2 Microwave-Induced Zero Resistance States

When the magnetic field applied to a quantum Hall system is switched on,[1] one first passes through a regime at weak magnetic fields ($B \lesssim 0.1$ T) where the transport properties follow the classical Drude behavior. Upon increasing B, Shubnikov-deHaas oscillations appear. At higher magnetic fields, IQHE states and, at even higher magnetic fields, also FQHE states are realized. If the system's mobility is extremely high, very recent experiments [36, 37] show that irradiation of the system by microwaves has surprising consequences on its behavior at weak magnetic fields (typically below the onset of the IQHE, inside or below the Shubnikov-deHaas regime): These systems can show unexpected zeros in their macroscopic resistance – the so-called zero resistance states (ZRS) – which are only present when the sample is irradiated by microwaves. This chapter is devoted to an exposition of such experiments on microwave-irradiated 2D electron systems of very high purity and to a brief introduction to the theoretical scenario liable to explain the phenomenon of ZRS.

2.1 Zero Resistance States and Microwave-Induced Resistance Oscillations

When irradiating ultrahigh-mobility 2DEG samples with microwaves [36, 37], for suitably chosen microwave power and frequency (usually in the GHz range) and sufficiently high mobility of the sample, there are regions in magnetic field where the diagonal resistance R_{xx} *vanishes* within experimental accuracy. Unlike in the QHE, the observed zeros in the diagonal resistance are not accompanied by plateaus in the Hall resistance R_{xy}; instead, the Hall resistance stays practically unaffected by the microwaves and approximately follows the classical Drude behavior.

These so-called zero resistance states (ZRS) are observed at weak magnetic fields, far below the typical magnetic fields needed for an observation of the QHE[2] and, often, also below the magnetic fields where the Shubnikov- deHaas oscillations set in. The regions of magnetic field, where the ZRS are observed lie in the vicinity of the cyclotron resonance $\omega = \omega_c$ and its lowest harmonics $\omega = k\omega_c$ (k integer), where ω is the microwave frequency. The microwave frequency thus has to be chosen to be of the order of the cyclotron frequency for ZRS to be observed.

[1] see, for example, Fig. 1.4

[2] ZRS are typically observed at magnetic fields $B \sim 0.1$ T, about one to two orders of magnitude smaller than the fields where the QHE is normally realized

