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Nonlinear Conductance of Long Quantum Wires at a Conductance Plateau Transition: Where Does the Voltage Drop?

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We calculate the linear and nonlinear conductance of spinless fermions in clean, long quantum wires, where short-ranged interactions lead locally to equilibration. Close to the quantum phase transition, where the conductance jumps from zero to one conductance quantum, the conductance obtains a universal form governed by the ratios of temperature, bias voltage, and gate voltage. Asymptotic analytic results are compared to solutions of a Boltzmann equation which includes the effects of three-particle scattering. Surprisingly, we find that for long wires the voltage predominantly drops close to one end of the quantum wire due to a thermoelectric effect.

Introduction.—A clean quantum wire with adiabatic contacts is characterized by a quantized conductance, $G = nG_0$ with $G_0 = e^2/h$. The integer n describes the number of conduction channels (including spin). The conductance quantization is closely related to charge quantization and survives (for sufficiently low temperatures T) even in the presence of interactions [1–3] as long as momentum relaxation by umklapp scattering can be neglected [4,5].

The transition from one conductance plateau to the next is an example of a quantum phase transition without order parameter, where only a topological property, the number of conducting channels, changes. While the thermodynamics of this quantum critical point (QCP) is quite well understood [6–9], a theory of the quantum critical conductance is much more challenging: how does the conductance change from one conductance plateau to the next at low but finite T? We will answer this question both in the linear and nonlinear regime for the most simple situation, i.e., the transition from $n = 0$ to $n = 1$ for spinless fermions with finite-ranged interactions. Here, the QCP describes the transition from zero to a finite fermion density. As interactions are irrelevant at this QCP, thermodynamic properties are well described by noninteracting fermions [5,10], but transport in long wires is still governed by collisions. Relaxation by collisions and nonequilibrium dynamics in one-dimensional (1D) systems have recently moved into the focus of theoretical [11–18] and experimental [19–21] research.

A general question is where the voltage drops when a finite current is driven through a clean 1D quantum wire by applying a bias voltage V. For diffusive (multichannel) quantum wires, one expects a linear drop of the voltage (i.e., of the electrochemical potential) across the wire, while for noninteracting, ballistic quantum wires the voltage drop occurs only close to the two contacts [22].

In clean *interacting* quantum wires with low fermion density (and therefore negligible umklapp scattering), the dc conductivity is infinite for an infinitely long wire due to momentum conservation, $\sigma(T) = \infty$. The vanishing resistivity strongly suggests that there is, again, no voltage drop inside the wire.

A recent series of papers [23–27] that studied the role of equilibration in long but finite quantum wires of length L found that in the linear response regime, $V \to 0$, there is a linear drop of voltage [25] along the wire. We resolve this apparent contradiction to $\sigma(T) = \infty$ by noting that the limits $V \to 0$ and $L \to \infty$ do not commute. The drop of voltage is governed by a new length scale ℓ_V which diverges for $V \to 0$. For $L \ll \ell_V$, a linear drop of voltage occurs. In the opposite limit, $L \gg \ell_V$; however, the voltage drops only within a distance of ℓ_V of the contacts. Surprisingly, the voltage drop is not symmetrical and occurs predominantly only at one of the two contacts!

Previous work on equilibrated quantum wires [23–27] focused on the limit $T \ll \epsilon_F$, where ϵ_F is the Fermi energy. As scattering processes equilibrating left-moving and right-moving fermions involve the bottom of the band [18,25–28], they are exponentially suppressed and, therefore, the corresponding equilibration length is exponentially large, $\ell_{eq} \sim e^{\Delta/T}$, where $\Delta \approx \epsilon_F$ for weak interactions (thermal equilibration relevant for heat conductance occurs on shorter length scales [29]). For $L \gg \ell_{eq}$, it was found that the quantized (linear) conductance obtains corrections of order $(T/\Delta)^2$. Large effects can therefore be expected close to the conductance plateau transition, where $\Delta \sim T$, as studied in this Letter.

