Chapter 3

The effects of heterogeneities on seismic images - Numerical Studies

Deep seismic reflection imaging improved the knowledge on the lithospheric structure and the understanding of geodynamic processes. A number of projects investigated the crustal structures throughout the past decades such as DEKORP (Deutsches Kontinentales Reflexionsseismisches Programm), COCORP (Consortium for Continental Reflection Profiling), BIRPS (The British Institutions Reflection Profiling Syndicate), Lithoprobe and others. Thereby, the lithosphere was revealed as a complex fabric containing heterogeneities and scattering structures over large scales. The influence of heterogeneities on seismic wave propagation was therefore studied intensively by numerical modelling experiments in combination the application to real data. From these studies it was revealed that the coherency loss of reflections in heterogeneous media becomes rapidly large with increasing frequencies (Raynaud, 1988). Some authors noted that seismic imaging in heterogeneous media suffer from apparent attenuation due to multiple scattering (Gibson and Levander, 1988). Other studies showed that multiple scattering can produce coherent signatures in migrated sections which cannot be correlated with real structures in the medium (Emmerich et al., 1993). Estimates of the reflectivity of deeper structures are significantly influenced by the transmission loss and amplitude fluctuation in heterogeneous layers (Henstock and Levander, 2000). It was also revealed that in the presence of complex reflector a polarity analysis becomes difficult and unreliable due to the superposition of reflected and scattered waves in the seismic response. Numerical studies investigating the coherency loss of seismic images due to scattering showed that the degree of the image distortion is mainly dependent on the standard deviation of the velocity fluctuations in the medium (Martini et al., 2001). They also showed that an excellent control of migration velocity and the application of prestack depth migration techniques are required to reduce the image distortion to an acceptable level.

Considering these results certain questions arise, e.g. "how severe is the image distortion if the target is located at large depths, where amplitude and travel time fluctuations become large?". Also, it seems of interest whether the image distortion of deep reflections is only dependent on the standard deviation of the velocity fluctuation or whether the distortion is also dependent on the correlation lengths of the heterogeneities. The frequency dependent reflection and scattering behaviour suggest that images obtained from different frequency bandwidths contained in reflection data will provide different reflection images (Imhof, 2003; Sato and Fehler, 1998; Hong, 2004). Certain frequencies are less affected by scattering and coherency loss and will provide less distorted reflector images compared to other frequency ranges. It is assumed that these narrow-frequency-band images will extract reflections, which are covered in the broadband frequency range. The resulting key questions, which by the means of numerical simulations are studied and analysed in the following, are:

- How severe is the effect of scattering in a heterogeneous layer on the reflectivity and the coherency of the reflections below?
- Do narrow frequency band images provide additional structural information on deep reflectors located below heterogeneous regions?
- How severe is the distortion of the reflector shape when wrong migration velocities are used?

To discuss these questions numerical experiments were carried out. Synthetic depth sections obtained from forward modelling and prestack depth migration were studied in the context of the stated problems. The velocity models consisted of a heterogeneous layer located above a deep reflector. The standard deviations of the velocity fluctuations and the correlation lengths of the heterogeneities in the medium were varied. The synthetic data were analysed with respect to the observed reflectivity in the heterogeneous layer and along the deep reflector. This analysis was carried out in dependence on the ratio between the dominant wavelength and the horizontal correlation length of the heterogeneities. The Reflection Image Spectroscopy (RIS) method was invented to recover reflections in narrow-frequency-band images, which were not recognised in the broadband frequency image. The reflection images are band-pass filtered and migrated. The reflection images are compared and analysed to reveal additional structural details on the medium, e.g. spatial distribution and concentration of the scatterers. Also, the correlation between the frequency content, the structural parameters in the heterogeneous layer and the apparently dominant structures in the depth images were analysed. Finally, the influence of errors in the migration velocity on the reflector was studied. Depth images were calculated for different velocities to reveal whether wrong migration velocities influence the coherency and the reflectivity of the image.

This chapter is structured as follows: First, an introduction to the numerical work tools will be given. In the second part, the numerical modelling studies, corresponding results and a discussion will be presented. Finally, the chapter will be closed by a summary and conclusion.

