excess dispersion of observed buy ratios due to stock idiosyncrasies as deviations from independent trading.

A second issue associated with the LSV approach is that $\hat{p}$ is based on all trades of the respective time interval of interest. The estimation, therefore, includes trades that may have deviated from the independent trading decision. In order for $\hat{p}$ not to be biased into the direction of the non-independent trades, those trades need to cancel each other out across the cross-section, i.e. $\sum_i \varepsilon_i = 0$. Yet, investor coordination may be aligned across stocks. Consider our model for two stocks and assume that $\widetilde{br}_1 = \widetilde{br}_2 = 0.5$ and $br_1 = br_2 = 0.7$, then there is a strong deviation from independent trading. Yet, calculating the respective LSV measures yields $LSV_i = -AF_i < 0$.[13] We cannot assume in general that the stock-specific deviations ε_i cancel each other out in the cross-section. As the example illustrates, the LSV measure tends to underestimate investor coordination if $\sum_i \varepsilon_i \neq 0$.

[12]Even more so during the recent financial crisis which is investigated in Paper 4.

[13]Lakonishok et al. (1992) themselves state that their measure does not capture the case when investors enter or leave the market as a whole.

This casts considerable doubt on whether the LSV measure reliably detects investor coordination or herding.

3.3.2 A New Measure for Investor Coordination

Based on the previous discussion, we propose to modify the LSV approach in the following two ways. (1) In line with our model, stock-specific buy ratios under independent trading should be taken into account. (2) The estimation of the independent buy-ratios should be based only on those trades which are indeed likely to be independent.

Modification (1) leads us to the following measure

$$\widetilde{LSV}_i = |br_i - \widetilde{br}_i| - AF_i^{\widetilde{LSV}}, \tag{3.3}$$

where br_i is the observed buy ratio in stock i as before and $\widetilde{br}_i$ is the stock-specific true unknown buy ratio under independent trading. $AF_i^{\widetilde{LSV}}$ is the corresponding adjustment factor and is given by

$$AF_i^{\widetilde{LSV}} := \mathbb{E}_{\zeta_k}|\frac{k}{T_i} - \widetilde{br}_i| = \sum_{k=0}^{T_i} \zeta(k|T_i, \widetilde{br}_i)|\frac{k}{T_i} - \widetilde{br}_i|. \tag{3.4}$$

$\zeta(k|\cdot)$ again stands for the Binomial distribution. Since we do not observe $\widetilde{br}_i$, it remains to be shown how we can estimate the $\widetilde{br}_i$ subject to modification (2). That is, we need to develop a view on which trades are likely to be independent.

Market microstructure theory tells us that at the outset of the trading process, deviations from independent trading are less likely to occur (see Proposition 7 in Avery and Zemsky (1998)). For a trader to change her opinion, there has to be a sufficient amount of information that she can infer from the preceding trades. This is unlikely to be the case if only a few trades have been executed. In the extreme, starting with the first trade, no other trade could have been observed that could have influenced the decision of the first trader.[14]

[14] The first trade is always the one that the researcher defines to be the first trade. Anything that happened before that trade will enter either S_t or p_0 in Figure 3.1 and, thus, is part of the traders' prior, compare again Park and Sabourian (2011).

Based on these insights we formulate the following

Assumption 3.1. *The first few τ_i trades in any stock i on a particular day, are carried out independently.*

In our model context, this means that given the number of trades in stock i, T_i, on a particular day, there exists a τ_i, $1 \leq \tau_i \leq T_i$, such that the number of buys in stock i until τ_i, $B_i^{\tau_i}|_{\widetilde{br}_i} \sim \text{Bino}(\tau_i, \widetilde{br}_i)$.

A naive estimator for $\widetilde{br}_i$ is e.g. $B_i^{\tau_i}/\tau_i$. According to Assumption 3.1, however, τ_i should be chosen as low as possible. Moreover, we cannot utilize the cross-section to increase the number of observations to estimate $\widetilde{br}_i$ as the independent buy-ratios are generally different across stocks. Yet, if based on small τ_i, estimators such as $B_i^{\tau_i}/\tau_i$ are too noisy to conduct meaningful inference on them. Consequently, $\widetilde{LSV}_i$ from Equation (3.3) cannot be estimated directly.

We will show, however, that the *distribution* of $\widetilde{br}_i$ can be estimated reliably under the following

Assumption 3.2. *On each day, the buy ratios under independent trading $\widetilde{br}_i$ are iid beta distributed, i.e. $\widetilde{br}_i \overset{iid}{\sim} \text{Beta}(\alpha, \beta)$ $\forall i \in \mathcal{I}$.*[15]

This assumption is equivalent to the number of buys under independent trading being iid Beta-Binomially distributed, i.e. $B_i^{\tau_i} \overset{iid}{\sim} \text{BetaBino}(\tau_i, \alpha, \beta)$ $\forall i \in \mathcal{I}$. We test whether the distributional assumption of $B_i^{\tau_i}$ implied by Assumption 3.2 holds for real-world trade data in Paper 4. The test results provide strong evidence that Assumption 3.2 is reasonable. Correspondingly, the test rejects the LSV implied assumption that $\widetilde{br}_i \equiv p$.[16]

Assumptions 3.1 and 3.2 together prompt us to modify $\widetilde{LSV}_i$ further to obtain

[15] Details on the Beta distribution are provided in the Appendix.

[16] We would like to stress that our method does not depend on any specific distributional assumption for the $\widetilde{br}_i$. If the data reject parametric models, one can describe the distribution of $\widetilde{br}_i$ by a non-parametric density estimator.

our new measure for investor coordination — the *expected* deviation from independent trading. It is defined by

$$\begin{aligned} H_i &= \mathbb{E}_{f_p}|br_i - p| - \tilde{AF}_i \\ &= \int_0^1 f(p|\alpha,\beta)|br_i - p|\mathrm{d}p - \tilde{AF}_i, \end{aligned} \tag{3.5}$$

where $br_i = B_i/T_i$ is the observed buy ratio as above and $f(\cdot|\alpha,\beta)$ is the Beta density.[17] $\tilde{AF}_i$ is an adjustment factor to center the expectation of H_i over zero if in fact all trades were carried out as under independent trading in line with Assumption 3.2. It is given by

$$\begin{aligned} \tilde{AF}_i &= \mathbb{E}_{f_p}\mathbb{E}_{g_k}|\frac{k}{T_i} - p| \\ &= \int_0^1 f(p|\alpha,\beta)\sum_k^{T_i} g(k|T_i,\alpha,\beta)|\frac{k}{T_i} - p|\mathrm{d}p \\ &= \int_0^1 f(p|\alpha,\beta)\int_0^1 f(\tilde{p}|\alpha,\beta)\sum_{k=0}^{T_i}\binom{T_i}{k}\tilde{p}^k(1-\tilde{p})^{T_i-k}|\frac{k}{T_i} - p|\mathrm{d}\tilde{p}\,\mathrm{d}p, \end{aligned} \tag{3.6}$$

where $g(k|\cdot)$ is the Beta-Binomial distribution. Note that $\tilde{AF}_i$ corrects for *two* sources of randomness. First, as for the LSV measure, we observe only a finite number of trades. That is, even if each single trade has been drawn from a Bernoulli distribution with the success probability of a buy equal to the independent buy-ratio, there is a positive chance that the observed B_i/T_i is not equal to the independent buy-ratio. In addition, the true independent buy-ratio $\widetilde{br}_i$ may deviate significantly from any $p \in [0;1]$, because according to Assumption 3.2, it's value is itself a random variable drawn from the Beta distribution.

Since we do not know the parameters of the independent buy ratio distribution, we need to estimate them. This leads to the question whether we can estimate our new measure H_i consistently and without bias even for finite cross-section sizes I.

[17]The Beta density is given by $p^{\alpha-1}(1-p)^{\beta-1}/\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\mathrm{d}u$ with $\alpha,\beta > 0$.

3.3.3 Estimating H_i

To estimate H_i, we have to estimate the distribution of the independent buy-ratios, that is, α and β. Since according to Assumption 3.1 and 3.2, $B_i^{\tau_i} \sim$ Beta-Bino(τ_i, α, β), we can obtain estimators for α and β via maximum likelihood (ML). More precisely, we get $\hat{\alpha}, \hat{\beta} = \arg\max_{\alpha,\beta} \mathscr{L}(B_i^{\tau_i}|\tau_i, \alpha, \beta)$, where $\mathscr{L}(\cdot)$ is the log-likelihood function on data $\{(B_i^{\tau_i}, \tau_i)\}$. A closed-form solution to the maximization problem does not exists, but one can use numerical methods such as Newton-Raphson.[18] Replacing α and β in equations (3.5) and (3.6) by their ML estimates yields $\hat{H}_i^I$, which has the following properties:

Proposition 3.1. *Let I be the number of stocks in the considered cross-section, then $\hat{H}_i^I$ consistently estimates H_i, i.e.* $\underset{I\to\infty}{plim}\ \hat{H}_i^I = H_i$.

Proof: The maximum likelihood estimators, $\hat{\alpha}, \hat{\beta}$, of the Beta-Binomial distribution are consistent estimators (Garren 2004, p. 240). Since moreover, $E_{\hat{f}_p}|br_i - p|$ and $\hat{\tilde{AF}}_i = \mathbb{E}_{\hat{f}_p}\mathbb{E}_{\hat{g}_k}|\frac{k}{T_i} - p|$ are both compositions of continuous functions in $(\hat{\alpha}, \hat{\beta})$, the continuous mapping theorem implies, that both quantities converge in probability to $E_{f_p}|br_i - p|$ and $\tilde{AF}_i$ respectively as $I \to \infty$. Hence, we have consistency of $\hat{H}_i^I$.

□

A similar consistency result can be derived if one does not want to restrict oneself to a particular family of distributions and uses kernel density estimation instead. A multitude of consistency results is available for kernel density estimators, see e.g. Parzen (1958), Silverman (1978) and Epanechnikov (2006). With such results at our disposal, Proposition 3.1 can be restated if the respective kernel density estimators are employed.

Also note that Proposition 3.1 also holds for the cross-sectional average of the $\hat{H}_i^I$, i.e. $\hat{\bar{H}} = \sum_{i=1}^{I} \hat{H}_i^I$.[19]

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[18] We use the fixed-point iteration algorithm of Minka (2012) with a maximal number of iterations equal to 3000.

[19] For the remainder of this paper we drop the index I for any estimator for notational convenience.

Putting our method into action requires a choice of τ_i. Pointing to the precise moment when traders start to go against their private information amounts to uncover the latent private information itself. In line with Assumption 3.1, however, a conservatively small choice of τ_i, but large enough for the Beta-Binomial estimation to make any sense should suffice.[20] By means of simulation, we find that $\tau_i = 10$ is already large enough to provide good estimates.

For $\tau_i = 10$, we find that $\hat{H}_i$ is unbiased even for finite I, i.e. $(\sum_{i=1}^{I} \hat{H}_i^I - H_i) \approx 0$ for $I < \infty$. This property has been shown by means of numerical simulations for $I \geq 75$. The simulation results are provided in the Appendix of this paper.

Now that we have an operational alternative to the LSV approach to measure deviations from independent trading, we want to provide a more detailed comparison of the two approaches.

3.4 LSV Versus H — a Comparison

The literature using the LSV measure typically interprets their findings based on the cross-sectional mean $\overline{LSV} = \sum_i LSV_i/I$ rather than the stock specific measures discussed in the previous section, compare e.g. Wermers (1999) or Dorn et al. (2008).[21] The reason for this is that the stock-specific measures LSV_i may become high (or low) due to randomness only. The same is true for our measure.[22] This section, thus, focuses on the comparison of the cross-sectional average measures, i.e. $\bar{H} = \sum_i H_i/I$ and $\overline{LSV}$.

[20] The precise meaning of "small", hereby, depends on the empirical context regarding, e.g. sampling frequency and the definition of a trade. One may be interested in counting each transaction as a single trade, others may be interested in aggregating single transactions into the orders that induced them, or even aggregating transactions of single traders into their net-positions over a certain time interval. Those choices affect the amount of data available at any point after the start of trading and, thus, after information starts to accumulate in the market.

[21] In fact, the corresponding works often not only average over the cross-section of stocks but also over time.

[22] See Section 3.6.B in the Appendix for a detailed discussion of this matter.

Our analysis is conducted in two steps. First, we show that our approach is a generalization of the LSV approach. Second, we quantify the differences between the two measurement approaches through a simulation study.

3.4.1 Our Approach is a Generalization of the LSV Approach

This result is formalized in the following

Proposition 3.2. *If the buy ratios under independent trading are the same for all stocks and deviations under dependent trading cancel each other out over the considered cross-section of stocks, then our approach and the LSV approach asymptotically render the same degree of dependent trading, i.e.:*
If $\widetilde{br}_i \equiv p^* \ \forall i \in \mathcal{I}$ *and* $\sum_{i=1}^{I} \varepsilon_i = 0$*, then* $\underset{I\to\infty}{plim}\ \hat{H}_i = \underset{I\to\infty}{plim}\ \widehat{LSV}_i$*, where* $\widehat{LSV}_i$ *is the estimated LSV measure.*

Proof: If $\widetilde{br}_i \equiv p^* \ \forall i \in \mathcal{I}$, then in distributional terms, we have $\widetilde{br}_i \sim \delta_{p^*}$ iid, where $\delta.$ is the dirac-measure. Noting that $\lim_{\alpha,\beta\to\infty} Beta(\alpha,\beta) = \delta.$ and re-invoking the consistency of the maximum-likelihood estimator $\arg\max_{\alpha,\beta} \mathscr{L}(B_i^{\tau_i}|\tau_i,\alpha,\beta) = (\hat{\alpha},\hat{\beta})$, we infer that $\hat{\alpha},\hat{\beta} \underset{I\to\infty}{\to} \infty$, and

$$\lim_{I\to\infty} \frac{\hat{\alpha}}{\hat{\alpha}+\hat{\beta}} = \lim_{I\to\infty} \frac{\sum_{i=1}^{I} B_i^{\tau_i}}{\sum_{i=1}^{I} \tau_i} = p^*.$$

As a consequence, we have that $f_p(\hat{\alpha},\hat{\beta}) \underset{I\to\infty}{\to} \delta_{p^*}$ and, hence,

$$\begin{aligned}\underset{I\to\infty}{plim}\ \hat{H}_i &= \mathbb{E}_{\hat{f}_p}|br_i - p| - \hat{\hat{AF}}_i \\ &= |br_i - p^*| - \sum_{k=0}^{T_i} \binom{T_i}{k} (p^*)^k (1-p^*)^{T_i-k} \left|\frac{k}{T_i} - p^*\right|. \qquad (3.7)\end{aligned}$$

Noting, that $\sum_{i=1}^{I} \varepsilon_i = 0$ implies, that $p^* = \underset{I\to\infty}{plim} \frac{\sum_i B_i}{\sum_i T_i}$, we conclude that the last line of Equation (3.7) equals $\underset{I\to\infty}{plim}\ \widehat{LSV}_i$, which is the desired result.

□

Proposition 3.2 states that our measure is equal to the LSV measure if the LSV assumptions hold. The reverse of the statement is also true for most of the cases.[23] If the LSV assumptions do not hold, our measure H is generally very different from the LSV measure.

Before we quantify these differences in a simulation study, however, two additional remarks are in order. First, the result of Proposition 3.2 generalizes to any distributional assumption for the $\widetilde{br}_i$, as long as we can estimate the distribution consistently. Second, Proposition 3.2 also holds for the cross-sectional averages of $\hat{H}_i$ and $\widehat{LSV}_i$.[24]

3.4.2 Quantifying the Differences Between our Measure H and the LSV Approach - A Simulation Study

3.4.2.1 Simulation Setup

We simulate trade data based on our trading model from Section 3.2. The simulation is designed to resemble the real-world conditions we face in Paper 4. We simulate a basket of 75 stocks over $D = 250$ trading days (=1 year). In line with Assumption 3.1, we assume that the first $\tau = 10$ trades are always conducted independently.

The stock-specific buy ratios under independent trading are $\widetilde{br}_i \overset{iid}{\sim} Beta(\alpha, \beta)$ with mean equal to 0.5, i.e. $\alpha = \beta$. For each trading day α and β are drawn from the uniform distribution $U[2; 10]$.[25] The number of observed trades T_i is randomly drawn from $\{50; 51; \ldots; 500\}$ for each stock and day reflecting that stocks are traded with (very) different intensity.

[23] One could, however, construct unlikely scenarios, where the reverse is not true. To see this, consider some T_i and $\alpha, \beta < \infty$. H_i attains it's minimum if $br_i = Median(\widetilde{br}_i)$. This minimum is less than minus the adjustment factor of the LSV measure, i.e. $< -AF_i$. Now note that $LSV_i = -AF_i$ if for any $c \in (0; 1)$, the observed buy ratios are $br_i \equiv c$ for all $i \in \mathcal{I}$. Moreover, $\exists br_i \in (0; 1)$ such that $H_i > 0$. Since H_i is also continuous in br_i, the intermediate value theorem implies that $\exists c^* \in (0; 1)$ such that $H_i = -AF_i$ if $br_i = c^*$ and, thus, $H_i = LSV_i$ even though the conditions of Proposition 3.2 are not met.

[24] A version of Proposition 3.2 can be formulated for the theoretical measures H_i and LSV_i and the respective cross-sectional averages as well.

[25] These parameterizations are in line with the empirical findings in Paper 4. We emphasize that the Beta distribution is supported by the real-world data. Moreover, this distribution is only moderately skewed ($\alpha \approx \beta$), yet highly disperse (α, β small) on most of the days. Independent trade behavior varies between different days for each stock.

For each day, actual trading after trade number 10 is either dependent for *all* stocks with 0.5 probability or independent for *all* stocks with 0.5 probability. This is done for illustrative purposes to avoid less interesting scenarios of low average deviations from independent trading. Those scenarios are considered in separate simulation setups, see below.

We conduct the described simulation for three stylized panels of dependent investor behavior - (A) herding, (B) shift in mean and (C) contrarianism.[26]

Panel (A), herding, is simulated by choosing the deviation from independent trading ε_i from $\min\{1-\widetilde{br}_i; U\,[0.2; 0.5)\}$ and $\max\{-\widetilde{br}_i; U\,(-0.5; -0.2]\}$ with 0.5 probability respectively given that there are deviations from independent trading.[27] Since $br_i \sim \widetilde{br}_i + \varepsilon_i$, this implies that the distribution of the br_i is more disperse than the distribution of the $\widetilde{br}_i$. In other words, we observe on average more extreme buy ratios than we would expect under independent trading. As outlined in the modeling section this is consistent with crowd-following behavior and, thus, herding.

We simulate Panel (B), shift in mean, in the same way with the only difference that $\varepsilon_i > 0$, i.e. $\varepsilon_i = \min\{1-\widetilde{br}_i; U\,[0.2; 0.5)\}$, now occurs with 0.8 probability given that there is a deviation from independent trading. Consequently, $0 > \varepsilon_i = \max\{-\widetilde{br}_i; U\,(-0.5; -0.2]\}$ only happens with 0.2 probability. Since $br_i \sim \widetilde{br}_i + \varepsilon_i$, the observed buy ratios are on average higher than the buy ratios under independent trading, i.e. $E[br_i] > E[\widetilde{br}_i]$.[28] By the same line of reasoning as before one could argue that such a deviation from independent trading is again due to investor herding. A broadly, cross-sectionally aligned change in trade behavior is, however, also an aspect in favor of market-wide effects that have changed the trading environment altogether. For instance, an unexpected decrease of interest rates or a terror attack may dramatically alter the traders' information structure S or even shift the stock's fundamental value V, which will lead to different trade behavior even if traders are not influenced by the ac-

[26] The different setups are also illustrated and discussed in further detail in Figure 3.5 in the Appendix.

[27] U again denotes the uniform distribution.

[28] Note that the dispersion of the br_i is also different from what is expected under independent trading. The dominating effect, however, is the shift in mean.

tions of others or recent stock price movements. Whether a shift in mean in fact constitutes dependent trading can only be revealed through additional analyses.

Finally, Panel (C), contrarianism, is simulated by setting $\varepsilon_i = (0.5 - \widetilde{br}_i) \times U\,[0.2; 0.5)$. This ensures that the distribution of the $\widetilde{br}_i$ constitutes a mean-preserving spread of the distribution of the br_i. More precisely, the observed buy ratios cluster much closer around 0.5 than the buy ratios under independent trading. In the model section, we have argued that this is consistent with some of the traders acting against the crowd and, hence, contrarianism.

In line with the literature, we evaluate the performance of the cross-sectional mean measures $\widehat{\overline{LSV}} = \sum_i \widehat{LSV}_i / I$ and $\hat{\bar{H}} = \sum_i \hat{H}_i / I$. They are calculated for each trading day in each panel based on the simulated trade data. For each panel, we analyze whether the measures separate the particular form of dependent trading from the null of independent trading. To gauge whether the measures pick up the extent of dependent trading, we analyze the correlation between the respective measure and the true deviation from independent trading, given there is dependent trading. Since we know the true $\widetilde{br}_i$ in our simualtion, we can use $\widetilde{\overline{LSV}} = \sum_{i=1}^{I} \widetilde{LSV}_i / I$, where $\widetilde{LSV}_i$ is from Equation (3.3), to proxy true deviation from independent trading.[29]

Further simulation setups include

- low deviations from independent trading, i.e. $\sum_{i=1}^{I} |\varepsilon_i|$ small,
- larger cross-sections, i.e. $I \in \{250, 500, 1000, 2000, 4000, 8000, 16000\}$,
- different distributional assumptions for $\widetilde{br}_i$, i.e.
  - skewed distributions, that is, $\alpha \neq \beta$,
  - less disperse distributions, that is, $\alpha, \beta > 20$,

[29] Note that as a proxy for true deviations from independent trading, $\widetilde{\overline{LSV}}$ is slightly downward biased due its adjustment factor. At the given degrees of ε and the number of observed trades per stock and day, however, this downward bias is vanishingly small. Indeed, we find correlations of > 0.99 between $\widetilde{\overline{LSV}}$ and the share of dependent trades.

- stronger shift in mean, that is, the probability that the observed buy ratio increases is chosen > 0.8.

They render qualitative results that are similar to the main study. A brief discussion of the most important particularities of the respective setups is provided in Section 3.4.2.3 after the main study the following section.

3.4.2.2 Simulation Results - Main Study

Table 3.1 reports the results of our simulation. Key insights regarding the LSV measure can be derived from columns 1 and 7. Column 1 shows, that the estimated cross-sectional mean LSV measure, $\widehat{LSV}$, is not centered over 0 under the null of independent trading. Column 7 shows, that the $\widehat{LSV}$ is not correlated with the degree of true deviation from independent trading. Both insights hold for all forms of dependent trading (i.e. Panels A-C). Hence, under the given assumptions the LSV measure does not reliably detect the days, where deviations from independent trading occur.[30]

The fact, that$\widehat{LSV}$ is always positive casts serious doubts on the validity of the findings published in the literature that uses the LSV measure. Our simulation study shows that a persistently positive $\widehat{LSV}$ is likely to be a statistical artifact of the violation of the LSV assumptions.

At face value the LSV measure compares the observed deviation from independent trading $|br_i - p|$ to the to be expected one due to random variations governed by a Binomial distribution, see Equation (3.2). Any excess dispersion of the observed buy ratios br_i that cannot be explained by the Binomial distribution is registered as investor coordination via positive $\widehat{LSV}$. In the present simulation study, however, we know by design that this excess dispersion is to a strong degree due to the idiosyncratic $\widetilde{br}_i$ (*solely* due to the idiosyncratic $\widetilde{br}_i$ under independent trading).

The performance of our measure is considerably better. A first striking finding

[30] Despite the fact that the LSV measure is significantly larger under dependent trading than under independent trading in Panels A and B we cannot derive reliable cut-off points to distinguish dependent from independent trading. This is because the realizations of the LSV vary under dependent as well as independent trading for different simulation setups.

Panel A - Herding	**Indep. Trading** Mean	Std	95%	**Dep. Trading** Mean	Std	5%	**Corr with True Dev.** ($\widetilde{\widetilde{LSV}}$)
$\widehat{LSV}$	0.097	0.028	0.148	0.194	0.018	0.168	−0.058 (0.497)
$\hat{\hat{H}}$	0.003	0.014	0.028	0.085	0.030	0.035	0.276 (0.000)
# Observations	109			141			141
Panel B - Shift in mean	**Indep. Trading** Mean	Std	95%	**Dep. Trading** Mean	Std	5%	**Corr with True Dev.** ($\widetilde{\widetilde{LSV}}$)
$\widehat{LSV}$	0.092	0.026	0.145	0.180	0.019	0.153	0.123 (0.169)
$\hat{\hat{H}}$	0.005	0.015	0.028	0.082	0.025	0.038	0.421 (0.000)
# Observations	124			126			126
Panel C - Contrarianism	**Indep. Trading** Mean	Std	5%	**Dep. Trading** Mean	Std	95%	**Corr with True Dev.** ($\widetilde{\widetilde{LSV}}$)
$\widehat{LSV}$	0.083	0.028	0.051	0.085	0.029	0.136	−0.054 (0.556)
$\hat{\hat{H}}$	-0.003	0.013	-0.021	-0.003	0.015	0.023	−0.472 (0.000)
# Observations	121			129			129

Table 3.1: Realizations of $\hat{\hat{H}}$ and $\widehat{LSV}$ for simulated trade data

Notes: This table reports summary statistics for our measures for coordinated trading $\hat{\hat{H}}$ and $\widehat{LSV}$. We simulate three stylized forms of dependent investor behavior and report the results in three separate panels - (A) herding, (B) shift in mean and (C) contrarianism. For each panel, columns 1 to 3 report mean, standard deviation and relevant cut-offs of the empirical distributions of the measures under independent trading. Columns 4 to 6 do so under the panel-specific type of dependent trading. Finally, column 7 reports the correlation between the measure and true deviation from independent trading, given that trading is dependent. P-values for significance of the correlation are reported in parentheses. Deviation from independent trading is approximated by $\widetilde{\widetilde{LSV}} = \sum_{i=1}^{I} \widetilde{LSV}_i / I$, where $\widetilde{LSV}_i$ is from Equation (3.3).

is that the cross-sectional mean estimate of our measure, $\hat{\hat{H}}$, is positive under herding and shifts in mean (Panels A and B). It tends to become negative under contrarianism (Panel C).[31]

Columns 1 and 7 of Table 3.1 show that $\hat{\hat{H}}$ is centered at 0 under the null and is significantly related to true deviations from independent trading. We note that in Panel C (contrarianism), the correlation with true deviation is negative. This is intuitive, since the (absolute) deviation from independent trading is non-negative by definition. Since contrarianism becomes manifest in values of $\hat{\hat{H}} < 0$, we expect $\hat{\hat{H}}$ to become more negative, the more pronounced the contrarianism, i.e. the higher the (absolute) deviation from independent trading.

We also observe that the correlation between $\hat{\hat{H}}$ and true deviation from independent trading is only half as strong in the Herding Panel as in Panels B and C. This is due to the fact that some stock-specific deviations from independent trading are more consistent with contrarian tendencies. This dilutes the measured cross-sectional average deviation to some extend and makes it prone to noise-induced distortions (if I is small). If we had ensured stock-wise herding, that is if we had imposed buy (sell) deviations if and only if $\widetilde{br}_i > 0.5$ ($\widetilde{br}_i < 0.5$) for all i, then the correlation in Panel A would be significantly higher.

Comparing columns 1-3 with 4-6 in Panels A and B, we see that $\hat{\hat{H}}$ does well in separating days with independent trading from those with dependent trading. In Panel C, however, this is not the case. This is due to the fact, that our simulation design implies much smaller deviations from independent trading in Panel C (0.005 on average) than in Panels A and B (0.147 on average). If low numbers of stocks in the cross-section (e.g. $I = 75$ as in Table 3.1) meet low degrees of deviation from independent trading, then $\hat{\hat{H}}$ has difficulties to distinguish between systematic deviation from independent trading and noise. This is not of great concern, however, since minor deviations from independent trading should bear little potential of distorting prices. Moreover, if the cross-section of stocks is increased to $\geq$ 500, $\hat{\hat{H}}$ separates dependent and independent trading days much better even if the true deviation from

[31] See Section 3.6.B in the Appendix of this paper for a more detailed explanation of this finding.

independent trading is as low as in Panel C.[32]

We conclude that under the specified assumptions of our simulation study our approach is much better suited to identify deviations from independent trading than the LSV approach. We stress that we obtain such favorable results using $\hat{\hat{H}}$ despite the fact that the distribution under independent trading is estimated from only 10 trades.

3.4.2.3 Qualitative Differences for Other Simulation Setups

Lower Average Deviation from Independent Trading: In line with the results of Panel C in the previous section, lower average deviation from independent trading impedes our measure H from separating dependent from independent trading, if I is also low. The correlation between $\hat{\hat{H}}$ and true deviation from independent trading is also lower or even insignificant, if the average deviation from independent trading is very low. The LSV measure performs comparable to Panel C from the previous section.

Different Cross-Section Sizes I: Both measures' capability to separate dependent from independent trading increases, as the cross-section I increases. If $I \geq 500$, even small deviations from independent trading are detected and both measures are significantly correlated with the true deviation of independent trading, given that there is dependent trading. Yet, regardless of I, the LSV measure remains centered over values $c > 0$ under independent trading.

Different Distributional Assumptions: The center of $\widehat{LSV}$ under independent trading shifts closer to 0 as α and β are chosen larger, i.e. if the dispersion of the buy ratios under independent trading becomes smaller. In line with our theoretical discussion, the LSV measure will only center over 0 under independent trading as α and β go to ∞.

If the distribution of the $\widetilde{br}_i$ is skewed, i.e. if $\alpha \neq \beta$, then the performance of our measure is slightly impaired while the correlation between $\widehat{LSV}$ and true

[32]Note that the cross-sections of stocks considered in the literature frequently exceed 1000 stocks, see for instance Wermers (1999) and Dorn et al. (2008).

deviation from independent trading improves slightly. This is because the opposite effects of the biases of the LSV measure work in it's favor. For skewed distributions under independent trading the likelihood of $|\sum_i \varepsilon_i| >> 0$ increases and so does the associated downward bias of the LSV measure. This partially compensates for the upward bias due to the wrong distributional assumption, compare Section 3.3.1.

Stronger Shift in Mean: The LSV measure has particular difficulties for the shift in mean case. If we assume an even stronger market wide shift, i.e. choose the probability of a buy side deviation > 0.9 instead of 0.8, $\widehat{LSV}$ attains the same range of values regardless of whether $|\varepsilon| = 0$ or $|\varepsilon| > 0$. In extreme cases, $\widehat{LSV}$ may even become smaller under dependent trading than under independent trading. This is because the downward bias due to $\sum_{i=1}^{I} \varepsilon_i \neq 0$ becomes stronger as the shift in mean is more pronounced negating or even over-compensating possible increases in the measure due to excess dispersion of the observed buy ratios.

The performance of our measure H is not affected by changes of this parameter.

We stress that the most important insights from the main study remain intact. That is, our measure is centered over zero under the null and separates well between dependent and independent trading. It is significantly correlated to the true deviation from independent trading. The LSV does not center over zero under the null and does not generally separate between dependent and independent trading. The LSV's relation to true deviation from independent trading depends on the particular setup.

3.4.2.4 Does $\bar{H}$ Indicate Market Inefficiencies?

Before we conclude, we want to briefly discuss whether $\bar{H}$ indicates potential price distortions or unstable markets. Park and Sabourian (2011) emphasize that contrarianism and herding alike may impair the efficient functioning of financial markets. We have argued, however, that contrarianism coincides with moderate trader accumulation on either side of the market. It would follow that stock price movements are also bounded in markets prone to contrarianism, compare Paper 5 of this thesis. We, thus, conjecture that $\bar{H} < 0$ hardly indicates economically relevant intra-day price-distorting investor behavior. If the considered time period is longer - for instance, quarters or even years - this assessment might have to be revised, as persistent contrarianism might prevent asset prices from adjusting sufficiently to correctly reflect the stock's fundamentals.

If $\bar{H} = 0$, this indicates that there is no deviation from independent trading, which we have argued to be informationally efficient in Section 3.2. Consequently, we do not expect adverse effects of investor behavior on market efficiency in this case.

This is different for the case of herding. Extreme buy ratios indicating strong trader accumulations on either side of the market have the potential to amplify stock price movements and, if decoupled from the assets' fundamentals, may lead to significant price distortions, again compare5 of this thesis. If $\bar{H}$ is positive due to a shift in mean, this suggests investor coordination across assets. Such a broad correlation of trade behavior bears particular potential for market destabilization. As a consequence, we operate under the hypothesis that only a $\bar{H} > 0$ warrants a more detailed look at what has been going on during a particular trading day.

3.5 Conclusion

This paper contributes to the literature on herding in financial markets by providing an improved measure to assess coordinated investor behavior. As a starting point we consider the highly celebrated measure for correlated trading of Lakonishok et al. (1992). We argue, however, that the assumptions implied by the LSV approach are too restrictive to hold in real-world trading settings. Based on a modification of the LSV assumptions, we design a new measure for investor coordination. Our theoretical analysis and a simulation study reveal that our measure can distinguish between herding (significantly positive sign), contrarianism (significantly negative sign) and independent trading (no significant deviation from 0), even when the considered cross-section of stocks is small.

We emphasize, however, that significant deviations of our measure from zero are only necessary for coordinated investor behavior but not sufficient. In other words, a realization of H of greater than 0 merely *indicates* correlated trading and, hence, potentially price destabilizing investor behavior. While such evidence warrants a closer look at the market, it is by no means sufficient proof that market efficiency or stability is threatened. In the case of $H \leq 0$, on the other hand, we can confidently claim that there is no conspicuous or dangerous investor coordination. $H = 0$ constitutes strong evidence for independent trading, while contrarianism — as indicated by a negative H — has a moderating effect on stock price movements and is unlikely to cause distortions in an intra-day context.

The view that independent trading is a desirable outcome is based on the assumption that the information used to make trade decisions is related to the asset's fundamental value. This is not to say, however, that one should generally rule out the possibility that information acquisition is inefficient (Froot et al. (1992)) or that the available information itself is flawed. These considerations, however, constitute a different question namely on the efficiency of investors' information management prior to trading, but not on the efficiency of the trading behavior. Developing statistical methods to answer this question could be an interesting avenue for future research.

When applying the LSV measure to the same simulated data as our measure, we find it to be significantly positive even if trades are carried out independently. This is due to the violation of the LSV implied assumption that under independent trading the investors' buy propensities are the same across all stocks in the considered cross-section. We show that the positivity of the LSV is solely due to stock idiosyncrasies but not due to investor coordination. This casts serious doubt on the validity of the results presented by the literature employing the LSV measure.

The natural next step is to apply our measure to real world transaction data. This is done in Paper 4. The key objective is to confirm the validity of the assumptions of our measurement approach and at the same time to check whether the Lakonishok et al. (1992) implied assumptions are violated. This can be used to solidify our doubts regarding the applicability of the LSV as a measure for coordinated trading. In addition, we analyze whether the theoretical intuition of an association of our measure with contrarianism and herding, respectively, translate to an actual trading environment.

3.6 Appendix

The Appendix is structured as follows. First, we provide numerical evidence that the estimators of our new measure H are unbiased even for small cross-sections, given that Assumption 3.1 and 3.2 hold.

Second, we provide a detailed discussion, why a positive H can be associated with herding while a negative one relates to contrarianism. We also highlight under which conditions establishing such a link is not feasible.

The Appendix concludes with a small fact pact regarding the Beta distribution summarizing it's most important properties for our application.

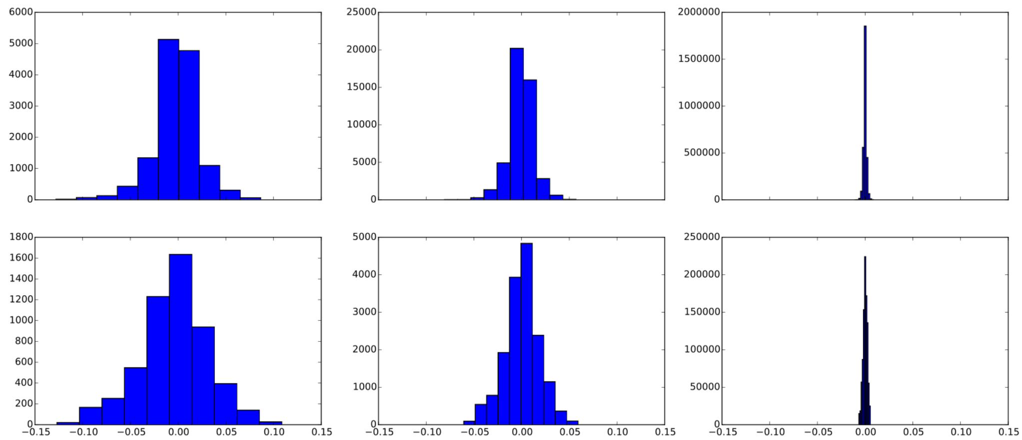
3.6.A Unbiasedness of $\hat{H}$

This section shows the unbiasedness of the estimators $\hat{H}_i$ and $\hat{\bar{H}}_i$ for the respective theoretical measures even for finite cross-sections I. The results are obtained by means of numerical simulation and summarized in Figure 3.2.

