

Analysis of Nutation Patterns in Fourier-Transform NMR of Non-Thermally Polarized Multispin Systems

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Dedicated to Prof. Dr. Hans-Heinrich Limbach on the occasion of his 70th birthday

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The complex spin order of hyperpolarized multispin systems giving rise to anomalous NMR spectral patterns that vary with the RF excitation angle is analyzed by decomposing its nutation behavior in a superposition of Fourier harmonics. The product operator formalism is applied to calculating the spectral contributions of the various mutual alignments of scalar coupled spins. Two cases are treated, namely systems exhibiting only differences in population of their spin states and systems showing in addition zero-quantum coherences between states, a situation often seen at hyperpolarization. After deriving the general solution a number of representative examples are discussed in detail. The theoretical treatment is applied to analyzing the spin order observed in a hyperpolarized two-spin system that is prepared in the singlet state by para-hydrogen induced polarization.

1. Introduction

Nuclear Magnetic Resonance (NMR) provides numerous powerful and informative spectroscopic methods. Modern NMR developments, most notably, pulsed techniques, multi-dimensional methods, and Magnetic Resonance Imaging (MRI), have lead to many useful applications in physics, chemistry, material science, biology, and medicine. The work of Prof. Limbach is a prominent example of these achievements. Thus, NMR is a very informative, versatile and multipurpose spectroscopy having many advantages and only a few disadvantages, the most adverse being its relatively low sensitivity. This is because Zeeman interaction is weak and the resonance frequency γB_0 of nuclear spin

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transitions is small (γ stands for the nuclear gyromagnetic ratio, B_0 for the external magnetic field). Moreover, NMR signals are directly proportional to population differences of spin levels (spin polarization), which at thermal equilibrium are always low: they are proportional to $f = \frac{\gamma \hbar B_0}{2kT}$ (Boltzmann polarization), which is a small number as the splitting of nuclear spin energy levels in a reasonable field B_0 is much smaller than the thermal energy, kT . For instance, at room temperature and $B_0 = 7$ T (corresponding to 300 MHz proton NMR frequency) spin polarization is approximately 5×10^{-5} . Low sensitivity is often a limitation for NMR applications. Although the sensitivity in NMR was considerably increased during the last decades by using, *e.g.*, higher magnetic fields, Fourier spectroscopy [1] cross-polarization methods [2,3], INEPT technique [4] remote detection techniques [5] and cryo-probes [6,7] there is still room for improvement because thermal spin polarization always stays low. Even at the presently highest usable magnetic field (approximately 23.5 T) and low temperature the factor f remains small: at a temperature of 4 K it is only approximately 5×10^{-3} for protons and even less for other nuclei. Thus, by using thermally polarized spins one always faces the problem of NMR signal reduction by a factor of $f \ll 1$. A remedy to this problem is using strong non-thermal spin polarization, also termed hyperpolarization. Hyperpolarization methods, such as Dynamic Nuclear Polarization (DNP) [8–10], Spin Exchange Optical Pumping (SEOP) [11], Optical Nuclear Polarization (ONP) [12–14], Chemically Induced Dynamic Nuclear Polarization (CIDNP) [15–17] and Para-Hydrogen Induced Polarization (PHIP) [18–22] allow one to shift spin systems from thermal equilibrium to achieve NMR enhancements of a few orders of magnitude (in the ideal case of $1/f$) thus making new NMR and MRI applications possible [23–32].

Spin hyperpolarization can not only lead to NMR signal enhancement but can also change the appearance of the spectral pattern. This is the case when a hyperpolarized spin carries multiplet polarization, which is characterized by mutual entanglement of spins. It is important to note that polarization of such a type can be observed only for hyperpolarized systems: at thermal equilibrium multiplet polarization is of the order of f^2 being by a factor of $f \ll 1$ smaller than the net polarization of spins, which is, in turn, also small being equal to f . This makes multiplet polarization practically non-observable for spin systems at thermal equilibrium. When multiplet polarization is present in the system it results in different intensity of NMR lines within a spin multiplet (which is also the reason for its name). Such unusually looking spectra can be often observed for spins polarized by CIDNP [16] and PHIP [19]. For some other hyperpolarization techniques such effects are practically absent as, *e.g.*, SEOP is dealing specifically with hyperpolarized nuclear spins of noble gases, which are not coupled to other nuclei and thus carry no multiplet polarization. In the DNP case, except for the situation of very strong polarization [33], spins are only net-polarized. For this reason, our treatment is of importance mainly for NMR spectroscopy of spin systems hyperpolarized by means of CIDNP and PHIP. The theoretical description of NMR patterns is nonetheless relatively simple in the case of cw-spectroscopy: the intensity of an NMR line, which corresponds to a transition between a pair of spin states, $|i\rangle$ and $|j\rangle$, is directly proportional to the difference in the corresponding state populations, P_i and P_j . However, nowadays cw-NMR spectra are rarely recorded, since modern NMR utilizes mainly pulsed methods. In pulsed NMR with non-selective excitation the multiplet polarization leads to more complicated effects. This is in contrast to thermally polarized

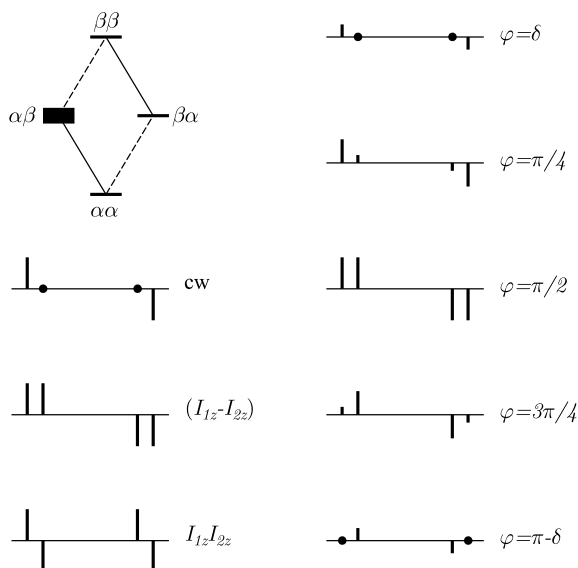


Fig. 1. Schematic representation of the peculiarities of FT-NMR of hyperpolarized spin systems. In the left column we show state populations assuming that only one spin state is populated (such population patterns can be obtained in CIDNP and PHIP experiments); ‘parallel’ NMR transitions are indicated by solid and dashed lines. We also show the cw-spectrum of such a system and spectra of pure net polarization, $(\hat{I}_{1z} - \hat{I}_{2z})$, and multiplet polarization, $\hat{I}_{1z}\hat{I}_{2z}$. In the right column we show FT-NMR spectra of the system obtained for different flip angles: δ (small angle), $\pi/4$, $\pi/2$, $3\pi/4$, $\pi - \delta$.

spin systems, for which cw and Fourier Transform (FT) NMR spectral patterns coincide. Even in the case of single-pulse FT-NMR there are new features in the spectra as has been pointed out by Ernst and coworkers [1,34]. First, the shape of the spectrum depends on the magnetization flip angle $\varphi = \gamma B_1 \tau_p$ (with B_1 being the strength of the oscillating RF-field and τ_p being the pulse duration) in a non-trivial way. Only for very small φ do cw- and FT-NMR spectra have the same shape. This is demonstrated in Fig. 1, in which we show FT-NMR spectra of a polarized two-spin system at different values of φ . In the case shown in Fig. 1 the spin system has both net polarization of the individual spins (of the same amplitude but of the opposite sign) and multiplet polarization. Such population patterns can, for example, be obtained by means of CIDNP [35] and PHIP [36]. Spectra exhibiting both types of spin order are also shown; they result in a dependence on the flip angle φ (see below). Second, an interesting effect reported earlier [1,34] is that in FT-NMR the intensity of the line, which corresponds to the $|i\rangle \rightarrow |j\rangle$ NMR transition, is determined not only by $(P_i - P_j)$ but also by population differences for so-called ‘parallel transitions’. Pairs of the parallel transitions are also indicated in Fig. 1. In more complex pulse sequences multiplet polarization can lead to additional features. For instance, in COSY experiments multiplet polarization leads to the formation of additional cross-peaks, which are absent for thermally polarized spin systems [37].

In our previous work published in the special issue of *Z. Phys. Chem.* dedicated to the 60-year anniversary of Prof. H.-H. Limbach [38] we suggested a new way of

how to analyze FT-NMR spectra of hyperpolarized spins and the φ -dependence of the NMR lines also termed ‘nutaton patterns’. We proposed to present nutation patterns as a Fourier series with respect to the flip angle, φ . We showed that this presentation enables a separation of different spin orders; the analysis of an experimental example for CIDNP-polarized multispin system revealed that higher spin orders (mutual entanglement of up to 10 spins polarized at low field) can be formed in hyperpolarization experiments. Although our method allows one to disentangle different spin orders and simplify the interpretation of the spectra and nutation patterns it requires a further development. In particular, we (i) did not establish the exact contributions of different spin orders to particular Fourier harmonics and (ii) did not analyze in detail the shape of NMR spectra resulting from specific spin orders. The goal of this work is to find amplitudes of all Fourier harmonics and determine the corresponding NMR lineshapes thus fully exploiting the potential of our idea of the Fourier decomposition of the nutation patterns. To reach this goal we will utilize the product operator formalism [1,39,40]. The previous treatment stemming from the density matrix description as suggested by Ernst *et al.* [1,34] was unable to give a simple solution to the problems we are going to solve. Although mathematically both approaches are equivalent, the product operator formalism (when applicable) gives a much simpler (and mathematically strict) way to understand the spin dynamics behind pulsed NMR experiments. Here we will consider only weakly coupled spins, which makes the product operator formalism applicable to our problem and allows us to obtain *analytical* results for the spectral patterns of hyperpolarized multispin systems. We will not consider strongly coupled spins because analytical results can be obtained only for two-spin systems (being already quite cumbersome), whereas the analytical treatment of higher-spin systems is not feasible.