Model.—We consider 1D spin-polarized electrons with quadratic dispersion $\epsilon_p = \frac{p^2}{2m}$, interacting via a short range potential. Close to the QCP, where filling of the first subband becomes small, interactions are strongly irrelevant in the

renormalization group sense [6,30] and a single-electron description becomes approximately valid. To study equilibration and its effect on transport, we may thus use the Boltzmann equation,

$$v_p \partial_x f_{x,p} = -I^{col}_{x,p}[f], \qquad (1)$$

where $f_{x,p}$ is the quasiclassical distribution function, $v_p = p/m$ the velocity, and the collision integral I^{col} describes collisions. The contacts of the quantum wire to the leads at $x = \pm\frac{L}{2}$ induce boundary conditions for electrons moving *into* the quantum wire,

$$f_{x=-(L/2),p>0} = \frac{1}{e^{\xi^l_p/T} + 1},$$
$$f_{x=(L/2),p<0} = \frac{1}{e^{\xi^r_p/T} + 1}, \qquad (2)$$

where $\xi^{l/r}_p = \epsilon_p - \mu \mp eV/2$ with $\mu = 0$ at the QCP. Here, we assume adiabatic and ballistic contacts, i.e., contacts which are smooth compared to the electronic wavelength but short in comparison to the scattering length.

In 1D systems, energy and momentum conservation severely restrict the phase space available for scattering: in a two-particle process, two particles of equal mass can only exchange their momenta [6,18] which leaves f_p unchanged. One therefore has to study the effects of three-particle collisions [18,28] described by

$$I^{col}_{x,p_1}[f] = \sum_{\substack{p_2 p_3 \\ p'_1 p'_2 p'_3}} W^{1'2'3'}_{123}[f_1 f_2 f_3(1 - f_{1'})(1 - f_{2'})(1 - f_{3'})$$
$$- f_{1'} f_{2'} f_{3'}(1 - f_1)(1 - f_2)(1 - f_3)], \qquad (3)$$

where the scattering rate $W^{1'2'3'}_{123}$ arises to fourth order in the bare two-particle interactions [31]. For low energies and spinless fermions, the Pauli principle ensures that it takes the universal form

$$W^{1'2'3'}_{123} = W((p_1 - p_2)(p_1 - p_3)(p_2 - p_3)(p'_1 - p'_2)$$
$$\times (p'_1 - p'_3)(p'_2 - p'_3))^2 \delta_{P_i,P_f} \delta(E_i - E_f), \qquad (4)$$

where $P_{i(f)} = p_1 + p_2 + p_3$ and $E_{i(f)} = \epsilon_1 + \epsilon_2 + \epsilon_3$ are the total momentum and energy of the three scattering particles before (after) the collision, respectively. A simple dimensional analysis allows us to identify a characteristic length scale of equilibration at the QCP ($\mu = 0$) by setting typical momenta to $\sqrt{2mT}$,

$$\frac{1}{\ell_{eq}} = \frac{2Wm^2L^4}{(2\pi\hbar)^4}(2mT)^{13/2}. \qquad (5)$$

Measuring all length scales in units of ℓ_{eq} and all momenta in units of $\sqrt{2mT}$ allows us to scale out the parameters W, m, and T and the only remaining parameters are L/ℓ_{eq}, eV/T, and μ/T. We have checked both numerically and analytically that close to the QCP, Hartree-Fock potentials [not included in Eq. (1)] can be neglected.

For a numerical solution of the Boltzmann equation (1), it is important to avoid discretization errors leading to a violation of conservation laws. We, therefore, use a conservative splitting method following Ref. [32] (see Supplemental Material [30]) to solve the time-dependent Boltzmann equation until a steady state has been reached. For the linear response calculation, we use a linearized collision integral.

Conservation laws.—Three conservation laws govern transport in long quantum wires: charge, energy, and momentum conservation. The corresponding currents are the charge current, $j_c = e\sum_p v_p f_p$, the energy current, $j_E \approx \sum_p \epsilon_p v_p f_p$, and the momentum current, $j_p \approx \sum_p p v_p f_p$. The latter can be identified with pressure. For sufficiently long quantum wires and far away from the contacts, the system will reach locally equilibrium with the distribution function

$$f^{eq}_p(u, \mu, T) = [1 + e^{(p-mu)^2/(2mT)-\mu/T}]^{-1} \qquad (6)$$

parametrized by three space-dependent Lagrange parameters $\mu(x)$, $T(x)$, and the velocity $u(x)$ reflecting the three conservation laws. For the distribution function (6) one can calculate the corresponding equilibrium currents j^{eq}_c, j^{eq}_E, and j^{eq}_p as function of μ, T and u.