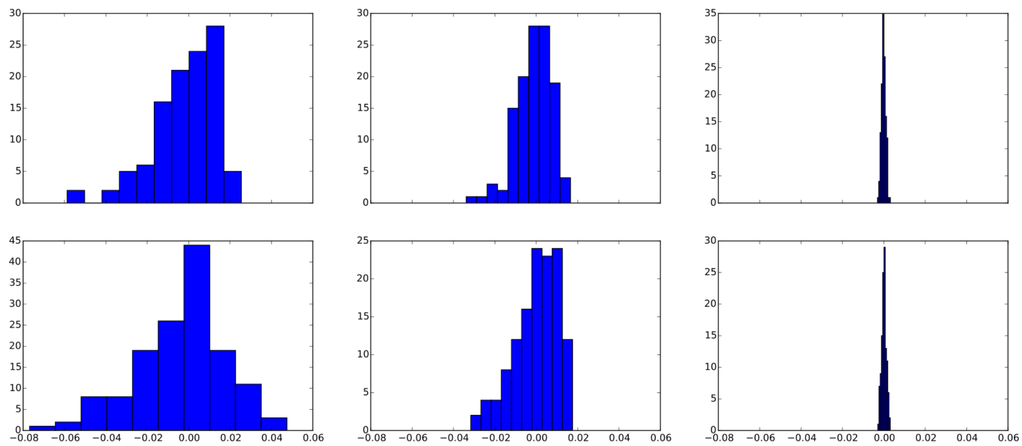
Figure 3.2 (a) shows the bias, $H_i - \hat{H}_i$, as a histogram for $D = 250$ times I observations. The upper panels do so for independent trading, the lower panels for dependent. The simulation setup is as in Section 3.4.2.1 for Panel A. The cross-section increases from the left to the right from $I = 75$, over $I = 250$ to $I = 16000$. As stated in Section 3.3.3, we see that the estimation is unbiased even in small samples and for both cases of zero and under dependent trading. Moreover, the variance in the estimator almost vanishes for $I = 16000$. The same observation applies to Figure 3.2 (b), which plots histograms of the bias $\bar{H} - \hat{\bar{H}}$ for those cross-sections where traders act independently in the upper panels and they trade dependently in the lower panels.[33]

We conclude that the finite sample unbiasedness of estimator $\hat{\bar{H}}$ holds numerically. We would like to emphasize that we obtain these results despite estimating α and β from only 10 trades.

[33] The results are the same for all other simulation setups described in Section 3.4.



(a) Bias of single stock estimators $\bar{H}_i$



(b) Bias of cross-sectional mean estimators $\hat{\bar{H}}$

Figure 3.2: Estimation bias for estimators of H

Notes: This figure shows histograms of the biases $H_i - \hat{H}_i$ in (a) and $\bar{H} - \hat{\bar{H}}$ in (b). The abscissas show the bins for the realizations of the mentioned differences. The number of bins is always 10, the bin size is determined by the most extreme realizations of the respective biases.
The upper panels (rows one and three) show the results under independent trading, the lower panels (rows two and four) the results under dependent trading. H_i and $\hat{H}_i$ are obtained from simulating $250 \times I$ observations where $I \in \{75, 250, 16000\}$ (left,middle,right). The simulation setup is as for Panel A of Table 3.1, see Section 3.4.2.1 for details.

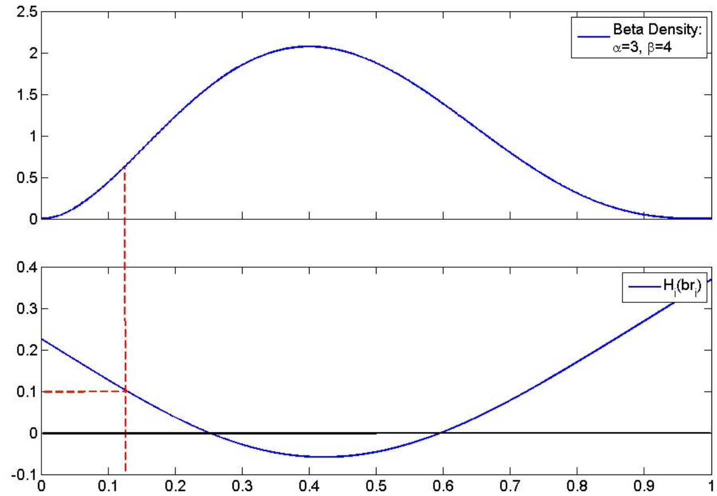


Figure 3.3: Distribution of $\widetilde{br}_i$ for $\alpha = 3$ and $\beta = 4$ and corresponding H_i

Notes: The figure shows a Beta distribution with $\alpha = 3$ and $\beta = 4$ and the resulting H_i, as well as $H_i + \widetilde{AF}_i$ for $T_i = 150$ as function of br_i. At $br_i = 0.13$, indicated by the red line, H_i is approx. 0.1.

3.6.B Distinguishing Between Herding and Contrarianism - Interpreting our Measure H

This section discusses in further detail why positive realizations of the cross-sectional mean measure $\bar{H}$ can be associated with herding and shift in means while negative realizations should be associated with contrarianism. We highlight that these mappings only work if the distribution of the buy ratios under independent trading is not too extremely skewed. We first discuss the stock-specific measure H_i before focusing on the cross-sectional average measure $\bar{H} = \sum_i H_i/I$.

3.6.B.1 Associating Stock-Specific H_i with Herding and Contrarianism

By considering the stock-specific measure we want to illustrate two things. First, we want to highlight for which realizations of br_i, H_i takes positive and negative values respectively. Second, we want to show under which conditions the different signs of H_i can be associated with herding and contrarianism.

In line with Asummption 3.2, H_i depends on four parameters — the distributional parameters of the buy ratios under independent trading, α and β, the number of trades T_i and the observed buy ratio br_i. We take α, β and T_i as given because we want to understand how H_i depends on br_i and, thus, how it can be linked to herding and contrarianism.

Let us assume that the buy ratios under independent trading $\widetilde{br}_i$ are iid Beta$(3, 4)$ distributed and that $T_i = 150$. The upper graph of Figure 3.3 shows the corresponding distribution of the $\widetilde{br}_i$. The lower graph depicts H_i as a function of the observed buy ratio br_i. H_i is negative for moderate br_i, i.e. br_i that are relatively close to 0.5. H_i is positive for extreme br_i, that is, observed buy ratios close to 0 or 1.

During our model discussion in Section 3.2 we argued that we expect extreme buy ratios under herding and moderate ones under contrarianism. This implies that a positive H_i indicates herding, while a negative H_i suggests that contrarianism is more likely given that there is deviation from independent trading.

While such an interpretation of H_i is appealing, we must stress that based on a single H_i we cannot infer that there is deviation from independent trading in that particular stock, let alone herding or contrarianism. Say, we observe a buy ratio of 0.13 in the case of Figure 3.3. Then $H_i = 0.1$ is positive. We also see from the Beta density in the upper graph that even under independent trading a $br_i = 0.13$ is not unlikely to occur. If the cross-section of stocks I is large, we would expect to see some extreme buy ratios even under independent trading. From a single stock perspective it is, thus, impossible to tell whether $H_i = 0.1$ due to systematic deviations from independent trading or because of to be expected random fluctuations. Only if the cross-sectional average measure $\bar{H}$ significantly differs from zero, we can infer that traders systematically deviate from independent trading, see the subsequent section. The same argument applies to the determination whether investors herd or act as contrarians.

Before we discuss the cross-sectional mean measure $\bar{H}$, however, we must point out that we need to be particularly careful with the interpretation of our

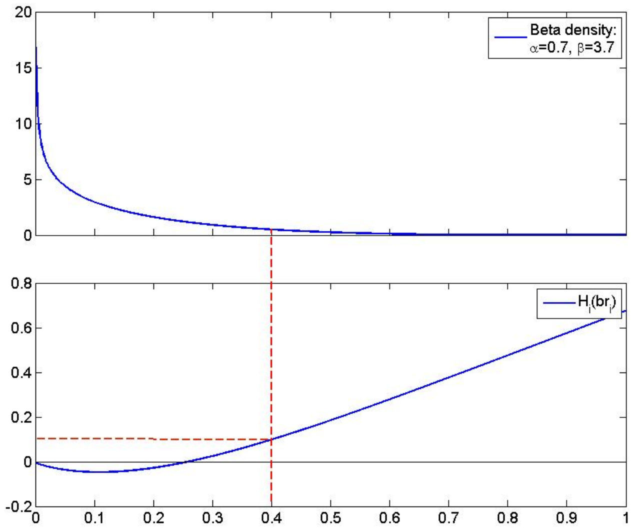


Figure 3.4: Distribution of $\widetilde{br}_i$ for $\alpha = 0.7$ and $\beta = 3.7$ and corresponding H_i

Notes: The figure shows a Beta distribution with $\alpha = 0.7$ and $\beta = 3.7$ and the resulting H_i, as well as $H_i + \widetilde{AF}_i$ for $T_i = 150$ as function of br_i.

measure H if the distribution of the buy ratios under independent trading is very skewed.[34]

Such a case is depicted in Figure 3.4. Here we have $\widetilde{br}_i \sim \text{Beta}(0.7, 3.7)$. If we observe $br_i = 0.4$, our model intuition implies that this indicates contrarian behavior since under independent trading we would expect to observe on average buy ratios of 0.16, which are much more extreme than the observed buy ratio br_i. Due to the strong skew of the Beta distribution, however, H_i is now positive for $br_i = 0.4$. By our previous line of reasoning, this would indicate herding.

Consequently, H_i cannot be related to herding or contrarianism if the skew in the distribution of the $\widetilde{br}_i$ is too strong. As a rule of thumb we suggest to consider the skew to be too strong if $H_i > 0$ for $br_i = 0.5$. In this case additional analyses have to be conducted to ascertain the particular type of investor behavior.[35]

[34] The skew of a beta distribution is given by $\frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{(\alpha+\beta+2)\sqrt{\alpha\beta}}$.

[35] In Paper 4 we find that the skewness of the estimated distributions under independent trading in the German stock market of 2008 are typically moderate. That is, in most of the cases of our application, a positive H can indeed be associated with herding while a negative H points at contrarianism. In the few cases where this is not possible a more detailed analysis of the events of that day is conducted.

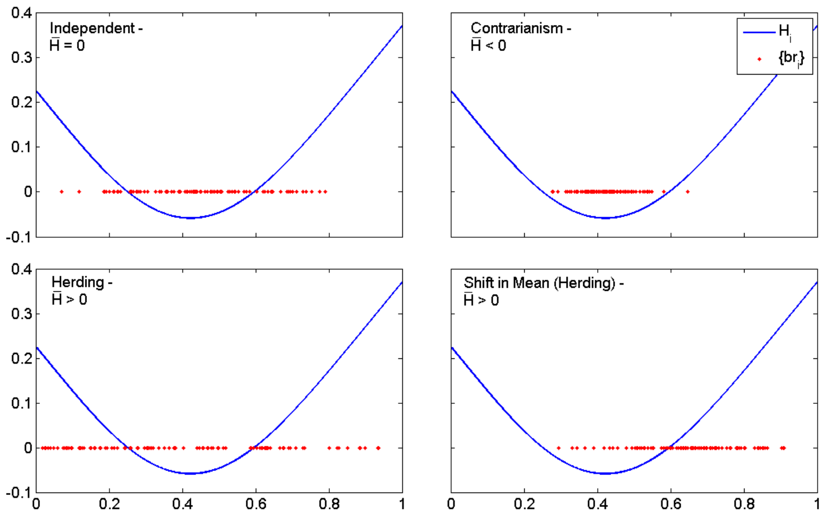


Figure 3.5: $\bar{H}$ under independent and dependent trading

Notes: The figure shows H_i as a function of $br_i = B_i/T$ for a Beta distribution with $\alpha = 3$, $\beta = 4$ and $T = 150$ for all stocks together with different realizations of br_i. That is, the abscissas show different realizations of br_i, while the y-axis shows the corresponding $H_i(br_i)$.
In the upper-left Panel each br_i, $i = 1, \ldots, 100$ is drawn from Beta$(3, 4)$ (independent trading). In the upper-right Panel each br_i is drawn from Beta$(18, 24)$ (contrarianism). In the lower-left Panel each br_i is drawn from Bino$(1, 4/3)$ (herding). In the lower-right Panel each br_i is drawn from Bino$(8, 4)$ (shift in mean).

We continue by discussing the interpretation of $\bar{H}$ under the assumption that the distribution of the $\widetilde{br}_i$ has a sufficiently low skew.

3.6.B.2 Translating the Stock-Specific Insights to $\bar{H}$

We conduct the discussion of $\bar{H}$ under the assumption that $\widetilde{br}_i$ are again iid Beta$(3, 4)$, i.e. the skew of the distribution is sufficiently low to associate positive H_i with herding and negative ones with contrarianism. We assume that the considered stock basket consists of $I = 100$ assets and that $T = 150$ for all stocks in the cross-section. In line with our previous discussion, we illustrate three stylized cases of investor behavior - (a) independent trading, (b) contrarianism and (c) herding. For independent trading we draw the observed buy ratios br_i from the same distribution as $\widetilde{br}_i$. For contrarianism we draw the br_i from Beta$(18, 24)$. Thus, the distribution of the $\widetilde{br}_i$ constitutes a mean-preserving spread of the distribution of the observed buy ratios.[36] This, in turn implies that the br_i are less extreme (closer to 0.5) on average than the buy ratios under independent trading. For herding, we draw br_i from Beta$(1, 4/3)$. Hence, the distribution of the br_i now constitutes a mean-preserving spread of the distribution of the $\widetilde{br}_i$, which means the br_i are more extreme (farther away from 0.5) on average than the buy ratios under independent trading.

In addition we consider the special case (d) shift-in-mean, where the observed buy ratios br_i differ on average from the buy ratios under independent trading. This case is modeled by drawing br_i from Beta$(8, 4)$.

Figure 3.5 illustrates the four stylized cases of investor behavior and the associated realizations of the H_i. Each Panel plots H_i as a function of br_i given the distribution of $\widetilde{br}_i$ and T. Note that since α, β and T are the same for all 100 stocks in the cross-section, so is $H_i(br_i)$. The $H_i(br_i)$ function here corresponds to the one depicted in Figure 3.3. It is the same in all four panels since α and β are the same across panels. The observed buy ratios br_i, $i = 1, ..., 100$ are depicted by the red dots and are drawn according to the previously described distributions.

(a) Under independent trading we have $\bar{H} = 0$: The upper-left Panel of Figure 3.5 depicts the case of independent trading, i.e. $br_i \sim$ Beta$(3, 4)$. The adjustment factor $\tilde{AF}_i$ from Equation (3.6) is designed to center H_i over 0 on average if trades are actually carried out as if under independent trading. Deviations indicated by the stock-specific H_i are merely due to random fluctuations which cancel each other out in the cross-section. Hence, it is not surprising, that the resulting $\bar{H}$ is close to zero in this case.[37]

(b) Under contrarianism we have $\bar{H} < 0$: The upper-right Panel shows the case of contrarianism. The observed buy ratios br_i are less disperse than we would have expected under independent trading. The concentration of the br_i around $E[\widetilde{br}_i] = 0.43$ causes H_i to be negative for almost all stocks in the cross-section.

[36] The variance of a beta distribution is given by $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

[37] As long as $br_i \sim \widetilde{br}_i$ holds, $\bar{H} \xrightarrow{\text{H}_0} 0$ almost surely for $I \to \infty$ by the law of large numbers.

Consequently, $\bar{H} < 0$ as well. Since, moreover, $H_i(0.5) < 0$, the concentration of br_i around $E[\widetilde{br}_i] = 0.43$ is in line with the intuition of contrarianism.

(c) Under herding we have $\bar{H} > 0$: The lower left Panel shows the case of herding. The observed buy ratios br_i are more disperse than we would have expected under independent trading. An unusually high number of br_i realizes close to 1 and 0 respectively and we observe only relatively few moderate br_i in the neighborhood of 0.5. Hence, only few H_i are negative and many H_i are much greater than 0. Consequently, $\bar{H} > 0$.

(d) A shift in mean leads to $\bar{H} > 0$: In the lower right Panel we have illustrated the case where the buy ratios under independent trading are lower on average than the observed buy ratios. This shift in mean causes many of the stock-specific H_i to be positive, which in turn yields a $\bar{H} > 0$.[38] By the same line of reasoning as before, one could argue that this apparent deviation from independent trading is due to investor herding. As already discussed in the main part, such a broadly aligned change in trade behavior may also indicate some market-wide effect that has changed the trading environment altogether. A characterization of the investors' trading behavior is only possible if one takes a closer look on the events of that particular day.

3.6.C Beta Distribution Fact Pack

The Beta distribution is a continuous distribution with support $[0;1]$. It is, thus, well-suited to model the realization of buy ratios. The Beta distribution has two parameters $\alpha > 0$ and $\beta > 0$ that determine the shape of it's density. The Beta density is given by $p^{\alpha-1}(1-p)^{\beta-1}/\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\mathrm{d}u$.

The expected value of the Beta distribution is given by $\alpha/(\alpha+\beta)$. It's variance equals to $(\alpha\beta)/[(\alpha+\beta)^2(\alpha+\beta+1)]$. Finally, it's skew is given by $2(\beta-\alpha)\sqrt{\alpha+\beta+1}/[(\alpha+\beta+2)\sqrt{\alpha\beta}]$.

Figure 3.6 illustrates how different parameters α, β affect the distributional shape. The larger α, β, the less disperse the distribution and vice versa. As long

[38] Note that the dispersion of the br_i is also different from what we would have expected under independent trading. The dominating effect, however, is the shift in mean.

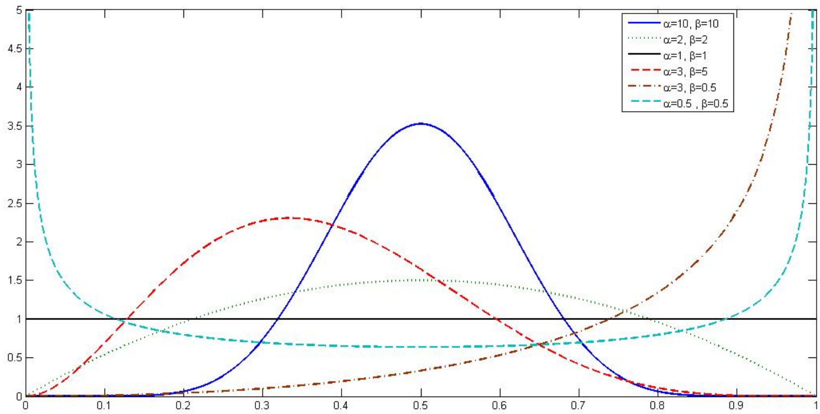


Figure 3.6: Different beta densities

Notes: The figure shows Beta densities for different parameter α and β.

as $\alpha = \beta$, the distribution is symmetric around it's mean 0.5. For $\alpha = \beta = 1$, the Beta distribution is identical with the Uniform distribution on $[0; 1]$.

The red dashed graph ($\alpha = 3, \beta = 5$) shows a right skewed distribution, while the brown dotted-dashed line ($\alpha = 3, \beta = 0.5$) shows a strongly left skewed beta distribution. If both parameters are less than 1, the density becomes u-shaped.

Finally note, that if the success probability of a binomially distributed random variable X is beta distributed, then X is in fact beta-binomially distributed.

Paper 4

Herding and Contrarianism in the German Stock Market During the Recent Financial Crisis[1]

4.1 Introduction

Our analyses in Paper 3 of this thesis cast serious doubts on the consensus that the highly celebrated LSV measure for investor coordination of Lakonishok et al. (1992) is a reliable test whether the necessary condition for herding is fulfilled. Based on theory-guided arguments and simulated trade data we show that if the rather restrictive assumptions of the LSV measure are violated, it will generally be positive due to statistical artifacts and will be unrelated to actual investor coordination.

Paper 3 proposes a new measure H to overcome the shortcomings of the LSV. It demonstrates that our measure H correctly distinguishes between independent trading and different forms of investor coordination in much more general settings than the LSV measure. The evidence in Paper 3, however, remains theory-based.

It is, therefore, the goal of this paper to investigate whether the insights developed in Paper 3 carry over to real world trading data from the German stock market in 2008.

[1]This paper was written in collaboration with my co-authors Simon Jurkatis and Pruiya Abbassi.

To that end, we calculate the LSV and H measures on a daily basis. For each day, we test the validity of the distributional assumptions underlying the respective measurement approaches by using Pearson's Goodness of Fit test.

We find strong evidence that supports the distributional assumption associated with our measure H. That is, the data suggest that the buy ratios under independent trading are iid beta distributed across stocks for the first 10 trades of a trading day. In contrast, the data reject in almost all cases the LSV implied assumption that a Binomial distribution accurately describes the independent trades.

As we use H to gauge investor behavior in the German stock market, we find that investors predominantly exhibit contrarian tendencies or trade independently.[2] Herd behavior, on the other hand, is a rare event. If investors do coordinate in such a fashion, however, they tend to destabilize the market.

In line with the simulation study of Paper 3, the LSV measure always becomes significantly positive suggesting there is a more or less constant degree of investor coordination. The overall means ranging from 4.8% to 6.5% are in line with previous results.[3] Given that the distributional assumptions of the LSV measure are violated, however, we consider these results to be a reflection of persistent stock idiosyncrasies rather than investor coordination, compare Paper 3.

Similar to Paper 2 of this thesis, we use high-frequency, investor-specific transaction data. We focus on data from the electronic limit order book XETRA, which is the largest trading platform for equity in Germany.

The reason, why such data are particularly suited to analyze investor coordination has been discussed at length in Paper 2. In the present paper, we refine

[2]Note that this is in line with the experimental literature which finds that traders have a natural tendency to act as contrarians above what could rationally be explained, compare Cipriani and Guarino (2005), Drehmann et al. (2005) and Park and Sgroi (2012).

[3]Dorn et al. (2008) who also calculate the LSV measure on a daily basis, find an average value of 4.4%. Studies of coordinated trading of mutual and pension funds trading on the American stock markets based on quarterly portfolio changes typically find LSV measures of 2.5% to 3.4%, see Lakonishok et al. (1992), Grinblatt et al. (1995),Wermers (1999) and Brown et al. (2014). Measures based on monthly data for individual investors on American stock exchanges are typically somewhat larger with 6.81% to 12.79%, see Barber et al. (2009a), Barber et al. (2009b).

the transaction data by matching it with XETRA quote data. The combination of the two data sets allows us to determine the initiator of a trade which greatly increases the informational value of our measure H, compare Section 3.2.2 of Paper 3.

Moreover, the data allow us to differentiate between proprietary trades of all securities services institutions which are permitted to trade on German exchanges and their customers' trades. A separate analysis of the trade behavior of these trader subgroups is conducted. Indeed, destabilizing investor coordination is only exhibited by the group of customer traders while financial institutions appear to act as moderators and display the tendency to stabilize markets through their trade behavior.

The remainder of this paper is structured as follows. Section 4.2 presents the employed data and methodology. Section 4.3 contains the results on independent trading behavior that strongly favor the assumptions associated with our measure H. Based on our measure H, Section 4.4 shows to what extent and in which manner investors coordinated in the German Stock market in 2008 and confirms that the LSV measure is inapt to assess investor coordination. Finally, Section 4.5 concludes.

4.2 Data and Methodology

We employ high-frequency and investor-specific transaction data from the German stock market in 2008. We combine them with corresponding quote data from the electronic trading platform XETRA to separate market from limit orders, i.e. to decide which side initiated the trade.

We obtained the transaction data from the Deutsche Bundesbank. Any financial service institution trading securities on a German stock exchange is required to report its transactions to the German Federal Financial Supervisory Authority (BaFin) under article 9 of the Securities Trading Act (WpHG).[4] The

[4]There are few exceptions to the reporting requirement such as home loan banks, and private as well as public insurance companies, as long as they are not themselves permitted to trade on a domestic exchange.

transaction data is available to the Deutsche Bundesbank under article 5 of the Financial Stability Act (FinStabG). The XETRA quote data was provided by the Collaborative Research Center (CRC) 649 and was purchased from the Deutsche Börse AG, which operates the XETRA platform.

4.2.1 The Transaction Data

The transaction data contain *all* trades of *all* stocks executed on German stock exchanges. For reasons of compatibility with the quote data, we focus on the trades executed on the electronic trading platform XETRA. Those trades represent a market share of $\sim 90\%$ of the total trading volume during continuous time trading, i.e. during 9h and 17.30h CET.

Moreover, we restrict our attention on the 233 stocks from today's German Prime Standard that were also Prime Standard in 2008.[5] The Prime Standard is a prerequisite for a stock to be listed in the most prominent German indices such as DAX, MDAX or TecDAX and, thus, comprises the most liquidly traded stocks of the German stock market. A total of 260 financial institutions subject to report to BaFin actively traded Prime Standard stocks in 2008.

For each trade, the transaction data specify the time of the transaction precise to a second, the type of the transaction (buy or sale), the number of shares traded, and the transaction price. Moreover, the data uniquely identify the trading institutions, and indicate whether the transaction was conducted on behalf of a customer of the institution (e.g. retail banks, other financial intermediaries, retail traders) or on the institution's own account.

Given the possibly different degree of trading sophistication and financial literacy of these groups we may expect different trading behavior. Hence, we analyze the trading patterns of customer traders and proprietary trading desks of financial institutions separately.

[5] Prime Standard companies adhere to stricter transparency standards than General Standard stocks. These standards are defined by the Deutsche Börse AG. To be listed in the Prime Standard, companies have to e.g. submit quarterly reports in addition to half-year and year-end reports and they have to adhere to tighter deadlines for doing so.

4.2.2 Combining the Transaction Data with the Quote Data

For each executed trade the transaction data show both counterparties, that is, the buyer as well as the seller. The data do not indicate, however, which of the counterparties initiated the trade.

A differentiation of the active trade side, i.e. Market Orders (MO), from the passive trade side, i.e. Limit Orders (LO), is, however, crucial for our analysis. Dorn et al. (2008) and Barber et al. (2009b) note that considering LO tends to bias the assessment of investor coordination. From a theoretic point of view, we are interested in the influence of the recent trade and price history on current trading decisions. Non-marketable LO, however, enter the order book before they are executed. LO may even be carried over from one trading day to the next. Thus, decisions behind these orders cannot have been influenced by any trade that has taken place in the meantime. Since the time-of-entry of LO cannot be inferred from the data, we cannot decide whether a passive transaction is conducted independently or in a dependent manner in the sense of Section 3.2 in Paper 3, and, hence, whether it contributes to investor coordination.

As a consequence, we match the transaction data set with the XETRA quote data to determine which trades are MO. The quote data contain the best bid- and ask-price, the number of shares that can be traded at the best quotes and the time of quote changes (including changes in the volume at the quotes) precise to one-hundredth of a second. It also flags quotes from call auction periods.

The lower record frequency of the transaction data set poses a problem. It prevents us from applying standard classification algorithms like the one of Lee and Ready (1991) to filter out the MO. We propose a new algorithm that deals with this issue. As it is not the focus of this study, we provide details in Section 4.6.A in the Appendix of this paper. All trades that are not classified as MO are excluded from consideration.

We apply two additional guidelines to exclude trades that might bias our results.

First, transactions from call auctions are excluded. These include the opening, mid-day and closing auction, as well as unscheduled call-auctions induced by e.g. volatility breaks.[6] The reason for this is that we cannot decide whether auction trades are carried out in a dependent or independent fashion since the data does not reveal in which order auction trades are submitted.

Second, we only consider Prime Standard stocks that each subgroup trades liquidly enough for our application on any given trading day. If less than 10 trades are initiated by traders of either subgroup within the first trading hour, we drop the respective stock from consideration on that particular day.[7] The average number of stocks that are considered each day, thus, amounts to 82 and 80 for financial institutions and their customers, respectively. That is, roughly 35% of the 233 Prime Standard Stocks are, in fact, analyzed each day. Consequently, the effective cross-section mainly consists of DAX30 stocks plus a combination of MDAX and TecDAX stocks and some foreign titles.

Overall, this leaves us about 34 million active trades to analyze, which amounts to roughly 55% of the single-counted equity trades executed on XETRA during 2008.[8]

4.2.3 Measuring Coordinated Trading

We assess coordinated trading with the LSV and H measures. The respective stock specific measures are defined in Equations (3.1) and (3.5) of Paper 3. In line with the literature on coordinated trading, we calculate the cross-sectional average measures, that is, $\overline{LSV} = \sum_{i=1}^{I} LSV_i/I$ and $\bar{H} = \sum_{i=1}^{I} H_i/I$, where I is the number of stocks in the considered cross-section.[9] This is done separately

[6] We use the XETRA quote data to identify auction periods. Transactions that are conducted at the end of these periods are excluded because they are part of the resolution of the auction.

[7] We require at least 10 trades to reliably estimate the distribution of the buy ratios under independent trading in accordance with our measure H. For the estimation, we also require that those trades are in fact carried out independently. If too much time passes, this casts doubt on the independence assumption as investors of a particular subgroup may be influenced by investors from other subgroups in their trading decision or spillovers from other stocks, compare discussion of Assumption 3.1 in Paper 3.

[8] More detailed trade statistics are provided in Section 4.6.B in the Appendix of this paper. Those statistics also reveal considerable similarities between the real world trade data and the simulated trade data from Paper 3, which supports the validity of our theoretical insights for our empirical application in the present paper.

[9] By an abuse of notation, we refer to the estimators of the theoretical measures even if we omit the "hat" throughout this paper.

for each of the 252 trading days of 2008 and the respective subgroups of customer traders and proprietary traders.

Now that we have a better understanding of the data, let us address the first key question of this paper: Are the distributional assumptions implied by either measurement approach for investor coordination supported by the data?

4.3 Analyzing Independent Trade Behavior

This section analyzes whether the distributional assumptions of the LSV and our approach are favored by the data. We find that the LSV implied assumption is rejected, while the distributional assumption associated with our measure H is found to be reasonable.

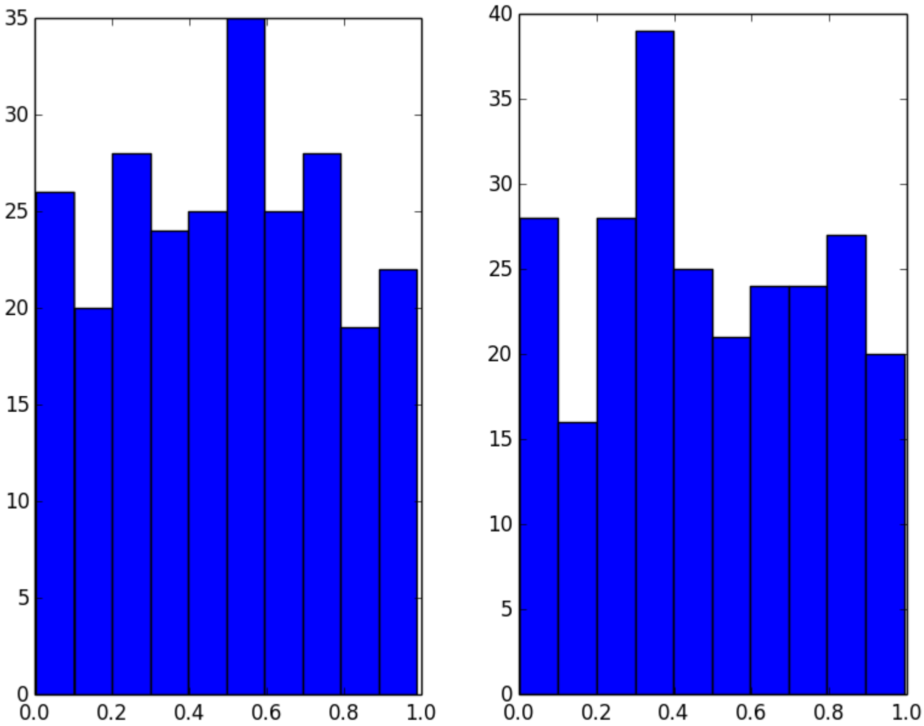
4.3.1 Testing the Distributional Assumptions of H and LSV

Before providing the test results, we briefly revisit the distributional assumptions associated with each measurement approach.

The LSV approach implicitly assumes that the number of buys under independent trading is binomially distributed with some fixed investor buy propensity for all stocks in the cross-section. Our approach, on the other hand, assumes that the number of buys under independent trading is beta-binomially distributed with idiosyncratic buy propensities $\widetilde{br}_i$ for the different stocks. In addition, our approach requires a convention which trades in the sample are carried out independently. In line with Paper 3 of this thesis, we assume that the first ten continuous trades are conducted independently for every stock on every day ($\tau_i = 10$ for all stocks i).[10]

We begin by testing the distributional assumption of H. That is, we check whether the number of buys from the first 10 trades can be described by the Beta-Binomial distribution. To do so, we estimate the Beta-Binomial distribution by maximum likelihood for all 252 days in both our samples,

[10] The first trade is the first MO of the respective subgroup from the continuous trading phase of the day.



(a) Customer Traders (b) Proprietary Traders

Figure 4.1: P-values from Pearson's Goodness of Fit test

Notes: This figure shows histograms of the p-values from Pearson's GoF tests. The test is applied to 252 estimated Beta-Binomial distributions. Each GoF test decides whether the observed distribution of the number of buys from the first 10 trades fits to what we should expect under the estimated distribution. The tests are applied to the subgroup of customer traders (a) and the subgroup of the proprietary trading desks of financial institutions subject to report to BaFin (b).

customers and proprietary trading desks, and apply to each Pearson's Goodness-of-Fit (GoF) test with 10 being the required minimum number of observations in each bin. In total, we conduct 504 GoF tests.

To get an overall impression whether the Beta-Binomial distribution is an appropriate description of the empirical distribution of the buys under independent trading, we can look at the distribution of the p-values from the 252 GoF tests for the respective subgroup.

The results are depicted in Figure 4.1. If all 252 test-statistics were drawn from the null hypothesis of beta-binomially distributed buys, the p-values would be uniformly distributed. That is, 5% of the 252 tests should have p-values of smaller or equal to 0.05, 10% should have p-values smaller or equal to 0.1, and so on. For instance, Figure 4.1 (a) shows that on 26 out of the 252 trading days, the p-value of the GoF test is ≤ 0.1. Put differently, the test rejects our null of beta-binomially distributed buys in 10.3% of the days at a 10% significance level. This is precisely what we expect the test to do if the null hypothesis is true.

To confirm that the the p-vales are uniformly distributed, we use a Kolmogorov-Smirnov test. The test does not reject the null that the 252 p-values of each investor group are uniformly distributed.[11] This provides strong support for our assumption that the independent buys follow a Beta-Binomial distribution.

We stress that this evidence in favor of our distributional assumption also supports the premise that the first 10 trades on each stock-day of the respective subgroup are carried out independently. If they had not been, it is unlikely that a standard distribution could have described the data so well.

Testing the fit of the binomial distribution via GoF, on the other hand, generally rejects it as an appropriate description of the data. More than 95% of the 252 of the GoF tests from each group reject the LSV-implied null at a significance level of 0.05.

[11] The p-values from the Kolmogorov-Smirnov tests are 0.28 for the group of customer trades and 0.31 for the group of proprietary trades.

We continue by providing detailed statistics for independent trade behavior of both considered trader subgroups. In line with the results of this section, we assume that the number of independent buys is beta-binomially distributed, which is equivalent to the fact that the buy ratios under independent trading are beta-distributed.[12]

4.3.2 Independent Trading Versus Actual Trading

Table 4.1 reports the statistics on independent trading allowing for the derivation of three key insights. First, the buy ratios under independent trading are much more disperse than the observed buy ratios. Second, the average of the buy ratios under independent trading as well as the observed buy ratios is close to 0.5. Third, as indicated by a minimal α_d of 0.64, there are days when customer traders strongly accumulate on the sell side of the market.

From rows one, two and four of each Panel, we can see that the buy ratios under independent trading are fairly dispersed. This is indicated by the fact that on more than 75% of the days α and β are less than 4.5 for both trader groups, compare column 4 (Q75). The corresponding variance of the buy ratios under independent trading is reflected by the respective beta moment Var_d. It ranges from 0.26 (Q25) to 0.66 (Max) for both trader groups with a slightly higher mean variance for customer traders of 0.038 than for proprietary traders 0.033 (column 6).[13] The observed buy ratios under actual trading $br_{i,d}$ are much less disperse. Column 7, which reports the standard deviation implies that the variance of the $br_{i,d}$ amounts to only 0.014 and 0.01, respectively.

The buy ratios under independent as well as actual trading are close to 0.5 on average. For independent trading, this is indicated by the $Mean_d$ beta moment (row 3 in each Panel). Column 6 shows that the mean expected buy ratio under independent trading over all 252 days is 0.48 for customer traders and 0.5 for proprietary traders, respectively. Moreover, the Q25 and Q75 values show that

[12]For details on the beta distribution, please refer to the Appendix of Paper 3 of this thesis.

[13]As a comparison note that for $\alpha = \beta = 1$ the Beta distribution equals the inherently disperse uniform distribution on $[0; 1]$ with variance 0.083. For $\alpha = \beta = 4.5$ the variance is still fairly high at 0.025.