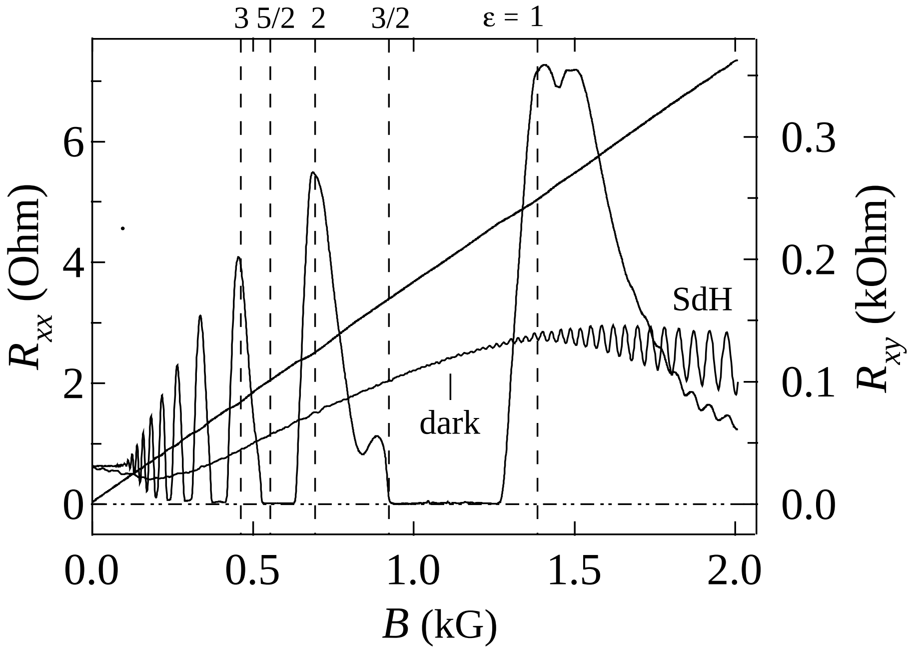

Figure 2.1: Measurement by Zudov *et al.* exhibiting regions (in magnetic field B) of zero diagonal resistance R_{xx}. This experiment has been performed at a temperature of approximately 1 K and with microwaves of frequency 57 GHz. Shown are the diagonal resistance in the absence ("dark") and in the presence of microwaves and the Hall resistance R_{xy} in the presence of microwaves, which turns out to be practically unaffected by the irradiation. In certain regions of magnetic field, below the regime of Shubnikov-deHaas oscillations, the diagonal resistance under irradiation drops to zero within experimental accuracy. The periodicity of these zero resistance states is determined by the ratio $\varepsilon = \omega/\omega_c$. Maxima are found at integer, minima at half-integer values of ε. The Shubnikov-deHaas oscillations (marked by "SdH" in the dark diagonal resistance) are still present, but are strongly suppressed under irradiation. This figure is a modified version of a picture taken from Ref. [37].

A typical experiment [37] revealing ZRS is depicted in Fig. 2.1. Shown are the longitudinal magnetoresistance in the absence of microwaves (indicated by the label "dark"), the longitudinal magnetoresistance in the presence of microwaves, exhibiting zones where its value drops to zero, and the transverse (or Hall) magnetoresistance which is virtually unaffected by the microwaves and does not show the characteristic plateaus present in the case of the quantum Hall effect. The Shubnikov-deHaas oscillations in the absence of microwaves are indicated by "SdH" and can also be found in the longitudinal resistance in the presence of microwaves, where they are suppressed below their dark values. The fact that, unlike in the QHE, these resistance minima do not coincide with quantized plateaus in the Hall resistivity, calls for a separate theoretical explanation.[3]

[3] There are other well-studied types of magnetooscillations than those appearing in the QHE (e.g. commensurability oscillations in the presence of a periodic modulation, known as Weiss oscillations, or oscillations due to the presence of interfacial acoustic phonons or surface acoustic waves) which also fail to provide a picture for understanding the ZRS.

The ZRS are periodic as a function of inverse magnetic field (or, equivalently, inverse cyclotron frequency).[4] In general, ZRS may occur whenever the microwave frequency ω is related to the cyclotron frequency ω_c as

$$\omega = (k + \alpha)\omega_c, \qquad k = 1, 2, 3, ... \tag{2.1}$$

where α is a constant (usually positive) phase shift which has been found to take different values in different experiments and thus seems to be nonuniversal. In the particular example shown in Fig. 2.1, three ZRS are observed at $\omega \simeq k\omega_c$ with $k = 1, 2, 3$ and no ZRS are observed for integers $k > 3$, i.e. for lower magnetic fields. Instead, oscillations in the longitudinal resistance appear at lower magnetic fields. We will comment on these oscillations shortly.

The ZRS strengthen with decreasing temperature T and the resistance shows seemingly activated behavior $\propto \exp(-E_A/k_B T)$. Surprisingly, the activation energy E_A turns out to be very high [42, 36], $E_A \sim 10 - 20$ K, almost an order of magnitude larger than the Landau-level spacing or the microwave photon energy. The reason for this unusually high activation energy remains unclear.

As the magnetic field weakens, the zero resistance states turn into oscillations in the longitudinal magnetoresistance known as MIRO (microwave-induced resistance oscillations). For samples of lesser quality (lower mobility), only MIRO and no ZRS are observed. The MIRO can thus be interpreted as precursors of the ZRS. The ZRS develop gradually out of the MIRO as the oscillation amplitude of the MIRO increases due to e.g. higher carrier mobility [36, 37, 43, 44, 45]. The conductivity oscillations leading to the ZRS (see Fig. 2.1) indeed appear to be truncated at zero where they would have swung over to negative values. Due to the close relationship between ZRS and MIRO, the periodicity of the MIRO is the same as for the ZRS and given by Eq. (2.1). Historically, MIRO have been observed [42, 36] before the discovery of ZRS (and predicted by Ryzhii [46] more than thirty years before their first observation).