2. Theory

Let us consider a system of N weakly coupled spins. We will consider only two types of interactions: (i) the Zeeman interaction with the field of the NMR spectrometer described by $\omega_i \hat{I}_i$ terms in the Hamiltonian and (ii) the secular part (zz-part) of the scalar spin-spin interactions, which are given by terms $2\pi J_{ij} \hat{I}_{iz} \hat{I}_{jz}$. Hereafter ω_i is the Zeeman interaction of spin i with the field (determined by its chemical shift); J_{ij} is the scalar spin-spin interaction constant for spins i and j ; \hat{I}_{iz} and \hat{I}_{jz} denote the z -projections of the corresponding spin operators. Considering only the secular parts of the interactions is an approximation assuming that the spins are coupled weakly, *i.e.*, $2\pi |J_{ij}| \ll |\omega_i - \omega_j|$. The full Hamiltonian of the system in the frequency units is then as follows:

$$\hat{H} = \sum_i \omega_i \hat{I}_i + 2\pi \sum_{i \neq j} J_{ij} \hat{I}_{iz} \hat{I}_{jz} \quad (1)$$

Now let us define the initial state of the system by introducing its density matrix, $\hat{\rho}$. In general, non-equilibrium spin states can be characterized using Ernst’s classification [1,34]. States with non-thermal populations (diagonal elements of the density matrix in the eigen-basis of \hat{H}) of their eigen-states and but no coherences (zero off-diagonal elements of the density matrix) are termed ‘non-equilibrium spin states of the first kind’; when spin coherences are present in the system such states are called

‘non-equilibrium spin states of the second kind’. Here we will consider both situations starting with the simpler case of non-equilibrium spin states of the first kind. When their spectroscopy is well understood we will extend our treatment to states of the second kind. Such a treatment becomes simple after a slight adaptation of the approach used for the non-equilibrium spin states of the first kind.

2.1 Non-equilibrium states of the first kind

When the spins are weakly coupled their Hamiltonian contains only the z -parts of spin operators with the consequence that the eigen-states are characterized by z -projections of all spins. As a result, the density matrix, which is diagonal in the eigen-basis of \hat{H} , can be expressed in the product operator formalism using only the z -parts, \hat{I}_{iz} , of spin operators. In turn, all spin coherences are expressed *via* the x - and y -parts of spin operators (containing at least one operator \hat{I}_{ix} or \hat{I}_{iy}). Let us define the starting density matrix for the non-equilibrium spin states of the first kind. In contrast to the treatment suggested by Schäublin *et al.* [34] where the elements of the density matrix, such as state populations, P_i and P_j , and spin coherences, ρ_{ij} ($i \neq j$), are specified

$$\hat{\rho}_0 = \sum_i P_i |i\rangle\langle i| + \sum_{i \neq j} \rho_{ij} |i\rangle\langle j| \quad (2)$$

we will present $\hat{\rho}_0$ as a sum of the product operators. In the case of the non-equilibrium spin states of the first kind all off diagonal elements, ρ_{ij} ($i \neq j$), are zero. In this situation the spin system is characterized by net polarizations of all spins, multiplet polarizations for each pair of spins, multiplet polarizations for each triplet of spins and so on. Contributions from net polarization are proportional to the spin operators \hat{I}_{iz} ; multiplet polarization of two spins (two-spin order) is given by $\hat{I}_{iz}\hat{I}_{jz}$ for all pairs $\{i, j\}$; three-spin order is given by products $\hat{I}_{iz}\hat{I}_{jz}\hat{I}_{kz}$ for all possible $\{i, j, k\}$; and so on. To write down all contributions from high-spin orders let us define a tensor $M^{i_1 i_2 \dots i_K}$ in the following way. Here K denotes the spin order ranging from 1 to N ; $i_1 i_2 \dots i_K$ cover all possible sets of K spins out of N ; each combination of $i_1 i_2 \dots i_K$ has a different weight described by the corresponding element of $M^{i_1 i_2 \dots i_K}$. This weight gives the K -th spin order for the corresponding combination of spins. In the definition of the tensor $M^{i_1 i_2 \dots i_K}$ we use double indices $i_1 i_2 \dots i_K$ because there are different possibilities (total amount of them is equal to the binomial coefficient C_N^K) for choosing K spins out of N . The definition of the tensor that we have chosen covers all these possibilities; however, as a result the indices get this level of complexity. For instance, when $N = 3$ and $K = 2$ there are three possible sets of spins: $i_1 i_2 = \{1, 2\}$, $\{1, 3\}$ or $\{2, 3\}$. The tensor gives the multiplet polarization for each pair of spins: $M^{i_1 i_2} = M^{1,2}$, $M^{1,3}$ or $M^{2,3}$. As a result, there are three contributions in the expression for the density matrix:

$$M^{1,2} \hat{I}_{1z} \hat{I}_{2z} + M^{1,3} \hat{I}_{1z} \hat{I}_{3z} + M^{2,3} \hat{I}_{2z} \hat{I}_{3z} \quad (3)$$

Accordingly, the complete expression for the initial density matrix, which accounts for all possible spin orders (from net polarization of individual spins to the entanglement of all N spins) in a weakly coupled spin system prepared in a non-equilibrium state of the

first kind, has the following form:

$$\hat{\rho}_0 = \frac{1}{2^N} \hat{E} + \sum_{K=1}^N \sum_{i_1 i_2 \dots i_K} M^{i_1 i_2 \dots i_K} \hat{I}_{i_1 z} \hat{I}_{i_2 z} \dots \hat{I}_{i_K z} \tag{4}$$

Finally, the weights, $M^{i_1 i_2 \dots i_K}$ (being the expectation values of the corresponding product operators) are determined from the density matrix in the following way:

$$M^{i_1 i_2 \dots i_K} = 2^K \text{Tr} \left\{ \hat{I}_{i_1 z} \hat{I}_{i_2 z} \dots \hat{I}_{i_K z} \hat{\rho}_0 \right\} \tag{5}$$

The unity operator \hat{E} is introduced in order to make sure that the trace of the density operator is equal to 1. Its presence does not affect any NMR property; for this reason it can be omitted in the following analysis. Since all operators, $\hat{I}_{i_1 z}, \hat{I}_{i_2 z}, \dots, \hat{I}_{i_K z}$, in Eq. (5) commute with each other it is possible to change arbitrarily the order of indices of $M^{i_1 i_2 \dots i_K}$. In the following analysis we will be usually interested in the shape of the NMR multiplet belonging to the i -th spin. Therefore, we will set one of the double indices to i and place it in the first position; the other $(K - 1)$ will remain double indices.

Let us now describe the evolution of the spin system. We will do it for the K -th spin order and show the spectral pattern of the spin multiplet of spin i . After that, the results can be easily generalized to the complete set of spin orders and all spin multiplets. The initial density matrix changes after a non-selective RF pulse is applied: we assume that it rotates all spins about the y -axis by an angle φ . Pulses are considered ideal, flipping all spins by the same angle; spin relaxation during the pulse application is neglected. We will assume that the phases of the RF-excitation (applied along the y -axis of the rotating frame) and the receiver coincide. As a result, for a spin system at thermal equilibrium the FT-NMR spectrum, which is obtained by performing the Fourier transform of the x -component of the spin magnetization, has purely Lorentzian lines. At the same time, the Fourier transform of the y -magnetization gives purely dispersive lines. Hereafter, we will name the x -channel as ‘in-phase channel’ and the y -channel as ‘out-of-phase channel’.

Since the spin operators of different spins commute with each other, the rotation of magnetization can be applied subsequently to all individual spins. As a consequence, each \hat{I}_{iz} changes to $(\hat{I}_{iz} \cos \varphi + \hat{I}_{ix} \sin \varphi)$, hence transverse magnetization components are created. Since the x - and y -components of the spin operators do not commute with the Hamiltonian (Eq. 1), coherent evolution of the system starts. We can describe it by subsequently applying the action of the terms $\omega_i \hat{I}_i$ and $2\pi J_{ij} \hat{I}_{iz} \hat{I}_{jz}$ because all of them commute with each other. This can be done because the evolution of the density matrix in terms of the Hamiltonian is described as follows:

$$\hat{\rho}(t) = \exp(-i \hat{H}t) \hat{\rho}(0) \exp(i \hat{H}t) \tag{6}$$

When \hat{H} contains two parts, \hat{H}_1 and \hat{H}_2 , which commute with each other we can rewrite this expression

$$\begin{aligned} \hat{\rho}(t) &= \exp(-i \hat{H}_2 t) \{ \exp(-i \hat{H}_1 t) \hat{\rho}_0 \exp(i \hat{H}_1 t) \} \exp(i \hat{H}_2 t) \\ &= \exp(-i \hat{H}_2 t) \hat{\rho}' \exp(i \hat{H}_2 t) \end{aligned} \tag{7}$$

where $\hat{\rho}' = \exp(-i\hat{H}_1 t)\hat{\rho}_0\exp(i\hat{H}_1 t)$. Thus, one can first calculate the evolution of the density matrix under the action of \hat{H}_1 obtaining $\hat{\rho}'$ from $\hat{\rho}_0$ and then apply the action of \hat{H}_2 . This approach is valid for an arbitrary number of terms in the Hamiltonian: it is only important that all of them commute with each other.

The quantity, which is usually measured in NMR is spin magnetization in the transverse plane, *i.e.*, the expectation values of the \hat{I}_{ix} and \hat{I}_{iy} . Thus, only contributions of the density matrix, which contain \hat{I}_{ix} and \hat{I}_{iy} , are observed, while all other terms are non-measurable. Mathematically this can be formulated as follows. The observable magnetization, for instance, its *x*-component, is written as

$$M_x(t) = \text{Tr} \left\{ \hat{I}_x \hat{\rho}(t) \right\} = \sum_{i=1}^N \text{Tr} \left\{ \hat{I}_{ix} \hat{\rho}(t) \right\} \tag{8}$$

Only when $\hat{\rho}(t)$ has terms containing single *x*-operators, \hat{I}_{ix} , will the Trace and, consequently, the magnetization be non-zero. For instance, the $\hat{I}_{ix}\hat{I}_{jx}$ is non-observable because $\{\hat{I}_{ix}(\hat{I}_{ix}\hat{I}_{jx})\}$ is proportional to \hat{I}_{jx} , which has zero trace. Similarly, the product of any two or more spin operators is non-observable. For this reason when describing the spin evolution we need to keep only the terms, which eventually contain pure transverse magnetization and can neglect all other contributions. The latter we will hereafter denote as Non-Observable Terms (NOT) and omit them in the analysis. The action of both kinds of interaction in the case under study is well known and can be treated by the product operator formalism in a very simple way. The Zeeman interaction mixes \hat{I}_x and \hat{I}_y operators for each individual spin and does not affect any of the *z*-operators [1,39,40]. The spin-spin interaction mixes \hat{I}_{ix} and \hat{I}_{iy} with $\hat{I}_{iy}\hat{I}_{jz}$ and $-\hat{I}_{ix}\hat{I}_{jz}$, respectively, and does not change the number of \hat{I}_{ix} and \hat{I}_{iy} in the product. More specifically, spin-spin interactions lead to the following evolution [1,39,40]:

$$\begin{aligned} \hat{I}_{ix}\hat{I}_{jz} &\xrightarrow{2\pi J_{ij}\hat{I}_{ix}\hat{I}_{jz}t} \hat{I}_{ix}\hat{I}_{jz} \cos(\pi J_{ij}t) + \frac{1}{2}\hat{I}_{iy} \sin(\pi J_{ij}t), \\ \hat{I}_{iy}\hat{I}_{jz} &\xrightarrow{2\pi J_{ij}\hat{I}_{ix}\hat{I}_{jz}t} \hat{I}_{iy}\hat{I}_{jz} \cos(\pi J_{ij}t) - \frac{1}{2}\hat{I}_{ix} \sin(\pi J_{ij}t) \end{aligned} \tag{9}$$

