

# Tricritical Point in Quantum Phase Transitions of the Coleman-Weinberg Model at Higgs Mass

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## Abstract

The tricritical point, which separates first and second order phase transitions in three-dimensional superconductors, is studied in the four-dimensional Coleman-Weinberg model, and the similarities as well as the differences with respect to the three-dimensional result are exhibited. The position of the tricritical point in the Coleman-Weinberg is derived and found to be in agreement with the Thomas-Fermi approximation in the three-dimensional Ginzburg-Landau theory. From this we deduce a special role of the tricritical point for the Standard Model Higgs sector in the scope of the latest experimental results, which suggests the unexpected relevance of tricritical behavior in the electroweak interactions.

## INTRODUCTION

Ever since the formulation in 1964 of the electroweak spontaneous symmetry breaking mechanism [1–5] to explain how elementary particles acquire mass, superconductivity and high-energy physics became intimately connected. For instance, the Ginzburg-Landau (GL) theory [1], proposed in 1950 to provide a local macroscopic description of superconductivity, makes use of a quartic potential of the same type that reappeared in the Higgs model<sup>1</sup>. In fact, the GL-theory is the three-dimensional predecessor of what is now called 3+1-dimensional scalar quantum electrodynamics, that was studied in detail by Coleman and Weinberg [6, 7]. Before the work of Ginzburg and Landau, the London theory explained the existence of a finite penetration depth of magnetic fields into a superconductor, the Meissner-Ochsenfeld effect [8, 9]. GL extended this theory by a local complex scalar order field  $\phi(x)$ , whose gradient terms in the energy density produces a finite length scale of fluctuations of the order field, the so-called coherence length  $\xi$ . In their theory, the Meissner-Ochsenfeld effect was explained by a local mass term of the vector potential, whose size is proportional to  $|\phi|^2$ .

The GL-theory possesses two length scales, the London penetration depth  $\lambda_L$ , and the coherence length  $\xi$ . The competition between the two is ruled by the GL parameter  $\kappa \equiv \lambda_L/\sqrt{2}\xi$ . This serves to distinguish two types of superconductors, type-I with  $\kappa > 1/\sqrt{2}$  and type-II with  $\kappa < 1/\sqrt{2}$ . The second type, possess bundles of vortices which confine magnetic flux in tubes of radius  $\lambda_L$  [10]. In this way, the GL-theory has become what may be called the *Standard Model* of superconductive phenomenology.

A similar Ginzburg-Landau-like scalar field theory with quartic interaction is successful in unifying the weak and the electromagnetic interactions, so that it has become the *Standard Model* of particle physics, also called the *Higgs Model*.

An important new aspect that arises at the transition from the three-dimensional GL-theory to the 3+1-dimensional scalar electrodynamic Higgs model is that the field possess canonical commutation rules. These call for the existence of a particle associated with each field. After all, this is the logic which led to the discovery of pions as the quantum of the forces of nuclear physics. In particle physics, it induced an intensive search for a *Higgs particle* for many years. The recent discovery of a new signal in the 124-126 GeV mass region by the ATLAS and CMS collaborations at the Large Hadron Collider [11, 12] is a hopeful candidate for such a particle.

In this note we want to put this mass value into context with a known fact in superconductivity, that the superconductive phase transition may occur in two different orders: a second order if the GL parameter  $\kappa$  lies deep in the type-II regime, and a first order in the type-I regime. For a long time, this issue was a matter of theoretical controversy after it had been argued by Halperin, Lubensky, and Ma (HLM) [13] that superconductors should really arise in a first-order transition. The issue was finally settled by the calculation of a tricritical point near the dividing line between type-II to type-I superconductivity. The approximate value of  $\kappa$  where this happens was predicted to be  $\kappa_{tr} \approx 0.81$  [14–16], a value later confirmed by Monte Carlo simulations to lie at  $\kappa = 0.76/\sqrt{2} \pm 0.04$  [17]. The important point in the theory was that the mass term of the electromagnetic potential was reliable only as long as it was big, which is the case in the type-I regime. If it is small, the mass is destroyed by fluctuating vortex lines [18, 19]. The precise position of the tricritical point is unknown and should be determined by Monte Carlo simulations as described in [20].

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<sup>1</sup> With the short name “Higgs” we abbreviate collectively all authors that contributed from [1–5].

