Essays on Monetary and Fiscal Policy in the Euro Zone

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Chapter 1

General Introduction

The past century has fundamentally changed the way we think about monetary policy, its tasks and economic consequences. At the beginning of the twentieth century, it was mostly taken for granted that monetary units should be backed by a specific quantity of some precious metal and that the task of central banks was basically to guarantee the convertibility of their currencies in gold. Today we live in a world with so-called fiat money, where the specific value of a currency is no longer determined by an amount of silver or gold reserves, but by the quality of "currency management" of the central bank.

Since the early 1980s the central banks of major industrialized countries have been able to considerably reduce inflation and the variability of inflation. Central banks have committed themselves to more straightforward objectives with respect to price stability, while macroeconomic stabilization has become a secondary task, if any at all. Despite this focus on price stabilization, countries with this type of monetary policy commitment have experienced stable economic growth. Today there is a wide consensus in favor of a monetary policy that is based on clear rules. With the development of a new generation of microfounded models in macroeconomics, the question of how these rules should optimally look has attracted a lot of research in recent years.

The introduction of the euro as the common currency in Europe has transferred the question of optimal monetary policy to the case of a currency union with a single central bank. In this context then a strand of research that has basically developed during the last decade has analyzed the consequences of fiscal policy actions for the conduct of monetary policy. This research has delivered new insights into how fiscal policy affects monetary policy. These insights have stimulated a discussion about the necessity of a rule-based fiscal policy.

The present dissertation was generally motivated by this strand of research. The dissertation consists of four papers, of which each may be read separately. Nonetheless, the four papers are not independent of each other. The general aim of this dissertation is to contribute to a better understanding on how fiscal policy affects monetary policy and what is required in terms of fiscal policy in order to guarantee a continuous success of the European Monetary Union (EMU).

The first paper is a purely theoretical contribution presenting the decisive impulses for the questions analyzed empirically in the remaining three papers of this dissertation. It is investigated, how fiscal policy actions affect the monetary policy of a central bank that is obliged to realize the welfare-maximizing policy plan. It is basically shown, using a simple dynamic game between monetary and fiscal policy, that the optimal behavior of monetary policy is non-trivially determined by fiscal behavior. Further, it is shown that an optimal monetary policy in a currency union gives member states with a larger degree of price rigidities a larger weight in its targeting rule.

In the second paper of this dissertation, we analyze how inflation has been transmitted among the five largest countries in the EMU during the last three decades. The motivation for this paper goes back to the finding of the first paper that optimal monetary policy in a currency union accounts for different degrees of price rigidities across members states. While the previous empirical research in this field has favored univariate approaches in measuring price rigidities and inflation persistence, we utilize a multivariate approach that allows for spillover effects in inflation persistence among the five countries under consideration. As the analysis covers the years from 1970 to 2006, which is quite a long period given the continuous economic integration of European countries. As a result, we allow for regime changes in the transmission process.

The third paper is devoted to the question of monetary and fiscal policy interaction. Using German and Spanish data as an example of two countries within the EMU, which performed very differently in terms of inflation in the recent past, we analyze, if these differences can be traced back to fiscal policy actions. In particular, we analyze, if the fiscal theory of the price level is a relevant mechanism in one of the two countries. In the econometric analysis we utilize a Bayesian approach. Generally, there has been a dramatic increase in the use of Bayesian techniques in econometrics in the past 15 years. While the potential of Bayesian methods have long been recognized, the recent popularity stems in parts from advances in computational power.

Of course, Bayesian methods are not a substitute for classical econometric approaches. Rather they should be seen as a supplement to these. Nonetheless, there are cases, when Bayesian methods seem to be more promising than classical approaches. As we will argue, one of these cases is the analysis of fiscal data, where classical methods exhibit some problems.

Finally, the fourth paper analyzes fiscal policy behavior in Germany and Spain using simple policy rules. It completes and extends the results obtained in the third paper. In particular, we investigate how fiscal policy in the two countries reacts to a rising level of debt. We use a Bayesian Markov-switching approach to find evidence for changes in the underlying regimes.

Chapter 2

Optimal Monetary Policy and Output Stabilization

In this paper we derive optimal monetary targeting rules under various assumptions about fiscal behavior. In contrast to the existing literature we use a linear-quadratic approach and allow for dynamic games between the two policy authorities. It is shown that optimal monetary behavior is non-trivially determined by fiscal behavior. It is argued that output stabilization is generally not a monetary task. Furthermore, the results point out that monetary policy can only be optimal, if fiscal policy is committed to a predetermined targeting rule.

2.1 Introduction

2.1.1 Motivation

15 Member States of the European Union (EU) have so far introduced the euro. These countries currently represent two-thirds of the EU's total population and more than 80 % of its GDP. A further enlargement of the euro area is virtually certain. Four Member States are currently participating in ERM II (European Exchange Rate Mechanism II)[1]. A country must successfully participate for at least two years in ERM II before being allowed to introduce the euro as the formal currency. Slovenia was the first "new" Member State that introduced the euro, Malta and Cyprus followed in January 2008.

The new members, mainly transition economies, differ sharply from the incumbents not only in terms of economic structures and per capita incomes, but also in terms of other variables such as inflation and growth rates. The introduction of the euro in these countries will make the European Monetary Union (EMU) even more heterogeneous, posing a non-trivial challenge to the European Central Bank (ECB).

This paper addresses the question of optimal monetary and fiscal policy in a currency union from the perspective of welfare. We start the analysis by deriving an optimal targeting rule under perfect cooperation of monetary and fiscal policy. It is shown that not only is the economic size of a member state a significant factor in the determination of optimal monetary policy, but so also are different degrees of price rigidities across regions. We then consider dynamic games between monetary and fiscal authorities and focus on the question of optimal monetary policy under various assumptions about fiscal behavior. The analysis points out that output stabilization is generally not a task for a welfare-maximizing monetary policy, as the central bank needs to offset output deviations arising from inefficiencies of fiscal policy.

2.1.2 Literature Review

The early works of Kydland and Prescott (1977) and Barro (1986) showed that monetary policy under discretion is suboptimal, as the resulting equilibrium is subject to inefficiently high inflation

[1] The purpose of ERM II is to link the currencies of the Member States outside the euro area to the euro in order to facilitate convergence between the Member States as a further step towards the adoption of the euro in the corresponding countries. With the participation in ERM II a Member State commits itself to keep its exchange rate fluctuations within a $\pm 15\%$ standard band vis-à-vis the euro.

even if the policy maker, i.e. the central bank, is absolutely benevolent. These results imply that binding the hands of the central bank in advance is necessary for a low inflation equilibrium to be feasible and moreover credible. This is usually done by committing the central bank to a specified policy rule. As argued by Barro (1986), these rules should generally be kept simple so that they become verifiable for the public and hence credible.

Taylor (1993) proposed that within the class of simple policy rules an interest-rate rule that places a positive weight on both real output and inflation is preferable for the central bank in comparison to other rules[2]. Since then these so-called Taylor rules have attracted a lot of attention. The reason for the popularity of these rules may lie in their simple form on the one hand, but also in their benchmark character for monetary policy evaluation on the other hand. But here is precisely where the weak point of Taylor rule specifications lies. Taylor rules are ad hoc in the sense that they are not derived on the basis of any welfare theoretic considerations. That means that they are indeed likely to perform better than other policy rule specifications in practice for the reasons mentioned above, but they do not allow one to make any statements about monetary policy in terms of optimality. Following Svensson (2005) this is true for any instrument rule[3]. The Taylor rule, linking the interest rate linearly to inflation and output, is just the best-known example for an instrument rule.

As a solution to the ad hoc nature of instrument rules recent literature has suggested the application of optimal targeting rules. These targeting rules result from a micro-foundation of central bank behavior. Thereby, the targeting rule is obtained from the first-order condition to the minimization of the central bank's intertemporal loss function.

Besides these weak points of instrument rules, and Taylor rules in particular, the researcher's choice for one or the other in his/her analysis should depend on the question highlighted. In particular, if one wants to model certain kinds of policy behavior, an instrument rule is frequently more appropriate than a targeting rule. Or to put it in the words of McCallum and Nelson (2005b), "if the intention is to work out the effects of a constant money growth rule, then the central bank should be modeled as following a constant money growth rule" instead of a

[2]Taylor showed that U.S. monetary policy behavior in the period 1987-1992 is well described by a policy rule linking nominal interest rates, inflation and real GDP. At the same time the rate of inflation is substantially declining which suggests an improvement of monetary policy compared to earlier periods.

[3]An instrument rule is defined as "a formula setting the central bank's instrument rate as a given function of observable variables" (Svensson, 2005).

targeting rule approach[4]. As our aim in this paper is to derive optimal monetary policy behavior in a currency union under different fiscal scenarios, our analysis must necessarily be based on a targeting rule approach.

In recent years, there have been some attempts to extend the micro-foundation of monetary policy to the case of a currency union or a multi-country framework in general, such as De Paoli (2004), Benigno (2004) and Ferrero (2005). All of these papers show that a pure targeting of the Harmonized Index of Consumer Prices (HICP) is not optimal for a central bank in a currency union, or even more generally for monetary policy coordination on an international level. It is shown that besides the economic size of a member state more fundamental values such as steady state values or the size of price rigidities should determine the importance a country has in the central bank's targeting rule.

The theoretical approach that is used in this paper refers back to Ferrero (2005) and is an extension of Benigno and Woodford (2003). The idea of this approach is to analyze the consequences of alternative policy rules in terms of welfare implications. For this purpose we derive a joint quadratic loss function for both monetary and fiscal policy which arises from a second-order Taylor approximation. Under the assumption that policy has the aim to maximize economy-wide welfare, we seek to minimize this loss function given private sector's optimal behavior. This then gives explicit targeting rules constituting optimal policy behavior.

In contrast to the existing literature this paper focuses on optimal monetary policy behavior under different fiscal policy scenarios by allowing for dynamic games between the two policy authorities. In particular, we try to answer the following questions

1. What constitutes optimal monetary and fiscal policy in a currency union under complete cooperation?

2. Does fiscal behavior have an impact on the behavior of a welfare-maximizing central bank?

3. Should a central bank try to stabilize the output around a certain target in a currency union such as the EMU?

The remainder of this paper is structured as follows. In section 2, we introduce the model of Ferrero (2005). We derive a fully micro-founded welfare criterion for both monetary and fiscal

[4] For a detailed discussion of the pros and cons of targeting and instrument rules, the interested reader is referred to McCallum and Nelson (2005a,b) and Svensson (2005).

policy and sketch the solution of the model. In section 3, we present the results of the model under full cooperation of the authorities. Furthermore, we consider dynamic games between monetary and fiscal policy and analyze, if and how optimal monetary behavior is dependent on fiscal behavior. Finally, section 4 summarizes the results and presents conclusions.

2.2 Model

Throughout this section, we first introduce the model of Ferrero (2005), which is an extension of Benigno and Woodford (2003). This model delivers optimal targeting rules under a joint policy institution. After providing the results for this cooperative solution, we differentiate between the two authorities and assume non-simultaneous decisions of monetary and fiscal policy. We then consider the differences between the optimal solution under cooperation and the optimal solution under this alternative set-up.

2.2.1 Set-Up

For reasons of simplicity we consider a currency union that consists of only two countries[5] H and F. The currency union's population is standardized to one with country H having a population of n and country F a population of $1-n$ with $0 < n < 1$. Furthermore, we assume that the population size is equal to the country's economic weight, which implies that income across households is identical. The private sector is separated in households and firms. The public sector is given by a monetary and fiscal authority. While financial markets are perfect, goods markets are assumed to be characterized by monopolistic competition and price rigidities. Labor markets are segmented, as households offer a specialized labor input for the production of a specific final good.

Households

The lifetime utility of household $j \in [0,1]$ is given by

$$u_0^j = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(C_t^j) - V(l_t^j) \right] \right\}, \tag{2.1}$$

[5] One could also think of H and F being two regions.

where E_t is the expectations operator conditional on information at time t. $U(C_t^j)$ denotes utility from consumption C at time t and $V(l_t^j)$ disutility of labor l at time t. $U(C)$ and $V(l)$ have the usual properties, i.e. $U(C)$ is increasing in C at a decreasing rate and $V(l)$ is convex and increasing in l. $\beta \in [0,1]$ is the intertemporal discount factor.

We assume that the nominal one-period budget constraint of household j is given by

$$P_t C_t^j + E_t \left\{ Q_{t,t+1} D_{t,t+1}^j \right\} = \mu_{i,t}^w w_t^j l_t^j + \Gamma_t^j + D_t^j \quad \text{for } i = \{H, F\}, \tag{2.2}$$

where P_t denotes the aggregate price level, w_t^j is the nominal wage household j obtains for each hour worked and Γ_t^j are profits net of taxation related to the ownership of firms. As previously stated, we assume that the labor market is segmented, as households offer a specialized labor input for the production of a specific final good. Therefore, we introduce an exogenous country-specific wage mark-up denoted by $\mu_{i,t}^w > 1$ for $i = \{H, F\}$, which allows for modeling monopolistic distortions in labor supply. The household trades in a set of one period state-contingent securities[6] that spans all states of nature. $D_{t,t+1}^j$ is a random variable describing the payoff of a portfolio of these state-contingent securities and $Q_{t,t+1}$ is the corresponding price.

The household's maximization problem is then given by lifetime utility (2.1) subject to the budget constraint (2.2). To prevent unsustainable paths of debt[7] from occurring a transversality condition is added to the system of equations, which is assumed to be of the form

$$\lim_{T \to \infty} E_t \left\{ Q_{t,T} D_{t,T}^j \right\} = 0. \tag{2.3}$$

From the household's maximization problem we obtain

$$E_t \left\{ Q_{t,t+1} \right\} = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \frac{U_C(C_{t+1}^j)}{U_C(C_t^j)} \right\}. \tag{2.4}$$

$$\frac{V_l(l_t^j)}{U_C(C_t^j)} = \mu_{i,t}^w \frac{w_t^j}{P_t}. \tag{2.5}$$

The no-arbitrage principle implies that the price of a security equals the reciprocal of the gross nominal interest rate, R_t,

[6] A state-contingent security is a security delivering one unit of numéraire in a given state of nature.

[7] Also referred to as Ponzi schemes.

$$R_t^{-1} = E_t\{Q_{t,t+1}\}. \tag{2.6}$$

To complete the description of the household's behavior, we introduce some additional definitional equations in the following, which describe the aggregate level of the economy and the relationship between countries H and F.

Consumption of household j is defined in terms of a consumption index of the form

$$C_t^j = \left[n^{\frac{1}{\theta}}(C_{H,t}^j)^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}}(C_{F,t}^j)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{2.7}$$

with $\theta > 1$ being the elasticity of substitution between consumption of goods by household j produced in country H, $C_{H,t}^j$, and in country F, $C_{F,t}^j$. This implies that each household consumes goods produced in both countries. As C_t^j, $C_{H,t}^j$ and $C_{F,t}^j$ are again defined as indexes in the form of Dixit-Stiglitz aggregators. This implies a demand structure with a constant elasticity of substitution for the differentiated products that are produced within a country. $C_{H,t}^j$ and $C_{F,t}^j$ then take the form

$$C_{H,t}^j = \left[\left(\frac{1}{n}\right)^{1/\sigma}\int_0^n c_t^j(h)^{\frac{\sigma-1}{\sigma}}dh\right]^{\frac{\sigma}{\sigma-1}} \tag{2.8}$$
$$C_{F,t}^j = \left[\left(\frac{1}{1-n}\right)^{1/\sigma}\int_n^1 c_t^j(f)^{\frac{\sigma-1}{\sigma}}df\right]^{\frac{\sigma}{\sigma-1}},$$

with the elasticity of substitution between goods produced within one country, σ, being larger 1 and $c_t^j(z)$, with $z = \{h, f\}$, denoting household's j consumption of good[8] z. Finally, union wide consumption is given by

$$C_{W,t} = \int_0^1 C_t^j dj. \tag{2.9}$$

For the aggregate price index it then follows from (2.7)

$$P_t = \left[nP_{H,t}^{1-\theta} + (1-n)P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{2.10}$$

[8]Note that h and f describe continuums of goods produced in the corresponding country, in contrast to H and F.

with $P_{H,t}$ and $P_{F,t}$ denoting the aggregate price indexes in countries H and F, which in turn are of the form

$$P_{H,t} = \left[\frac{1}{n}\int_0^n p_t(h)^{1-\sigma}dh\right]^{\frac{1}{\sigma-1}}$$
$$P_{F,t} = \left[\frac{1}{1-n}\int_n^1 p_t(f)^{1-\sigma}df\right]^{\frac{1}{\sigma-1}}. \tag{2.11}$$

$p_t(h)$ represents the price of a good in the interval $[0, n]$ produced in H, and correspondingly $p_t(f)$ the price of a good in the interval $[n, 1]$ produced in F.

Households minimize total consumption expenditures subject to a minimum level of C_t^j. It then can be shown that consumption of good z by household j has the functional form

$$c_t^j(z) = \left[\frac{p_t(z)}{P_{i,t}}\right]^{-\sigma} C_t^j, \tag{2.12}$$

with $p_t(z)$ being the price for z. Since each household consumes a positive amount of any good produced, we obtain for the total consumption of goods produced in country i

$$C_{i,t} = \left[\frac{P_{i,t}}{P_t}\right]^{-\theta} C_{W,t}. \tag{2.13}$$

This implies that consumption of goods produced in country i is determined by overall consumption $C_{W,t}$ and the ratio of domestic prices to overall prices. From this it follows that ceteris paribus an increase in prices in country i will make consumption of goods produced in country i fall.

Total demand for country i is then obtained by taking into account government demand, $G_{i,t}$, leading to the following functional form

$$Y_{i,t} = \left[\frac{P_{i,t}}{P_t}\right]^{-\theta} C_{W,t} + G_{i,t} \tag{2.14}$$

Furthermore, we define the terms of trade as being the ratio of the price indexes in H and F

$$T_t = \frac{P_{F,t}}{P_{H,t}}. \tag{2.15}$$

It is now possible to make up a direct relation between prices in country H and the terms of trade by starting with the aggregate price level (2.10). After some minor rearrangements the price index (2.10) may be written as

$$\left[\frac{P_{H,t}}{P_t}\right]^{\theta-1} = n + (1-n)T_t^{1-\theta}. \tag{2.16}$$

Correspondingly, we obtain for country F

$$\left[\frac{P_{F,t}}{P_t}\right]^{\theta-1} = nT_t^{\theta-1} + (1-n). \tag{2.17}$$

Firms

Similarly to the households, we assume a continuum of firms for countries H and F, where we assume $h \in [0,n]$ to be the number of firms located in H, and $f \in [n,1]$ being the number of firms producing in F.

Each firm produces a single-differentiated consumption good at time t denoted by $y_t(z)$. The production technology is assumed to be of the general form

$$y_t(z) = a_{i,t} l_t(z), \tag{2.18}$$

with $a_{i,t}$ being an exogenous technology shock taking place in country i at time t and $l_t(z)$ the labor input to produce $y_t(z)$. As mentioned before, labor markets are segmented, and we assume labor to be immobile across countries[9].

Firms maximize their profits, given wages set by the households according to (2.5) and taxes $\tau_{i,t}$ set by fiscal authorities. We assume prices to be sticky, which is implemented in the model by Calvo-type price stickiness. That means that each period a firm can change its price with a probability of $1-\alpha_i$, where i emphasizes that this probability may differ across countries. The firms' maximization problem in the event of a price change at time t is then given by net discounted profits

$$\max_{p_t(z)} E_t \left\{ \sum_{T=t}^{\infty} \alpha_i^{T-t} Q_{t,T} \left[(1-\tau_{i,T})p_t(z)y_{t,T}(z) - w_T(z)l_T(z)\right] \right\}, \tag{2.19}$$

[9]For the EU this should be a reasonable assumption. Cf. Nickell (1997).

subject to

$$y_{t,T}(z) = \left[\frac{p_t(z)}{P_{i,T}}\right]^{-\sigma} Y_{i,T} \quad \text{with } T \geq t,$$

with $y_{t,T}(z)$ being the demand of good z at time T conditional on no further price change after period t.

Making use of the production function (2.18), we are now able to build up a direct relationship between wages, as given by equation (2.5), and disutility of labor, as assumed in the utility function (2.1). This relationship will turn out to be very useful during the further analysis of the firms' price-setting behavior. We start by defining disutility of labor in terms of inverse production $y_t(z)$, i.e. we solve (2.18) for l_t and take the resulting expression to the power of $1+\eta$, where η is the inverse of the Frisch elasticity of labor supply, which is a measure for the sensitivity of labor supply to real wages. Furthermore, we divide both sides by $1+\eta$ for reasons of simplicity. Disutility of labor is then given by

$$V(l_t) \equiv V(y_t(z), a_{i,t}) = a_{i,t}^{-(1+\eta)} \frac{(y_t(z))^{1+\eta}}{1+\eta}. \tag{2.20}$$

Taking the derivative with respect to $y_t(z)$ and then using the demand function (2.12) to substitute for $y_t(z)$, yields the marginal disutility of labor

$$V_y(y_t(z), a_{i,t}) = \left[\frac{p_t(z)}{P_{i,t}}\right]^{-\sigma\eta} V_y(Y_{i,t}, a_{i,t}). \tag{2.21}$$

This result can be used to substitute for $V_l(l_t^j)$ in the wage-setting equation (2.5), which then changes to

$$\frac{w_t(z)}{P_t} = \mu_{i,t}^w \frac{a_{i,t} V_y(Y_{i,t}, a_{i,t})}{U_C(C_{W,t})}. \tag{2.22}$$

One can show that the first order condition of the firms can be rearranged to

$$\frac{p_t(z)}{P_{i,t}} = \left(\frac{K_{i,t}}{F_{i,t}}\right)^{\frac{1}{1+\sigma\eta}}, \tag{2.23}$$

where F_t and K_t are functions of current aggregate output, current exogenous variables, and the expected future evolution of prices, output, and disturbances, defined as

$$
\begin{aligned}
K_{i,t} &= E_t \left\{ \sum_{T=t}^{\infty} (\alpha_i \beta)^{T-t} k_{i,t} \left(\frac{P_{i,T}}{P_{i,t}} \right)^{\sigma(1+\eta)} \right\} \\
k_{i,T} &= \left(\frac{\sigma}{\sigma - 1} \right) \mu^w_{i,T} V_y(Y_{i,T}, a_{i,T}) Y_{i,T} \\
F_{i,t} &= E_t \left\{ \sum_{T=t}^{\infty} (\alpha_i \beta)^{T-t} (1 - \tau_{i,T}) \frac{P_{i,T}}{P_T} f_{i,T} \left(\frac{P_{i,T}}{P_{i,t}} \right)^{\sigma-1} \right\} \\
f_{i,T} &= U_C(C_{W,T}) Y_{i,T}.
\end{aligned}
$$

Furthermore, we define the rate of inflation for country i as

$$
\pi_{i,t} = \frac{P_{i,t}}{P_{i,t-1}}. \tag{2.24}
$$

Rearranging the country-based price indexes (2.11) yields

$$
\begin{aligned}
P_{H,t}^{1-\sigma} &= \frac{1}{n} \int_0^n p_t(h)^{1-\sigma} dh \\
P_{F,t}^{1-\sigma} &= \frac{1}{1-n} \int_n^1 p_t(f)^{1-\sigma} df.
\end{aligned}
$$

From Calvo prices it follows

$$
P_{i,t}^{1-\sigma} = \alpha_i P_{i,t-1}^{1-\sigma} + (1 - \alpha_i) p_t(z)^{1-\sigma}. \tag{2.25}
$$

Dividing both sides by $P_{i,t}^{1-\sigma}$ and making use of (2.24), we obtain

$$
1 = \alpha_i \pi_{i,t}^{-(1-\sigma)} + (1 - \alpha_i) \left(\frac{p_t(z)}{P_{i,t}} \right)^{1-\sigma}. \tag{2.26}
$$

Finally, substituting the second term on the right-hand side by (2.23), we end up with the following functional form after some minor rearrangements

$$
\left[\frac{1 - \alpha_i \pi_{i,t}^{-(1-\sigma)}}{1 - \alpha_i} \right]^{\frac{1+\sigma\eta}{\sigma-1}} = \frac{F_{i,t}}{K_{i,t}}. \tag{2.27}
$$

As both $F_{i,t}$ and $K_{i,t}$ depend on future expected prices and output and thus on future expected rates of inflation, (2.27) describes a form of expectations-augmented Phillips curve.

Policy Authorities

Economic policy is assumed to be described by a monetary and fiscal policy authority, which differ in their policy instruments. While the monetary policy instrument is the nominal interest rate, R_t, fiscal policy actions are determined by choosing the mix between taxes and one-period nominal risk-free bonds to finance its expenditures, which are assumed to follow an exogenous process.

As the model does not include any monetary frictions, seigniorage is not a source of national government revenues. Furthermore, monetary policy has a direct impact on fiscal variables, as it determines the net present value of public debt by setting the interest rate.

Fiscal authorities in country H are subject to the flow budget constraint

$$nB_{H,t} = R_{t-1}nB_{H,t-1} - \int_0^n p_t(h)\left[\tau_{H,t}y_t(h) - g_t(h)\right]dh, \tag{2.28}$$

where $B_{H,t}$ denotes nominal public debt of country H at time t in per-capita terms and $g_t(h)$ per-capita government expenditures, which are assumed to be driven by an exogenous process. Similarly, the government budget constraint for country F is given by

$$(1-n)B_{F,t} = R_{t-1}(1-n)B_{F,t-1} - \int_n^1 p_t(f)\left[\tau_{F,t}y_t(f) - g_t(f)\right]df. \tag{2.29}$$

Furthermore, we define real surpluses of country i as

$$s_{i,t} \equiv \frac{P_{i,t}}{P_t}\left(\tau_{i,t}Y_{i,t} - G_{i,t}\right). \tag{2.30}$$

Using this definition, it is obvious that the integral in (2.28) and (2.29) is equivalent to the nominal surplus of the government. Thus, we can easily simplify these two equations using the definition of real surpluses. This yields for country i

$$B_{i,t} = R_{t-1}B_{i,t-1} - P_t s_{i,t} \tag{2.31}$$

$$\tag{2.32}$$

The consolidated government budget constraint is then given by

$$nB_{H,t} + (1-n)B_{F,t} = R_{t-1}(nB_{H,t-1} + (1-n)B_{F,t-1}) - P_t(ns_{H,t} + (1-n)s_{F,t}), \qquad (2.33)$$

where consolidated refers to the union-wide budget constraint for the fiscal authorities. The corresponding transversality condition to (2.33) would therefore be given by

$$\lim_{T\to\infty} E_t \left\{ Q_{t,T}[nB_{H,t} + (1-n)B_{F,t}] \right\} = 0. \qquad (2.34)$$

In the following we will use this transversality condition on a union-wide level for government authorities instead of a country-based level. This means that we are generally ruling out Ponzi-games on the union-wide level. The motivation for using a union-wide transversality condition goes back to the assumption that financial markets clear on an international level but not necessarily on a country level, as financial markets are highly integrated.

Iterating (2.31) one period ahead and substituting R_t^{-1} by $Q_{t,t+1}$, the consolidated budget constraint (2.33) may be rewritten as

$$\begin{aligned} & R_{t-1}(nB_{H,t-1} + (1-n)B_{F,t-1}) \\ = \;& n\left(E_0 \left\{ Q_{t,t+1}(B_{H,t+1} + P_{t+1}s_{H,t+1}) \right\} + P_t s_{H,t}\right) + (1-n)\left(E_t \left\{ Q_{t,t+1}(B_{F,t+1} + P_{t+1}s_{F,t+1}) \right\} + P_t s_{F,t}\right). \end{aligned}$$

Proceeding this iteration procedure infinitively often and applying the transversality condition (2.34) yields

$$R_{t-1}(nB_{H,t-1} + (1-n)B_{F,t-1}) = E_t \left\{ \sum_{t=T}^{\infty} Q_{t,T} P_t \left[ns_{H,t} + (1-n)s_{F,t} \right] \right\}, \qquad (2.35)$$

which is the intertemporal budget constraint of the government on a union level. This condition has to be fulfilled at any point of time by fiscal authorities. As it is written in nominal terms, changes on the left-hand side, i.e. in period $t-1$, can be compensated either by adjustments in the sequence of surpluses or by changes of the price level or the discount factor Q. The latter case is usually referred to as non-Ricardian fiscal policy.

Finally, if one uses the Euler equation (2.4) to substitute for Q in (2.35) and applies the definition of inflation, one ends up with the following functional form

$$\frac{U_C(C_{W,t})}{\pi_t}\left[nb_{H,t-1}+(1-n)b_{F,t-1}\right]=E_t\left\{\sum_{t=T}^{\infty}\beta^{T-t}U_C(C_{W,t})\left[ns_{H,t}+(1-n)s_{F,t}\right]\right\}, \qquad (2.36)$$

where $b_{i,t-1}$ denotes real public debt of country i in per-capita terms, which we defined as $b_{H,t-1}=(R_{t-1}B_{H,t-1})/P_{t-1}$.

2.2.2 Equilibrium

After having specified the sectors of the economies in the two countries, we now turn to the conditions describing the equilibrium. In doing so, it will be particularly useful to define an index of price dispersion for each of the two countries. Thereby, price dispersion measures the distribution of prices across sellers and hence can be viewed as a measure of market inefficiency. Following Canzoneri, Cumby and Diba (2005), we define the index of price dispersion as

$$\begin{aligned}\Xi_{H,t} &\equiv \frac{1}{n}\int_0^n\left[\frac{p_t(h)}{P_{H,t}}\right]^{-\sigma(1+\eta)}dh\\ \Xi_{F,t} &\equiv \frac{1}{1-n}\int_n^1\left[\frac{p_t(f)}{P_{F,t}}\right]^{-\sigma(1+\eta)}df. \qquad (2.37)\end{aligned}$$

Similarly, we summarize the ratio of $P_{i,t}$ and P_t as the relative price in country i denoted by $\wp_{i,t}$. In the following we derive the equations describing the equilibrium. We then group the equations according to their origin in the model's set-up as conditions belonging to

- The aggregate demand block
- The supply side
- Policy authorities
- Definitional equations.

Aggregate Demand Block

We start with the first-order conditions of the households. The Euler equation (2.4) may be rearranged to

$$1 = \beta R_t E_t \left\{ \frac{1}{\pi_{t+1}} \frac{U_C(C_{W,t+1})}{U_C(C_{W,t})} \right\}. \tag{2.38}$$

Aggregate economic activity of country i, as given in (2.14), reduces to

$$Y_{i,t} = \wp_{i,t}^{-\theta} C_{W,t} + G_{i,t}. \tag{2.39}$$

Finally, applying the definition of relative prices to (2.16) and (2.17) yields

$$\begin{aligned} \wp_{H,t}^{\theta-1} &= n + (1-n) T_t^{1-\theta} \\ \wp_{F,t}^{\theta-1} &= n T_t^{\theta-1} + (1-n) \end{aligned} \tag{2.40}$$

Equations (2.38) to (2.40) are the equilibrium conditions describing the aggregate demand block of the economy.