3.1 Seismic modelling and imaging - The work tools

In the following a description of the numerical work tools will be given. First, the finite difference forward modelling scheme and modelling constraints will be discussed. Second, an introduction to Kirchhoff migration and travel time calculation will be given.

3.1.1 FD forward modelling using the rotated staggered grid

Numerical modelling of seismic wave propagation in real media is an important tool used in earthquake and exploration seismology. It has been used to support interpretations of field data, to provide synthetic data for testing of processing techniques and acquisition parameters, and to improve the understanding of seismic wave propagation for seismologists as well as for geologists. A general overview of different techniques and applications can be found in Carcione et al. (2002) and references therein. Since the widely used finite-difference (FD) approaches are based on the wave equation without physical approximations, the methods account not only for direct waves, primary reflected waves, and multiply reflected waves, but also for surface waves, head waves, converted reflected waves, and waves observed in ray-theoretical shadow zones (Kelly et al., 1976).

Staggered grid FD operators are commonly used for the computation of derivatives in the wave equations for elastic, viscoelastic and anisotropic media [e.g. Virieux (1986); Levander (1988); Robertsson et al. (1994); Igel et al. (1995)]. One disadvantage is that the standard staggered FD grid scheme according to Virieux (1986) can become unstable in the case where high contrasts of material properties are present. The boundary conditions of the elastic wave field at a free surface, i.e. the high contrast discontinuity between vacuum and rock, have to be defined in the FD algorithm [e.g. Robertsson (1996); Graves (1996); Hestholm and Ruud (1998); Oprsal and Zahradnik (1999)]. By using the so-called rotated staggered grid (RSG) technique (Saenger et al., 2000), discontinuities can be incorporated without applying explicit boundary conditions and without averaging elastic moduli. For this reason, the RSG is a powerful tool for studying strong multiple scattering effects of wave propagation in highly heterogeneous media. All synthetic seismograms presented in the following were calculated using the RSG with 2nd order time update operator and 8th order spatial differentiation operator.

The applicability of FD modelling is limited due to three major numerical problems, i.e. grid dispersion, stability and boundary reflections. During finite-difference modelling of the wave field numerical phase and amplitude errors can occur. This is because numerical operators that use Taylor polynomials are implemented to approximate the time and space derivatives of the wave field. Phase errors, also called *Numerical dispersion*, occur in the case of spatial undersampling of the wave field. Numerical errors due to interpolation bias the calculated travel times towards lower values. To minimise these effects to an acceptable level the spatial grid layout has to fulfill the following dispersion criterion:

$$10 \cdot max(\Delta x, \Delta z) < \frac{v_{min}}{f_{dom}} = \lambda_{min(dom)} \tag{3.1}$$

Even if the dispersion criterion is fulfilled a small amount of dispersion effects still might be present. A correction of dispersion effects was not applied to the data throughout this thesis, as travel time errors due to numerical dispersion effects are assumed to be negligibly small by applying the dispersion criterion (eq. 3.1).

Temporal undersampling can cause an exponential increase of amplitudes with every time step and thus leads to amplitude errors. To avoid these numerical errors the sampling rate has to be defined larger than a threshold value. The *stability criterion* defines the minimum sampling interval Δt in time according to the grid spacing Δh and the P-wave velocity v_P in the velocity model. For the RSG the critical time step can be obtained by (Saenger et al., 2000):

$$\frac{\Delta t v_P}{\Delta h} \leq 1/(\sum_{k=1}^{n} \mid c_k \mid) \leq 1. \tag{3.2}$$

Here, $\mid c_k \mid$ are the used finite difference coefficients (Holberg, 1987).

3.1.2 Imaging using Kirchhoff prestack depth migration

Seismic migration consists of two steps: *wave field extrapolation* and *imaging* of the reflected field. Wave field extrapolation is the process where the wave field recorded at the surface is back propagated in time and down into the subsurface. The propagated seismic energy is *imaged* at the reflection point at the time of incident, i.e. the travel time between the source and subsurface point. Back propagation methods of the wave field are commonly based on three approaches: the finite difference method (Claerbout, 1976), the frequency-wavenumber method (Stolt, 1978; Gazdag, 1978; Gazdag and Squazzero, 1984), and the Kirchhoff integral migration method (Schneider, 1978). In this thesis Kirchhoff prestack depth migration was used to obtain the synthetic and the real data depth sections. A short introduction to the Kirchhoff method will be given in the following. For detailed theory of the Kirchhoff migration the reader is referred to e.g. Schneider (1978), Müller (1989), and Schleicher et al. (1993).