It turns out that the MIRO and ZRS are very sensitive to sample mobility: While unresolved[5] at mobilities $\sim 1.0 \times 10^6$ cm^2/Vs, they are of considerable magnitude in samples of mobility $\sim 3.0 \times 10^6$ cm^2/Vs (at electron densities of the order of 2×10^{11} cm^{-2}) and evolve into ZRS at mobilities higher by about one order of magnitude.

2.2 Theoretical Explanation of ZRS

The theoretical explanation of ZRS rests on two pillars. First, the *microscopic* mechanism by which the microwaves interact with the 2DEG has to be identified. It turns out that the local diagonal conductivity σ_d of a 2DEG can be reduced considerably from its dark value σ_d^{dark} by microwave irradiation, and that it may even assume negative values, $\sigma_d < 0$. The microwave irradiation can affect transport in essentially two possible ways:

[4] In contrast to the behavior of the ZRS, the periodicity of the Shubnikov - deHaas oscillations is governed by the ratio $E_F/(\hbar\omega_c)$, where E_F is the Fermi energy of the 2D electron system.

[5] In samples of low mobility, only a single cyclotron resonance peak of comparatively low magnitude is observed upon irradiation with microwaves (see, e.g., Ref. [47]).

- via opening of new scattering channels,
- via a redistribution of electrons.

Indeed, both ways may lead to an oscillatory photoconductivity and negative diagonal conductivity, defining two relevant microscopic mechanisms, the displacement (DP) mechanism and the distribution function (DF) mechanism. The DP mechanism, suggested by Durst *et al.* [48], is a scattering mechanism involving simultaneous photoexcitation and disorder scattering of electrons, while the DF mechanism, identified by Dmitriev and co-workers [49, 50] to likely be the dominant mechanism in the experimental systems studied, relies on the fact that the distribution function is altered by the microwave irradiation.

Second, the emergence of zero resistance observed in *macroscopic* transport measurements has to be explained. The main ingredient here has been provided by Andreev *et al.* [51] who pointed out that absolute negative conductivity leads to an instability of the zero-current state with respect to formation of an inhomogeneous state of nonvanishing current. Within the parameter regions of negative conductivity, this inhomogeneous state triggers the formation of the zero resistance states observed in macroscopic current measurements. In Section 2.2.1, we first discuss this instability before proceeding to a discussion of the microscopic mechanisms leading to absolute negative diagonal conductivity in Section 2.2.2.

2.2.1 Instability of the Zero-Current State

We now present a somewhat simplified version of the argument in Ref. [51] for the instability due to negative local conductivity. The main idea behind this argument is to study charge density fluctuations in the regime of negative local conductivity and to show that they grow exponentially with time, thus inevitably leading to an instability. To do so, we start from the continuity equation, which relates the change in electron charge density n to the local current $\mathbf{j}$ via

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{j} = 0 \tag{2.2}$$

or, equivalently, in Fourier space,

$$\omega n - \mathbf{q} \cdot \mathbf{j} = 0 \quad . \tag{2.3}$$

The current is related to the electric field $\mathbf{E}$ via Ohm's law

$$\mathbf{j}(\mathbf{q}) = \hat{\sigma}(\mathbf{q})\mathbf{E}(\mathbf{q}) \quad , \tag{2.4}$$

where $\hat{\sigma}(\mathbf{q})$ is the conductivity tensor and $\mathbf{E}(\mathbf{q})$ can be expressed through the potential $\phi(\mathbf{q})$ as

$$\mathbf{E}(\mathbf{q}) = -i\mathbf{q}\phi(\mathbf{q}) \quad . \tag{2.5}$$

The Coulomb potential $\phi(\mathbf{q})$ arising from the charge density $n(\mathbf{q})$ takes the form