Aggregate Supply Block

For the derivation of the equations constituting the behavior of the supply side in equilibrium, we start with the index of price dispersion for country i as defined in (2.37). We then exploit the implications resulting from Calvo prices, i.e. all $1-\alpha_i$ firms that get to set a new price in period t will actually choose the same, namely the optimal one. Therefore, we can drop the integral in the index of price dispersion in period t so that we may write

$$\begin{aligned} \Xi_{H,t} &= (1-\alpha_H) \left[\frac{p_t(h)}{P_{H,t}} \right]^{-\sigma(1+\eta)} + \alpha_H \left\{ \frac{1}{n} \int_0^n \left[\frac{p_{t-1}(h)}{P_{H,t}} \right]^{-\sigma(1+\eta)} dh \right\} \\ \Xi_{F,t} &= (1-\alpha_F) \left[\frac{p_t(f)}{P_{F,t}} \right]^{-\sigma(1+\eta)} + \alpha_F \left\{ \frac{1}{1-n} \int_n^1 \left[\frac{p_{t-1}(f)}{P_{F,t}} \right]^{-\sigma(1+\eta)} df \right\}. \end{aligned} \tag{2.41}$$

Expanding the term inside the integral by $P_{i,t-1}$ allows us to express price dispersion in period t in terms of inflation and its own lagged value

$$\Xi_{i,t} = (1-\alpha_i) \left[\frac{p_t(i)}{P_{i,t}} \right]^{-\sigma(1+\eta)} + \alpha_i \Xi_{i,t-1} \pi_{i,t}^{\sigma(1+\eta)}. \tag{2.42}$$

Next we combine the first-order condition of the firms (2.23) with the Phillips curve (2.27) to obtain

$$\left(\frac{p_t(i)}{P_{i,t}}\right)^{-\sigma(1+\eta)} = \left(\frac{1-\alpha_i\pi_{i,t}^{\sigma-1}}{1-\alpha_i}\right)^{\frac{\sigma(1+\eta)}{\sigma-1}}. \tag{2.43}$$

Combining these two equations $\Xi_{i,t}$ takes the form

$$\Xi_{i,t} = (1-\alpha_i)\left(\frac{1-\alpha_i\pi_{i,t}^{\sigma-1}}{1-\alpha_i}\right)^{\frac{\sigma(1+\eta)}{\sigma-1}} + \alpha_i\Xi_{i,t-1}\pi_{i,t}^{\sigma(1+\eta)}, \tag{2.44}$$

which is the first equation for the supply side of the economy. Furthermore, we need to incorporate the Phillips curve (2.27), as it links inflation to future prices and output. The Phillips curve relationship then completes the description of the supply side.

Policy Authorities

The equilibrium path for government actions is generally given by the government budget constraint for country i subject to the transversality condition on the consolidated level. After some minor rearrangements we may write

$$\frac{U_C(C_{W,t})}{\pi_t}b_{i,t-1} = U_C(C_{W,t})\wp_{i,t}(\tau_{i,t}Y_{i,t} - G_{i,t}) + \beta E_t\left\{\frac{U_C(C_{W,t+1})}{\pi_t+1}b_{i,t}\right\}, \tag{2.45}$$

where we reduced the constraint to a two-periods equation, exploiting the fact that $b_{i,t}$ may be used to summarize the terms for all future points of time. As already mentioned before, fiscal policy is constrained in the long run by the transversality condition on the consolidated level (2.34).

As stated in the beginning, policy is described by both a fiscal and a monetary authority. So-far, we have neither specified a fiscal nor a monetary policy rule. What we have specified are accounting identities for fiscal policy. Furthermore, we have assumed that government expenditures are driven by exogenous shocks. It should not be surprising that rules or further patterns for the behavior of the public sector are not determined a priori, as the aim of this section is to derive rules which turn out to be optimal for policy behavior in a currency union with a focus on optimality in terms of monetary policy.

Definitional Equations

To complete the description of the competitive equilibrium, we need to take into account some of the definitional equations given before.

Since the above equations refer to the terms of trade, we need to incorporate its definition as a further equilibrium condition. For the analysis it will be more useful to consider the ratio of terms of trade in period t and $t-1$, as it enables us to give a direct relationship between terms of trade and the rate of inflation

$$\frac{T_t}{T_{t-1}} = \frac{\pi_{F,t}}{\pi_{H,t}}. \tag{2.46}$$

Finally, from the definition of the price level in (2.10) it follows

$$\pi_t^{1-\theta} = n(\pi_{H,t}\wp_{H,t-1})^{1-\theta} + (1-n)(\pi_{F,t}\wp_{F,t-1})^{1-\theta}, \tag{2.47}$$

which is the last equation required to determine the equilibrium path of the economy.

Besides the equations given above, the variables $a_{i,t}$, $\mu_{i,t}^w$ and $G_{i,t}$ are assumed to follow exogenous processes so that any dynamics in the system off the steady state path would be induced by fundamental shocks in one of these variables.

2.2.3 Deriving the Welfare Criterion

We now solve the model in such a way that it allows one to obtain targeting rules for both monetary and fiscal policy, which may be considered as being optimal from a welfare-theoretic point of view. This is basically done with the help of a linear-quadratic (LQ) approximation introduced by Benigno and Woodford (2003). As this exercise involves a lot of algebra, we stick to the major steps in the derivation to impart the idea and intuition behind the LQ approximation. The LQ approach allows for a microfoundation of policy analysis with the aim of policy being the maximization of economy-wide welfare. This means that the primary objective of the central bank is not to fight inflation, but to maximize the welfare within the currency union. Fighting or targeting inflation then becomes only a criterion that may be derived from the policy authorities' minimization problem.

We start with welfare in per-capita terms given by the difference between utility from consumption and disutility of labor, i.e. for the two countries separately in infinite horizons

$$u_{H,0} \equiv E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(C_{W,t}) - \frac{1}{n} \int_0^n V(y_t(h), a_{H,t}) dh \right] \right\} \tag{2.48}$$

$$u_{F,0} \equiv E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(C_{W,t}) - \frac{1}{1-n} \int_n^1 V(y_t(f), a_{F,t}) df \right] \right\} \tag{2.49}$$

Union-wide welfare is then given by

$$u_{W,0} \equiv n u_{H,0} + (1-n) u_{F,0}. \tag{2.50}$$

As mentioned before, under perfect cooperation the central bank as well as the fiscal authorities are obliged to maximize $u_{W,0}$ subject to the equilibrium conditions as given in the previous section. The idea of the LQ approximation is to derive a quadratic objective function for the policy authorities. This is done by approximating (2.50) around its steady state. This yields an objective function that is usually referred to as the quadratic loss function. It expresses the stabilization objective of policy authorities in terms of the key variables such as inflation and output. Hence, the LQ approach may be seen as a way to bring microfoundation into policy analysis, whereby the loss function provides the objective of policy authorities, analogously to the utility function in the analysis of the households.

For the model given, one can show that the joint loss function is given by[10]

$$\begin{aligned} u_{W,0} = -\frac{1}{2} U_C \bar{C} E_0 \Bigg\{ \sum_{t=0}^{\infty} \beta^t \Bigg[& \lambda_y (\hat{Y}_{W,t} - \tilde{Y}_{W,t})^2 + n(1-n)\lambda_q (\hat{T}_t - \tilde{T}_t)^2 \\ & + n\lambda_{\pi,H} \hat{\pi}_{H,t}^2 + (1-n)\lambda_{\pi,F} \hat{\pi}_{F,t}^2 \Bigg] \Bigg\} + t.i.p. + J_{W,0} + \circ(\|\hat{\varepsilon}_t\|^3), \end{aligned} \tag{2.51}$$

where variables with a hat denote steady state deviations and variables with a tilde target values. The first two terms represent deviations of the corresponding variable from its target value[11]. $t.i.p.$ denotes variables that are totally independent of policy actions. $J_{W,0}$ summarizes terms

[10] Further details of the derivation may be found in the appendix.

[11] Note that the (theoretical) target value for inflation is zero and hence not separately given. It can be shown that a rate of inflation of zero is welfare maximizing.

that are given in period 0 and therefore are exogenous and unaffected by policy actions[12]. An important result one obtains from (2.51) is the identification of the properties of the weights in front of inflation $\lambda_{\pi,H}$ and $\lambda_{\pi,F}$. All $\lambda-$values entering (2.51) are defined as linear combinations of the structural parameters. Thereby, $\lambda_{\pi,H}$ and $\lambda_{\pi,F}$ substantially depend on price rigidities in the two countries, i.e. the size of the parameters α_H and α_F. One can show that with increasing price rigidities, the weight for inflation in country i also increases. In mathematical terms we may say

$$\frac{\partial \lambda_{\pi,i}}{\partial \alpha_i} > 0.$$

This means that, in addition to the economic size of a country, price rigidities play a significant role in determining the weights a country should have in the objective function of policy authorities. Only in cases, when the countries are characterized by equal price rigidities, is the weight in the objective function solely determined by the country's economic size, as $\lambda_{\pi,H}$ and $\lambda_{\pi,F}$ cancel out.

The complete policy problem under cooperation of the two policy authorities is finally given by the loss function (2.51), which is minimized over all endogenous variables, including the fiscal variables of debt and taxes, subject to the log-linearized equilibrium conditions. These can be reduced to a set of six equations by appropriate substitutions

$$\begin{aligned}
\hat{\pi}_{H,t} &= \kappa_H[\delta_y y_t + \omega_\tau(\hat{\tau}_{H,t} - \tilde{\tau}_{H,t}) + (1-n)\delta_q q_t] + \beta E_t \hat{\pi}_{H,t+1} && (2.52)\\
\hat{\pi}_{F,t} &= \kappa_F[\delta_y y_t + \omega_\tau(\hat{\tau}_{F,t} - \tilde{\tau}_{F,t}) - n\delta_q q_t] + \beta E_t \hat{\pi}_{F,t+1} && (2.53)\\
\hat{b}_{H,t-1} - \rho s_c^{-1} y_t - \hat{\pi}_t + \psi_{H,t} &= (1-\beta)[b_y y_t + (1-n) b_q q_t + (1+\omega_g)(\hat{\tau}_{H,t} - \tilde{\tau}_{H,t})] && (2.54)\\
&\quad + \beta E_t(\hat{b}_{H,t} - \rho s_c^{-1} y_{t+1} - \hat{\pi}_{t+1} + \psi_{H,t+1}) &&\\
\hat{b}_{F,t-1} - \rho s_c^{-1} y_t - \hat{\pi}_t + \psi_{F,t} &= (1-\beta)[b_y y_t - n b_q q_t + (1+\omega_g)(\hat{\tau}_{F,t} - \tilde{\tau}_{F,t})] && (2.55)\\
&\quad + \beta E_t(\hat{b}_{F,t} - \rho s_c^{-1} y_{t+1} - \hat{\pi}_{t+1} + \psi_{F,t+1}) &&\\
\hat{\pi}_t &= n\hat{\pi}_{H,t} + (1-n)\hat{\pi}_{F,t} && (2.56)\\
q_t &= q_{t-1} + \hat{\pi}_{F,t} - \hat{\pi}_{H,t} - \Delta \tilde{T}_t, && (2.57)
\end{aligned}$$

[12] These terms are summarized under $J_{W,0}$ and not *t.i.p.* to emphasize that the exogeneity is induced by the fact that they are given a priori.

where

$$\begin{aligned}
\delta_y &= \eta + \rho s_c^{-1} \\
\delta_q &= 1 + \eta\theta s_c^{-1} \\
\omega_\tau &= \frac{\bar{\tau}}{1-\bar{\tau}} \\
\omega_g &= \frac{\bar{G}}{\bar{s}} \\
b_y &= (1+\omega_g) - \rho s_c^{-1} \\
b_q &= (1+\omega_g)\theta s_c^{-1} - 1 \\
y_t &= \hat{Y}_{W,t} - \tilde{Y}_{W,t} \\
q_t &= \hat{T}_t - \tilde{T}_t.
\end{aligned}$$

$\psi_{i,t}$ summarizes further terms in target values and exogenous government expenditures and can be thought of as an indicator for the exogenous environment in which policy decisions take place. We, therefore, refer to $\psi_{i,t}$ as fiscal stress in country i.

In the following we will provide the results in form of the targeting rules obtained from this policy problem. We will start with the cooperative solution, before considering dynamic games between the two policy authorities.

2.3 Results

2.3.1 Optimal Policy Behavior under Cooperation

Proposition 1. *Under perfect cooperation optimal policy behavior is described by flexible inflation targeting.*

Under complete cooperation of monetary and fiscal policy the solution of the joint policy problem satisfies the following three conditions

$$n\lambda_{\pi,H}\hat{\pi}_{H,t} + (1-n)\lambda_{\pi,F}\hat{\pi}_{F,t} = 0 \tag{2.58}$$

$$\lambda_y \Delta y_t = 0 \tag{2.59}$$

$$\begin{aligned} 2n(1-n)\lambda_q \Delta q_t = {} & \beta[(1-n)\lambda_{\pi,F} E_t \Delta\hat{\pi}_{F,t+1} - n\lambda_{\pi,H} E_t \Delta\hat{\pi}_{H,t+1}] \\ & -[(1-n)\lambda_{\pi,F}\Delta\hat{\pi}_{F,t} - n\lambda_{\pi,H}\Delta\hat{\pi}_{H,t}] \end{aligned} \tag{2.60}$$

with Δ denoting the first difference of the corresponding variable. (2.58)-(2.60) may be interpreted as the optimal targeting rules from a central planner's perspective. (2.58) and (2.59) perfectly conform to targeting rules reported in standard text books[13]. At first glance, (2.58) seems to contradict traditional macroeconomic literature, as it does not require one to target expected future inflation directly. But one should keep in mind that $\hat{\pi}_{H,t}$ and $\hat{\pi}_{F,t}$ themselves are processes subject to the Phillips curve (2.27), which again involves expected future rates of inflation. Therefore, (2.58) does incorporate future inflation indirectly via the Phillips curve condition. In accordance with other literature such as Benigno (2004) or Benigno and Woodford (2003), the weights for the country-specific inflation rates are substantially determined by the degree of price rigidities in H and F, as $\lambda_{\pi,i}$ is increasing in α_i. (2.59) describes the optimal policy plan with respect to output deviations. And finally, (2.60) is the optimal targeting rule for deviations of the terms of trade from their targeted values.

In the absence of terms of trade deviations, (2.58) and (2.59) can be interpreted jointly in an additive relationship

$$n\lambda_{\pi,H}\hat{\pi}_{H,t} + (1-n)\lambda_{\pi,F}\hat{\pi}_{F,t} + \lambda_y(y_t - y_{t-1}) = 0, \tag{2.61}$$

which allows for a more flexible interpretation of the two targeting rules. To interpret (2.61), let us suppose a negative technology shock in period t, which pushes output below its targeted value so that $y_t < 0$. This then leads to a negative value for $y_t - y_{t-1}$, if the economy was in a steady state in period $t-1$. To restore equality of (2.61), inflation in the two countries needs to move upwards, which in turn stimulates output in the next period via the Phillips curve relationship. But from period $t+1$ onwards $y_t - y_{t-1}$ becomes positive, as the output gap is gradually closing and moving back towards zero. As for the entire period of this catch-up, the difference between

[13] E.g. Woodford (2003), p. 523.

y_t and y_{t-1} remains positive, inflation has to stay below its targeted value. This implies that the output deviation from the targeted value should be reduced gradually.

(2.60) has a less obvious interpretation. For this purpose let us assume that there occurs a positive deviation of the terms of trade from their targeted value in period t. This implies then a positive value for both q_t and Δq_t, if the economy was in a steady state in $t-1$. By definition of the terms of trade a positive q_t necessarily implies a positive rate of inflation in country F at time t[14]. This yields a positive value for $\Delta\hat{\pi}_{F,t}$. As $\Delta\hat{\pi}_{F,t}$ enters the equation with a negative sign on the right-hand side, it is not able to compensate for the increase on the left-hand side of (2.60). Furthermore, we may say that after the increase in q_t has occurred, it requires $E_t\Delta\hat{\pi}_{F,t+1}$ to become negative, as otherwise we would consider an unstable solution. But this decrease in $E_t\Delta\hat{\pi}_{F,t+1}$ does also not contribute to reducing the imbalance between the two sides of (2.60). Therefore, equality in (2.60) can only be restored by an increase in $\Delta\hat{\pi}_{H,t}$ combined with a decrease in $E_t\Delta\hat{\pi}_{H,t+1}$. Hence, we may conclude from (2.60) that the central bank should not reduce deviations in the terms of trade between H and F by focusing only on one or the other country. Instead, both countries should go through the same development. In this sense then, the central bank should seek to synchronize the run of inflation rates across regions.

In the following, we will give up the assumption that there is a social planner, who has perfect control over both monetary and fiscal policy. Instead we allow for non-cooperation between the policy authorities with simultaneous and non-simultaneous decisions. This gives us the possibility to study the implications of different fiscal policy scenarios on optimal monetary policy behavior. Thereby, the joint optimality condition (2.61) will serve as a benchmark for all other scenarios.

2.3.2 Optimal Policy Behavior under Non-Cooperation

We now want to consider the case in which monetary and fiscal policy seek to maximize welfare separately. This requires additional assumptions about policy behavior in the sense that we need to define which endogenous variables each policy authority seeks to control or is able to control. In particular, we assume that monetary policy solves the minimization problem in $\hat{\pi}_t$, $\hat{\pi}_{H,t}$, $\hat{\pi}_{F,t}$ and q_t, which reflects the central bank's responsibility for price stability. Fiscal policy is assumed to solve the minimization problem in its endogenous instruments $\hat{\tau}_{H,t}$, $\hat{\tau}_{F,t}$, $\hat{b}_{H,t}$ and $\hat{b}_{F,t}$. The

[14] Alternatively, one could consider the case of a negative rate of inflation in country H.

only endogenous variable remaining is output, $\hat{Y}_{W,t}$.

Proposition 2. *Both monetary and fiscal policy are perfectly able to realize any path for* $\hat{Y}_{W,t}$.

By appropriate substitutions one can show that the expected change in output steady state deviations is given in terms of policy variables by

$$E_t\Delta\hat{Y}_{H,t+1} = -\rho^{-1}s_c(1-\beta)E_t\hat{R}_{t+1} + E_t\Delta\hat{G}_{H,t+1} - \rho^{-1}s_c\hat{R}_t \tag{2.62}$$

$$E_t\Delta\hat{Y}_{F,t+1} = -\rho^{-1}s_c(1-\beta)E_t\hat{R}_{t+1} + E_t\Delta\hat{G}_{F,t+1} - \rho^{-1}s_c\hat{R}_t, \tag{2.63}$$

which implies then

$$E_t\Delta\hat{Y}_{W,t+1} = -\rho^{-1}s_c(1-\beta)E_t\hat{R}_{t+1} + nE_t\Delta\hat{G}_{H,t+1} + (1-n)E_t\Delta\hat{G}_{F,t+1} - \rho^{-1}s_c\hat{R}_t. \tag{2.64}$$

From (2.64) it follows that both monetary and fiscal policy have perfect control over changes in output. That means that both are able to realize any path for $\hat{Y}_{W,t}$ using their instruments. From this it follows that output stabilization can be either a monetary or a fiscal task.

Simultaneous Policy Decisions

We assume that monetary policy maximizes social welfare given the path of endogenous fiscal instruments $\hat{\tau}_{i,t}$ and $\hat{b}_{i,t}$, while fiscal policy maximizes welfare given the path of inflation rates both on the union and country level and the terms of trade. Furthermore, we let fiscal policy maximize welfare with respect to output deviations.
In a first step, we assume that neither monetary nor fiscal policy can anticipate what the other policy authority does. This means that both policy institutions decide simultaneously.

Proposition 3. *Under non-cooperation of policy institutions the optimal monetary policy plan is given by strict inflation targeting.*

The resulting optimal monetary and fiscal targeting rules are given by

$$n\lambda_{\pi,H}\hat{\pi}_{H,t} + (1-n)\lambda_{\pi,F}\hat{\pi}_{F,t} = 0 \tag{2.65}$$

$$\lambda_y(y_t - y_{t-1}) = 0, \tag{2.66}$$

implying that the trade-off between inflation and output deviations disappears. The reason for this is quite simple. Each policy authority has an externality on the other one. While in the cooperative solution these externalities are incorporated, the optimal non-cooperative solution requires that these externalities do not occur.

Non-Simultaneuous Policy Decisions

In practice, fiscal policy basically is set on an annual basis. Monetary policy, however, is more frequently set, usually on a monthly basis. This allows monetary policy to anticipate fiscal actions in its decisions. This implies that a welfare-maximizing monetary authority will incorporate the fiscal policy plan implemented in period t. As before we assume that both monetary and fiscal policy seek solely to maximize union-wide welfare with respect to the same variables.

Proposition 4. *With monetary policy anticipating fiscal behavior, the optimal solution is equivalent to the one under complete cooperation.*

Using backwards induction we start with the fiscal policy problem, which not surprisingly delivers the following condition guaranteeing optimal fiscal behavior

$$\lambda_y(y_t - y_{t-1}) = 0. \tag{2.67}$$

After having substituted optimal fiscal behavior in the monetary policy problem and minimized the policy problem in the relevant monetary variables, one can again solve for the corresponding targeting rules, which then turn out to be equivalent to those in the cooperative solution given by (2.58)-(2.60). With respect to (2.58) and (2.60) the conclusions and implications remain the same, i.e. in order to achieve a welfare maximum the central bank should target inflation with the weight for national inflation rates being determined by the economic size of the countries and their price rigidities, similar to the cooperative solution.

The distinction from the cooperative solution arises from the output stabilization problem. It follows that, if monetary policy seeks to stabilize output in order to maximize union-wide welfare, the entire policy problem has no unique solution, as the monetary targeting rule with respect to output deviations is the same as for fiscal policy. The reason for this is quite simple. We know that the fiscal targeting rule is given by (2.67) and that this targeting rule implies $y_t = y_{t-1}$.

From this it follows directly that the system of first-order conditions resulting from the monetary policy problem has infinitely many solutions, as fiscal policy has already implemented the optimal solution with respect to output deviations. Hence, as long as fiscal policy is perfectly committed to this optimal behavior, monetary policy can basically do anything with respect to the output gap, since fiscal policy will restore the welfare maximum by following the optimal targeting rule. Therefore, we may say that if fiscal policy is optimal in the sense that it is obliged and able to maximize welfare in the relevant variables[15] and monetary policy may anticipate fiscal behavior, output stabilization is a purely fiscal task.

In the following, we will turn to the case in which monetary policy authorities are still committed to maximizing welfare, but fiscal policy follows a suboptimal targeting rule.

2.3.3 Optimal Monetary Policy Behavior under Suboptimal Fiscal Policy

Let us suppose now that monetary policy is still able to anticipate policy behavior. But in contrast to the previous section fiscal policy no longer implements the optimal policy plan. One reason for such a scenario to occur could be that the fiscal target is misspecified and fiscal actions are persistently wrong. E.g. this could be caused by recognition or implementation lags of fiscal actions.

Proposition 5. *Under suboptimal fiscal policy behavior, monetary policy will restore optimality by implementing the optimal policy plan.*

Suboptimal fiscal policy implies that the optimal targeting rule (2.67) is no longer fulfilled. In particular, we assume that fiscal policy follows an arbitrary suboptimal targeting rule. For the sake of simplicity we assume that once fiscal policy has committed itself to some targeting rule, it may not deviate from it. This implies that as before monetary policy can perfectly anticipate fiscal behavior. Generally, fiscal policy may be considered suboptimal, if

$$\lambda_y(y_t - y_{t-1}) \neq 0. \tag{2.68}$$

[15] Within the next section we will consider the case in which this condition is not satisfied.

From (2.68) it directly follows that there exists some ι such that

$$\lambda_y(y_t - y_{t-1}) - \iota = 0. \tag{2.69}$$

Again, monetary policy seeks to maximize social welfare. In contrast to the case considered before, the central bank anticipates that fiscal behavior no longer follows the optimal rule but instead follows (2.69). This yields an optimality condition for monetary policy of the following form

$$n\lambda_{\pi,H}\hat{\pi}_{H,t} + (1-n)\lambda_{\pi,F}\hat{\pi}_{F,t} + \lambda_y\iota = 0. \tag{2.70}$$

This rule, which has a similar interpretation as (2.61), depends not only on the rates of inflation, but also on the size of ι. To make (2.70) hold, the weighted sum of country-specific inflation rates has to compensate for the constant effect of ι. This implies that if fiscal policy is consistently misspecified or unable to implement the optimal policy plan, a welfare-maximizing monetary policy will be required to offset these inefficiencies by allowing for permanent non-zero rates of inflation.

2.4 Conclusions and Policy Implications

Using the optimal policy plan under a central planner's decision as a benchmark, the analysis has shown that optimality of monetary policy generally depends on the nature of fiscal policy. Output stabilization only turns out to be a monetary task, if fiscal policy does not implement the optimal policy plan. The larger these fiscal inefficiencies are, the larger the need will be for output stabilization by the central bank in order to restore the optimal policy plan.

The results imply that output stabilization is generally not a monetary task. Instead it should be ensured that fiscal policy is committed to the optimal policy plan. This requires efficient institutions on the one hand, but also a well-defined commitment of fiscal institutions on the other hand, since otherwise fiscal policy inefficiencies are compensated for by monetary policy as far as possible.

In contrast to the previous analysis, one could also think of a scenario with monetary policy being committed to maximize welfare both with respect to inflation and the output gap, while fiscal policy is discretionary. This then would imply that fiscal policy may well anticipate the behavior of monetary policy, when making decisions. With monetary policy having perfect control over the output path, there is no unique fiscal policy targeting rule. This means then that fiscal policy may realize any policy plan, as it knows that monetary policy will restore optimality due to its commitment to the optimal rule. The result would be that there is a potential free rider opportunity for fiscal policy, as it expects monetary policy to restore optimality as far as possible. Restricting fiscal policy by a predetermined targeting rule can solve this problem.

The analysis highlights the need for a commitment of fiscal authorities to a predetermined rule, as the optimal behavior of monetary policy is determined by fiscal actions. In order to make monetary policy independent of fiscal actions and guarantee the steady state inflation rate in the long run, fiscal policy must be efficient, because otherwise monetary policy has to allow for permanent non-zero rates of inflation to compensate for these inefficiencies.

2.5 Appendix

2.5.1 Derivation of the Welfare Criterion

Second-Order Approximation of the Utility Function

We assume the utility function to be isoelastic, i.e. the household's consumption path is characterized by a constant elasticity of intertemporal substitution. The utility function is then given by

$$U(C_{W,t}) \equiv \frac{C_{W,t}^{1-\varphi}}{1-\varphi}, \tag{2.71}$$

where φ denotes the intertemporal elasticity of substitution.
The Taylor approximation of the utility function around the steady state is given by

$$U(C_{W,t}) \approx U(\bar{C}) + U_C(C_{W,t} - \bar{C}) + \frac{1}{2}U_{cc}(C_{W,t} - \bar{C})^2 + \circ(\|\varepsilon_t - \bar{\varepsilon}\|^3), \tag{2.72}$$

where $\circ(\|\varepsilon_t - \bar{\varepsilon}\|^3)$ includes terms that are of order three or higher.
One can show that with a constant steady state tax rate, $\bar{\tau}$, steady state consumption remains unaffected by any policy actions. Therefore, we may say that $\bar{C}$ is totally independent of policy. Terms that are independent of policy are summarized by $t.i.p.$
Using a second-order Taylor approximation for consumption $C_{W,t}$ yields the following relationship

$$\frac{C_{W,t} - \bar{C}}{\bar{C}} = \hat{C}_{W,t} + \frac{1}{2}\hat{C}_{W,t}^2, \tag{2.73}$$

where we defined $\hat{C}_{W,t}$ as absolute deviation from the steady state, $\hat{C}_{W,t} = C_{W,t} - \bar{C}$. (2.73) gives an expression for percentage deviations from steady state and allows us to reformulate (2.72) in terms of steady state deviations, which finally yields

$$U(C_{W,t}) \approx U(\bar{C})\bar{C}\left[\hat{C}_{W,t} + \frac{1}{2}(1-\varphi)\hat{C}_{W,t}^2\right] + t.i.p. + \circ(\|\hat{\varepsilon}_t\|^3), \tag{2.74}$$

where we made use of the fact that $\frac{U_{cc}\bar{C}}{U_C} = -\rho$.

Second-Order Approximation of the "Disutility" Function

We assume disutility of labor to be isoelastic. It is given by

$$V(y_t(z), a_{i,t}) \equiv \frac{1}{1+\eta}\left(\frac{y_t(z)}{a_{i,t}}\right)^{1+\eta}.$$

In the functional form of (2.48) and (2.49) we obtain

$$\frac{1}{n}\int_0^n V(y_t(h), a_{H,t})dh = \frac{(Y_{H,t}/a_{H,t})^{1+\eta}}{1+\eta}\Xi_{H,t}, \tag{2.75}$$

where we made use of the demand equation (2.12) to substitute for $y_t(h)$. A similar relationship is obtained for country F

$$\frac{1}{1-n}\int_n^1 V(y_t(f), a_{F,t})df = \frac{(Y_{F,t}/a_{F,t})^{1+\eta}}{1+\eta}\Xi_{F,t}. \tag{2.76}$$

These two expressions may be summarized to $V(Y_{i,t}, a_{i,t})\Xi_{i,t}$. The corresponding Taylor approximation is given by

$$\begin{aligned} V(Y_{i,t}, a_{i,t})\Xi_{i,t} = V_y\bar{Y}\Bigg[&\hat{Y}_{i,t} + \frac{1}{2}(1+\eta)\hat{Y}_{i,t}^2 - (1+\eta)\hat{Y}_{i,t}\hat{a}_{i,t} + \left(\frac{1}{1+\eta}\right)(\Xi_{i,t}-1) \\ &+\hat{Y}_{i,t}(\Xi_{i,t}-1) - (\Xi_{i,t}-1)\hat{a}_{i,t}\Bigg] + t.i.p. + \circ(\|\hat{\varepsilon}_t\|^3), \end{aligned} \tag{2.77}$$

where we exploited the fact that all terms depending solely on $a_{i,t}$ or steady state values are independent of policy and summarized as $t.i.p.$ Furthermore, one should note that $\bar{\Xi}_i = 1$, as there are no differences in prices across goods in the steady state.