Diffraction stack and Kirchhoff migration

The diffraction stack provided the basis of the Kirchhoff migration method and was presented by Hagedoorn (1954). The diffraction stack is based on the assumption that any reflector can be regarded as a group of closely spaced scattering points. Reflections caused by a single diffraction element in the subsurface are located on the so called diffraction surface (Fig. 3.1). The shape of the diffraction surface is given by the sum of the travel time t_s from the source to the diffraction point and the travel time t_r from the subsurface point to each receiver. For a constant velocity medium the shape of the diffraction surface is a hyperbola in 2D or a hyperboloid in 3D, respectively. The shape depends on the velocity and the position of the regarded subsurface point. The diffraction curves caused by several diffraction points will superimpose and the envelope of those diffraction curves gives the respective reflector (Fig. 3.2). Diffraction stack is then carried out by calculation of the diffraction surfaces for any given subsurface point, summation of the recorded amplitudes along these surfaces and assigning the value to the respective subsurface point. Physically, diffraction stack is a process where the scattered energy is led back to its scattering point.

Kirchhoff migration is a weighted diffraction stack, where amplitudes are adjusted for obliquity and geometrical spreading before the summation process. Mathematically the Kirchhoff method is based on the Kirchhoff integral solution to the wave equation. The Kirchhoff integral states that the wave field $U(x, z, t)$ at any point in the subsurface $p(x, z)$ is given by spatial integration of the time differentiated and weighted wave field observed at the surface. The migrated section $M(x, z)$ can be obtained by taking the back propagated

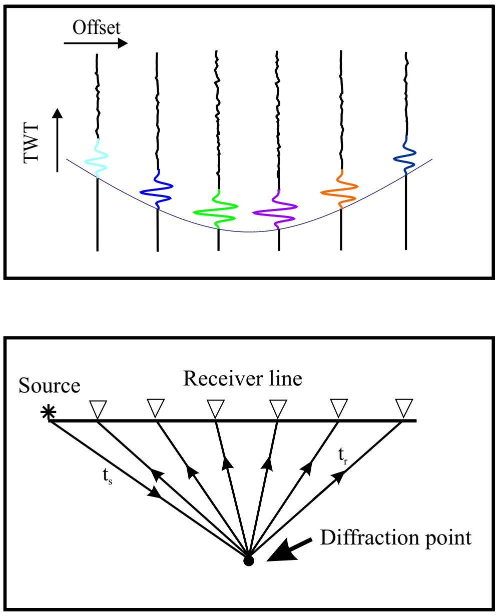

Figure 3.1: Principle of diffraction stack. The summation of the amplitudes along the diffraction hyperbola in a finite-offset time section and assigning this value to the respective subsurface point yields an image of the subsurface.

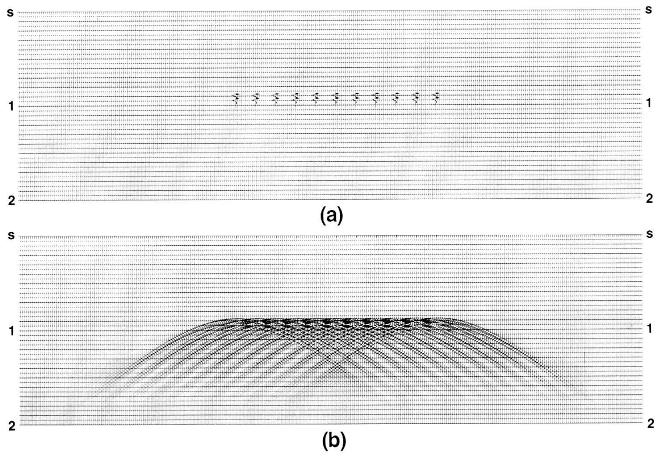

Figure 3.2: The diffraction envelope (Yilmaz, 1987). The envelope of diffraction curves generated by a reflector consisting of several diffraction points yields the reflector itself.

wave field $U(x, z, t = t_I)$ at time t_I, when the wave is being reflected at the considered subsurface point $p(x, z)$:

$$M(x,z) = U[(x,z,t_I(x,z)] = -\frac{1}{2\pi v}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial U(x',0,t')}{\partial t'}W(x'-x,t'-t_I)dt'dx'. \quad (3.3)$$

In this equation W denotes the weighting function which adjusts the amplitudes for geometrical spreading and obliquity. v presents the constant migration velocity.