As it is usually written in the product operator formalism, above the arrow we specify a quantum mechanical operator, which leads to the corresponding spin evolution. Thus, when we start from the $\hat{I}_{ix}\hat{I}_{jz}$ operator spin-spin interaction can produce terms, which do not contain the \hat{I}_{jz} operator, resulting in the formation of the pure spin magnetization, \hat{I}_{iy} . The reason why the action of spin-spin interaction can reduce the number of the *z*-operators can be understood from a presentation shown in Fig. 2. The product operator $2\hat{I}_{ix}\hat{I}_{jz}$ can be presented as a combination of two magnetization vectors of spin *i*, which are parallel or anti-parallel to the *y*-axis, each of them representing a different state of spin *j* (namely, its *z*-projections of $\pm 1/2$). In this case all magnetization components of the *i*-th spin are equal to zero. Under the action of the coupling $2\pi J_{ij}$ the state of the *j*-th spin does not change, while the *i*-th spin starts to precess around the *z*-axis. The direction of the precession is different depending on the state of the *j*-th spin. For this reason at an instant of time $t = 1/2J_{ij}$ both vectors are pointing along the *y*-axis meaning that the initial spin order has been converted into the pure magnetization. Thus,

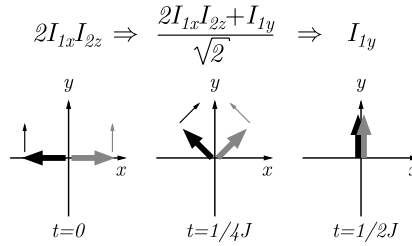


Fig. 2. Schematic representation of the evolution of the product operator $2\hat{I}_{1x}\hat{I}_{2z}$ under the action of J -coupling. The starting spin order ($t = 0$) can be depicted by two arrows (black and grey), which are parallel and anti-parallel to the x -axis. Each arrow shows the direction of the first spin and corresponds to a particular state of the second spin (with z -projection of $\pm 1/2$) Directions of the precession are shown for both components. Due to the different directions of the precession, the time evolution creates net polarization, \hat{I}_{1y} , of the first spin along the y -axis. This can be seen for $t = 1/4J$ (angle between the vectors is equal to 90°), the effect is maximal for $t = 1/2J$ when both vectors are pointing along the y -axis; spin order at different instants of time is indicated.

the spin-spin interaction leads to two consequences: (i) the z -part in the product operator can disappear and (ii) the magnetization phase rotates by 90° . Disappearance of the spin operators in the product can occur only for the z -components whereas the number of the transverse components always remains the same. The fact that J -couplings lead to terms with a reduced number of z -operators in the product is crucial for our treatment, since it opens a way to obtain pure transverse spin magnetization from the starting spin order. Let us demonstrate how spin magnetization can be obtained in our case.

After the action of the pulse φ_y the starting spin order changes in the following way [1,39,40]:

$$M^{i_1 i_2 \dots i_K} \hat{I}_{i_1 z} \hat{I}_{i_2 z} \dots \hat{I}_{i_K z} \xrightarrow{\varphi_y} \sum_{l=1}^K M^{i_1 i_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi \hat{I}_{i_l x} \hat{I}_{i_2 z} \dots \hat{I}_{i_K z} + \text{NOT} \quad (10)$$

The starting spin order can be converted into the NMR observable in only one way [1, 39,40]: (i) in the product of all terms $(\hat{I}_{iz} \cos \varphi + \hat{I}_{ix} \sin \varphi)$ we need to consider only contributions containing a single x -operator; (ii) we need to select only a single pathway of the total spin evolution, in which J -couplings subsequently reduce the number of z -operators to zero. This is the only way how to form pure transverse magnetization, which can be measured in a single-pulse NMR experiment; the remaining terms are NOT. In the evolution under Zeeman interaction we can exclude NOT as well because they are not mixed with observable product operators [1]. Let us first see how the action of spin-spin interactions shifts all terms, which contain z -operators to NMR observables or to NOT. For this purpose let us set $i_1 = i$ and analyze the evolution of a term $\hat{I}_{ix} \hat{I}_{i_2 z} \dots \hat{I}_{i_K z}$ containing \hat{I}_{ix} , thereby choosing a particular spin multiplet belonging to the spin i . The results for all other spins will follow directly from the analysis below by redefining the indices of spins. Let us now analyze how the action of J -couplings reduces the number of observable terms in the product. The product operator $M^{i_1 i_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi \hat{I}_{ix} \hat{I}_{i_2 z} \dots \hat{I}_{i_K z}$ will evolve as follows under the action of

$$2\pi \sum_{i \neq j} J_{ij} \hat{I}_{iz} \hat{I}_{jz}:$$

$$M^{ii_2 \dots i_K} \hat{I}_{ix} \hat{I}_{i_2z} \dots \hat{I}_{i_Kz} \sin \varphi \cos^{K-1} \varphi \xrightarrow{2\pi \sum_{l=2}^K J_{il} \hat{I}_{iz} \hat{I}_{lzt}; \text{even } K} \frac{1}{2^{K-1}} (-1)^{\frac{K}{2}-1} M^{ii_2 \dots i_K} \\ \times \sin \varphi \cos^{K-1} \varphi \hat{I}_{iy} \prod_{l=2}^K \sin(\pi J_{il} t) \prod_{m \neq l} \cos(\pi J_{im} t) + \text{NOT} \tag{11}$$

$$M^{ii_2 \dots i_K} \hat{I}_{ix} \hat{I}_{i_2z} \dots \hat{I}_{i_Kz} \sin \varphi \cos^{K-1} \varphi \xrightarrow{2\pi \sum_{l=2}^K J_{il} \hat{I}_{iz} \hat{I}_{lzt}; \text{odd } K} \frac{1}{2^{K-1}} (-1)^{\frac{K-1}{2}} M^{ii_2 \dots i_K} \\ \times \sin \varphi \cos^{K-1} \varphi \hat{I}_{ix} \prod_{l=2}^K \sin(\pi J_{il} t) \prod_{m \neq l} \cos(\pi J_{im} t) + \text{NOT} \tag{12}$$

The formulas have been obtained by subsequently reducing the number of the z -operators in the product by the action of spin-spin interactions and thus eventually obtaining pure spin magnetization. Each such ‘annihilation’ of \hat{I}_{iz} also causes phase changes, *i.e.*, $\hat{I}_{ix} \rightarrow \hat{I}_{iy}$ and $\hat{I}_{iy} \rightarrow -\hat{I}_{ix}$; an odd number of such ‘annihilations’ thus changes \hat{I}_{ix} to \hat{I}_{iy} ; an even number of ‘annihilations’ keeps the starting operator \hat{I}_{ix} . In addition, each phase change of the type $\hat{I}_{iy} \rightarrow -\hat{I}_{ix}$ changes the sign of the product operator. For instance:

$$\hat{I}_{1x} \hat{I}_{2z} \hat{I}_{3z} \xrightarrow{2\pi J_{12} \hat{I}_{1z} \hat{I}_{2z} t} \frac{1}{2} \sin(\pi J_{12} t) \hat{I}_{1y} \hat{I}_{3z} + \text{NOT} \\ \xrightarrow{2\pi J_{13} \hat{I}_{1z} \hat{I}_{3z} t} -\frac{1}{4} \sin(\pi J_{12} t) \sin(\pi J_{13} t) \hat{I}_{1x} + \text{NOT} \tag{13}$$

This is exactly the way how in Eqs. (11) and (12) we have obtained terms containing no z -operators. We have also taken into account interactions with other spins, which do not belong to the chosen group $\{i i_2 \dots i_K\}$. They are described by products of $\cos(\pi J_{im} t)$ for $m \neq l$. The results, see Eqs. (11) and (12), are thus different for even or odd K : either \hat{I}_{ix} or \hat{I}_{iy} is left in the formula. Now let us see how the evolution proceeds under the action of the Zeeman term $\omega_i \hat{I}_{iz}$ for the cases of even and odd K .

2.1.1 Case of even K

The evolution under the Zeeman interaction is as follows:

$$\frac{1}{2^{K-1}} (-1)^{\frac{K}{2}-1} M^{ii_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi \hat{I}_{iy} \prod_{l=2}^K \sin(\pi J_{il} t) \prod_{m \neq l} \cos(\pi J_{im} t) \\ \xrightarrow{\omega_i \hat{I}_{iz} t} \frac{1}{2^{K-1}} (-1)^{\frac{K}{2}-1} M^{ii_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi (\hat{I}_{iy} \cos \omega_i t - \hat{I}_{ix} \sin \omega_i t) \\ \times \prod_{l=2}^K \sin(\pi J_{il} t) \prod_{m \neq l} \cos(\pi J_{im} t) \tag{14}$$

The observed x -magnetization (in-phase magnetization) is written as follows by using the Euler formulas for the sines and cosines

$$M_x(t) = \frac{1}{2^{K-1}} M^{ii_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi \frac{e^{i\omega_1 t} - e^{-i\omega_1 t}}{2^N} \sum_{\pm} \prod_{l=2}^K (-1)^Q e^{\pm i\pi J_{ii_l} t} \prod_{m \neq l} e^{\pm i\pi J_{im} t} \quad (15)$$

Here the notation \sum_{\pm} means a summation over all complex exponential terms with positive and negative arguments; Q is the number of terms with negative sign in the exponents coming from the sine terms, $\sin(\pi J_{ii_l} t)$. Thus, spins from the group $\{i_2 \dots i_K\}$ change the phase of NMR lines (A-absorptive or E-emissive) within a spin multiplet (this is what multiplet polarization is); whereas all other spins only give additional NMR lines of the same phase. The phase of the NMR line is positive or negative depending on whether Q is odd or even. The observed y -magnetization (out-of-phase magnetization) is as follows

$$M_y(t) = -i M^{ii_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi \frac{e^{i\omega_1 t} + e^{-i\omega_1 t}}{2^{N+K-1}} \times \sum_{\pm} \prod_{l=2}^K (-1)^Q e^{\pm i\pi J_{ii_l} t} \prod_{m \neq l} e^{\pm i\pi J_{im} t} \quad (16)$$

2.1.2 Case of odd K

The evolution under the Zeeman interaction is as follows:

$$\begin{aligned} & \frac{1}{2^{K-1}} (-1)^{\frac{K-1}{2}} M^{ii_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi \hat{I}_{ix} \prod_{l=2}^K \sin(\pi J_{ii_l} t) \prod_{m \neq l} \cos(\pi J_{im} t) \\ & \xrightarrow{\omega_1 \hat{I}_{iz} t} \frac{1}{2^{K-1}} (-1)^{\frac{K-1}{2}} M^{ii_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi \left(\hat{I}_{ix} \cos \omega_1 t + \hat{I}_{iy} \sin \omega_1 t \right) \\ & \times \prod_{l=2}^K \sin(\pi J_{ii_l} t) \prod_{m \neq l} \cos(\pi J_{im} t) \end{aligned} \quad (17)$$