As (2.77) still includes terms in $\Xi_{i,t}$, we again approximate these terms by a Taylor expansion. We start this approximation with (2.44). After plugging in for the corresponding derivatives, the approximation of $\Xi_{i,t}$ is given by

$$\Xi_{i,t} = \bar{\Xi} + \alpha_i(\Xi_{i,t-1} - \bar{\Xi}) + \frac{\alpha_i\sigma(1+\eta)(1+\sigma\eta)}{2(1-\alpha_i)}(\pi_{i,t} - \bar{\pi})^2 + \alpha_i\sigma(1+\eta)(\Xi_{i,t-1} - \bar{\Xi})(\pi_{i,t} - \bar{\pi}) + \circ(\|\varepsilon_t - \bar{\varepsilon}\|^3). \tag{2.78}$$

For $\Xi_{i,t}$ and $\pi_{i,t}$ we may write

$$\Xi_{i,t} \;=\; e^{\hat{\Xi}_{i,t}} \approx 1 + \hat{\Xi}_{i,t}$$

$$\pi_{i,t} \quad = \quad e^{\hat{\pi}_{i,t}} \approx 1 + \hat{\Xi}_{i,t},$$

Applying these expressions to (2.78), we obtain

$$\hat{\Xi}_{i,t} = \alpha_i \hat{\Xi}_{i,t-1} + \frac{\alpha_i \sigma(1+\eta)(1+\sigma\eta)}{2(1-\alpha)} \hat{\pi}_{i,t}^2 + \circ(\|\hat{\varepsilon}_t\|^3), \tag{2.79}$$

where we assumed that the product of $\hat{\Xi}_{i,t}$ and $\hat{\pi}_{i,t}$ is approximately equal to zero. It can be shown that the discounted sum over all $\hat{\Xi}_{i,t}$ is given by

$$\sum_{t=0}^{\infty} \beta^t \hat{\Xi}_{i,t} = \frac{\alpha_i \sigma(1+\eta)(1+\sigma\eta)}{2(1-\alpha_i)(1-\alpha_i\beta)} \sum_{t=0}^{\infty} \beta^t \hat{\pi}_{i,t}^2 + t.i.p. + \circ(\|\hat{\varepsilon}_t\|^3). \tag{2.80}$$

Approximation of the Welfare Criterion and the Loss Function

(2.77)and (2.80) may now be used to obtain a complete approximation of the welfare criteria (2.48) and (2.49). This yields

$$\begin{aligned} u_{i,0} = U_C \bar{C} E_0 \Bigg\{ \sum_{t=0}^{\infty} \beta^t \Bigg[& \hat{C}_{W,t} + (1-\varphi)\hat{C}_{W,t}^2 \\ & - (s_c\bar{\mu})^{-1}(\hat{Y}_{i,t} + \frac{1}{2}(1+\eta)\hat{Y}_{i,t}^2 - (1+\eta)\hat{\alpha}_{i,t}\hat{Y}_{i,t} + \frac{1}{2}\sigma\kappa_i^{-1}\hat{\pi}_{i,t}^2) \Bigg] \Bigg\} + t.i.p. + \circ(\|\hat{\varepsilon}_t\|^3), \end{aligned} \tag{2.81}$$

with

$$\begin{aligned} \kappa_i &= \frac{(1-\alpha_i)(1-\alpha_i\beta)}{\alpha_i(1+\sigma\eta)} \\ \bar{\mu} &= \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{\bar{\mu}^w}{1-\bar{\tau}}\right) = \frac{U_C}{V_y} \\ s_c &= \frac{\bar{C}}{\bar{Y}}. \end{aligned}$$

(2.81) expresses welfare in country i in terms of quadratic consumption, output and inflation. We now eliminate the linear terms in (2.81) to obtain an expression depending purely on quadratic terms of the endogenous variables. This expression then is referred to as the loss function.
We reformulate (2.81) in terms of a vector including all endogenous variables except for inflation, denoted by x_t, and a vector including all exogenous variables, ξ_t. x_t and ξ_t are defined as

$$x_t = \left[\begin{array}{cccccccc} \hat{Y}_t & \hat{C}_{H,t} & \hat{\wp}_{H,t} & \hat{\tau}_{H,t} & \hat{C}_{F,t} & \hat{\wp}_{F,t} & \hat{\tau}_{F,t} & \hat{T}_t \end{array} \right]'$$

$$\xi_t = \left[\begin{array}{cccccc} \hat{a}_{H,t} & \hat{\mu}^w_{H,t} & \hat{G}_{H,t} & \hat{a}_{F,t} & \hat{\mu}^w_{F,t} & \hat{G}_{F,t} \end{array} \right]'.$$

This gives for (2.81) the following relationship

$$u_{i,0} = U_C \bar{C} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[z'_{i,x} x_t - \frac{1}{2} x'_t Z_{i,x} x_t - x'_t Z_{i,\xi} \xi_t - \frac{1}{2} z_{i,\pi} \hat{\pi}^2_{i,t} \right] \right\} + t.i.p. + \circ(\|\hat{\varepsilon}_t\|^3), \qquad (2.82)$$

where $z_{i,x}$ denotes a column vector with eight elements, $Z_{i,x}$ a matrix of dimension (8×8), $Z_{i,\xi}$ a matrix of dimension (8×6), and finally $z_{i,\pi}$ is just a scalar. Using second order Taylor approximations to the equilibrium conditions, we may eliminate all terms that enter (2.82) linearly. As we already formulated the welfare criterion (2.82) in terms of vectors of the endogenous and exogenous variables, we also do this for the approximations of the equilibrium conditions. Using these equations then gives a welfare criterion formulated only in quadratic terms. The solution method then involves the following steps

1. We derive second order approximations of the equilibrium conditions.
2. We then formulate the approximated equilibrium conditions in terms of endogenous and exogenous vectors and solve for the linear term.
3. We define a matrix Γ that includes all vectors in front of the endogenous linear terms in the approximated equilibrium conditions and multiply it with ζ_i, which is a vector of weights. These weights are chosen so that we obtain the following relationship

$$\Gamma \zeta_i = z_{i,x}. \qquad (2.83)$$

4. It is then possible to substitute for the linear terms in (2.82) using the previous results.

To substitute for the linear terms in (2.82), we define a vector of weights, ζ, which satisfies (2.83), where Γ is a (9×9) matrix collecting the vectors in front of the linear terms in the structural equilibrium conditions. That means, we apply a second order Taylor approximation to the equilibrium conditions and bring them into a similar form as the welfare criterion itself by using the definition of the vector of endogenous variables x. This yields for Γ

$$\Gamma = \begin{bmatrix} \eta & 0 & 1+\omega_g & 0 & -1 & 0 & 0 & 0 & 0 \\ \rho & 0 & -\rho & 0 & ns_c & ns_c & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -\theta s_c & 0 & -1 & 0 & 0 \\ \omega_\tau & 0 & 1+\omega_g & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta & 0 & 1+\omega_g & 0 & -1 & 0 & 0 & 0 \\ 0 & \rho & 0 & -\rho & (1-n)s_c & (1-n)s_c & 0 & 0 & -1 \\ 0 & -1 & 0 & 1 & 0 & -\theta s_c & 0 & -1 & 0 \\ 0 & \omega_\tau & 0 & 1+\omega_g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(1-n) & -n & 0 \end{bmatrix}.$$

Next, we define

$$z_{W,x} = nz_{H,x} + (1-n)z_{F,x},$$

and correspondingly for ζ

$$\zeta_W = n\zeta_H + (1-n)\zeta_F.$$

We then may solve the system of equations resulting from

$$\Gamma\zeta_W = z_{W,x}$$

for each component of ζ.

The solution satisfies the following conditions

$$\begin{aligned} \zeta_{W,1} &= n(\bar{\mu}-1)(1+\omega_g)(\bar{\mu}d_\zeta)^{-1} \\ \zeta_{W,2} &= n^{-1}(1-n)\zeta_{W,1} \\ \zeta_{W,3} &= -n\omega_\tau(\bar{\mu}-1)(\bar{\mu}d_\zeta)^{-1} \\ \zeta_{W,4} &= n^{-1}(1-n)\zeta_{W,3} \\ \zeta_{W,5} &= n\varphi(\bar{\mu}s_c d_\zeta)^{-1} \end{aligned}$$

$$
\begin{aligned}
\zeta_{W,6} &= n^{-1}(1-n)\zeta_{W,4} \\
\zeta_{W,7} &= -n((\bar{\mu}-1)(1+\omega_g+\omega_\tau)+\theta\varphi)(\bar{\mu}d_\zeta)^{-1} \\
\zeta_{W,8} &= n^{-1}(1-n)\zeta_{W,7}
\end{aligned}
$$

with

$$
\begin{aligned}
d_\zeta &= s_c(1+\omega_g)(\rho s_c^{-1}+\eta-\omega_\tau)+\rho\omega_\tau \\
\varphi &= \bar{\mu}s_c(1+\omega_g)(\eta-\omega_\tau)+\rho(1+\omega_g+\omega_\tau).
\end{aligned}
$$

Furthermore, we set $\zeta_9 = 0$ to obtain a unique solution.

We then multiply each of the second order Taylor approximations to the equilibrium conditions with the corresponding component of ζ and sum up over all equilibrium conditions. As Γ collects the vectors in front of the linear terms in the structural equilibrium conditions, and each of these equilibrium conditions is multiplied by the corresponding component of ζ, we may apply the relationship between Γ and the vector of weights ζ to obtain $z_{H,x}$. We solve for $z_{H,x}$ and plug the resulting expression into the welfare criterion, which then gives a purely quadratic expression

$$
u_{i,0} = -\frac{1}{2}U_C\bar{C}E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[x_t'Q_{i,x}x_t+2x_t'Q_{i,\xi}\xi_t+q_{i,\pi,H}\hat{\pi}_{H,t}^2+q_{i,\pi,F}\hat{\pi}_{F,t}^2\right]\right\}+J_{i,0}+t.i.p.+\circ(\|\hat{\varepsilon}_t\|^3),
$$

where

$$
\begin{aligned}
Q_{i,x} &= Z_{i,x}+\zeta_{i,1}A_{H,x}+\zeta_{i,2}A_{F,x}+\zeta_{i,3}W_{H,x}+\zeta_{i,4}W_{F,x}+\zeta_{i,5}D_{H,x}+\zeta_{i,6}D_{F,x}+\zeta_{i,7}H_{H,x}+\zeta_{i,8}H_{F,x} \\
Q_{i,\xi} &= Z_{i,\xi}+\zeta_{i,1}A_{H,\xi}+\zeta_{i,2}A_{F,\xi}+\zeta_{i,3}W_{H,\xi}+\zeta_{i,4}W_{F,\xi}+\zeta_{i,5}D_{H,\xi}+\zeta_{i,6}D_{F,\xi} \\
q_{H,\pi,H} &= z_{H,\pi}+\zeta_{H,1}a_{\pi,H}, \quad q_{H,\pi,F}=\zeta_{H,2}a_{\pi,F} \\
q_{F,\pi,H} &= \zeta_{F,1}a_{\pi,H}, \quad q_{F,\pi,F}=z_{F,\pi}+\zeta_{F,2}a_{\pi,F} \\
a_{\pi,i} &= \sigma(1+\eta)\kappa_i^{-1}.
\end{aligned}
$$

Next, we reduce the size of the vector of endogenous variables, x. For this purpose, we construct second order Taylor approximations to the following equations constituting the relationship between terms of trade and relative prices

$$\begin{aligned}\wp_{H,t}^{\theta-1} &= n + (1-n)T_t^{1-\theta} \\ \wp_{F,t}^{\theta-1} &= (1-n) + nT_t^{\theta-1},\end{aligned}$$

which then gives

$$\hat{\wp}_{H,t} = -(1-n)\hat{T}_t \tag{2.1}$$

$$\hat{\wp}_{F,t} = n\hat{T}_t. \tag{2.2}$$

Similarly, we approximate the demand equation, which yields the following relationship

$$\hat{Y}_{i,t} = \underbrace{\frac{\bar{C}_W}{\bar{Y}}}_{\equiv s_c}(-\theta\hat{\wp}_{i,t} + \hat{C}_{W,t}) + \frac{\bar{G}}{\bar{Y}}\hat{G}_{i,t}.$$

When combining these two expressions, we obtain

$$\begin{aligned}\hat{Y}_{H,t} &= s_c((1-n)\theta\hat{T}_t + \hat{C}_{W,t}) + \frac{\bar{G}}{\bar{Y}}\hat{G}_{H,t} \\ \hat{Y}_{F,t} &= s_c(-n\theta\hat{T}_t + \hat{C}_{W,t}) + \frac{\bar{G}}{\bar{Y}}\hat{G}_{F,t}\end{aligned}$$

Furthermore, we define union wide output deviations as

$$\hat{Y}_{W,t} = n\hat{Y}_{H,t} + (1-n)\hat{Y}_{F,t}.$$

Substituting for $\hat{Y}_{H,t}$ and $\hat{Y}_{F,t}$, using the results obtained above, gives for union wide output deviations

$$\hat{Y}_{W,t} = s_c\hat{C}_{W,t} + \frac{\bar{G}}{\bar{Y}}(n\hat{G}_{H,t} + (1-n)\hat{G}_{F,t}). \tag{2.3}$$

Finally, we solve this equation for $s_c\hat{C}_{W,t}$ and plug in the resulting expression in the equations for $\hat{Y}_{H,t}$ and $\hat{Y}_{F,t}$ given above. This yields

$$\hat{Y}_{H,t} = \hat{Y}_{W,t} + (1-n)s_c\theta\hat{T}_t + \frac{\bar{G}}{\bar{Y}}[(1-n)\hat{G}_{H,t} - (1-n)\hat{G}_{F,t}] \tag{2.4}$$

$$\hat{Y}_{F,t} = \hat{Y}_{W,t} + ns_c\theta\hat{T}_t + \frac{\bar{G}}{\bar{Y}}[-n\hat{G}_{H,t} + n\hat{G}_{F,t}]. \tag{2.5}$$

The reduced vector of endogenous variables will be defined as

$$\hat{y}_t' = \left[\begin{array}{cccc} \hat{Y}_{W,t} & \hat{\tau}_{H,t} & \hat{\tau}_{F,t} & \hat{T}_t \end{array}\right].$$

The relationship between x_t and $\hat{y}_t$ is generally described by

$$x_t = N_x\hat{y}_t + N_\xi\xi_t$$

where N_x is a matrix of size 9×4 and N_ξ of size 9×6. These two matrices are derived from the system of equations relying on (2.2)-(2.5). By rewriting these equations in matrix notation, we obtain the following forms for N_x and N_ξ

$$N_x = \left[\begin{array}{cccc} 1 & 0 & 0 & (1-n)\theta s_c \\ s_c^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1-n) \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & ns_c\theta \\ s_c^{-1} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$N_\xi = \left[\begin{array}{cccccc} 0 & 0 & (1-n)\bar{G}/\bar{Y} & 0 & 0 & -(1-n)\bar{G}/\bar{Y} \\ 0 & 0 & -ns_c^{-1}\bar{G}/\bar{Y} & 0 & 0 & -(1-n)s_c^{-1}\bar{G}/\bar{Y} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -n\bar{G}/\bar{Y} & 0 & 0 & n\bar{G}/\bar{Y} \\ 0 & 0 & -ns_c^{-1}\bar{G}/\bar{Y} & 0 & 0 & -(1-n)s_c^{-1}\bar{G}/\bar{Y} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

We then replace for x, which gives a relationship for welfare in country i depending only on the reduced vector of endogenous variables, the vector of exogenous variables and the rates of inflation in the two countries

$$u_{i,0} = -\frac{1}{2}U_C\bar{C}E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\hat{y}_t'\tilde{Q}_{i,x}\hat{y}_t + 2\hat{y}_t'\tilde{Q}_{i,\xi}\xi_t + q_{i,\pi,H}\hat{\pi}_{H,t}^2 + q_{i,\pi,F}\hat{\pi}_{F,t}^2\right]\right\} + J_{i,0} + t.i.p. + \circ(\|\hat{\varepsilon}_t\|^3),$$

where

$$\tilde{Q}_{i,x} = N_x' Q_{i,x} N_x, \quad \tilde{Q}_{i,\xi} = N_x' Q_{i,x} N_\xi + N_x' Q_{i,\xi}.$$

We assigned the quadratic term in the vector of exogenous variables to $t.i.p.$, as it is not affected by any policy actions. Given this expression for country-specific welfare, union-wide welfare is by definition obtained from

$$u_{W,0} = nu_{H,0} + (1-n)u_{F,0} = -\frac{1}{2}U_C\bar{C}E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\hat{y}_t'\tilde{Q}_{W,x}\hat{y}_t + 2\hat{y}_t'\tilde{Q}_{W,\xi}\xi_t + q_{W,\pi,H}\hat{\pi}_{H,t}^2 + q_{W,\pi,F}\hat{\pi}_{F,t}^2\right]\right\}$$
$$+J_{W,0} + t.i.p. + \circ(\|\hat{\varepsilon}_t\|^3),$$

where the matrices entering $u_{W,0}$ are defined as

$$\begin{aligned}
\tilde{Q}_{W,x} &= n\tilde{Q}_{H,x} + (1-n)\tilde{Q}_{F,x} = N_x' Q_{W,x} N_x = N_x'(nQ_{H,x} + (1-n)Q_{F,x})N_x \\
\tilde{Q}_{W,\xi} &= n\tilde{Q}_{H,\xi} + (1-n)\tilde{Q}_{F,\xi} = N_x' Q_{W,x} N_\xi + N_x' Q_{W,\xi} \\
&= N_x'(nQ_{H,x} + (1-n)Q_{F,x})N_\xi + N_x'(nQ_{H,\xi} + (1-n)Q_{F,\xi}) \\
q_{W,\pi,H} &= nq_{H,\pi,H} + (1-n)q_{F,\pi,H} \\
q_{W,\pi,F} &= nq_{H,\pi,F} + (1-n)q_{F,\pi,F}.
\end{aligned}$$

By successively substituting for the terms included in $\tilde{Q}_{W,x}$, one can show that the matrix has the general form

$$\tilde{Q}_{W,x} = \begin{bmatrix} (\tilde{Q}_{W,x})_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (\tilde{Q}_{W,x})_{44} \end{bmatrix},$$

showing that all terms that are multiplied with the tax rate, $\hat{\tau}_{i,t}$, become zero. What remains, is a quadratic term in output and the terms of trade, where the corresponding factors are defined as

$$
\begin{aligned}
(\tilde{Q}_{W,x})_{11} &= (Q_{W,x})_{11} + (Q_{W,x})_{55} + s_c^{-2}\left[(Q_{W,x})_{22} + 2(Q_{W,x})_{26} + (Q_{W,x})_{66}\right] + 2s_c^{-1}\left[(Q_{W,x})_{12} + (Q_{W,x})_{56}\right] \\
(\tilde{Q}_{W,x})_{44} &= (\theta s_c)^2\left[(1-n)^2(Q_{W,x})_{11} + n^2(Q_{W,x})_{55}\right] + \left[(1-n)^2(Q_{W,x})_{33} + n^2(Q_{W,x})_{77}\right] + (Q_{W,x})_{99} \\
&\quad -2\theta s_c\left[(1-n)^2(Q_{W,x})_{13} + n^2(Q_{W,x})_{57}\right],
\end{aligned}
$$

where

$$
\begin{aligned}
(Q_{W,x})_{11} + (Q_{W,x})_{55} &= n(s_c\bar{\mu})^{-1}(1+\eta) + \eta(2+\eta)(\zeta_{W,1} + \zeta_{W,2}) + (1+\omega_g)(\zeta_{W,3} + \zeta_{W,4}) \\
(Q_{W,x})_{22} + 2(Q_{W,x})_{26} + (Q_{W,x})_{66} &= (1-n)(s_c\bar{\mu})^{-1}(1+\eta) - n(1-\rho) \\
&\quad -\rho^2\left[(\zeta_{W,1} + \zeta_{W,2}) - (\zeta_{W,3} + \zeta_{W,4})\right] + (\zeta_{W,5} + \zeta_{W,6})s_c(1-s_c) \\
(Q_{W,x})_{12} + (Q_{W,x})_{56} &= \rho(\zeta_{W,1} + \zeta_{W,2}) - (1+\omega_g)(\zeta_{W,3} + \zeta_{W,4}) \\
(1-n)^2(Q_{W,x})_{11} + n^2(Q_{W,x})_{55} &= (1-n)^2\left[(Q_{W,x})_{11} + (Q_{W,x})_{55}\right] - (1-2n)(Q_{W,x})_{55} \\
(Q_{W,x})_{55} &= \eta(2+\eta)\zeta_{W,2} + (1+\omega_g)\zeta_{W,4} \\
(1-n)^2(Q_{W,x})_{33} + n^2(Q_{W,x})_{77} &= n(1-n)[-(\zeta_{W,1} + \zeta_{W,2}) + (\zeta_{W,3} + \zeta_{W,4}) \\
&\quad +\theta^2 s_c(1-s_c)(\zeta_{W,5} + \zeta_{W,6})] \\
(1-n)^2(Q_{W,x})_{13} + n^2(Q_{W,x})_{57} &= -n(1-n)\rho^{-1}\left[(Q_{W,x})_{12} + (Q_{W,x})_{56}\right] \\
(Q_{W,x})_{99} &= n(1-n)(\theta-1)(\zeta_{W,7} + \zeta_{W,8}).
\end{aligned}
$$

Similarly, one can show for the matrix $\tilde{Q}_{W,\xi}$ that all elements multiplied with the tax rates become zero, i.e.

$$
(\tilde{Q}_{W,\xi})_{i2} = (\tilde{Q}_{W,\xi})_{i3} = 0.
$$

The remaining elements are given by

$$
\begin{aligned}
(\tilde{Q}_{W,\xi})_{11} &= -n(s_c\bar{\mu})^{-1}(1+\eta) - (1+\eta)^2\zeta_{W,1} \\
(\tilde{Q}_{W,\xi})_{33} &= -\frac{\bar{Y}}{\bar{s}}\zeta_{W,3} + \theta s_c\zeta_{W,5} \\
(\tilde{Q}_{W,\xi})_{14} &= n^{-1}(1-n)(Q_{W,\xi})_{11} \\
(\tilde{Q}_{W,\xi})_{12} &= (1+\eta)\zeta_{W,1}
\end{aligned}
$$

$$\begin{aligned}
(\tilde{Q}_{W,\xi})_{15} &= n^{-1}(1-n)(\tilde{Q}_{W,\xi})_{12} \\
(\tilde{Q}_{W,\xi})_{13} &= s_c^{-1}\left[(\tilde{Q}_{W,\xi})_{23} + (\tilde{Q}_{W,\xi})_{63}\right] - ns_c^{-2}\left[(Q_{W,\xi})_{22} + 2(Q_{W,\xi})_{26} + (Q_{W,\xi})_{66}\right] \\
&\quad + ns_c^{-1}\left[(Q_{W,\xi})_{12} + (Q_{W,\xi})_{56}\right] \\
(\tilde{Q}_{W,\xi})_{16} &= n^{-1}(1-n)(\tilde{Q}_{W,\xi})_{13}
\end{aligned}$$

with

$$\begin{aligned}
(\tilde{Q}_{W,\xi})_{23} + (\tilde{Q}_{W,\xi})_{63} &= \rho\frac{\bar{Y}}{\bar{s}}\zeta_{W,3} - s_c\zeta_{W,5} \\
(\tilde{Q}_{W,\xi})_{41} &= (1-n)\theta s_c(\tilde{Q}_{W,\xi})_{11} \\
(\tilde{Q}_{W,\xi})_{44} &= -(\tilde{Q}_{W,\xi})_{41} \\
(\tilde{Q}_{W,\xi})_{42} &= (1-n)\theta s_c(\tilde{Q}_{W,\xi})_{12} \\
(\tilde{Q}_{W,\xi})_{45} &= -(\tilde{Q}_{W,\xi})_{42} \\
(\tilde{Q}_{W,\xi})_{43} &= -(1-n)(\tilde{Q}_{W,\xi})_{33} - \left[(1-n)^2(\tilde{Q}_{W,\xi})_{13} + n^2(\tilde{Q}_{W,\xi})_{57}\right] \\
&\quad + \theta s_c\left[(1-n)^2(\tilde{Q}_{W,\xi})_{11} + n^2(\tilde{Q}_{W,\xi})_{55}\right] \\
(\tilde{Q}_{W,\xi})_{46} &= -(\tilde{Q}_{W,\xi})_{43}.
\end{aligned}$$

Following Benigno and Woodford (2003) we define the target values of the remaining variables in y as a function of exogenous disturbances. This yields a function for the welfare criterion, which only depends on purely quadratic and observable variables given by

$$\begin{aligned}
u_{W,0} = -\frac{1}{2}U_C\bar{C}E_0\Bigg\{\sum_{t=0}^{\infty}\beta^t\Big[&\lambda_y(\hat{Y}_{W,t} - \tilde{Y}_{W,t})^2 + n(1-n)\lambda_q(\hat{T}_t - \tilde{T}_t)^2 \\
&+ n\lambda_{\pi,H}\hat{\pi}_{H,t}^2 + (1-n)\lambda_{\pi,F}\hat{\pi}_{F,t}^2\Big]\Bigg\} + t.i.p. + J_{W,0} + \circ(\|\hat{\varepsilon}_t\|^3),
\end{aligned}$$

where

$$\begin{aligned}
\tilde{Y}_{W,t} &= -(n\lambda_y)^{-1}\left[(\tilde{Q}_{W,\xi})_{11}\hat{a}_{W,t} + (\tilde{Q}_{W,\xi})_{12}\hat{\mu}_{W,t}^{w} + (\tilde{Q}_{W,\xi})_{13}\hat{G}_{W,t}\right] \\
\tilde{T}_t &= -(\lambda_q)^{-1}\left[(\tilde{Q}_{W,\xi})_{41}\hat{a}_{R,t} + (\tilde{Q}_{W,\xi})_{42}\hat{\mu}_{R,t}^{w} + (\tilde{Q}_{W,\xi})_{43}\hat{G}_{R,t}\right].
\end{aligned}$$

Exogenous variables that are denoted with the subscript R represent relative shocks, while shocks with the subscript W affect both countries similarly. These are generally defined as

$$\begin{aligned} \xi_{W,t} &= n\xi_{H,t} + (1-n)\xi_{F,t} \\ \xi_{R,t} &= \xi_{H,t} - \xi_{F,t}. \end{aligned}$$

Finally, the weights in the objective function have the following form

$$\begin{aligned} \lambda_y &= (\tilde{Q}_{W,x})_{11} \\ \lambda_q &= n^{-1}(1-n)^{-1}(\tilde{Q}_{W,x})_{44} \\ \lambda_{\pi,H} &= n^{-1}q_{W,\pi,H} \\ \lambda_{\pi,F} &= (1-n)^{-1}q_{W,\pi,F}. \end{aligned}$$

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Chapter 3

Inflation Transmission in the EMU: A Markov-Switching VECM Analysis

This paper analyzes the transmission of inflation across the five largest economies in the European Monetary Union, i.e. France, Germany, Italy, Netherlands and Spain. We use monthly CPI inflation rates for the period 1970-2006. Given the long observation period and the continuing economic integration of Europe's economies, we first try to investigate, whether there were changes in inflation dynamics in these countries using univariate Markov-switching models. To assess the inflation transmission mechanism, we first establish a long-term relationship between the five countries using cointegration methods. As implied by the results of the univariate models, we allow for changes in the adjustment coefficients of the cointegrating relationships and the short-term dynamics. Using a Markov-switching vector error correction model we find evidence for multiple regime switches from the early 1970s up through the mid 1980s. It is precisely during this period that we find evidence for Germany being weakly exogenous, which clearly demonstrates the dominance of German monetary policy at this time. Since the mid-1980s we find evidence for a stable transmission mechanism both in the long- and the short-term characterized by a low degree of inflation persistence.

3.1 Introduction

3.1.1 Motivation

A huge strand of economic literature has dealt with the question of optimal monetary policy, particularly since the introduction of the euro as the common currency in the European Monetary Union (EMU). One result of this field of research is that price rigidities across regions determine the optimal behavior of monetary policy in a currency union such as the EMU. Price rigidities in turn determine inflation persistence or more generally inflation dynamics. Recent research has focused on estimating country-specific price rigidities within the EMU using various methods ranging from simple univariate models for aggregate data to disaggregated models measuring product- and country-specific frequencies of price adjustments with heterogeneous results.

Besides the knowledge of price-setting behavior across countries and sectors a central bank like the ECB needs to know how inflation dynamics differ across regions within the monetary union. Further, a central bank in a monetary union needs to have sufficient knowledge about the transmission of inflation across its member states in order to conduct monetary policy optimally.

This paper analyzes the transmission of inflation across the five largest economies in the EMU, i.e. France, Germany, Italy, Netherlands and Spain. We use monthly CPI inflation rates for the period 1970-2006. Given the long observation period and the continuing economic integration of Europe's economies, we first try to investigate, if there were changes in inflation dynamics in these countries. Using a univariate Markov-switching model, we obtain evidence for an almost simultaneous regime change in all five countries during the mid-1980s. To assess the inflation transmission mechanism, we first establish a long-term relationship between the five countries using cointegration methods. As implied by the results of the univariate models, we allow for changes in the adjustment coefficients of the cointegrating relationships and the short-term dynamics. Using a Markov-switching vector error correction model (MS-VECM), we find evidence for multiple regime switches from the early 1970s through the mid 1980s. It is precisely during this period that we find evidence for Germany being weakly exogenous, which clearly demonstrates the dominance of German monetary policy at this time. Since the mid-1980s we find evidence for a stable transmission mechanism both in the long- and the short-term. Further, the analysis shows that the decrease of inflation persistence in the euro area, which is found in recent studies such as Altissimo, Ehrmann and Smets (2006), has its origin in French and German inflation dynamics and their importance for the transmission process[1].

3.1.2 Literature Review

Country-specific inflation dynamics and persistence are basically determined by country-specific price rigidities. For this reason a thorough understanding of the patterns and determinants of inflation persistence is important for policy-makers, as inflation persistence has immediate consequences for the conduct of monetary policy, e.g. the appropriate response to a rise in inflation depends on the degree to which the shock itself is persistent (cf. Altissimo, Ehrmann and Smets, 2006). Country-specific inflation dynamics are in turn affected by inflation dynamics in other countries, particularly in a currency union such as the EMU with highly integrated economies. From this it follows that a country's inflation is not only affected by its own inflation persistence but also by other countries' inflation persistence.

