Abstract

The Fréchet distance is a metric for parameterized curves and surfaces. It is used in shape matching for measuring the similarity of geometric shapes. For polygonal curves, it can be computed in polynomial time. For triangulated surfaces, deciding whether the Fréchet distance between two surfaces is less than or equal a given threshold is NP-hard. It is not known, whether the Fréchet distance between triangulated surfaces is computable.

In this thesis, we study the computability of the Fréchet distance between triangulated surfaces. We give three partial answers to the question whether it is computable. For triangulated surfaces, we show that the Fréchet distance is semi-computable, a weaker notion of computability. For a variant of the Fréchet distance, the weak Fréchet distance, we show that it is polynomial time computable for triangulated surfaces. For a restricted class of surfaces, simple polygons, we show that the Fréchet distance is polynomial time computable.

Finally, we study a related question, the definition of a summed or average Fréchet distance between curves. We show that none of several intuitive definitions fulfill the triangle inequality.

Zusammenfassung

Der Fréchet-Abstand ist eine Metrik für parametrisierte Kurven und Flächen. Er wird benutzt, um die Ähnlichkeit geometrischer Formen zu messen. Für polygonale Kurven kann er in polynomieller Zeit berechnet werden. Für triangulierte Flächen ist es NP-schwer zu entscheiden, ob der Fréchet-Abstand zwischen zwei Flächen kleiner oder gleich einem gegebenen Wert ist. Es ist nicht bekannt, ob der Fréchet-Abstand zwischen triangulierten Flächen berechenbar ist.

In dieser Arbeit wird die Berechenbarkeit des Fréchet-Abstandes zwischen triangulierten Flächen untersucht. Wir geben drei Teilantworten auf die Frage, ob dieser berechenbar ist. Für triangulierte Flächen zeigen wir, dass der Fréchet-Abstand semi-berechenbar ist, eine schwächere Form der Berechenbarkeit. Für eine Variante des Fréchet-Abstandes, den schwachen Fréchet-Abstand, zeigen wir, dass er in polynomieller Zeit berechenbar ist für triangulierte Flächen. Für eine eingeschränkte Klasse von Flächen, einfache Polygonen, zeigen wir, dass der Fréchet-Abstand in polynomieller Zeit berechenbar ist.

Schließlich betrachten wir eine verwandte Fragestellung, die Definition eines summierten oder durchschnittlichen Fréchet-Abstandes zwischen Kurven. Wir zeigen, dass mehrere intuitive Definitionen nicht die Dreiecksungleichung erfüllen.

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