Panel A		Customer Traders						
		Min	Q25	Median	Q75	Max	Mean	Std
	α_d	0.64	2.03	2.68	3.40	9.32	2.87	1.19
	β_d	1.51	2.39	2.84	3.47	8.58	3.03	0.98
Beta Moments	Mean_d	0.18	0.43	0.49	0.54	0.67	0.48	0.08
	Var_d	0.013	0.032	0.037	0.043	0.066	0.038	0.009
	Skewness_d	-0.40	-0.10	0.02	0.19	1.27	0.07	0.24
	$br_{i,d}$	0.02	0.43	0.50	0.57	1.0000	0.50	0.12
Panel B		Proprietary Traders						
		Min	Q25	Median	Q75	Max	Mean	Std
	α_d	1.30	2.64	3.30	4.15	14.65	3.53	1.45
	β_d	1.42	2.56	3.32	4.34	15.05	3.56	1.49
Beta Moments	Mean_d	0.36	0.45	0.50	0.55	0.66	0.50	0.07
	Var_d	0.008	0.026	0.032	0.040	0.065	0.033	0.010
	Skewness_d	-0.39	-0.13	0.01	0.13	0.42	0.00	0.17
	$br_{i,d}$	0.03	0.44	0.50	0.56	1.0000	0.50	0.10

Table 4.1: Independent and actual trading statistics of proprietary trading desks of financial service institutions and their customers on XETRA in 2008

Notes: This table shows summary statistics on the estimated distributional parameters of the buy ratios under independent trading. We us ML estimators in line with Paper 3. Panel A shows the results for customer trades, Panel B for traders from proprietary trading desks of the financial institutions. α and β refer to the parameters of the Beta distribution of the buy ratios under independent trading estimated over the cross-section of stocks for each day in our sample. That is, we have 252 observations for α and β in each Panel. Mean, Var, i.e. Variance, and Skewness refer to the moments of the Beta distribution over the independent buy-ratios implied by the estimated α and β. br is the buy-ratio computed as the number of buys over the number of trades calculated for each stock on every day. For customer trades we, thus, have 20218 observations, for proprietary trades 20589.

on 50% of the trading days the expected buy ratio under independent trading is contained on $[0.43; 0.55]$ for both trader groups. Similar values are found for the $br_{i,d}$.[14] This implies that on average, traders of neither subgroup moved in or out of Prime Standard stocks as a whole.

In conjunction with the variance statistics, these numbers suggest a very particular trading pattern. The high dispersion of the independent buy ratios indicates that well-informed traders incorporate new stock-specific information into the respective stocks early in the trading day by accumulating on either the buy or the sell side, compare e.g. Park and Sgroi (2012). Once the price accurately reflects this information, traders become aware of this and stop trading the asset for informational reasons. In terms of e.g. Park and Sabourian (2011), this means they essentially revert to noise trading, i.e. buy and sell with equal probability.[15] Consequently, the observed (rest-of-day) buy ratios $br_{i,d}$ cluster much stronger around 0.5 than the independent buy ratios.

The fact that distributions of the independent buy ratios have largely low skews and are centered around 0.5 also implies that the interpretation of H suggested in Paper 3 of this thesis is applicable to the real-world data in most cases. That is, $\bar{H} > 0$ suggests herding and $\bar{H} < 0$ suggests contrarianism, compare Section 4.6.C in the Appendix of Paper 3 for details.

Finally, we point out that for customer traders we observe very skewed distributions under independent trading at least occasionally. The maximal skewness value of 1.27 and the minimal α of 0.64 both suggest that sometimes customer traders accumulate on the sell side of most if not all stocks in the considered cross-section. That is, they collectively move out of the market under independent trading. Such extreme concentration on the buy side is not observed. This should not come as a surprise, however, since we are anaylzing investor behavior during 2008 when the recent fiancial crisis took hold globally.

Interestingly, proprietary traders never coordinate across stocks in such an extreme fashion.

[14] Since the $br_{i,d}$ are stock specific, we observe buy ratios close to 0 and 1 in the extreme. This is partially driven by a very low number of trades executed in the respective stock on that day.

[15] This also in line with findings of Paper 2 of this thesis that document that information risk, i.e. informed trading is most pronounced in the morning.

	LSV		*H*	
	Median	Mean	Median	Mean
Customer Traders	0.047	0.065	-0.045	-0.026
Observations	20218			
Proprietary Traders	0.033	0.048	-0.045	-0.033
Observations	20589			

Table 4.2: Coordinated trading statistics of proprietary trading desks of financial service institutions and their customers on XETRA in 2008

Notes: The mean and median statistics for investor coordination of the year 2008 are presented in accordance with the LSV measure and our new measure H for both, proprietary and customer traders. The mean is calculated as the weighted average of the daily cross-sectional mean mesures, i.e. $LSV = \sum_{d=1}^{252}(I_d\overline{LSV}_d)/\sum_{d=1}^{252} I_d$ and $H = \sum_{d=1}^{252}(I_d\bar{H}_d)/\sum_{d=1}^{252} I_d$, where I_d is the number of sufficiently liquidly traded stocks for the respective investor subgroup on day d. The Median is calculated over all stock-day observations.

4.4 Investor Coordination in the German Stock Market in 2008

This section provides the results on coordinated investor behavior in accordance with the LSV and H measures respectively. In line with the literature, we first discuss the mean measures that are aggregated over the whole year of 2008, compare Lakonishok et al. (1992). We continue by looking at the day-to-day dynamics of both herd measures. The section concludes with an event study of two trading days when investor coordination among customer traders was particularly high, highlighting when our measure indicates price distortions.

4.4.1 Aggregate Results

Table 4.2 presents the aggregated results for investor coordination across German Prime Standard stocks in 2008 according to both, LSV and H. Because the samples of the respective trader subgroups are large, the results are statistically significant. The mean LSV is 0.065 for customer traders and 0.048 for financial institutions. The median LSV measures are somewhat smaller but still clearly positive. The typical interpretation of the mean LSV measure is that, for the average stock 6.5% of the trades initiated by customers (5% of the trades initiated by financial institutions) are more buys or sales than what we had expected under independent trading. That is, both subgroups herd in and out of the same stocks respectively.

The result for LSV compares to the one of Dorn et al. (2008) (0.044 for market orders of retail traders on a daily basis). Barber et al. (2009b) find slightly higher LSV ranging from 0.068 to 0.128 when assessing market orders of retail traders with monthly frequency.[16]

Given, however, that the LSV implied assumption of binomially distributed buys under independent trading is violated and that we find evidence in support of the more disperse beta-binomial distribution we know that there are stock-idiosyncratic buy propensities under independent trading for both subgroups. We would, thus, argue that the positive LSV does not indicate coordinated trading or herding but rather is a reflection of the bespoken idiosyncratic buy propensities.

While our measure also finds significant deviations from independent trading, the negative sign is more consistent with contrarianism than herding. With -0.045 the median deviations are the same for both subgroups of traders. With -0.026 the mean H of the customer traders is larger than for the proprietary trading desks, indicating that financial services institutions exhibit stronger contrarian tendencies than their customers on average. When compared to the median values, the larger mean of the customer traders suggests that there might be more positive outliers than for the financial institutions subject to report to BaFin.[17] Whether these positive outliers cluster in an interesting

[16]Differences may stem from the fact that we consider the rather special time period of the recent financial crisis and that our group of customer traders might be more heterogeneous than in the mentioned papers since it is likely to comprise actual households as well as fund managers and other banking institutions trading through intermediaries. In the case of Barber et al. (2009b), the lower record frequency may also contribute to higher realization of the LSV measure.

[17]Indeed, we find that for about 20% of the stock-days, H_i is positive with a maximum of 0.48.

fashion pointing towards herding at least on some days is revealed by the analysis of the day-to-day dynamics of $\overline{LSV}$ and $\bar{H}$ in the next section.

4.4.2 Dynamics of Daily Investor Coordination

This section shows that for the subgroup of customer traders, $\bar{H}$ becomes positive occasionally when considering the daily measures. The indication from Table 4.2 in the previous section that proprietary traders tend to act as contrarians essentially carries over one-to-one to the daily horizon.

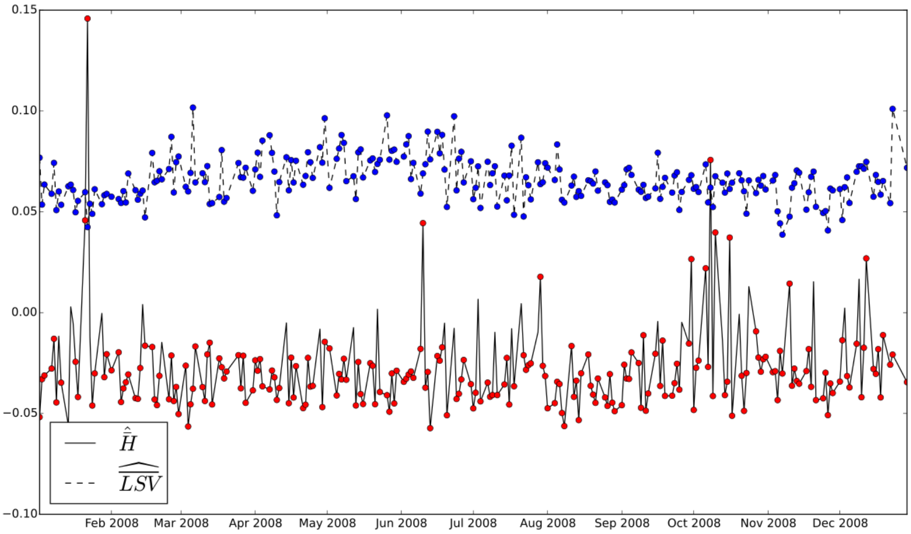
Figure 4.2 plots the time series of $\bar{H}_d$ and $\overline{LSV}_d$, for all 252 trading days in 2008 for both subgroups of investors. For our measure $\bar{H}$ we find that among the 252 averages for the group of financial institutions subject to report to BaFin only 2 are significantly positive, while 229 are significantly negative. On the remaining 21 days H does not differ significantly from 0 indicating independent trading.

For customer traders we find that $\bar{H}$ is significantly positive on 11 days, while it tests significantly negative on 215 days. The null of independent trading is not rejected on the remaining 26 days.[18]

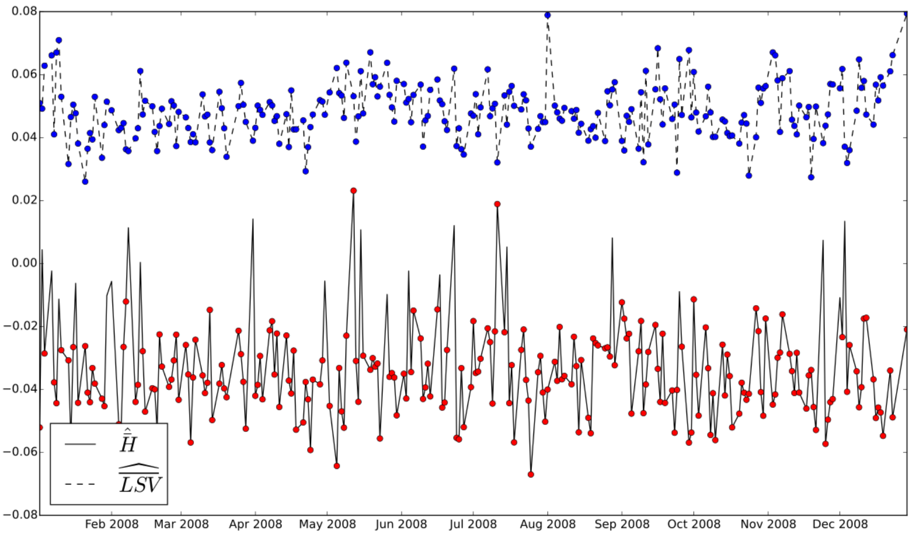
For the means of the LSV measure, on the other hand, all estimates are significantly positive for both groups. Significant estimates are indicated by red and blue dots for H and LSV respectively.

The day-to-day dynamics largely confirm the insights from Table 4.2. Apparently, there are sufficient idiosyncrasies across stocks to cause the LSV measure to become positive on every given day. Hence, the fact that the LSV measures are larger for customers rather than financial institutions is by no means an indication that coordination is stronger among the first group. The simulation study in Paper 3 of this thesis shows that under dependent trading the LSV measure is not correlated with true investor coordination if it's

[18] Our simulation results from Paper 3 of this thesis indicate that the estimator of our measure might be slightly skewed if the number of considered stocks I_d is small. To be on the safe side, we correct the t-values from our tests for skewness and kurtosis as suggested by Yanagihara and Yuan (2005). Moreover, as for each group we conduct 252 hypotheses tests on zero means we control the False Discovery Rate at 0.05 by the procedure of Benjamini and Hochberg (1995) to account for the multiple hypotheses testing problem. We applied the procedure to the one-sided alternatives ($\hat{\bar{H}} > 0$ and $\hat{\bar{H}} < 0$) separately.



(a) Customer Trades



(b) Proprietary Trades

Figure 4.2: Time series for daily LSV and H measures for coordinated trading of proprietary and customer traders on XETRA, 2008

Notes: This figure shows the time series of $\bar{H}_d$ and $\overline{LSV}_d$ for the group of customer trades in (a) and the group of proprietary trades in (b). The red and blue dots indicate significant estimates by the adjusted t-test of Yanagihara and Yuan (2005) for the one-sided alternatives and accounted for the multiple hypotheses testing problem by the procedure of Benjamini and Hochberg (1995).

distributional assumption is violated. Hence, a relative assessment of degrees of investor coordination via the LSV measure is very likely to be misleading.

The predominantly negative signs of $\bar{H}$ indicate contrarian tendencies for both trader subgroups on most days. We would argue, however, that this contrarianism is not likely to be price distorting as it coincides with a pattern of very pronounced stock-specific trade behavior in the morning and more or less noise trading for the remainder of the day. We have argued in Section 4.3.2 that such a pattern — while a deviation from independent trading — is likely to be price and informationally efficient.

For customer traders there are, however, some days when $\bar{H}$ is not only positive but also abnormally large. This may be an indication for investor herding. Yet, we find that these positive $\bar{H}$ are generally triggered by a shift in mean of the buy ratios under independent trading. In addition, the distributions of the buy ratios under independent trading are strongly right-skewed on these days. As a consequence, a more detailed analysis of the events of those days is required to determine the type of investor coordination.

As a representative example, we conduct an event study for the days of January 21 and 22 in the subsequent section.

4.4.3 Event Study for January 21 and 22 - When Investor Herding Destabilized the German Stock Market

This section sheds some light on the events of January 21 and 22, 2008. On these days $\bar{H} > 0$ is abnormally large compared to the rest of the sample. Taking a closer look at the trading pattern, we find that the distribution of the buy ratios under independent trading is strongly right-skewed. Indeed, Table 4.3 shows that $\alpha << \beta$ on both days for customer traders coinciding with a skew of 0.88 and 1.27 on January 21 and 22, respectively.[19] This implies strong accumulation of customer traders on the sell side for all stocks at the beginning of the trading day, i.e. customer traders collectively move out of Prime Standard Stocks under

[19] Compare Table 4.1 for skews on other days

	January 21		January 22	
	Customers	Proprietary	Customers	Proprietary
α	1.04	3.14	0.64	4.06
β	3.23	5.02	2.90	4.14
$\overline{br}$	0.44	0.46	0.50	0.53
$\bar{H}$	0.05	-0.03	0.15	-0.04
$\overline{LSV}$	0.06	0.02	0.04	0.03
DAX Return	-6.9%		2.7%	
NIKKEI Return	-2.7%		-4.2%	

Table 4.3: Coordinated trading statistics for proprietary and customer traders as well as stock index returns on January 21 and 22, 2008

Notes: α and β refer to the parameters of the Beta distribution of the buy ratios under independent trading estimated over the cross-section of stocks for each day in our sample. $\overline{br}$ is the cross-sectional mean of the observed buy ratios. $\bar{H}$ and $\overline{LSV}$ report the respective statistics for investor coordination. The returns are calculated open-to-close and are taken from www.finanzen.net.

independent trading. The expected buy ratio under independent trading is 0.24 on January 21 and 0.18 on January 22.

The average observed buy ratio $\overline{br}$ of 0.44 on January 21 and 0.5 on January 22, however, suggests that through the course of the day the outright panic of the customer traders in the morning is extenuated. Hence, we observe an upward shift in mean, which causes $\bar{H}$ to be positive.[20] Interestingly, the trade behavior of the financial service institutions is much less extreme. On January 21, there is a slight sell side accumulation in the morning. The corresponding α of 3.14 and β of 5.02 imply an expected independent buy ratio of 0.38 with a moderate skew of 0.28. On January 22, $\alpha \approx \beta$ for proprietary traders, indicating that they neither move in nor out of the market as a whole. On both days, proprietary traders at least partially lean against the panic sales of their customers.

[20]Details on the distributions of the buy ratios under independent trading and the observed buy ratios are found in Section 4.6.C in the Appendix of this paper.

The return figures reported in Table 4.3 show that the German Stock Index of the 30 largest companies DAX plummeted on Monday, January 21. That date is known as a Black Monday for the German stock market. One week earlier, Citigroup - the largest bank in the US - had announced a loss of 9.8 billion USD for the last quarter of 2007 mainly due to defaults of sub-prime mortgages. The resulting uncertainty culminated in a drop of more than 3 % of US stock exchanges on Friday, January 18. Yet, this happened only after German stock exchanges had already closed. Over the weekend the fear of long term recessions and bankruptcies particularly in the German financial sector grew. Many German banks like the HSH Nordbank and the Hypo Real Estate were largely invested in CDOs backed by US sub-prime mortgages. The sharp drop of the Japanese NIKKEI Index of 2.7% on January 21 was an additional catalyst for these fears.[21] An outright panic among customer traders on the German stock market was subsequently observed. As investors withdrew their money from the German stock market the DAX crashed more than 6.9 % — the sharpest drop since September 11, 2001, the date of the terror attack on the World Trade Center in New York. The panic carried over to the next day when the Nikkei dropped by another 4.2%. The DAX, however, rebounded on January 22. It went back up by 2.7%. What happened?

In the morning the panic among customer traders was even stronger. This is indicated by both the strong right-skew of the distribution of the buy ratios under independent trading as well as the highly positive $\bar{H} = 0.15$. Indeed, the unreported low of the DAX on January 22 was 2.6% *below* the opening price. Interestingly, the traders of the financial institutions appear to have kept their calm even in the morning. This might have been caused by rumors regarding a drastic cut in US interest rates by the FED. Indeed, the FED announced a cut of 75 basis points to 3.5 % later that day. This had a globally stabilizing effect. Customer traders in the German stock market returned to a more balanced trade behavior ($\overline{br} = 0.5$) and the DAX recovered.

A look at Figure 4.2 reveals that $\bar{H}$ for customer traders drops below 0 again during the subsequent days. On the one hand this indicates that the intervention

[21] Note that the Japanese stock exchanges close right before XETRA opens.

of the US Fed had a longer lasting calming effect causing investors to trade independently or engage in benign contrarianism. The negative $\bar{H}$ might, however, also be driven by the fact that many investors belonging to the customer trader subgroup had already dissolved their stock portfolios on January 21 and 22.

For the sake of completeness, let us note that the LSV measure did not pick up on any of these dynamics. If anything, the daily LSV measure became smaller on days with particular strong investor coordination and herding.

Based on these insights, we conjecture that on January 21, customer traders, indeed, herded on the depreciation of Asian and US stock markets. On the following day their herding tendencies increased even further due the historical plummet of the German and international stock markets on January 21.[22] What is more, the market wide herding of customer traders while rare, apparently has the potential to destabilize markets.

4.5 Conclusion

This paper confirms our doubts regarding the validity of the LSV implied assumptions in favor of the assumptions associated with our new measure for coordinated trading H. In line with the simulation study in Paper 3 of this thesis we find that the LSV measure is always significantly positive. We conclude that this is solely due the fact that the LSV measure picks up idiosyncrasies of trade behavior across stocks and falsely registers them as investor coordination.

When applying our measure H to actual transaction data from the German stock market, we find that traders predominantly act as contrarians or trade independently. This might appear somewhat surprising, given that the year 2008 was rife with uncertainty due to the evolving financial crisis. Only the less proficient customer traders herd occasionally according to our measure. When they do, however, they tend to destabilize the German stock market. Interestingly even on those days, financial institutions with presumably higher trading expertise acted

[22] In the second week of October, the positive values of $\bar{H}$ are again driven by a shift in mean of the buy ratios. Again news reports indicate, that this particular type of customer trade behavior was mainly driven by record losses at other exchanges, particularly by the Nikkei index.

far less extreme. Indeed, as our event study for January 22, 2008, indicates traders from financial institutions seem to correctly anticipate when the German stock market rebounds.

Interesting avenues for future research include an application of $\bar{H}$ to an extended version of the data set introduced in this paper. We could, thus, verify whether our approach to detect deviations from independent trading and it's associated assumptions are valid under different economic environments. It would be particularly intriguing to investigate, whether $\bar{H}$ has predictive power regarding future price reversals or regimes of increased volatility. For this, it might be interesting to consider different time horizons such as weeks, months or quarters. Conversely, a stronger focus on intra-day patterns might prove illuminating as to when information reaches the market, how long it takes the market to incorporate it into the price and whether there exist mini-herds that cause intra-day prices to overshoot and reverse. A stronger exploitation of the investor specificity of the transaction data should benefit the analysis.

Given the apparent spill-over effects from other markets a corresponding analysis might also be quite intriguing.

Papers 3 and 4 are motivated by the assumption that investor herding has adverse effects on the functioning of financial markets. Models such as the ones of Park and Sabourian (2011) and Avery and Zemsky (1998) that are part of the rational herding literature, however, rarely produce outcomes where herds cause price distortions or destabilize markets, compare Eyster and Rabin (2010). On the contrary, Eyster and Rabin (2010) show that rational herds on average *contribute towards price efficiency.* Corresponding discussions in Park and Sabourian (2011) and Avery and Zemsky (1998) are in line with their assessment. If herd behavior is generally benign, however, then why should we put efforts into assessing it empirically at all? One answer to this question is provided in Paper 5 of this thesis, where we derive conditions under which investor coordination in general and herd behavior in particular may move prices away from fundamentals.

4.6 Appendix

This Appendix consists of three sections.

Section 4.6.A details the classification algorithm used to filter out MO by matching the transaction data set with the quote data set. Section 4.6.B provides detailed trade statistics that supplement Section 4.2 of the main part and highlight similarities between the data investigated in this paper and the simulated trade data of Paper 3. Finally, Section 4.6.C details the distributional findings discussed in the event study for January 21 and 22, 2008.

4.6.A Classification of Trades as Market Orders (MO)

As outlined in the main text, we classify trades as MO and LO by matching the transaction data with the quote data. Yet, the different record frequencies of the two data sets (second for transaction data, one hundredths of a second for quote data) prevents us from using standard classification algorithms. In the following, we provide details of our newly developed classification algorithm.

To classify a transaction as buyer- or seller-initiated we need to compare the transaction price to all quotes within the second of the transaction. We classify a trade as buyer initiated, i.e. as a Market Buy Order if one of the following criteria is met:

1. The transaction price hits any ask quote, but none of the bid quotes within the second of the transaction.
2. The transaction price is strictly larger than every mid-quote, i.e. $(bid + ask)/2$, within the second of the transaction.
3. The transaction price hits both ask and bid quotes within the second of the transaction, but only the ask side changes as though it was hit. That is, for instance, an increase in the best ask quote or a decrease in the number of shares offered at the best ask quote.
4. The transaction price differs from all ask and bid quotes during the respective second. Yet, it lies between two tic-by-tic best ask quotes $ask_t <$

ask_{t+1}, while it does not lie between between two tic-by-tic best bid quotes $bid_t > bid_{t+1}$.[23]

The criteria are tested in the given order. Once one of the criteria is met the classification algorithm breaks and continues with the next trade. We classify a trade Sell Market Order analogously. If none of the MO criteria is met, then the trade remains unclassified and is removed from consideration. This happened in less than 20 % of the cases.

[23]The last criterion was necessary, because the quote data did not record when a large market order ate through the order book matching with several quotes from the transition of the best quote to the last one until the order size was filled.

	Customer Traders	Proprietary Traders
Number of trading days	252	252
Number of stock-days	20218	20589
Number of MO Trades (in million)	13.6	20.6

Table 4.4: Trade statistics of proprietary trading desks of financial service institutions and their customers on XETRA, 2008

4.6.B Trade Statistics of Financial Institutions and their Customers on XETRA, 2008

This section presents detailed trade statistics highlighting similarities of the employed real-world data and our simulated data from Paper 3.

We start by discussing summary trade statistics to highlight what share of MO trades on XETRA is investigated. In a second step, trading intensity of the respective trader subgroups is analyzed.

4.6.B.1 Summary Trade Statistics

Table 4.4 shows trade statistics for Prime Standard Stocks traded on XETRA in 2008 for customer and proprietary traders, respectively. In total, we count more than 34 million active trades, 60% of which stem from proprietary traders and 40% from their customers. The number of single-counted equity trades executed on XETRA during 2008 is 62.2 million.[24] That is, in our analysis we consider roughly 55% of all active trades on XETRA in 2008.

There are three reasons why not all active trades enter our analysis. First, we cannot unequivocally classify all trades as MO or LO. Second, we exclude auction trades from consideration. Finally, we only consider a sub-sample of the equity stocks traded on XETRA, compare the main part of this paper. The number of stock-days corresponds to the number of stocks that are traded liquidly enough on each trading day.

[24]Source: Cash Market: Monthly Statistics, Deutsche Börse AG, January 5, 2016; Product-ID: STX-0024-1 1.1.

	Customer Traders			Proprietary Traders		
	Median	Mean	Std	Median	Mean	Std
I_d	79	80	12	81	82	8
Observations	252			252		
(Time at $\tau)_{d,i}$	800	1129	n/a	743	1041	n/a
Observations	20218			20589		
$T_{d,i}$	347	673	960	441	1002	1390
Observations	20218			20589		

Table 4.5: Trading intensity of proprietary trading desks of financial service institutions and their customers on XETRA, 2008

Notes: I_d refers to the number of Prime Standard stocks that are traded sufficiently liquidly by the respective trader subgroup on any given day. Time at τ refers to the number of seconds that pass before 10 trades are initiated by the respective subgroup per stock and day. $T_{i,d}$ refers to the total number of MO trades carried by the respective subgroup in any given stock on any given day.

The number of trading days corresponds to the number of days XETRA was open for trading in 2008. Similar to physical stock exchanges, XETRA is closed on weekends and on legal holidays.

4.6.B.2 Trading Intensity and the Number of Liquidly Traded Stocks

Table 4.5 reports statistics on trading intensity. Row one shows the number of sufficiently liquidly traded stocks per day, I_d. Since I_d is calculated on a daily basis, the number of observations corresponds to the number of trading days in Table 4.4. The average I_d amounts to 80 and 82 for the respective trader subgroups, that is, less than 35% of the 233 Prime Standard Stocks. The effectively considered cross-section mainly consists of DAX30 stocks plus a combination of MDAX and TecDAX stocks and some foreign titles (e.g. Airbus Group (NL), Rofin-Sinar Technologies (US)).[25]

[25]On each day, the foreign stocks constitute less than 10% of the considered cross-section I_d.

I_d corresponds the number of stocks that are used to estimate the distribution of the buy ratios under independent trading as well as to calculate $\bar{H}$ and $\overline{LSV}$ and, thus, is of crucial importance. We have shown in Paper 3 that $\bar{H}$ can be estimated without bias for $I_d > 75$. The simulation study also employs only 75 stocks which supports the validity of it's findings for our application.

Row three of Table 4.5 shows the time in seconds that passes before the respective trader subgroup initiates 10 trades in each stock and on each day.[26] Traders from both subgroups execute their first 10 trades in less than 20 minutes on average. Indeed, for more than half of the stocks, this happens in less than 15 minutes. Since market micro-structure theory proclaims that trades at the outset of the trading process are likely to be independent, the fact that they are carried out fairly quickly provides support for the assumption that the first 10 trades are carried out independently, compare Assumption 3.1 from Paper 3.

Finally, $T_{d,i}$ reports the number of active trades we observe for each stock-day. The $T_{d,i}$ confirm the notion that financial institutions are more active in the stock market than their customers. The $T_{d,i}$ also indicate that the number of trades per stock-day in the simulation study of Paper 3 is sufficiently conservative to make it's findings valid for our empirical application.

Since (Time at $\tau)_{d,i}$ as well as $T_{d,i}$ are calculated on a stock-day basis, their number of observations coincides with the corresponding numbers of row two in Table 4.4.

4.6.C Details on the Distribution of Buy Ratios on January 21 and 22, 2008

Figure 4.3 depicts the estimated distribution of the buy ratios under independent trading as a blue line. The estimation is conducted in line with our confirmed distributional assumption that the number of buys under independent trading is beta-binomially distributed. The empirical distribution of the observed buy ratios is represented by the red dots. Figure 4.3, thus, serves as a detailing of Table 4.3.

[26] Since we exclude stock-days for which the respective subgroups trades less than 10 times during the first trading hour, the theoretical maximum of (Time at $\tau)_{d,i}$ is 3599. Due to this upper boundary, we have excluded the standard deviation statistic of (Time at $\tau)_{d,i}$.

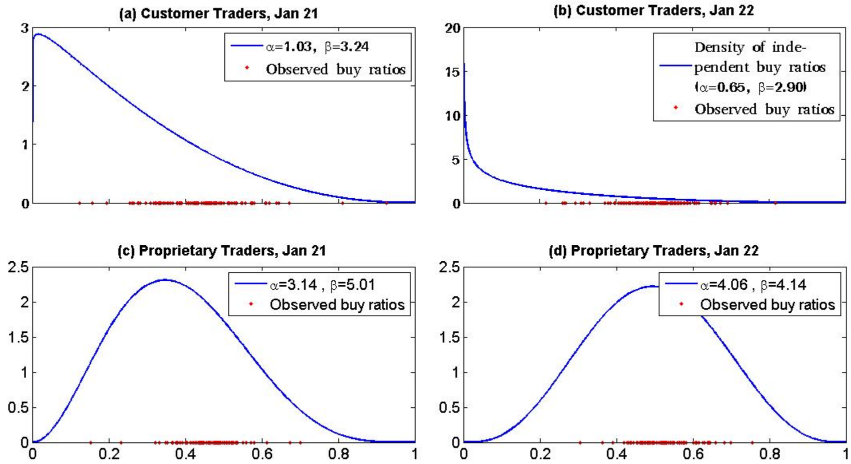


Figure 4.3: Distributions of the buy ratios under independent and actual trading on January 21 and 22, 2008

Notes: The blue line in each figure depicts the beta density associated with the distribution of the buy ratios under independent trading. The red dots show the observed buy ratios for all stocks on that particular day.

Figures 4.3 (a) and (b) illustrate the shift in mean of the highly right-skewed buy ratios under independent trading toward the middle for the customer traders. Figures 4.3 (c) and (d) show that there is no such shift for proprietary traders. For them, the observed buy ratios cluster closely to the expected buy ratio under independent trading, yet with a much smaller dispersion. This results in a $\bar{H} < 0$ which in turn indicates contrarian tendencies.

Paper 5

Irrational Exuberance and Herding in Financial Markets - How Investors Facing Ambiguity Drive Prices away from Fundamentals

5.1 Introduction

Throughout the past decade financial markets exhibited strong degrees of volatility and were characterized by the formation and subsequent burst of bubbles. The prevalent view in the economic literature is that herding among investors is an important driver for such undesirable market phenomena.

The intuition behind this claim is appealing: Investors face a decision whether or not to buy (or sell) a risky asset. As they observe other investors accumulating on one side of the market, they choose to ignore their own noisy information regarding the asset's true value and follow the crowd instead. If the crowd is wrong, such herding on the crowd's action drives prices away from fundamentals contributing towards the formation of bubbles (or excessive downturns). This argument, however, breaks down for most of the existing financial market herd models, see e.g. the seminal works of Avery and Zemsky (1998) and Park and Sabourian (2011). Their models assume that upon the arrival of new information, investors update their beliefs according to Bayes' Rule and that investor choices are based on subjective expected utility theory, i.e. that ambiguity over probabilities does not matter for financial decision makers. Together, these assumptions essentially prevent the existence of wrong crowds and, thus, wrong herds, compare Eyster and Rabin (2010) and Brunnermeier (2001).[1]

In this paper, we want to study how individual investment decisions and the resulting crowd behavior are affected if financial choices are made under ambiguity (Knightian uncertainty). We are particularly interested if ambiguity contributes towards potentially price-distorting herding (contrarianism) and may, thus, support the intuition that investor coordination and bubbles are linked.[2]

We apply the concept of ambiguity to the two-state, two-trader version of the rational market microstructure herd model of Avery and Zemsky (1998). We assume that investors facing ambiguity make decisions in line with non-extreme-outcome-additive (neo-additive) Choquet preferences which were first introduced by Chateauneuf et al. (2007).

The principal model in this paper is related to the one proposed by Ford et al. (2013).[3] Yet, we modify and extend their framework in many important ways. First and most importantly, in our framework investor preferences are part of the common knowledge structure of the model. Second, in line with Brunnermeier (2001) and the bulk of the theoretical herding literature, we define herding (contrarianism) as a *switch* in an agent's opinion toward (against) that

[1]Bayesian updating and preferences in accord with subjective expected utility theory are in line with Barberis and Thaler (2003)'s notion of investor rationality. They argue that the assumption of rationality precludes financial market models from explaining "basic facts about the aggregate stock market, the cross-section of average returns and individual trading behavior" (Barberis and Thaler (2003), p.3).

[2]A departure from belief updating according to Bayes' Rule as proposed by e.g. Eyster and Rabin (2010) would also explain the existence of wrong herds. Yet, as Daniel et al. (1998) explicate, any such behavioral bias of investor behavior requires an in-depth experimental and empirical foundation, lest it will be subject to criticism that it is arbitrary. Evidence supports biases such as overconfidence, see Weizsäcker (2010) and Daniel et al. (1998), or probability weightings and loss aversion in line with prospect theory, see Tversky and Kahneman (1992). Applied to herd models these biases cast additional doubt on the rationale that herding may be the cause for market inefficiencies, compare Huber et al. (2015).

[3]Another paper that modifies the model of Avery and Zemsky (1998) to reflect investment choices under ambiguity is the one of Dong et al. (2010). They use smooth ambiguity functions as introduced by Klibanoff et al. (2005) to model ambiguity stemming from multiple priors regarding the distribution of the risky asset. They find that herding is possible if the degree of ambiguity aversion differs between market maker and informed traders. This is in line with Décamps and Lovo (2006), who obtain a similar result for different *risk* preferences among traders and market maker.

of the crowd *that has to be induced by the crowd.*[4] Third, we consider a more general setup as we depart from the General Bayesian Updating (GBU) rule for Choquet preferences proposed by Eichberger et al. (2010). More precisely, we assume that the individual degree of optimism, i.e. investor's ambiguity preference, may vary with the asset price.[5] Finally, we study a whole class of perturbed versions of our model where the market exhibits marginal uncertainty regarding the true investor preferences. Indeed, Ford et al. (2013)'s assumption that the market is fully ignorant of the true investor preferences can be seen as an extreme special case of the perturbed model setup.

The key insights developed in this paper can be grouped in two categories. First, we characterize conditions under which herding and contrarianism are possible. Second, we discuss how such investor behavior affects market outcomes.

With respect to the first category, we find that informed traders with neo-additive Choquet preferences never herd but show strong contrarianistic tendencies, when beliefs are updated according to the GBU rule.[6] As we depart from GBU, we specify necessary and sufficient conditions for investor herding. We find that herding becomes possible if high degrees of perceived ambiguity coincide with (potentially irrational) exuberance and despair among informed traders.

Second, in terms of market outcomes, we find that in our two-state, two-trader setup informational cascades occur as soon as investors herd or act as contrarians. Since prices stop moving during an informational cascade we find that both, herding and contrarianism prevent the market from learning the asset's true value. In addition, they have an equal potential to drive prices away from fundamentals. A comprehensive comparative static analysis of the probability of such price distortions is provided.

[4]Indeed, Ford et al. (2013) do not require that herding and contrarian behavior are crowd-induced.

[5]To motivate this assumption, we appeal to a growing finance literature that assumes that *risk* aversion is subject to change, see for instance Campbell and Cochrane (1999) or Bekaert et al. (2009). Indeed, standard approaches to measure risk aversion via volatility premia, abundantly show that risk aversion depends on market sentiment and recent price trajectories, see e.g. Jurado et al. (2015), Bekaert et al. (2013) and Bollerslev et al. (2011). We posit that if risk aversion is assumed to move with prices, so should ambiguity aversion.

[6]The impossibility of herding derived here contradicts the findings of Ford et al. (2013). This is due to their different definition of herd behavior.

Informational cascades due to herding and contrarianism, however, exhibit an important qualitative difference, which is revealed by the analysis of the pertubed version of our model. In the perturbed model we still assume that all informed traders have neo-additive Choquet preferences. Yet, market participants think that informed traders have Choquet preferences only with probability $1-\epsilon$ and that they are expected utility maximizers with probability ϵ. In this case, social learning continues even as investors engage in herd or contrarian behavior.

We find that ambiguity in conjunction with strong exuberance or desperation may cause investors to confidently herd on the wrong state of the world with economically relevant probability in the perturbed model. Markets prone to contrarianism show similar outcomes as in the non-perturbed model, i.e. no learning about the asset's true value and limited long-term price distortions.

The literature of decision making under ambiguity can be grouped in two main approaches that are closely related. First, the multiple prior approach explicitly models a range of probability distributions of the states of the world an individual considers possible (her set of priors) and from which she chooses according to some specified decision rule such as maxmin, compare Gilboa and Schmeidler (1989). Second, the Choquet Expected Utiliy (CEU) approach models decision making under ambiguity through non-additive probability measures or so-called capacities, compare Schmeidler (1989). If no objective probabilities are available as e.g. in Ellsberg (1961)'s famous mind experiment, CEU agents assign individual likelihoods to different outcomes.