$$\phi(\mathbf{q}) = v(\mathbf{q})n(\mathbf{q}) \quad , \tag{2.6}$$

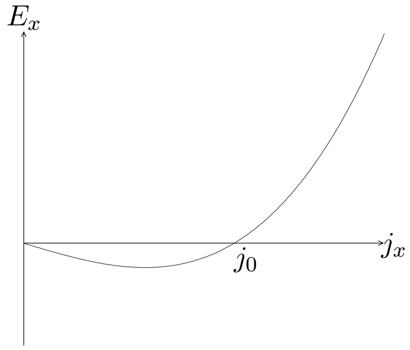

Figure 2.2: Conjectured current-voltage characteristic based on the assumption of a continuous dependence of the dissipative resistivity on the current. In addition to the trivial zero of E_x at $j_x = 0$, the assumption of a region of negative conductivity necessitates a second zero at a critical current j_0. It turns out that homogeneous, time-independent states of current density $|j| < j_0$ are unstable with respect to inhomogeneous current fluctuations.

where, in two dimensions,

$$v(\mathbf{q}) = \frac{2\pi}{q} \quad . \tag{2.7}$$

Inserting Eqs. (2.4-2.5) into the continuity equation, Eq. (2.3), leads to

$$\left[\omega + iq^2\sigma_d v(q)\right] n = 0 \quad , \tag{2.8}$$

where use was made of the fact that, due to spatial isotropy, the components of the conductivity tensor fulfill the following symmetries: $\sigma_{xx} = \sigma_{yy} = \sigma_d$ and $\sigma_{yx} = -\sigma_{xy}$. For finite charge density n, one thus finds

$$\omega = -iq^2 v(q)\sigma_d \quad , \tag{2.9}$$

so that a charge density fluctuation evolves in time as

$$\Delta n \sim e^{-i\omega t} \sim e^{-q^2 v(q)\sigma_d t} \quad . \tag{2.10}$$

For positive diagonal conductivity, $\sigma_d > 0$, charge density fluctuations decay as expected. For negative diagonal conductivity $\sigma_d < 0$, however, a charge density fluctuation grows *exponentially.* Thus, the zero-current state is unstable with respect to small fluctuations in the charge density. It is this instability which will be ultimately responsible for the formation of the zero resistance state.

Assuming that the diagonal conductivity is a continuous function of the current and that there is indeed a region of magnetic fields where this diagonal conductivity assumes absolute negative values, the current-voltage characteristics of our systems must assume the qualitative form depicted in Fig. 2.2, which shows the dissipative component of the local electric field E_x as a function of the dc current j_x. At sufficiently large values of the dc current, the microwave field has to be considered as a weak perturbation, so that the electric field reverts to its dark value. The assumed continuity of the diagonal conductivity then implies the generic shape of the current-voltage characteristics shown

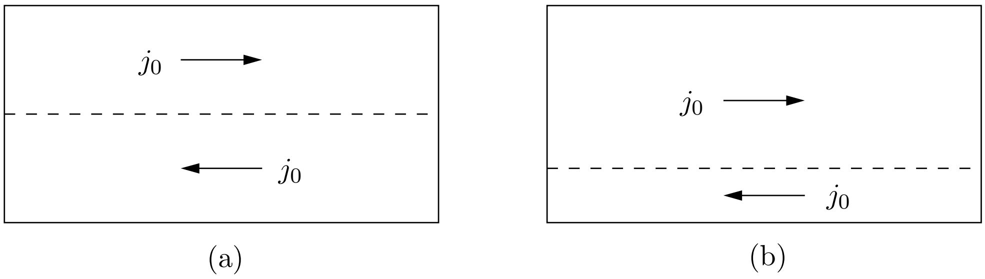

Figure 2.3: The simplest possible pattern of the current distribution is a linear domain wall (dashed line) separating a domain of local current density of magnitude j_0 pointing to the right (top) from a domain of local current density of magnitude j_0 pointing to the left (bottom). The total macroscopic current measured is zero in (a) since the domains have the same spatial extension. The magnitude of the current density is thus the same everywhere except at isolated points or lines, implying vanishing dissipation, $\mathbf{j} \cdot \mathbf{E}$. In (b), the domain wall is shifted down by a certain amount, accommodating for a finite macroscopic current pointing to the right.

in Fig. 2.2. Besides the trivial zero of the electric field at zero current, there must be an additional value j_0 of the current at which the dissipative electric field vanishes. A homogeneous, time-independent state with current density of magnitude $|j| < j_0$ then turns out to be unstable, since it lies in the negative conductivity regime. The only possible time-independent (stationary) state is one in which the magnitude $|j|$ of the current density is j_0 everywhere except at isolated points (vortices) or lines (domain walls) in the sample. The simplest such configuration is depicted in Fig. 2.3.