The calculation of HLM had an interesting parallel in 3+1-dimensional scalar QED, where Coleman and Weinberg<sup>2</sup> calculated that a massless field would acquire a mass from the fluctuations of the electromagnetic field. In the language of superconductive, this implies that scalar QED has a first-order phase transition<sup>3</sup>. After the calculation of the tricritical point in superconductive it was proposed that a similar tricritical point should come up in 3+1-dimensional QED [21]. The Coleman-Weinberg result was derived without considering the fluctuating vortex sheets which are the 3+1-dimensional analogs of the vortex lines in superconductors. These should modify the CW-result in the small  $e^2$  regime. One should therefore expect a tricritical value of  $\kappa$  also in 3+1-dimensional scalar QED, and the present letter gives further support for this expectation with experimental consequences. Moreover, the tricritical point is predicted and interpreted in the Standard Model as the absolute stability boundary of the Higgs potential, by analogy with superconductivity. The latest theoretical predictions on the meta-stability and instability boundaries up to the Planck scale of the Standard Model Higgs potential are discussed in the context of the recent results on the observed signal at the LHC.

#### QUARTIC INTERACTION AND TRICRITICAL POINT

The Ginzburg-Landau theory of superconductivity is characterized by the following energy density:

$$\begin{aligned} \mathcal{H}(\psi, \nabla\psi, \mathbf{A}, \nabla\mathbf{A}) &= \frac{1}{2} (\nabla + ie\mathbf{A}) \psi^* (\nabla - ie\mathbf{A}) \psi \\ &+ \frac{\tau}{2} |\psi|^2 + \frac{g}{4} |\psi|^4 \\ &+ \frac{1}{2} (\nabla \times \mathbf{A})^2, \end{aligned} \quad (1)$$

with the order parameter  $\psi(x) = \rho(x)e^{i\theta(x)}$ , where  $\rho(x)$  and  $\theta(x)$  are real fields. The vector field is represented by  $\mathbf{A}$ ,  $e$  is the electric charge of the Cooper pairs<sup>4</sup>, and the real constants  $\tau$  and  $g$  give the strength of the quadratic and quartic terms, respectively. If the mass parameter  $\tau$  drops below zero, the ground state of the potential,  $V(\psi) = \frac{1}{2}\tau|\psi|^2 + \frac{g}{4}|\psi|^4$ , is obtained for an infinite number of degenerate states satisfying:

$$\langle \psi \rangle^2 = \rho_0^2 = -\frac{\tau}{g}, \quad (2)$$

and corresponds to a second-order phase transition. After the spontaneous symmetry breaking, *i.e.* fixing the gauge to  $\theta(x) = 0$ , the Hamiltonian becomes,

$$\begin{aligned} \mathcal{H}(\psi, \nabla\psi, \mathbf{A}, \nabla\mathbf{A}) &= \frac{1}{2} (\nabla\rho)^2 + V(\rho) + \frac{e^2\rho^2}{2} \mathbf{A}^2 \\ &+ \frac{1}{2} (\nabla \times \mathbf{A})^2. \end{aligned} \quad (3)$$

The mass term of the vector field,  $m_A = e\nu$ , which appeared with the spontaneous symmetry breaking, and the scalar field mass term, can be associated with two characteristic lengths of a superconductor, the London penetration length,  $\lambda_L = 1/m_A = 1/e\rho_0$ , and the coherence length,  $\xi = 1/\sqrt{-2\tau}$ , respectively.

The first-order phase transition can be achieved in the Ginzburg-Landau theory by considering quantum corrections, which in the Thomas-Fermi approximation [22], neglecting fluctuations in  $\rho$ , leads to an additional cubic term in the potential,

$$V(\psi) = \frac{1}{2}\tau^2\rho^2 + \frac{g}{4}\rho^4 - \frac{c}{3}\rho^3, \quad c = \frac{e^3}{2\pi}. \quad (4)$$

As shown in Fig. 1, the cubic term generates a second minimum for  $\tau < c^2/4g$ , at the minimum,

$$\tilde{\rho}_0 = \frac{c}{2g} \left( 1 + \sqrt{1 - \frac{4\tau g}{c^2}} \right). \quad (5)$$

At the specific point  $\tau_1 = 2c^2/9g$ , the minimum lies at the same level as the origin for  $\rho_1 = 2c/3g$ , where the phase transition becomes of first-order (tricritical point). Therefore, in this point, the coherence length of the  $\rho$ -field fluctuations becomes,

$$\xi_1 = \frac{1}{\sqrt{\tau + 3g\rho_1^2 - 2c\rho_1}} = \frac{3}{c} \sqrt{\frac{g}{2}}, \quad (6)$$

which is the same as the fluctuations around  $\rho = 0$ . Finally, the Ginzburg parameter at the tricritical point is,

$$\kappa = \frac{1}{2} \sqrt{\frac{g}{e^2}}. \quad (7)$$

#### TRICRITICAL POINT IN THE COLEMAN-WEINBERG MODEL

The intriguing question now is how this result changes in the four-dimensional version of the Ginzburg-Landau theory, the Coleman-Weinberg model. The effective potential of the Coleman-Weinberg model at one-loop level is [7],

$$V(\phi_c) = \frac{1}{2}m^2\phi_c^2 + \frac{\lambda}{4}\phi_c^4 + \frac{3e^4}{64\pi^2}\phi_c^4 \left( \log \frac{\phi_c^2}{M^2} - \frac{25}{6} \right), \quad (8)$$

<sup>2</sup> Their work was done almost simultaneously with [13] on the same floor at Harvard University.