For our application, we choose neo-additive capacities over multiple prior setups as well as general Choquet preferences for three reasons. First, our analysis requires a parametric separation of the degree of perceived ambiguity and the individual attitude towards ambiguity. This makes neo-additive capacities the superior choice when compared to general capacities, compare Eichberger et al. (2005), Eichberger et al. (2007) and Chateauneuf et al. (2007). Second, focusing on neo-additive capacities is particularly appealing in the Avery and Zemsky (1998) framework, since it rids us of investor beliefs that are unintuitive. For instance, neo-additive beliefs prevent investors from assigning higher likelihoods to the state that is objectively less likely. Finally, in line with Chateauneuf et al. (2007), neo-additive capacities allow us to relate individual degrees of optimism and pessimism to other components of the Avery and Zemsky (1998) model such as the bid and ask price. We can, thus, intuitively describe if and when irrational exuberance (despair) may lead to herding that moves prices away from fundamentals.[7]

We should mention, however, that the theoretical finance literature investigating investor behavior under ambiguity outside social learning settings gravitates towards the multiple prior framework.

Examples of static investment and portfolio choices include the works of Bossaerts et al. (2010), Gollier (2011) and Schröder (2011). They use multiple prior setups such as smooth ambiguity functions as introduced by Klibanoff et al. (2005) (KMM approach) or α-maxmin decision rules to model ambiguity and ambiguity preference.[8] Intertemporal financial choices under ambiguity are discussed in e.g. Klibanoff et al. (2009). They generalize the KMM framework to an intertemporal setting, deriving a recursive representation for ambiguity preferences. Ju and Miao (2012) employ the generalized KKM framework to model intertemporal asset pricing and investment choices under ambiguity.

There is, however a very strong unifying assumption underlying the KMM, α-maxmin and neo-additive Choquet frameworks. That is, the decision makers' ambiguity attitude is not necessarily limited to aversion but may also reflect lovingness for ambiguous gambles.[9] Indeed, robustness checks reveal that the results in this paper can be replicated when employing a multiple prior setup with smooth ambiguity preferences or α-maxmin decision rules.

[7] To the best of our knowledge, the concepts of optimism and pessimism have not yet been associated with the mentioned multiple prior frameworks.

[8] The α-maxmin framework is introduced by Ghirardato et al. (2004) and can be seen as the multiple prior counterpart of neo-additive Choquet preferences.

[9] Recent applications of the multiple prior framework include variational and multiplier preferences and are particularly designed to apply ambiguity aversion to intertemporal optimization problems, compare Ghirardato et al. (2004) and Hansen and Sargent (2001) respectively. Since they exclude ambiguity lovingness by definition, they are not suited for our application.

Ambiguity in our framework can be seen as an agent's lack of confidence in the validity of her information (informational ambiguity). Hence, it is natural to choose a market model, where herding (and contrarianism) is triggered by *information externalities* that an investment decision by one agent imposes on subsequent agents' expectations about the asset value, compare the seminal work of Bikhchandani et al. (1992).[10]

We choose the two-state, two-trader version of Avery and Zemsky (1998) as the baseline model over more recent and complex market microstructure herd models such as Park and Sabourian (2011) or Cipriani and Guarino (2014) since we want to avoid unnecessary distractions due to complex model features.[11] Indeed, in the baseline model without ambiguity neither herding nor contrarianism are possible, compare Avery and Zemsky (1998). This constitutes a sharp and, hence, illustrative contrast to investor behavior under ambiguity. Having said that, we will also argue that the insights from this paper are conveniently transferred to more complex setups.

The remainder of this paper is organized as follows: In Section 5.2, we revisit the model of Avery and Zemsky (1998) and discuss investor behavior if there is no ambiguity. In Section 5.3, we apply ambiguity to the model of Avery and Zemsky. We derive the necessary and sufficient conditions for herding and contrarianism under ambiguity and discuss corresponding market outcomes in Section 5.4. In Section 5.5, we introduce the perturbed model and highlight differences of price-dynamics under herding and contrarianism. Section 5.6 is devoted to the discussion of the robustness of our findings, while Section 5.7 concludes. Technical proofs as well as additional material and deep dive analyses are found in the Appendix.

[10] Alternative drivers for herd behavior include reputational concerns as well as investigative herding. Reputational herd models modify the agents' objective functions such that their decisions are affected by positive externalities from a good reputation, see e.g. Scharfstein and Stein (1990), Graham (1999) and Dasgupta et al. (2011). Investigative herd models examine conditions under which investors may choose to base their decisions on the same information resulting in correlated trading behavior, see e.g. Froot et al. (1992) and Hirshleifer et al. (1994).

[11] Other financial market herd models such as Lee (1998), Chari and Kehoe (2004), and Cipriani and Guarino (2008), investigate how investor herding is related to transaction costs, endogenous timing of trading decisions, and informational spillovers between different assets, respectively.

5.2 The Baseline Herd Model Without Ambiguity

This section reviews the two-state, two-trader version of the model of Avery and Zemsky (1998) and presents it's key property: Without ambiguity no herding and no contrarianism are possible.

5.2.1 The model setup

Avery and Zemsky (1998) consider a sequential trading model in the spirit of Glosten and Milgrom (1985), consisting of a single asset, informed as well as noise traders and a market maker. The model assumes rational expectations and common knowledge of it's structure. Moreover, all decisions in the model are decisions under risk, i.e. there is no ambiguity. We refer to the model specified in this section as the baseline model.

The Asset: There is a single risky asset with unknown fundamental value $V \in \{V_0, V_1\}$, where $V_0 < V_1$. We refer to V_1 as the high state and V_0 as the low state. Without loss of generality, let $V_0 = 0$ and $V_1 = 1$. The prior is fully characterized by the prior probability for the high state $\pi_0 := P(V = V_1)$ and assumed to be non-degenerate, i.e. $0 < \pi_0 < 1$. The asset is traded over T consecutive points in time. After T, the true state of the world is revealed and traders receive their payment accordingly.

The Market Maker: Trading takes place in interaction with a market maker who quotes a bid and an ask price at every time $t = 1, ..., T$. The market maker only has access to public information, consisting of the history of trades H_t and the risky asset's prior distribution π_0. The trade history is defined as $H_t := \{(a_1, p_1), ..., (a_{t-1}, p_{t-1})\}$, where $a_i \in \{\text{buy, sell, hold}\}$ is the action of a trader in period $i \geq 1$ and p_i is the price at which that action is executed.

The relevant public information is fully reflected by the public belief regarding the asset's true value, which is given by $E[V \mid H_t] = P(V = 1|H_t) =: \pi_t$.[12]

[12] π_t uniquely identifies the history of trades up to the number of holds. In particular, it can be bijectively mapped to any order imbalance in the trade history. We will, hence, also refer to π_t as the market's sentiment or degree of optimism. This argument is discussed formally in Section 5.8.E the Appendix of this paper, see Proposition 5.8.

In line with Avery and Zemsky (1998), we also refer to π_t as the asset's price in period t. The market maker is subject to Bertrand competition and, thus, quotes bid and ask prices according to a zero-profit condition. Formally, we have $ask_t = E[V|H_t \cup \{a_t = buy\}]$ and $bid_t = E[V|H_t \cup \{a_t = sell\}]$.

The Traders: Traders arrive at the market one at a time in a random exogenous order and decide to buy, sell, or not to trade one unit of the asset at the quoted bid and ask prices. Traders are either risk neutral informed traders or noise traders. The fraction of informed traders is denoted by μ. Informed traders base their decision to buy, sell, or not to trade on their expectations regarding the asset's true value. In addition to publicly available information, informed traders form their beliefs based on a private signal $S \in \{S_0, S_1\}$. We refer to S_0 as the low signal and S_1 as the high signal.[13] Informed traders buy (sell) one unit of the asset if their expected value of the asset $E[V \mid S, H_t] = P(V = 1|S, H_t)$ is strictly greater (smaller) than the ask (bid) price quoted by the market maker.[14] Otherwise, they choose not to trade. In contrast to informed traders, noise traders choose their action randomly, that is, they decide to buy, sell, or not to trade with equal probability of 1/3. Consequently, the probability that a noise trader arrives at the market and either buys, sells or holds the asset is equal to $(1 - \mu)/3$. For notational convenience, we define $(1 - \mu)/3 =: \theta$.

The Private Signal: The distribution of the private signals S_0, S_1 is conditional only on the true state of the world and is denoted by $P(S|V)$. In particular, it does not depend on the trading history H_t. Without loss of generality, we assume symmetric binary signals (SBS) with precision $1 > q > 0.5$, i.e. $P(S_i|V_i) = q$ for $i = 1, 2$. Assuming $q > 0.5$ ensures, that signals are informative in the sense, that they point an informed trader towards the true state of the world. If the low state realizes, then it is more likely to receive a low signal than receiving a high signal (and vice versa if the high state realizes). The larger q, the less noisy and more informative the signal gets.

[13]Throughout this paper, by an abuse of notation, we also label the informed trader who receives signal S, by S.

[14]We can think of traders being endowed with one unit of money. In that sense, selling the asset really means to short-sell it. The investors' endowment is risk and ambiguity free.

Updating: Belief updating follows Bayes' rule. Public beliefs are updated from π_t to π_{t+1} when a trading decision a_t is observed in $t+1$. Similarly, the public belief π_t is updated to a private belief $E[V|S, H_t]$ if a trader arriving at the market at time t has received a private information signal S.[15]

The updating rules imply that for any fixed model parameterization the market maker's bid and ask quotes as well as the informed traders' asset valuations in t only depend on the price π_t. We can, thus, view ask_t and bid_t as well as $E[V|S, H_t]$ as functions of π_t. As a notational convention we write $E[V|\cdot, H_t] = E[V \mid \cdot, \pi_t] = E_{\pi_t}[V \mid \cdot]$ and $ask_t = ask(\pi_t)$ and $bid_t = bid(\pi_t)$. We will sometimes omit the time index for convenience.

Herding and Contrarianism: In line with Avery and Zemsky (1998), we define herding (contrarianism) as a "history-induced switch of opinion of a certain informed trader in (against) the direction of the crowd", compare Brunnermeier (2001). For instance, if an informed trader S sells the asset initially based on her asset valuation $E[V \mid S]$ but decides to buy the asset at $t \geq 1$ after she has observed a price increase (decrease), she is said to engage in buy herding (contrarianism).[16]

Informational Cascade: Following Avery and Zemsky (1998) we say that an informational cascade occurs at time t if and only if $P(a_t|V, H_t) = P(a_t|H_t),\ \forall a_t$. This characterizes a situation where the public cannot or does not infer any information from the observation of a trade, i.e. if $P(V|H_{t+1}) = P(V|H_t)$. To see this, note that during an informational cascade Bayes' Rule implies

$$P(V|H_{t+1}) = P(V|H_t, a_t) = \frac{P(a_t|V, H_t)P(V|H_t)}{P(a_t|H_t)} = P(V|H_t),$$

where the last equality holds due to the informational cascade definition.

Sometimes an informational cascade is also defined as a situation when all informed traders take the same action irrespective of their information signal,

[15] For the readers convenience, we have stated the formulas for the informed traders' and the market's beliefs as well as bid and ask prices with respect to model parameters in Lemma 5.10 in Section 5.8.E in the Appendix of this paper.

[16] The definition for sell herding and contrarianism is symmetric if S buys initially. For formal definitions, see Avery and Zemsky (1998) or Park and Sabourian (2011).

compare e.g. Cipriani and Guarino (2008).[17] We note, that as long as the whole model structure is common knowledge, this alternative definition is equivalent to the one we use here. We will, however, also consider a perturbed version of our model, where the market is uncertain regarding the true investor preferences. In this case it is conceivable that all traders take the same action, yet, the market still infers information from observed trades. Hence, the more general definition of Avery and Zemsky (1998) prevents us from wrongly identifying a situation as an informational cascade, while social learning still continues.[18]

Having revisited the two-state, two-trader version of the Avery and Zemsky (1998) model, we now state the key result regarding investor behavior.

5.2.2 Investor Behavior in the Baseline Model

Informed traders in the Avery and Zemsky (1998) model never change their initial trade decision. Low signals always sell the asset while high signals always buy the asset. This fact is summarized in the following

Proposition 5.1. ***Avery And Zemsky***
Informed traders in the two-state, two-trader model of Avery and Zemsky always follow their private signals, i.e. $\forall t$ *and histories* H_t*:*

$$0 < E[V \mid S_0, H_t] < bid_t < \pi_t < ask_t < E[V \mid S_1, H_t] < 1.$$

Proof: Avery and Zemsky (1998).

□

An immediate consequence is that neither herding nor contrarianism is possible. Both types of investor behavior require that traders change their initial trade decision, which never happens due to Proposition 5.1. This is illustrated in Figure 5.1. For any price $\pi_t \in (0;1)$ (and thus any conceivable history H_t) the expectation of the high signal remains above the ask price while the expectation

[17]The intuition behind this is appealing. If all informed traders take the same action independent of their signal, the market cannot infer any information from their actions any more. Consequently, social learning and price updating stop.

[18]A formal discussion of the different definitions of informational cascades is provided in Proposition 5.9 in the Appendix.

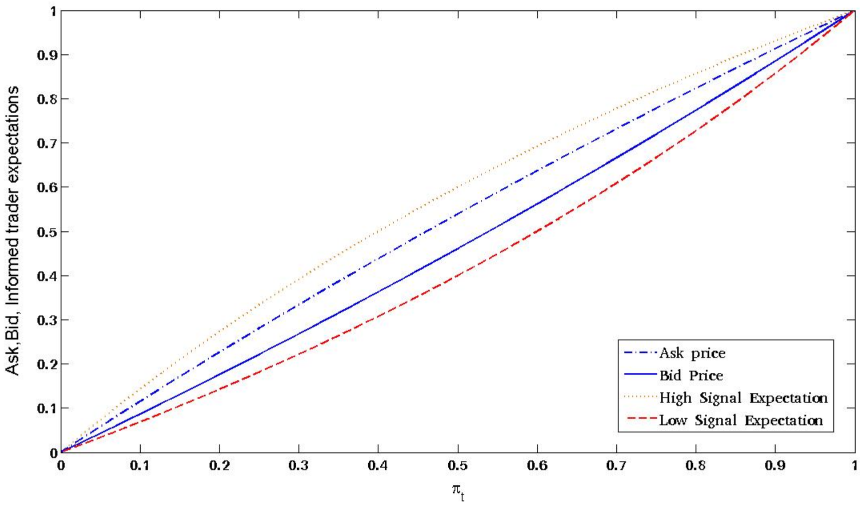


Figure 5.1: Trading decisions of informed traders in Avery and Zemsky (1998).

Notes: Informed trader expectations $E[V \mid S, \pi_t]$, bid price bid_t and ask price ask_t are depicted with respect to the public prior belief at time t, π_t. The informed trader share $\mu = 0.3$ and the signal precision $q = 0.6$.

of the low signal remains below the bid price. Analyses in Avery and Zemsky (1998) and Chamley (2004) show that the market confidently learns about the true value of V in this case. The higher the signal precision q and the informed trader share μ, the faster the market learns.

These clearcut results are an important reason for choosing the two-state, two-trader version of Avery and Zemsky (1998) as our baseline model. It allows us to highlight that introducing informational ambiguity to the model in the next section, indeed, has game changing effects on investor behavior and social learning.

5.3 Introducing Ambiguity to the Baseline Herd Model

In this section we apply the concept of ambiguity to the model framework of Avery and Zemsky (1998). We show how the assumption that informed traders have non-extreme-outcome-additive (neo-additive) Choquet Preferences affects their asset valuation based on the insights provided by Chateauneuf et al. (2007).[19] We particularly focus our discussion on the role of the perceived ambiguity δ as well as the informed traders' attitude towards ambiguity α. In line with Eichberger et al. (2010), we provide updating rules for the neo-additive Choquet Expected Utility (CEU) beliefs. Finally, formal definitions for herding and contrarianism for investors with CEU preferences are provided. For the remainder of this paper, we refer to this model as the CEU model.

5.3.1 Investors with NEO-Additive Preferences

As we introduce ambiguity to the model of Avery and Zemsky, we make three general assumptions. First, in order to isolate the effects of ambiguity on investor decisions and social learning, we assume that informed traders as well as the market maker remain risk neutral. Second, the market maker does not perceive ambiguity. We may think of the market maker as an invisible hand that enforces a normatively acceptable price mechanism. Bid and ask prices as well as the public belief π_t should, therefore, be inherently unambiguous. Third, we consider investor preferences to be part of the common knowledge structure of the model.[20]

To incorporate ambiguity, we assume that informed traders have neo-additive CEU preferences. An individual with this type of preference assigns additive probabilities to every event that does not include the best and the worst outcome. For extreme outcomes neo-additive agents assign a weighted average of additive probabilities and non-additive likelihoods. Since in the two-state world of Avery and Zemsky every outcome is extreme, the resulting neo-additive CEU valuation is particularly simple to derive. In line with Chateauneuf et al.

[19] A similar exercise has been conducted by Ford et al. (2013). A toolbox of the mathematical objects and results related to the neo-additive ambiguity concept is provided Section 5.8.G in the Appendix.

[20] This is a key distinguishing feature from the model of Ford et al. (2013). In Section 5.5 we relax the common knowledge assumption to study differences of stylized price dynamics in markets prone to herding and markets prone to contrarianism.

(2007), we infer that an informed trader with neo-additive CEU preferences and signal S values the asset at

$$CEU[V \mid S, H_t] = (1 - \delta_S)E[V \mid S, H_t] + \delta_S \alpha, \tag{5.1}$$

where $\alpha, \delta_S \in [0; 1]$.[21]

$CEU[V \mid S, H_t]$ is essentially a weighted average of the subjective expected utility (SEU) valuation $E[V \mid S, H_t]$ and a subjectively assigned likelihood α that $V = 1$ is the true state of nature. In line with Chateauneuf et al. (2007), we regard α as the individual degree of optimism. Indeed, the higher α, the more likely the investor considers the high state to be true, the more optimistic, excited or exuberant she gets regarding the investment prospect and it's pay-off (and vice versa).[22] The weighting parameter δ_S is the degree of perceived ambiguity and can be viewed as the investor's lack of confidence in her ability to form a SEU belief.The higher δ_S the more the investor relies on her gut feeling α as to whether the low or the high state is true.[23]

Throughout this paper we assume that the asset valuations of the different informed trader types are monotone in the sense that $CEU[V \mid S_0, H_t] \leq CEU[V \mid S_1, H_t]$. From an economical perspective this can be seen as the ambiguity version of a weak form of the Monotone Likelihood Ratio Property (MLRP) of private signals. Indeed, Park and Sabourian (2011) show that MLRP signal structures imply that the order of the informed traders' asset valuations is the same for all histories H_t. Our monotonicity assumption constitutes a corresponding property under ambiguity. Park and Sabourian (2011) label MLRP and associated trade behavior as "well-behaved". We presume that this "well-behavedness" is preserved under ambiguity.[24]

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[21] The same result has been obtained by Ford et al. (2013). In Section 5.8.G in the Appendix we provide a more detailed and formal derivation of Equation (5.1).

[22] From a decision theoretic perspective, α is primarily a preference parameter describing the investor's attitude towards ambiguity. In line with Ghirardato and Marinacci (2002), the investor is absolutely ambiguity loving (averse) if and only if $\alpha > E[V \mid S, H_t]$ ($\alpha < E[V \mid S, H_t]$. She is absolutely ambiguity neutral if and only if $\alpha = E[V \mid S, H_t]$. Technical details on this are provided in Proposition 5.10 in Section 5.8.G of the Appendix of this paper.

[23] For an intuitive example of an investor facing ambiguity, see Section 5.8.A in the Appendix.

[24] We stress that the results of this paper do not hinge on this assumption. Yet, stating the results and discussing them is facilitated.

For the remainder of this paper, whenever we speak of CEU, we actually mean CEU with respect to neo-additive capacities unless explicitly stated otherwise.

5.3.2 Updating CEU Beliefs

5.3.2.1 General Bayesian Updating (GBU)

The GBU rule of Eichberger et al. (2010) implies that upon the arrival of new information, i.e. the observation of a trade, the additive part of neo-additive beliefs in Equation (5.1), $E[V \mid S, H_t]$, is updated according to Bayes' rule as usual. In addition, the degree of ambiguity δ_S is also updated, while the degree of individual optimism α remains fixed.

The updating rule for δ_S is given by

$$\delta_S = \frac{\delta_0}{(1-\delta_0)P(S|H_t)+\delta_0}. \tag{5.2}$$

We note again that the dynamics of δ_S solely depend on π_t and that we can, hence, view δ_S as function of π_t.[25]

The parameter δ_0 can be interpreted as a degree of primary ambiguity that investors perceive when deciding to trade the risky asset. It may stem from the complexity of the asset or from the fact that erratic asset price movements elude established forecasting methods. For instance, derivatives like options, swaps or Collateralized Debt Obligations (CDOs) might trigger a higher primary ambiguity than actual stocks, because they are more difficult to understand and their future values are more difficult to predict accurately. By the same line of reasoning the degree of primary ambiguity should depend on the expertise of the investor. A retail trader perceives much higher degrees of primary ambiguity than a professional investment banker. We assume that $\delta_0 > 0$ is constant across informed traders and during the trading period under consideration $[0; T]$.

Figure 5.2 illustrates that there are two additional sources of ambiguity that

[25] To see this, note that $P(S|H_t) = \pi_t P(S|V_1) + (1-\pi_t)P(S|V_0)$ by the law of total probability and that $P(S|V_i)$ are time-invariant parameters.

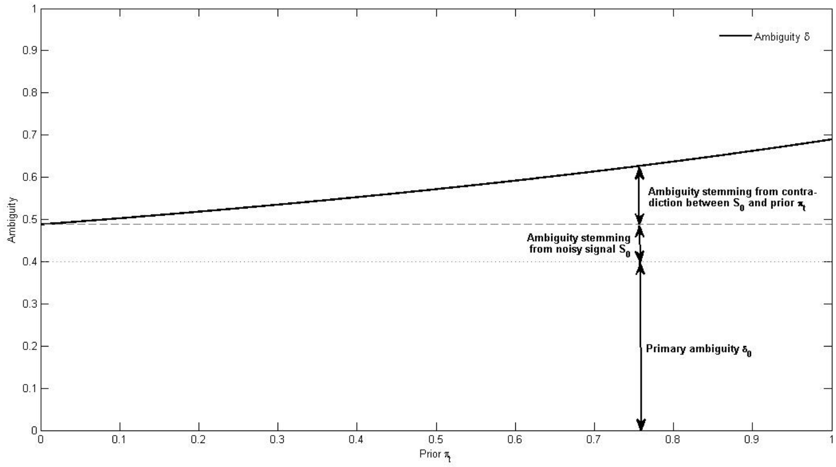


Figure 5.2: Sources of ambiguity for a low signal

Notes: The graph shows the degree of ambiguity δ_{S_0} with respect to prior π_t for S_0. The primary ambiguity level is $\delta_0 = 0.4$, the informed trader share is $\mu = 0.3$, the signal precision is $q = 0.7$. δ_{S_0} is computed according to Equation (5.2).

contribute towards the degree of total perceived ambiguity δ_S. It depicts δ_S with respect to the price π_t for informed traders receiving a low signal S_0 as induced by GBU.

In addition to the fixed level of δ_0, total perceived ambiguity also includes ambiguity stemming from the fact that the private information signal S_0 may contradict the public information reflected in the price π_t and the noisiness of the private signal. For instance, a high price π_t indicates strong market confidence that the high state is the true state. The low signal S_0, however, suggests that the low state is more likely to be true than the high state, thus contradicting the public information reflected in π_t. Indeed, the greater π_t the more the low signal contradicts the public information and the higher the low signal's total degree of perceived ambiguity δ_{S_0}. Yet, even if public and private information are aligned, i.e. if $\pi_t \to 0$, the noise in S_0 prevents the informed trader from fully discounting the possibility that the high state is true. As a consequence, the δ_{S_0} remains strictly above the degree of primary ambiguity δ_0 for all prices π_t.[26]

[26]If a trader receives a perfect signal (no noise) there is no informational ambiguity on top of primary ambiguity. In that case, we have $P(S|H_t) \equiv 1$ which implies that $\delta_S \equiv \delta_0$ for all π_t.

Eichberger et al. (2010) argue that α is an individual ambiguity preference parameter which should not be affected by the arrival of new information. Yet, since we want to study whether potentially irrational exuberance and outright panics drive investor herding, allowing α to vary might prove insightful. Moreover, the following section shows that the economic literature has readily documented that the conceptually related *risk* preference in fact is subject to change.

5.3.2.2 Departing from GBU - Varying Degrees of Optimism

It is fairly common in the finance literature to assume that the degree of *risk* aversion depends on the market sentiment, see e.g. Campbell and Cochrane (1999). This is supported by a large body of empirical evidence showing that during crisis periods risk aversion increases, while it tends to vanish during boom phases, see e.g. Jurado et al. (2015), Bekaert et al. (2013) and Bollerslev et al. (2011). Given the conceptual similarities of risk and ambiguity aversion, we argue that the individual degree of optimism α (ambiguity aversion) should also depend on the general market sentiment.

In our model framework market sentiment is best captured by the price π_t. The higher π_t the stronger the degree of optimism exhibited by the market as a whole that V is a lucrative investment opportunity. Since it is reasonable to assume that market-wide optimism affects individual optimism, allowing α to vary with π_t is a feasible generalization of GBU.

Formally, we set $\alpha = \alpha_S(\pi_t \mid q, \pi_0, \cdot)$, i.e. it varies with the price but may also depend on the signal precision, the information signal S or exogenous events. A low signal S_0, for example, may dampen optimism or boost panic. Likewise, strong and accurate information signals, i.e. a high q, might prevent investors from overreacting to changes in market sentiment, while low q could make the CEU trader particularly susceptible for such mood swings. Finally, we observe that the informed trader share μ plays no explicit role when informed traders form additive beliefs in the baseline model without ambiguity.[27] Consequently, we would argue that μ should not directly affect CEU beliefs

[27] For example, note that $E[V \mid S_0, H_t] = \frac{(1-q)\pi_t}{(1-q)\pi_t + q(1-\pi_t)}$.

neither. To ensure this, α must be independent of μ.

We make three additional assumptions regarding $\alpha(\pi_t \mid \cdot)$ for convenience. These assumptions are not crucial for the results derived in this paper, yet, they allow us to state them in a lean and intuitive way.[28]

(A1) For π_0 both informed trader types act as if they were ambiguity neutral, i.e. $\alpha_S(\pi_0) = E[V \mid S]$.

(A1) implies that S_0 and S_1 type informed traders have different ambiguity functions. While such an assumption may seem ad hoc, we stress that it is made without loss of generality and only to focus our discussion on the case where the low (high) signal sells (buys) initially. (A1) prevents us from being distracted from less interesting scenarios. For example, it precludes the possibility that the two informed trader types take the same action in $t = 0$, which would cause an informational cascade right at the beginning of trading. Moreover, conditions for herding and contrarianism derived under (A1), i.e. conditions under which traders switch their initial trade decision from selling to buying and vice versa, also hold for weaker forms of switching behavior, i.e. from holding into buying and selling.

(A2) $\alpha(\cdot)$ is sufficiently regular in π_t and the change in α is marginal as the market becomes confident about either state, i.e. $\frac{\partial \alpha}{\partial \pi}(1) = \frac{\partial \alpha}{\partial \pi}(0) = 0$.

(A3) The individual degree of optimism is identical for all low signal traders and all high signal traders respectively.

The updating of δ_S as well as the additive belief component of CEU remains as under GBU. Since δ_S and α_S as well as the additive component $E[V \mid S, H_t]$ can be viewed as functions of π_t, we may also consider CEU as a function of the

[28] Section 5.6 highlights the effects of generalizing (A3) to obtain a framework where investor preferences follow a random distribution. Section 5.8.B in the Appendix discusses technical effects of dropping (A1) to (A3) on the stated Lemmas and Theorems.

price. In line with Section 5.2, we write $CEU[V \mid S, H_t] = CEU_S(\pi)$ for notational convenience.

Now that we have formalized how traders with neo-additive preferences facing ambiguity value the risky asset, we can provide appropriately adjusted definitions for herd and contrarian behavior.

5.3.3 Herding and Contrarianism in the CEU Model

We modify Avery and Zemsky (1998)'s definition of herding and contrarianism to account for the fact that investor perceive ambiguity and have neo-additive preferences.

Definition 5.1. ***Herding With NEO-Additive preferences***
An informed trader with neo-additive CEU preferences and signal S ***buy herds*** *in $\hat{t}$ at history $H_{\hat{t}}$ if the following three conditions hold:*

*(*BH1)* $CEU[V \mid S] < bid_0$, *i.e. an informed trader with signal S and neo-additive CEU preferences sells at $t = 0$,*

*(*BH2)* $CEU[V \mid S, H_{\hat{t}}] > ask_{\hat{t}}$, *i.e. an informed trader with signal S and neo-additive CEU preferences buys in $t = \hat{t}$.*

*(*BH3)* $\pi_{\hat{t}} > \pi_0$, *i.e. the asset price has increased during $[0; \hat{t}]$.*

Analogously, an informed trader ***sell herds*** *in period $\hat{t}$ at history $H_{\hat{t}}$ if and only if (*SH1) $CEU[V \mid S] \geq bid_0$, (*SH2) $CEU[V \mid S, H_{\hat{t}}] < bid_{\hat{t}}$, and (*SH3) $\pi_{\hat{t}} < \pi_0$ hold simultaneously.*

These modifications ensure that in line with Brunnermeier (2001) and the bulk of the theoretical herding literature, an informed trader's switch in opinion is still induced by the observed trade history. More precisely, herding would by definition be impossible if the trade decisions of other investors were not observable, compare opaque market in Park and Sabourian (2011).[29]

The corresponding definition for contrarian behavior is

[29] This is not the case in Ford et al. (2013). They consider any buy (sell) decision of a low (high) CEU signal to be a corresponding herding trade as long as the price has increased (decreased).

Definition 5.2. ***Contrarianism With NEO-Additive preferences***
An informed trader with with neo-additive preferences and signal S acts as a ***buy contrarian*** *in $\hat{t}$ at history $H_{\hat{t}}$ if the following three conditions hold:*

*(*BC1) $CEU[V \mid S] < bid_0$, i.e. an informed trader with signal S and neo-additive CEU preferences sells in $t = 0$.*

*(*BC2) $CEU[V \mid S, H_{\hat{t}}] > ask_{\hat{t}}$, i.e. an informed trader with signal S and neo-additive CEU preferences buys in $t = \hat{t}$.*

*(*BC3) $\pi_{\hat{t}} < \pi_0$, i.e. the asset price has decreased during $[0; \hat{t}]$.*

Analogously, an informed trader acts as a ***sell contrarian*** *in period $\hat{t}$ at history $H_{\hat{t}}$ if and only if (*SC1) $CEU^S_{\delta_0,\alpha}[V] > ask_0$, (*SC2) $CEU^S_{\delta_0,\alpha}[V|H_{\hat{t}}] < bid_{\hat{t}}$, and (*SC3) $\pi_{\hat{t}} > \pi_0$ hold simultaneously.*

With these definitions at hand, we are now prepared to investigate investor behavior in the model of Avery and Zemsky under ambiguity.

5.4 Investor Behavior and Social Learning under Ambiguity

In this section we present the main results. We will first solve the CEU model by providing equilibrium prices and updating rules. We then investigate investor behavior in the CEU model under GBU and varying α, respectively. As we derive necessary and sufficient conditions for herding and contrarianism, we note that both types of investor behavior will inevitably lead to informational cascades. As we study the characteristics of the corresponding market outcomes, we find that herders and contrarians are equally likely to cause prices to move away from fundamentals. This probability is derived analytically and comparative statics are conducted.

5.4.1 Solving the CEU Model

We conjecture that unlike in the baseline model without ambiguity in Section 5.2, informed traders with neo-additive CEU preferences may change their initial trade decision after having observed certain histories of trades. If S_0 and S_1 take the same action at any time t, an informational cascade occurs. Social learning stops and the market maker quotes ask and bid prices equal to π_t.[30] Even if there is no informational cascade, any decision change of an informed trader affects the market maker's price setting as well as the public belief updating. We shall begin by formalizing the market maker's price setting.

Lemma 5.1. ***Equilibrium Prices in the CEU Model***
Under the assumptions of the CEU model, let bid_t and ask_t be the bid and ask prices that are quoted in the Avery and Zemsky model at any time t. If there is no informational cascade in t, then the market maker quotes

$$bid_t^{CEU} = \min\{\max\{bid_t; CEU[V \mid S_0, H_t]\}; \pi_t\}$$

and

$$ask_t^{CEU} = \max\{\min\{ask_t; CEU[V \mid S_1, H_t]\}; \pi_t\}.$$

If there is an informational cascade in t, then the market maker quotes $bid_t^{CK} = ask_t^{CK} = \pi_t$

Proof: We have already established pricing given that S_0 and S_1 take the same action at t, i.e. if there is an informational cascade. As long as S_0 sells and S_1 buys, prices are as in the Avery and Zemsky model. Indeed, since $CEU[V \mid S_0, H_t] < bid_t < \pi_t$ in this case, we have that $bid_t^{CEU} = bid_t$. The same argument applies for $ask_t^{CEU} = ask_t$.
Moreover, monotonicity of the CEU-beliefs implies, that S_1 never sells if S_0 does not sell, and that S_0 never buys if S_1 does not buy. This leaves only two additional cases to consider. First, the case where S_0 holds and S_1 buys and second, the case where S_1 holds and S_0 sells. For symmetry reasons, we will only prove the first case.

Let ask_t and bid_t denote the ask and bid prices the market maker quotes in the Avery and Zemsky model. Assume that at some time t, the high signal still buys

[30] Compare Avery and Zemsky (1998) and Cipriani and Guarino (2008) for detailed discussions of informational cascades as well as Proposition 5.9 in Section 5.8.F in the Appendix of this paper.

and we have $bid_t \leq CEU[V \mid S_0, H_t] \leq \pi_t < ask_t$. This implies that the low signal with CEU preferences holds in t.

Since the market maker (and his fictive Bertrand competition) are aware of this, the zero-profit condition implies an increase of the quoted bid price to $bid_t^{CEU} = CEU[V \mid S_0, H_t]$. If the market maker set $bid_t^{CEU} < CEU[V \mid S_0, H_t]$, then he would make an average gain on every sell of $\pi_t - bid_t^{CEU}$. The market maker's competition's best response is to quote a bid price $\widetilde{bid}_t^{CEU} > bid_t^{CEU}$ such that $\widetilde{bid}_t^{CEU} < CEU[V \mid S_0, H_t]$ and $\pi_t - \widetilde{bid}_t^{CEU} > 0$. In other words, the competition can quote a better bid price, thereby drawing away all noise traders that sell the asset, while still making profits. In turn, the optimal response of the market maker then is to increase bid_t^{CEU} correspondingly. This price war continues until $bid_t^{CEU} = \widetilde{bid}_t^{CEU} = CEU[V \mid S_0, H_t]$. If the market maker quoted $bid_t^{CEU} > CEU[V \mid S_0, H_t]$, then the low signal would sell at t. Consequently, the market maker would make an average loss of $bid_t - bid_t^{CEU}$ for each sell he fills and would, therefore, eventually go out of business. Since the competition is in the same situation, no market maker has an incentive to deviate from the equilibrium bid price of $CEU[V \mid S_0, H_t]$.

If $\pi_t < CEU[V \mid S_0, H_t]_t \leq ask_t$, then the market maker quotes an equilibrium bid price of $bid_t^{CEU} = \pi_t$. A downward deviation is not possible due to Bertrand competition by the same reasoning as above. An upward deviation would cause the market maker to make average losses of at least $\pi_t - bid_t^{CEU}$ for each sell he fills and, therefore, would again lead to bankruptcy.

□

There are two important implications of Lemma 5.1.

First, as soon as an informed trader switches into holding, the market maker can make profits. As long S_1 holds and $CEU_{S_1}(\pi)$ remains above the market price π (S_0 holds and $CEU_{S_0}(\pi)$ remains below π), the market maker on average profits from every buy (sell) he fills. This is in line with the intuition that if traders depart from rationality in the sense of Barberis and Thaler (2003), there will be opportunities for other market participants to make money additional money. In all other cases the zero-profit condition holds.

Second, note that the quoted ask (bid) price in the CEU model remains the same as in the Avery and Zemsky model as long as the high signal S_1 buys and the low signal S_0 does not buy (the low signal S_0 sells and the high signal S_1 does not sell). This is crucial for the derivation of the result on the possibility of herding and contrarianism and it ensures that the corresponding results transfer to the perturbed model setup of Section 5.5.

Next, we derive the updating rules for the public belief π_t in the CEU model.

Lemma 5.2. ***Public Belief Updating in the CEU Model***
Under the assumptions of the CEU model, let bid_t and ask_t be the bid and ask prices that are quoted in the Avery and Zemsky model at any time t.

(1) If the high signal buys and the low signal sells at time t, then π_{t+1} is as in the Avery and Zemsky model.

(2) If both signals take the same action in t, then there is an informational cascade and $\pi_{t+1} = \pi_t$.