Voltage drop.—Figure 1 shows the density profile and the local chemical potential (insets) of long quantum wires, $L \gg \ell_{eq}$, obtained from our Boltzmann simulations (the Supplemental Material [30] discusses how $\mu(x)$ and $T(x)$ can be measured by tunneling contacts). For small V [Fig. 1(a)], there is both a linear drop of the chemical potential along the wire and a finite jump directly at the two contacts (the separate points at $x = \pm L/2$ show μ in the leads). In an experiment, this jump will occur on the length scale describing the crossover from the 1D lead to the higher-dimensional contacts. This jump is also present for larger V [Fig. 1(b)], where, however, the linear voltage drop is absent. Surprisingly, there is instead a large *asymmetric* voltage drop that occurs only close to one of the two contacts. This behavior occurs for sufficiently long wires not only directly at the QCP but also away from it. Interestingly, one observes cooling instead of heating close to the left contact. Though unexpected at first glance, it can be related [25] to the finite boost u in (6), which is partially compensated by a reduced temperature to match the boundary condition for right-movers.

The qualitative difference between small and larger voltage can be understood from a simple argument based on matching currents. The steady state for $V > 0$ is characterized by the three currents j_c, j_E, and j_p. From the three equations $j_\alpha = j^{eq}_\alpha$, $\alpha = c, E, p$, one can, for sufficiently long wires, determine the three parameters μ, T, and u, which will be constant along the wire as $j_\alpha = $ const. Therefore, for a sufficiently long wire and finite V, a voltage drop can occur only close to the contacts. In the linear response regime, i.e., for small V, the situation is, however, different. By setting only one of the three
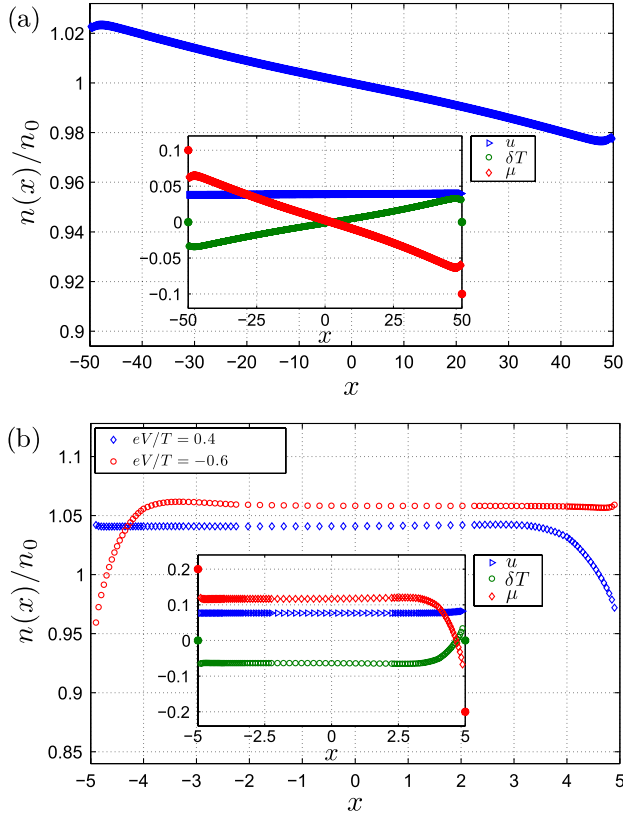
FIG. 1 (color online).   Electron density, $n(x) = n_0(1 + \delta n(x))$, at the QCP ($\mu = 0$) calculated from a solution of the Boltzmann equation (1), using (a) a linearized collision integral for $L = 100\ell_{\text{eq}}$ and $V \to 0$ and (b) the nonlinearized collision integral for $L = 10\ell_{\text{eq}}$ and larger voltages, $eV/T = 0.4$ and $eV/T = -0.6$ (inset: $eV/T = 0.4$). The inset shows $\mu(x)$, $\delta T(x) = T(x) - T$, and $u(x)$ (in units of T and $\sqrt{2T/m}$) obtained from fitting the local charge, energy, and momentum densities to Eq. (6). While in the linear response regime, $L \ll |\ell_V|$, there is a linear voltage drop across the wire, the voltage (and the density) drops predominantly close to one of the two contacts for $L \gg |\ell_V|$. Note that due to a finite drift u, the chemical potentials close to the contacts do not match the chemical potential $\mu \pm eV/2$ in the leads (shown as separate dots at $x = \pm L/2$ in the insets).

parameters, u, to zero, two currents, j_c^{eq} and j_E^{eq}, vanish in equilibrium. As both j_c^{eq} and j_E^{eq} are linear in u, their ratio $j_c^{\text{eq}}/j_E^{\text{eq}}$ is—in the limit of small V—fixed by the average μ and T. This is used below when calculating the linear-response conductance analytically.