<sup>3</sup> On a hiking excursion into the mountains near Geneva with Sid Coleman, H.K. once asked him whether this was really what they proved, he said “yes, but we foolishly did not put it that way”.

<sup>4</sup> The Euler number is represented by  $e$ , and shall not be confused with the electric charge  $e$ .

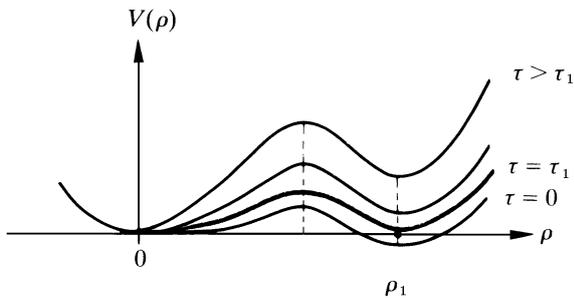


FIG. 1: Potential for the order parameter  $\rho$  with cubic term. A new minimum develops around  $\rho_1$  causing a first-order transition for  $\tau = \tau_1$ .

where the corrected scalar (spin-0) field is represented by  $\phi_c(x)$ , with a mass term  $m^2$ . Here  $\lambda$  gives the strength of the quartic term, and  $M$  is the value of  $\phi_c$  at which the renormalizations are done. Note that we assumed  $\lambda$  to be of the same order of  $e^4$ , and therefore, in the one-loop approximation, the scalar loop diagrams were neglected, since they are of the same order of magnitude as the diagrams with two photon loops. For convenience, a new variable  $\mu$  can be defined as,

$$\frac{\lambda}{4} = \frac{3e^4}{64\pi^2} \left( \log \frac{M^2}{\mu^2} + \frac{11}{3} \right), \quad (9)$$

which turns the effective potential into,

$$V(\phi_c) = \frac{1}{2}m^2\phi_c^2 + \frac{3e^4}{64\pi^2}\phi_c^4 \left( \log \frac{\phi_c^2}{\mu^2} - \frac{1}{2} \right). \quad (10)$$

As described in [7], for a positive  $m^2$ , the effective potential has a maximum and minimum, for  $m^2 < 3e^4\mu^2e^{-1}/16\pi^2$ . In particular, the minimum of the potential lies at the same level as the origin if  $m^2 = 3e^4\mu^2e^{-1/2}/32\pi^2$ , for  $\langle\phi_c\rangle^2 = \mu^2e^{-1/2}$ . The mass of the scalar field in the tricritical point is, therefore,

$$\begin{aligned} m^2(\phi_c) &= \left. \frac{\partial^2 V}{\partial \phi_c^2} \right|_{\phi_c = \langle \phi_c \rangle} \\ &= m^2 + \frac{e^4 \langle \phi_c \rangle^2}{16\pi^2} \left( 6 + 9 \log \frac{\langle \phi_c \rangle^2}{\mu^2} \right) \\ &= \frac{3e^4\mu^2}{16\pi^2} e^{-1/2} = \frac{\lambda}{\alpha} \langle \phi_c \rangle^2, \end{aligned} \quad (11)$$

where  $\alpha = \left( \log \frac{M^2}{\mu^2} + \frac{11}{3} \right)$  gives the size of the renormalization scale. Consequently, at the tricritical point, the Ginzburg parameter becomes,

$$\kappa = \frac{1}{\sqrt{2}} \frac{1/e \langle \phi_c \rangle}{1/m(\phi)} = \frac{1}{\sqrt{2\alpha}} \sqrt{\frac{\lambda}{e^2}}. \quad (12)$$

The result has the same form as the previously obtained 3-dimensional result, and becomes the same with an

appropriate choice of the renormalization scale. Even though these results were computed using only the stability boundary of the corrected quartic potential, without making any use of the dual disorder field theory [14, 22], the position of the tricritical point does not change from 3 to 3+1 dimensions, thus justifying the applicability of the Thomas-Fermi approximation in the tricritical regime.