(3) If the high signal buys and the low signal holds at time t, then

$$\pi_{t+1} = \begin{cases} ask_t, & \text{if } a_t = \{buy\} \\ \pi_t, & \text{if } a_t = \{sell\} \\ bid_t, & \text{if } a_t = \{hold\}. \end{cases} \tag{5.3}$$

(4) If the high signal holds and the low signal sells at time t, then

$$\pi_{t+1} = \begin{cases} \pi_t, & \text{if } a_t = \{buy\} \\ bid_t, & \text{if } a_t = \{sell\} \\ ask_t, & \text{if } a_t = \{hold\}. \end{cases} \tag{5.4}$$

Proof: Case (2) is directly implied by the definition of an informational cascade. To see, that cases (1), (3) and (4) hold, note, that informed traders still reveal their fundamentally driven signal through their action. Since in case (1), informed traders decide as in the Avery and Zemsky model price updating also coincides. In case (3) - where S_1 buys and S_0 holds - a sell would contain no informational value, because it is a noise trade for sure. Consequently, the price remains constant. If a buy is observed, the market maker knows, that it is conducted by either the high signal or a noise trader. Consequently, updating after a buy is exactly the same as in the Avery and Zemsky model. Finally, if a hold is observed, the market maker knows, that it is due to the low signal or a noise trader. This is equivalent to the situation of an observed sell in the Avery and Zemsky model. Consequently, the market maker sets $\pi_{t+1} = bid_t$, when observing a hold. The argument is symmetric for case (4), i.e. if S_1 holds and S_0 sells. □

Lemma 5.2 implies that the probability to observe a price increase (decrease), i.e. $\pi_{t+1} > \pi_t$ ($\pi_{t+1} < \pi_t$) remains constant as long as there is no informational cascade.[31] This is crucial for deriving a closed formula for the probability of price-distorting market outcomes below.

Having solved the model dynamics, we turn to the analysis of investor behavior in the CEU model. We will first focus on the case where informed traders update their CEU belief in accordance with the GBU rule. We will then consider the general case, where α may vary with the price π_t.

5.4.2 Investor Behavior in the CEU Model under GBU

This section shows that if informed traders perceive ambiguity and update their neo-additive CEU preferences according to the GBU rule, i.e. they exhibit invariant ambiguity preference α, then there is *no herding* in the CEU model. At the same time, informed traders show *strong contrarian tendencies* that prevent the market from becoming confident about either state.

Theorem 5.1.
In the CEU with $\alpha \in [0; 1]$ fixed, ***no herding*** *can occur. If, in addition, S_0 sells initially and S_1 buys initially, then* ***contrarianism occurs*** *with positive probability.*

[31] In particular, note that the probability of a price increase (decrease) is the same as in the baseline model, where it coincides with the probability of buy (sell), see Lemma 5.10 in Section 5.8.E in the Appendix of this paper for the respective formulas.

Proof: Since this proof is fairly technical, it is left to the Appendix.

□

We require, that S_0 (S_1) sells (buys) initially to avoid situations in which both informed trader types take the same action initially, thereby causing an informational cascade right at the beginning of trading.[32] Note, that Theorem 5.1 contradicts some of the key results in Ford et al. (2013). This is because we apply different definitions for herding and contrarianism. Although the mathematical proof is left to the Appendix, we would like to provide some intuition for our result.

The impossibility of herd behavior stems from the fact that neo-additive CEU traders' beliefs are anchored around α. As a result, their belief updating process exhibits a strong degree of sluggishness. CEU traders show particular reluctance in following the crowd. Consider for instance a low signal type trader. Since she sells initially by assumption, she can only engage in buy herding. When she observes a price increase, the additive part of the CEU belief, $E_{\pi_t}[V|S_0]$, increases in line with Bayes' rule as in Avery and Zemsky (1998). Since, however, she perceives ambiguity regarding the validity of her Bayesian asset valuation she tends to rely on her gut feeling to some extent, i.e. her individual degree of optimism. Under GBU, α is, however, unaffected by the price increase. As a consequence, the upward revision of $CEU[V \mid S_0, H]$ turns out be smaller than the corresponding belief revision of her SEU counterpart from the model of Avery and Zemsky. Consequently, since $E_\pi[V|S_0] < bid(\pi) \leq bid^{CEU}(\pi)$ for all π, S_0 keeps selling a fortiori after having observed a price increase given that she sells to begin with.[33] This precludes her from ever engaging in buy herding behavior. The argument is symmetric for the high signal.

The mechanism preventing herding is, at the same time, the key driver for contrarianism. The sluggish belief updating makes neo-additive CEU traders prone to act against the crowd. As the asset price approaches one of the possible true states, i.e. if $\pi_t \to 1$ (or $\pi_t \to 0$), both informed trader types will eventually

[32] We can enforce this condition by setting $\alpha_S = E[V \mid S]$. The effects of dropping this assumption are discussed as we prove Theorem 5.1 in the Appendix.

[33] To see that $E_\pi[V|S_0] < bid(\pi) \leq bid^{CEU}(\pi)$ holds, review Proposition 5.1 and Lemma 5.1.

start selling (or buying) the asset, regardless of their initial trading decision. This is due to the fact, that their non-additive belief component bounds away their asset valuation from 1 and 0, as the public becomes increasingly confident about either state. Assume for instance a price decrease. As $\pi_t \to 0$, the quoted bid and ask prices also approach zero. At the same time, the non-additive part of the informed traders CEU beliefs $\delta_S \alpha_S$ is bounded away from 0.[34] Given their initial trading decisions the low signal can only engage in *buy* contrarianism while the high signal can only engage in *sell* contrarianism.

5.4.3 Investor Behavior in the CEU Model with Varying α

We now investigate investor behavior under the assumption that the individual degree of optimism α varies with the price π_t.

5.4.3.1 Irrational Exuberance and Herd Behavior

In this section we derive necessary and sufficient conditions for herd behavior in the CEU model with varying α. We begin our analysis by deriving a necessary condition.

Neccessary Condition: The essential finding is that the degree of optimism α has to move pro-cyclically, i.e. increase with the market price. In addition, the individual reaction to market-wide optimism (pessimism) needs to be strong enough, i.e. informed CEU traders need to become particularly exuberant (desperate).

Theorem 5.2. ***Necessary Condition for Herding***
Consider the CEU model with varying α.
*If **buy herding** occurs with positive probability, then $\exists \pi \in (\pi_0; 1) : \alpha_{S_0}(\pi) > ask(\pi)$.*
*If **sell herding** occurs with positive probability, then $\exists \pi \in (0; \pi_0) : \alpha_{S_1}(\pi) < bid(\pi)$.*

[34] There are some peculiarities if $\alpha = 1$ or $\alpha = 0$. These cases of pure optimism and pessism are discussed in the Appendix.

Proof: Due to symmetry reasons, we show only the buy herding statement. (A1) implies that S_0 sells initially while S_1 buys initially. Hence, only S_0 can buy herd (*BH1). If S_0 buy herds at some price $\pi > \pi_0$ (*BH3), then $CEU_{S_0}(\pi) > ask(\pi)$ (*BH2). Consequently,

$$(1 - \delta_{S_0})E_\pi[V \mid S_0] + \delta_{S_0}\alpha_{S_0}(\pi) > ask(\pi).$$

Solving this inequality for $\alpha_{S_0}(\pi)$ after having added and subtracted $\delta_{S_0}ask(\pi)$ on the r.h.s. of the inequality, yields

$$\alpha_{S_0}(\pi) > \frac{\delta_{S_0}\, ask(\pi) + (1 - \delta_{S_0})\, ask(\pi) - (1 - \delta_{S_0})E_\pi[V \mid S_0]}{\delta_{S_0}}.$$

Now noting that $ask(\pi) > E_\pi[V \mid S_0]$, we infer, that the r.h.s. of the inequality is greater than $ask(\pi)$, which proves the statement.

□

In line with the market maker's price setting derived in Lemma 5.1, *bid* and *ask* in Theorem 5.2 refer to the bid and ask prices that are quoted in the similarly parameterized baseline herd model without ambiguity.

Let us provide some additional intuition regarding the buy herding condition. (A1) implies that only S_0 sells initially. In particular, the low signal initially values the asset at $E_{\pi_0}[V \mid S_0]$, i.e. she is neither particularly optimistic nor pessimistic. Even as the asset price π appreciates, the additive part of the low signal's asset valuation $E_\pi[V \mid S_0]$ remains well below the ask price, compare Section 5.2. Hence, S_0 will only decide to buy at some price $\pi_t > \pi$ if her degree of optimism $\alpha_{S_0}(\pi)$ is large enough to compensate for this fact. Since $\alpha_{S_0}(\pi_0) = E[V \mid S_0] < E_\pi[V \mid S_0]$, this means that it is necessary for buy herding, that $\alpha_{S_0}(\pi) >> \alpha_{S_0}(\pi_0)$, i.e. S_0's degree of optimism has to increase with the asset price. An incremental rise of α_{S_0} would, however, be insufficient.

The minimum requirement for S_0 to value the asset above the ask price is $\alpha_{S_0} > ask > \pi$. Noting that $\alpha_{S_0}(\pi_0) < \pi_0$, this implies that buy herding requires that $\alpha_{S_0}(\pi) - \alpha_{S_0}(\pi_0) >> \pi - \pi_0$. Such a disproportionate surge in individual optimism compared to the increase of optimism exhibited by the market, can well be interpreted as (possibly unwarranted) exuberance on the part of the S_0 type trader. The intuition is similar for sell herding of S_1. We would, however, label the required disproportionate increase in individual pessimism as (potentially exaggerated) desperation of the S_1 type traders.

Sufficient Condition: Before stating the formal sufficient condition for herding, let us develop some intuition first. Sufficiency for e.g. buy herding requires that at some point the degree of optimism α surmounts the ask price for good, i.e. $\exists \pi^* > \pi_0$, where $\pi^* < 1$ such that $\alpha(\pi) > ask(\pi)$ for all prices $\pi \in (\pi^*; 1)$. If such an optimism function coincides with high primary ambiguity δ_0, then this is sufficient for S_0 to buy the asset, at least for prices in a neighborhood of 1.

If δ_0 is large the trader's lack of confidence in her additive belief component is strong. Consequently, her asset valuation is strongly biased towards the non-additive component α. If her faith in her gut-feeling is sufficiently strong, $\alpha(\pi) > ask(\pi)$ over-compensates the fact that her additive belief component $E_\pi[V \mid S_0] < ask(\pi)$ driving her asset valuation $CEU_{S_0}(\pi)$ above the ask price for π close to 1. If primary ambiguity is too low, then no amount of optimism will ever drive the low signal's valuation of the asset above the ask price.[35]

This sufficiency condition is in line with the intuition that for example retail traders are more prone to herding than professionals. Indeed, the worse a trader's understanding about financial markets in general and the functioning of a particular financial asset, the higher her degree of perceived primary ambiguity and the more likely, that she will eventually engage in herd behavior.

Theorem 5.3. ***Sufficient Condition For Herding***
Consider the CEU model with varying α.
Let the level of primary ambiguity

$$\delta_0 > 1 - \frac{\mu(1-q)+\theta}{q(\mu+2\theta)},$$

where q denotes the signal precision, μ the informed trader share and $\theta := (1-\mu)/3$ is the probability that a noise trader buys, sells or holds one unit

[35] Compare the case of a pure optimist in the Appendix.

of the asset.
If $\alpha_{S_0}(1) = 1$, *then* ***buy herding*** *occurs with positive probability.*
If $\alpha_{S_1}(0) = 0$, *then* ***sell herding*** *occurs with positive probability.*

Proof: The mathematical proof is again left to the Appendix. The principal idea of the proof has been outlined prior to the statement of Theorem 5.3.

□

A few remarks regarding Theorem 5.3 are in order.

First, the requirement that $\alpha_{S_0}(1) = 1$ ($\alpha_{S_1}(0) = 0$) has intuitive appeal. In the limiting case, when the market becomes confident about either state of the world beyond any doubt, i.e. if risk vanishes completely, then even CEU traders who perceive ambiguity should value the asset at 1 and 0 respectively.

Second, we note that the minimum required amount of primary ambiguity δ_0 depends on the informed trader share μ and the signal precision q. Comparative static analyses reveal that $\delta_0^* = 1 - \frac{\mu(1-q)+\theta}{q(\mu+2\theta)}$ increases in both μ and q.[36] That is, an increase in μ and q tends to reduce investor proclivity to engage in herd behavior. This appears to be intuitive in the case of q. Better informed traders should ceteris paribus be less easily swayed by the crowd to change their trade decisions. In the case of μ the result is driven by the fact that the market maker faces a higher risk that his counter-party is informed. To compensate for that risk, he quotes a higher bid-ask spread which makes extreme switches of traders from selling to buying and vice versa less likely, compare the discussion of the impact of information risk on herding intensity in Paper 1 of this thesis.

Finally, we note that the way Theorem 5.3 is stated, it hinges on (A2). A general version of the sufficiency result that does not require (A2) is provided in Section 5.8.B in the Appendix of this paper.

The question remains, whether we expect to observe herding implied by Theorems 5.2 and 5.3 in the real world.

[36] The formal derivation of these results is based on elementary calculus and has, thus, been omitted from the paper.

Since there is strong evidence that *risk* aversion moves pro-cyclically, see e.g. Bollerslev et al. (2011), we conjecture that ambiguity aversion or optimism should exhibit similar features given the conceptual proximity of these preference parameters.

Moreover, we would argue that there are abundantly many real-world examples where investor behavior showed corresponding characteristics. Popular precedents are the dot-com bubble at the turn of the millenium, the recent US house price bubble as well as the bubble of the Shanghai Composite Index in 2015.

Finally, the relevance of investor herding under ambiguity can be motivated micro-economically by the findings of Heath and Tversky (1991). They provide experimental evidence that once the judged probability of an ambiguous event is high, individuals tend to become ambiguity loving. If the price π_t is in a neighborhood of 1, even an S_0 type trader is fairly confident that the high state is true. Hence, according to Heath and Tversky (1991), S_0 should assign a higher probability to the high state than prescribed by her additive belief component.[37] This would support the idea of Theorem 5.3 that (potentially irrational) exuberance and despair may drive investor herding at least for prices close to 1 and 0.

A Class of Optimism Functions Allowing for Investor Herding: We provide a class of optimism functions $\alpha(\pi|q,\pi_0)$ that are sufficient for herd behavior if $\delta_0 > 1 - \frac{\mu(1-q)+\theta}{q(\mu+2\theta)}$.

For the low signal, we have

$$\alpha_{S_0}(\pi_t) = \begin{cases} E[V \mid S_0, \pi_t] + E[V \mid S_0, \pi_t]\frac{\pi_t - \beta_1}{\beta_1}, & \text{if } \pi_t \leq \beta_1 \\ E[V \mid S_0, \pi_t] + (1 - E[V \mid S_0, \pi_t])\frac{\pi_t - \beta_1}{1-\beta_1}, & \text{if } \pi_t > \beta_1, \end{cases} \tag{5.5}$$

where $\beta_1 \geq \pi_0$. If $\beta_1 = \pi_0$, then in line with (A1) the corresponding CEU asset valuation of the low signal is initially equal to the additive component, i.e.

[37] Note that strong pessimism also reflects ambiguity lovingness in the sense of Heath and Tversky (1991). The mere difference is that a high degree of confidence regarding the low state causes CEU traders to assign a higher probability to the low state, than their additive belief would dictate.

$CEU_{S_0}(\pi_0) = E[V \mid S_0]$. The larger β_1, the more pessimistic S_0 and the longer it takes before the low signal eventually becomes exuberant. S_0 needs to observe a much stronger buy side accumulation of traders before she will decide to follow the crowd and buy the asset. Indeed, note that for $\beta_1 > \pi_0$, we have $CEU_{S_0}(\pi_0) < E[V \mid S_0]$ which means that S_0 initially has a pessimistic view on the investment opportunity. She needs to overcome this a priori skepticism before she becomes inclined to invest in V. As long as $\beta_1 < 1$, sufficiency for buy herding as implied by Theorem 5.3 holds.

Similarly for the high signal, we have

$$\alpha_{S_1}(\pi_t) = \begin{cases} E[V \mid S_1, \pi_t] + E[V \mid S_1, \pi_t]\frac{\pi_t - \beta_2}{\beta_2}, & \text{if } \pi_t \leq \beta_2 \\ E[V \mid S_1, \pi_t] + (1 - E[V \mid S_1, \pi_t])\frac{\pi_t - \beta_2}{1-\beta_2}, & \text{if } \pi_t > \beta_2, \end{cases} \tag{5.6}$$

where $\beta_2 \leq \pi_0$. The interpretation of β_2 for the high signal is symmetric to the interpretation of β_1 for the low signal.

5.4.3.2 Contrarianism With Varying α

The intuition and mechanisms driving contrianism are the same for varying α as under GBU. Thus, we do not provide formal necessary and sufficient conditions. We point out, however, that necessary conditions for contrarianism can be stated in a similar fashion as for herding. One simply has to exchange the intervals from which π is chosen in Theorem 5.2. For sufficiency, consider a CEU trader, whose degree of optimism α essentially stays constant for all $\pi \in [\epsilon; 1-\epsilon]$ or even changes in a countercyclical fashion, then by similar arguments as in Section 5.4.2, the initial valuation of the low (high) signal would rise above (drop below) the ask (bid) price as the price decreases (increases).

Now, that we have derived fairly general conditions for herding and contrarianism in the CEU model, we shall shift our focus on the analysis of market outcomes and social learning.

5.4.4 Market Outcomes and Social Learning in the CEU Model

This section illustrates that herding and contrarianism in the CEU model have an equal potential to prevent the market from learning about the asset's fundamental value and to move prices away from fundamentals.

Monotonicity of the informed trader expectations, i.e. $CEU[V \mid S_0, H_t] < CEU[V \mid S_1, H_t]$, implies that whenever an informed trader type engages in herding or contrarianism all informed traders take the same action. That is, both types of investor behavior necessarily lead to an informational cascade. If an informational cascade occurs at t, social learning stops and the price fixes at π^* until the end of trading, i.e. $\pi_\tau \equiv \pi^*$ for all $t < \tau \leq T$. Such a price consensus is inherently inefficient since it prevents the market from learning about the asset's true value and consequently from pricing the asset at it's fundamental, compare Chamley (2004).

In that sense, herding as well as contrarianism in the CEU model lead to inefficient market outcomes. This would be of minor concern if prices generally moved towards the asset's fundamental until the informational cascade takes place. That is, if $V = 1$ ($V = 0$), we observed $\pi^* > \pi_0$ ($\pi^* < \pi_0$) with high probability. This is, however, not necessarily the case as we illustrate by discussing Figures 5.3 and 5.4.

Figure 5.3 depicts a situation, where the low signal S_0 (the high signal S_1) engages in buy (sell) herding with positive probability. We focus only on the buy herding case. Initially, at $\pi_0 = 0.5$, the low signal values the asset as if she was an SEU maximizer, i.e. $CEU_{S_0}(\pi_0) = E_{\pi_0}[V \mid S_0]$. The low signal's optimism function α_{S_0} ensures that her asset valuation $CEU_{S_0}(\pi)$ is highly elastic with respect to the degree of optimism exhibited by the market. Indeed, as S_0 observes an increasingly strong price upsurge, she not only contracts the optimistic market sentiment but really becomes *overly* enthusiastic regarding the prospect of investing into the risky asset V.

As the price π rises above π^1, the low signal changes her trading decision from selling to holding. In line with Lemma 5.1, the quoted bid price is equal to

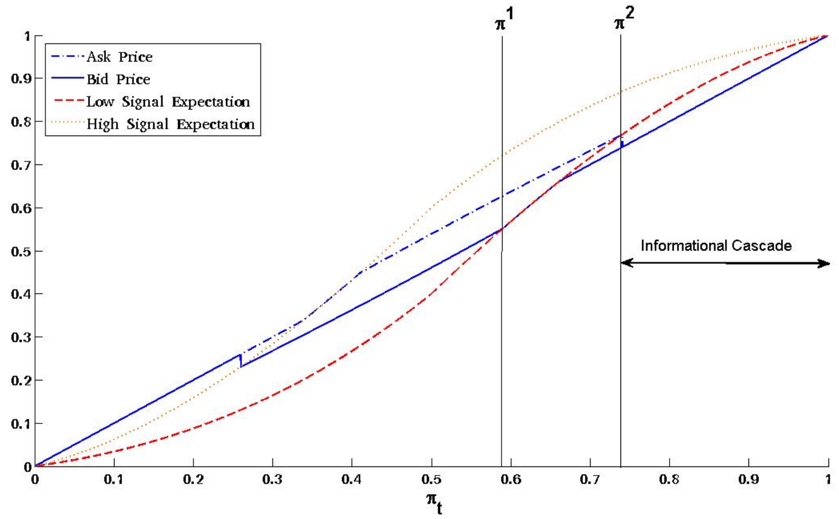


Figure 5.3: Irrational exuberance and buy herding

Notes: Informed trader asset valuations CEU_S, bid price bid^{CEU} and ask price ask^{CEU} are depicted with respect to the price π_t. The primary ambiguity is $\delta_0 = 0.5$, the informed trader share is $\mu = 0.3$, the initial prior is $\pi_0 = 0.5$ and the signal precision is $q = 0.6$. $\alpha.(\pi_t)$ is given by Equations (5.5) and (5.6) with $\beta_1 = \beta_2 = 0.5$.

S_0's valuation of the asset. As $CEU_{S_0}(\pi)$ becomes greater than π_t, the market maker quotes a bid price equal to π_t. If additional buys are observed, the price π_t eventually rises above π^2. At that point, S_0's exuberance causes her to start buy herding. An informational cascade occurs, since all informed trader types buy at that point. Social learning stops and the price is fixed at $\pi^* = 0.75$.[38]

To see that $\pi^* = 0.75$ becomes the price consensus with relevant probability even if the low state is true, i.e. if $V = 0$, note that S_0 engages in buy herding if the trade history contains at least 7 more buys than sells. Even though $V = 0$, there is ample potential for a buy side accumulation of traders. All high signals and one third of the noise traders buy the asset upon arrival. As long as there is no informational cascade, the buy probability, i.e. the probability of a price increase, is $\mu(1-q)+\theta = 0.353$ for the model parameterization of Figure 5.3. Similarly, the probability to observe a price decrease is $\mu q + \theta = 0.413$. Given the high short-term dispersion of a

[38] Note, that the discontinuity of the ask price at $\pi = 0.26$ indicates that the symmetrically modeled high signal starts selling. An informational cascade occurs, because at that point all informed traders sell and the price is fixed at 0.25.

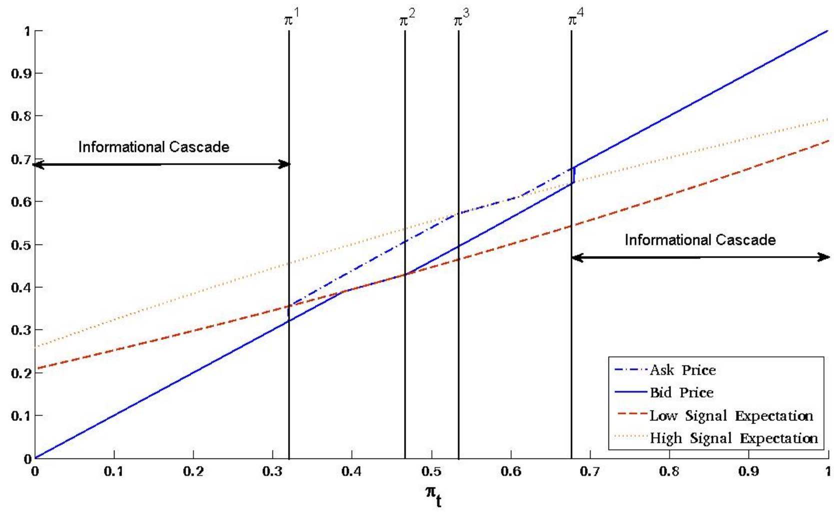


Figure 5.4: Market outcomes under contrarianism

Notes: Informed trader asset valuations CEU_S, bid price bid^{CEU} and ask price ask^{CEU} are depicted with respect to the price π_t. The degree of optimism is $\alpha = 0.5$ for both informed trader types, the primary ambiguity is $\delta_0 = 0.3$, the initial prior is $\pi_0 = 0.5$, the informed trader share is $\mu = 0.3$ and the signal precision $q = 0.6$.

price-process that is governed by these probabilities, it is apparent that the likelihood of prices moving away from fundamentals due to herding is far from negligible. The situation is symmetric for sell herding if $V = 1$.

Figure 5.4 illustrates the case where the low signal (high signal) may become a buy (sell) contrarianist. In line with the GBU rule, $\alpha = 0.5$ for both informed trader types. Assuming that $\pi_0 = 0.5$, S_0 sells initially and S_1 buys initially. Let us focus on the buy contrarianism case. As sells are observed, the asset price π decreases. As π falls below π^2, S_0 stops selling and starts holding. If additional sells cause the price to drop below π^1, S_0 switches from holding into buying, thus acting as a buy contrarianist. Since at that point both informed trader types buy the asset, an informational cascade occurs. Social learning stops and the asset price remains constant at $0.3 = \pi^* < \pi^1$ for all remaining trade periods.

Now, assume that $V = 1$ is the true state. The probability of a price decrease in t is $\mu(1-q) + \theta = 0.353$ as long as there is no informational cascade.[39] This

[39]The probability of price increase is $\mu q + \theta = 0.413$.

is due to the fact that the low signal reveals her private information through her action regardless of whether she sells or holds, compare price updating in Lemma 5.2. If S_0 sells (holds), prices decrease after an observed sell (hold).

If the trade history contains at least 4 more sells than buys, S_0 engages in buy contrarianism. The market "agrees" with considerably positive probability to depreciate the asset value to $0.3 = \pi^*$ even though $V = 1$ is the true state.

To make these insights more precise, we derive a formula for the probability of such wrong cascades due to herding as well contrarianism in the following section. We can use this formula to conduct comparative statics on the likelihood that herders and contrarians move prices away from fundamentals.

5.4.5 Investors Moving Prices away from Fundamentals - Deriving the Probability of a Wrong Cascade

We have illustrated that herding and contrarianism in the CEU model may lead to price distortions. The aim of this section is to quantify the probability of such an event and investigate how this probability is related to other model parameters.

To keep things tractable, let us assume without loss of generality that either herding or contrarianism are possible but not both. In addition, we assume symmetry of α_{S_0} and α_{S_1} in the sense that the minimum number of price increases (decreases) after which the informed traders start herding or acting as contrarians are the same for the high and low signals.[40] We denote this number as n^*.

Under these assumptions the probability of prices moving away from fundamentals coincides with the probability of a buy herding (sell contrarianism) induced cascade if $V = 0$ and vice versa a sell herding (buy contrarianism) induced cascade given $V = 1$. We obtain

[40]This also includes the GBU case of fixed α. The result easily generalizes to the case where the symmetry assumption is dropped. A look at the proof of Lemma 5.3 in the Appendix will reveal why this is true.

Lemma 5.3. ***Probability Of A Wrong Informational Cascade***
Consider the CEU model, where either herding or contrarianism is possible. Let n^* be the minimum number of price increases (decreases) after which the low (high) signal starts herding or acting as a contrarian. Then, the probability of an informational cascade where all informed traders buy herd (act as sell contrarians), given that $V = 0$, is equal to the probability of an informational cascade where all informed traders sell herd (act as buy contrarians), given that $V = 1$. This probability is given by

$$P_{n^*} = \frac{1}{\exp\left(-\ln\left(\frac{\mu(1-q)+\theta}{\mu q+\theta}\right) n^*\right) + 1}. \tag{5.7}$$

Proof: The proof is fairly technical and, thus, details are left to the Appendix. The idea of the proof is that the probabilities to observe a price increase, decrease or constant prices are the same as long as there is no informational cascade. This makes the problem of calculating P_{n^*} equivalent to a two-sided gambler's ruin problem with $2n^*$ possible states. We can specifiy the transition probabilities in each state. Then standard techniques of linear algebra yield the desired result.

□

To get a broader perspective of how the probability of wrong cascades reacts to shifts in model parameters, let us analyze the comparative statics of P_{n^*}.

Lemma 5.4. ***Comparative Statics Of Price Distortions***
In the CEU model, the probability of prices moving away from fundamentals P_{n^*}

- *Decreases with the informed trader share μ;*
- *Decreases with the signal precision q;*
- *Increases with the degree of primary ambiguity δ_0;*
- *Increases with the degree of individual optimism α if investors are prone to herding;*

- *Decreases with the degree of individual optimism α if investors are prone to contrarianism;*

Proof: Since the proof is fairly technical, it is left to the Appendix.

□

The results of Lemma 5.4 are fairly intuitive. Since μ can be seen as the quantity of fundamentally relevant information in the market and q can be viewed as the quality of the same, it is straight forward that an increase of both should reduce the likelihood that prices move away from fundamentals. If there are more and better informed traders, then prices should more accurately reflect the asset's true value.

Likewise an increase in primary ambiguity δ_0 will make investors more prone to irrational exuberance (desperation) in the case of herding and will make the belief updating of potential contrarian traders even more sluggish. In both cases, higher ambiguity will cause CEU traders to rely more on their gut feeling which may advise them to take wrong actions. Hence the probability of prices moving away from fundamentals increases. The different results for a change in α stem from the fact that we see α as a function in π. We define an increase in α as a general increase of individual optimism elasticity with respect to a change in market sentiment π.[41] Now, consider a CEU trader who is prone to contrarianism. If her individual optimism reacts more elastically to price changes, then her belief updating tends to be less sluggish, thus, reducing her contrarian tendencies. Similarly if the CEU trader is prone to herding, a higher α implies that her willingness to ignore her private signal and follow the crowd increases.

We have investigated the conditions under which herding and contrarianism are possible in the CEU model. Both lead to informational cascades and, thus, prevent the market from confidently learning about the true state and may cause price distortions. In fact, we find that price distortions are equally likely

[41] Formally, α^1 is said to be greater than α^2 if and only if $\alpha^1(\pi) < \alpha^2(\pi)$ $\forall \pi \in (0; \pi_0)$ and $\alpha^1(\pi) > \alpha^2(\pi)$ $\forall \pi \in (\pi_0; 1)$.

under contrarianism and under herding. Given the antithetical nature of the two types of behavior, however, we would have expected that herding and contrarianism result in different market outcomes. To carve out these differences we shall depart from the assumption that investor preferences are common knowledge and consider a perturbed version of the model. We will discuss the details of this approach and it's insights in the next section.

5.5 Price Dynamics under Herding and Contrarianism - the Perturbed CEU Model

This section provides insights regarding differences of stylized price dynamics under herding and contrarianism, respectively. In the CEU model, however, herding and contrarianism lead to informational cascades, i.e. constant prices. To circumvent this issue, we assume that market participants exhibit marginal uncertainty regarding investor preferences. The resulting setup is called the perturbed CEU model because it resembles in spirit the concept of a perturbed game.

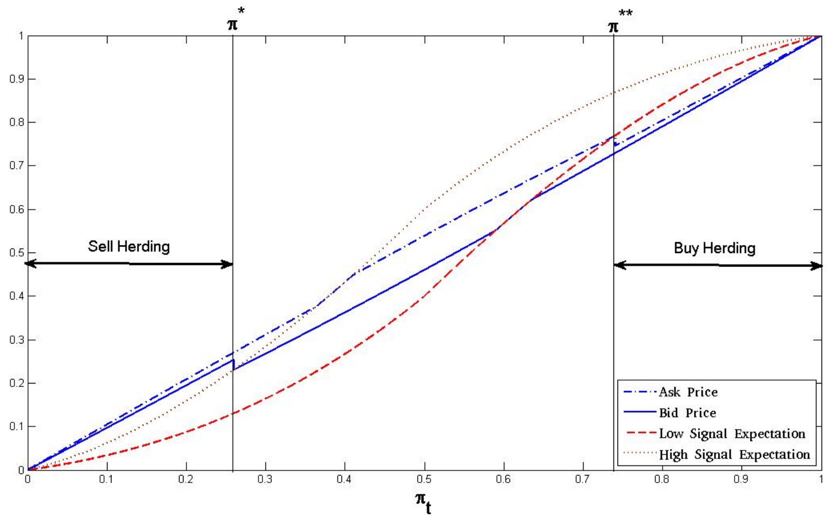
5.5.1 The Perturbed CEU Model

All definitions and assumptions from Sections 5.2 and 5.3 hold. In particular, all informed traders are CEU maximizers. Yet, now we assume that market participants perceive ambiguity regarding informed traders' preferences. That is, they do not fully discount the possibility that informed traders are ambiguity neutral. More precisely, the market believes that informed traders have neo-additive CEU preferences with probability $1 - \epsilon$ and are expected value maximizers as in the baseline model with probability ϵ for some arbitrarily small $\epsilon > 0$.[42]

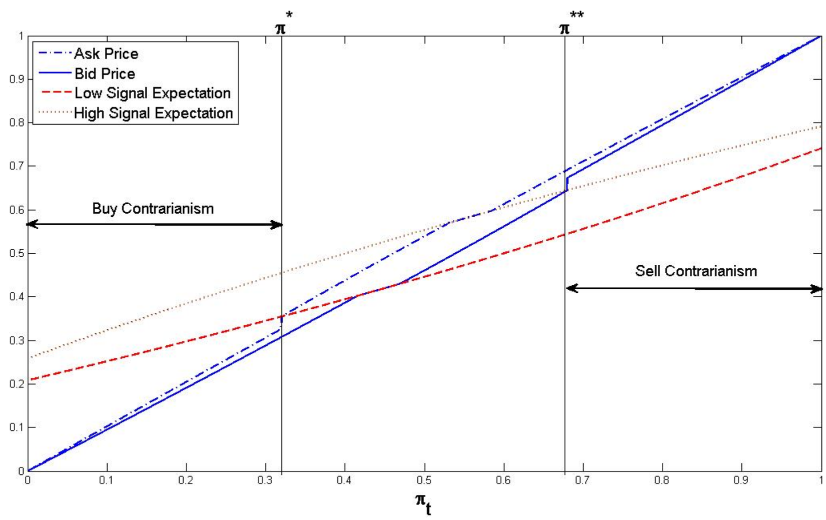
The most important property of the perturbed CEU model is the absence of

[42]The fact that market participants perceive ambiguity regarding other informed traders' preferences is in line with the game theoretic literature, compare e.g. Eichberger and Kelsey (2014) and Eichberger and Kelsey (2000). In the context of these frameworks an agent's belief regarding the other players' ambiguity preferences may deviate from the truth.

We note that Ford et al. (2013) focus their analysis exclusively on the somewhat extreme case of $\epsilon = 1$.



(a) Herding



(b) Contrarianism

Figure 5.5: Herding and contrarianism in the perturbed model

Notes: Figure (a) depicts informed trader asset valuations CEU_S, bid price $bid^{\epsilon-CEU}$ and ask price $ask^{\epsilon-CEU}$ with respect to the price π_t in a market prone to herding. For illustrative purposes, we set $\epsilon = 0.25$. The primary ambiguity is $\delta_0 = 0.5$, the informed trader share is $\mu = 0.3$, the initial prior is $\pi_0 = 0.5$ and the signal precision is $q = 0.6$. $\alpha_\cdot(\pi_t)$ is given by Equations (5.5) and (5.6) with $\beta_1 = \beta_2 = 0.5$. Figure (b) depicts informed trader asset valuations CEU_S, bid price $bid^{\epsilon-CEU}$ and ask price $ask^{\epsilon-CEU}$ with respect to the price π_t prone to contrarianism. For illustrative purposes, we set $\epsilon = 0.25$. The primary ambiguity is $\delta_0 = 0.3$, the informed trader share is $\mu = 0.3$, the initial prior is $\pi_0 = 0.5$ and the signal precision is $q = 0.6$. $\alpha = 0.5$ for both informed trader types in line with GBU.

(full) informational cascades. Figure 5.5 illustrates this fact by showing that even under herding and contrarianism the market maker quotes a positive bid-ask spread. This implies that the market still infers information from observed trade decisions a_t even if all informed traders take the same action in t. As a consequence, the price π_t continues to evolve under herding and contrarianism in the perturbed model.[43]

The absence of informational cascades stems from the market's belief that a share of ϵ of the informed traders are expected value maximizers and, thus, behave as in the baseline model. Consider for instance a situation where both signals buy the asset (e.g. buy herding in Figure 5.5 (a)). The market believes that a share of ϵ of the S_0 type traders still acts as in the baseline model without ambiguity and sells the asset, compare Proposition 5.1. Hence, an observed sell is viewed to contain some information as the market does not fully discount the possibility that the trade is carried out by an S_0 type informed trader. Consequently, π_t decreases in line with Bayes' rule after a sell is observed. The argument is similar for a price increase after an observed buy.

Note, however, that the amount of information inferred from a trade under herding and contrarianism may be very small, particularly if ϵ is small. Consequently, herding and contrarian regimes can be seen as partial informational cascades in the sense of Avery and Zemsky (1998) and Park and Sabourian (2011).

The second key property of the perturbed model is that it inherits the results regarding the necessary and sufficient conditions for herding and contrarianism derived in the CEU model, see Theorems 5.1, 5.2 and 5.3.

To see why this is true, assume that in line with (A1) the low signal sells initially while the high signal buys initially. As long as S_1 buys and S_0 does not, the market maker quotes the same ask price as in the CEU and baseline model without ambiguity.[44] Since S_0's asset valuation is also not affected by the

[43] The corresponding formal results on market maker price setting and price updating are notationally tedious in the perturbed CEU model and, thus, left to Section 5.8.C in the Appendix of this paper, see Propositions 5.4 and 5.5.