The observed x -magnetization (in-phase magnetization) is written as

$$M_x(t) = \frac{1}{2^{K-1}} M^{ii_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi \frac{e^{i\omega_1 t} + e^{-i\omega_1 t}}{2^N} \times \sum_{\pm} \prod_{l=2}^K (-1)^Q e^{\pm i\pi J_{ii_l} t} \prod_{m \neq i_l} e^{\pm i\pi J_{im} t} \quad (18)$$

The observed y -magnetization (out-of-phase magnetization) is written as

$$M_y(t) = -\frac{i}{2^{K-1}} M^{ii_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi \frac{e^{i\omega_1 t} - e^{-i\omega_1 t}}{2^N} \times \sum_{\pm} \prod_{l=2}^K (-1)^Q e^{\pm i\pi J_{ii_l} t} \prod_{m \neq l} e^{\pm i\pi J_{im} t} \quad (19)$$

The time dependences, $M_x(t)$ and $M_y(t)$, we have obtained are, in fact, the Free Induction Decay (FID) kinetics. In order to obtain the NMR spectrum we only have to multiply the FIDs by a decaying exponent e^{-t/T_2} (with T_2 being the transverse relaxation time) and perform the Fourier transformation, which is simple because all the results are already presented as linear combinations of complex exponential terms. However, before doing this let us find out to what Fourier harmonics in the nutation patterns, that is the φ -dependence, the K -th spin order contributes. Thus, we present the angular part [1,39,40] in the expressions as a Fourier series:

$$\sin \varphi \cos^{K-1} \varphi = \sum_{l=1}^K A_l \sin l\varphi \tag{20}$$

In this expression there are only sine terms; the cosine harmonics are zero. Using the Euler formulas for sine and cosine and the binomial expansion for $\cos^{K-1} \varphi$ we obtain:

$$\begin{aligned} \sin \varphi \cos^{K-1} \varphi &= \frac{1}{2^{K-1}} \sum_{l=0}^{K-1} C_{K-1}^l e^{i\varphi(K-2l)} \frac{e^{i\varphi} - e^{-i\varphi}}{2i} \\ &= \begin{cases} \frac{1}{2^{K-1}} C_{K-1}^{\frac{K-1}{2}} \sin \varphi + \frac{1}{2^{K-1}} \sum_{m=1}^{\frac{K-3}{2}} C_{K-1}^{\frac{K-3}{2}-m} \{\sin(2m+3)\varphi - \sin(2m+1)\varphi\}; & \text{odd } K \\ \frac{1}{2^{K-1}} \sum_{m=0}^{\frac{K-1}{2}-1} C_{K-1}^{\frac{K}{2}-m-1} \{\sin(2m+2)\varphi - \sin 2m\varphi\}; & \text{even } K \end{cases} \end{aligned} \tag{21}$$

Thus, each spin order contributes to several Fourier harmonics, which gives some complications of the nutation analysis. However, our formulas minimize these difficulties, since coefficients for all contributions are well defined. For instance, the Fourier harmonics are as follows for small K values:

$$\begin{aligned} \sin \varphi \cos^{K-1} \varphi &= \sin \varphi && \text{for } K = 1 \quad (\text{single-spin order, net polarization}) \\ \sin \varphi \cos^{K-1} \varphi &= \frac{1}{2} \sin 2\varphi && \text{for } K = 2 \quad (\text{two-spin order}) \\ \sin \varphi \cos^{K-1} \varphi &= \frac{1}{4} \sin \varphi + \frac{1}{4} \sin 3\varphi && \text{for } K = 3 \quad (\text{three-spin order}) \\ \sin \varphi \cos^{K-1} \varphi &= \frac{1}{4} \sin 2\varphi + \frac{1}{8} \sin 4\varphi && \text{for } K = 4 \quad (\text{four-spin order}) \\ \sin \varphi \cos^{K-1} \varphi &= \frac{1}{8} \sin \varphi + \frac{3}{16} \sin 3\varphi + \frac{1}{16} \sin 5\varphi && \text{for } K = 5 \quad (\text{five-spin order}) \end{aligned} \tag{22}$$

Thus, the K -th spin order always gives the K -th harmonics; in addition, it can contribute to lower harmonics, but never to higher ones. Odd spin order gives only odd harmonics; even spin order gives only even harmonics. Thus, the Fourier analysis is now clear: we have found to what Fourier harmonics each spin order contributes.

Now let us obtain the NMR spectrum, $S(\omega)$, of the polarized spins. The Fourier transformation of Eqs. (15) and (16) and also of Eqs. (18) and (19) over time gives the following result for the in-phase and out-of-phase spectra, $S_{in}(\omega)$, and $S_{out}(\omega)$, respectively, obtained by the Fourier transform of $M_x(t)$ and $M_y(t)$:

$$S_{in}(\omega) = \frac{1}{2^{N+K-1}} M^{ii_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi \sum_{\pm} (-1)^Q L \left(\omega - \omega_i + \sum_{l=2}^K (\pm \pi J_{ii_l}) \right) \quad (23)$$

$$S_{out}(\omega) = \frac{1}{2^{N+K-1}} M^{ii_2 \dots i_K} \sin \varphi \cos^{K-1} \varphi \sum_{\pm} (-1)^Q D \left(\omega - \omega_i + \sum_{l=2}^K (\pm \pi J_{ii_l}) \right) \quad (24)$$

Here, as previously, the summation over all configurations of spins 1/2 is taken; we also introduced the Lorentzian and dispersive lineshapes, which are as follows:

$$L(\omega) = \frac{T_2}{1 + T_2^2 \omega^2}, \quad D(\omega) = \frac{T_2^2 \omega}{1 + T_2^2 \omega^2} \quad (25)$$

Thus, for non-equilibrium states of the first kind the in-phase spectrum (x -channel) consists of Lorentzian lines, whereas in the out-of-phase spectrum (y -channel) all lines are dispersive. Spectra for all other spin multiplets can be obtained from our formulas by assigning i to other spins.

2.2 Non-equilibrium states of the second kind

Now let us describe the effects of coherences in the FT-NMR spectrum, *i.e.*, consider non-equilibrium states of the second kind. Here we will not treat arbitrary coherences and describe only effects coming from zero-quantum coherences (ZQC). These are coherences between spin states characterized by the same value of I_z (z -projection of the total spin). ZQCs evolve relatively slowly (with low frequencies equal to frequency differences $(\omega_i - \omega_j)$ given by chemical shift differences) in contrast to single-quantum coherences (which evolve with the NMR frequencies, ω_i, ω_j). In addition ZQCs can be formed by hyperpolarization methods, for instance, by PHIP [18,19,41], where the polarized spin system is prepared in the singlet state and the initial density matrix of a two-spin system is as follows:

$$\hat{\rho}_0 \propto \frac{1}{4} \hat{E} - (\hat{I}_{1x} \hat{I}_{2x} + \hat{I}_{1y} \hat{I}_{2y} + \hat{I}_{1z} \hat{I}_{2z}) \quad (26)$$

In the simplest case of two spins it very easy to determine what the actions of the product operators $\hat{I}_{1x} \hat{I}_{2x}$ and $\hat{I}_{1y} \hat{I}_{2y}$ are. Pulses applied along the y -axis do not affect the yy -operator (which itself cannot produce observable spin magnetization); the spectrum coming from the xx -contribution is exactly the opposite of that from the zz -contribution, which we analyzed before. This is because a system prepared in its singlet state is silent to non-selective NMR excitation and does not give a spectrum. Since all three contributions to the spectrum are additive and the yy -one is zero, we immediately conclude that the spectra from the xx - and zz -operators cancel each other giving spectral lines of the same intensity but of opposite phase. This result can also be obtained in an easy

way using the product operator formalism. For brevity, we do not perform this treatment here.

For a spin system prepared in the singlet spin state at high field there is the need to consider not only spectra determined by the $\hat{I}_{1x}\hat{I}_{2x}$ and $\hat{I}_{1y}\hat{I}_{2y}$ product operators but also by the $\hat{I}_{1x}\hat{I}_{2y}$ and $\hat{I}_{1y}\hat{I}_{2x}$ operators. This is because the difference in frequencies ($\omega_1 - \omega_2$) $\neq 0$ (i.e., the $(\omega_1 - \omega_2)(\hat{I}_{1z} - \hat{I}_{2z})$ term in the Hamiltonian) causes the following spin evolution [1,39,40].

$$\hat{I}_{1x}\hat{I}_{2x} + \hat{I}_{1y}\hat{I}_{2y} \xrightarrow{\delta\omega\delta\hat{I}_z} \hat{I}_{1x}\hat{I}_{2y} - \hat{I}_{1y}\hat{I}_{2x} \tag{27}$$

Here $\delta\omega = (\omega_i - \omega_j)$ and $\delta\hat{I}_z = (\hat{I}_{1z} - \hat{I}_{2z})$. Thus, when there is a delay between the preparation of polarization and an RF-pulse, it becomes necessary to consider also these additional terms. As we will show below, they result in dispersive NMR lines in the in-phase channel and Lorentzian lines in the out-of-phase channel.