Univariate models using the sum of autoregressive coefficients as a measure of persistence, as proposed by Andrews and Chen (1994), offer a simple way to estimate inflation persistence both at an aggregated and disaggregated level. Applications of this approach to EMU data, including both aggregated and country-specific estimations, may be found in Gadzinski and Orlandi (2004), Batini (2006) and Levin and Piger (2004). The results of these contributions substantially differ depending on the inflation definition chosen, i.e. annual, monthly, the proxy chosen for the price level, i.e. CPI, HCPI or GDP deflator, and the time frame of the analysis. Another reason for the heterogeneity of the results may be due to country-specific restrictions of the estimates, as univariate models do not allow for interactions of inflation dynamics across countries. Instead it is more appropriate to assume, as mentioned above, that a persistent inflation shock in country A will lead to a persistent run of inflation in country B, if both A and B are highly integrated. Univariate models would then suggest that inflation is persistent in both country A and B, although the origin of inflation persistence arises from country A.

The transmission of inflation is analyzed in few empirical papers. Motivated by the period of high inflation in Western countries during the 1970s Darby et al. (1983) and Darby and Lothian

[1] In contrast to Altissimo, Ehrmann and Smets (2006) we find evidence that the decline in inflation persistence has already taken place during the 1980s.

(1989) were among the first to analyze the transmission of inflation across countries empirically. In particular, they investigate the sources and origins of high inflation in the industrialized West with a focus on the economic linkages between these countries, while the quantification of the inflation transmission mechanism itself is not considered. Yang, Guo and Wang (2006) investigate the international transmission of inflation among G-7 countries in a vector error correction model (VECM). They show that U.S. inflation has a less dominant role than is usually assumed. Generally, they find a broad linkage of inflation among G-7 countries for the period 1973-2003. Cheung and Yuen (2002), who assess inflation dynamics across the U.S., Hong Kong and Singapore, find evidence for the inflation rates in the small economy being caused by the large economy.

In this paper we analyze how inflation is transmitted across euro area's member states. Starting with univariate models, which offer a simple way to measure inflation persistence[2], and in turn determine inflation dynamics, we first investigate, if one can find evidence for parameter instabilities in the sample. We then try to establish a long-term relationship between the five countries' inflation rates, as implied by the relative purchasing power parity. Given that there exists at least one cointegrating relationship, we formulate a VECM in the five inflation rates. In contrast to the existing literature, we allow for changes in the short-term dynamics and the adjustments coefficients. We then compute impulse response for each of the regimes of the MS-VECM.

3.2 Analysis

3.2.1 A Linear Univariate Approach

Methodology

As a first step in the analysis we start with estimating simple univariate autoregressive (AR) models for each of the five countries. We do this to investigate, if regime shifts have possibly

[2] Of course, this simplicity has its costs, as interactions in inflation dynamics between countries cannot be considered. More sophisticated models such as structural ones, would probably deliver estimated parameters that differ from the ones given in the next section. Nonetheless, we think that for the purpose of assessing parameter changes, it is sufficient to use a reduced form univariate model.

played a role in describing the inflation persistence in the euro area, as this would in turn have an impact on the inflation dynamics within a country and would most likely affect the transmission process in the entire euro area.

We follow the line of Andrews and Chen (1994) and use the sum of AR coefficients as a measure for persistence of a time series. That means we start with an AR(p) model of the form

$$\pi_{i,t} = c + \alpha_{1,i}\pi_{i,t-1} + \alpha_{2,i}\pi_{i,t-2} + \ldots + \alpha_{p,i}\pi_{i,t-p} + \nu_t, \tag{3.1}$$

where $\pi_{i,t}$ denotes the inflation rate in country i at time t, c represents a constant and ν_t is an *i.i.d.* error term. (3.1) may be easily rewritten in the augmented Dickey-Fuller (ADF) form as

$$\pi_{i,t} = c + \alpha_i^*\pi_{i,t-1} + \psi_{1,i}\Delta\pi_{i,t-1} + \ldots + \psi_{1,p-1}\Delta\pi_{i,t-p+1} + \nu_t, \tag{3.2}$$

where Δ denotes the difference operator. One can show that α_i^* equals the sum of the p AR coefficients, which is the measure of persistence proposed by Andrews and Chen (1994).

For the estimation of (3.2), a particular number of lagged differences have to be chosen. We decided to choose the lag length such that the residuals exhibited no autocorrelation. As the least squares estimator frequently underestimates[3] α_i^*, in particular, when α_i^* approaches unity, we apply the grid bootstrap method of Hansen (1999). The grid bootstrap method yields confidence intervals for α_i^* with correct coverage[4]. The estimate for α_i^* is then given by the median of the confidence interval.

Data

We use monthly non-seasonally adjusted CPI data for France, Germany, Italy, the Netherlands and Spain for the period[5] 1970-2006. The data is seasonally adjusted with help of the Census X12 method[6]. To compute the inflation rate, we use the first difference of the logged CPI series. All

[3]Cf. Andrews and Chen (1994).

[4]The grid bootstrap method simulates the sampling distribution of the t-statistic over a grid of possible true values for α_i^*.

[5]We decided to use this period, since in 1967 a harmonization in European tax law was taking place, which also had impacts on CPI data.

[6]We decided to apply this procedure, as the seasonality in the data was not entirely captured by an auxiliary regression on a set of seasonal dummies.

	Sample	5	50	95
France	1970-2006	0.75	0.84	1.02
	1970-1988	0.75	0.86	1.07
	1989-2006	0.45	0.66	1.05
Germany	1970-2006	0.88	0.93	1.05
	1970-1988	0.67	0.81	1.07
	1989-2006	0.58	0.74	1.07
Italy	1970-2006	0.93	0.96	1.03
	1970-1988	0.87	0.92	1.05
	1989-2006	0.24	0.51	1.09
Netherlands	1970-2006	0.90	0.94	1.02
	1970-1988	0.79	0.87	1.02
	1989-2006	0.76	0.84	1.00
Spain	1970-2006	0.87	0.92	1.05
	1970-1988	0.88	0.94	1.09
	1989-2006	0.25	0.52	1.06

Table 3.1: *Inflation Persistence Measured as the Sum of AR Coefficients in Different Periods. Values Shown Are the 5th, 50th and 95th Percentiles for* α_i^* *Obtained from the Grid Bootstrap Procedure. The Results are Based on 1000 Bootstrap Simulations for each of the 100 Gridpoints within the Range of Six Standard Deviations above and below the Least-Squares Estimates.*

data is taken from OECD statistics. In the case of Germany the data relates to West Germany before 1991 and to unified Germany afterward.

Results

Table 3.1 gives the estimates for the sum of AR coefficients for the five countries in different periods. One can basically see that, when considering the entire sample from 1970-2006, we find median-unbiased estimates (namely, the 50th percentile of the distribution) for the sum of AR coefficients which are all larger than 0.8. Furthermore, the 95th percentile exceeds unity in all cases, suggesting that the null hypotheses of a unit root cannot be rejected at the 5% significance level[7].

When splitting the sample right in the middle, we obtain values for the inflation persistence in

[7]As a robustness check, we also checked the time series properties of the data with an ADF test. In none of the cases considered in table 3.1 the null of a unit root could be rejected.

the period 1970-1988 that are slightly lower or of almost equal size compared with the entire sample. The results change, when we analyze the second half of the sample, i.e. the period after 1988. Except for the Netherlands we find sums of AR coefficients that are considerably lower than during the first half of the sample, which suggests a change in inflation persistence.

The choice for the split of the sample is purely arbitrary and not economically founded, of course. Furthermore, the model considered here is kept very simple and neglects structural elements arising from economic theory as well as international linkages between inflation rates. Another source of problems stems from the possibility of the variables being fractionally integrated. Hassler and Wolters (1995) analyze monthly inflation rates for five industrialized countries including France, Germany and Italy. They find evidence for monthly inflation rates being fractionally integrated with a difference parameter of about 0.5 for all countries. With fractionally integrated variables the absolute t-values of the ADF test decrease monotonically, when the number of lagged differences increases. That means the higher the number of lagged differences in (3.2), the more likely it becomes that we cannot reject the null of a unit root, when applying the usual ADF tests.

To summarize, the results may be interpreted as a first hint for a change in inflation persistence during the observation period. A change in inflation persistence does in turn indicate a change in inflation dynamics.

3.2.2 A Non-Linear Univariate Approach

Methodology

Empirical evidence suggests that many macroeconomic variables behave differently during upswings and downturns, i.e. the underlying data generating process (DGP) is subject to non-linearities (Hamilton, 1989). The previous results suggest that this may also be the case for euro area's inflation rates. For this reason we consider a non-linear version of (3.2) given by

$$\pi_{i,t} = c(S_t) + \alpha_i^*(S_t)\pi_{i,t-1} + \psi_{1,i}(S_t)\Delta\pi_{i,t-1} + \ldots + \psi_{1,p-1}(S_t)\Delta\pi_{i,t-p+1} + \sigma(S_t)\nu_t, \tag{3.3}$$

where S_t denotes the state of the parameters at time t. σ^2 is the variance of the error term, which is generally assumed to be state-dependent[8]. Since the parameters depend on the regime S, which is assumed to be unobservable and stochastic, for the data generating process to be complete it is necessary that we formulate a regime generating process. For notational reasons we define θ as the vector of all parameters in (3.3).

In Markov-switching models the states are assumed to follow a Markov chain with transition probability p_{uv}, where p_{uv} is defined as the probability for state v at time t given state u in $t-1$. The number of states is finite, i.e. $S_t = 1, \ldots, M$. We may write this as

$$p_{uv} = Pr(S_t = v | S_{t-1} = u), \quad \sum_{v=1}^{M} p_{uv} = 1 \quad \forall u, v \in \{1, \ldots, M\}.$$

The estimation is carried out with the Expectation-Maximization (EM) algorithm introduced by Dempster, Laird and Rubin (1977) and described in Krolzig (1996). The EM algorithm is an iterative maximum-likelihood estimation technique. It allows one to estimate models with the observed time series depending on some unobservable stochastic variables, as given by the states S here in our case. In each iteration of the EM algorithm two steps are carried out:

1. The expectation step
2. The maximization step.

For notational purposes let us define an indicator variable for the states

$$I(S_t = u) = \begin{cases} 1 & \text{if } S_t = u \\ 0 & \text{otherwise,} \end{cases}$$

for $u = 1, \ldots, M$. We then may summarize the vector of all indicator variables at a given time t as

$$\xi_t = \begin{bmatrix} I(S_t = 1) \\ \vdots \\ I(S_t = M) \end{bmatrix}.$$

[8] This should be a reasonable assumption in the case of inflation rates, which are among others determined by monetary policy behavior. Sims and Zha (2006) have emphasized the importance of heteroscedastic error terms in describing U.S. monetary policy.

The expectation step in iteration j estimates the unobserved states ξ_t by their smoothed probabilities conditional on the data and the estimated vector of parameters of the previous iteration, which we denote by λ^{j-1}. Thereby, λ includes the parameters in (3.3) that need to be estimated as well as the parameters determining the Markov process, i.e. the initial state ξ_0 and the transition probabilities p_{uv}. Formally, the expectation maximization gives

$$Pr(\xi|\pi_i, \lambda^{j-1}).$$

In the maximization step the parameter vector λ is estimated with the help of the first-order condition of the likelihood function given by

$$p(\pi_i|\lambda) = \int p(\pi_i, \xi|\lambda) d\xi, \tag{3.4}$$

This may be rewritten as

$$p(\pi_{i,t}|\lambda) = \int p(\pi_i|\xi, \theta) Pr(\xi|p_{uv}, \xi_0) d\xi. \tag{3.5}$$

The conditional regime probabilities $Pr(\xi_t|\pi_i, \lambda)$ are replaced by smoothed regime probabilities obtained from the last iteration of the expectation step.

We start the analysis for each country with two regimes. We then test the non-linear model with two regimes against the linear alternative using a Davies test[9]. If the Davies test cannot reject the null, we will also consider more than two states in the regression. The number of differenced lags is chosen such that the residuals are not autocorrelated. We start with the most general model (MSIAH), which allows for a state-dependent intercept (I), parameters (A) and variance (H). Using LR tests[10] we investigate, whether further restrictions are appropriate (i.e. MSIH, MSAH, MSH). The estimation is carried out with Hans-Martin Krolzig's MSVAR package.

Coefficient	**France**	**Germany**	**Italy**	**Netherlands**	**Spain**
c_1	0.11	0.11	-0.10	-0.05	0.14
	(0.01)	(0.02)	(0.04)	(0.01)	(0.02)
c_2	0.35	0.06	0.03	0.23	—
	(0.04)	(0.13)	(0.03)	(0.09)	(—)
α_1^*	0.24	0.61	0.57	0.84	0.24
	(0.06)	(0.05)	(0.19)	(0.04)	(0.11)
α_2^*	—	0.98	0.90	0.67	0.73
	(—)	(0.12)	(0.07)	(0.12)	(0.09)
p_{11}	0.99	0.99	0.99	0.99	0.97
p_{22}	0.97	0.98	0.99	0.97	0.95
σ_1	0.16	0.18	0.19	0.15	0.15
σ_2	0.28	0.51	0.22	0.44	0.28
ln$L$	91.71	-98.14	71.60	28.05	68.16
# lags	1	9	11	11	2
Model	MSIH	MSIAH	MSIAH	MSIAH	MSAH

Table 3.2: *Inflation Persistence Measured as the Sum of AR Coefficients - Evidence from Non-Linear Models. Numbers in Parentheses Indicate the Standard Errors.*

Results

The numerical results of the estimation are reported in table 3.2. For all countries we estimated a model with two regimes, as proposed by the Davies test. We also tried specifications with a higher number of regimes, which did not deliver any reasonable results[11]. The temporal distributions of the regime probabilities are given in section 3.4.1 of the appendix. We can basically observe a dominance of regime 2 during the first half of the sample in all five countries, while regime 1 is more appropriate in describing the data during the second half of the sample. This regime change is first observed in France, followed by the Netherlands, Spain and finally Germany and Italy. While we find evidence for lower inflation persistence in Germany, Italy and Spain in regime 1, as indicated by α_1^*, the opposite is true for the Netherlands. Using an LR test, France provides

[9]The Davies test is a modified likelihood ratio (LR) test and has the null hypothesis $M-1$ regimes vs. $\geq M$ regimes. In the case when $M = 1$ we basically test for a linear vs. a non-linear specification. For more details see Davies (1987).

[10]The LR test can be based on the LR statistic $LR = 2(lnL(\lambda) - lnL(\lambda_0))$, where λ_0 denotes the restricted ML estimate of the parameter vector λ. Under the null, LR has an asymptotic χ^2 distribution with r degrees of freedom, where r is the number of restrictions.

[11]The usual information criteria did also favor two-state specifications instead of more complex ones.

no evidence at all for a change in the autoregressive coefficients so that we estimated an MSIH model. The LR test does also suggest a regime-invariant intercept for Spain. For all countries we find a decrease in variance from regime 2 to regime 1.

Despite the differences in the model specifications and results, we observe a simultaneity in the regime switches across countries, as the switch toward regime 1 occurs in all five countries around 1985. Given these results and the fact that all five time series provide evidence for non-stationary behavior, as shown in table 3.1, we attempt in the following analysis to identify one or more cointegrating relationships between the variables. Given that there exists at least one cointegrating relationship, we estimate an MS-VECM, which allows for parameter changes in the adjustment coefficients and the short-run dynamics.

3.2.3 Evidence for Changes in Inflation Transmission

Economic Fundamentals

For an analysis of the dynamics and transmission of inflation in the euro area, it is necessary to have a theoretic framework on which the econometric model can be based. A simple but effective model framework is offered by the relative purchasing power parity (PPP), which may be found in many standard macroeconomics textbooks[12]. We define the real exchange rate ρ as

$$\rho = E\frac{P}{P^*}, \tag{3.6}$$

where E denotes the nominal exchange rate, P is the price of domestic goods in domestic currency and P^* the price of foreign goods in foreign currency. When we now consider rates of change in the real exchange rate, we end up with

$$\frac{\Delta\rho}{\rho} = \frac{\Delta E}{E} + \frac{\Delta P}{P} - \frac{\Delta P^*}{P^*}, \tag{3.7}$$

which is the sum of the change in the nominal exchange rate, i.e. nominal appreciation, plus the change in the domestic price level, i.e. domestic inflation, less foreign inflation. While absolute PPP demands the real exchange rate to be equal to 1 in the long run, the relative PPP requires

[12] E.g. Burda and Wyplosz (2005).

it to be constant in the long run, i.e. $\rho = const$. From this it follows that $\frac{\Delta\rho}{\rho} = 0$, which finally gives

$$\frac{\Delta E}{E} = \pi^* - \pi. \tag{3.8}$$

This relationship will be helpful for the empirical analysis presented below, as it gives a theoretical relationship for the long-term behavior of the inflation rates of the countries under consideration. This means that (3.8) offers us a way to test for a long-term relationship between the rates of inflation using cointegration methods given that the inflation rates may be considered as $I(1)$ variables, which has already been analyzed in section 3.2.1.

Methodology

We formulate a simple vector autoregression (VAR) in the five inflation rates with lag length $q+1$, in which we should expect a positive relationship between the inflation rates, as implied by the relative purchasing power parity. The VAR takes the following form

$$y_t = c^* + \sum_{s=1}^{q+1} A_s^* y_{t-s} + u_t, \tag{3.9}$$

where y_t is a 5×1 vector in the inflation rates, A_i^* are the 5×5 matrices of coefficients and u_t is a 5-dimensional vector of Gaussian errors. c^* denotes a 5-dimensional vector of constants.

Given that the five endogenous variables are cointegrated, we may formulate a VECM as implied by Granger's representation theorem

$$\Delta y_t = \alpha(\beta' y_{t-1} + c) + \sum_{s=1}^{q} A_s \Delta y_{t-s} + u_t. \tag{3.10}$$

The non-linear version of (3.10) would be given by

$$\Delta y_t = \alpha(S_t)(\beta' y_{t-1} + c) + \sum_{s=0}^{q} A_s(S_t) \Delta y_{t-s} + u_t, \tag{3.11}$$

where S_t indicates the state of the coefficients at time t. Further, we consider the covariance matrix of the error terms Σ to be state-dependent. In particular, we assume $u_t | S_t \sim N(0, \Sigma(S_t))$. The unobservable regime variable S_t is determined by a Markov chain with M states.

H_0	Trace Stat.	5% Critical Val.	Max. Eigenvalue Stat.	5% Critical Val.
$r = 0$	207.54	59.46	78.87	30.04
$r = 1$	128.67	39.89	57.20	23.80
$r = 2$	71.47	24.31	42.96	17.89
$r = 3$	28.51	12.53	25.61	11.44
$r = 4$	2.90	3.84	2.90	3.84

Table 3.3: *Results Trace Test and Maximum Eigenvalue Test.*

For the estimation of (3.11) we use the two-step procedure proposed in Krolzig (1996). The first step uses the Johansen maximum likelihood procedure to determine the cointegration rank r of the system (3.10) and the corresponding cointegrating relationships (Johansen, 1995). In a second step we estimate the remaining coefficients in (3.11) given the cointegrating relationships obtained in the first step using the EM algorithm.

Cointegration Analysis

Given that the five inflation series were all found to be integrated of order one for the entire observation period, we want to analyze in the following, if we are able to identify common stochastic trends among them, using a cointegration analysis. Table 3.3 reports the results of the trace and maximum eigenvalue test[13]. Both tests indicate four cointegrating relationships[14]. The results given in the following were produced by the Johansen procedure[15]. In particular, we find[16]

[13] We did not include a constant or trend in the test specification. We first allowed for a constant inside the cointegrating relationship, which turned out to be not significant in any cointegrating equation. We included five differenced lags, as suggested by the Akaike criterion.

[14] These results were also supported by pairwise country-specific cointegration tests.

[15] We also applied the simple two step (S2S) method described in Lütkepohl and Krätzig (2004), which left the results basically unchanged.

[16] Numbers in parentheses indicate the standard errors.

$$z_{1,t} = \pi_{FR,t} - \underset{(0.06)}{0.28}\,\pi_{GER,t} \quad (3.12)$$

$$z_{2,t} = \pi_{IT,t} - \underset{(0.06)}{0.79}\,\pi_{GER,t} \quad (3.13)$$

$$z_{3,t} = \pi_{NL,t} - \underset{(0.09)}{1.11}\,\pi_{GER,t} \quad (3.14)$$

$$z_{4,t} = \pi_{ES,t} - \underset{(0.08)}{0.40}\,\pi_{GER,t}. \quad (3.15)$$

We decided to order Germany last, as it is the largest economy within the group of countries under consideration. This allows an easier interpretation and comparison of the estimated coefficients. All coefficients head in the expected direction. Interestingly, we find coefficients for Germany, which are all smaller than one in absolute values except for the one in equation (3.14). This result contradicts the implications of the relative purchasing power parity (PPP), which would require a cointegrating vector of $(1, -1)$. A possible explanation for this strong deviations from the theoretically implied relationship could be the fact that the PPP is built on true prices and true inflation, while we use CPI inflation, which is of course just a proxy for true inflation. The underlying baskets of the five CPIs include different products with different weights, which may cause deviations from the theoretical relationships. Furthermore, the cointegrating relations come from a linear model that may be regarded as a first approximation to the non-linear one, which we think is more appropriate to describe the data. Since we think that the relative PPP is a plausible mechanism for these five economies, we continue not only with the results given by equations (3.12)-(3.15), but also with the theory-consistent cointegrating vectors[17] $(1, -1)$. Furthermore, a cointegrating vector $(1, -1)$ would be also consistent with fractionally integrated inflation rates. In the following we will report the results of both the restricted and the unrestricted version.

Model Specification

We estimated a MS-VECM with two regimes[18]. We allowed for regime-dependent error variance, which we think is reasonable in the case of inflation rates and which is also supported by formal

[17] We tested for cointegration with pairwise Engle-Granger tests using the correct asymptotic critical values of MacKinnon (1991). The null of no cointegration was clearly rejected at the 5-percent level in all cases.

[18] The Davies test could clearly reject the null hypothesis of a linear system. We also estimated the model with three regimes, which did not deliver any reasonable results. Further, one should note that the degrees of freedom substantially decrease with the number of regimes in a MS-VECM.

LR tests. We included one differenced lag, which turned out to be sufficient for the residuals exhibiting no significant autocorrelation.

Results

We obtain the following numerical results for the matrix of adjustment coefficients in the two regimes[19]:

$$\alpha_{unrestr}(S_t = 1) = \begin{bmatrix} \mathbf{-0.70} & 0.13 & 0.05 & 0.16 \\ \mathbf{0.41} & -0.19 & \mathbf{0.13} & -0.12 \\ \mathbf{0.55} & \mathbf{0.64} & \mathbf{-0.50} & -0.14 \\ 0.08 & \mathbf{0.32} & 0.12 & \mathbf{-0.94} \\ -0.05 & 0.31 & 0.34 & -0.14 \end{bmatrix}$$

$$\alpha_{unrestr}(S_t = 2) = \begin{bmatrix} \mathbf{-1.01} & -0.06 & \mathbf{0.25} & \mathbf{0.25} \\ \mathbf{-0.35} & \mathbf{-0.73} & \mathbf{0.51} & \mathbf{0.30} \\ \mathbf{-0.29} & \mathbf{0.26} & \mathbf{-0.19} & \mathbf{0.17} \\ \mathbf{0.37} & -0.08 & \mathbf{0.27} & \mathbf{-0.68} \\ \mathbf{-0.54} & \mathbf{0.42} & \mathbf{0.58} & \mathbf{0.51} \end{bmatrix}$$

$$\alpha_{restr}(S_t = 1) = \begin{bmatrix} \mathbf{-0.32} & -0.10 & -0.04 & \mathbf{0.47} \\ \mathbf{0.48} & \mathbf{-0.42} & 0.06 & -0.01 \\ 0.28 & \mathbf{0.67} & \mathbf{-0.50} & -0.30 \\ \mathbf{0.39} & 0.30 & 0.33 & \mathbf{-0.84} \\ 0.20 & 0.15 & 0.24 & 0.09 \end{bmatrix}$$

$$\alpha_{restr}(S_t = 2) = \begin{bmatrix} \mathbf{-0.64} & 0.18 & 0.15 & \mathbf{0.41} \\ \mathbf{-0.16} & \mathbf{-0.62} & \mathbf{0.44} & \mathbf{0.37} \\ \mathbf{-0.12} & \mathbf{0.28} & \mathbf{-0.36} & \mathbf{0.17} \\ \mathbf{0.58} & 0.02 & 0.11 & \mathbf{-0.63} \\ \mathbf{-0.54} & \mathbf{0.44} & \mathbf{0.61} & \mathbf{0.52} \end{bmatrix}.$$

[19]Numbers in bold face indicate significance at the 5-percent level.

Regardless of the specification chosen, we find the adjustment coefficients for Germany being insignificant at the 5-percent level in regime 1. This implies a weak exogeneity for Germany within regime 1, which reveals the dominance of Germany's monetary policy. The cointegrating equation between Germany and France has a negative and significant adjustment coefficient in the equation for France, which is a reasonable result. The same is true for the other countries respectively. The only exception is Italy in the unrestricted version of the model, where we obtain an insignificant adjustment coefficient for the cointegrating relation between Germany and Italy, while both the cointegrating relation between France and Germany and the one between the Netherlands and Germany are significant, but with a positive sign. Restricting those coefficients that are not statistically significant at the 5-percent level to zero, would imply an unstable system. In the version with the restricted cointegrating relationships we find basically the same results for the adjustment coefficients in regime 1, except for the fact that Italy exhibits a significant and negative adjustment coefficient for the cointegrating equation between Italy and Germany, which seems to be a more reasonable result than the one in the unrestricted version.

The most substantial result we may find in regime 2 is the fact that Germany is no longer weakly exogenous, regardless of how the cointegrating relationships are specified. All four adjustment coefficients become significant. Interestingly, we see that the overall number of significant adjustment coefficients has substantially increased, which would be consistent with a system of more strongly integrated economies. Again, we find that a cointegrating relationship between Germany's rate of inflation and a specific country does turn out to be significant for the specific country, which should be a reasonable result.

Figure 3.1 reports the temporal distributions of the two regimes for both the unrestricted and restricted version of the model. Again, we see that the results are hardly affected by the specification chosen. For the period 1970-1973 we find inflation transmission being described by regime 2 with Germany being endogenous. With the breakdown of the Bretton Woods system in 1973 we observe a shift toward regime 1 with Germany becoming weakly exogenous. Regime 1 played the dominant role in the years 1973-1983. Except for some brief changes in 1986, 1987 and 1989 regime 1 played basically no role after 1983. Finally, from 1989 onwards we see that the inflation transmission mechanism is solely described by regime 2.

Hence, regardless of how the cointegrating relations are specified, we find a similar distribution of the regimes over time. Furthermore, the regime shifts seem to be consistent with the evidence

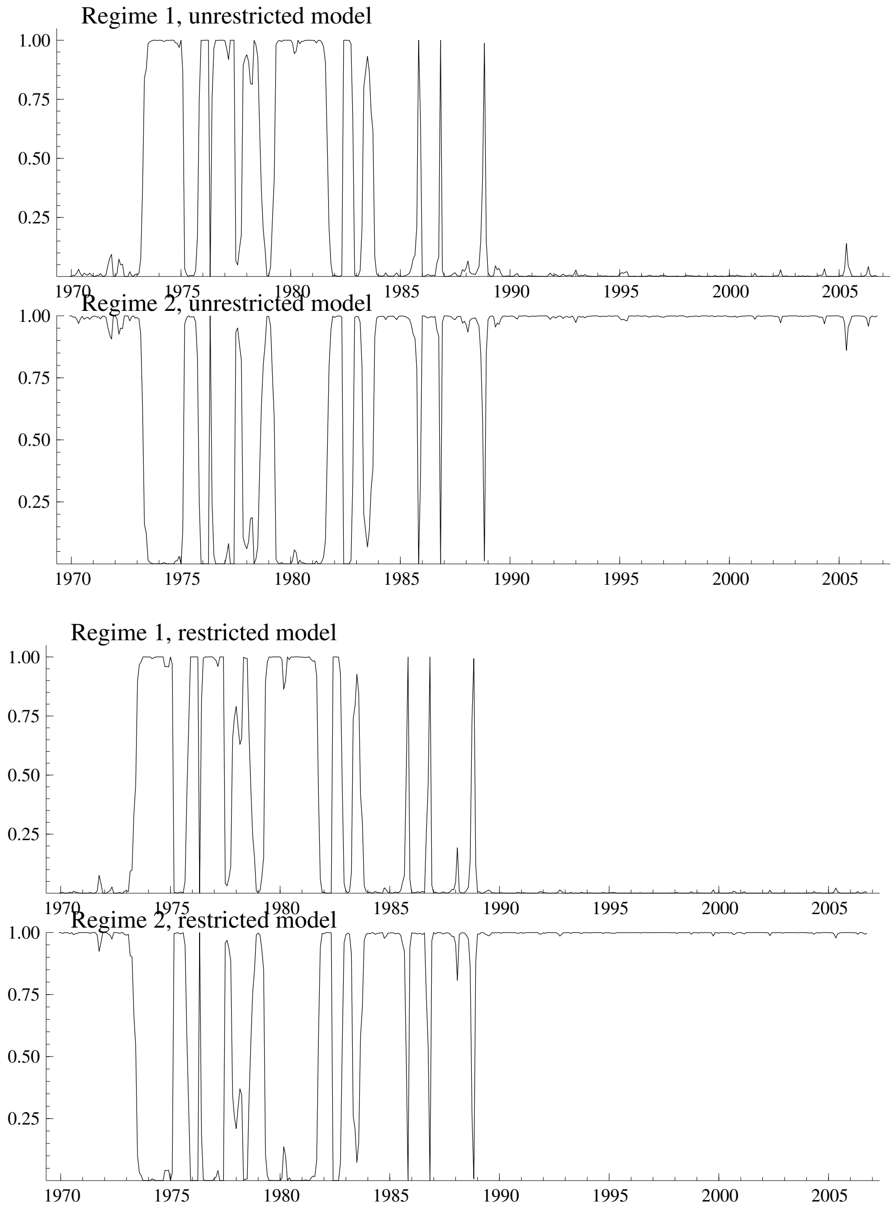

Figure 3.1: *Temporal Distribution of Regime Probabilities in the Unrestricted (Upper Panel) and Restricted Model (Lower Panel), 1970-2006.*

obtained from the univariate models in section 3.2.2. With respect to the adjustment coefficients we also obtain consistent results over the two specifications except for Italy, where we find more plausible estimates in the case of the restricted version of the model. For this reason, we will continue the analysis on the basis of the restricted model.
The results indicate that the dominance of German monetary policy already ended around 1984 with Germany being no longer weakly exogenous.