For the Standard Model Higgs potential, the relation between the boundary of absolute stability and the tricritical point will be discussed in the next section.

## HIGGS BOSON MASS AND VACUUM STABILITY

On 4th July 2012, the CMS and ATLAS experiments announced the discovery of a new boson, compatible with the SM Higgs boson. The observed signal currently lies at  $125.3 \pm 0.4$  (stat.)  $\pm 0.5$  (sys.) GeV (CMS) and  $126 \pm 0.4$  (stat.)  $\pm 0.4$  (sys.) GeV (ATLAS), with global statistical significances of 5.8 sigma (CMS) and 5.9 sigma (ATLAS), and no significant deviations from the predicted SM Higgs boson properties were observed to the present date.

Assuming the Standard Model to be valid up to the Planck scale, the Higgs potential develops a new local minimum for a positive value of the running quartic coupling with the renormalization scale. However, for a negative quartic coupling, the potential becomes unbounded from below and, therefore, unstable. Thus the absolute stability of the electroweak vacuum has its boundary where the quartic coupling flips sign. This feature of the SM Higgs potential corresponds precisely to the previously discussed tricritical point in a quartic interaction, which separates first and second order phase transitions in superconductors. The phenomenology associated with the two physical situations is, of course, quite different. While in superconductivity, the spontaneous symmetry breaking appears as a result of the radiative corrections overcoming the effect of a positive mass term in the Coleman-Weinberg model, the SM Higgs potential is characterized by the existence of two non-zero vacua. Nevertheless, it is clear that the vanishing of the quartic coupling and the degeneracy of the vacuum states correspond to a tricritical behavior in the two scenarios.

The determination of the SM vacuum stability has been studied in detail in the past two decades [25]. The latest and most precise next-to-next-to-leading-order (NNLO) prediction of the absolute stability boundary was established this year by Degrossi *et al.* [26], using two-loop renormalization-group equations, one-loop threshold corrections at the electroweak scale (possibly improved with two-loop terms in the case of pure QCD corrections), and one-loop improved effective potential. Assuming a top quark mass of  $m_t = 173.1 \pm 0.7$  GeV [27–29], and the strong coupling constant at  $\alpha_s(M_Z) = 0.1184 \pm 0.0007$  [30], the absolute stability boundary up

to the Planck scale was predicted for a Higgs boson mass,

$$\begin{aligned}
m_H[\text{GeV}] &= 129.4 \\
&\pm 1.4 \left( \frac{m_t[\text{GeV}] - 173.1}{0.7} \right) \\
&\pm 0.5 \left( \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \\
&\pm 1.0 \text{ (theoretical)}. \tag{13}
\end{aligned}$$

By combining in quadrature the theoretical and experimental uncertainties, the result becomes  $m_H = 129.4 \pm 1.8$  GeV, distanced by roughly 2 sigma from the LHC results. Therefore, one cannot state that the Higgs boson lies precisely on the tricritical point of the electroweak interactions, nor exclude that possibility. The allowed regions, up to 3 sigma, on the top quark and Higgs boson masses measurements, seem to indicate a significant preference for meta-stability of the SM potential when compared with the latest experimental results from the Tevatron and LHC. This tells us there is a non-zero probability of quantum tunneling into the global minimum, lying deeper than the electroweak vacuum. As the new vacuum appears at a very high-energy scale, the probability of tunneling is very small, with a mean lifetime larger than the age of the Universe. Nonetheless, the absolute stability boundary strongly depends on the Higgs boson and top quark masses: slight variations may have dramatic implications. The possible improvement of the precision of these observables at the LHC and in future linear colliders, and further progress on the theoretical understanding of the vacuum stability, may provide further insights into the nature of the Higgs boson mass.

## SUMMARY

In this letter, we argue that the tricritical point, obtained for the three-dimensional Ginzburg-Landau theory with the help of a duality transformation to a disorder version, is not expected to significantly change when analyzed in the context of the 3+1-dimensional Coleman-Weinberg model. This leads us to conclude that the absolute stability boundary of the Higgs potential is a tricritical point of the electroweak interaction, by analogy with superconductivity. The recently obtained result on the NNLO prediction of the absolute stability boundary, up to the Planck scale, at a Higgs boson mass of  $m_H \approx 129.4 \pm 1.8$  GeV, compatible with the observed signals at LHC in the 124-126 GeV mass region, suggests that the electroweak interactions make use of the tricritical behavior as its natural working point. To validate this statement, we must wait for a greater precision of the experimental measurements and theoretical predictions. Finally, and more strategically, this interpretation may enhance the bridge between the physics of elementary particles and superconductivity, that has

led to many important insights since Nambu's pioneering work on the chiral phase transition.

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