[44] Compare market maker pricing results for the CEU model (Lemma 5.1) and the perturbed model (Proposition 5.4 in Section 5.8.C in the Appendix).

	Probability of a Price Increase	Probability of a Price Decrease	Probability of a Constant Price
Buy Herding	$\mu + \theta$	θ	θ
Sell Contrarianism	θ	$\mu + \theta$	θ
Sell Herding	θ	$\mu + \theta$	θ
Buy Contrarianism	$\mu + \theta$	θ	θ

Table 5.1: Transition probabilities for π_t under herding and contrarianism

Notes: This table reports the transition probabilities of the price process in the perturbed CEU model given that CEU traders herd or act as contrarians respectively. μ is the share of informed traders. $\theta = (1 - \mu)/3$ is the probability for a noise trader to either buy, sell or hold.

perturbation assumption, the conditions for CEU_{S_0} surpass the ask price, i.e. for S_0 to buy herd or act as a buy contrarian are the same as in the CEU model.

Indeed, since the model parameterizations in Figures 5.5 (a) and (b) coincide with the ones used for Figures 5.3 and 5.4, respectively (aside from the perturbation parameter ϵ), the cut-off prices for herding and contrarianism (π^*, π^{**}) from Figures 5.5 (a) and (b) are precisely equal to the corresponding cut-off prices depicted in Figures 5.3 and 5.4.

Since we have established that herding and contrarianism are possible in the perturbed CEU framework and that learning always continues we can now study how prices evolve under herding and contrarianism.

5.5.2 Prices under Herding and Contrarianism in the Perturbed CEU Model

Before deriving formal results we want to develop some intuition for the price dynamics by looking at the price process' transition probabilities under the different regimes.

Table 5.1 reports the probabilities of whether π_t moves up, down or remains constant given that investors herd or act as contrarians. We note that under all four regimes a price increase (decrease) coincides with an observed buy (sell).

As a consequence, the probability for a price increase (decrease) is the same as the probability of a buy (sell). The same argument applies for constant prices and observed holds.[45]

Consider for instance the case of buy herding. The total probability of observing a buy is the probability of an informed buy plus the probability of a noise buy. Since under buy herding all informed traders buy the asset, the probability of an informed buy is μ. Similarly, since one third of the noise traders buys the asset, the probability of observing a noise buy is $\theta = (1 - \mu)/3$. Thus, the total probability of observing a buy under buy herding and, hence, a price increase is $\mu + \theta$. Likewise, since the only traders selling under buy herding are noise traders, the probability of observing a sell and, thus, a price decrease is θ.

By a similar line of reasoning the probability of a price decrease under sell contrarianism is $\mu + \theta$. Since only noise traders buy under sell contrarianism, the probability of a price increase is θ.

Avery and Zemsky (1998) show that the price in their model eventually converges to 1 if the majority of the informed traders buys while it converges to 0 if the majority of the informed traders sells.

The same principal should govern the price process in the perturbed CEU model under herding and contrarianism. We conjecture that buy herds in the perturbed CEU model push prices towards 1 (the majority buys), while sell contrarianists pull it back towards π_0 (the majority sells). Similarly, sell herds should push the price towards 0 while buy contrarians pull it back up towards π_0.

These hypotheses are confirmed by the following two propositions.

Proposition 5.2. ***Prices in the Perturbed Model - Herding***
In the perturbed CEU model, let $\pi^* < \pi_0$ ($\pi^{**} > \pi_0$) be the cut-off prices, such that S_1 sell herds for all $\pi < \pi^*$ (S_0 buy herds for all $\pi > \pi^{**}$).
Then the market will become confident regarding the low (high) state with positive probability, regardless of the true state of V.

[45] Compare the price updating rules in the perturbed model summarized in Proposition 5.5 in Section 5.8.C in the Appendix for details.

Proof: We discuss the proof for the buy herding case only. The sell herding case is symmetric. For the buy herding statement we need to show that $P(\lim_{t\to\infty} \pi_t = 1) > 0$. First note that by the law of total probability, we have

$$\begin{aligned}&P(\lim_{t\to\infty} \pi_t = 1)\\&= P(\lim_{t\to\infty} \pi_t = 1 | \exists \tau \geq 0 : \pi_\tau > \pi^{**}) P(\exists \tau \geq 0 : \pi_\tau > \pi^{**}).\end{aligned} \tag{5.8}$$

The second probability on the r.h.s. of Equation (5.8) is greater zero by the assumption that buy herding is possible. For the first probability on the r.h.s. of Equation (5.8), we define $\pi_t^{BH} := (\pi_t | \pi_t > \pi^{**})$ to be the price process under buy herding. Observe that π_t^{BH} is a sub-martingale with respect to the history H_t, i.e. $E[\pi_{t+1}^{BH} | H_t] > \pi_t^{BH}$.[46] Then, the martingale convergence theorem implies that $\pi_t^{BH} \to \Pi$ for $t \to \infty$ almost surely, i.e. $P(\lim_{t\to\infty} \pi_t^{BH} = \Pi) = 1$.
Since, the sub-martingale property implies that $\pi_t^{BH} \in (\pi^{**}; 1)$ increases almost surely for $t \to \infty$ and since $\pi_t^{BH} < 1$ for all t by definition, it follows that $\Pi = 1$. This implies that $P(\lim_{t\to\infty} \pi_t = 1 | \exists \tau \geq 0 : \pi_\tau > \pi^{**}) > 0$ and, thus, concludes the proof that a market prone to buy herding will become confident regarding the high state with positive probability regardless of the true state of V.

□

Proposition 5.3. ***Prices in the Perturbed Model - Contrarianism***
In the perturbed CEU model, let $\pi^* < \pi_0$ ($\pi^{**} > \pi_0$) be the cut-off prices, such that S_0 acts as a buy contrarian for all $\pi < \pi^*$ (such that S_1 acts as a sell contrarian for all $\pi < \pi^*$).
Then the price will rise above π^* (drop below π^{**}) again almost surely.

Proof: Consider the sell contrarian case. We need to show that $P(\exists \tau > t : \pi_\tau \leq \pi^{**} | \pi_t > \pi^{**}) = 1$.
Similar to before, we define $\pi_t^{SC} := (\pi_t | \pi_t > \pi^{**})$. We observe that π_t^{SC} is a super-martingale with respect to the history H_t, i.e. $E[\pi_{t+1}^{SC} \mid H_t] < \pi_t^{SC}$.[47] In other words, prices fall almost surely as long as there is sell contarianism. Moreover, there exists a $\tilde{\pi}_t > \pi^{**}$ such that $\tilde{\pi}_{t+1} < \pi^{**}$ if there is a sell in t.

[46] The proof for this is left to Section 5.8.C in the Appendix.
[47] The proof for this is left to Section 5.8.C in the Appendix.

Both arguments together yield that the price will drop below π^{**} almost surely given there is sell contrarianism at some time t. The argument is symmetric for buy contrarianism.

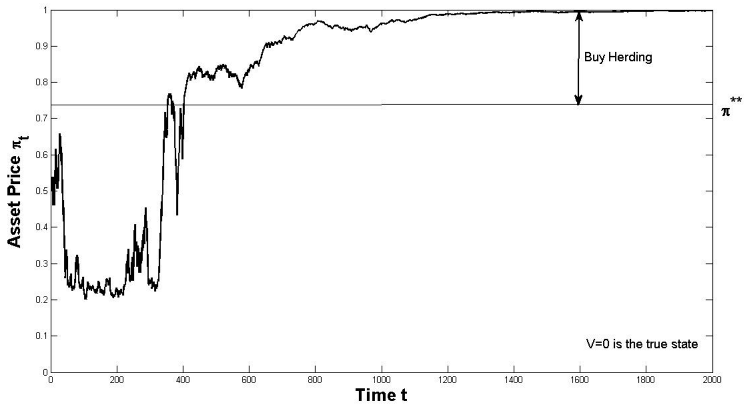
□

Propositions 5.2 and 5.3 state that the price evolves quite differently under herding and contrarianism in the perturbed CEU model. According to Proposition 5.2, herding causes the market to become confident about one of the states. A buy herd eventually drives the price towards 1 while a sell herd drives the price towards 0. Since this may happen regardless of the true state of V, the perturbed CEU model predicts that the market herds on the wrong state with positive probability.[48]
In line with Proposition 5.3, contrarianism prevents the market from learning about the true state and anchors the price on some interval $\pi^* < \pi_0 < \pi^{**}$. Hence, both types of investor behavior still distort prices but they do so in very dissimilar ways.

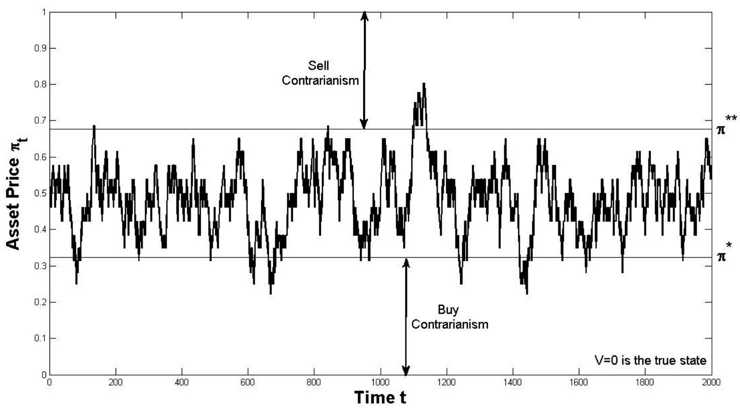
The different stylized price movement under herding and contrarianism in the perturbed CEU model are illustrated in Figure 5.6.
The herding case is depicted in Figure 5.6 (a). In line with the previously illustrated model outcome in Figure 5.5 (a), S_0 engages in buy herding as soon as the price surpasses π^{**}.[49] The first buy herd forms right before period 400. We observe, that this buy herd is broken shortly thereafter. This is due to a random arrival of noise traders selling the asset who push the price below π^{**}. When the second buy herd starts after period 400, however, it persists until period $T = 2000$. The sub-martingale property now governs the price process resulting in a long term price increase towards 1. Thus, the longer the herd persists, the less likely it is broken. In line with Park and Sabourian (2011) we refer to this

[48]Note that this result partly driven by the assumption that the market operates under the wrong assumption that a share of ϵ traders are SEU maximizers. Still, it provides qualitatively valuable insights as it is fair to assume that investor preferences are typically not accurately estimated in the real world. Since, moreover, there is a strong consensus regarding the validity of the efficient market hypothesis, we conjecture that markets tend to underestimate the share of investors deviating from SEU.

[49]Note, that the model parameterizations in Figure 5.6 are exactly as the corresponding model parameterizations in Figure 5.5.



(a) Confident Herding On The Wrong State



(b) Anchored Price Movement und Contrarianism

Figure 5.6: Prices under herding and contrarianism

Notes: Figure (a) shows a simulated price path under herding for $T = 2000$ time periods. As before, the primary ambiguity is $\delta_0 = 0.5$, the informed trader share is $\mu = 0.3$, the initial prior is $\pi_0 = 0.5$ and the signal precision is $q = 0.6$. $\alpha.(\pi_t)$ is given by Equations (5.5) and (5.6) with $\beta_1 = \beta_2 = 0.5$. The pertubation term is $\epsilon = 0.25$. Figure (b) shows a simulated price path under contrarianism for $T = 2000$ time periods. The primary ambiguity is $\delta_0 = 0.3$, the informed trader share is $\mu = 0.3$, the initial prioris $\pi_0 = 0.5$ and the signal precisionis $q = 0.6$. $\alpha = 0.5$ for both informed trader types in line with GBU. The pertubation term is $\epsilon = 0.25$.

as the self-enforcing nature of herding. By period 800, the price is well above 0.9. For it to fall below $\pi^{**} = 0.74$ again, many consecutive (noise trader) sells would have to be observed. The probability for such an event is already very small. Indeed, since we expect π_t to increase further on average (sub-martingale property), the probability of the herd being broken vanishes as $t \to T$. Note that the market becomes confident that the asset's true value is 1, while V in fact is 0, i.e. the market confidently herds on the wrong state.

Also observe that the price is considerably less volatile when S_0 type informed traders buy herd compared to when they do not. This is in line with the characteristics of a partial informational cascade. Since the market is certain that most informed traders have CEU preferences and knows that they engage in buy herding as $\pi > \pi^{**}$, the informational content the market infers from an observed trade drops significantly under buy herding. Since the price dynamics are mainly determined by the inferable information from an observed trade, prices become less volatile when S_0 buy herds, compare Avery and Zemsky (1998) and Park and Sabourian (2011).[50]

The price evolution under contrarianism is depicted in 5.6 (b). In line with Figure 5.5 (b), S_0 engages in buy contrarianism (S_1 engages in sell contrarianism) as soon as the price surpasses π^{**} (falls below π^{*}). In line with Proposition 5.3, the price mainly stays on $(\pi^{*}; \pi^{**})$. The contrarian regimes are always very short due to the self-defeating nature of contrarianism, compare Park and Sabourian (2011). As soon as the price exceeds π^{**}, for instance, sell contrarians pull the price below π^{**} again causing the regime to end rather quickly. As a consequence, contrarians prevent the market from becoming confident regarding either state and, hence, from learning. In $T = 2000$, the asset price is still very far away from the asset's fundamental value $V = 0$.

While for given model parameterization the price evolution is always similar in a market prone to contrarianism, the outcome could have been different under herding. A sell herd could have driven the price *towards* the asset's fundamental

[50] In the case of an informational cascade the information content of a trade is 0 and, thus, the price remains constant, compare the CEU model.

value of 0. As a consequence, we want to study the a priori probability of a wrong herd similar to the CEU model.

5.5.3 Price-Distorting Herding in the perturbed model

Quantifying the probability of price-distorting herds in the perturbed model is more complicated than in the CEU model. When informed traders change their trade decisions in the perturbed model, the probability and the extend of price increases and decreases varies. Yet, our ability to derive an analytical formula for the probability of wrong cascades in the CEU model in Lemma 5.3 hinges on the fact that price changes and their probabilities are fix as long as there is no cascade and trivial when there is an informational cascade. Hence, we are not able to provide a similarly appealing result for the perturbed model. We can, however, leverage Lemma 5.3 to infer upper and lower boundaries for the probability of wrong herds:

Lemma 5.5. ***Probability of Wrong Herds in the Perturbed Model***
Consider the perturbed CEU model, where only herding is possible and assume symmetry between the signals as in Lemma 5.3. Let P^{ϵ}_{WH} denote the probability of a wrong herd in the perturbed CEU model. Let n^* be the minimum number of price increases (decreases) after which the low (high) signal starts herding. Let $k^* < n^*$ be the minimum number of observed price increases (decreases) after which the low (high) signal starts holding. Then $P^{\epsilon}_{WH} \in [P_{n^*}; P_{k^*}]$, where $P_.$ is as in Equation (5.7)*.*

Proof: See Appendix.

□

In the perturbed model, the probability of wrong buy herds tends to be even larger than in the CEU model. Driver for this result is the market's erring assumption that a share of ϵ of the informed traders are SEU maximizers. As long as preferences are common knowledge (CEU model), the price process is a martingale and cannot exhibit long term trends away from the asset's fundamental. In the perturbed model, on the other hand, such a wrong trend is possible.

Let $V = 0$ be the true state and assume for illustrative purposes that $\epsilon = 1$, that is, the market assumes that all informed traders are SEU maximizers although they have CEU preferences. Once S_0 switches into holding, the market does not accurately adjust the asset price downward anymore. Instead of depreciating the price after an observed hold (CEU model), the market depreciates the price after a sell. Yet, the probability to observe a sell is only θ (noise trader sell) since S_0 holds. For π_t to be a martingale, however, the probability of a downward revision would need to be $\mu + \theta$.

Hence, price decreases occur less often than they should. This results in an upward drift in the price process, i.e. a trend away from $V = 0$ (sub-martingale property). This effect is amplified if S_0 engages in buy herding.

These trends have the overall effect to drive prices away from fundamentals with greater probability in the perturbed model than in the CEU model.

We note, that the comparative statics of the lower boundary of P^{ϵ}_{WH} are readily provided by Lemma 5.4. For the comparative statics of the upper boundary P_{k^*}, we find that the effect of an increase in μ is, indeed. An increase in q has weaker effects on P_{k^*} than on P_{n^*}.

5.5.4 The Burst Of A Bubble

Given the initial motivation of this paper, we would like to conclude this section by illustrating how the CEU model can be leveraged to explain the formation and subsequent burst of bubbles. As we have seen in the previous sections, the perturbed CEU model can explain the formation of a bubble. Yet, it cannot endogenously produce the burst of bubble before the asset's true value is revealed after the final period T.

To overcome this issue, we will allow ambiguity in the perturbed CEU model to be exogenously removed at some period $\tau < T$. Indeed, it is conceivable that unexpected events like the September 11 attacks on the United States in 2001 or Mario Draghi's "whatever it takes" speech in 2012 may cause jumps in primary ambiguity δ_0 or remove it altogether. The result is illustrated in Figure 5.7.

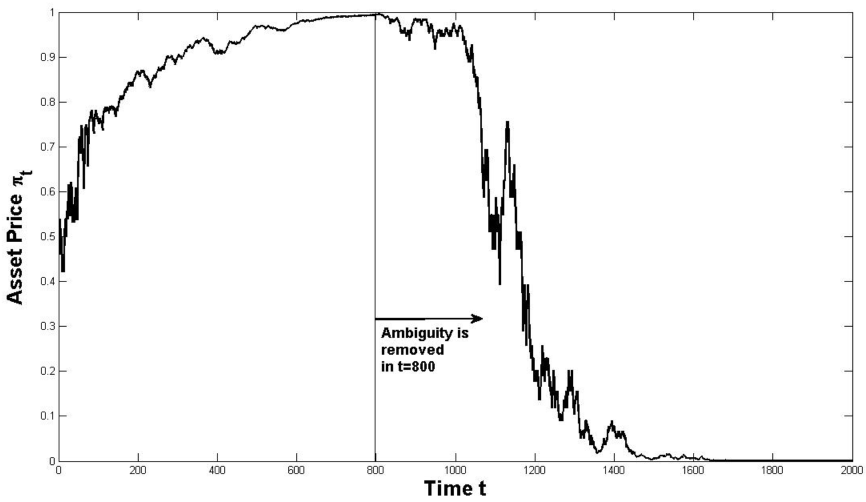


Figure 5.7: The formation and burst of a bubble

Notes: The figure shows a simulated price path for $T = 2000$ periods. The primary ambiguity $\delta_0 = 0.5$, the informed trader share is $\mu = 0.3$, the initial prior $\pi_0 = 0.5$ and the signal precision $q = 0.6$. $\alpha.(\pi_t)$ is given by Equations (5.5) and (5.6) with $\beta_1 = \beta_2 = 0.5$. The perturbation parameter $\epsilon = 0.25$ for illustrative purposes.
Primary ambiguity δ_0 is set equal to 0 after $t = 800$ periods. The true state is $V = 0$.

The model parameterization used to simulate the price path in Figure 5.7 allows for (wrong) herding. The asset's fundamental value is $V = 0$. Due to noise trading and trading of high signals, we observe an increase in the price in the first 70 periods. Shortly after $t = 70$, the low signal type traders engage in buy herding for the first time. The buy herd is broken a few times. A persistent buy herd forms only after period 110. Prices increase further towards 1 in line with Proposition 5.2. This continues until period 800, where some exogenous event removes the ambiguity, thereby bringing the herd to a halt.

Once ambiguity is removed, informed traders become expected value maximizers. Hence, S_0 type traders sell the asset. Since $V = 0$ is the true state, there are more S_0 type traders in the market than traders with a high signal. In line with the baseline model, we expect a price correction and that π_t eventually converges towards 0.

Since the informed traders' signal precision $q = 0.6$ is relatively low, it takes the market some time to learn that $V = 0$ is the true state. It is not before period 1000 that the market starts learning that the asset is wrongly priced. From period 1000 to 1300 the market corrects it's assessment. We observe a sharp decline in the asset's value accompanied by regimes of increasing volatility. After period 1300 volatility diminishes as the market becomes increasingly confident regarding the low state and the standard learning mechanisms of Avery and Zemsky (1998) take predominant effect.

Event Study - The Subprime Mortgage Crisis: Let us apply these theoretical insights to real world events in the years 2002 to 2007. Consider Collateralized Debt Obligations (CDOs), highly complex and non-transparent credit derivatives that enjoyed unprecedented popularity among investors in the years leading up to the recent global financial crisis in 2007 and became essentially worthless once the underlying collaterals (sub-prime mortgages) defaulted in large numbers.

In a noteworthy statement of Warren Buffet in the annual report of Berkshire Hathaway in 2002, he outlaws complex derivatives such as CDOs as time bombs and financial weapons of mass destruction. He claims that these products depend on too many variables and have far too long times-to-maturity to be valued accurately. To make things worse, sub-prime mortgages, i.e. credits with a very high default risk, became a predominant collateral for CDOs between 2002 and 2007.

If we think of Warren Buffet's view as rational in an economic sense and take into account the high risk associated with the CDOs' collaterals, a rational assessment of CDOs should result in a low probability for the event "I earn money with CDOs (in the long run)". At the same time, Mr. Buffet's argument implies that the perceived ambiguity associated with CDOs should be high even among professional traders, i.e. δ_0 should be large. Indeed, since there was insufficient information regarding the actual default risks of the CDO's collaterals, let alone correlation structures of defaults, there was no way for investors to accurately assess the value of a CDO.

The incessantly high AAA-rating of CDOs by US rating agencies added to investors' perceived ambiguity as it contradicted their objective belief that CDOs with sub-prime mortgages as collaterals are inherently risky. In conjunction with increasing evidence of high returns and the general market sentiment - everybody was buying them - this contributed to a highly optimistic view of investors on the event "I earn money with CDOs (in the long run)". In neo-additive terms, this means that investors not only had a high δ but also a tremendously elastic α. These are the required ingredients for price distorting herding under ambiguity.

Only after default rates increased across subprime mortgages in the end of 2006 and the beginning of 2007, investors realized that they had erred. To speak in model terms, the ambiguity was removed - it then was clear that many CDOs were essentially worthless. Interestingly, this did not affect the CDO prices at first. Only as time passed and default rates skyrocketed, the market depreciated CDO prices to reflect their true value.

5.6 Robustness Discussion

5.6.1 Multiple Prior Setup With Smooth Ambiguity Functions

In this section we want address the question, whether our results hold if we choose a different approach to model ambiguity. For this, we translate the CEU model into a multiple prior setup and investigate whether similar results hold under the assumption that informed investors form beliefs according to smooth ambiguity functions as proposed by Klibanoff et al. (2005).

To model informational ambiguity in a multiple prior context, we follow the ideas provided by Gollier (2011) and assume that investors find it plausible that the precision of their signal is either q with probability 1 or it is uniformly distributed across $[0; 1]$. In addition, we assume that either distribution for q is considered to be equally likely, i.e. occurs with probability $d = 0.5$. This captures informational ambiguity in the sense that the informed trader has a lack of confidence that her private information signal is fundamentally driven. Thus, the informed trader's second-order belief d is closely related the degree of perceived ambiguity in the neo-additive CEU model. As a suitable family of smooth ambiguity functions, consider

$$\Phi(x) = \begin{cases} \exp(-ax), & \text{if } a \neq 0 \\ x & \text{if } a = 0 \end{cases}.$$

The parameter a would reflect the informed trader's degree of optimism similar to α in the CEU setup. If $a > 0$, the informed trader is pessimistic, for $a < 0$, she is optimistic. For $a = 0$, she is ambiguity neutral. The informed traders' asset valuation is given by

$$\begin{aligned} V_S = -\frac{1}{a}\log\Big(d\int_0^1 \exp\left(-aE_q[V \mid S, \pi]\right)\, dq \\ + (1-d)\exp\left(-aE_q[V \mid S, \pi]\right)\Big) \end{aligned} \tag{5.9}$$

if $a \neq 0$. While we can obtain results similar to Theorems 5.1, 5.2 and 5.3 as well as Lemmas 5.3 and 5.4 by the same arguments as in the neo-additive CEU model, we would loose some of the intuition provided in this paper. The parameters d and a cannot be linked as nicely to the parameters in the Avery and Zemsky (1998) framework, as δ_0 and α.

To gain some intuition how a pro-cyclical time varying a in the multiple prior setup can drive (potentially price-distorting) herd behavior, we suggest to set $a(p) = -K\tan\left(\pi(p-0.5)\right)$ for some $K > 0$, where in this case p represents the price and π the actual number π.

5.6.2 Risk Preferences

By the same line of reasoning as for the multiple prior setup with smooth ambiguity functions, we argue that the results of this paper can be reproduced in a framework where informed traders have varying risk preferences. If risk-preferences move pro-cyclically (counter-cyclically) in the same way ambiguity preferences do, we find that herding (contrarianism) becomes possible in the Avery and Zemsky (1998) framework.

To see this for herding, take Φ and $a(p)$ as before and note that the informed traders asset valuation becomes $V_S = \Phi^{-1}(E[\Phi(V) \mid S, \cdot])$. For contrarianism, repeat the exercise with fixed a.[51]

This is appealing from a theoretical perspective, in the sense that it generalizes the results presented in this paper. Indeed, experimental evidence from Drehmann et al. (2005) and Cipriani and Guarino (2005) suggests that investors in the Avery and Zemsky (1998) baseline model, do engage in contrarian behavior to some extent. This indicates that latent (or even counter-cyclical) risk preferences may play a role in decision-making underrisk in social learning settings. The same experiments, however, generally find no evidence of herd behavior casting strong doubt on the economic relevance of pro-cyclical changes in risk preference for risky investment decisions.

We would argue that this does not impede the validity of the herding results derived in this paper. The choices in Drehmann et al. (2005) and Cipriani and Guarino (2005) are not choices under ambiguity. In line with Heath and Tversky (1991), we conjecture that investors facing ambiguity, exhibit ambiguity lovingness but not risk lovingness as they become confident about either state.

When viewing the ambiguity preference parameter α as a measure for optimism and pessimism it's link to investor herding is also intuitively more appealing than for risk lovingness. While it is conceivable that investors contract optimism exhibited by the market and become overly enthusiastic regarding an investment opportunity, it is difficult to argue, why they would all of a sudden like the risk associated with a particular investment.

5.6.3 More States, More Different Types Of Traders

The results derived in this paper carry over to the more complex versions of Avery and Zemsky (1998) or the frameworks presented by Park and Sabourian (2011) and Cipriani and Guarino (2014). The important thing to note is that traders that perceive informational ambiguity may become prone to herding (contrarianism), even if they receive monotone private signals in the sense of Park and Sabourian

[51] This does not come as a surprise, since Décamps and Lovo (2006) show that differences in risk preferences between informed traders and market maker are sufficient for herding in a setup closely related to the Avery and Zemsky (1998) framework.

(2011).[52] Hence, herd (contrarian) behavior is not limited anymore to certain signal types like the u-shaped (hill-shaped) trader in Park and Sabourian (2011) or the low precision signal in Cipriani and Guarino (2014). As a consequence, herds or contrarianists in those models may consist of much larger shares of informed traders and, thereby, have the potential to drive prices away from fundamentals even in the long run.

5.6.4 Heterogeneous Attitudes Towards Ambiguity and Degrees Of Perceived Ambiguity

Instead of assuming a particular α for the informed traders, one could assume that the degree of optimism is distributed randomly across the population of informed traders. As the market grows more or less confident about either state, one could shift the mean and the variance of the distribution. If the mean of that distribution was to shift in a way that it fulfills the sufficient conditions for herding of Theorem 5.3 and it's variance was connected to e.g. the variance of the public belief π_t, then herding would still be possible. In that case not all informed traders with a particular private signal would change their trading decision simultaneously and there would always be a certain amount of traders that do not change their decision to follow or act against the crowd. If the share of traders engaging in herd or contrarian behavior is large enough, however, price distortions are still possible in principle. Yet, if the distribution of the informed traders is part of the common knowledge structure, the price process will remain a martingale even under herding or contrarianism, compare e.g. Avery and Zemsky (1998). Hence, herds, while potentially persistently price-distorting, will not drive the price towards the wrong state in the long run. A detailed analysis of such a model would be an interesting avenue for future research.

Similarly, we could consider a heterogeneous population of informed traders that perceive different degrees of primary ambiguity. If the distribution of the primary ambiguity levels is part of the common knowledge structure of the model, the same argument applies as before for the likely model outcomes and, thus should be included in future analyses.

[52] In that sense, every signal in a two-state world is monotone.

5.7 Conclusion

We have provided a comprehensive framework to explain how ambiguity may affect investor behavior and social learning in the two-state, two-trader version of the Avery and Zemsky (1998) model. As we review Avery and Zemsky's model setup without Knightian uncertainty, we observe that it implies that neither herding nor contrarianism are possible. We find that ambiguity and an invariant degree of optimism result in strong contrarian tendencies among informed traders but still preclude herd behavior. When allowing the individual degree of optimism to vary with market prices, herding becomes possible. It is necessary for herding, that the individual degree of optimism increases with market prices, i.e. with the degree of optimism exhibited by the market as a whole. If informed traders become overly exuberant or gloomy regarding an investment prospect they perceive a sufficiently high amount of primary ambiguity, then this is sufficient for herding to occur with positive probability.

We find, that herding and contrarianism alike cause informational cascades, thereby, preventing the market from pricing the asset at it's fundamental value. Indeed, contrarianism as well herding moves prices away from fundamentals with positive probability. Such wrong cascades are qualitatively different for herding and contrarianism. If the market is marginally uncertain regarding investor preferences, informational cascades are only partial. In that case, the self-defeating nature of contrarianism bounds prices away from the asset's potential fundamental values in the long run. The self-enforcing nature of herding, on the other hand, has the potential to drive price towards the wrong state.

Aside from the already mentioned theoretical extensions, the natural next step is to put the theoretical predictions of the CEU model to the test. Conducting experiments in a similar spirit as Drehmann et al. (2005) and Cipriani and Guarino (2005), where we add ambiguous components to the information signals to reflect the assumptions of the CEU model presented in this paper, would be an exciting avenue for future research. Similarly, one could test experimentally whether ambiguity regarding the distribution of the risky asset as proposed by Dong et al. (2010) leads to herding or contrarianism respectively.

From an empirical perspective, it may be interesting to investigate possibilities to measure the degree of primary ambiguity. If we assume for instance, that the absolute number of professional or expert traders for a particular asset is constant, then an increase in the number of traders would hint at an increase of average primary ambiguity. Similarly, one could investigate investor behavior in the aftermath of unexpected events, that are relevant to the valuation of the asset. It would be interesting to analyze whether joint evidence of higher primary ambiguity and investor coordination have predictive power regarding future price reversals, that would be consistent with price-distorting herding.

5.8 Appendix

This Appendix is structured as follows. Section 5.8.A provides an intuitive example why it is reasonable to assume that an economically relevant share of market participants perceives ambiguity when facing investment decisions. Section 5.8.B contains the all proofs omitted from the main part. Section 5.8.C provides the formal results for the perturbed model discussed in Section 5.5 in the main part. Section 5.8.D discusses market outcomes of a purely optimistic market under GBU, thus detailing the results of Section 5.4.2 in the main part. Section 5.8.E collects results from Avery and Zemsky (1998) that are relevant to this paper. Section 5.8.F discusses in further detail the relationship between different definitions for informational cascades and why we chose the one provided by Avery and Zemsky (1998). Section 5.8.G is a repository of the most important definitions and results from Chateauneuf et al. (2007) that are needed to confirm that our application of NEO-additive CEU preferences to the model of Avery and Zemsky (1998) is correct. Finally, Section 5.8.H discusses some inconsistencies of the GBU rule as prices approach 0 or 1, thereby, supporting the idea that it is reasonable that α varies with the price.

5.8.A Example of an Investor Facing Informational Ambiguity

As an example for informational ambiguity, consider a risk-neutral rational retail investor who has to decide whether or not to buy a particular stock (e.g. BMW - a German car manufacturer). She will make money on the investment if the price of the stock goes up, she will lose money if the price goes down.

She receives a recommendation from her online broker to buy the BMW stock but she has little knowledge about the German automotive industry (and BMW in particular). She knows her online broker is right 60 % of the times but she is not fully sure about her broker's agenda in this case because she sees that the BMW stock price has depreciated during the past month.

Given that she is rational and risk-neutral, she should buy the stock if and only if she believes that the price of the BMW stock will rebound with a probability greater than 50%. In the present case, however, the information does not enable her to determine the relevant probability exactly.[53]

There are many different ways the retail trader might process the online broker's recommendation and the stock price information to arrive at a probability assessment. She could think that her broker would not make such a recommendation if he did not have some valid information that the BMW stock price will go up. If such recommendations were made arbitrarily, the broker would risk to alienate and eventually loose customers who make losses as they follow the broker's recommendation. Despite BMW's recent downturn, she assigns a probability greater than 50% that the BMW stock price will go up again. Put differently, she retains an optimistic attitude towards the investment prospect. On the other hand, she might also think that the broker recommends buying the BMW stock because he wants to sell out it's own BMW shares before the price drops further. This pessimistic view would lead to a probability assignment of less than 50% to an increasing stock price. Finally, she might retain a skeptical view on the online broker's recommendation but she might be less pessimistic than in the previous scenario. Since she knows little about cars and nothing about the broker's motivation for the recommendation, she might conclude that she could as well toss a fair coin to decide what to do, i.e. she assigns a probability of precisely 50% to an increasing stock price.

5.8.B Mathematical Proofs

In this section provide the remaining proofs of the main part of this paper.

Proof of Theorem 5.1: To proof this, we restate some of the results of Ford et al. (2013):

Lemma 5.6. *In the CEU model with $\delta_0 > 0$ and $\alpha \in [0;1]$ fixed and $\pi_t \in [0;1]$, we have*

[53] One could argue that she would only need to update an uninformative prior with a noisy signal according to Bayes' rule to conclude that the price will go up with 60% probability. Yet, this would only be true if the retail trader was confident that the online broker's success probability of 60% does apply to the current situation.

- *The ask price in the Avery and Zemsky model is increasing and concave in* π_t.
- *The bid price in the Avery and Zemsky model is increasing and convex in* π_t.
- $CEU_{S_1}(\pi_t)$ *is increasing and concave in* π_t.
- $CEU_{S_0}(\pi_t)$ *is increasing and convex in* π_t.

All properties hold in a strict sense.

Proof: The proof is provided in Ford et al. (2013).

□

With this, we can immediately prove the impossibility of herding in the CEU model with fixed α. We will conduct the proof for the impossibility of herding for the low signal S_0. The proof is symmetric for S_1.

Let us start with the impossibility of buy herding. For buy herding to be possible, S_0 has to sell initially, i.e.

$$x_1 := CEU_{S_0}(\pi_0) < bid^{CEU}(\pi_0) < \pi_0.$$

Moreover, we have that

$$y_1 := CEU_{S_0}(1) = (1 - \delta_{S_0}(1)) + \delta_{S_0}(1)\alpha = 1 - \delta_{S_0}(1)(1 - \alpha) \leq 1.$$

Now assume that the market price has increased, i.e. $\pi_t > \pi_0$. Then π_t can be written as a convex combination of π_0 and 1. That is, $\exists\lambda \in (0; 1)$ such that $\pi_t = \lambda\pi_0 + (1 - \lambda)$. In line with Lemma 5.6, convexity of $CEU_{S_0}(\pi_t)$ implies that $\forall\lambda \in [0; 1]$, we have

$$\lambda x_1 + (1 - \lambda)y_1 > CEU_{S_0}(\pi_t).$$

Since $x_1 > \pi_0$ and $y_1 \leq 1$, it immediately follows that

$$\lambda x_1 + (1 - \lambda)y_1 < \lambda\pi_0 + (1 - \lambda) = \pi_t \leq ask^{CEU}(\pi_t),$$

which in turn implies that $CEU_{S_0}(\pi_t) < ask^{CEU}(\pi_t)\ \forall \pi_t \in [\pi_0; 1]$. This is equivalent to the fact that S_0 never buy herds.

The assumption of monotonicity of the CEU asset valuation implies that if S_0 buys so does S_1. This constitutes an informational cascade and, thus, S_0 can never sell herd.

For argument's sake, however, let us drop the monotonicity assumption for a moment. Sell herding would then still be impossible: Assume that S_0 buys initially (and S_1 does not). Hence, we have

$$x_1 := CEU_{S_0}(\pi_0) > \pi_0.$$

Moreover, we have again

$$y_1 := CEU_{S_0}(1) \leq 1.$$

Let $m_1 := \frac{y_1 - x_1}{1-\pi_0}$. Then m_1 denotes the average slope of $CEU_{S_0}(\pi_t)$ on $[\pi_0; 1]$. Since $x_1 < \pi_0$ and $y_1 \leq 1$, it follows that $m_1 < 1$. Hence, since $CEU_{S_0}(\pi_t)$ is convex and increasing, it follows that

$$\frac{\partial CEU_{S_0}}{\partial \pi_t}(\pi_0) \leq m_1 < 1.$$

Again invoking that $x_1 > \pi_0$, this implies that the tangent Θ_1 of CEU_{S_0} in π_0 lies above π_t on $[o; \pi_0]$. Moreover, convexity of CEU_{S_0} implies that $CEU_{S_0} \geq \Theta_1$ for all π_t. Hence, we conclude that $bid^{CEU}(\pi_t) < \pi_t < \theta_1 \leq CEU_{S_0}$ for all $\pi_t \in [0; \pi_0]$, which precludes the possibility that S_0 engages in sell herding.