Impulse Responses

Impulse responses offer a fairly simple way to analyze the inflation dynamics and inflation transmission for the five countries under consideration. The impulse response analysis describes the response of $\pi_{i,t+\tau}$ to a one-time impulse shock in $\pi_{j,t}$ with all other variables held constant in period t. One problematic assumption in the impulse response analysis is that a shock in t occurs only in one variable. This demands the error terms summarized in the vector u_t in (3.11) to be uncorrelated. This assumption will be hardly fulfilled, when considering the inflation rates of the five largest economies in the euro area. Therefore, we need a way to orthogonalize the impulse responses. We do this using a Cholesky decomposition of the covariance matrix. The Cholesky decomposition requires an appropriate causal choice of the variables' ordering, as the impulse responses may be substantially affected by it. We decided here to order the inflation rates by the economic size of the countries, i.e. we put Germany first followed by France, Italy, Netherlands and Spain. A covariance that turned out to be insignificant at the 5-percent level was restricted to zero[20].
Figure 3.7-3.11, given in section 3.4.2 of the appendix, show the country-specific impulse responses to a one-unit shock. For the impulse response analysis we restricted those coefficients in the short-term dynamics that turned out to be insignificant at a 5-percent level to zero. For Germany it then follows that its rate of inflation during regime 1 is described by a random walk, as both long-term and short-term dynamics are not significantly different from zero in the VECM representation. Therefore, it should not be surprising in the following, when we do not find any response of Germany in regime 1 to inflation shocks taking place in one of the other countries.

[20]We used simple t-tests to test for the corresponding correlation coefficient being significantly different from zero. The impulse responses are computed with a Matlab code that is available upon request.

Generally, one can see that the initial shock has become less persistent in regime 2 than it used to be in regime 1 in case of the two largest countries France and Germany, while the shocks in Italy, the Netherlands and Spain exhibit a persistence of almost equal size across the two regimes. With the initial shocks in France and Germany in regime 1 being more persistent than in regime 2, we find that the responses of the other countries show a higher degree of persistence. Furthermore, one can see that the importance of the three smaller countries for the transmission mechanism has increased in regime 2, particularly the importance of Italy and the Netherlands. In regime 1 in figure 3.8 we observe an initial negative impact of the German inflation shock on France, Italy and the Netherlands. The same is true for regime 2 in figure 3.7 for the reaction of Germany to the French inflation shock. This result is rather unexpected in that an inflationary shock can actually elicit a deflationary reaction. Other studies on inflation transmission have found similar results[21]. Eun and Jeong (1999) explain this result by an overshooting depreciation of the country's currency, in which the initial shock takes place. This then leads to lower import prices in the other countries despite the inflation shock. A theoretical foundation for this may be found in Dornbusch (1976) known as Dornbusch's overshooting result.

Due to the lower persistence of the inflation shock itself in France and Germany, we may say that inflation persistence has reached a lower level since late 1980s, when regime 1 occurred for the last time in the sample period. This result is particularly remarkable, as it offers an explanation for the decrease in inflation persistence in the euro area, as described in Altissimo, Ehrmann and Smets (2006). According to the impulse responses presented here, the fall in inflation persistence within the euro area during the last decade is substantially related to a fall in the persistence of inflation shocks in Germany and France.

Furthermore, one can easily see that the impact of an inflation shock in Germany and France on the other countries is almost negligible in regime 2, particularly in the case of Germany, while the three smaller countries, i.e. Italy, Netherlands and Spain, become more important in regime 2.

[21] Cf. Yang, Guo and Wang (2006) and Eun and Jeong (1999).

Robustness and Plausibility of the Results

As the results suggest that regime 2 alone describes the inflation transmission process since 1987 except for one spike in 1988, we separately estimated non-regime dependent VECMs for the periods[22] 1970-1986 and 1987-2006. When using the S2S method[23] as described in Lütkepohl and Krätzig (2004), we basically obtained the same results as in section 5.2.3 with respect to the adjustment coefficients. For the later period almost all adjustments coefficients headed in the same direction with almost equal size in comparison to regime 2. For the first subsample, we again found evidence for Germany being weakly exogenous. Furthermore, the estimations delivered adjustments coefficients, which headed in the same direction compared to regime 1, but with some differences in size. This should not be surprising, as the period 1970-1986 is not characterized by one regime alone.

A recent paper by Bataa et al. (2007) supports the results presented in this paper. They analyze structural breaks in inflation and causality in the international transmission process of price shocks. Their results provide evidence for structural breaks in inflation transmission between France, Germany and Italy at the end of the Bretton-Woods era and in the mid-1980s, which are consistent with the regime changes found in our analysis here.

3.3 Conclusions

This paper has empirically analyzed the inflation transmission mechanism in the euro area for the period 1970-2006 using Markov-regime switching models.

The analysis offers evidence for stable inflation dynamics and a stable transmission mechanism in the euro area since the mid-1980s. Interestingly, it turns out that the period in which Germany is weakly exogenous already ended 1984. This indicates that the period of Germany's monetary policy being dominant for today's euro area already ended earlier than is usually assumed. Using impulse responses, we find that inflation persistence across countries has substantially decreased since that time, which is in accordance with recent research on inflation persistence. Our analysis

[22] Again, we checked the variables for their properties, using the usual ADF tests. All inflation rates turned out to be integrated of order one in any of the two subsamples. The usual cointegration tests indicated four cointegrating relationships for both periods.

[23] We restricted the cointegrating vector to $(1, -1)$ for each of the four cointegrating relationships.

finds the reason for this decline in the decrease of inflation persistence in France and Germany, leading to less persistent responses of the other countries to a shock in inflation in one of these countries. For the period since the mid-1980s it is found that country-specific inflation shocks are less strongly transmitted across the euro area than in the period before. Instead we find that the country in which the shock takes place returns quite rapidly back to the long-term equilibrium, which contradicts conventional findings that inflation in the small country is caused by the large country[24].

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[24]Cf. Cheung and Yuen (2002).

3.4 Appendix

3.4.1 Regime Probabilities - Univariate Approach

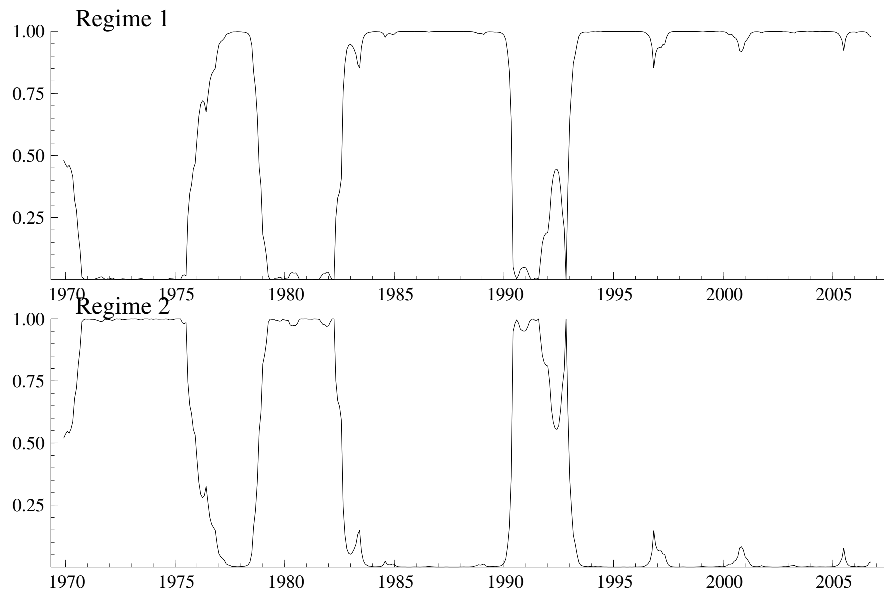

Figure 3.2: *France, Regime Probabilities, 1970-2006.*

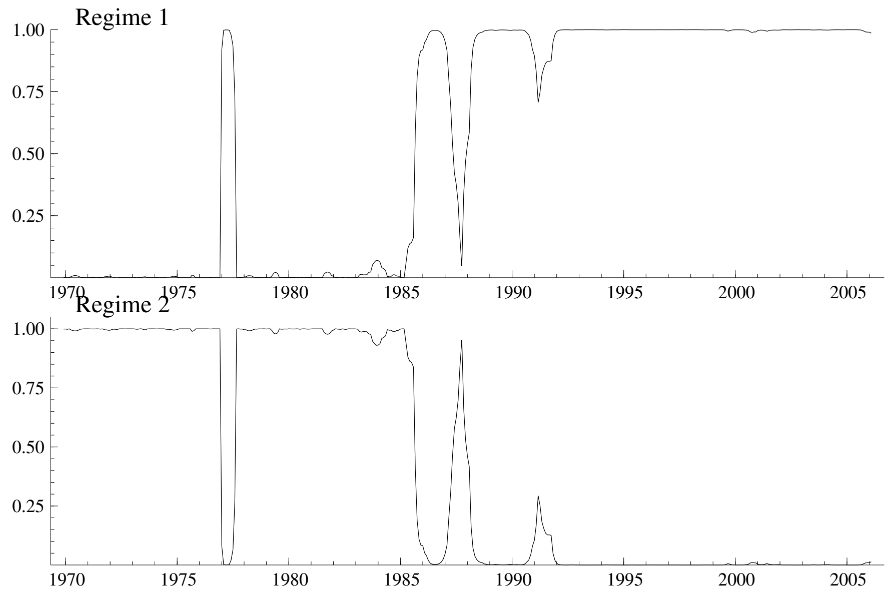

Figure 3.3: *Germany, Regime Probabilities, 1970-2006.*

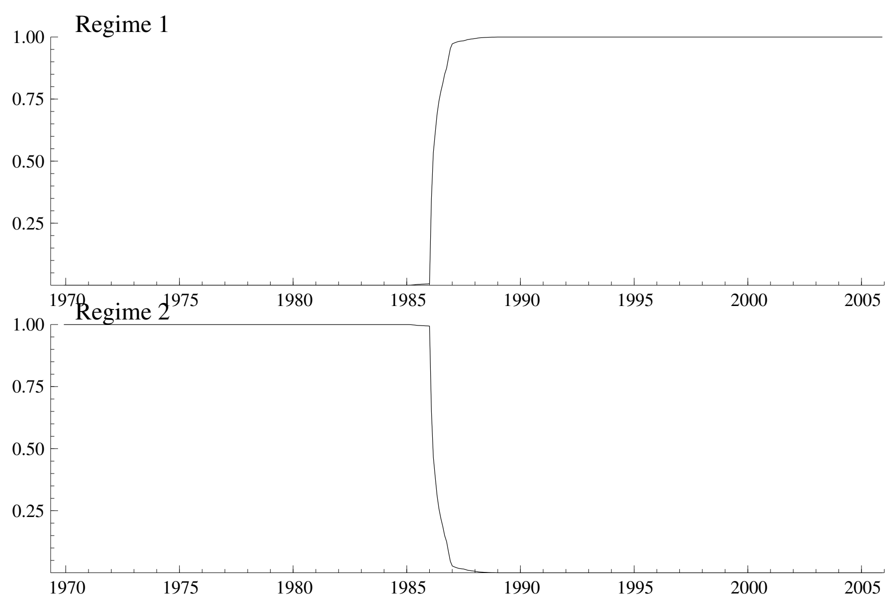

Figure 3.4: *Italy, Regime Probabilities, 1970-2006.*

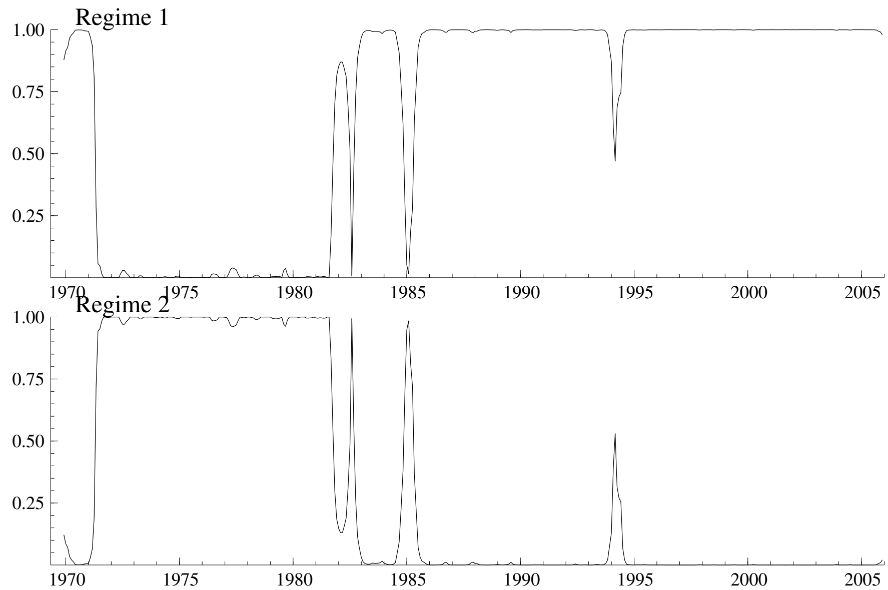

Figure 3.5: *Netherlands, Regime Probabilities, 1970-2006.*

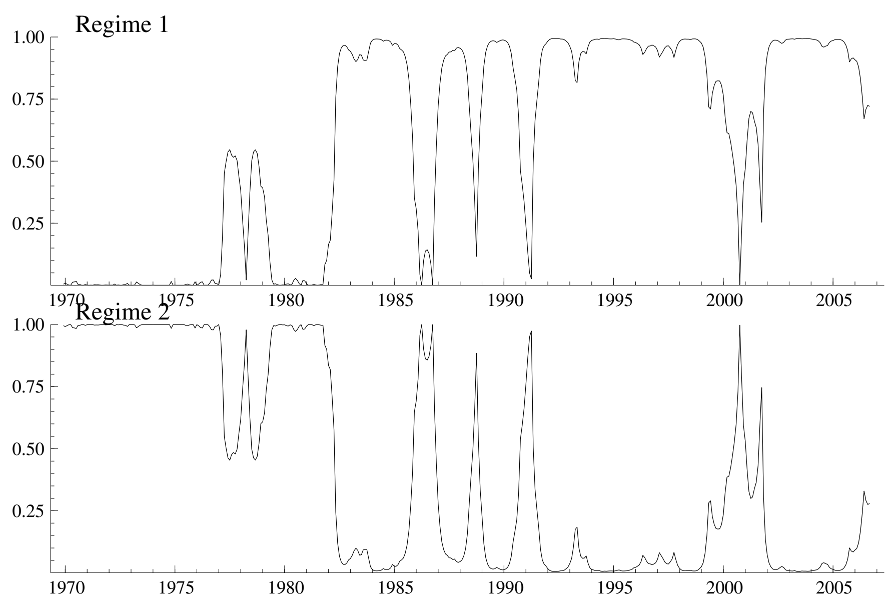

Figure 3.6: *Spain, Regime Probabilities, 1970-2006.*

3.4.2 Impulse Responses

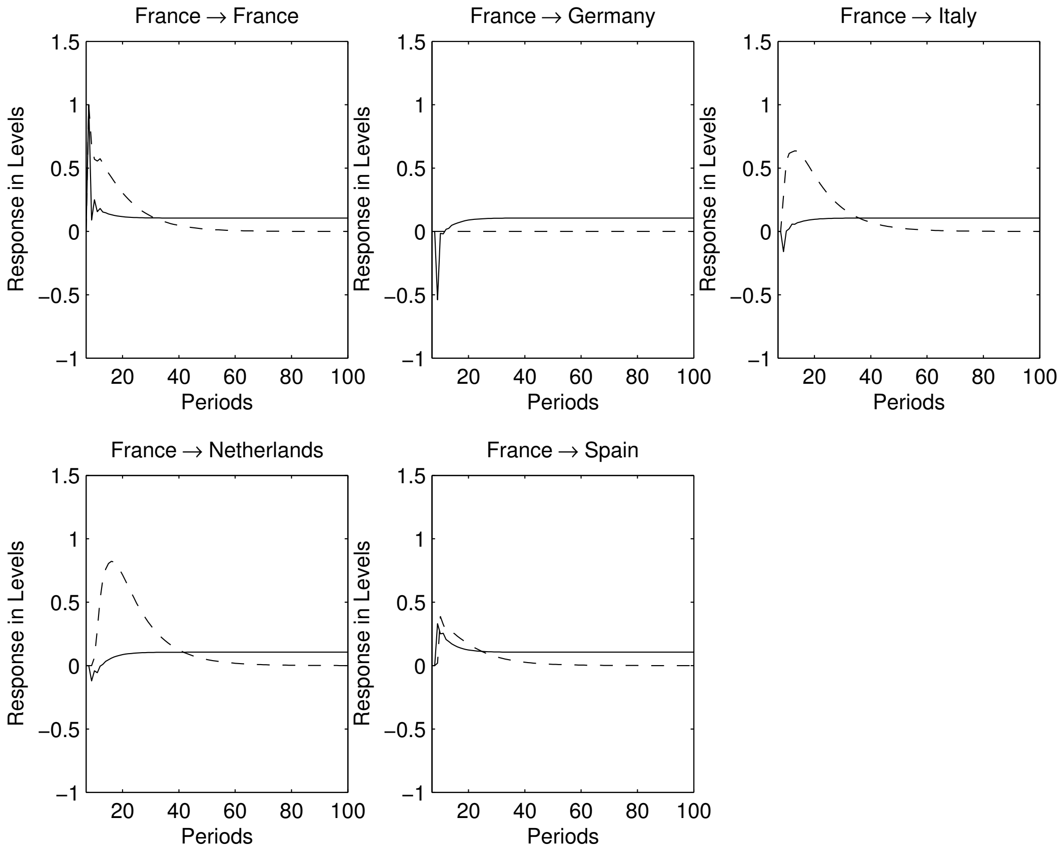

Figure 3.7: *Impulse Responses to a One Unit Shock in France (Dashed Line: Regime 1, Solid Line: Regime 2).*

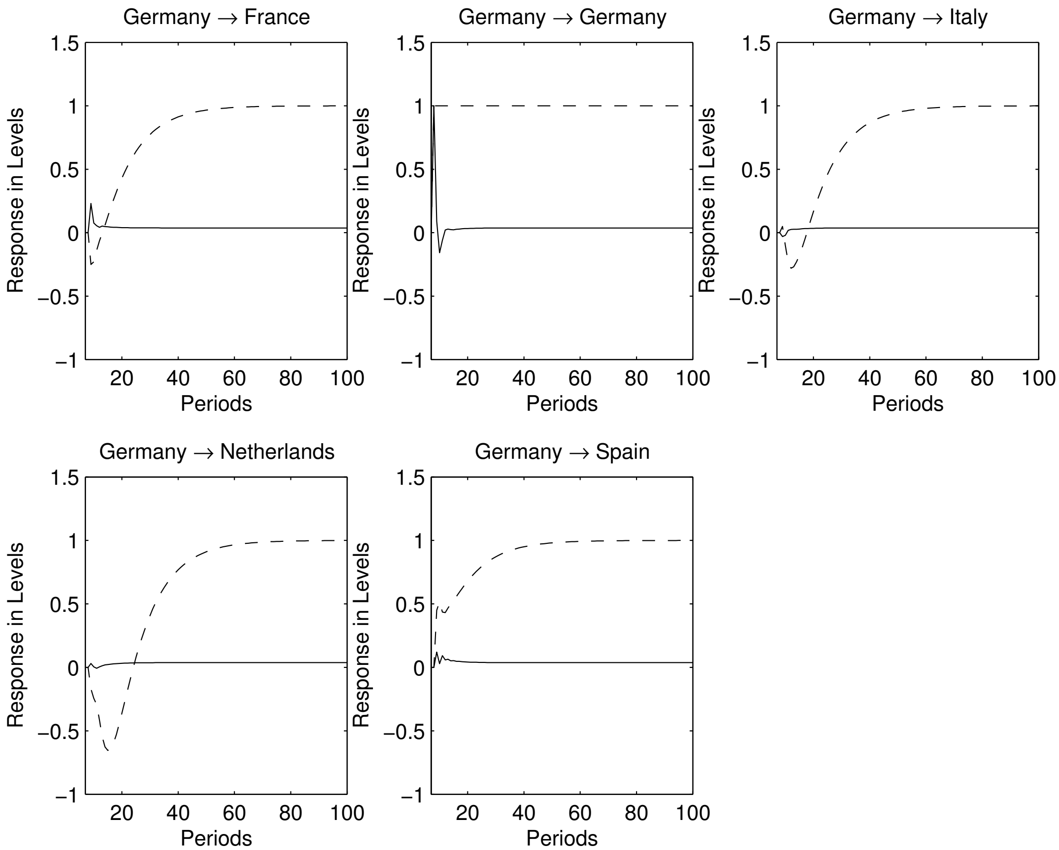

Figure 3.8: *Impulse Responses to a One Unit Shock in Germany (Dashed Line: Regime 1, Solid Line: Regime 2).*

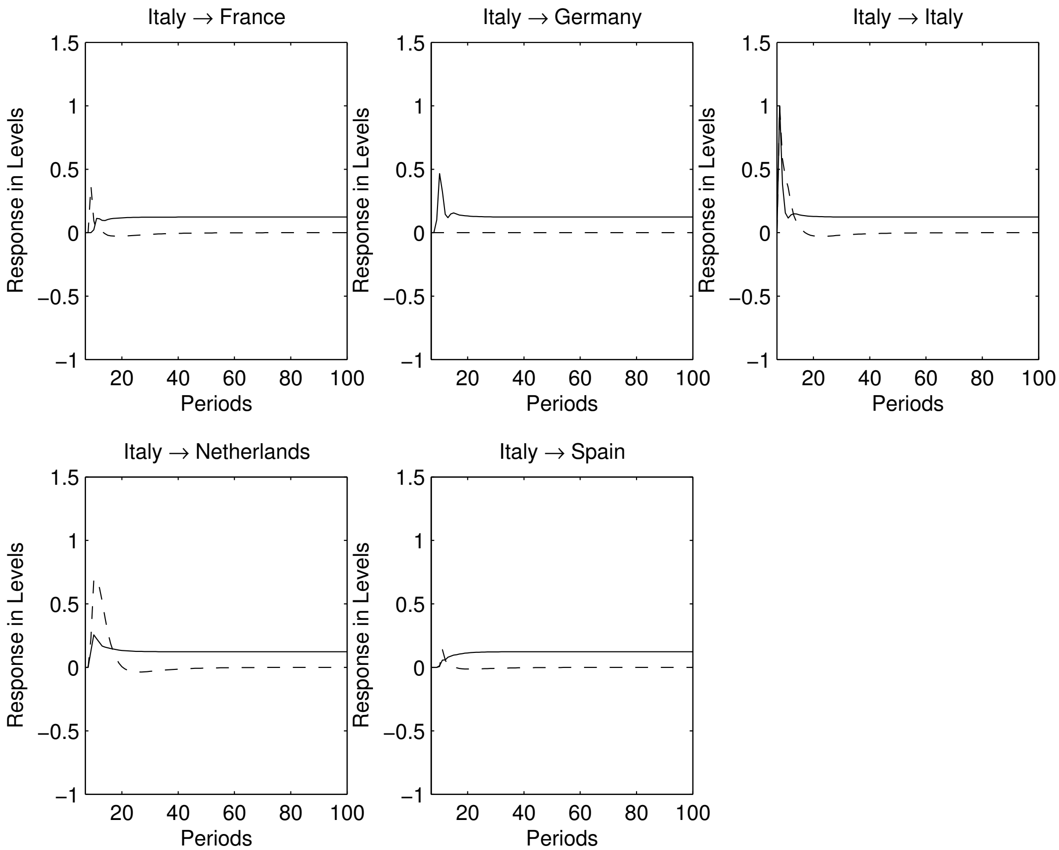

Figure 3.9: *Impulse Responses to a One Unit Shock in Italy (Dashed Line: Regime 1, Solid Line: Regime 2).*

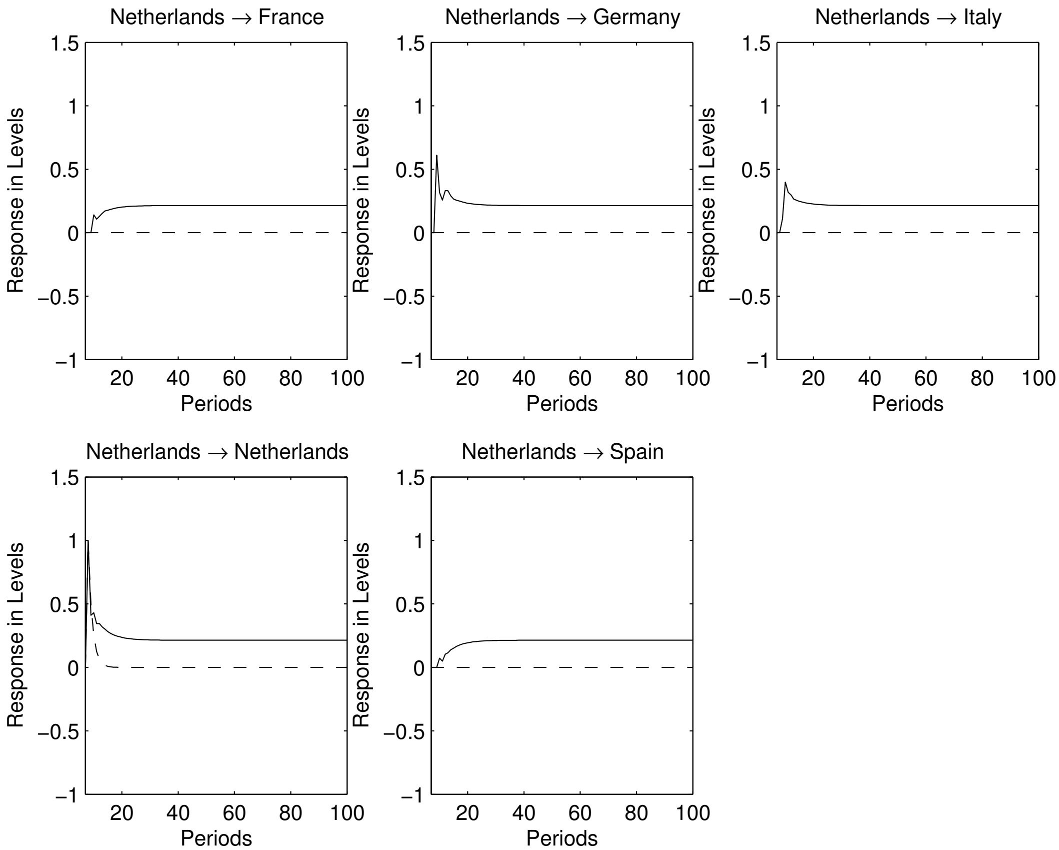

Figure 3.10: *Impulse Responses to a One Unit Shock in the Netherlands (Dashed Line: Regime 1, Solid Line: Regime 2).*

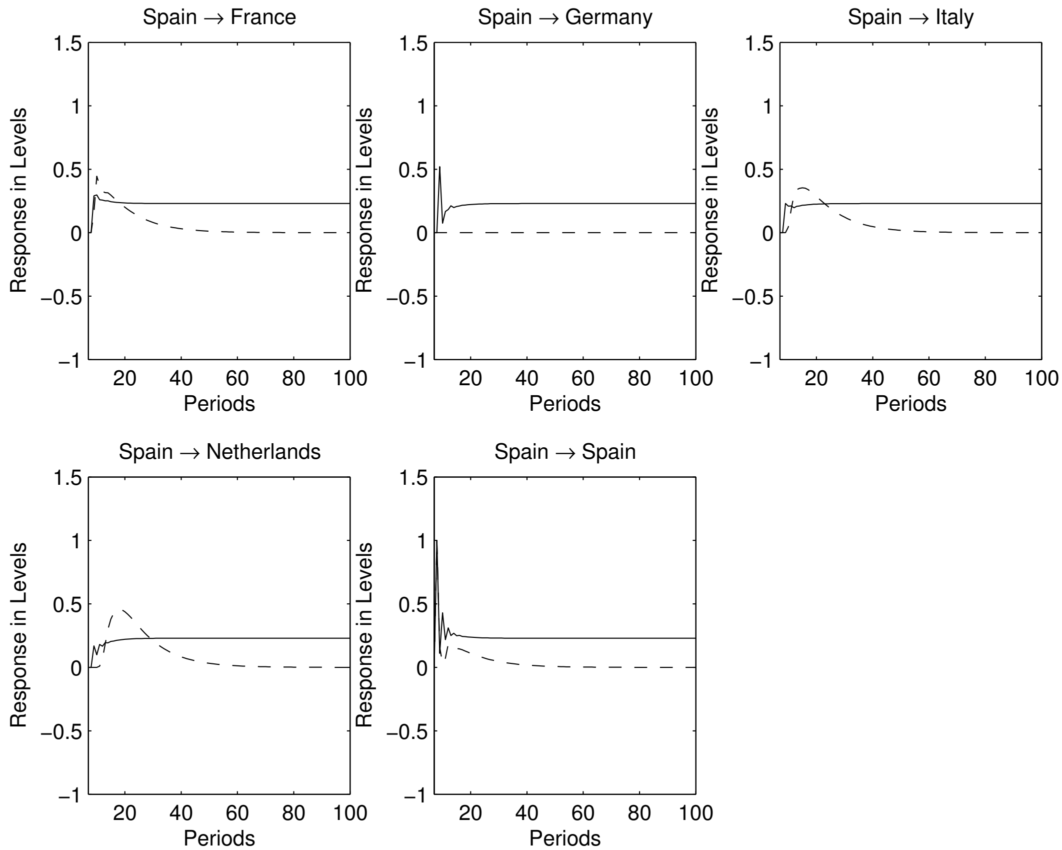

Figure 3.11: *Impulse Responses to a One Unit Shock in Spain (Dashed Line: Regime 1, Solid Line: Regime 2).*

3.4.3 Data

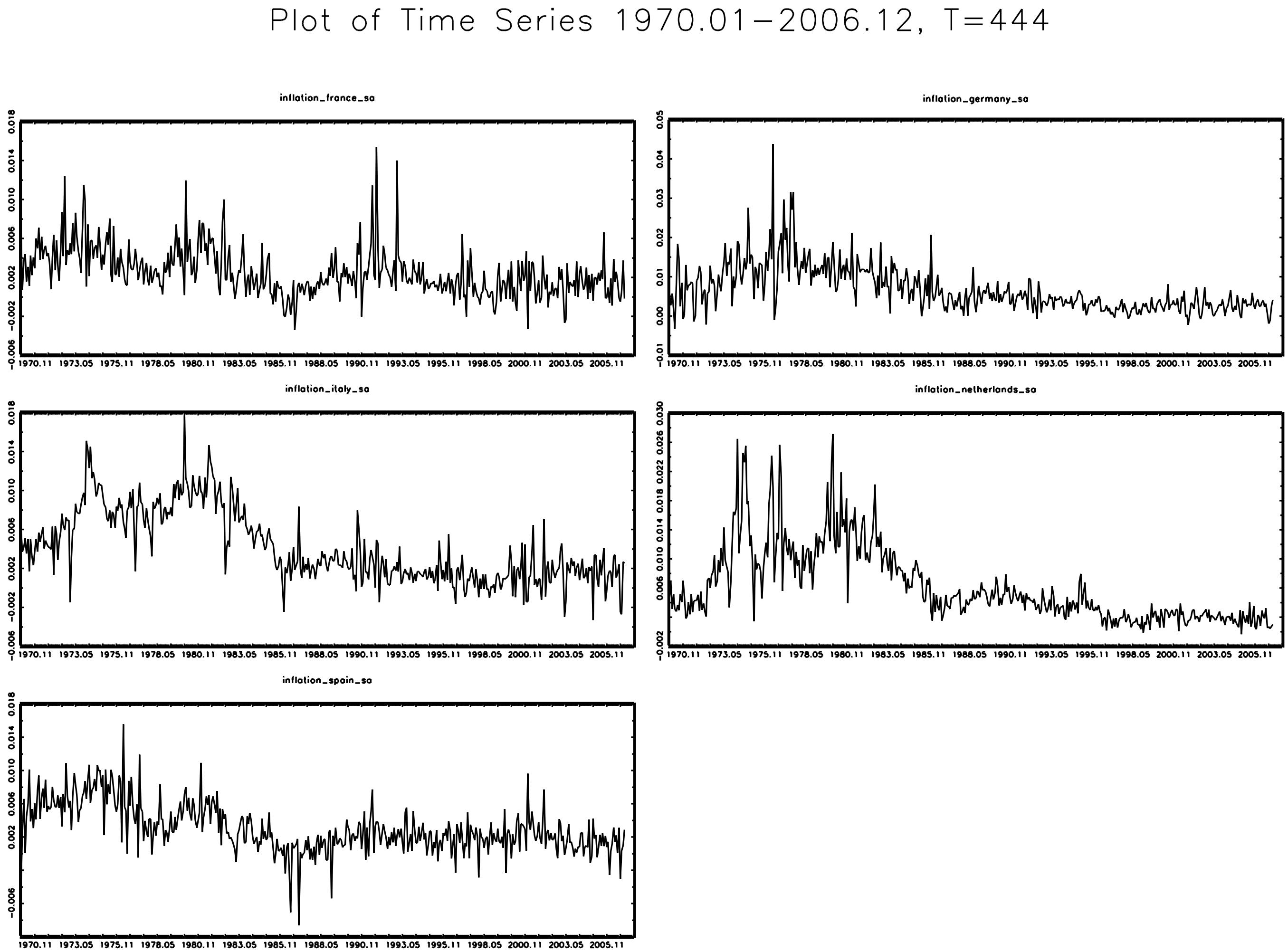

Figure 3.12: *Monthly Inflation Rates, Seasonally Adjusted, 1970-2006.*

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Chapter 4

The Relevance of the Fiscal Theory of the Price Level Revisited

This paper analyzes empirically the impact of fiscal policy on the price level for Germany and Spain. We investigate, whether the fiscal theory of the price level is able to deliver a reasonable explanation for the different evolutions of the price levels in these two countries during recent years. We apply a Bayesian vector autoregressive model with sign restrictions on the impulse responses to assess the relation between surpluses and public debt. The analysis basically evidences non-Ricardian equilibria in Spain, while the opposite is true for Germany. We interpret this as evidence for the inflation differences in these two countries being partially induced by fiscal policy shocks.