To develop an approximate analytical theory valid in both regimes, we consider small but finite voltages V and parametrize $f_{x,p}$ by

$$f_{x,p} = f_{x,p}^{\text{eq}} + \delta f_{x,p}, \tag{7}$$

where $\delta f_{x,p}$ accounts for deviations from local equilibrium $f_{x,p}^{\text{eq}} = f_p^{\text{eq}}(u(x), \mu(x), T(x))$. Here, it is convenient to determine $\mu(x)$, $T(x)$, and $u(x)$ from the two equations $j_c = j_c^{\text{eq}}$ and $j_p = j_p^{eq}$ while the third parameter is fixed by fitting the local density $n(x) = n^{\text{eq}}(x)$.

By linearizing the Boltzmann equation (1) in δf, one obtains that δf is proportional to $\partial n/\partial x$. For the energy current, one therefore obtains

$$j_E = j_E^{\text{eq}} + \tilde{D}\frac{\partial n}{\partial x} = \text{const}, \tag{8}$$

where $\tilde{D}$ is the thermoelectric diffusion constant describing how density gradients generate energy currents. Using Kubo's formula and Einstein relations, $\tilde{D}$ can be calculated from the product of a correlation function of the heat- and particle current and the compressibility.

For small voltages, $\tilde{D}$ is approximately constant across the wire. Using Galilei invariance which implies $u = j_c^{\text{eq}}/en^{\text{eq}} = j_c/en$, we obtain $j_E^{\text{eq}} = \frac{3j_pj_c}{2en} - \frac{mj_c^3}{e^3n^2}$. For small V, the last term can be neglected and one can linearize the density $n = n_0 + \delta n$ to obtain

$$-\frac{3j_pj_c}{2en_0^2}\delta n + \tilde{D}\frac{\partial\delta n}{\partial x} \approx j_E - \frac{3j_pj_c}{2en_0} = \text{const}. \tag{9}$$

This equation introduces a new length scale,

$$\ell_V = \frac{2\tilde{D}en_0^2}{3j_pj_c}, \tag{10}$$

which diverges for $V \to 0$ as j_c vanishes in this limit while $\tilde{D}$, n_0, and j_p remain finite. For $|eV| \ll T$ and $\mu = 0$, i.e., at the QCP, a simple dimensional analysis gives $\ell_V \sim \ell_{\text{eq}}\frac{T}{eV}$.

For $L \ll |\ell_V|$, one obtains from Eq. (9) $\frac{\partial\delta n}{\partial x} = \text{const}$ and, therefore, a linear drop in density and local chemical potential as in our numerical results, Fig. 1(a). In the other limit, $L \gg |\ell_V|$, δn obtains an exponential x dependence,

$$n(x) = n_L + (n_R - n_L)\exp\left[\frac{x - L/2}{\ell_V}\right], \tag{11}$$

with $n_{L/R} \approx n(\mp L/2)$. The direction of the current determines whether the drop of density and voltage occur at the right ($\ell_V > 0$) or left ($\ell_V < 0$) lead; see Fig. 1(b). This shows that the strongly asymmetric drop of voltage arises from a thermoelectric effect captured by the simple hydrodynamic equation (9).

Linear response regime.—Interestingly, it is possible to calculate in the linear response regime the quantum critical conductance for long quantum wires ($\ell_{\text{eq}} \ll L \ll |\ell_V|$) analytically. We use the approach developed in Ref. [23,25] (where only $T \ll \mu$ was considered) and keep track of the change of the charge and energy current carried by right-moving electrons with $p > 0$, $j_c^R(x) = e\sum_{p>0}v_pf_{x,p}$ and $j_E^R(x) = \sum_{p>0}v_p\epsilon_pf_{x,p}$, respectively. We use that, far away from the contacts, the distribution function obtains local equilibrium, $f_{x,p} \approx f_{x,p}^{\text{eq}}$, described by Eq. (6) with $T(x) = T + \delta T(x)$, $\mu(x) = \mu + \delta\mu(x)$ and $u(x)$. This allows us to directly calculate j_c, j_c^R, j_E, $j_E{}^R$ and j_p in terms of three unknown functions, $u(x)$, $\delta\mu(x)$, and $\delta T(x)$. Current conservation implies $u(x) = \text{const}$. Furthermore, the ratio $r_1 = j_E/j_c$ is to linear order just a simple function of μ and T independent of V and u. The condition of constant momentum current fixes another