We continue the proof by showing the possibility of contrarian behavior for S_0. Again, the argument is symmetric for the high signal.

Let S_0 sell initially, then only buy contrarianism is possible. Hence, we have to find a $\pi_t \in (0; \pi_0)$, such that $CEU_{S_0}(\pi_t) > ask^{CEU}(\pi_t)$. Noting that

$$CEU_{S_0}(0) = \delta_{S_0}(0)\alpha > 0 = ask^{CEU}(0),$$

continuity of CEU_{S_0} and ask^{CEU} implies that $CEU_{S_0} > ask^{CEU}$ in a whole neighborhood of zero, i.e. $\exists \epsilon > 0 : CEU_{S_0}(\pi_t) > ask(\pi_t)\ \forall \pi_t \in [0; \epsilon]$. But this already implies that S_0 engages in buy contrarianism $\forall \pi_t \in [0; \epsilon]$.
As before, monotonicity of CEU beliefs actually prevents sell contrarianism of S_0 from being possible. For argument's sake, let us drop this assumption for a moment. Then sell contrarianism of S_0 is indeed possible.

Let S_0 buy initially. For S_0 to act as a sell contrarian, we have to find a $\pi_t \in (\pi_0; 1)$, such that $CEU_{S_0}(\pi_t) < bid^{CEU}(\pi_t)$. Noting that

$$CEU_{S_0}(1) = (1 - \delta_{S_0}(1)) + \delta_{S_0}(1)\alpha = 1 - \delta_{S_0}(1)(1 - \alpha) < 1 = bid^{CEU}(1)$$

and invoking a continuity argument as before implies that there exists $\tilde{\epsilon} > 0$ such that $CEU_{S_0}(\pi_t) < bid(\pi_t)\ \forall \pi_t \in [1 - \tilde{\epsilon}; 1]$. Hence, S_0 may engage in sell contrarianism.

□

Note that for the impossibility of buy herding, we could have actually shown that $CEU_{S_0} < bid$ for all $\pi \in [\pi_0; 1]$. Intuitively, the argument would be that the increase of $CEU_{S_0}(\pi)$ in π is lower than the increase of the low signal's SEU belief $E[V \mid S_0, \pi]$ from the baseline model. Monotonicity in conjunction with convexity of CEU_{S_0} and $E[V \mid S_0, \pi]$ as well as the fact that $E[V \mid S_0, 1] = 1$ imply that CEU_{S_0} must remain below the bid price. That is, even weak forms of herding, i.e. switches from selling (buying) to holding are impossible if α is fixed.

Proof of Theorem 5.3: Again due to symmetry, we show only the buy herding statement. First, we note that $\alpha_{S_0}(1) = 1$, implies that $CEU_{S_0}(1) = 1 = ask(1)$, where $ask(\cdot) \geq ask^{CEU}(\cdot)$ denotes the ask price from the baseline model, see Equation (5.35).

Noting that (A2) implies that CEU_{S_0} is regular, we get

$$\begin{aligned} \frac{\partial CEU_{S_0}}{\partial \pi} &= \frac{\partial}{\partial \pi}\left[(1 - \delta_{S_0}(\pi))E_\pi[V \mid S_0] + \delta_{S_0}(\pi)\alpha_{S_0}(\pi)\right] \\ &= (1 - \delta_{S_0}(\pi))\frac{\partial}{\partial \pi}E_\pi[V \mid S_0] + \delta'_{S_0}(\pi)\left(\alpha_{S_0}(\pi) - E_\pi[V \mid S_0]\right) \end{aligned}$$

$$+\delta_{S_0}(\pi)\alpha'_{S_0}(\pi)$$

and evaluating it at $\pi = 1$, we get

$$\frac{\partial CEU_{S_0}}{\partial \pi}(1) = (1-\delta_{S_0}(1))\frac{\partial}{\partial \pi}E_1[V \mid S_0] + \delta_{S_0}(1)\alpha'_{S_0}(1), \tag{5.10}$$

where we used the fact that $\alpha_{S_0}(1) = E_1[V \mid S_0] = 1$. Since $\alpha'_{S_0}(1) = 0$ due to (A2), we infer that CEU_{S_0} must be strictly increasing in 1 and thus also in a neighborhood of 1.

Moreover, $ask(\pi)$ is also strictly increasing in 1 and in a neighborhood of 1 (compare Lemma 5.6). For CEU_{S_0} to be greater than ask in a neighborhood of 1 it is thus sufficient, if

$$\frac{\partial CEU_{S_0}}{\partial \pi}(1) < \frac{\partial ask}{\partial \pi}(1). \tag{5.11}$$

Plugging in the right hand side of Equation (5.10) into the left hand side of Inequality (5.11) and using that $\alpha'_{S_0}(1) = 0$, we find that Inequality (5.11) is equivalent to

$$(1-\delta_{S_0}(1))\frac{\partial}{\partial \pi}E_1[V \mid S_0] < \frac{\partial ask}{\partial \pi}(1). \tag{5.12}$$

Solving for $\delta_{S_0}(1)$ yields

$$\delta_{S_0}(1) > \frac{\frac{\partial}{\partial \pi}E_1[V \mid S_0] - \frac{\partial ask}{\partial \pi}(1)]}{\frac{\partial}{\partial \pi}E_1[V \mid S_0]}. \tag{5.13}$$

Now observing that

$$\begin{aligned}
\delta_{S_0}(1) &= \frac{\delta_0}{(1-\delta_0)(1-q)+\delta_0}, \\
\frac{\partial}{\partial \pi}E_1[V \mid S_0] &= \frac{q}{1-q}, \\
\frac{\partial ask}{\partial \pi}(1) &= \frac{\mu(1-q)+\theta}{\mu q+\theta},
\end{aligned}$$

we can solve Inequality (5.13) for δ_0 and obtain

$$\delta_0 > 1 - \frac{\mu(1-q)+\theta}{q(\mu+2\theta)},$$

which according to our initial argument is sufficient for the low signal to buy at prices in a neighborhood of 1.

□

We note that if we were to drop assumption (A1), then Theorem 5.3 would hold trivially if the low (high) signal were to buy (sell) initially given that there is no informational cascade. For this, we would only have to exchange α_{S_0} and α_{S_1} in the buy and sell herding conditions. Then, the sufficient condition for sell herding implies that $\alpha_{S_0}(\pi) < bid(\pi) \leq bid^{CEU}$ after a sufficiently strong price drop and, thus, $CEU_{S_0}(\pi) < bid^{CEU}(\pi)$, which implies sell herding on the part of the low signal. The argument for S_1 is symmetric. A similar argument can be made if the definition for herding included switches from hold to buy and sell and the informed traders held initially.

If the second part (A2) is dropped (i.e. regularity still holds), then we require an additional condition regarding $\alpha'_{S_0}(1)$ and $\alpha'_{S_1}(0)$.

Corollary 5.1. ***General Sufficient Condition For Herding***
Consider the CEU model with varying α, where (A1) and (A3) hold and α is sufficiently regular in π.
Let

$$C := \frac{K_1}{K_2} + \frac{(1-\delta_0)}{\delta_0 K_2}(\mu + \theta)(1 - 2q),$$

where q denotes the signal precision, μ the informed trader share and $\theta := (1-\mu)/3$ and $K_1 := \mu(1-q) + \theta$ and $K_2 := \mu q + \theta$.
*If $\alpha_{S_0}(1) = 1$ and $\alpha'_{S_0}(1) < C$, then **buy herding** occurs with positive probability.*
*If $\alpha_{S_1}(0) = 0$ and $\alpha'_{S_1}(0) < C$, then **sell herding** occurs with positive probability.*

Proof: For the buy herding case, simply note that the $\alpha'_{S_0}(1)$ term does not disappear in Inequality (5.12). Then, solving it for $\alpha'_{S_0}(1)$ yields the condition $\alpha'_{S_0}(1) < C$. The argument for sell herding is identical.

□

The boundary C implies that high primary ambiguity δ_0 still contributes towards the possibility of herding. The higher δ_0, the less negative the second summand of C, the larger C and the less binding the slope condition for the optimism function. If, however, α increases too strongly in 1, i.e. $\alpha'_{S_0}(1) \geq K_1/K_2$, that is, the degree of optimism is rather inelastic with respect to changes in market sentiment for moderate prices, then no amount of primary ambiguity will lead to herd behavior.

In that sense, herding in the CEU model requires a departure from the certainty effect implied by propsect theory, compare e.g. Barberis and Thaler (2003). The certainty effect implies that individuals facing risk tend to undervalue probabilities close to 1 and overvalue probabilities close to 0. We would again appeal to the finding of Heath and Tversky (1991) that this is not necessarily the case for probability judgments under ambiguity.

Proof of Lemma 5.3: We will focus the proof on the herding case. The arguments are identical for the contrarian case.

Before we start the actual proof, let us state a supporting Lemma that will also help with our subsequent comparative static analysis.

Lemma 5.7. *Consider the CEU model. Let herding be possible and let wlog* $\exists!\, \pi^* > \pi_0, \pi^{**} < \pi_0$ *such that* S_0 *engages in buy herding for* $\pi > \pi^*$ *and* S_1 *engages in sell herding for* $\pi < \pi^{**}$.[54] *Then Equations*

$$\pi^* = \frac{(\mu q + \theta)^x \pi_0}{(\mu q + \theta)^x \pi_0 + (\mu(1-q) + \theta)^x (1 - \pi_0)} \tag{5.14}$$

and

$$\pi^{**} = \frac{(\mu(1-q) + \theta)^x \pi_0}{(\mu(1-q) + \theta)^x \pi_0 + (\mu q + \theta)^x (1 - \pi_0)} \tag{5.15}$$

have unique solutions $x^* > 0$ *and* $x^{**} > 0$ *respectively. Then* $n^* := \lfloor x^* \rfloor + 1$ *defines the minimum number of price increases the low signal has to observe before engaging in buy herding. Similarly,* $n^{**} := \lfloor x^{**} \rfloor + 1$ *defines the minimum number of price decreases the low signal has to observe before engaging in sell herding.*

[54] This assumption is made for convenience. The sufficient condition only implies that there exist such π but not that they are unique. We would then have to consider the respective minimum or maximum over all such π.

Proof: Uniqueness and positivity of the solutions of Equations (5.14) and (5.15) with respect to x follow from the fact that r.h.s of Equation (5.14) is π_0 for $x = 0$, goes to 1 as $x \to \infty$ and is strictly increasing in x, while the r.h.s of Equation (5.15) is π_0 for $x = 0$, goes to 0 as $x \to \infty$ and is strictly decreasing in x. The definitions of n^* and n^{**} are immediately implied by Corollary 5.3.

□

We note that there are no closed-form solutions for x^* and x^{**} in general. Moreover, we have $n^{**} = n^*$, if we assume symmetry between α_{S_0} and α_{S_1} in the sense of Section 5.4.5. Finally, note that we can state a similar result for the contrarian case with the mere difference that π^* and π^{**} need to be exchanged.

Under the symmetry assumption for α_{S_0} and α_{S_1}, the problem of calculating P_{n^*} in Lemma 5.3 essentially reduces to a common ruin problem. To see this, note that price updating in the CEU model (Lemma 5.2) immediately implies that the probabilities $P(\{\pi \text{ increases in } t\}|\pi_{t-1}, V)$,
$P(\{\pi \text{ decreases in } t\}|\pi_{t-1}, V)$ and $P(\{\pi \text{ remains constant in } t\}|\pi_{t-1}, V)$ remain constant as long as $\pi_{t-1} \in [\pi^{**}; \pi^*]$, where $t \geq 1$ and π^*, π^{**} are from Lemma 5.7. In particular, we have

$$\begin{aligned} P(\{\pi \text{ increases in } t\}|\pi_{t-1}, V) &= P(a_t = \{buy\} \mid V) =: p_b, \\ P(\{\pi \text{ decreases in } t\}|\pi_{t-1}, V) &= P(a_t = \{sell\} \mid V) =: p_s, \\ P(\{\pi \text{ remains constant in } t\}|\pi_{t-1}, V) &= P(a_t = \{hold\} \mid V) =: p_h. \end{aligned} \tag{5.16}$$

Now, we define for $t \geq 0$

$$i_{t+1} := \begin{cases} i_t + 1 & \text{if } \pi \text{ increases in } t \\ i_t - 1 & \text{if } \pi \text{ decreases in } t \\ i_t & \text{if } \pi \text{ remains constant in } t \end{cases}, \tag{5.17}$$

where $i_0 = 0$. Then $\pi < \pi^{**}$ is equivalent to $i_t = -n^*$ and $\pi > \pi^*$ is equivalent to $i_t = n^*$ under the symmetry assumption. Let π_{i_t} denote the price process.

We derive the probability of an informational cascade under buy herding given that $V = 0$. The case of sell herding when $V = 1$ is symmetric.

Let A^{π}_{ICB} denote the event of a buy side informational cascade given that the price is π. Then, initially, we have

$$\begin{aligned} P(A^{\pi_0}_{ICB}|V=0) =& P(A^{\pi_1}_{ICB} \cap \{i_1 - i_0 = 1\}|V=0) \\ &+ P(A^{\pi_{-1}}_{ICB} \cap \{i_1 - i_0 = -1\}|V=0) \\ &+ P(A^{\pi_0}_{ICB} \cap \{i_1 - i_0 = 0\}|V=0). \end{aligned} \tag{5.18}$$

We have decomposed $A^{\pi_0}_{ICB}$ disjointly and then used the additivity of probability measures for disjoint events. Now noting that the events $A^{\pi_\cdot}_{ICB}$ and $\{i_1 - i_0 = -1\}$ are independent and incorporating Equations (5.16), Equation (5.18) becomes

$$\begin{aligned} P(A^{\pi_0}_{ICB}|V=0) =& p_b P(A^{\pi_1}_{ICB}|V=0) + p_s P(A^{\pi_{-1}}_{ICB}|V=0) \\ &+ p_h P(A^{\pi_0}_{ICB}|V=0). \end{aligned} \tag{5.19}$$

Denoting $P(k) := P(A^{\pi_k}_{ICB}|V=0)$ and solving (5.19) for $k = 0$, we get

$$(1 - p_h)P(0) = p_b P(1) + p_s P(-1).$$

Since this holds for all integers $k \in [-n^* + 1; -n^* + 1]$ and moreover, $P(n^*) = 1$ and $P(-n^*) = 0$, shifting variables to $j = k + n^*$ yields the following system of linear equations

$$\begin{aligned} & (1 - p_h)P(j) && = p_b P(j+1) + p_s P(j-1) \quad \forall j = 1, \ldots, 2 * n^* - 1 \\ \wedge \quad & P(0) && = 0 \\ \wedge \quad & P(2n^*) && = 1. \end{aligned} \tag{5.20}$$

Since, on the other hand $1 = p_s + p_b + p_h$, we have that

$$(1 - p_h)P(j) = p_b P(j) + p_s P(j).$$

Equations $j = 1, \ldots 2n^* - 1$ from Equation System (5.20) are, therefore, equivalent to

$$(P(j+1) - P(j))\frac{p_b}{p_s} = (P(i) - P(i-1)).$$

Let $K := \frac{p_b}{p_s}$. By backwards induction starting at $j = 2n^* - 1$ we then get

$$P(j) - P(j-1) = K^{2n^*-j}(P(2n^*) - P(2n^*-1)) \tag{5.21}$$

for $j = 1, \ldots, 2n^* - 1$. Now noting that

$$\begin{aligned} 1 = P(2n^*) - P(0) &= \sum_{j=0}^{2n^*} (P(j+1) - P(j)) \\ &= \sum_{j=0}^{2n^*} K^{2n^*-j}(P(2n^*) - P(2n^*-1)) \\ &= (1 - P(2n^*-1)) \sum_{j=0}^{2n^*} K^j \\ &= (1 - P(2n^*-1)) \frac{1 - K^{2n^*}}{1 - K}, \end{aligned}$$

where the last equation holds because $\sum_{j=0}^{2n^*} K^j$ is a geometric sum. Solving this for $P(2n^* - 1)$ yields that

$$P(2n^* - 1) = \frac{K - K^{2n^*}}{1 - K^{2n^*}}.$$

Now noting that $P(j-1) = (P(j-1) - P(j)) + P(j)$ for $j = 1, \ldots, 2n^* - 1$ and inserting Equations (5.21) allows us to invoke another backward induction argument to conclude that

$$P(j) = \frac{K^j - K^{2n^*}}{1 - K^{2n^*}}. \tag{5.22}$$

Setting $j = n^*$ in Equation (5.22), basic algebra to transform the fraction yields

$$\begin{aligned} P(n^*) &= K^{n^*} \frac{1 - K^{n^*}}{1 - K^{2n^*}} \\ &= K^{n^*} \frac{1}{1 + K^{n^*}} \end{aligned}$$

$$= \frac{1}{\exp(-\ln(K)n^*) + 1}.$$

Noting that $p_b = \mu(1-q) + \theta$ and $p_s = \mu q + \theta$ if $V = 0$, and plugging these quantities in for K yields the formula for P_{n^*} in Equation (5.7).

□

We state an immediate consequence of Lemma 5.3.

Corollary 5.2. *Under the same conditions as in Lemma 5.3, the probability of a correct informational cascade is given by*

$$\widetilde{P}_{n^*} = \frac{1}{\exp\left(-\ln\left(\frac{\mu q+\theta}{\mu(1-q)+\theta}\right) n^*\right)}. \tag{5.23}$$

The prove is identical to the one of Lemma 5.3. It implies, in particular that Equation (5.22) becomes

$$\widetilde{P}(j) = \frac{K^{-j} - K^{-2n^*}}{1 - K^{-2n^*}} \tag{5.24}$$

for arbitrary $j \in [0; 2n^*]$, where j is the number of price decreases that needs to be observed before the correct informational cascade occurs.

□

Proof of Lemma 5.4: To develop an understanding for the idea of the proof note that a parameter shift can have two effects on P_{n^*}. First, it may affect $P_.$ directly. Second, it may cause n^* to vary. Noting that P_{n^*} decreases in n^*, this has an indirect effect on the probability of wrong herds. Changes in n^* occur as discrete jumps. Locally, this indirect effect on $P_.$ is, therefore, zero. It becomes relevant only for larger parameter shifts.

We first summarize the relevant calculus in a support lemma. Note that under the assumptions of Section 5.4.5, we have that $ask^{CEU} = ask$ if the market is prone to buy herding and buy contrarianism and that $bid^{CEU} = bid$ if the market is prone to sell herding and sell contrarianism. Hence, we can rely on the analytics of *ask* and *bid* from the Avery and Zemsky model to derive the results of Lemma 5.8.

Lemma 5.8. *Consider the CEU model. Let P_{n^*} be as in Equation (5.7). Then the following hold*

(i) $\frac{\partial P.}{\partial \mu} < 0$, $\frac{\partial P.}{\partial q} < 0$

(ii) $\frac{\partial P_x}{\partial x} < 0$

(iii) $\frac{\partial ask}{\partial \mu} > 0$, $\frac{\partial ask}{\partial q} > 0$

(iv) $\frac{\partial bid}{\partial \mu} < 0$, $\frac{\partial bid}{\partial q} < 0$

(v) $\frac{\partial CEU_S}{\partial \mu} = 0$

(vi) $\frac{\partial CEU_S}{\partial \delta_0} > 0$ *iff* $\alpha_S > E[V \mid S, \pi]$;
$\frac{\partial CEU_S}{\partial \delta_0} < 0$ *iff* $\alpha_S > E[V \mid S, \pi]$

(vii) $\frac{\partial CEU_S}{\partial q} = \frac{\partial E_q[V|S,\pi]}{\partial q} + \delta_q \left(\frac{\partial \alpha_q}{\partial q} - \frac{\partial E_q[V|S,\pi]}{\partial q} \right) + \frac{\partial \delta_q}{\partial q} \left(\alpha_q - E_q[V \mid S, \pi] \right)$

(viii) $\frac{\partial \delta_q}{\partial q} > 0$ *if* $\delta = \delta_{S_0}$ *and* $\pi > 0.5$ *or if* $\delta = \delta_{S_1}$ *and* $\pi < 0.5$.

(ix) $\frac{\partial E_q[V|S_0,\pi]}{\partial q} < 0$, $\frac{\partial E_q[V|S_0,\pi]}{\partial q} > 0$

Proof: For (i) note that

$$\exp\left(-\ln\left(\frac{\mu(1-q)+\theta}{\mu q + \theta} \right) \right) = \frac{\mu q + \theta}{\mu(1-q)+\theta}$$

Differentiating the r.h.s. with respect to μ and q yields quantities > 0. Applying the quotient rule, therefore implies (i).
For (ii) note that

$$\frac{\partial P_x}{\partial x} = \frac{-\ln\left(\frac{\mu q + \theta}{\mu(1-q)+\theta} \right) \exp(\cdot)}{f^2}.$$

Noting that $\mu q + \theta > \mu(1-q) + \theta$ implies that $\ln(\cdot) > 0$ and, thus (ii).
For (iii) and (iv) we refer to the reader to the market microstructure literature, e.g. Glosten and Milgrom (1985).
(v) follows from the assumption that α is independent of μ.
For (vi) note that

$$\frac{\partial \delta_{\delta_0}}{\partial \delta_0} = \frac{1}{f^2} > 0.$$

The remainder follows from the definition of CEU_S.
(vii) is a mere application of differentiation rules.
For (viii) note that e.g.

$$\delta_{S_0} = \frac{\delta_0}{(1-\delta_0)\left[\pi(1-q)+q(1-\pi)\right]+\delta_0}.$$

Differentiating with respect to q yields that the sign of $\frac{\partial \delta_q}{\partial q}$ is determined by $-\delta_0(1-\delta_0)(1-2\pi)$, which is greater than 0 if and only if $\pi > 0.5$. The argument is symmetric for δ_{S_1}.
Finally, (ix) follows from the literature, see e.g. Chamley (2004).

□

For the main proof we make the assumption that the immediate effects of q on n^* dominate the ancillary effects transmitted through changes in the ambiguity parameters. That is, we assume that the increasing effect of q on the ask price and the decreasing effect of q on the additive component of S_0's CEU belief

$$\frac{\partial ask}{\partial q} - (1-\delta_q)\frac{\partial E_q[V \mid S_0, \pi]}{\partial q}$$

dominate the ancillary effect that an increase in q actually increases CEU_{S_0} due to it's effect on δ and α

$$\delta_q \frac{\partial \alpha_q}{\partial q} + \frac{\partial \delta_q}{\partial q}\left(\alpha_q - E_q[V \mid S_0, \pi]\right).$$

That is, the difference of these terms should be positive.

Similarly, the difference between

$$(1-\delta_q)\frac{\partial E_q[V \mid S_1, \pi]}{\partial q} - \frac{\partial bid}{\partial q}$$

and

$$\delta_q \frac{\partial \alpha_q}{\partial q} + \frac{\partial \delta_q}{\partial q}\left(E_q[V \mid S_1, \pi] - \alpha_q\right)$$

should also be positive.

Then, (i)-(v) together imply that an increase in μ decreases $P_.$ and increases n^*. For the increase in n^* note that an increase in μ increases the ask price and decreases CEU_{S_0} (decreases the bid price and increases CEU_{S_1}). The decrease in $P_.$ and the increase n^* together imply unambiguously that P_{n^*} decreases in μ.

(i)-(iv) and (vii) in conjunction with our previously made assumptions imply the effects of an increase in q on the probability of a wrong cascade. The assumptions are required to ensure that q unambiguously increases n^*. Beyond that, the argument is identical to the one of the increase of μ.

(vi) implies that an increase in δ_0 causes investors to more heavily rely on their gut feel parameter α. In the case of contrarianism this means that CEU beliefs become less elastic to changes in π, which shifts the cut-off points towards π_0 and, hence decreases n^*, which in turn implies an increase in P_{n^*}. In the case of herding CEU beliefs become more elastic, which again shifts the cut-off points towards π_0.

For the increase in α note that we define such an increase as follows: α^1 is said to be greater than α^2 if and only if $\alpha^1(\pi) < \alpha^2(\pi)$ $\forall \pi \in (0; \pi_0)$ and $\alpha^1(\pi) > \alpha^2(\pi)$ $\forall \pi \in (\pi_0; 1)$. Now consider some $\pi > \pi_0$. An increase in α then implies an in increase in CEU_S regardless of the signal type. For the low signal prone to buy herding this means that the cut-off point π^* moves left, i.e. n^* decreases and P_{n^*} increases. For the high signal, who is prone to contrarianism this means that π^* moves right, i.e. n^* increases and P_{n^*} decreases. The argument is symmetric if $\pi < \pi_0$.

□

Proof of Lemma 5.5: We start by proving that $P_{n^*} \leq P^{\epsilon}_{WH}$. We can again focus on the buy herding case given $V = 0$ due to symmetry. For ease of notation we assume without loss of generality that $\epsilon = 1$. The line of reasoning is identical if $0 < \epsilon < 1$.

Let i_t be defined as in Equation (5.17) and let π_{i_t} be the corresponding price process. Moreover, let p_s and p_b be the true buy and sell probabilities in the perturbed model given $V = 0$. Note that p_b and p_s correspond to the respective buy and sell probabilities in the CEU model. Finally, let $\tilde{p}_s$ and $\tilde{p}_b$ be the corresponding buy and sell probabilities as perceived by the market.

We note that for $i_t \in [-k^*+1; k^*-1]$, we have $p_s = \tilde{p}_s$ and $p_b = \tilde{p}_b$ since both informed traders act as in the baseline model without ambiguity.

For $i_t \in [k^*; n^*-1]$, we have that S_0 holds and S_1 buys and, thus,

$$\frac{p_b}{p_s} = \frac{\mu(1-q)+\theta}{\mu q+\theta} < \frac{\mu(1-q)+\theta}{\theta} = \frac{\tilde{p}_b}{\tilde{p}_s}. \tag{5.25}$$

Hence, the CEU model assumptions imply a lower probability for strong buy side accumulations than in the perturbed model. This indicates that P_{n^*} underestimates P^{ϵ}_{WH}.

Correspondingly, for $i_t \in [-n^*+1; -k^*]$, we have that S_1 holds and S_0 sells, thus yielding

$$\frac{p_b}{p_s} = \frac{\mu(1-q)+\theta}{\mu q+\theta} > \frac{\theta}{\mu q+\theta} = \frac{\tilde{p}_b}{\tilde{p}_s}. \tag{5.26}$$

Hence, the CEU model assumptions imply a higher probability for strong sell side accumulations than the perturbed model. This indicates that P_{n^*} overestimates P^{ϵ}_{WH}.

Now, aggregating the net underestimation and net overestimation for some $i_{t,1} \in [-n^*+1; -k^*]$ and some $i_{t,2} \in [k^*; n^*-1]$ respectively, we get

$$\begin{aligned} & \left[\frac{\mu(1-q)+\theta}{\mu q+\theta} - \frac{\mu(1-q)+\theta}{\theta}\right] + \left[\frac{\mu(1-q)+\theta}{\mu q+\theta} - \frac{\theta}{\mu q+\theta}\right] \\ = \; & \mu\frac{\theta(1-2q)-\mu q(1-q)}{\theta(\mu q+\theta)} < 0. \end{aligned}$$

Due to symmetry we can consequently infer that for $i_t \in [-n^*+1; n^*-1]$, P_{n^*} in total underestimates P^{ϵ}_{WH}.

For $i_t \leq -n^*$ P_{n^*} underestimates P^{ϵ}_{WH} in the sense that in the CEU model social learning stops and the sell herd can never be broken to result in a buy herd after all.

For $i_t \geq n^*$, P_{n^*} overestimates P^{ϵ}_{WH} in the sense that in the CEU model social learning stops and the buy herd can never be broken to result in a sell herd after all.

Since, however, for $i_t \geq n^*$, we have that

$$\frac{\tilde{p}_s}{\tilde{p}_b} = \frac{\theta}{\mu q + \theta}, \tag{5.27}$$

and for $i_t \leq -n^*$, we have

$$\frac{\tilde{p}_b}{\tilde{p}_s} = \frac{\theta}{\mu q + \theta}, \tag{5.28}$$

we conclude that the probability to observe a sell herd given that there is currently buy herding is exactly equal to the probability to observe a buy herd given that there is currently sell herding. Hence, the associated over- and underestimation of P_{n^*} versus P^{ϵ}_{WH} cancel out.

In total P_{n^*} is, thus, a lower boundary for P^{ϵ}_{WH}.

We move to show that $P_{k^*} \geq P^{\epsilon}_{WH}$. For this to see, note that as above the probability of i_t becoming $\geq |k^*|$ in the CEU model and the perturbed CEU model is the same. Again, we focus on the buy herding case, where $V = 0$ and when $\epsilon = 1$ without loss of generality.
In the perturbed model, the probability of observing a price reversal given that $i_t \geq k^*$ is relatively higher than the probability of observing a price reversal into the opposite direction given that $i_t \leq -k^*$. Note that if $i_t \geq k^*$, then $\frac{\tilde{p}_s}{\tilde{p}_b} = \frac{\theta}{\mu(1-q)+\theta}$. Similarly, if $i_t \leq -k^*$, then $\frac{\tilde{p}_b}{\tilde{p}_s} = \frac{\theta}{\mu q+\theta}$. Observing that the second ratio is smaller than the first one, this confirms our claim.
Now, invoking a similar symmetry argument as in the lower boundary case, we conclude that P_{k^*} is, indeed an upper boundary of P^{ϵ}_{WH}.

□

If $k^* = n^*$, then $P^{NCK}_{WH} = P_{n^*}$. This implies in particular that probabilities of wrong herds and wrong learning coincide if there is no bid-ask spread.

5.8.C Formal Results for the Perturbed Model

The perturbed CEU model is described by the following two Propositions.

Proposition 5.4. ***Equilibrium Prices in the Perturbed CEU Model***
For any time t, let bid_t and ask_t be the bid and ask prices that are quoted in the

Avery and Zemsky model. Let π_t be the public belief in the perturbed CEU model. Moreover, let

$$\begin{aligned} bid_t^{\epsilon} &:= \frac{(\mu(1-q)\epsilon+\theta)\pi_t}{(\mu(1-q)\epsilon+\theta)\pi_t+(\mu q\epsilon+\theta)(1-\pi_t)}, \\ bid_t^{1-\epsilon} &:= \frac{(\mu(q(1-\epsilon)+(1-q))+\theta)\pi_t}{(\mu(q(1-\epsilon)+(1-q))+\theta)\pi_t+(\mu((1-q)(1-\epsilon)+q)+\theta)(1-\pi_t)}, \\ ask_t^{\epsilon} &:= \frac{(\mu q\epsilon+\theta)\pi_t}{(\mu q\epsilon+\theta)\pi_t+(\mu(1-q)\epsilon+\theta)(1-\pi_t)}, \\ ask_t^{1-\epsilon} &:= \frac{(\mu(q+(1-q)(1-\epsilon))+\theta)\pi_t}{(\mu(q+(1-q)(1-\epsilon))+\theta)\pi_t+(\mu((1-q)+q(1-\epsilon))+\theta)(1-\pi_t)}. \end{aligned}$$

Then, the market maker quotes the following bid and ask prices

$$bid_t^{\epsilon-CEU} = \begin{cases} \min\{bid_t^{\epsilon}; CEU_{S_0}(\pi_t)\}, & \textit{if } CEU_{S_0}(\pi_t) \geq bid_t \\ bid_t, & \textit{if } CEU_{S_0}(\pi_t) < bid_t \wedge CEU_{S_1}(\pi_t) \geq bid_t \\ bid_t^{1-\epsilon}, & \textit{if } CEU_{S_0}(\pi_t) < bid_t \wedge CEU_{S_1}(\pi_t) < bid_t \end{cases}$$

and

$$ask_t^{\epsilon-CEU} = \begin{cases} \max\{ask_t^{\epsilon}; CEU_{S_1}(\pi_t)\}, & \textit{if } CEU_{S_1}(\pi_t) \leq ask_t \\ ask_t, & \textit{if } CEU_{S_1}(\pi_t) > ask_t \wedge CEU_{S_0}(\pi_t) \leq ask_t \\ ask_t^{1-\epsilon}, & \textit{if } CEU_{S_1}(\pi_t) > ask_t \wedge CEU_{S_0}(\pi_t) > ask_t. \end{cases}$$

Proof: The proof is essentially a repeated application of Bayes' rule and game theoretic arguments as in the proof of Lemma 5.1. We outline the proof for $bid_t^{\epsilon-CEU}$ when $CEU_{S_0}(\pi_t) \geq bid_t$. The arguments for the other cases are similar.

First note that by monotonicity of the informed traders asset valuation, i.e. $CEU_{S_0}(\pi_t) < CEU_{S_1}(\pi_t)$, the high signal does not sell if the low signal does not sell. Consequently, by Bayes' rule the bid price under the zero-profit condition for the market maker is

$$bid_t^{\epsilon-CEU} = \frac{\tilde{P}(a_t = \{sell\}|H_t, V=1)P(V=1|H_t)}{\tilde{P}(a_t=\{sell\}|H_t)} = bid_t^{\epsilon},$$

since $P(V=1|H_t)=\pi_t$, $\tilde{P}(a_t=\{sell\}|H_t, V=1) = \mu(1-q)\epsilon+\theta$ and $\tilde{P}(a_t = \{sell\}|H_t) = (\mu(1-q)\epsilon+\theta)\pi_t + (\mu q\epsilon+\theta)(1-\pi_t)$. Thus, noting that $bid_t^{\epsilon} > bid_t$, we have to distinguish between the case where $CEU_{S_0}(\pi_t) < bid_t^\epsilon$ and where $CEU_{S_0}(\pi_t) \geq bid_t^\epsilon$.

If $CEU_{S_0}(\pi_t) < bid_t^\epsilon$, then quoting $bid_t^{\epsilon-CEU} = bid_t^\epsilon$ would cause the low signal to sell at π_t. Yet, if S_0 sells, the market maker makes zero-profit only when quoting $bid_t < bid_t^\epsilon$. Hence, he makes an average loss on every sell he fills of $bid_t - bid_t^\epsilon$, causing him to eventually go out of business. As long as the market maker quotes $bid_t^{\epsilon-CEU} > CEU_{S_0}(\pi_t)$ and S_0 sells at π_t a similar argument applies.

Consequently, it must be that $bid_t^{\epsilon-CEU} \leq CEU_{S_0}(\pi_t)$ to ensure that S_0 holds. When quoting $bid_t^{\epsilon-CEU} < CEU_{S_0}(\pi_t)$, then the market maker makes an expected profit of $bid_t^\epsilon - bid_t^{\epsilon-CEU}$. His Bertrand competition can then quote a more competitive bid price, $\widetilde{bid}$, where

$$bid_t^{\epsilon-CEU} < \widetilde{bid} < CEU_{S_0}(\pi_t).$$

By this the competition draws away all noise traders from the market maker, making a slightly smaller but still positive expected profit $bid_t^\epsilon - \widetilde{bid}$ on every sell they fill.

The market maker's best response to $\widetilde{bid}$ would then be a similar increase in the quoted bid price. This price war continues until

$$bid_t^{\epsilon-CEU} = \widetilde{bid} = CEU_{S_0}(\pi_t).$$

Hence, for $CEU_{S_0}(\pi_t) < bid_t^\epsilon$, $bid_t^{\epsilon-CEU} = CEU_{S_0}(\pi_t)$ is the equilibrium bid price.

If $CEU_{S_0}(\pi_t) \geq bid_t^\epsilon$, then the market maker quotes $bid_t^{\epsilon-CEU} = bid_t^\epsilon$ according to the zero profit condition. Hence,

$$bid_t^{\epsilon-CEU} = \min\{bid_t^\epsilon; CEU_{S_0}(\pi_t)\},$$

if the low signal stops selling.

□

Key thing to note here is that $ask_t^{\epsilon-CEU} = ask_t$ ($bid_t^{\epsilon-CEU} = bid_t$) as long S_1 buys (S_0) sells. As a consequence, the analytics of a switch from selling to buying and vice versa are the same as in the unperturbed CEU model. This, in turn implies that Theorems 5.1, 5.2 and 5.3 also hold in the perturbed model.

Moreover, since $bid_t^{\epsilon} < ask_t^{1-\epsilon}$ ($bid_t^{1-\epsilon} < ask_t^{\epsilon}$), there is a positive bid-ask spread even if all informed traders take the same action. This is because the market (maker) believes there is a share of ϵ SEU traders in the market. Hence, there is no complete informational cascade in the perturbed CEU model since the market still infers information from the trades and updates prices accordingly. This is formalized in

Proposition 5.5. ***Public Belief Updating in the perturbed CEU Model***
Under the assumptions and with the notation of Proposition 5.4, let

$$\begin{aligned} h_t^1 &:= \frac{(\mu(1-q)(1-\epsilon)+\theta)\pi_t}{(\mu(1-q)(1-\epsilon)+\theta)\pi_t + (\mu q(1-\epsilon)+\theta)(1-\pi_t)}, \\ h_t^2 &:= \frac{(\mu q(1-\epsilon)+\theta)\pi_t}{(\mu q(1-\epsilon)+\theta)\pi_t + (\mu(1-q)(1-\epsilon)+\theta)(1-\pi_t)}. \end{aligned}$$

Then:

(1) If the high CEU-signal buys and the low CEU-signal sells at time t, then π_{t+1} is as in the model of Avery and Zemsky, see Equation (5.39).