Let us consider only two types of coherences: (1) an even number of x -operators and an arbitrary number of z -operators; (2) an odd number of x -operators, a single y -operator and arbitrary number of z -operators. The number of the x - and y -operators must be even to ensure that the coherences are indeed ZQCs: for instance, a single x -operator will result in single-quantum coherences, which are associated with transverse magnetization. Including more than one y -operator will result in non-observable spin evolution because the RF-pulse having y -phase will keep the number of the y -operators and the combination of two or more y -operators will never give pure spin magnetization. For the first type of state the starting spin order is determined in the following way:

$$R^{i_1\dots i_{2L}j_{2L+1}\dots j_K} \prod_{l=1}^{2L} \hat{I}_{i_l x} \prod_{l=2L+1}^K \hat{I}_{j_l z} \tag{28}$$

Here we introduce the spin tensor, $R^{i_1\dots i_{2L}j_{2L+1}\dots j_K}$, by analogy with the M -tensor introduced earlier:

$$R^{i_1\dots i_{2L}j_{2L+1}\dots j_K} = 2^K \text{Tr} \left\{ \left(\prod_{l=1}^{2L} \hat{I}_{i_l x} \prod_{l=2L+1}^K \hat{I}_{j_l z} \right) \hat{\rho}_0 \right\} \tag{29}$$

This tensor characterizes the spin order in a system where the first $2L$ spins have a spin order described by $\hat{I}_{i_l x}$ operators; the states of the remaining $(K - 2L)$ spins are characterized by $\hat{I}_{j_l z}$ operators. When an NMR pulse is applied it rotates all spins around the y -axis. As previously, in the resulting density matrix we need to consider only contributions having a single x -operator; the rest will be NOT. There are two possibilities for this: (i) all but a single spin from the first group (x -operators) are rotated while all spins from the second group (z -operators) are not rotated; (ii) all spins from the first group (x -operators) are rotated while from the second group (z -operators) only a single spin

is rotated. Thus, the effect of the pulse is as follows:

$$\begin{aligned}
 & R^{i_1 \dots i_{2L} j_{2L+1} \dots j_K} \prod_{l=1}^{2L} \hat{I}_{i_l x} \prod_{l=2L+1}^K \hat{I}_{j_l z} \\
 \xrightarrow{\varphi \hat{I}_y} & - \sum_m R^{i_1 \dots i_{2L} j_{2L+1} \dots j_K} \sin^{2L-1} \varphi \cos^{K-2L+1} \varphi \delta_{i_m} \hat{I}_{i_m} \prod_{\substack{l=1 \\ l \neq m}}^{2L} \hat{I}_{i_l z} \prod_{l=2L+1}^K \hat{I}_{j_l z} \\
 & + \sum_q R^{i_1 \dots i_{2L} j_{2L+1} \dots j_K} \sin^{2L+1} \varphi \cos^{K-2L-1} \varphi \delta_{i_q} \hat{I}_{i_q} \prod_{l=1}^{2L} \hat{I}_{i_l z} \prod_{\substack{l=2L+1 \\ l \neq q}}^K \hat{I}_{j_l z} + \text{NOT} \quad (30)
 \end{aligned}$$

Here δ_{ij} is the Kronecker delta, which is equal to 1 when its indices coincide and is zero otherwise. Our previous results for the non-equilibrium states of the first kind have already described the spin evolution of the product of one x -operator and an arbitrary number of z -operators; the only difference in this case is the φ -part of the result. Thus, as previously, we will obtain a Lorentzian line in the in-phase channel; they will exhibit multiplet polarization. In the out-of-phase channel there will be only dispersive lines. The products $\sin^{2L-1} \varphi \cos^{K-2L+1} \varphi$ and $\sin^{2L+1} \varphi \cos^{K-2L-1} \varphi$, which determine the flip angle dependence, can also be presented as a Fourier series in the same way as before in Eqs. (20) and (21). Thus, the results are clear in this case. In the second situation (single y -operator) the initial spin order is as follows:

$$R^{ii_2 \dots i_{2L} j_{2L+1} \dots j_K} \hat{I}_{i_y} \prod_{l=2}^{2L} \hat{I}_{i_l x} \prod_{l=2L+1}^K \hat{I}_{j_l z} \quad (31)$$

This tensor describes a spin order, in which the i -th spin is characterized by its y -operator, while the other spins are characterized by their x -operators in a number of $(2L - 1)$ and their z -operators in a number of $(K - 2L)$. To obtain the observable magnetization it is necessary to rotate all x -operators and keep all z -operators while the y -operator is not affected by the y -pulse:

$$\begin{aligned}
 & R^{ii_2 \dots i_{2L} j_{2L+1} \dots j_K} \hat{I}_{i_y} \prod_{l=2}^{2L} \hat{I}_{i_l x} \prod_{l=2L+1}^K \hat{I}_{j_l z} \\
 \xrightarrow{\varphi \hat{I}_y} & - R^{ii_2 \dots i_{2L} j_{2L+1} \dots j_K} \sin^{2L-1} \varphi \cos^{K-2L} \varphi \hat{I}_{i_y} \prod_{l=2}^{2L} \hat{I}_{i_l z} \prod_{l=2L+1}^K \hat{I}_{j_l z} + \text{NOT} \quad (32)
 \end{aligned}$$

The problem can again be reduced to the previous consideration. The only difference from the previous results for the non-equilibrium states of the first kind is, however, the need of exchanging the x - and y -indices. As a consequence, what was expected in the in-phase channel for the non-equilibrium states of the first kind will come out in the out-of-phase channel in the present case and vice versa. Thus, we can use the previous results only exchanging the two NMR detection channels. As a consequence, in the in-phase channel there will be only dispersive lines, while in the out-of-phase channel there will be Lorentzian lines of different sign. The angular dependence given by

$\sin^{2L-1} \varphi \cos^{K-2L} \varphi$ can also be decomposed in a combination of Fourier harmonics. To gain a more clear insight into the NMR of such a spin order we will present an example (see below in the following section).

Now let us discuss representative examples of multiplet-polarized spin systems. This will serve for a better understanding of the general results that we obtained. We will mainly focus on the non-equilibrium states of the first kind; however, an example of an NMR spectrum of a non-equilibrium state of the second kind will close the discussion section.

3. Discussion

Example 1, single spin: $N = 1$. In this case there can be only a single-spin order present, the initial state is thus $M^1 \hat{I}_{1z}$. It results in the following spin evolution:

$$M^1 \hat{I}_{1z} \xrightarrow{\varphi \hat{I}_y} M^1 \sin \varphi \hat{I}_{1x} \xrightarrow{\omega_1 \hat{I}_{1z} t} M^1 \sin \varphi (\hat{I}_{1x} \cos \omega_1 t + \hat{I}_{1y} \sin \omega_1 t) e^{-t/T_2} \quad (33)$$

Thus, the y -pulse forms transverse magnetization, which rotates around the z -axis due to the Zeeman interaction. In this simplest case there will be only one line in the spectrum:

$$S_{in}(\omega) = \frac{1}{2} M^1 \sin \varphi L(\omega - \omega_1), \quad S_{out}(\omega) = \frac{1}{2} M^1 \sin \varphi D(\omega - \omega_1) \quad (34)$$

The line is Lorentzian for the in-phase channel and dispersive for the out-of-phase channel. The in-phase part of the spectrum is shown in Fig. 3 (spectrum 1).

Example 2, two spins: $N = 2$. In this case there can be net polarization and second-order multiplet polarization in the system. Thus, the initial state is $M^1 \hat{I}_{1z} + M^2 \hat{I}_{2z} + M^{12} \hat{I}_{1z} \hat{I}_{2z}$. The second term does not give lines at ω_1 and will be omitted for brevity. The first two terms and the third term correspond to $K = 1$ and $K = 2$, respectively. They can be easily separated because the net polarization contributes only to the first Fourier harmonics with respect to the flip angle while the multiplet polarization contributes only to the second harmonics. Let us specify what NMR signals will be obtained from the two types of spin order.

For $K = 1$ (net polarization) we obtain the following evolution of magnetization:

$$M^1 \hat{I}_{1z} \xrightarrow{\varphi \hat{I}_y} M^1 \sin \varphi \hat{I}_{1x} \xrightarrow{\pi J_{12} 2 \hat{I}_{1z} \hat{I}_{2z} t} M^1 \sin \varphi \hat{I}_{1x} \cos \pi J_{12} t + 2M^1 \sin \varphi \hat{I}_{1y} \hat{I}_{2z} \sin \pi J_{12} t \quad (35)$$

Here $\hat{I}_{1y} \hat{I}_{2z}$ is NOT and can be omitted. As a consequence, we obtain the following in-phase spectrum:

$$S_{in}(\omega) = \frac{1}{2} M^1 \sin \varphi (L(\omega - \omega_1 - \pi J_{12}) + L(\omega - \omega_1 + \pi J_{12})) \quad (36)$$

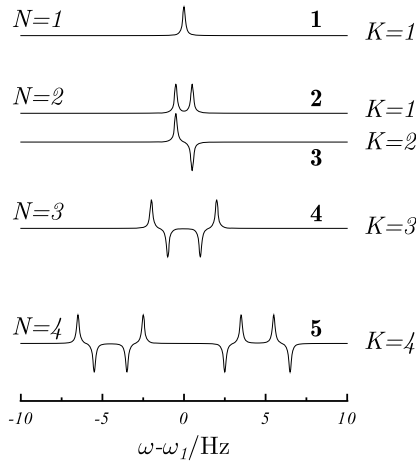


Fig. 3. FT-NMR spectra for different spin orders; spin multiplet of spin 1 centered at a frequency ω_1 is shown. Number of spins, N , and spin-order, K , are specified. Here we show cases of single spin ($N = K = 1$, spin order is \hat{I}_{1z} , **spectrum 1**), two-spin system ($N = 2$, $K = 1$ and 2, spin order is \hat{I}_{1z} and $\hat{I}_{1z}\hat{I}_{2z}$, respectively, **spectra 2 and 3**), three-spin system (for $N = K = 3$, spin order is $\hat{I}_{1z}\hat{I}_{2z}\hat{I}_{3z}$, **spectrum 4**), four-spin system (for $N = K = 4$, spin order is $\hat{I}_{1z}\hat{I}_{2z}\hat{I}_{3z}\hat{I}_{4z}$, **spectrum 5**). Flip angle dependencies for different spin orders are described in the text. Spin-spin interactions are: $J_{12} = 1$ Hz, $J_{13} = 3$ Hz; $J_{14} = 9$ Hz.

This multiplet of the first nucleus centered at $\omega = \omega_1$ is shown in Fig. 3 (spectrum 2): it contains two lines of the same phase (absorptive in our case) and intensity. The spectrum is thus an even function of frequency when the origin of the frame placed at ω_1 . The flip angle dependence is trivial being the same as that for thermally polarized spins.

For $K = 2$ (multiplet polarization) we obtain:

$$M^{12} \hat{I}_{1z} \hat{I}_{2z} \xrightarrow{\varphi \hat{I}_y} \frac{1}{2} \sin 2\varphi M^{12} (\hat{I}_{1x} \hat{I}_{2z} + \hat{I}_{1z} \hat{I}_{2x}) + \text{NOT} \tag{37}$$

Here the first and the second terms are observed at frequencies ω_1 and ω_2 , respectively, giving the same spectral pattern. Let us consider only the spectrum at the frequency of the first spin and describe only the evolution of the in-phase magnetization:

$$\begin{aligned} & \frac{1}{2} M^{12} \sin 2\varphi \hat{I}_{1x} \hat{I}_{2z} \xrightarrow{2\pi J_{12} \hat{I}_{1z} \hat{I}_{2z} t} \frac{1}{4} M^{12} \sin 2\varphi \hat{I}_{1y} \sin(\pi J_{12} t) \\ & + \text{NOT} \xrightarrow{\omega_1 \hat{I}_{1z} t} -\frac{1}{4} M^{12} \sin 2\varphi \hat{I}_{1x} \sin(\omega_1 t) \sin(\pi J_{12} t) + \text{NOT} \end{aligned} \tag{38}$$

The in-phase spectrum at the frequency of the first spin is as follows

$$S_{in}(\omega) = \frac{1}{8} M^{12} \sin 2\varphi (L(\omega - \omega_1 + \pi J_{12}) - L(\omega - \omega_1 - \pi J_{12})) \tag{39}$$

This spectrum (Fig. 3, spectrum 3) has two lines, which have the same intensity but opposite phase. Thus, the spectrum exhibits multiplet polarization of the type absorption/emission (AE). The spectrum is an odd function of frequency when the origin of the coordinate frame is placed at ω_1 . The flip angle dependence of the spectrum

coming from this spin order is simple and is given by sine of 2φ in accordance with previous works [38,42,43].