An immediate consequence of this instability is that any net dc current (less than some threshold value [51]) can be sustained at vanishing dissipative electric field by adjusting the details of the current-domain pattern, so that any microscopic mechanism leading to negative local conductivity results in zero dissipative macroscopic resistance. To accommodate for the observed zero resistance states, it is thus sufficient to show that there are microscopic mechanisms leading to a locally negative diagonal conductivity.

2.2.2 Microscopic Mechanisms leading to Absolute Negative Conductivity

The well-established conventional cyclotron resonance consists of microwave absorption by electrons at a microwave frequency ω matching closely the cyclotron frequency ω_c, via a direct transition between neighboring Landau levels. This process, which is of first order in the scattering matrix, does not contribute any additional current. Instead, it might lead to a resonance peak in the longitudinal resistivity due to resonant heating of the electron gas. A microscopic mechanism liable to explain the observed ZRS, however, must be able to reduce or even reverse the current in the system through the emergence of an additional irradiation-induced photocurrent.

Essentially two microscopic mechanisms have been identified to produce such an additional current, the displacement (DP) mechanism and the distribution

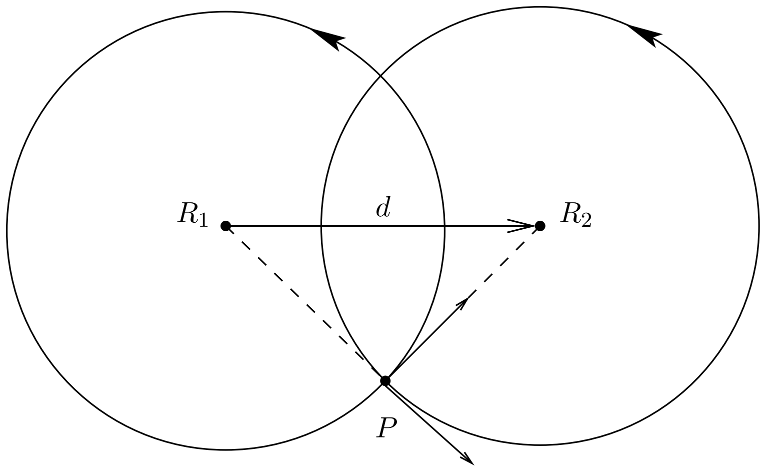

Figure 2.4: Intuitive classical picture for the relation between momentum transfer and real space displacement. An electron, initially gyrating on a circular trajectory of cyclotron radius R_c centered at R_1 is scattered (due to disorder) at a point P. The change in momentum takes the electron to a new cyclotron trajectory, centered at R_2. The average real space position of the electron is thus shifted by a distance $d = |\mathbf{R}_2 - \mathbf{R}_1|$, which can be interpreted as a real space jump.

function (DF) mechanism. The latter seems to prevail in the experimental systems studied, while under certain conditions, also the DP mechanism may become important.

Displacement Mechanism (DP)

Disorder-assisted microwave absorption and emission are able to alter the momentum of the electrons via a mechanism known as displacement mechanism [48, 52, 53, 54] and thus may lead to an additional current in the system.[6] Under specific circumstances, this photocurrent can become negative and may even exceed the dark current, leading to negative diagonal conductivity.