4.1 Introduction

4.1.1 Motivation

Since the introduction of the euro as the common currency in twelve Member States of the European Union (EU) in 2002, there has been a steady debate about the effectiveness of the Stability and Growth Pact (SGP), which requires all countries in the euro zone to have a budget deficit of less than 3% of their nominal GDP. This so-called 3-percent criterion has the aim to prevent excessive government deficits from occurring, which theoretically may lead to substantial increases in the overall price level.

In 2004 France, Germany, Greece, Italy and Portugal had a deficit ratio of 3% or higher, while the euro zone average was with 2.7% not considerably smaller than the limit imposed by the SGP. At the same time the rate of inflation in the EMU was quite modest not exceeding 2.2%. This raises the question, whether control of public debt is really a requirement for price stability? Or to put it differently: Is the fiscal theory of the price level (FTPL) a relevant mechanism?

This paper aims to answer this question by investigating German and Spanish data as an example of two countries which performed very differently in terms of inflation during recent years, although both countries were subject to the same monetary policy. We analyze, whether these inflation differences are related to fiscal policy.

We base the analysis on an extension of Canzoneri et al. (2001). We modify this approach by using Bayesian techniques and identify fiscal shocks by imposing sign restrictions on the impulse responses[1]. Furthermore, we include discount factors based on short- and long-term interest rates in the analysis to model changes in expectations about future fiscal policy.

To our knowledge, this is the first attempt to test for the relevance of the FTPL with German and Spanish data. The results show that the FTPL is able to explain differences in inflation rates between the two countries.

[1] The advantages and the motivation of this approach are discussed in section 4.1.2.

4.1.2 Literature Review

During the 1990s there has been a considerable amount of theoretical literature about the impact of fiscal policy on inflation. Cornerstones of this theory are the works of Leeper (1991), Sims (1994), Woodford (1994, 1995, 1996 and 2001) and Cochrane (1998, 2000). While traditional theory regards the stock of money as the sole determinant of the price level, the FTPL argues that if fiscal policy is free to set primary surpluses independently of government debt, fiscal shocks may well have an impact on the price level. Whereas traditional theory assumes that fiscal authorities adjust primary surpluses to guarantee solvency of the government for any price level[2], the FTPL considers the possibility that fiscal policy is able to set primary surpluses independently of government debt accumulated. As a result the price level will adjust to make the government's intertemporal budget constraint hold at any point in time. Woodford (1995) refers to these two cases of fiscal policy behavior with the terms "non-Ricardian" and "Ricardian". While Ricardian fiscal policy describes the case in which primary surpluses may not be set independently of government debt, "non-Ricardian" refers to the opposite case. In both cases the intertemporal budget constraint holds in equilibrium. The crucial difference between the two scenarios is the causal link between prices and surpluses.

Woodford (1996, 1998a) argues that fiscal shocks affect aggregate demand in non-Ricardian environments. This is induced, as he says, by the fact that households regard government debt as net wealth affecting their future path of consumption due to the exogeneity of government deficits.

Sims (1997) states that government commitments to stable prices can easily turn out to be unsustainable. Furthermore, the fact that there are practical bounds for governments on primary surpluses and unpredictable disturbances to fiscal balance, highlights the possibility of an exogenous path of government deficits. Sims (1997) concludes that generally an interest-rate-pegging policy, which is the fundamental idea of a monetary union, can only work, if each country with an initial level of public debt larger than zero commits itself to some positive level of primary surpluses in the future. From a game theoretic perspective each government has an incentive to deviate from this strategy to increase welfare of its own citizens leading to an upward jump in the price level. The costs of this policy have to be paid by all members of the monetary union.

[2] Cf. Barro (1974).

This implies that a monetary union can only succeed, if national governments are required to commit themselves to a deficit or surplus rule, i.e. a limit on borrowing as done by the existing SGP, or as Sims argues to a path of some positive primary surpluses.

Hence, theoretically there seems to be some evidence for a causal link between public debt and prices[3]. Empirically the evidence for the FTPL is less clear-cut.

Cochrane (1998) states that the FTPL per se has no implications for the time series of debt, surplus and price level that are directly testable. The budget constraint of the government written in nominal terms holds in both Ricardian and non-Ricardian regimes. Whether this equilibrium is restored by price or surplus adjustments remains unclear[4]. Hence, all we observe are equilibrium points, but not the fundamentals behind them. Woodford (1995) supports this view saying that it does not make much sense to test the FTPL in empirical terms. Heading in the same direction, Buiter (1999) states that “the government's intertemporal budget constraint is a constraint on the government's instruments that must be satisfied for all admissible values of the economy-wide endogenous variables.” So what really matters for the characterization of fiscal policy behavior is, whether prices or future surpluses adjust to make the government budget constraint hold. In empirical terms we may say that the FTPL exhibits a severe identification problem.

In recent years, there have been some attempts to measure empirically the effect of fiscal policy on the price level. Canzoneri et al. (2001) investigate U.S. data for the period 1951-1995 with a bivariate vector autoregressive model (VAR) in surplus-GDP ratio and liabilities-GDP ratio. This VAR specification allows one to analyze, whether prices or surpluses adjust in order to make the intertemporal government budget constraint hold. They come to the conclusion that fiscal policy in the U.S. may be considered to be Ricardian rather than non-Ricardian.

Bohn (1998) finds that U.S. fiscal surpluses have responded positively to debt. He argues that this provides evidence that U.S. fiscal policy has been sustainable, and although he does not directly comment on the FTPL, his results are consistent with those of Canzoneri et al. (2001). Janssen et al. (2002) analyze the impacts of monetary and fiscal policy on the path of inflation in the UK. This paper is remarkable, as it is built on almost 300 years of data starting in 1705. They also conclude that there is little econometric evidence that fiscal policy has significantly

[3]For the sake of completeness, we want to mention at this point that the theoretical relevance of the FTPL is also doubted by some contributions such as Buiter (2002).

[4]We will comment on this issue in greater detail within the next section of this paper.

affected the price level.

For the EMU, Afonso (2002) demonstrates, applying a panel data approach, that the FTPL is not supported for the EU-15 countries during the period 1970-2001. The Member States of the EMU tend to react with larger future surpluses to increases in the government liabilities. Therefore, fiscal policy may considered to be Ricardian.

Hence, there seems to be empirical evidence that Ricardian fiscal policies are possible and likely. More recent papers by Davig et al. (2006) as well as Davig and Leeper (2005) analyze regime switches in both fiscal and monetary policy for the U.S. They distinguish between active and passive behavior for monetary and fiscal authorities[5]. Their work shows that tax cuts always generate wealth effects and non-Ricardian outcomes as long as there is a positive probability for an active fiscal policy in the next period. Therefore, their work may be interpreted as evidence in favor of the FTPL mechanism.

Another attempt to examine fiscal policy regimes in the light of Markov-switching processes is carried out by Favero and Monacelli (2005). They investigate U.S. data for the period 1960-2002 and come to similar conclusions, namely that fiscal policy has switched between active and passive regimes.

With respect to the reliability of all these results, one should consider a point raised by Woodford (1998b) in the context of analyzing the FTPL empirically. He emphasized that a single-equation approach is unlikely to be sufficient to disentangle the impact of monetary and fiscal variables on the price level. Therefore, an empirical analysis of the FTPL demands a multivariate approach that controls for endogeneity of the variables. Canzoneri et al. (2001) follow such a multivariate approach using a VAR model. Janssen et al. (2002) pursue a very similar strategy in their analysis.

All of the empirical papers mentioned so far, use classical estimation methods. These raise specific problems in the context of analyzing fiscal variables. Classical econometric approaches relying on asymptotic test theory require the data to be stationary to avoid spurious regression results. In classical time series econometrics, the first step of the analysis usually involves to test for the stationarity of the variables, which are going to be included in the model. If the data turns out to be stationary, the model is estimated in levels, if not the model is estimated in differences, when no meaningful cointegrating relationships between the variables are found. But

[5] The terms active and non-Ricardian fiscal policy are basically equivalent as well as passive and Ricardian.

is this procedure appropriate, when analyzing fiscal variables? Bohn (2006) shows that theoretically fiscal variables, particularly debt and deficit series, can be integrated of any finite order without violating the sustainability condition imposed by the intertemporal budget constraint of the government. Further, fiscal policy may be even sustainable, when no appropriate cointegrating relationship between revenues and spending is found. This has important implications for applied work in fiscal policy evaluation in general. From the results of Bohn (2006) a repeated sequential strategy of differencing and unit root testing seems to be a possible solution for an empirical researcher working in this field. But this is, as Bohn (2006) states, challenging, since his results imply an open-ended sequence of differencing and testing, if fiscal policy were indeed unsustainable. In practice, many economic time series including public debt and deficits turn out to be stationary in first or second differences. This would immediately imply a sustainable fiscal regime, as otherwise the testing procedure would have been repeated infinitively. Therefore, classical econometric approaches do not seem to be promising, when studying the reaction and behavior of fiscal variables in general, as they fail in identifying unsustainable fiscal policy behavior.

Therefore, this paper embarks on a different empirical strategy. In section 2, we introduce a Bayesian version of the approach developed by Canzoneri et al. (2001), which does not require the data to be stationary. Following the method of Uhlig (1999) we identify fiscal shocks by imposing sign restrictions on the impulse responses. After the interpretation of the results obtained for Germany and Spain, we briefly summarize the findings and comment on the policy implications arising from the analysis in section 3. Finally, we attempt to answer the question, whether the FTPL is able to explain the different processes of inflation in these two countries.

4.2 Analysis

In the following, we introduce a Bayesian version of the method suggested by Canzoneri et al. (2001) to test for the FTPL empirically and apply it to German and Spanish data. We use Bayesian techniques, as, on the one hand they do not require the underlying time series to be stationary, while on the other hand they also allow one to formulate prior beliefs about the parameters in question[6]. To identify fiscal shocks we impose sign restrictions on the impulse responses.

4.2.1 Theoretical Foundation of the Model

The fundamentals of the FTPL can be explained by the government's budget constraint alone. The government's budget constraint in nominal terms for period t is naturally given by

$$B_t = (T_t - G_t) + (M_{t+1} - M_t) + \frac{B_{t+1}}{1+i_t}, \tag{4.1}$$

where M_t denotes the stock of base money and B_t the stock of government debt outstanding at the beginning of period t. At this point it is important to notice that B_t and M_t are quoted in nominal terms and their values are fixed at the beginning of each period. The difference between taxes T_t and government expenditures G_t in period t yields the primary surplus. i_t is the nominal interest rate at time t.

(4.1) states that government liabilities outstanding in period t have to be repaid by either running a surplus in the same period, monetized by increasing the stock of base money, or financed by issuing new debt at the beginning of the next period.

We divide (4.1) by nominal GDP $P_t y_t$, where P_t denotes the price level in t and y_t real GDP in t. After some rearrangements using simple algebra we obtain

$$\frac{M_t + B_t}{P_t y_t} = \frac{T_t - G_t}{P_t y_t} + \frac{M_{t+1}}{P_t y_t}\frac{i_t}{1+i_t} + \frac{y_{t+1}/y_t}{(1+i_t)P_t/P_{t+1}}\frac{M_{t+1}+B_{t+1}}{P_{t+1}y_{t+1}}. \tag{4.2}$$

On the left-hand side of (4.2) we find the sum of base money and government debt outstanding divided by nominal GDP, which corresponds to the ratio of total government liabilities and GDP. As a short form of writing we will use L_t to denote total government liabilities in period t in the following.

On the right-hand side of (4.2), $\frac{T_t - G_t}{P_t y_t}$ describes the primary surplus of the government in period t scaled on nominal GDP. When we think of the government as renting the money supply to the private sector[7] charging $\frac{i_t}{1+i_t}$, the second term represents the central bank transfers scaled on current nominal GDP. Thus, the first two terms on the right-hand side of (4.2) add up to

[6] We will comment on this issue in greater detail later on.

[7] Cf. Obstfeld and Rogoff (1996), p. 537.

the total surplus-GDP ratio of the government, which we will denote in the following by S_t/Y_t. Using the notation introduced above, $\frac{M_{t+1}+B_{t+1}}{P_{t+1}y_{t+1}}$ reduces to L_{t+1}/Y_{t+1}. Finally, the numerator in $\frac{y_{t+1}/y_t}{(1+i_t)P_t/P_{t+1}}$ corresponds to real growth of GDP and the denominator gives the real interest rate, when applying the well-known Fisher equation. Using the standard Euler equation relationship, we may interpret the whole term as a discount factor of next period's total government liabilities. In the following we will refer to this discount factor as β_t.

This enables us to simplify (4.2) to

$$\frac{L_t}{Y_t} = \frac{S_t}{Y_t} + \beta_t \frac{L_{t+1}}{Y_{t+1}}. \tag{4.3}$$

Iterating this equation forward and recursively substituting $\frac{L_{t+1}}{Y_{t+1}}$, we obtain

$$\frac{L_t}{Y_t} = \frac{S_t}{Y_t} + E_t \sum_{j=t+1}^{\infty} \left(\prod_{k=t}^{j-1} \beta_k \right) \frac{S_j}{Y_j}, \tag{4.4}$$

which implies the transversality condition

$$\lim_{T\to\infty} E_t \left(\prod_{k=t}^{T+t-1} \beta_k \right) \frac{L_{t+T}}{Y_{t+T}} = 0, \tag{4.5}$$

with E_t being the expectations operator conditional on information available at time t. The flow budget constraint (4.4) has to be fulfilled at any point of time. This can basically be achieved in two ways:

1. Consider the case in which the surpluses follow an endogenous process so that (4.4) is fulfilled by adjustments in the sequence of S_t, whereas the values of the discount factor β_t and nominal GDP Y_t are determined outside the system. We refer to this type of fiscal policy behavior as Ricardian, since both real GDP and the price level remain unaffected by changes of the fiscal variables.

2. Let the sequence of primary surpluses be determined by an arbitrary exogenous process. Now, to make (4.4) hold, either the discount factor or the liabilities-GDP ratio have to move. As mentioned before, we assume nominal government liabilities to be fixed at the beginning of each period. Hence, equality of (4.4) can only be restored through Y_t in the numerator, which also implies an impact on the discount factor β_t. Fiscal policy is said to be non-Ricardian.

That means, whenever surpluses are set independently of the stock of government debt accumulated, nominal income is determined by fiscal policy actions. By definition, nominal GDP is the product of real GDP y_t and the price level P_t. Thus, an increase in nominal GDP will generally affect both real GDP as well as the price level[8], which induces a change in the discount factor. Using these basic insights in the FTPL we now try to figure out which of the variables considered above, responds to changes in the fiscal variables using German and Spanish data.

4.2.2 Model

In the following, we use the implications of the FTPL that follow from the government's budget constraint. In particular, we investigate, how the liabilities-GDP ratio reacts to changes in the surplus-GDP ratio.

If we suppose that S_t/Y_t increases in period t, then, if fiscal policy is Ricardian, we should either expect future surpluses to decrease or to use the surplus to repay the debt, if possible. Thus, an indicator for a Ricardian policy behavior would be a negative or zero-response of L_t/Y_t to a positive shock in S_t/Y_t. This Ricardian interpretation would only be reasonable, if the surplus shock is persistent in a sense that it does not immediately change in sign to a deficit so that the impact is immediately diminished. For this reason it will be important to regard the pattern of S_t/Y_t for conclusions about the character of the shock. Furthermore, the discount factor β should remain unaffected, if the response of L/Y is strong enough to leave the price level unaffected.

The non-Ricardian case is somewhat easier to describe in terms of the results we should expect. A non-Ricardian fiscal policy is definitively at work, if the reaction of L_t/Y_t is positive following a positive shock in S_t/Y_t for reasons which should be obvious from equation (4.4). Furthermore, a negative response of L_t/Y_t should also considered to be non-Ricardian, if S_t/Y_t is significantly negatively autocorrelated, i.e. the shock is not positively autocorrelated and quickly changes in sign, or if the discount factor reacts negatively to a significant shock in S_t/Y_t combined with a negative reaction of L_t/Y_t.

Formally, we analyze a VAR of the form

[8] A theoretical quantification of the impact fiscal policy has on both real GDP and inflation can be found for instance in Woodford (1996).

$$\begin{bmatrix} S_t/Y_t \\ L_t/Y_t \\ \beta_t \end{bmatrix} = \sum_{s=1}^{p} \underbrace{\begin{bmatrix} B_{11}(s) & B_{12}(s) & B_{13}(s) \\ B_{21}(s) & B_{22}(s) & B_{23}(s) \\ B_{31}(s) & B_{32}(s) & B_{33}(s) \end{bmatrix}}_{B(s)} \begin{bmatrix} S_{t-s}/Y_{t-s} \\ L_{t-s}/Y_{t-s} \\ \beta_{t-s} \end{bmatrix} + CD_t + \underbrace{\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix}}_{u_t}, \tag{4.6}$$

where the $B(s)$ are a set of p $(m \times m)$ coefficient matrices with m denoting the number of dependent variables included $(m = 3)$. D_t contains all deterministic variables, C is the corresponding parameter matrix. u_t is Gaussian with zero mean and

$$E[u_t u_t' | (S_{t-s}/Y_{t-s}; L_{t-s}/Y_{t-s})] = \Omega \tag{4.7}$$

with Ω being the positive definite symmetric and time-invariant covariance matrix of size $(m \times m)$. The VAR is an approximation of the non-linear relationship given in (4.3).

4.2.3 Data

All data used corresponds to statistics of the International Monetary Fund except for German GDP, which is taken from the Federal Statistical Office Germany. All data is denoted in nominal terms and has a quarterly frequency. For monetary liabilities, M, we take the monetary base including both money in circulation and reserves. Government debt, B, is represented by total government debt, which in the case of Germany includes both debt of federal and federal state authorities. L is then defined as the sum of total government debt, B, and the monetary base, M. For S we decided to take the difference of total government revenues and expenditures. As all other variables GDP enters in nominal terms. As the data is not seasonally-adjusted and the seasonal pattern is not completely captured by a regression on dummy variables, we applied the Census X-12 method for the seasonal adjustment. To capture movements in β, we included the inverse of the gross interest rate as a proxy for the discount factor. We analyze separately the impact of a fiscal shock on short-term and long-term discount factors to capture expectations about future policy behavior. To model the short-term discount factor, we use the average 3-month money market rate and for the long-term factor we take the yield on a 10-year government bond. The data has a quarterly frequency and starts for Germany with the 1^{st} quarter 1970 and ends with the 4^{th} quarter 1998. As the approach requires the monetary base to be included, the analysis is restricted to this period[9]. To take the German reunification into account we introduced a shift dummy in the case of Germany, which is zero before 1991 and one from 1991 onwards[10]. Unfortunately, the corresponding data for Spain is only partially available before 1986 so that the analysis of Spanish fiscal policy has to rely on the period 1986-1998.

4.2.4 Estimation Method

In opposite to Canzoneri et al. (2001) we choose a Bayesian instead of a classical approach. The reasoning for this analytic strategy was outlined in section 4.1.2. Generally, Bayesian inference has the advantage that it generally does not raise specific difficulties, as classical inference does, when the data analyzed is non-stationary, since the application of Bayes' theorem does not require the data to be stationary. This is particularly helpful, when the statistical properties of the data with respect to stationarity issues contradict economic theory.

When examining the data for both Germany and Spain it should be quite obvious that government liabilities as well as government liabilities divided by GDP are steadily increasing for almost the entire observation period. That means that the sample data is not mean reverting and hence not stationary in a common sense. When we apply an ADF test to the four series in the two countries, i.e. S/Y, L/Y and short- and long-term interest rates, the null of a unit root may not be rejected[11]. It turns out that all variables are $I(1)$. The non-stationarity property is statistically reasonable, but from an economic point of view it is dubious for the reasons given in section 4.1.2. By applying a Bayesian approach we do not have to make any statements about the order of integration, as Bayes' theorem holds regardless of the data being stationary or non-stationary. This is particularly helpful, when analyzing fiscal variables, as any statement about the order of integration would induce sustainability of fiscal policy[12]. Therefore, Bayesian techniques seem to be the natural approach, when analyzing fiscal variables.

In a Bayesian analysis we aim at finding the posterior probability density function (pdf) of

[9] One should note at this point that already since 1994, with Stage Two of the EMU, European monetary policy was coordinated. Cooperation between the national banks of the member states was encouraged by the European Monetary Institute.

[10] Alternatively, we did the regression with an impulse dummy for the years 1991-1993, which left the results basically unchanged.

[11] It should be obvious that allowing for a deterministic trend is not reasonable from the perspective of sustainable fiscal policy. The lag length was chosen according to the Akaike criterion.

[12] Cf. Bohn (2006).

the parameters. This posterior pdf is obtained in two steps. First, we choose a prior pdf, which expresses our prior beliefs about the coefficients in $B(s)$ and the covariance matrix Ω. Afterwards, we compute the likelihood function, i.e. the joint pdf of the data, conditional on the unknown parameters.

As Uhlig (1994) suggests, it is reasonable to assume a Normal Wishard distribution for the prior and the posterior pdf, $\phi_{NW}(B, \Omega^{-1}|\bar{B}, N, \Lambda, v)$, with $\bar{B}$ being the mean coefficient matrix of size $(p\times m)$, Λ the positive definite mean covariance matrix of size $(m\times m)$, N a semi-positive definite matrix of size $(p\times p)$ and finally $v \geq 0$ denotes the degrees of freedom to describe the uncertainty about B and Ω around $(\bar{B}, \Lambda)$. The prior distribution of the inverse of the covariance matrix Ω^{-1} follows a Wishard distribution of the form $W_m(\Lambda^{-1}/v, v)$. For the specification of the prior we have to choose values for $\bar{B}, \Lambda, N, v$.

We assume that our prior information is diffuse so that basically the parameters in $B(s)$ may take any value in the interval $-\infty$ to ∞ with equal probability. This implies that our prior beliefs are best represented by a flat prior. We obtain a flat prior by setting $N_0 = v_0 = 0$ and $\bar{B}_0$ as well as Λ_0 arbitrarily under the restriction that Λ_0 has to be positive definite. This prior specification implies that inferences are unaffected by information external to the current data[13].

As stated above, we aim at examining the reaction of L/Y to a positive shock in S/Y. For the sake of completeness we want to mention at this point that following the method of Dolado and Lütkepohl (1996) the null of no Granger causality from S/Y to L/Y may be rejected at the 5-percent level in both countries.

A meaningful impulse response analysis of (4.6) requires further identifying assumptions such that u_t is represented by a linear combination of orthogonalized structural shocks,

$$u_t = A\upsilon_t, \text{ with } E[\upsilon_t\upsilon_t'] = I,$$

where I denotes the identity matrix. Generally, the reaction or impulse response of L/Y to a shock in S/Y may be both positive and negative in sign. We know from the theoretical considerations given above that a positive response of L/Y leads to a non-Ricardian interpretation of the data in the corresponding period. Therefore, it is of little interest to consider those impulse responses with a positive impact on L/Y. Instead, we identify those impulse responses that

[13]See Gelman et al. (1995). For a general discussion on the appropriate prior pdf choice, the interested reader is referred to Zellner (1971).

are candidates for a Ricardian fiscal policy behavior. This is done with the pure-sign-restriction approach by Uhlig (1999). Using this approach we only consider those cases in which the orthogonalized impulse responses head for the desired direction during the period in which the shock takes place[14]. By applying the sign-restriction approach, we basically divide the draw of all impulse responses from the posterior distribution in those impulse responses which are candidates for a Ricardian interpretation and those which are not, i.e.

1. A positive (negative) shock in S_t/Y_t immediately leads to a negative (positive) impact on L_t/Y_t.
2. A positive (negative) shock in S_t/Y_t immediately leads to a positive (negative) impact on L_t/Y_t.

The discount factor remains unrestricted. We will then focus on determining, whether the further process, i.e. the process of the three variables after the shock has occurred, also matches a Ricardian pattern in case of scenario 1 and how many of the draws generally match scenario 1 and 2. We made 1,000 draws from the reduced-form posterior density and for each reduced-form draw 50 draws of the α-vector[15]. The lag length p is set to[16] 2. The model includes a constant and the deterministic terms described in section 4.2.3.

4.2.5 Results

In the following we provide the results for the two countries in form of impulse responses to a one-standard-deviation shock in S/Y. All impulse responses show the median response as well as the 16% and 84% quantiles corresponding to a one standard deviation band, if the distribution was normal.

Germany

Figures 4.1 and 4.2 show the results for Germany in the period 1970-1998, when the inverse of gross short-term and long-term interest rates are alternatively used to model changes in the

[14] The sign restriction is binding for only one period. The orthogonalized impulse responses are obtained from a Cholesky decomposition with S/Y being ordered first.
[15] Details about the meaning of the α-vector may be found in section 4.4.1 of the appendix.
[16] The model was also estimated with a higher number of lags, which left the results basically unchanged.