Example 3, three spins: $N = 3$. In this case in the system there can be three net polarizations, three two-spin orders and one three-spin order. We already know what happens for $K = 1$ and $K = 2$. In the present case the only complication is that the spectrum coming from the two-spin order is a linear combination of multiplet polarization of different spins: in the NMR multiplet of the first spin contributions from both $\hat{I}_{1z}\hat{I}_{2z}$ and $\hat{I}_{1z}\hat{I}_{3z}$ will appear. However, with the use of our previous results such a generalization becomes trivial as it requires a straightforward summation of spectra coming from these spin orders. Thus, let us describe the results for the three-spin order, *i.e.*, for $K = N = 3$. Again, it is sufficient to characterize the spin multiplet centered at ω_1 as the multiplets of the other two nuclei can be determined in the same way. The relevant terms in the spin evolution are as follows:

$$M^{123} \hat{I}_{1z} \hat{I}_{2z} \hat{I}_{3z} \xrightarrow{\varphi \hat{I}_y} M^{123} \sin \varphi \cos^2 \varphi (\hat{I}_{1x} \hat{I}_{2z} \hat{I}_{3z} + \hat{I}_{1z} \hat{I}_{2x} \hat{I}_{3z} + \hat{I}_{1z} \hat{I}_{2z} \hat{I}_{3x}) + \text{NOT} \quad (40)$$

Here only the first term contributes to NMR signals of the first spin, thus, the relevant part of the spin evolution takes the form:

$$M^{123} \sin \varphi \cos^2 \varphi \hat{I}_{1x} \hat{I}_{2z} \hat{I}_{3z} \xrightarrow{\text{J-couplings}} -\frac{1}{4} M^{123} \sin \varphi \cos^2 \varphi \hat{I}_{1x} \sin(\pi J_{12}t) \sin(\pi J_{13}t) + \text{NOT} \quad (41)$$

The in-phase spectrum is as follows:

$$S_{in}(\omega) = \frac{1}{16} M^{123} \sin \varphi \cos^2 \varphi (L(\omega - \omega_1 + \pi J_{12} + \pi J_{13}) - L(\omega - \omega_1 - \pi J_{12} + \pi J_{13}) - L(\omega - \omega_1 + \pi J_{12} - \pi J_{13}) + L(\omega - \omega_1 - \pi J_{12} - \pi J_{13})) \quad (42)$$

This spectrum is shown in Fig. 3 (spectrum 4). The spin multiplet comprises four lines of different phase with an equal number of absorptive and emissive lines. The spectrum of the three-spin order is an even function of frequency because K is odd. The phase of the lines is given by the factor $(-1)^Q$ in Eqs. (15), (18) and (23), which is present in the general formulas; the spectral pattern is thus of an AEEA type. The flip angle dependence of the resulting spectrum is given by the φ -dependent factor $\sin \varphi \cos^2 \varphi$, which results in contributions to the first and third Fourier harmonics.

Example 4. Finally, let us describe the spectrum of a four-spin order. We will consider here a four-spin system with all spins entangled, *i.e.*, $N = K = 4$. When $K < N$ all results for this system can be easily obtained from the considerations presented above. The spectrum coming from the four-spin order is shown in Fig. 3 (spectrum 5). Again the number of lines with positive and negative phase is the same; the spectrum is an odd function of frequency with the spectral pattern of the type AEEAEAAE. The flip angle dependence of the spin order under consideration has two Fourier harmonics, namely, the second and the fourth harmonics.

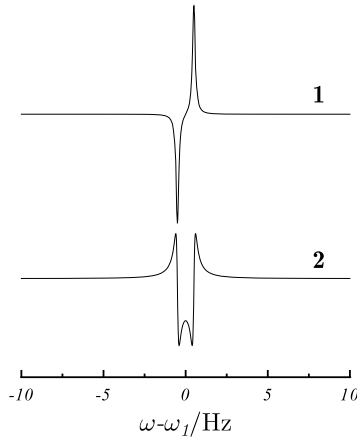


Fig. 4. FT-NMR spectra of a two-spin system in a non-equilibrium state of the second kind, which is $\hat{I}_{1x} \hat{I}_{2x}$ (**spectrum 1**) or $(\hat{I}_{1x} \hat{I}_{2y} - \hat{I}_{1y} \hat{I}_{2x})$ (**spectrum 2**). Here the spin multiplet of spin 1 centered at a frequency ω_1 is shown. Flip angle dependencies for different spin orders are described in the text; spin-spin interaction is $J_{12} = 1$ Hz.

Spectra of higher-spin orders can be constructed in a similar way. Spin multiplets are even or odd functions of frequency (which is dictated by the factors $(-1)^Q$ for odd and even K , respectively). The flip angle dependence of the spectrum is determined by factors $\sin \varphi \cos^{K-1} \varphi$, which contribute to well-defined sine harmonics in the Fourier series expansion.

Example 5, (non-equilibrium states of the second kind). We consider a two-spin system having ZQCs. In this case there are two possibilities for the ZQCs, which are given either by xx -operators or by xy - and yx -operators.

Case (a): coherence of an xx -type. The initial state of the system is $R^{12} \hat{I}_{1x} \hat{I}_{2x}$; its spin evolution is written as follows (here we omit the evolution of \hat{I}_{1y}):

$$\begin{aligned}
 R^{12} \hat{I}_{1x} \hat{I}_{2x} \xrightarrow{\varphi \hat{I}_y} & -\frac{1}{2} R^{12} \sin 2\varphi (\hat{I}_{1x} \hat{I}_{2z} + \hat{I}_{1x} \hat{I}_{2z}) + \text{NOT} \xrightarrow{2\pi J_{12} \hat{I}_{1z} \hat{I}_{2z} t + \omega_1 \hat{I}_{1z} t} \\
 & -\frac{1}{4} R^{12} \sin 2\varphi \hat{I}_{1x} \sin(\pi J_{12} t) \sin(\omega_1 t) + \text{NOT}
 \end{aligned}
 \tag{43}$$

The spectrum is shown in Fig. 4 (spectrum 1); it is just the inversion of the spectrum coming from the zz -order, which was considered in detail in the previous examples. The flip angle dependence of the NMR spectrum of the xx -spin order is given by $\sin 2\varphi$ as it was for the zz -spin order.

Case (b): coherence of the xy - and yx -type. The initial state of the system is $R^{12} (\hat{I}_{1x} \hat{I}_{2y} - \hat{I}_{1y} \hat{I}_{2x})$; spin evolution is written as follows (here we omit the evolution

of \hat{I}_{1y}):

$$R^{12} \left(\hat{I}_{1x} \hat{I}_{2y} - \hat{I}_{1y} \hat{I}_{2x} \right) \xrightarrow{\varphi \hat{I}_y} -R^{12} \hat{I}_{1y} \hat{I}_{2z} \sin \varphi + \text{NOT} \xrightarrow{\pi J_{12} 2 \hat{I}_{1z} \hat{I}_{2z} t + \omega_1 \hat{I}_{1z} t} \\ -\frac{1}{2} R^{12} \hat{I}_{1x} \sin \varphi \sin(\pi J_{12} t) \cos(\omega_1 t) + \text{NOT} \quad (44)$$

The spectrum has only dispersive lines in the in-phase channel (see Fig. 4, spectrum 2). Interestingly, for the spin order considered the flip angle dependence is not the same as for the xx - and zz -orders. Although we started with a two-spin order, the result is proportional to $\sin \varphi$, as it was the case for the single-spin order. Thus, the ZQC of the type $(\hat{I}_{1x} \hat{I}_{2y} - \hat{I}_{1y} \hat{I}_{2x})$ behaves quite differently in pulsed NMR as compared to the longitudinal order.

In principle, further examples for the non-equilibrium states of the second kind could be shown; however, our treatment already makes the general laws clear that govern the spin evolution.

4. Experiments

To illustrate and confirm our general theoretical results we performed the analysis of the nutation patterns for a hyperpolarized system of two protons, which we prepared in the singlet state by means of PHIP [18,19]. In the PHIP experiment a substrate molecule catalytically reacts with *para*-hydrogen (*i.e.*, dihydrogen, H_2 , in its singlet state); when the two protons occupy non-equivalent positions in the reaction product its NMR lines are strongly enhanced. Thus, in principle, PHIP enables the preparation of singlet order of the two-spins. However, to prepare the desired spin order special care has to be taken because at high field the singlet state is not an eigen-state of the two spins. As a consequence, in the PHIP experiment the spin system acquires not only non-equilibrium state populations but also a ZQC. In most cases this ZQC is washed out because of the finite preparation period, which is typically much longer than the inverse frequency difference $1/(\omega_1 - \omega_2) = 1/\delta\omega$. Here, to preserve the ZQC we changed the standard protocol [41] of the high-field PHIP experiment (see Fig. 5). We prepared polarization under a sufficiently strong resonant RF-field so that the singlet state of the two protons was an eigen-state of the Hamiltonian under these conditions. As a consequence, we got rid of the fading of the ZQC because PHIP produced polarized molecules in a spin eigen-state. When a sufficient amount of polarization was accumulated, we switched off the strong RF-field instantaneously so that the spin state of the molecules did not change. This strategy allowed us to form the initial singlet spin order, see Eq. (26).

Immediately after the switching the terms $(\hat{I}_{1x} \hat{I}_{2x} + \hat{I}_{1y} \hat{I}_{2y})$ start evolving under the action of $\delta\omega \cdot \delta \hat{I}_z$ as it is described in Eq. (27). Thus, the spin order changes with time and the detected NMR spectrum does not only depend on the flip angle but also on the delay, τ , between the switch-off of the strong RF-field and the RFpulse used for the NMR detection. In our experiments we measured how the observed spectrum depends on both quantities. Using this preparation the spin state under study is a non-equilibrium state of the second kind, since not only the state populations differ from those at thermal equilibrium but coherences are present also.