A sketch of the basic idea behind the DP mechanism is given in Figs. 2.5 and 2.6. If the electric field is applied in the x-direction, the LL energy has a spatial gradient

$$\epsilon_n \simeq n\omega_c + eE_{dc}x \quad . \tag{2.11}$$

Imagine now that the microwave frequency ω is tuned above an integer multiple of ω_c (see Fig. 2.5). Then, direct microwave absorption is impossible since there are no available final states which obey energy conservation. If, however, the electron was simultaneously displaced a distance Δx against the electric field, final states would be available. By contrast, there would be no corresponding process with displacement along the field direction, leading to an effective photocurrent *against* the field direction. Negative conductivity would ensue when this photocurrent exceeds the dark current.

[6] An idea closely related to the displacement mechanism was formulated long time ago by Ryzhii *et al.* [46, 55] in the context of a strong dc electric field.

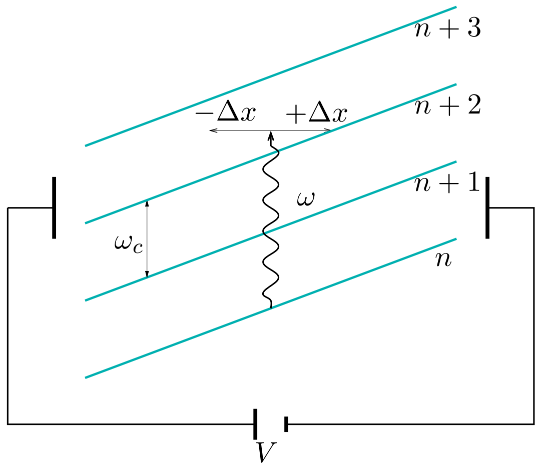

Figure 2.5: Sketch of disorder-assisted microwave absorption, which lies at the heart of the displacement (DP) mechanism. The Landau levels, separated in energy by ω_c, are tilted due to a dc bias and, for simplicity, the disorder-broadening of the LLs is neglected. By absorption of a microwave photon of frequency ω, slightly above a multiple of ω_c, an electron in the nth LL is excited and, subsequently, scattered to the left or right by a distance $\pm\Delta x$. If the final density of states to the left surpasses that on the right, the dc current is enhanced, while in the opposite case, it is reduced. In the case depicted here, the electron is disorder-scattered "uphill" into an available state in the $(n+2)$nd LL to the right, which effectively leads to to a reduction of the dc current. See also Fig. 2.6.

To complete this picture, we need to identify a mechanism which leads to a real-space displacement of the electrons. Indeed, in high magnetic fields, any scattering mechanism with finite momentum transfer is accompanied by such a displacement.[7] The underlying reason for this is the close association of position and momentum in the presence of a magnetic field. An intuitive classical picture for this is presented in Fig. 2.4, which shows that any scattering of an electron gyrating on a cyclotron orbit results in a displacement of the guiding center (i.e., of the center of the cyclotron orbit).

The dominant scattering mechanism at low temperatures is disorder scattering. The displacement mechanism arises from a second-order scattering event, involving *simultaneous* microwave absorption and disorder scattering.

So far, we considered the situation when the microwaves are detuned just above an integer multiple of ω_c. In the opposite situation of detuning below an integer multiple of ω_c, we find an enhanced density of final states along the electric field direction. In this case, the photoconductivity has the same sign as the dark conductivity. This shows that the displacement mechanism indeed yields a photoconductivity that oscillates as a function of the ratio ω/ω_c, as observed in experiment.

Durst *et al.* [48], who first advanced the DP mechanism as a possible scenario for the explanation of ZRS, performed a diagrammatic calculation and found radiation-induced resistivity oscillations of the correct period which they

[7] Note that the momentum transfer involved in the microwave absorption process is essentially zero due to the largeness of the speed of light (since $k = \omega/c$).