ratio, $r_2 = \partial_x \delta T / \partial_x \delta \mu$. This result is used to eliminate all unknowns from the ratio $r_3 = \partial_x j_E^R / \partial_x j_c^R$. To leading order, r_1, r_2, and r_3 are space-independent functions of μ and T (calculated in the Supplemental Material [30]). Finally, one identifies [23] the difference in the charge (energy) current of the interacting and noninteracting system as the total change in the right-moving charge (energy) current along the wire, $j_c = j_c^0 + \int \partial_x j_c^R dx$ ($j_E = j_E^0 + \int \partial_x j_E^R dx$), respectively. If we now assume (as we checked numerically), that these integrals are dominated by their bulk contribution, we obtain the equation

$$r_3 = \frac{r_1 j_c - j_E^0}{j_c - j_c^0} \qquad (12)$$

from which one can calculate directly j_c. Combining all results [30], we find for the linear-response conductance up to corrections of $\mathcal{O}(\ell_{eq}/L)$, i.e., for $\ell_{eq} \ll L \ll |\ell_V|$

$$G(z) = \frac{e^2}{h} \frac{\alpha_0(z)\alpha_2(z) - \alpha_1^2(z)}{\alpha_2(z) + \alpha_0(z)\kappa^2(z) - 2\alpha_1(z)\kappa(z)}, \qquad (13)$$

where $z = \mu/T$, $\langle \ldots \rangle_z = -\int_{-z}^{\infty} d\xi (\ldots) \frac{df_\xi^0}{d\xi}$, with $f_\xi^0 = \frac{1}{1+e^\xi}$ and $\alpha_k = \langle \xi^k \rangle_z$, $\kappa = \frac{\langle \xi \sqrt{z+\xi} \rangle_z}{\langle \sqrt{z+\xi} \rangle_z}$. At the QCP, i.e., for $\mu = 0$, this gives

$$\frac{G_{QCP}}{e^2/h} = \frac{\frac{\pi^2}{6} - 2\ln^2 2}{\frac{\pi^2}{3} + \frac{9}{8}\frac{\zeta^2(3/2)}{\zeta^2(1/2)} + \frac{6}{\sqrt{2}}\frac{\zeta(3/2)}{\zeta(1/2)}\ln 2} \approx 0.420, \qquad (14)$$

with $\zeta(x)$ the Riemann zeta function. G_{QCP} is about 16% below the noninteracting result $e^2/2h$.

Figure 2 displays the linear response conductance as functions of μ/T for noninteracting (see below) and fully equilibrated electrons that have a clearly different shape.

Our analytical formula (13) fits very well the numerical result (symbols).

Shorter wires.—Upon lowering T, ℓ_{eq} rapidly increases; see Eq. (5). For quantum wires, where $L/\ell_{eq} \ll 1$, one can neglect the effects of equilibrating interactions. Half of the voltage drops at the left and right contact, respectively, and there is no voltage drop inside the wire as $f_{x,p} = f_p$ is independent of x. For j_c one obtains the well-known non-interacting result $j_c^0 = \frac{eT}{h} \ln[\frac{1+e^{(\mu+eV/2)/T}}{1+e^{(\mu-eV/2)/T}}]$. The conductance plateau transition in linear response is therefore described by $G(\mu/T) = G_0/(1 + e^{-\mu/T})$, while at $\mu = 0$ the current is for arbitrary eV/T given by $j_0 = G_0 V/2$. In the inset of Fig. 2, we have calculated numerically the crossover from the interacting quantum critical conductance (14) to the non-interacting one, which occurs upon lowering T when $\ell_{eq} \sim L$.

Nonlinear response.—Figure 3 shows the nonlinear conductance j_c/V at the QCP, i.e., for $\mu = 0$. It interpolates between the linear-response value (14) and the noninteracting result obtained for $|eV|/T \to \infty$. For $|eV| \gg T$ and $\mu = 0$ all states with $p > 0$ and $\epsilon_p < |eV|/2$ are occupied. As this is also an equilibrium distribution function with $u = \sqrt{|eV|/4m}$ and $\mu = mu^2/2$, collisions have no effects in this limit.