The experiments were performed in the following manner (Fig. 5). To create PHIP we used the catalytic hydrogenation reaction of ethyl phenylpropiolate, which produces

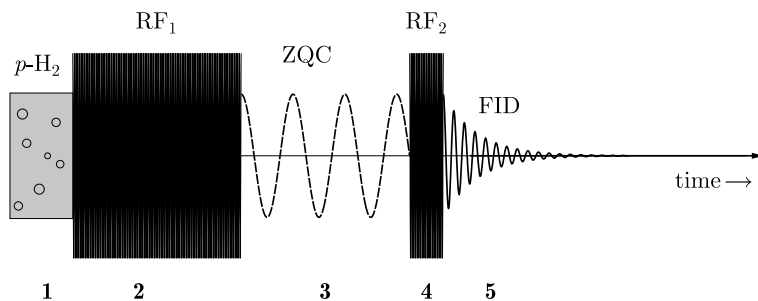
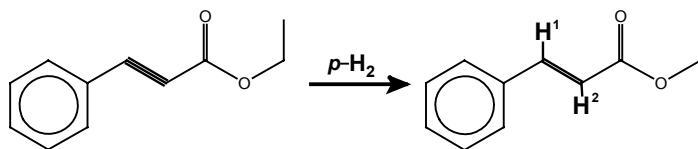


Fig. 5. Experimental protocol used for studying nutation patterns of a two-spin system in a non-equilibrium spin state of the second kind. The protocol comprises 5 steps: bubbling H_2 gas enriched in its *para*-component through the solvent (**step 1**); switching on a strong RF-field denoted as RF_1 (**step 2**); free evolution of the ZQC during a variable time τ (**step 3**); applying a non-selective RF-pulse denoted as RF_2 (**step 4**); recording the FID (**step 5**). Duration of step 1 and 2 was 2 s and 5 s, respectively; delay τ was varied on the millisecond timescale; detecting NMR pulses had a duration of $90 \mu s$ (for $\varphi = 2\pi$) or less.



Scheme 1. Scheme of the hydrogenation reaction used; the chemical structures of ethyl phenylpropiolate and ethyl cinnamate are shown; the polarized protons, H^1 and H^2 of ethyl cinnamate are indicated.

ethyl cinnamate [44]. The sample containing 200 mM of ethyl phenylpropiolate and 6 mM of the hydrogenation catalyst in deuterated methanol used as a solvent was placed in the NMR sample tube. We used the same sample tubes as before and performed bubbling in the way described earlier [45]. To prepare PHIP we bubbled dihydrogen gas enriched in its *para*-component through the solvent for 2 s (step 1). Then a strong RF field with an amplitude of 10 kHz was switched on for 5 s (step 2). During this time the dissolved *para*-hydrogen reacted with ethyl phenylpropiolate to produce ethyl cinnamate having two protons in the H^1 and H^2 positions (see Scheme 1) prepared in their singlet spin state. The chemical shifts of the protons are equal to 6 and 7 ppm; their scalar spin-spin coupling constant is 12.8 Hz. Then the RF field was switched off during approximately $100 \mu s$; after that the two polarized protons evolved under the action of the operator $\delta\omega \cdot \delta\hat{I}_z$ (step 3). Then, after a precisely set delay we applied a non-selective RF-pulse (step 4) to obtain the FID (step 5); the Fourier transform of the FID gave us the NMR spectrum of the system. The frequency of the evolution of the ZQC is given by the difference in NMR frequency, $\delta\omega/2\pi$ of the protons, which was about 300 Hz. The time of switching off the strong RF-field and the NMR pulse length ($90 \mu s$ for $\varphi = 2\pi$ and less for smaller flip angles) are much shorter than a period of the ZQC oscillations, which enabled the precise setting of the delay τ . Delays were taken relatively short so that spin relaxation did not affect the evolution of the ZQCs.

Let us first describe how the observed spectrum depends on the delay τ . At $\tau = 0$ the spin system is in its singlet spin state, hence its NMR spectrum disappears in the case

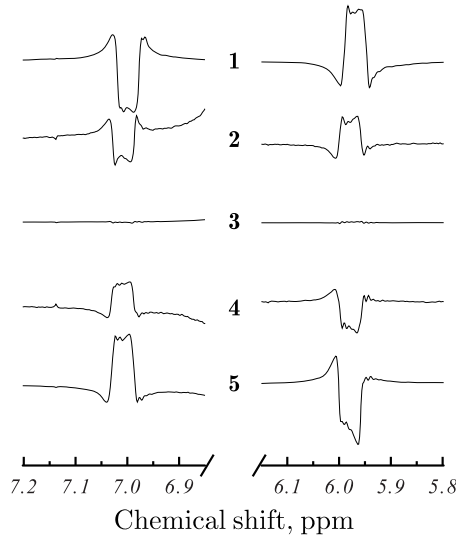


Fig. 6. Spectrum of the two-spin system prepared in its singlet state detected at different delays τ after the preparation. Here $\tau = n \cdot T + \delta\tau$, with $T \approx 3.3$ ms being the period of oscillations of the ZQC, $n = 61$ being the number of periods and $\delta\tau$ being equal to: $-T/4$ (spectrum 1); $-T/8$ (spectrum 2); $\delta\tau = 0$ (spectrum 3); $T/8$ (spectrum 4); $T/4$ (spectrum 5). Here, the in-phase spectra are shown; 90° pulses were used for NMR detection.

of non-selective excitation. Indeed, the singlet state is an eigen-state of the Hamiltonian $\gamma B_1 (\hat{I}_{1y} + \hat{I}_{2y})$, therefore the RF-pulse does not induce any transitions between the spin states. As the ZQC evolves the situation changes, since the spin order oscillates between $(\hat{I}_{1x} \hat{I}_{2x} + \hat{I}_{1y} \hat{I}_{2y})$ and $(\hat{I}_{1x} \hat{I}_{2y} - \hat{I}_{1y} \hat{I}_{2x})$. When the spin system goes from the pure singlet state to a state

$$\hat{\rho}_0 \propto \frac{1}{4} \hat{E} - \hat{I}_{1z} \hat{I}_{2z} - (\hat{I}_{1x} \hat{I}_{2y} - \hat{I}_{1y} \hat{I}_{2x}) \tag{45}$$

the NMR spectrum appears. Indeed, the longitudinal $\hat{I}_{1z} \hat{I}_{2z}$ spin order gives rise to the so-called PASADENA-type spectrum [41] (see spectrum 3 in Fig. 3; the flip angle dependence is given by $\sin 2\varphi$) while the coherence $(\hat{I}_{1x} \hat{I}_{2y} - \hat{I}_{1y} \hat{I}_{2x})$ gives a spectrum with dispersive lines (see spectrum 2 in Fig. 4) and the flip angle dependence is given by $\sin \varphi$.

Our experimental results are in full agreement with these expectations. In Fig. 6 a series of spectra are shown for various values of τ . Here, $\varphi = \pi/2$ detection pulses are used with the consequence that all components varying with $\sin 2\varphi$ are suppressed. As can be seen from Fig. 6, when the delay $\delta\omega \cdot \tau = 2n\pi$ (here n is an integer number) the spin system returns back to the singlet state and no NMR signals are seen (cf. spectrum 3 in Fig. 6). When $\delta\omega \cdot \tau = (2n \pm 1/2)\pi$ the contributions giving dispersive lines are maximal in amplitude (see spectra 1 and 5 in Fig. 6).

Now let us consider the flip angle dependence of the spectra and separate spin orders in the system by performing the Fourier analysis of the nutation patterns. Fig. 7

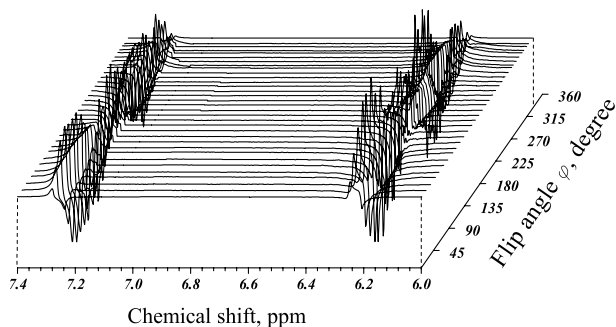


Fig. 7. Spectra of the two-spin system prepared in its singlet state detected at different flip angles. Here, the in-phase spectra are shown; the delay τ was the same as for spectrum 1 in Fig. 6, corresponding to the maximal contribution from $(\hat{I}_{1x}\hat{I}_{2y} - \hat{I}_{1y}\hat{I}_{2x})$.

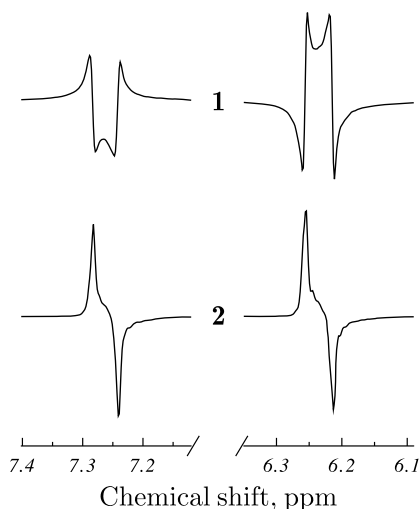


Fig. 8. Nutation analysis of the NMR of the two-spin system prepared in the singlet state: first Fourier harmonics (spectrum 1) and second Fourier harmonics (spectrum 2). Here the in-phase spectra are shown; the delay τ is the same as for spectrum 1 in Fig. 6, corresponding to the maximal contribution from the $(\hat{I}_{1x}\hat{I}_{2y} - \hat{I}_{1y}\hat{I}_{2x})$ term.

shows the NMR spectra as function of the flip angle φ . In Fig. 8 the result of the Fourier decomposition of the nutation patterns is presented.

The first Fourier harmonics (Fig. 8, spectrum 1) corresponds to the spectrum of the $(\hat{I}_{1x}\hat{I}_{2y} - \hat{I}_{1y}\hat{I}_{2x})$ spin order. In full agreement with the theory, in this spectrum there are only dispersive NMR lines although the NMR detection was performed in-phase. Thus, both spin multiplets have two dispersive lines of the opposite phase as it is expected (see spectrum 2 in Fig. 4). The contribution from the corresponding spin order is maximal when $\varphi = \pi/2$ is used. The second harmonics (Fig. 8, spectrum 2) gives the spectrum coming from the longitudinal two-spin order, $\hat{I}_{1z}\hat{I}_{2z}$. The spectral shape is standard for

the PHIP experiments done at high field: there are two AE-multiplets in the spectrum. Since both types of spin order depend on the flip angle in a different way, the Fourier analysis enables to separate them and to characterize fully the non-equilibrium state of the second kind. The experimental results are in full agreement with our theoretical treatment. Thus, our general theoretical consideration gives a correct description of the experimentally observed NMR spectra of non-equilibrium spin systems. It reproduces not only the flip angle dependence of the spectral patterns but also such unusual effects as dispersive lineshapes in the in-phase detection channel, which are present when the spin system has ZQCs.