The Kirchhoff prestack depth migration scheme as applied in this thesis is suitable for data with irregular spatial sampling, e.g. irregular receiver spacing and crooked line geometry. Furthermore it handles large differences in the elevation. The topography is taken into account during travel time calculation and migration process, so that static corrections are not necessary during data processing. Thus, errors due inappropriate corrections and biasing of the data are minimised.

3.1.3 Travel time calculation

Computation of diffraction surfaces

As mentioned in the previous section, Kirchhoff migration is a weighted stack of the time derivatives of the wave field measured at the surface along diffraction surfaces. For a constant velocity medium the diffraction surfaces are given by hyperbolas (2D) or by a hyperboloids (3D) and can be computed analytically. When the velocity model becomes complex the calculation of the diffraction surfaces has to be carried out numerically.

Calculation of travel times

Fast and accurate computation of travel times is a central task of imaging. Besides classical ray based methods, e.g. paraxial and dynamic ray tracing, Gaussian beam methods (Červený and Hron, 1980; Beydoun and Keho, 1987; Červený, 1986), finite difference solutions to the eikonal equation (Vidale, 1988; van Trier and Symes, 1991; Podvin and Lecomte, 1991) provide the base for a fast travel time calculation.

The eikonal equation describes seismic wave propagation based on a high frequency approximation of the wave equation. The approximation is valid if the wavelength is small compared to the change of the elastic parameters or to the curvature of the reflector, so that the latter can be approximated by a locally plane surface. Classical ray tracing techniques use an asymptotic approximation of the wave equation at infinite frequency to propagate rays. In complex media ray tracing becomes a difficult task, as a large number of rays must be considered for one given subsurface point in order to obtain the first arrival time. Moreover, there are subsurface regions that will not be illuminated by rays propagating from the source, thus diffraction signals from secondary sources located in those shadow zones have to be analysed in a very computing intensive procedure. Consequently, these techniques often suffer from very expensive computing times. A finite difference approximation of the eikonal equation was used in this thesis to calculate the travel times. A comprehensive review of travel time computation methods and discussion of their advantages and weaknesses is given in Leidenfrost et al. (1999).

The finite difference computation was performed using an algorithm proposed by Podvin and Lecomte (1991). This approach improved Vidale's finite difference algorithm (Vidale, 1988), which only considered the existence of single plane wave fronts. The algorithm is based on a systematic application of Huygen's principle, which considers the contribution of three different wave propagation modes: transmitted, diffracted and head waves. The first arrival is chosen using a minimum-time criterion. Travel time tables were calculated for each surface-subsurface distance.

3.2 Numerical modelling studies

In this section the numerical modelling studies will be presented. The velocity models, the experimental set-up and the results will be described separately for each experiment. The analysis of the image distortion due to the heterogeneous overburden will be discussed first. Then the RIS method, which analysis reflection images in narrow-frequency bands, will be introduced and applied to synthetic data. Finally, the impact of the velocity model on the accuracy of deep reflections images will be studied and discussed.

General remarks

A sufficient suppression of boundary reflections from the model side could not be realised with the implemented standard exponential damping boundaries (Clayton and Engquist, 1977; Karrenbach, 1995). To minimise this numerical noise the model size was enlarged and the receiver arrays were positioned in the central part of the model, such that boundary reflections were recorded at later arrival times. The experimental set-up, i.e. the receiver and the shot point spacing, the model size and its structural layout as well as the elastic parameters were chosen following the ANCORP'96 experiment. A detailed description of the ANCORP experiment and the data set will be given in section 4.2.2. The parameters