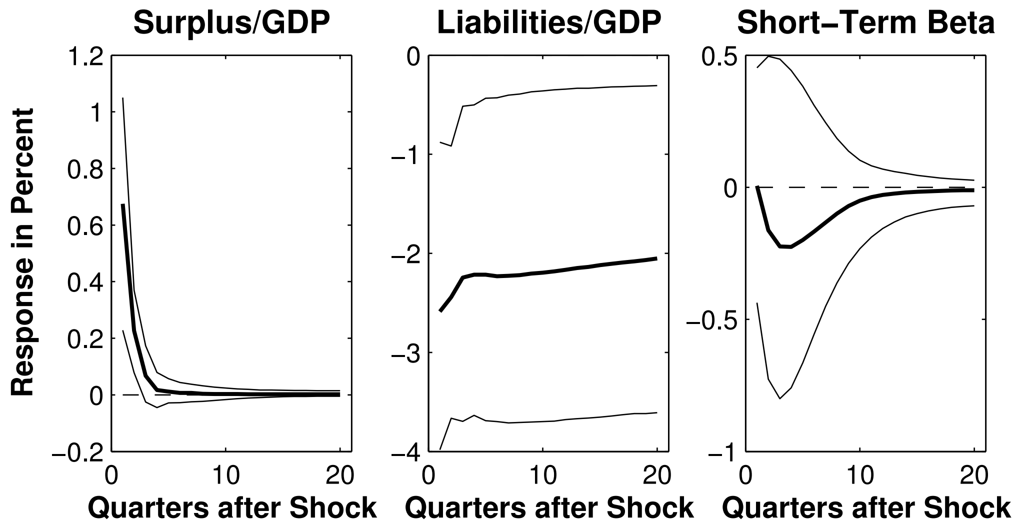

Figure 4.1: *Germany, 1970-1998, Response to a Surplus/GDP Shock in % with 68% Error Bands and Short-Term Discount Factor Included.*

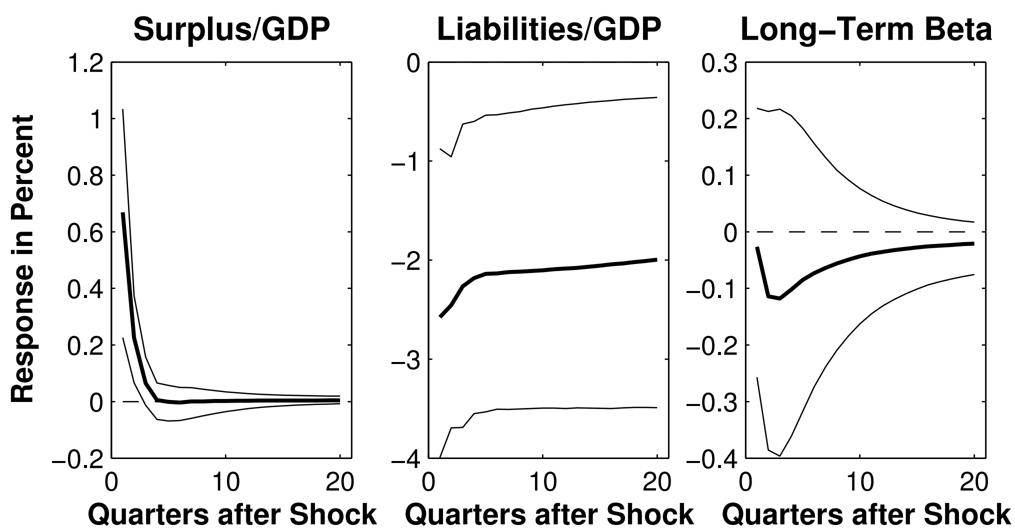

Figure 4.2: *Germany, 1970-1998, Response to a Surplus/GDP Shock in % with 68% Error Bands and Long-Term Discount Factor Included.*

discount factor. Basically, we can see that a positive shock in S/Y leads to a significant[17] and negative response of L/Y in the first period. This should not be surprising, as we used a sign restriction on the impulse responses to exclude all cases in which a positive surplus-GDP shock leads to a positive impact on L/Y. As the impact of the shock is significant and almost persistent in the process of L/Y for a horizon of at least 5 years, the results so far fit to a Ricardian interpretation of figures 4.1 and 4.2. The initial shock disappears after about one year, but does not change in sign, which gives further evidence for a Ricardian fiscal policy behavior. Furthermore, the median response of both the short-term and long-term discount factor is hardly significant with the upper and lower quantiles being symmetrically distributed around zero. With respect to the long-term discount factor the impulse responses are even more closely distributed around zero. That means that following a fiscal shock the liabilities-GDP ratio reacts strong enough to leave the discount factor unaffected. Hence, we may say that a fiscal shock has no influence on the discount factor. Taking the three pictures in figures 4.1 and 4.2 together, the analysis provides strong evidence that German fiscal policy was characterized by a Ricardian behavior during the sample period. In addition, this result is also confirmed by the fact that more than 60% of the overall number of impulse responses drawn from the posterior distribution match the prior sign restrictions, i.e. they follow a potential Ricardian pattern, so that the FTPL mechanism does not seem to be relevant for Germany in the period 1970-1998. The fact that both short- and long-term discount factors remain unaffected by the fiscal shock highlights that individuals expect fiscal policy to remain Ricardian in the future. Were this not the case, we should expect a significant reaction of the long-term discount factor.

Finally, regarding the quality of the model, we shall mention that the fiscal shock accounts for about 50% of the variance in L/Y and less than 20% of the variation in β. Thus, we find that a sizable fraction of the variation in L/Y can be attributed to fiscal shocks, while the opposite is true for the interest rates.

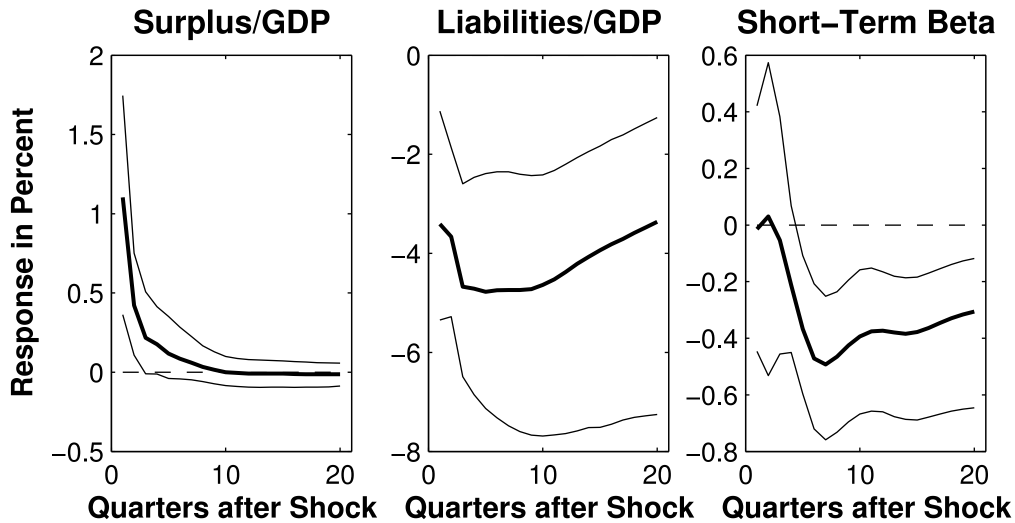

Figure 4.3: *Spain, 1986-1998, Response to a Surplus/GDP Shock in % with 68% Error Bands and Short-Term Discount Factor Included.*

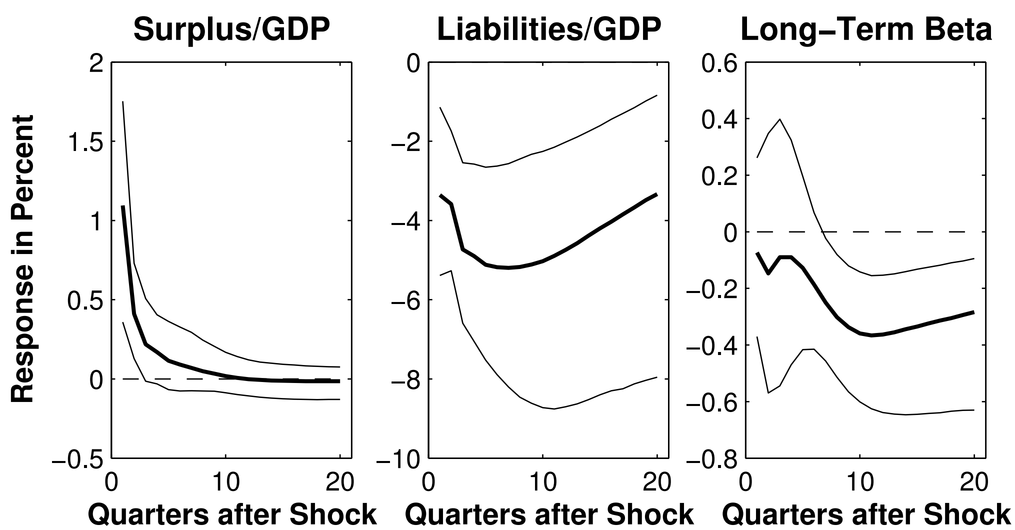

Figure 4.4: *Spain, 1986-1998, Response to a Surplus/GDP Shock in % with 68% Error Bands and Long-Term Discount Factor Included.*

Spain

The results for Spain are given in figures 4.3 and 4.4. In terms of the fiscal variables we can basically observe a similar pattern as for Germany. That means that both S/Y and L/Y generally fulfill the requirements for fiscal policy to be Ricardian also after the first period, for which we specified a Ricardian behavior. Again, about 60% of the overall number of impulse responses drawn from the posterior distribution match the prior sign restrictions. For a Ricardian interpretation, we should expect the discount factor to remain unaffected by the fiscal shock. Surprisingly, this is not the case. Regardless of the definition chosen, we see a strong and negative response of the discount factor. The median response is almost equal in size for both short- and long-term discount factors with the long-term discount factor showing a somewhat smaller response. Basically, there are two different explanations for a situation like the one depicted in figures 4.3 and 4.4 to occur. One would be that the reaction of future liabilities is not strong enough to leave the discount factor unaffected. Another would be that Spanish fiscal policy was indeed Ricardian during the period analyzed but individuals place a positive probability on fiscal policy to switch to a non-Ricardian regime in the near future, as both short- and long-term discount factors are affected by the fiscal shock. This in turn would imply that remaining in or returning to a Ricardian regime in the long run is not credible, as indicated by the reaction of the long-term discount factor.

As the response of the discount factor becomes significant after five quarters in the case of the short-term discount factor and after about eight quarters in the case of the long-term discount factor, which basically coincides with the turning point in the run of L/Y, we conclude that the former explanation is more reasonable, i.e. the reaction of liabilities is not strong enough to leave the discount factor unaffected. The fact that both the short- and long-term discount factors react similarly, shows that individuals do not expect fiscal policy to switch to a Ricardian regime in the future.

The non-Ricardian interpretation of the results is confirmed by the fraction of variation in both L/Y and the discount factor that is explained by the fiscal shock. With 50% of the impulse responses matching the sign restriction, a sizeable fraction of variation in L/Y can be attributed to the fiscal shock. In contrast to Germany we now find that also 40% of the variation in the

[17] Here, significance does not refer to hypothesis tests in a classical econometric sense. Instead, it describes a response of meaningful size.

discount factor is attributable to the fiscal shock, which should not be the case, if fiscal policy were indeed Ricardian leaving β unaffected.

Plausibility of the Results

In contrast to the results for Germany, Spain exhibits impulse responses that seem to be more consistent with a non-Ricardian fiscal policy. But is this really the case? Another possible explanation for the Spanish results could be a fiscal policy rule that aims to reinstate the pre-shock level of debt. Suppose there is an exogenous positive shock to the surplus. Under a Ricardian fiscal policy this will lead to a fall in real debt and the liabilities-GDP ratio respectively. In order to reinstate debt equilibrium, fiscal policy will raise expenditures. This fiscal expansion may then have an inflationary impact because of expansion in demand and the effect on marginal costs. A central bank committed to stable prices, will then increase interest rates, which in turn will have an impact on the discount factor. Therefore, it is worthwhile to consider the impact of the fiscal shock on the expenditure-GDP ratio for Spain. The impact of the fiscal shock on the expenditure-GDP ratio is given in figure 4.5 in the appendix[18]. It is obvious that with the median response of the expenditure-GDP ratio being initially negative the scenario described above does not seem to be a relevant alternative explanation for Spain. This means that the interpretation of the results for Spain remain unchanged, when conditioning on government expenditures.

Another possibility that we should consider and that might indeed lead to a scenario as depicted in figures 4.3 and 4.4, is the Samuelson effect. During the observation period Spain experienced a substantial increase in productivity, which then should lead to an increase in the price level. This in turn should have a positive impact on the interest rate. Therefore, we should also condition on productivity to check, if our results are induced by the Samuelson effect. The results are depicted in figure 4.6 in the appendix[19]. We can basically see that the results do not change with respect to the fiscal variables and the discount factor. The median response to the fiscal shock is considerably small. The upper and lower probability bands of the impulse responses are almost symmetrically distributed around zero. This implies that the results for Spain are not induced by a Samuelson-type effect.

[18]We just report the impulse response with the short-term discount factor being included. The results remained unchanged, when choosing the long-term discount factor.

[19]As a measure of productivity we used seasonally-adjusted GDP per hour worked taken from statistics of the OECD.

4.3 Conclusions and Policy Implications

The analysis has provided deeper insights in the relevance of the FTPL. It is shown that:

1. Despite increasing debt-output ratios during the sample period, Germany's fiscal policy clearly follows Ricardian patterns. We find evidence that this Ricardian policy behavior is credible, as the long-term discount factor provides no significant response.

2. Spanish fiscal policy exhibits non-Ricardian characteristics with discount factors being substantially affected by fiscal policy shocks. With the long-term discount factor offering almost the same reaction to the fiscal shock as the short-term discount factor, the analysis suggests that a change in Spanish policy behavior toward Ricardian characteristics does not seem to be credible.

The analysis yields evidence for the existence of the FTPL mechanism in Spanish data. The findings suggest that the FTPL is one relevant factor among others in explaining the differences in inflation rates between Germany and Spain. With the distinction between short- and long-term discount factors, the results provide evidence that not only the current characteristics of fiscal policy matter for the price level to be determined by fiscal actions, but moreover individuals' beliefs about future fiscal policy behavior. In this context then, in order for the price level to be unaffected by fiscal policy actions, it is essential that individuals believe fiscal policy to remain Ricardian or to return to a Ricardian regime in the future. Theoretically, this rationale has been elaborated by Davig et al. (2006).

The results highlight the necessity of a debt on borrowing as imposed by the SGP, since it is shown that there may be a causal link between prices and public debt. Furthermore, fiscal policy needs to be credible, as even the possibility of a change in the underlying policy regime may have substantial effects on the economy.

4.4 Appendix

4.4.1 Pure-Sign-Restriction Approach

The basic idea of the pure-sign-restriction approach is to consider only those impulse responses heading in the desired direction for at least Z periods. Let $a \in \mathbf{R}^m$ be an impulse vector, if there exists a matrix A such that $\Omega = AA'$ with a being a column of A. Following the notation used throughout the paper, Ω denotes the covariance matrix, m the number of variables in the vector of dependent variables, X_t, and p the lag length. Furthermore, let e_i for $i = 1, \ldots, n$ be the eigenvectors of Ω, normalized to form an orthonormal basis of $\mathbf{R}^m$, and υ_i the corresponding eigenvalues. Then, if there are coefficients α_i for $i = 1, \ldots, n$ such that $\sum_{i=1}^{m} \alpha_i^2 = 1$, the impulse vector a is given by

$$a = \sum_{i=1}^{m} (\alpha_i \sqrt{\upsilon_i})\, e_i. \tag{4.8}$$

To obtain the corresponding impulse responses we define $\mathbf{a} = \left[a', 0_{1,m(p-1)}\right]$. Given the impulse vector a, the impulse response of variable j with $j = 1, \ldots, m$ at horizon z may be computed as

$$r_{z,j} = (\Gamma^z \mathbf{a}), \tag{4.9}$$

$$\text{where } \Gamma = \begin{bmatrix} & B & \\ I_{m(p-1)} & & 0_{m(p-1),m} \end{bmatrix}.$$

For the application of the sign-restriction approach we take joint draws from both the posterior distribution of the VAR parameters and a uniform distribution over the $(m-1)$-dimensional sphere $(\alpha_1, \ldots, \alpha_{m-1})$. It is then possible to obtain the impulse vector a according to (4.8), which then may be used to calculate the impulse responses. Then, if the impulse response fulfills the sign restrictions imposed, we keep the draw. Otherwise we drop it from the further analysis.

4.4.2 Impulse Responses

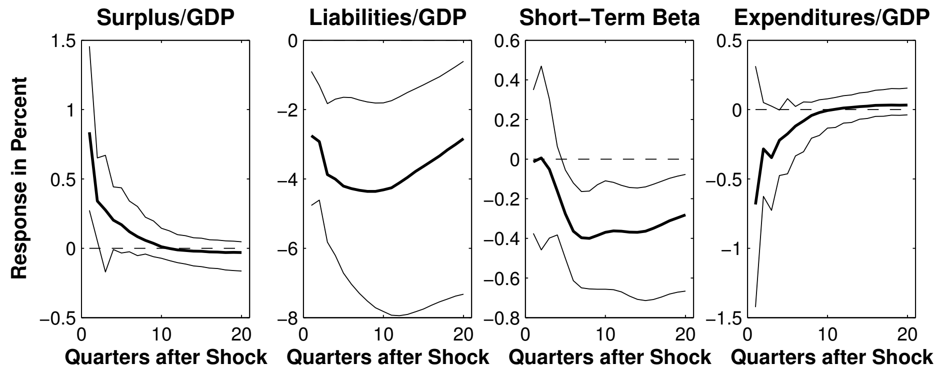

Figure 4.5: *Spain, 1986-1998, Response to a Surplus/GDP Shock in % with 68% Error Bands and Short-Term Discount Factor and Expenditure/GDP Included.*

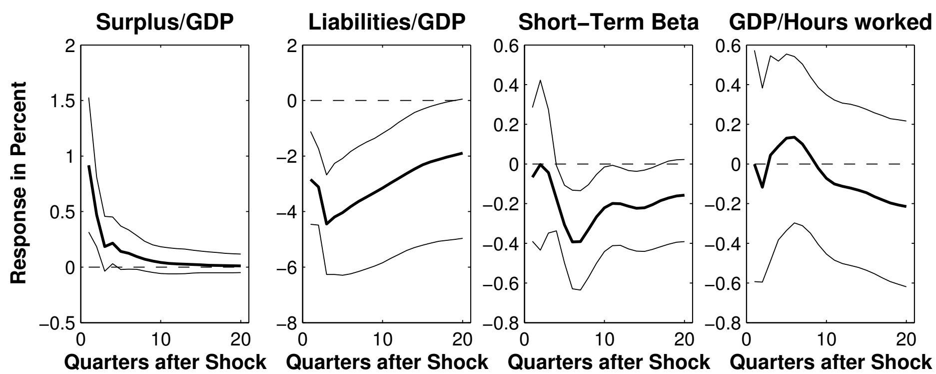

Figure 4.6: *Spain, 1986-1998, Response to a Surplus/GDP Shock in % with 68% Error Bands and Short-Term Discount Factor and GDP per Hours Worked Included.*

4.4.3 Data

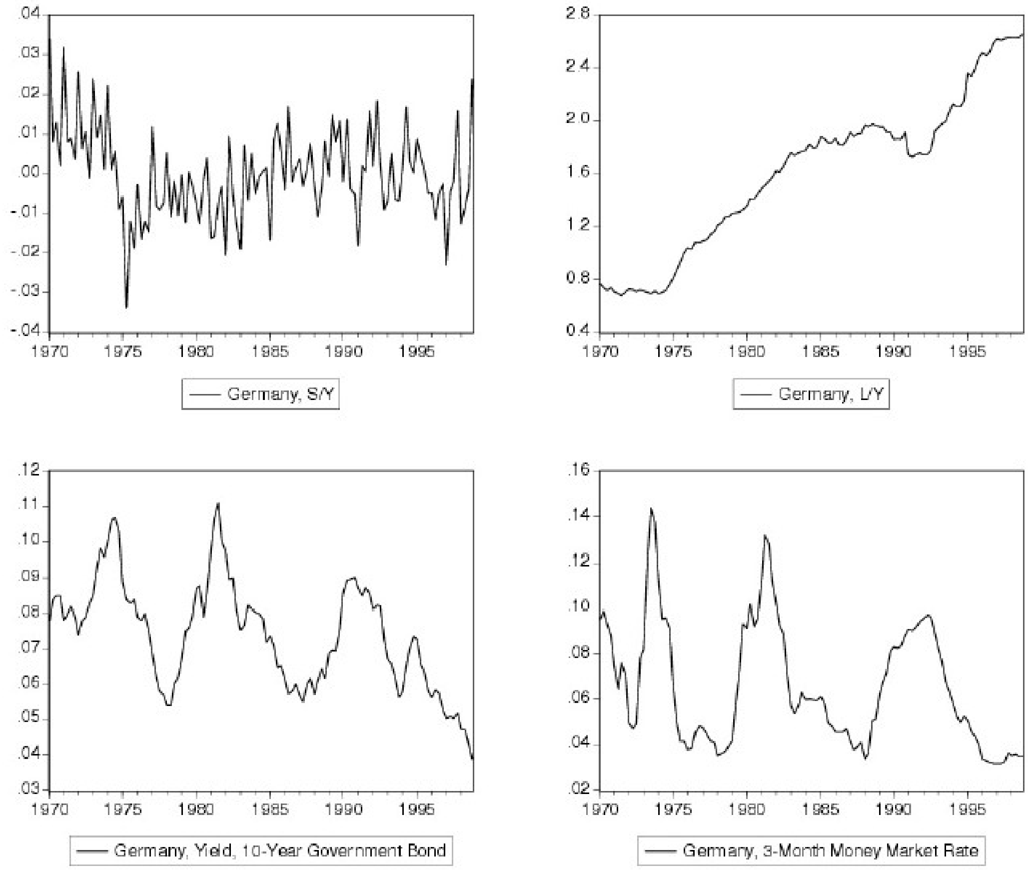

Figure 4.7: *Germany, Data, 1970-1998.*

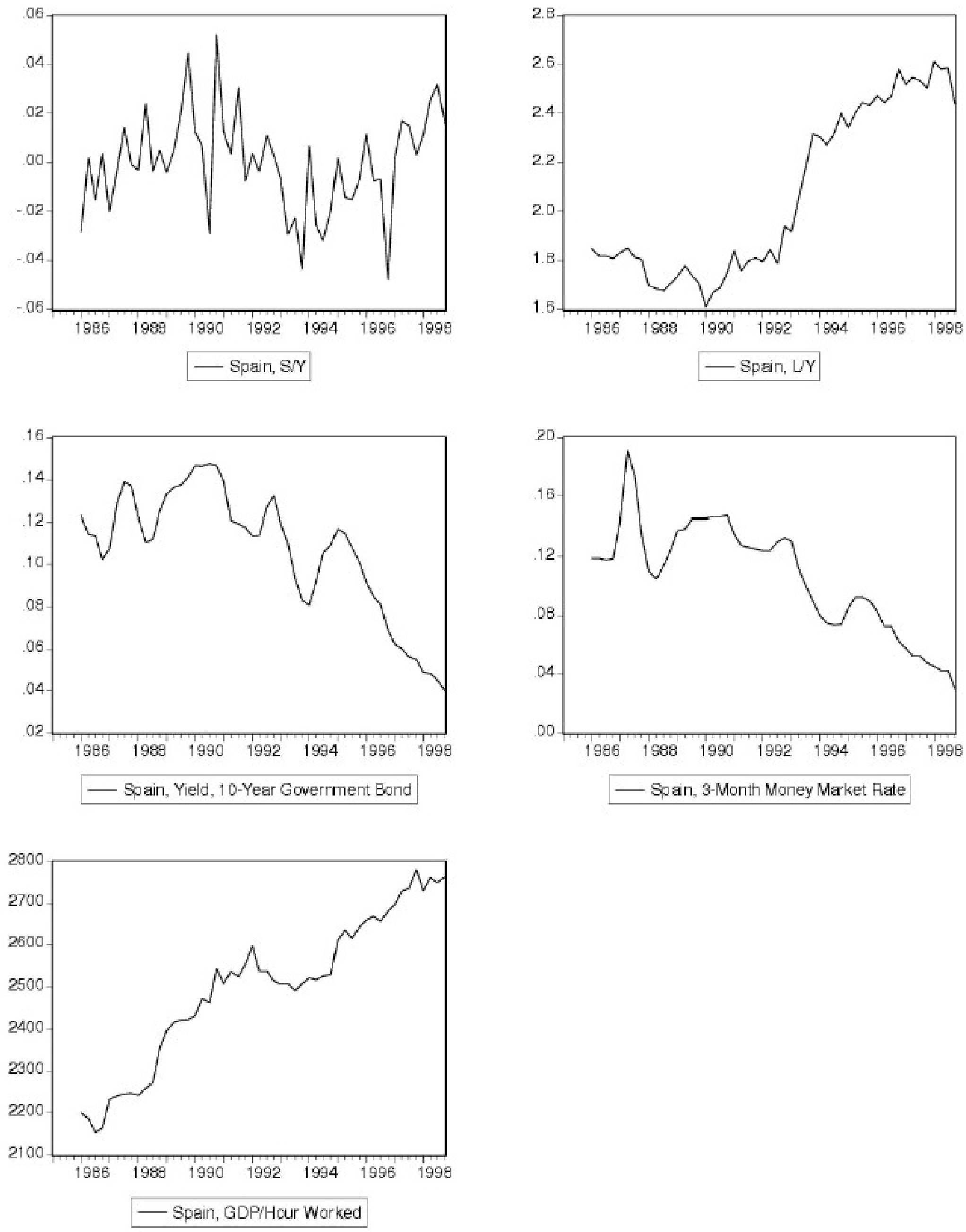

Figure 4.8: *Spain, Data, 1986-1998.*

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Chapter 5

Fiscal Policy Rules in Practice

This paper analyzes German and Spanish fiscal policy using simple policy rules. We choose Germany and Spain, as both are Member States in the European Monetary Union and underwent considerable increases in public debt in the early 1990s. We focus on how fiscal policy behaves under rising public debt ratios. It is found that both Germany and Spain generally exhibit a positive relationship between government revenues and debt. Using Bayesian Markov-switching techniques, we show that both countries underwent a change in policy behavior in light of rising debt/output ratios at the end of the 1990s. Interestingly, this change in policy behavior differs in its characteristics across the two countries and seems to be non-permanent in the case of Germany.

5.1 Introduction

5.1.1 Motivation

The theoretical and empirical evaluation of monetary policy with the help of simple policy rules has been subject of extensive research in recent years. This research has shown that monetary policy under discretion is suboptimal compared to a rule-based policy behavior. As a consequence of this strand of research monetary policy has substantially changed over the last three decades. Interest rate decisions of central banks have generally become more explicit and systematic.

In contrast to monetary policy rules, fiscal policy rules have received much less scrutiny in economics. Nonetheless, the design and performance of different fiscal policy rules remains an important element of macroeconomic policy analysis for a variety of reasons. One particular reason is that recent literature has discovered a link between fiscal policy and the price level. The relevance of this link between fiscal policy and prices depends crucially on the design of the policy rule.

This paper analyzes German and Spanish fiscal policy. Thereby, the principal objective of this paper is to investigate fiscal policy empirically in these two countries using simple policy rules. We choose Germany and Spain, as both are Member States in the European Monetary Union (EMU) and underwent considerable increases in public debt outstanding, particularly in the early 1990s. While other studies such as Taylor (2000) focus on the role of automatic stabilizers in fiscal policy behavior, we want to highlight the link between public debt and fiscal instruments. In particular, we want to determine how fiscal policy behaves under rising public debt ratios.

It is found that both Germany and Spain generally exhibit a positive relationship between government revenues and debt. Using Bayesian Markov-switching techniques, we show that both countries underwent a change in policy behavior in light of rising debt/output ratios at the end of the 1990s. Interestingly, this change in policy behavior differs in its characteristics across the two countries and seems to be non-permanent in the case of Germany.

5.1.2 Literature Review

In 1993 John B. Taylor proposed a simple monetary policy rule linking the instrument of the central bank, i.e. interest rates, positively to inflation and output deviations. Since then this so-called Taylor rule has attracted a lot of attention. One reason for the popularity of Taylor rules is obviously their simple form and their potential to differentiate between discretionary and rule-based policy behavior easily. In this sense Taylor rules may serve as a benchmark for monetary policy evaluation. Unfortunately, they do not allow for any statements in terms of optimality, as they are ad-hoc and not derived from any welfare-theoretic considerations. What is also often criticized is the fact that a central bank with dozens of well-trained economists is unlikely to follow a simple decision rule such as that proposed by Taylor (1993). Actually, central banks have developed complex decision processes based on numerous variables. They make considerable effort in collecting information directly from a large number of businesses and organizations[1]. A simple mechanical concept like the Taylor rule is hardly compatible with such a decision process. Taylor (1993) already mentioned that policymakers do not follow policy rules mechanically. Central banks need more than a simple policy rule to conduct policy. Additional judgment is particularly required, when dealing with special scenarios, which are not captured in a mechanical formula like the Taylor rule. But unlike pure discretion, the settings for the instruments are not determined from scratch each period. In this sense, policy rules in general are neither more nor less than a tool in identifying the basics behind policy actions, as it is neither desirable nor likely that a central bank starts from scratch each period.

We begin this literature review with monetary policy issues, because policy rules have generally found less application in fiscal policy analysis. Nonetheless, they offer a way to think about fiscal policy systematically. Numerous papers deal with the question of cyclical properties of fiscal policy and its ability to stabilize the economy using simple policy rule specifications. Examples would be Gali and Perotti (2003), who assess the cyclical properties of fiscal policy in the EMU before and after the introduction of the Maastricht Treaty, Taylor (2000), who investigates the reaction of automatic stabilizers in the United States, and Fatas and Mihov (2001), who analyze the relationship between government size and business cycle volatility in OECD countries.

Another strand of literature uses fiscal rules to test for the link between prices and public debt,

[1]For further details the interested reader is referred to Svensson (2001).

as induced by the fiscal theory of the price level (FTPL) and for the sustainability of fiscal policy in general. Bohn (1998) finds out that U.S. fiscal surpluses have responded positively to debt. He argues that this provides evidence that U.S. fiscal policy has been sustainable. For the EMU, Afonso (2002) demonstrates, applying a panel data approach, that the FTPL was not supported for the EU-15 countries during the period 1970-2001, as Member States tend to react with larger future surpluses to increases in government liabilities. A recent paper by Davig and Leeper (2005) analyzes regime switches in fiscal policy for the U.S. They show that there have been periods of time, when government revenues have been positively and negatively affected by changes in the debt-output ratio.

This paper follows the approach of Davig and Leeper (2005). We investigate the relationship between fiscal instruments and public debt in a Markov-switching model, as a crucial difference between the analysis of monetary and fiscal rules arises from the heterogeneity of fiscal policy over time. In contrast to monetary policy fiscal policy is substantially affected by political flavors. With different political responsibilities we may expect at least some change in fiscal policy behavior. For this reason, we propose that any econometric analysis of fiscal rules should allow for changes in the underlying coefficients, as these are generally unlikely to be stable over time.

The remainder of this paper is organized as follows. In section 5.2 we introduce a simple framework for fiscal policy analysis using policy rules. We then give a brief survey of the methodology that is applied in this paper. After a description of the data used in the analysis, we provide the reader with the results in section 5.2.3. We then make a systematic comparison of the country-specific results and check for their plausibility in section 5.2.4. Finally, section 5.3 summarizes the results and concludes.

5.2 A Simple Framework for Analyzing Fiscal Policy

The fundamental idea of policy rules is to evaluate and recommend certain types of policy behavior. This essentially means that we want to identify rules, which link the instrument of policy authorities to some exogenous variables and finally turn out to be advantageous over other rules. We are then left to inquire, what the instrument of fiscal policy is. Until now, there is no comprehensive framework to analyze fiscal policy rules empirically. Basically fiscal policy has two instruments, the tax rate and the benefit rate. The tax rate determines government revenues, while the benefit rate determines government spending. We decided to follow a similar approach as Davig and Leeper (2005) and use government revenues as the dependent variable for the policy rule, since we think that it best serves for our purpose of investigating short-run reactions of fiscal policy to rising debt/GDP ratios.