5. Conclusions

Here, we performed a theoretical treatment of FT-NMR spectra of hyperpolarized multispin systems, which have multiplet polarization of high order. We analyzed in detail the flip angle dependence of the spectra (so-called nutation patterns) and revealed to what Fourier harmonics with respect to the flip angle the different spin orders contribute. We also obtained a general expression for the shape of the NMR spectrum coming from a particular spin order, which allowed us to formulate rules for the spectral patterns, *i.e.*, to understand whether spin multiplets are even or odd functions of frequency. Bearing in mind the complexity of the problem in its general formulation we can state that our treatment is still relatively simple. This became possible because we used the product operator formalism. In addition to the non-equilibrium spin states of the first kind, which are characterized by longitudinal spin orders, we extended our treatment to non-equilibrium states of the second kind, in particular, to states with zero-quantum coherences. We have shown that for certain types of coherences there can be Lorentzian or dispersive lines resulting from the Fourier transform of the in-phase channel spin magnetization.

Our treatment makes determination of spin order possible in multi-spin systems by performing Fourier decomposition of the nutation patterns. First, odd and even spin orders contribute only to odd and even harmonics. Second, the highest odd/even observable harmonics will immediately give the highest odd/even spin order in the system. This spin order also contributes to lower harmonics; however, this fact does not create a big problem because the weights of these contributions are precisely known from Eqs. (21) and (22). Therefore all contributions of high spin order to lower harmonics can be easily separated. As a consequence, it becomes possible to decompose the nutation patterns in contributions arising from different spin orders. Thus, our strategy based on the Fourier series expansion of the nutation patterns allows for a straightforward determination of the full spin order in multispin system.

Taking a two-spin system prepared in its singlet spin state as an example we also demonstrated that our treatment indeed opens a way to a precise characterization of the spin order. It is important to emphasize that the system presented is a non-equilibrium system of the second kind corresponding to a more complicated case as compared to the non-equilibrium system of the first kind. We studied the evolution of the zero-quantum coherence in the system caused by the difference in Zeeman interaction of the spins with the external magnetic field. We presented the NMR nutation patterns as a sum of

two Fourier harmonics: the first harmonics results from the zero-quantum coherences and gives dispersive NMR lines, whereas the second harmonics comes from the longitudinal two-spin order and results in NMR spectra having the standard shape of the PASADENA-type spectrum with two AE multiplets.

The present treatment allowed us to understand both the flip angle dependence and the NMR spectral patterns of spin systems carrying multispin order. In fact, our treatment exhausts the theoretical description of both of these problems. We illustrated the general results by several theoretical and experimental examples, which make our treatment clear and give an impression of how typical NMR multiplet and nutation patterns look like when they are resulting from high-order multiplet polarization. Our treatment is particularly important to describe CIDNP and PHIP spectra as for both kinds of polarization it is typical that not only net but also strong multiplet polarization is formed.

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References

1. R. R. Ernst, G. Bodenhausen, and A. Wokaun, *Principles of Nuclear Magnetic Resonance in One and Two Dimensions*, Clarendon Press, Oxford (1978).
2. S. R. Hartmann and E. L. Hahn, *Phys. Rev.* **128** (1962) 2042.
3. A. Pines, M. G. Gibby, and J. S. Waugh, *J. Chem. Phys.* **56** (1972) 1776.
4. G. A. Morris and R. Freeman, *J. Am. Chem. Soc.* **101** (1979) 760.
5. A. J. Moule, M. M. Spence, S. I. Han, J. A. Seeley, K. L. Pierce, S. Saxena, and A. Pines, *P. Natl. Acad. Sci. USA* **100** (2003) 9122.
6. H. Kovacs, D. Moskau, and M. Spraul, *Prog. Nucl. Mag. Res. Sp.* **46** (2005) 131.
7. P. Styles, N. F. Soffe, C. A. Scott, D. A. Cragg, F. Row, D. J. White, and P. C. J. White, *J. Magn. Reson.* **60** (1984) 397.
8. K. H. Hausser and D. Stehlik, *Adv. Magn. Reson.* **3** (1968) 79.
9. C. Griesinger, M. Bennati, H.-M. Vieth, C. Luchinat, G. Parigi, P. Hofer, F. Engelke, S. J. Glaser, V. Denysenkov, and T. F. Prisner, *Prog. Nucl. Mag. Res. Sp.* **64** (2012) 4.
10. T. Maly, G. T. Debelouchina, V. S. Bajaj, K.-N. Hu, C.-G. Joo, M. L. Mak-Jurkauskas, J. R. Sirigiri, P. C. A. van der Wel, J. Herzfeld, R. J. Temkin, and R. G. Griffin, *J. Chem. Phys.* **128** (2008) 052211.
11. W. Happer, *Rev. Mod. Phys.* **44** (1972) 169.
12. D. Stehlik, *The Mechanism of Optical Nuclear Polarization in Molecular Crystals*, in: *Excited States*, Vol. 3, E. C. Lim (Ed.), Academic Press, New York (1977), 203.
13. H. M. Vieth, V. Macho, and D. Stehlik, *Chem. Phys. Lett.* **60** (1979) 368.
14. D. Stehlik and H.-M. Vieth, *Time Evolution of Electron-Nuclear Cross-Polarization in Radio-frequency Induced Optical Nuclear Spin Polarization*, in: *Pulsed Magnetic Resonance: NMR, ESR, and Optics: a Recognition of E. L. Hahn*, D. M. S. Bagguley (Ed.), Oxford University Press, Oxford (1992), 446.
15. J. Bargon and H. Fischer, *Z. Naturforsch. Pt. A* **22** (1967) 1556.
16. R. Kaptain, *J. Chem. Soc. Chem. Comm.* **14** (1971) 732.

17. K. M. Salikhov, Y. N. Molin, R. Z. Sagdeev, and A. L. Buchachenko, *Spin Polarization and Magnetic Effects in Chemical Reactions*, Elsevier, Amsterdam (1984).
18. R. A. Green, R. W. Adams, S. B. Duckett, R. E. Mewis, D. C. Williamson, and G. G. R. Green, *Prog. Nucl. Mag. Res. Sp.* **67** (2012) 1.
19. J. Natterer and J. Bargon, *Prog. Nucl. Mag. Res. Sp.* **31** (1997) 293.
20. J. Matthes, T. Pery, S. Gruendemann, G. Buntkowsky, S. Sabo-Etienne, B. Chaudret, and H.-H. Limbach, *J. Am. Chem. Soc.* **126** (2004) 8366.
21. G. Buntkowsky, J. Bargon, and H.-H. Limbach, *J. Am. Chem. Soc.* **118** (1996) 8677.
22. G. Buntkowsky and H.-H. Limbach, Dihydrogen Transfer and Symmetry: The Role of Symmetry in the Chemistry of Dihydrogen Transfer in the Light of NMR Spectroscopy, in: *Hydrogen-Transfer Reactions*, Vol. 2, R. L. Schowen (Ed.), Wiley-VCH Verlag, Weinheim (2007), 639.
23. H. Jóhannesson, O. Axelsson, and M. Karlsson, *C. R. Phys.* **5** (2004) 315.
24. W. Köckenberger and J. Matysik, Magnetic Resonance: Hyperpolarization Methods and Applications in NMR, in: *Encyclopedia of Spectroscopy and Spectrometry, 2nd edition*, J. C. Lindon (Ed.), Elsevier, Oxford (2010), p. 963.
25. T. Theis, P. Ganssle, G. Kervern, S. Knappe, J. Kitching, M. P. Ledbetter, D. Budker, and A. Pines, *Nat. Phys.* **7** (2011) 571.
26. T. Theis, M. P. Ledbetter, G. Kervern, J. W. Blanchard, P. J. Ganssle, M. C. Butler, H. D. Shin, D. Budker, and A. Pines, *J. Am. Chem. Soc.* **134** (2012) 3987.
27. M. H. Lerche, S. Meier, P. R. Jensen, S. O. Hustvedt, M. Karlsson, J. O. Duus, and J. H. Ardenkjaer-Larsen, *NMR in Biomed.* **24** (2011) 96.
28. M. D. Lingwood and S. I. Han, Solution-State Dynamic Nuclear Polarization, in: *Annual Reports on NMR Spectroscopy*, Vol. 73, G. A. Webb (Ed.), Elsevier Academic Press INC, San Diego (2011), 83.
29. M. Lelli, D. Gajan, A. Lesage, M. A. Caporini, V. Vitzthum, P. Mieville, F. Heroguel, F. Rascon, A. Roussey, C. Thieuleux, M. Boualleg, L. Veyre, G. Bodenhausen, C. Coperet, and L. Emsley, *J. Am. Chem. Soc.* **133** (2011) 2104.
30. D. Trease, V. S. Bajaj, J. Paulsen, and A. Pines, *Chem. Phys. Lett.* **503** (2011) 187.
31. P. Berthault, G. Huber, and H. Desvaux, *Prog. Nucl. Mag. Res. Sp.* **55** (2009) 35.
32. H. E. Moller, X. J. Chen, B. Saam, K. D. Hagspiel, G. A. Johnson, T. A. Altes, E. E. de Lange, and H. U. Kauczor, *Magn. Reson. Med.* **47** (2002) 1029.
33. M. C. D. Tayler, I. Marco-Rius, M. I. Kettunen, K. M. Brindle, M. H. Levitt, and G. Pileio, *J. Am. Chem. Soc.* **134** (2012) 7668.
34. S. Schäublin, A. Höhener, and R. R. Ernst, *J. Magn. Reson.* **13** (1974) 196.
35. K. L. Ivanov, H.-M. Vieth, K. Miesel, A. V. Yurkovskaya, and R. Z. Sagdeev, *Phys. Chem. Chem. Phys.* **5** (2003) 3470.
36. M. G. Pravica and D. P. Weitekamp, *Chem. Phys. Lett.* **145** (1988) 255.
37. R. Boelens, A. Podoplelov, and R. Kaptein, *J. Magn. Reson.* **69** (1986) 116.
38. K. L. Ivanov, K. Miesel, H.-M. Vieth, A. V. Yurkovskaya, and R. Z. Sagdeev, *Z. Phys. Chem.* **217** (2003) 1641.
39. H. Kessler, M. Gehrke, and C. Griesinger, *Angew. Chem. Int. Edit.* **27** (1988) 490.
40. O. W. Sørensen, G. W. Eich, M. H. Levitt, G. Bodenhausen, and R. R. Ernst, *Prog. Nucl. Mag. Res. Sp.* **16** (1983) 163.
41. C. R. Bowers and D. P. Weitekamp, *J. Am. Chem. Soc.* **109** (1987) 5541.
42. J.-K. Vollenweider and H. Fischer, *Chem. Phys.* **108** (1986) 365.
43. R. Hany, J.-K. Vollenweider, and H. Fischer, *Chem. Phys.* **120** (1988) 169.
44. S. Aime, W. Dastrù, R. Gobetto, and A. Viale, *Org. Biomol. Chem.* **3** (2005) 3948.
45. A. S. Kiryutin, K. L. Ivanov, A. V. Yurkovskaya, R. Kaptein, and H.-M. Vieth, *Z. Phys. Chem.* **226** (2012) 1343.