(2) If the high CEU-signal buys and the low signal holds at time t, then

$$\pi_{t+1} = \begin{cases} ask_t, & \text{if } a_t = \{buy\} \\ bid_t^{\epsilon}, & \text{if } a_t = \{sell\} \\ h_t^1, & \text{if } a_t = \{hold\}. \end{cases} \tag{5.29}$$

(3) If the high CEU-signal holds and the low CEU-signal sells at time t, then

$$\pi_{t+1} = \begin{cases} ask_t^{\epsilon}, & \text{if } a_t = \{buy\} \\ bid_t, & \text{if } a_t = \{sell\} \\ h_t^2, & \text{if } a_t = \{hold\}. \end{cases} \tag{5.30}$$

(4) If both CEU-signals buy at time t, then

$$\pi_{t+1} = \begin{cases} ask_t^{1-\epsilon}, & \textit{if } a_t = \{\textit{buy}\} \\ bid_t^{\epsilon}, & \textit{if } a_t = \{\textit{sell}\} \\ \pi_t, & \textit{if } a_t = \{\textit{hold}\}. \end{cases} \tag{5.31}$$

(5) If both CEU-signals sell at time t, then

$$\pi_{t+1} = \begin{cases} ask_t^{\epsilon}, & \textit{if } a_t = \{\textit{buy}\} \\ bid_t^{1-\epsilon}, & \textit{if } a_t = \{\textit{sell}\} \\ \pi_t, & \textit{if } a_t = \{\textit{hold}\}. \end{cases} \tag{5.32}$$

Proof: The proof is similar to the one of Lemma 5.2. For the sake of completeness let us consider case (4), which corresponds e.g. to a buy herding regime.
By Bayes' rule the updated prior belief after action a_t is observed is given by

$$\pi_{t+1} = P[V = 1|H_{t+1}] = \frac{\tilde{P}(a_t|H_t, V = 1)P(V = 1|H_t)}{\tilde{P}(a_t|H_t)}.$$

When choosing $a_t = \{buy\}$ and $a_t = \{sell\}$, then the updating rule for π_{t+1} after an observed buy and sell immediately follow from Proposition 5.4. That the price remains stable after a hold can also be inferred from the above formula. Intuitively, it reflects that the market considers a hold to be conducted by a noise trader for sure. All informed traders that are considered to be present on the market either buy (high signal S_1 or CEU preferences) or sell (low signal S_0 with SEU preferences) the asset. As a consequence, a hold bears no informational value.

□

To prove Propositions 5.2 and 5.3 in the main part, we need the following support propositions

Proposition 5.6. *In the perturbed CEU model, let $\pi^* < \pi_0$ ($\pi^{**} > \pi_0$) be the cut-off prices, such that S_1 sell herds for all $\pi < \pi^*$ (S_0 buy herds for all $\pi > \pi^{**}$). Then $\pi_t^{BH} := (\pi_t | \pi_t > \pi^{**})$ is a sub-martingale with respect to H_t and $\pi_t^{SH} := (\pi_t | \pi_t < \pi^*)$ is a super-martingale with respect to H_t.*

Proof: We only show that π_t^{BH} is a sub-martingale. The proof is symmetric for π_t^{SH}. To show that π_t^{BH} is a sub-martingale with respect to H_t, we need to prove that $E[\pi_{t+1}^{BH} \mid H_t] \geq \pi_t^{BH}$. Since π_{t+1}^{BH} is bounded by definition other martingale properties follow immediately.
Note that showing $E[\pi_{t+1}^{BH} \mid H_t] \geq \pi_t^{BH}$ is equivalent to showing

$$E[\pi_{t+1} \mid H_t, \, \pi_t > \pi^{**}] \geq \pi_t.$$

Let $p_b := P[a_t = \{buy\} | H_t, \, \pi_t > \pi^{**}]$, $p_s := P[a_t = \{sell\} | H_t, \, \pi_t > \pi^{**}]$ and $p_h := P[a_t = \{hold\} | H_t, \, \pi_t > \pi^{**}]$ be the actual probabilities to observe a buy, sell and hold respectively given that S_0 buy herds. Then from Case (4) of Proposition 5.5, we infer that

$$E[\pi_{t+1} \mid H_t, \, \pi_t > \pi^{**}] = p_b ask_t^{1-\epsilon} + p_s bid^{\epsilon} + p_h \pi_t. \tag{5.33}$$

Now define

$$\tilde{p}_b := (\mu(q + (1-q)(1-\epsilon) + \theta)\pi_t + (\mu((1-q) + q(1-\epsilon) + \theta)(1-\pi_t)$$

and

$$\tilde{p}_s := (\mu(1-q)\epsilon + \theta)\pi_t + (\mu q \epsilon + \theta)(1-\pi_t)$$

to be the unconditional probabilities of a buy and sell respectively as perceived by market participants. Observe that $p_h = \theta$ and that $p_b = \tilde{p}_b + x$ and $p_s = \tilde{p}_s - x$, where $x = \epsilon\mu((1-q)\pi_t + (1-\pi_t)q)$.
Moreover, we set

$$\tilde{p}_b^1 := \mu(q + (1-q)(1-\epsilon) + \theta$$

and

$$\tilde{p}_s^1 := \mu(1-q)\epsilon + \theta$$

to be the corresponding buy and sell probabilities conditional on $V = 1$.
In line with Proposition 5.4, we can then rewrite the r.h.s. of Equation (5.33) so that we have

$$\begin{aligned} E[\pi_{t+1} \mid H_t, \, \pi_t > \pi^{**}] &= p_b \frac{\tilde{p}_b^1}{\tilde{p}_b} \pi_t + \frac{\tilde{p}_s^1}{\tilde{p}_s} \pi_t + p_h \pi_t \\ &= \left((\tilde{p}_b + x) \frac{\tilde{p}_b^1}{\tilde{p}_b} + (\tilde{p}_s - x) \frac{\tilde{p}_s^1}{\tilde{p}_s} + \theta \right) \pi_t \\ &= \left(\tilde{p}_b^1 + \tilde{p}_s^1 + \theta + x \left(\frac{\tilde{p}_b^1}{\tilde{p}_b} - \frac{\tilde{p}_s^1}{\tilde{p}_s} \right) \right) \pi_t \\ &= \left(1 + x \left(\frac{\tilde{p}_b^1}{\tilde{p}_b} - \frac{\tilde{p}_s^1}{\tilde{p}_s} \right) \right) \pi_t, \end{aligned}$$

where for the last step observe that $\tilde{p}_b^1 = \mu q + \mu(1-q)(1-\epsilon) + \theta$ and $\tilde{p}_s^1 = \mu(1-q)\epsilon + \theta$ and, thus, $\tilde{p}_b^1 + \tilde{p}_s^2 = \mu + 2\theta$. Since moreover $\mu + 3\theta = 1$ by definition the last equality holds. Based on this, however, showing $E[\pi_{t+1} \mid H_t, \, \pi_t > \pi^{**}] \geq \pi_t$ is equivalent to showing that $\tilde{p}_b^1 \tilde{p}_s - \tilde{p}_s^1 \tilde{p}_b > 0$. Noting that $\tilde{p}_s = \theta + x$ and that $\tilde{p}_b = \mu + \theta - x$ and plugging in the respective formulae, basic manipulations of the l.h.s. of the last inequality yield

$$\begin{aligned} & \tilde{p}_b^1 \tilde{p}_s - \tilde{p}_s^1 \tilde{p}_b \\ = \; & \mu x - \mu^2 \epsilon (1-q) + 2\theta x - 2\mu\epsilon(1-q)\theta \\ = \; & \mu^2 \epsilon \left[(1 + 2\theta\epsilon)((1-q)\pi_t + (1-\pi_t)q - (1-q)) \right] \\ = \; & \mu^2 \epsilon \left[(1 + 2\theta\epsilon)(1-\pi_t)(2q-1) \right] > 0, \end{aligned}$$

since $q > 0.5$, which concludes the proof.

□

Proposition 5.7. *In the perturbed CEU model, let $\pi^* < \pi_0$ ($\pi^{**} > \pi_0$) be the cut-off prices, such that S_0 is a buy contrarian for all $\pi < \pi^*$ (S_1 is a sell contrarian for all $\pi > \pi^{**}$).*
Then $\pi_t^{SH} := (\pi_t | \pi_t > \pi^{**})$ is a super-martingale with respect to H_t and

$\pi_t^{SH} := (\pi_t | \pi_t < \pi^*)$ *is a sub-martingale with respect to* H_t.

Proof: The proof is point-symmetric to the one for Proposition 5.6.

□

5.8.D Discussion of a Purely Optimistic Market in the CEU Model

We limit our attention to the purely optimistic case since the purely pessimistic case is symmetric. The subsequent analysis holds in the CEU model as well as in the perturbed CEU model, yet we state formal results only for the CEU model.

We start by noting that in a purely optimistic market, the high signal always buys. Since $1 = \alpha > E_{\pi_t}[V \mid S_1, H_t]$ for all $\pi_t \in (0;1)$, it follows that $CEU_{S_1}(\pi_t) > E_{\pi_t}[V \mid S_1] > ask(\pi_t)$ for all $\pi_t \in (0;1)$.

For the low signal, one of three cases is possible depending on the primary ambiguity δ_0. If $\delta_0 > \delta^*$, i.e. if it is high enough, then the purely optimistic low signal always buys, too. If $\delta_0 < \delta^{**}$, i.e. if it is low enough, then S_0 essentially behaves as in the case where $\alpha \in (0;1)$. If δ_0 is between the two cut-off points, then S_0 will buy at low prices but eventually switch into holding as π_t approaches 1.

Lemma 5.9. *In the CEU model with* $\alpha = 1$ *and* $\delta_0 > 0$*, the high signal always buys. For the low signal:* $\exists \delta^*, \delta^{**} \in (0;1)$ *with* $\delta^* > \delta^{**}$*, such that*

- S_0 *always buys if and only if* $\delta_0 > \delta^*$*,*
- $\exists \pi^* < 1$ *such that* S_0 *sells* $\forall \pi_t \in (\pi^*;1)$ *if and only if* $\delta_0 < \delta^{**}$*,*
- $\exists \pi^{**} < 1$ *such that* S_0 *holds* $\forall \pi_t \in (\pi^{**};1)$ *if and only if* $\delta^* \geq \delta_0 \geq \delta^{**}$*,*

where $\delta^* = \frac{-K_1+\sqrt{K_1^2+4K_2}}{2}$ *and* $\delta^{**} = \frac{-K_3+\sqrt{K_3^2+4K_4}}{2}$ *with*

$$K_1 = \frac{2(1-q)\left(\mu(1-q)+\theta\right) - q(\mu q+\theta)}{(\mu q+\theta) + q\left(\mu(1-q)+\theta\right)}$$

$$K_2 = \frac{(1-q)\left[q(\mu q+\theta) - (1-q)\left(\mu(1-q)+\theta\right)\right]}{q\left[(\mu q+\theta) + q\left(\mu(1-q)+\theta\right)\right]}$$

$$K_3 = \frac{2(1-q)(\mu q+\theta) - q\,(\mu(1-q)+\theta)}{(\mu(1-q)+\theta)+q(\mu q+\theta)}$$
$$K_4 = \frac{(1-q)\,[q\,(\mu(1-q)+\theta)-(1-q)(\mu q+\theta)]}{q\,[(\mu(1-q)+\theta)+q(\mu q+\theta)]}$$

Proof: For S_1, there is nothing left to show.
For S_0, note that by a calculus argument similar to the ones used in the proof of Theorem 5.3, we have that S_0 always buys if and only if

$$\frac{\partial CEU_{S_0}}{\partial \pi_t}(1) < \frac{\partial ask}{\partial \pi_t}(1).$$

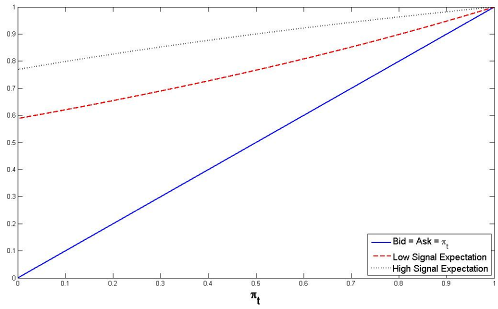
To see that this inequality holds if and only if $\delta_0 > \delta^*$, plug in the respective formulas to obtain a quadratic inequality of the form $\delta_0^2 + K_1\delta - K_2 > 0$. Observe that the l.h.s. of this inequality has two roots one which is < 0. Consequently, for this inequality to hold, δ_0 must be greater than the larger root, which is given by δ^*.
Observe again that the calculus arguments from the proof of Theorem 5.1 yield that S_0 sells the asset when the price is in a neighborhood of 1 if and only if

$$\frac{\partial CEU_{S_0}}{\partial \pi_t}(1) > \frac{\partial ask}{\partial \pi_t}(1).$$

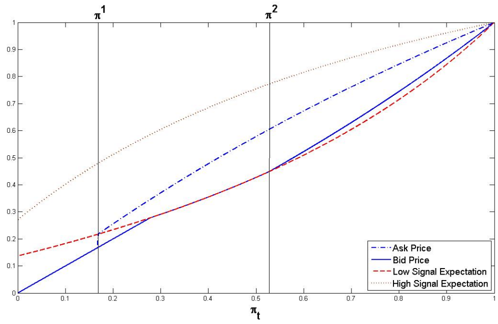
As before, this inequality holds if and only if $\delta_0^2 + K_3\delta - K_4 < 0$. A similar argument as in the previous case yields that for $\delta_0 > 0$, this holds if and only if $\delta_0 < \delta^{**}$.
With both cut-off points given, it follows immediately that S_0 holds in a neighborhood of 1 (but never buys) if and only if $\delta^* \geq \delta_0 \geq \delta^{**}$.

□

Figure 5.8 illustrates the trade behavior of the CEU traders if they are purely optimistic. In Figure 5.8 (a), the low signal always buys. Indeed, the primary ambiguity level $\delta_0 = 0.5$ is above the cut-off point $\delta^* = 0.47$ from Lemma 5.9. Since S_1 always buys as well, there is no social learning in a purely optimistic market. The market by default is in the state of an informational cascade.



(a) High primary ambiguity $\delta_0 = 0.5$



(b) Low primary ambiguity $\delta_0 = 0.1$

Figure 5.8: Low signal trading decisions in case of pure optimism

Notes: Low signal CEU beliefs as well as bid and ask prices with respect to π_t for high and low primary ambiguity δ_0. The degree of optimism is $\alpha = 1$, the informed trader share is $\mu = 0.3$ and the signal precision $q = 0.7$. The cut-off points according to Lemma 5.9 are $\delta^* = 0.47$ and $\delta^{**} = 0.22$.

In the perturbed model, the informed traders would drive the asset price towards $\alpha = 1$, regardless of the initial prior and independently of the true value of V.

This is the paragon of irrational exuberance driving the asset price away from it's fundamental value. Observe, however, that this is not a case of investor herding in the sense of Definition 5.1. While informed traders accumulate on the buy side of the market, they never change their initial trading decision. The herd-like behavior exhibited by the informed traders is due only to the high degree of perceived ambiguity in conjunction with the fundamentally unrelated pure optimism.

In Figure 5.8 (b), the low signal sells if $\pi_t > \pi^2$ and buys only if $\pi_t < \pi^1$ due to a low degree of primary ambiguity. Indeed, her additive belief component dominates, making her act as though in the baseline model if prices are high. This case is covered by Theorem 5.1. Once the price drops below π^1 a buy contrarian cascade occurs. This happens with positive probability even if $V = 1$. So even a purely optimistic market does not necessarily become confident regarding $V = 1$. Contrary to the case where $\alpha < 1$, there is, however, at least the possibility that the market confidently learns about $V = 1$, since high signal traders always buy.

In the perturbed model, there is no informational cascade. If $V = 0$ is the true state, the majority of low signals in the market will prevent the price from remaining above π^2 for extended periods of time. If the price is below π^2 or even below π^1, however, the likelihood of observing a price increase is greater than that of a further price decrease. Hence, we would assume, the price to always revert towards π_2. If $V = 1$, the market will confidently learn about the correct true state, as the majority of the high signals eventually pushes the price arbitrarily close to 1.

5.8.E Collection of Additional Results from Avery and Zemsky

For the reader's convenience we have collected relevant formulas for the Avery and Zemsky (1998) model in the following.

Lemma 5.10. ***Formulas of the Avery and Zemsky Framework***
In the Avery and Zemsky (1998) framework with initial prior π_0, informed trader share μ, symmetric binary signals $P[S|V]$ with signal precision q and noise traders that buy, sell or hold with equal probability θ, the following equations hold.

(i) Buy and sell probabilities conditional on V:

$$\begin{aligned} P(a_t = \{buy\}|V = 0) = P(a_t = \{sell\}|V = 1) = \mu(1-q) + \theta \\ P(a_t = \{sell\}|V = 0) = P(a_t = \{buy\}|V = 1) = \mu q + \theta \end{aligned} \tag{5.34}$$

(ii) Ask price in t:

$$ask_t = \frac{(\mu q + \theta)\pi_t}{(\mu q + \theta)\pi_t + (\mu(1-q) + \theta)(1-\pi_t)} \tag{5.35}$$

(iii) Bid price in t:

$$bid_t = \frac{(\mu(1-q)+\theta)\pi_t}{(\mu(1-q)+\theta)\pi_t + (\mu q+\theta)(1-\pi_t)} \tag{5.36}$$

(iv) Expected value asset valuation by low signal in t:

$$E[V \mid S_0, H_t] = \frac{(1-q)\pi_t}{(1-q)\pi_t + q(1-\pi_t)} \tag{5.37}$$

(v) Expected value asset valuation by high signal in t:

$$E[V \mid S_1, H_t] = \frac{q\pi_t}{q\pi_t + (1-q)(1-\pi_t)} \tag{5.38}$$

(vi) Price updating after observing trade action in t:

$$\pi_{t+1} = \begin{cases} ask_t, & \textit{if } a_t = \{buy\} \\ bid_t, & \textit{if } a_t = \{sell\} \\ \pi_t, & \textit{if } a_t = \{hold\} \end{cases} \tag{5.39}$$

The proofs can be found in Avery and Zemsky (1998).

In addition, we provide some interesting properties of the public belief π_t in the Avery and Zemsky model. The following proposition shows that the public belief can be uniquely identified with any order imbalance in the trade history H_t.

Proposition 5.8. ***Public Belief And Order Imbalance***
Let (μ, q, π_0) be some model parameterization of the Avery and Zemsky (1998) and let noise traders buy, sell or hold with equal probability θ. Moreover, let H_t be some trade history containing b buys, s sells and h holds, where $z := b - s$ denotes the trade imbalance. Then, π_t does not depend on the order at which traders arrive at the market if b, s, h remain unchanged. In particular, π_t only depends on the model parameters and z.

Before we provide the proof we would like to state a few implications of Proposition 5.8. Indeed, it suggests that we can view π_t as a measure for general market

sentiment. The larger π_t, the larger the buy side accumulation of traders, the more optimistic the market as a whole and vice versa.
Based on Proposition 5.8, we can also derive the following formula for π_t based on model parameters μ, q and π_0 as well as the order imbalance z.

Corollary 5.3. *With the same notation as in Proposition 5.8, we have*

$$\pi_t(z) = \begin{cases} \frac{(\mu q+\theta)^z \pi_0}{(\mu q+\theta)^z \pi_0 + (\mu(1-q)+\theta)^z (1-\pi_0)}, & \textit{if } z > 0 \\ \frac{(\mu(1-q)+\theta)^{-z} \pi_0}{(\mu(1-q)+\theta)^{-z} \pi_0 + (\mu q+\theta)^{-z} (1-\pi_0)}, & \textit{if } z < 0 \\ \pi_0, & \textit{if } z = 0 \end{cases} \tag{5.40}$$

Proof of Proposition 5.8 and Corollary 5.3 To prove Proposition 5.8, we first show the following

Lemma 5.11. *In the Avery and Zemsky (1998) framework with prior probability π_0, at any time $\tau \in [1; T]$, we have*

$$\pi_t = P(V=1|H_t) = \frac{\prod_{t=1}^{\tau} P(a_t|V=1)\pi_0}{\prod_{t=1}^{\tau} P(a_t|V=1)\pi_0 + \prod_{t=1}^{\tau} P(a_t|V=0)(1-\pi_0)} \tag{5.41}$$

Proof: We show this via induction over τ. Let $\tau = 1$. Using Equation (5.39), Bayes' rule and the law of total probability readily imply that

$$\pi_1 = P(V=1|H_1) = \frac{P(a_1|V=1)\pi_0}{P(a_1|V=1)\pi_0 + P(a_1|V=0)(1-\pi_0)}.$$

Now let us assume that the statement from Lemma 5.11 is true for any $\tau \geq 1$, then as for $\tau = 1$, we get

$$\pi_{\tau+1} = \frac{P(a_{\tau+1}|V=1)\pi_\tau}{P(a_{\tau+1}|V=1)\pi_\tau + P(a_{\tau+1}|V=0)(1-\pi_\tau)}.$$

Now using the induction assumption, we can plug in the r.h.s. of Equation (5.41) for π_τ and get:

$$\pi_{\tau+1} = \frac{P(a_{\tau+1}|V=1) \prod_{t=1}^{\tau} P(a_t|V=1)\pi_0}{C},$$

where

$$C := P(a_{\tau+1}|V=1)\prod_{t=1}^{\tau} P(a_t|V=1)\pi_0 + P(a_{\tau+1}|V=0)\prod_{t=1}^{\tau} P(a_t|V=0)(1-\pi_0).$$

As we absorb the terms $P(a_{\tau+1}|\cdot)$ into the respective products, we have shown that Equation (5.41) holds for $\tau+1$, which concludes the proof. □

Now continuing the proof of Proposition 5.8, we consider any history H_t of length ≥ 2 (if H_t contains less than two actions, then there is nothing to show). Now let $\sigma(H_t)$ denote an arbitrary permutation of actions contained in H_t, then $\sigma(H_t) =: \tilde{H}_t$ defines a second history with equal length as well as equal number of buys, sells and holds as H_t. Applying Equation 5.41, we see that $P(V=1|H_t)$ and $P(V=1|\tilde{H}_t)$ are identical up to a commutation within the product terms $\prod(\cdot)$. Consequently, we have $P(V=1|H_t) = P(V|\tilde{H}_t)$, which prooves that π_t does not depend on the order of arrival of traders as long as their trading decisions remain unchanged.

For the second part of Proposition 5.8 assume that history H_t contains b buys s sells and h holds. Let us assume without loss of generality that $b \geq s$. Moreover, for notational convenience, let us denote $p_{\cdot}^{B} = P(a_t = \{buy\}|V=\cdot)$, $p_{\cdot}^{S} = P(a_t = \{sell\}|V=\cdot)$ and $p_{\cdot}^{H} = P(a_t = \{buy\}|V=\cdot)$ for the remainder of the proof. Since the order of the actions is not important, we can rewrite Equation (5.41) as

$$\pi_t = P(V=1|H_t) = \frac{(p_1^B)^b(p_1^S)^s(p_1^H)^h\pi_0}{(p_1^B)^b(p_1^S)^s(p_1^H)^h\pi_0 + (p_0^B)^b(p_0^S)^s(p_0^H)^h(1-\pi_0)}.$$

Now noting that $p_0^S = p_1^B$ and vice versa and that $p_0^H = p_1^H$, we can factorize the denominator so that we get

$$\pi_t = P(V=1|H_t) = \frac{(p_1^B)^b(p_1^S)^s(p_1^H)^h\pi_0}{(p_1^B)^s(p_1^S)^s(p_1^H)^h[(p_1^B)^{b-s}\pi_0 + (p_0^B)^{b-s}(1-\pi_0)]}.$$

Setting $b-s=z$ and reducing the fraction, we get

$$\pi_t = P(V=1|H_t) = \frac{(p_1^B)^z\pi_0}{(p_1^B)^z\pi_0 + (p_0^B)^z(1-\pi_0)}. \tag{5.42}$$

For $s > b$, symmetry implies that we can simply replace the buy probabilities in Equation (5.42) with the corresponding sell probabilities, which concludes the proof that π_t only depends on z and the model parameters. To see that Corollary 5.3 holds, use the formulas for $P(a_t|V)$ according to Lemma 5.10.

□

5.8.F Informational Cascades

Proposition 5.9. *In the two-state, two-trader version of the Avery and Zemsky (1998) framework, an informational cascade occurs if and only if all informed traders take the same action.*

The "if" part of the result generalizes to any number and even to a continuum of states and different informed traders as long as all model parameters are common knowledge and the conditional signal distribution $P(S|V)$ is not constant in V. In general settings, e.g. confounded learning, informational cascades may occur if agents take different actions, compare Exercise 4.6 in Chamley (2004).

Proof:

"$\Leftarrow$": Let us assume all informed traders take the same action at t. Let this action without loss of generality be a buy. Then:

$$\begin{aligned} & P(a_t = \text{buy}|H_t, \text{“trade is informed”}) \\ = & P(S = S_0|H_t) + P(S = S_1|H_t) = 1 \end{aligned} \tag{5.43}$$

and

$$\begin{aligned} & P(a_t = \text{buy}|H_t, V, \text{“trade is informed”}) \\ = & P(S = S_0|H_t, V) + P(S = S_1|H_t, V) = 1 \end{aligned} \tag{5.44}$$

as well. We also note that the corresponding conditional probabilities for any informed trader action other than a buy are zero. Hence, we get

$$\begin{aligned} & P(a_t = \text{buy}|H_t, V) \\ = \quad & P(\text{“informed buy”}|H_t, V) + P(\text{“uninformed buy”}|H_t, V) \\ = \quad & P(a_t = \text{buy}|H_t, V, \text{“trade is informed”})P(\text{“trade is informed”}|H_t, V) \end{aligned}$$

$$+ P(\text{“uninformed buy”}|H_t, V).$$

Now noting that the probability μ that a trade is informed and the probability θ that an uninformed trader buys do not depend on the state of the world and applying equations (5.43) and (5.44), we get

$$\begin{aligned}
& P(a_t = \text{buy}|H_t, V) \\
= \quad & P(a_t = \text{buy}|H_t, \text{“trade is informed”})P(\text{“trade is informed”}|H_t) \\
& + P(\text{“uninformed buy”}|H_t) \\
= \quad & P(a_t = \text{buy}|H_t).
\end{aligned}$$

For any action other than buy, we have that $P(a_t|H_t, V) = \theta = P(a_t|H_t)$ and, therefore, the probability is independent of the state of the world, which concludes this part of the proof.

We note that common knowledge is crucial to the proof since Equations (5.43) and (5.44) would not necessarily hold if the informed traders' actions were obscured by some unobservable preference parameters.

“⇒”: We proof this indirectly by assuming that without loss of generality the low signal sells at t while the high signal buys. Then:

$$\begin{aligned}
& P(a_t = \text{buy}|H_t, \text{“trade is informed”}) \\
& = P(S = S_1|H_t) = \pi_t q + (1 - \pi_t)(1 - q)
\end{aligned} \tag{5.45}$$

and

$$\begin{aligned}
P(a_t = \text{buy}|H_t, V, \text{“trade is informed”}) &= P(S = S_1|H_t, V) \\
&= \begin{cases} \pi_t q, & V = V_1 \\ (1 - \pi_t)(1 - q) & V = V_0 \end{cases}.
\end{aligned} \tag{5.46}$$

We infer from equations 5.45 and 5.46 that

$$P(a_t = \text{buy}|H_t, \text{“trade is informed”}) \neq P(a_t = \text{buy}|H_t, V, \text{“trade is informed”}),$$

which readily implies that $P(a_t = \text{buy}|H_t, V) \neq P(a_t = \text{buy}|H_t)$ and, therefore, concludes the second part of the proof.

□

5.8.G CEU and NEO-Additivity Toolbox

This section is a summary of the most important concepts and results of Chateauneuf et al. (2007) and Eichberger et al. (2010). It is the mathematical foundation for Section 5.3.

Capacities and Choquet Expected Utility (CEU): Let $S \subset \mathbb{R}$ denote a non-empty set of possible states of the world. Let $\sigma(S) =: \mathcal{E}$ denote the corresponding Borel Sigma-Algebra of all possible subsets of S. Note that by definition $\forall s \in S : \{s\} \in \mathcal{E}$.

Definition 5.3. ***Capacity***
A ***capacity*** *is a mapping $\nu : \mathcal{E} \to [0;1]$ that assigns likelihood values to events in a way that it fulfills the following properties:*

(i) Monotonicity: $\forall E, F \in \mathcal{E}$, where $E \subseteq F : \nu(E) \leq \nu(F)$

(ii) Normalization: $\nu(\emptyset) = 0$ and $\nu(S) = 1$.

We note that a capacity defines a normalized measure. A special case of capacities are probability measures. Yet, capacities in general are not additive with respect to $\mathcal{E}$. This non-additivity implies in particular that for some event E, where $0 < \nu(E) < 1$, we do not necessarily have that $\nu(E) = 1 - \nu(E^C)$, where E^C denotes the complement of E. Therefore, capacities are suited to model agent behavior under ambiguity.

Capacities are designed to explain the Ellsberg paradox of Ellsberg (1961). In Ellsberg's experiment individuals are confronted with the choice of drawing a ball from one of two urns. They know that the first urn contains 50 white balls and 50 black balls while the composition of the second urn is unknown. Subjects win money, if they draw a white ball. Most participants choose to draw from urn 1, where the composition is known. This implies that they assign a probability of less than 50% of drawing a white ball from urn 2. Now, Savage's sure thing principle would predict that when the winning condition is changed to drawing a black ball, subjects should prefer urn 2 to urn 1. A corresponding repetition of the experiment shows, however, that subjects still tend to prefer urn 1 to urn 2, thus, violating Savage's SEU framework.

Next we define the Choquet integral with respect to capacities for a set of simple functions:

Definition 5.4. ***Choquet Integral***
Let $f : S \to B \subset \mathbb{R}$, where B has a finite number of elements. The Choquet integral with respect to the capacity ν is defined as

$$\int f\, d\nu := \sum_{t \in f(S)} t \cdot [\nu(\{s \mid f(s) \geq t\}) - \nu(\{s \mid f(s) > t\})].$$ [55]

The Choquet integral is interpreted as the expected value under ambiguity. If we think of f being a utility function, it is natural to denote $CEU := \int f\, d\nu$ as the Choquet Expeted Utility of an individual that perceives ambiguity and has ambiguity preferences that are captured by ν.

NEO-Additive Capacities: We provide a simplified definition for neo-additive capacities that is sufficient for this study.

[55] Note that the term $\nu(\{s \mid f(s) \geq t\}) - \nu(\{s \mid f(s) > t\})$ very much reminds us of decision weights from prospect theory according to Tversky and Kahneman (1992). Yet, while probability weightings are merely distortions of objective probabilities designed to capture individuals' tendencies to wrongly assess given probabilities, capacities model how individuals assign likelihoods to outcomes, for which no (single) probability is available.

Definition 5.5. ***NEO-Additive Capacity***
Let π be a probability measure on $(S, \mathcal{E})$ and let

$$\eta_\alpha = \begin{cases} 0 & E = \emptyset \\ \alpha & E \neq \emptyset \ \wedge \ E \neq S \\ 1 & E = S \end{cases}$$

be the Hurwicz capacity and let $\delta, \alpha \in [0;1]$, then a neo-additive capacity $\nu(\cdot \mid \pi, \delta, \alpha)$ is defined as

$$\nu(E \mid \pi, \delta, \alpha) := (1-\delta)\pi(E) + \delta\eta_\alpha(E).$$

The CEU with respect to a neo-additive capacity is shown by Chateauneuf et al. (2007) to be

$$\begin{aligned} CEU_{neo}[f] = (1-\delta)E_\pi[f] + \delta(\alpha \cdot \max_{x \in B}\{f^{-1}(x)\} \\ +(1-\alpha) \cdot \min_{x \in B}\{f^{-1}(x)\}). \end{aligned} \tag{5.47}$$

The function f again is a simple function in the sense of Definition 5.4. When assuming that informed traders have neo-additive CEU preferences in the Avery and Zemsky (1998) framework, f is the identity as we maintain the assumption of risk neutrality. Since there are only two states $V_0 = 0$ and $V_1 = 1$, the non-additive part simplifies to $\delta(\alpha \cdot 1 + (1-\alpha) \cdot 0) = \delta\alpha$. The parameter δ describes the degree of perceived ambiguity, while the parameter α measures the attitude towards ambiguity.

The absolute ambiguity attitude in the sense Ghirardato and Marinacci (2002) for individuals with neo-additive CEU preferences is then given by the following

Proposition 5.10. ***Absolute Ambiguity Attitude***
Let $\succ_{neo}$ denote a preference relation that can be represented by a neo-additive capacity $\nu(E \mid \pi, \delta, \alpha)$. Then, $\succ_{neo}$ is ambiguity averse (loving) in the sense of Ghirardato and Marinacci (2002) if and only if $\alpha < (>) E_\pi[\cdot]$. It is ambiguity neutral if and only if $\alpha = E_\pi[\cdot]$.

Proof: According to Proposition 15 in Ghirardato and Marinacci (2002), a preference relation is ambiguity neutral if and only if it is SEU. In the case of neo-additive capacities this would mean that $CEU_{neo} = E_{\pi}$. Now, let us assume that without loss of generality utilities are normalized (or canonical), that is, $\max\{u^{-1}(x)\}$ from Equation (5.47) equals 1 and $\min\{u^{-1}(x)\} = 0$. Now solving Equation (5.47) for α, we get $\alpha = E_{\pi}$. Hence, neo-additive preferences are ambiguity neutral if and only if $\alpha = E_{\pi}$.
If $\tilde{\alpha} > E_{\pi}$, it follows that $CEU^{\tilde{\alpha}}_{neo} > CEU^{\alpha}_{neo}$. This, in turn, implies that CEU^{α}_{neo} is more ambiguity averse than $CEU^{\tilde{\alpha}}_{neo}$ according to Definition 4 in Ghirardato and Marinacci (2002). Since we have already shown that CEU^{α}_{neo} is SEU, Definition 9 in Ghirardato and Marinacci (2002) implies that $CEU^{\tilde{\alpha}}_{neo}$ ambiguity loving. The argument for absolute ambiguity aversion is symmetric. □

General Bayesian Updating (GBU) Rule: The following GBU rule for neo-additive capacities is derived and discussed by Eichberger et al. (2010).

Proposition 5.11. ***General Bayesian Updating***
Let $E \subseteq S$ be some conditioning event and let $\nu(\cdot \mid \pi, \delta, \alpha)$ be an unconditional neo-additive capacity. Let $\pi(E) > 0$. Then:

- *The capacity $\nu_E(\cdot \mid \pi, \delta, \alpha)$ that is conditioned on E is neo-additive as well;*
- *The additive probability π is updated to π_E according to Bayes' rule, i.e. $\pi_E(A) = \pi(A \cap E)/\pi(E)$ for $A \in \mathcal{E}$;*
- *$\alpha_E = \alpha$;*
- *$\delta_E = \frac{\delta}{(1-\delta)\pi(E)+\delta}$.*

5.8.H Inconsistencies of GBU in the CEU Model

An important reason why α should vary with π is that it allows consistent assumptions regarding asymptotic ambiguity attitudes as the market becomes confident about either state.

If $\alpha \in (0;1)$ is constant, then for both informed trader types

$$\alpha_t^{rel} = \frac{\alpha}{E_{\pi_t}[V \mid H_t, S]} \to \alpha < 1, \text{ as } \pi_t \to 1 \tag{5.48}$$

and

$$\alpha_t^{rel} = \frac{\alpha}{E_{\pi_t}[V \mid H_t, S]} \to \infty, \text{ as } \pi_t \to 0. \tag{5.49}$$

In other words, informed traders become pessimistic as the market becomes confident about the *high state*. Similarly, traders become absolute optimistic as the market becomes confident about the *low state*. Hence, a fixed α in the CEU model does not guarantee that the informed traders' preference for ambiguity is invariant. Indeed, if an informed trader is optimistic at $t = 0$ and the market confidently learns that the high state is true, the informed trader will eventually become pessimistic. Yet, why of all times, would traders become pessimistic, when the market expresses strong or even full confidence about the high state and vice versa? If the market gets confident about either state of the world, *risk* becomes vanishingly small. In the limiting case that $\pi \in \{0; 1\}$, there is no uncertainty, Knightian or otherwise. Hence, CEU-investors should value the asset at 0 or 1 respectively just like their SEU counterparts from the baseline model. This is guaranteed if $\alpha(0) = 0$ and $\alpha(1) = 1$.

One might argue that there should, indeed, be no perceived ambiguity in the case of full confidence. That is, the degree of perceived ambiguity δ should go $\to 0$ as $\pi_t \to \{0; 1\}$. We would agree that such an assumption would be feasible as well but it would be an altogether different model. The way we understand ambiguity in this paper is that it cannot be learned away. The level of primary ambiguity δ_0, for instance, is associated with the complexity of the financial product or the level of expertise of the trader. Therefore, it does not vanish, even if the market becomes confident about the true state of V. Moreover, the ambiguity stemming from the informed trader's private information is highest when it contrasts to the view of the market, see Figure 5.2. For a low signal, the perceived informational ambiguity is, indeed, highest if the market confidently believes that $V = 1$. Hence, there is ambiguity if the market is confident, but it's effect on the informed traders' decision making should become marginal. If we assume regularity of α in π, this also implies that neo-additive Choquet preferences in the CEU model are consistent with smooth ambiguity preferences in the Klibanoff et al. (2005) approach.

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Ehrenwörtliche Erklärung

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