Our fiscal policy rule takes the following form

$$\tau_t = \text{constant}(S_t) + \gamma_Y(S_t)Y_t + \gamma_G(S_t)G_{t-1} + \gamma_B(S_t)B_{t-1} + \gamma_\tau(S_t)\tau_{t-1} + \sigma(S_t)\varepsilon_t, \tag{5.1}$$

where τ_t denotes the ratio between government revenues and GDP in period t, Y_t represents the output gap, G_{t-1} is the expenditure/GDP ratio in period $t-1$ and B_{t-1} stands for the debt/GDP ratio in period $t-1$. We decided to use lagged values of G and B for two reasons. On the one hand we would run into an endogeneity problem, when including G and B in period t, and on the other hand it is extremely unlikely that fiscal policy can immediately react to a change in one of the variables due to lags in the decision process of fiscal authorities. Therefore, we think that this is a plausible specification. Finally, to allow for revenue smoothing, we also included the dependent variable lagged by one period. S_t denotes the state of fiscal policy at time t. It emphasizes the fact that the coefficients and the variance of the error term, ε_t, are state dependent. We allow for regime switches to occur in fiscal policy behavior for the reasons given in the last section. We assume fiscal regimes evolve according to a Markov chain with transition matrix P. We allow for two different states of the parameters, which should be sufficient for the purpose of the analysis[2].

In terms of the parameters in (5.1), for fiscal policy to be sustainable, it is necessary that $\gamma_B > 0$. The value of $\gamma_B > 0$ must also be large enough so that a larger stock of public debt outstanding significantly increases government revenues and thus the path of government debt itself is stabilized.

We decided to use Bayesian techniques, since classical econometric approaches do not seem to be promising, when studying the reaction and behavior of fiscal variables, due to the fact that they fail to identify unsustainable fiscal policy behavior[3].

[2] We also did the analysis with a higher number of regimes, which did not deliver any reasonable results.
[3] Cf. Thams (2007) and Bohn (2006).

5.2.1 Bayesian Analysis of Markov-Switching Models

The Bayesian analysis of Markov-switching models goes back to McCulloch and Tsay (1994). They show that Bayesian estimation of Markov-switching models is kept relatively simple when using the Gibbs sampler, since it solves the problem of drawing samples from a multivariate density function by drawing successive samples from the corresponding univariate density functions. The exposition given in the following is based on Harris (1999) and Krolzig (1996).

We consider the following simple univariate model, where the parameters can take on M different states S,

$$y_t = c(S_t) + B(S_t)X_t + \varepsilon_t(S_t), \tag{5.2}$$

where X_t is the vector of explanatory variables and $\varepsilon_t(S_t)$ a normally distributed *i.i.d.* error term with mean zero and regime-dependent covariance matrix $\Omega(S_t)$. $c(S_t)$ denotes the constant in state S_t and $B(S_t)$ is the vector of coefficients in state S_t. Furthermore, we define the transition probabilities for a switch from regime u to regime v as $p_{uv} = p(S_t = v|S_{t-1} = u)$. We summarize these probabilities in the transition matrix P with size $(M \times M)$.

Let λ denote the set of all unknown parameters, i.e.

$$\lambda = [c(1), \ldots, c(M), B(1), \ldots, B(M), \Omega(1), \ldots, \Omega(M), P] \,.$$

In partitioned notation this boils down to $\lambda = [\theta, P]$. Inference on λ depends on the posterior distribution

$$p(\lambda|Y) \propto \pi(\lambda)p(Y|\lambda), \tag{5.3}$$

where $Y' = (y_1, \ldots, y_T)$ is the vector of observations and $\pi(\lambda)$ the prior for the parameter vector. As we are in a Markov-regime switching environment, we have additional unknown parameters given by the unobservable states. Therefore, the posterior density (5.3) is obtained by the integration of the joint probability distribution with respect to the state vector S, i.e.

$$p(\lambda|Y) = \int p(\lambda, S|Y)dS. \tag{5.4}$$

The problem arising from (5.4) is that the posterior distribution of λ depends on an unknown multivariate distribution $p(\lambda, S|Y)$. The Gibbs sampler offers a solution to this problem, as it allows to draw successive samples from univariate distributions for λ and S, namely $p(S|Y,\lambda)$ and $p(\lambda|Y,S)$, instead of the multivariate distribution $p(\lambda,S|Y)$. The Gibbs sampler constructs a Markov chain on (λ,S) such that the limiting distribution of the chain is the joint distribution of $p(\lambda,S|Y)$. There are two types of Gibbs sampler, single-move and multi-move, which differ in the way the states S are generated. We apply multi-move sampling as it - according to Liu et al. (1994) - will lead to a faster convergence than single-move sampling.

The idea of multi-move Gibbs sampling is to draw all states in S at once conditional on the observations. The starting point is to make use of the structure of the underlying Markov chain, i.e.

$$p(S|Y,\lambda) = p(S_T|Y,\lambda)\prod_{t=1}^{T-1} p(S_t|S_{t+1},y_t,\lambda). \tag{5.5}$$

The probabilities $p(S_T|Y,\lambda)$ can be calculated using the filter introduced by Hamilton (1989), after having chosen initial values for $p(S_0|Y)$. As we are not able to say anything about S_t for $t<1$, we assume that the economy was in a steady state in $t=0$. This enables us to choose steady-state probabilities for $p(S_0|Y)$, which are easy to compute[4]. We then may generate $p(S_T|Y,\lambda)$, which allows us to compute $p(S_t|S_{t+1},y_t,\lambda)$ by

$$p(S_t|S_{t+1},y_t,\lambda) = \frac{p(S_t,S_{t+1}|y_t,\lambda)}{p(S_{t+1}|y_t,\lambda)} = \frac{p(S_{t+1}|S_t)p(S_t|y_t,\lambda)}{p(S_{t+1}|y_t,\lambda)}. \tag{5.6}$$

The regimes can now be jointly generated according to (5.5). It is then possible to draw the unknown parameters from the conditional densities

$$p(\theta_v|S,\theta_{-v},Y) \propto L(Y|S,\lambda)\cdot p(\theta_v) \tag{5.7}$$

$$p(P|S,\theta,Y) \propto p(S_q|P)\prod_{t=q+1}^{T} p(S_t|S_{t-1},P)\cdot p(P), \tag{5.8}$$

where θ_{-v} denotes the set of parameters except for θ_v.

Some further details on the mathematical backgrounds of Bayesian analysis of Markov-switching models may be found in the appendix.

[4]The procedure is explicitly described in section 5.4.1 of the appendix.

5.2.2 Data

All data used corresponds to statistics of the International Monetary Fund other than German GDP, which is taken from the Federal Statistical Office Germany. All data is denoted in nominal terms and has a quarterly frequency. For τ_t and G_t we use total government revenues and expenditures. Government debt B_t is represented by total government debt, which includes in the case of Germany both debt of federal and federal state authorities. We use the Hodrick-Prescott filter with a smoothing parameter of 1600 to detrend GDP data. Output deviations Y are then given by the percentage deviation of GDP from its trend component[5]. The data starts for Germany with the 1^{st} quarter 1970 and ends with the 4^{th} quarter 2003. Unfortunately, the corresponding data for Spain is only partially available before 1986 so that the analysis of Spanish fiscal policy has to rely on the period[6] 1986-2003. As all data is not seasonally adjusted and the seasonal pattern is not completely captured by a regression on dummy variables, we applied the Census X-12 method for the seasonal adjustment.

5.2.3 Results

In the following we provide the estimated coefficients of the fiscal policy rule as well as the temporal distribution of the regimes. For the estimation we use a Matlab code, which takes 10,000 draws from the corresponding posterior distribution. We allowed for two regimes to occur. The prior probability density function (pdf) of the transition probabilities p_{uv} is assumed to follow a β-distribution, as it is restricted to the interval $[0, 1]$. For the prior pdf we decided to take a normal distribution with mean 0 in state 1 for the slope coefficients, which would depict an active policy regime as revenues become exogenous. For state 2 we use a normal distribution with mean 1, which would correspond to a passive policy regime, especially since the debt/output ratio has an impact on the revenue/GDP ratio. For the two prior pdfs we choose the same variance $\Sigma_{\gamma,0} = 1$ so that they are strongly overlapping each other[7]. This prior specification implies that we initially believe that there are no fundamental changes in policy behavior.

In the case that the coefficients are not significantly different across the two regimes, i.e. the

[5]We applied the Hodrick-Prescott filter in the case of Germany separately to the period before and after its reunification to avoid a bias in the trend component.

[6]One should note at this point that Bayesian techniques perform well even in small samples.

[7]We also did the estimation with other prior specifications, which left the results basically unchanged.

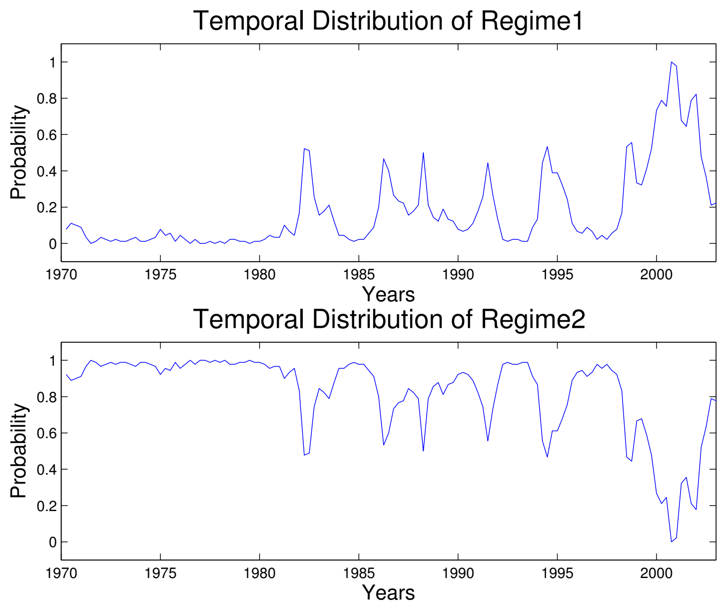

Figure 5.1: *Germany, Fiscal Policy Rule, Temporal Distribution of Regime Probabilities, 1970-2003.*

credible intervals given by the 2.5% and 97.5% quantiles are strongly overlapping each other, we decided to regard the corresponding coefficient as being not state-dependent. This approach has the advantage of requiring the estimation of fewer coefficients, while it is easier to identify changes in the underlying regimes. We do the same with the initially state-dependent variance of the error terms. Finally, the error terms were checked for their properties and may be considered as white noise.

Germany

Figure 5.1 shows the probability for each of the two potential fiscal regimes in Germany during the period 1970-2003. We can basically see that regime 1 hardly played any role in describing Germany's fiscal policy till the late 1990s. The estimated coefficients given in section 5.4.2 of the appendix show that the two regimes differ in the influence of the debt/GDP ratio, expenditure/GDP ratio, the size of the constant and the size of the variance. The other two coefficients are not regime-dependent. Starting with the regime-invariant coefficients, we may say that Germany's fiscal policy is mostly countercyclical, as the median coefficient is positive indicating that the revenue/GDP ratio increases with positive output deviations. This reflects the role of automatic stabilizers[8]. Further, we observe some degree of revenue smoothing, as indicated by the positive coefficient on τ_{t-1}.

Interesting insights in the fundamentals of Germany's fiscal policy are given by the regime-dependent parameters. In regime 2 we see that higher expenditure/GDP ratios are matched by growing revenue/GDP ratios, since the credible interval for γ_G is strictly positive, while in regime 1 the influence of expenditures on revenues is less clear-cut with the credible interval of the corresponding coefficient being distributed around zero. Further, in regime 1 we observe a much larger constant combined with a stronger reaction of the revenue/GDP ratio to increases in the debt/output ratio. The opposite is true for regime 2. Here we find a constant with a lower median value for the constant and for γ_B. Thus, we may say that regime 1 describes a more sustainable fiscal policy behavior. Interestingly, regime 1 is also associated with a higher variance, which implies an increase in uncertainty of fiscal policy.

Generally, we may say that there seems to be a tendency toward a more sustainable fiscal policy in Germany with respect to debt in the late 1990s. Nonetheless, this shift in fiscal policy behavior is not persistent, as shown by the temporal distribution of the regimes in figure 5.1. The analysis suggests so far that German fiscal policy underwent changes in its fundamentals, since the switch to regime 1 means an increase in autonomous government expenditures, as depicted by the constant, combined with a more reactive behavior to increases in government debt.

Spain

Figure 5.2 shows the temporal distribution of the two fiscal regimes for Spain. As indicated by the regime probabilities, we see a one-time shift in the fiscal regime during the mid-1990s. In contrast to Germany, revenue smoothing seemed to play no role in any of the two regimes. Therefore, we omitted the lagged dependent variable for Spain.

The difference between regime 1 and 2 is founded by the size of the constant. All other coefficients as well as the variance of the error term turned out not to differ across the two regimes. Regime 1 is characterized by a constant with a median value of almost 5, while the constant in regime 2 takes on a median value of about 3 with strictly positive credible intervals in both cases. Hence,

[8] One should note at this point that this analysis is built on total government revenues and expenditures, which include social security contributions.

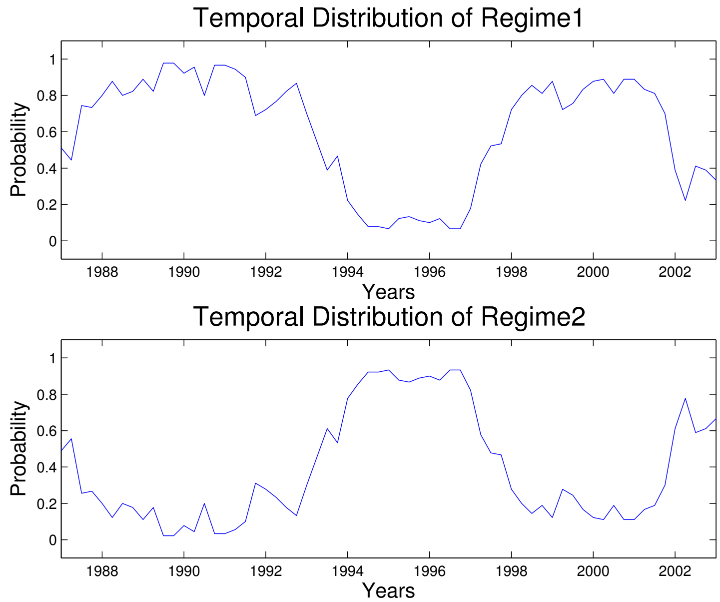

Figure 5.2: *Spain, Fiscal Policy Rule, Temporal Distribution of Regime Probabilities, 1986-2003.*

regime 1 leads to larger revenue/GDP ratios given everything else. As in the case of Germany we find a mostly countercyclical behavior of fiscal policy, as indicated by the estimated γ_Y, combined with a positive reaction of the revenue/GDP ratio to increases in the expenditure/GDP ratio. The debt/output ratio has also a positive impact on the revenue/output ratio. The estimated coefficient takes a median value of about 0.01 and thus lies between Germany's regime 1 and 2. Also the Spanish results suggest that there has been a shift toward a more sustainable fiscal policy toward the end-1990s. Unlike in Germany this shift seems to be rather persistent, as depicted in figure 5.2. The estimated variance of the error terms is considerably larger than in Germany's regime 1 or 2. This means that in average our policy rule specification (5.1) fits Spanish data worse than German data. One could interpret this as greater uncertainty in Spain's fiscal policy. When considering the regime switch in 1992 together with the data, we can see that it occurred simultaneously with the rise in the debt/output ratio. That means that the accumulation of government debt was caused by a drop in revenues as the substantially smaller constant in regime 2 suggests. With the regime switch in 1997 toward regime 1 the debt/output ratio starts falling again. This means that Spain did not undergo a fundamental change in policy behavior like Germany, instead it increased its autonomous government revenue to return to a

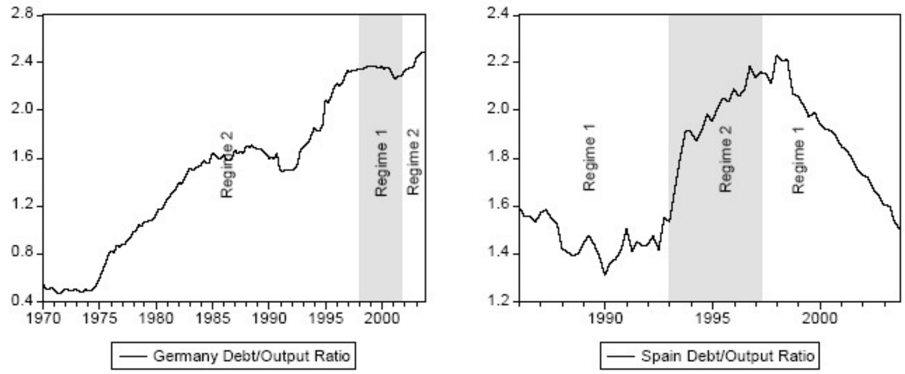

Figure 5.3: *Germany and Spain, Debt/GDP Ratios related to Regime Classification.*

sustainable debt/output path.

5.2.4 Plausibility of the Results

Figure 5.3 relates the run of debt/GDP ratios in the two countries to the underlying fiscal policy regimes[9]. The graphical representation shows that the regime changes also translate into the process of debt/GDP ratios. We see a stabilization of public debt in relation to GDP in Germany, during the more sustainable periods of fiscal policy, while the debt/GDP ratio starts shrinking in the case of Spain.

5.3 Conclusions

The analysis has uncovered changes in fiscal policy behavior using simple policy rules. In Spain we find a stronger response of revenues to changes in government expenditures than in Germany. Generally, Spain's fiscal policy deviates more strongly from the policy rule specification than Germany. This may be interpreted as evidence for higher uncertainty in Spain's fiscal policy.

In both countries we find evidence for a change toward a more sustainable fiscal policy at the end of the 1990s. This change only turns out to be persistent in the case of Spain. Nonetheless,

[9]The choice for one regime or the other is determined by the corresponding regime probabilities. We say that fiscal policy is in regime 1 at time t, if the probability for regime 1 at time t is larger than 0.5.

the difference between the two regimes seems to be more drastic in Germany than in Spain, as we find an increase in autonomous government revenues combined with a stronger response to changes in public debt. Spain exhibits a one-time shift in autonomous government revenues leaving the general relationship between revenues and debt unchanged.

Hence, we may say that both Germany and Spain underwent a switch in fiscal behavior in the light of rising debt/output ratios. Interestingly, this change in fiscal behavior exhibits different characteristics. While Germany tries to stabilize the debt/output ratio by a more fundamental change in fiscal behavior, Spain embarks on a different strategy by raising the overall revenue/output ratio. This result is remarkable, as both countries are subject to the same restrictions imposed by the Stability and Growth Pact. The analysis shows that the way fiscal authorities deal with these restrictions may well differ across countries.

5.4 Appendix

5.4.1 Bayesian Analysis of Markov-Regime Switching Models

Generating the Regimes S using Gibbs-Sampling

We generate the regimes S with the help of multi-move Gibbs sampling. The idea is to obtain the T elements in S within one draw conditional on λ and the observed data X. The starting point is to make use of the structure of the underlying Markov chain. The density of the regimes $p(S|X,\lambda)$ can easily be rearranged in a multiplicative relationship as

$$\begin{aligned} p(S|X,\lambda) &= p(S_1,\ldots,S_T|X,\lambda) \\ &= p(S_T|X,\lambda)p(S_{T-1},\ldots,S_1|S_T,X_{T-1},\lambda) \\ &= p(S_T|X,\lambda)p(S_{T-1}|S_T,X_{T-1},\lambda)p(S_{T-2},\ldots,S_1|S_{T-1},X_{T-2},\lambda) \\ &= p(S_T|X,\lambda)\prod_{t=1}^{T-1} p(S_t|S_{t+1},X_t,\lambda). \end{aligned} \tag{5.9}$$

Knowing $p(S_T|X_T,\lambda)$ and $p(S_t|S_{t+1},X_t,\lambda)$, we could first draw S_T. Conditional on S_T it would then possible to obtain S_{T-1}, and again conditional on S_{T-1} we could draw S_{T-2} etc.
With some algebra one can show that

$$p(S_t = u|S_{t+1} = v, X_t, \lambda) = \frac{p_{uv}p(S_t = u|X_t,\lambda)}{\sum_{z=1}^{k} p_{zj}p(S_t = z|X_t,\lambda)}. \tag{5.10}$$

That means that given the matrix of transition probabilities P, it only requires $P(S_t|X_t,\lambda)$ to compute $p(S_t|S_{t+1},X_t,\lambda)$. $p(S_t|X_t,\lambda)$ can in turn be determined using the filter proposed by Hamilton (1989). This procedure demands initial values for S_0. In what follows, we will briefly outline how these may reasonably be chosen.

Deriving the Initial Probabilities

Using the filter of Hamilton (1989) to compute $p(S_T|X_T,\lambda)$ requires initial values for $p(S_0|X)$. By assuming that the economy was in a steady state in $t = 0$, we may use steady-state probabilities for $p(S_0|X)$. The general condition for a steady-state probability is given by

$$P \cdot p(S_0|X) = p(S_0|X), \tag{5.11}$$

where P denotes the matrix of transition probabilities. This equation can be rearranged to

$$(I - P)p(S_0|X) = 0, \tag{5.12}$$

with I being a $(M \times M)$ identity matrix. We know that by construction the M probabilities in the vector of $p(S_0|X)$ add up to one. Thus, with $\iota = (1, \ldots, 1)'$ we may express this fact in vector notation as

$$\iota' p(S_0|X) = 1. \tag{5.13}$$

In matrix notation (5.12) and (5.13) can be rewritten as

$$\underbrace{\begin{bmatrix} I - P \\ \iota \end{bmatrix}}_{\equiv H} p(S_0|X) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{5.14}$$

We premultiply this expression by $(H'H)^{-1}H'$ and obtain for the initial probabilities

$$p(S_0|X) = (H'H)^{-1}H' \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{5.15}$$

Generating the Parameters

After having generated S, we are now able to formulate the conditional density of the parameters, which is generally given by

$$p(\lambda_j|S, \lambda_{-j}, X) \propto L(X|S, \lambda) \cdot p(S|\lambda) \cdot p(\lambda_j), \tag{5.16}$$

where λ_{-j} denotes the set of all parameters except for λ_j.

5.4.2 Numerical Results of the Regime-Switching Approach

Germany

- Fiscal Policy Rule: 1970-2003, $(-)$ indicates that the coefficient is not state-dependent.

	2.5%	Median	97.5%
Constant$(S_t = 1)$	3.7211	5.0136	6.4395
Constant$(S_t = 2)$	2.9490	3.6906	4.9055
$\gamma_Y(-)$	-0.0108	0.0262	0.0859
$\gamma_G(S_t = 1)$	-1.5193	-0.0627	0.1708
$\gamma_G(S_t = 2)$	0.0680	0.1495	0.2142
$\gamma_B(S_t = 1)$	0.0027	0.0160	0.1218
$\gamma_B(S_t = 2)$	0.0023	0.0038	0.0051
$\gamma_\tau(-)$	0.4125	0.5155	0.6171
$\sigma^2(S_t = 1)$	0.6596	1.0967	2.2666
$\sigma^2(S_t = 2)$	0.1089	0.2188	0.2936
p_{11}	0.5437	0.7678	0.9311
p_{12}	0.0689	0.2322	0.4563
p_{21}	0.0078	0.0484	0.1384
p_{22}	0.8616	0.9516	0.9922

Spain

- Fiscal Policy Rule: 1986-2003, $(-)$ indicates that the coefficient is not state-dependent.

	2.5%	Median	97.5%
Constant$(S_t = 1)$	3.8447	4.7078	5.9325
Constant$(S_t = 2)$	1.8146	2.8091	4.1594
$\gamma_Y(-)$	-0.0687	0.1690	0.3239
$\gamma_G(-)$	0.4826	0.5902	0.6625
$\gamma_B(-)$	0.0026	0.0117	0.0258
$\sigma^2(-)$	1.3786	1.7884	2.2395
p_{11}	0.6915	0.9092	0.9759
p_{12}	0.0241	0.0908	0.3085
p_{21}	0.0260	0.1543	0.3569
p_{22}	0.6431	0.8457	0.9740

5.4.3 Data

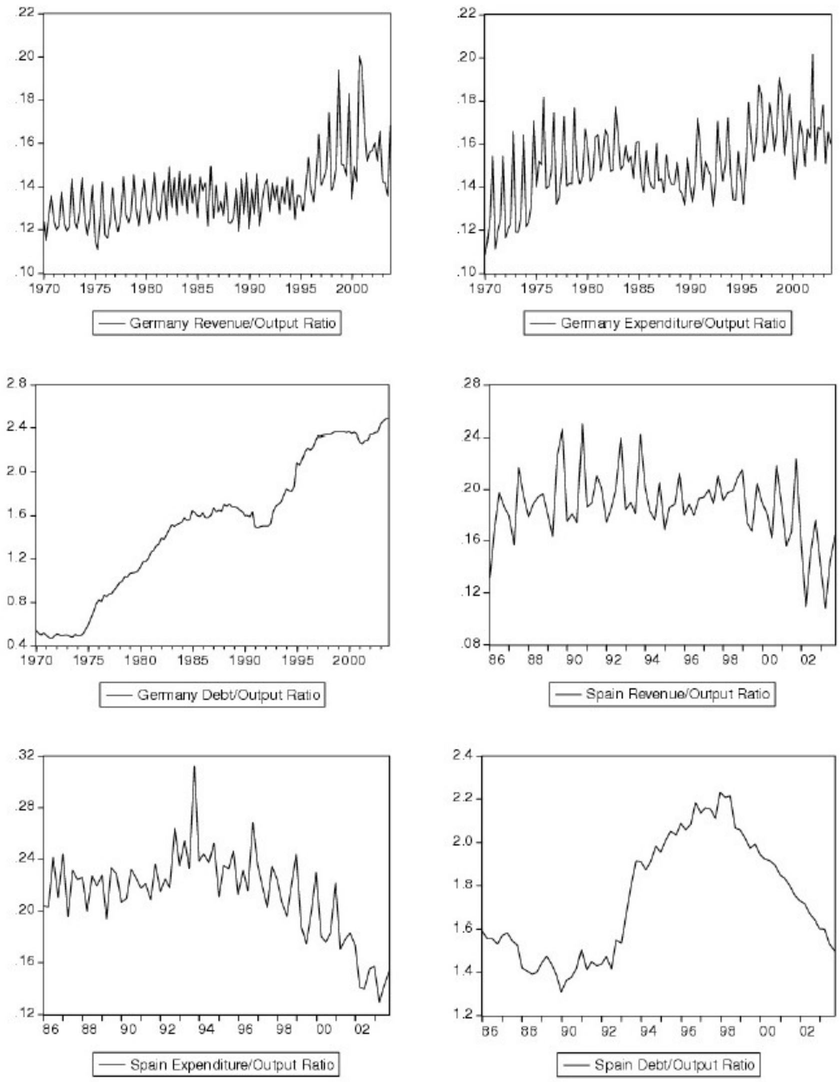

Figure 5.4: *Germany and Spain, Data, 1970-2003 (Germany), 1986-2003 (Spain).*

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Deutsche Zusammenfassung

Die vorliegende Dissertation besteht aus vier Artikeln, die zwar grundsätzlich einzeln gelesen werden können, jedoch in einem gewissen Zusammenhang zueinander stehen. Das Ziel dieser Dissertation soll es sein, zu einem besseren Verständnis der Interaktionseffekte zwischen Geld- und Fiskalpolitik beizutragen.

Der erste Artikel ist ein theoretischer Beitrag. In ihm wird der Frage nachgegangen, inwiefern Fiskalpolitik einen Einfluss auf das Verhalten einer wohlfahrtsoptimierenden Zentralbank hat. Es wird gezeigt, dass das optimale Verhalten der Geldpolitik durch das Verhalten der Fiskalpolitik beeinflusst wird.

In dem zweiten Artikel wird empirisch analysiert, wie Inflationsschocks innerhalb des Euroraumes übertragen werden. Motiviert ist die Analyse durch theoretische Arbeiten, in denen länderspezifische Unterschiede in der Stärke der Preisrigiditäten als entscheidende Determinanten eines optimalen Zentralbankverhaltens bestimmt wurden. Die Analyse baut auf einem Vektorfehlerkorrekturmodell auf, das Regimeveränderungen in der Kurzfristdynamik und in den Anpassungskoeffizienten berücksichtigt. Auf Grundlage der empirischen Ergebnisse werden Aussagen zum Transmissionsprozess von Inflationsschocks getroffen. Anschließend werden die sich daraus ableitenden Implikationen in Bezug auf die Höhe der Preisrigiditäten innerhalb des Euroraumes erörtert.

Der dritte Artikel widmet sich wiederum der Interaktion von Geld- und Fiskalpolitik, diesmal jedoch aus empirischer Perspektive. Am Beispiel von Deutschland und Spanien wird untersucht, ob die in jüngster Vergangenheit zu verzeichnenden Inflationsunterschiede durch fiskalpolitische Handlungen erklärt werden können. Den theoretischen Rahmen für die Analyse liefert die fiskalische Theorie des Preisniveaus. Der empirische Ansatz basiert auf einem bayesianischen vektorautoregressiven Modell. Es werden die Probleme klassischer ökonometrischer Verfahren bei der Analyse von fiskalpolitischen Variablen erörtert.

Der vierte Artikel schließlich beschäftigt sich mit der Analyse des fiskalpolitischen Verhaltens in Deutschland und Spanien auf der Grundlage einfacher fiskalpolitischer Regeln. Dabei liegt der Fokus der Analyse auf dem Zusammenhang zwischen Staatsverschuldung einerseits und Staatseinnahmen andererseits. Ausgehend von den Ergebnissen des dritten Artikels wird wiederum ein bayesianischer Ansatz verwendet. Um Änderungen im fiskalpolitischen Verhalten zu identifizieren, werden Regimewechsel über ein Markov-Modell berücksichtigt.

Erklärung zur Urheberschaft

Hiermit erkläre ich, dass ich mich noch keinem Promotionsverfahren unterzogen oder um Zulassung zu einem solchen beworben habe, und die Dissertation in der gleichen oder einer anderen Fassung bzw. Überarbeitung einer anderen Fakultät, einem Prüfungsausschuss oder einem Fachvertreter an einer anderen Hochschule nicht bereits zur Überprüfung vorgelegen hat. Ich habe diese Arbeit auf Grundlage der angegebenen Hilfsmittel selbständig verfasst.

Liste verwendeter Hilfsmittel

- Ecowin Pro 5.0
- Eviews 5.0
- JMulti 4.0
- Matlab 6.5
- Microsoft Excel
- MSVAR in Ox