For small V, the nonlinear conductance appears to be nonanalytic,

$$j_c(V) = G_{QCP}V + \gamma|V|V + \cdots, \qquad (15)$$

which can be traced back to the asymmetric voltage drop for $L \gg |\ell_V|$. As $n_R - n_L$ in Eq. (11) varies linear in V, the density in the center, $n(0) \approx \max(n_R, n_L)$ according to Eq. (11), obtains for $L \gg |\ell_V|$ a correction proportional to $|V|$. As $j_c \approx eun$, this implies a correction proportional to $|V|V$ to the current as soon as $L \gg |\ell_V|$. Due to numerical
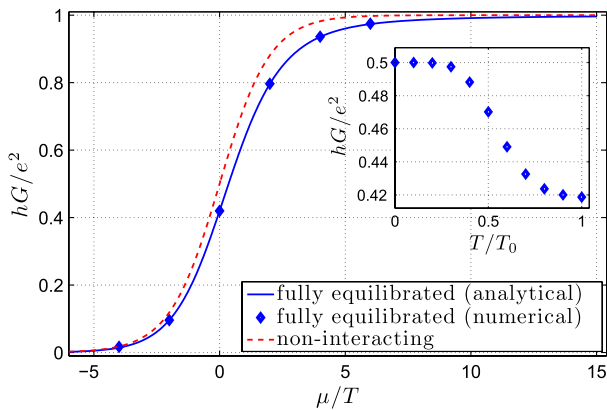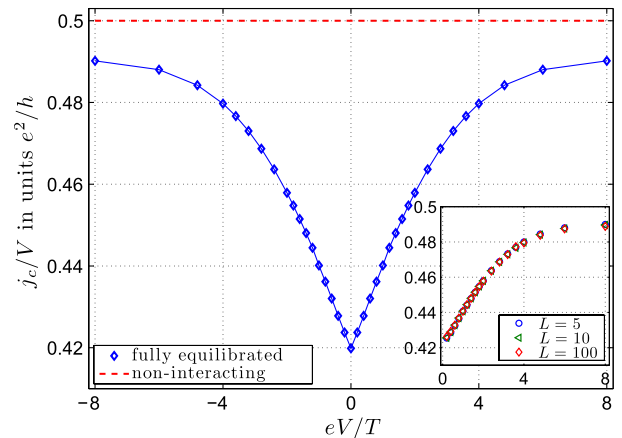

FIG. 2 (color online). Conductance of fully equilibrated (solid line) and noninteracting electrons (dashed line) in the linear response regime ($\ell_{eq} \ll L \ll |\ell_V|$). Numerical results (symbols) agree with Eq. (13). Inset: upon lowering T, ℓ_{eq} grows rapidly and a crossover from the equilibrated to the noninteracting conductance is observed for $\ell_{eq} \sim L$ ($L = 10\ell_{eq}$, $\mu = 0$).

FIG. 3 (color online). Nonlinear conductance, j_c/V obtained for $L = 7.5\ell_{eq}$ from a numerical solution of the Boltzmann equation. For $eV/T \to 0$ a linearized collision integral was used. Inset: within our numerical precision, there is no finite size dependence of the nonlinear conductance for $L \gg \ell_{eq}$ (L measured in units ℓ_{eq}).

problems, we were not able to obtain reliable numerical results in the small-V regime $L \lesssim |\ell_V|$ where we expect a rounding of the nonanalytic correction. Overall, the finite size corrections to the nonlinear conductance are smaller than our numerical resolution for $L \gg \ell_{\rm eq}$; see inset of Fig. 3.

Outlook.—While our results have been derived only for spinless fermions with short ranged interactions, we expect that our main qualitative results are also of direct relevance for quantum wires made of electrons with spin and long-ranged Coulomb interactions. For these systems, the hydrodynamic equation (9) should also be valid (with strongly modified parameters) at least if gates provide screening. The highly asymmetric voltage drop predicted by us will probably be even much easier to observe, as the stronger interactions imply that the regimes $L \gg \ell_{\rm eq}$ and $L \gg |\ell_V|$ are much easier to reach. In recent experiments with ultracold atoms [33], an atomic current was driven through a long quantum channel connecting two reservoirs and the resulting density profile (and therefore the drop of the chemical potential) was directly measured. This opens new exciting possiblities to verify our predictions also in cold-atom experiments.

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