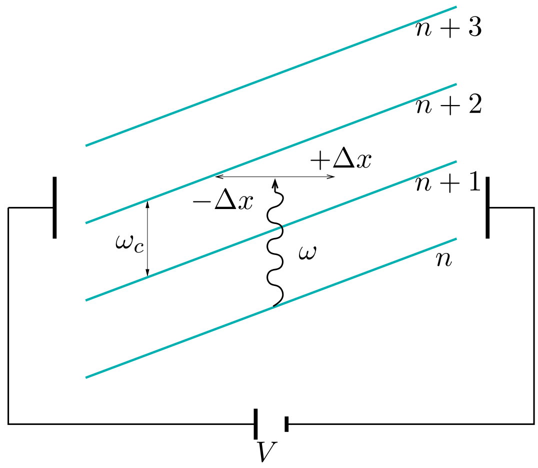

Figure 2.6: Corresponding sketch for $\omega < 2\omega_c$. In this case, the electron is photoexcited below the $(n+2)$nd LL and subsequently disorder-scattered to the left, thereby increasing the net total current. See also Fig. 2.5.

interpreted within the above picture. They found that the displacement mechanism may indeed lead to a locally negative conductivity, since there are regions in magnetic field where, for suitably chosen microwave power and frequency, the longitudinal linear response conductivity assumes absolute negative values, $\sigma_d < 0$. The conductivity oscillations leading to ZRS (see Fig. 2.1) indeed appear to be truncated at zero where they would have swung over to negative values.

Distribution Function Mechanism (DF)

The distribution function mechanism, later advanced by Dorozhkin [44] and Dmitriev *et al.* [56, 49, 50], however, seems to play the dominant role in the systems studied experimentally. The microwave irradiation drives the electronic distribution function away from equilibrium and may lead to a population inversion, which is able to produce a locally negative conductivity through the emergence of a negative photocurrent. The simplest example is sketched in Fig. 2.7. Shown on the left-hand side is a completely filled, disorder-broadened Landau band. This situation corresponds to the case without irradiation. Under irradiation with microwaves, some electrons are promoted to the upper Landau band. Inelastic relaxation then leads to the dynamically equilibrated electron distribution shown on the right of Fig. 2.7. If the microwave frequency ω slightly exceeds the LL center-to-center distance ω_c, this redistribution may lead to a population inversion in both the higher and lower Landau band, as it is shown in the top right of Fig. 2.7. This population inversion is then able to trigger negative conductivity as described in the next paragraph. For microwave frequencies below the cyclotron frequency (bottom right in Fig. 2.7), the partial filling of the higher Landau band is conventional, leading to a positive contribution to the photoconductivity.

The remainder of this section is devoted to the connection between population inversion of the topmost Landau band and absolute negative conductivity. The

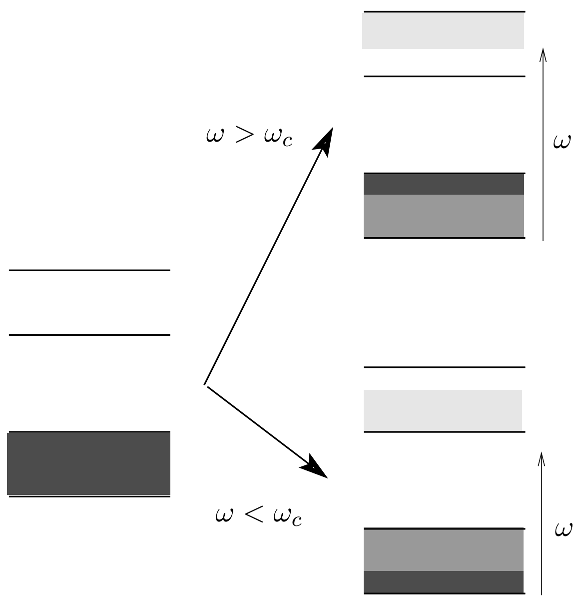

Figure 2.7: Simplest example of the change in electronic distribution in the presence of a driving microwave field. The total occupation is normalized to unity. Dark gray indicates complete filling of the corresponding states, while light gray indicates a small occupation ε. Intermediate gray then represents an occupation $(1-\varepsilon)$. To the left, corresponding to the equilibrium situation without microwave irradiation, a completely filled and a totally empty (disorder-broadened) Landau band are shown. To the right, the occupation of these bands is sketched for $\omega > \omega_c$ (top) and $\omega < \omega_c$ (bottom). For $\omega > \omega_c$, the Landau bands show a population inversion, which may trigger the formation of a ZRS as described in the main text.

total conductivity of the system can be written as

$$\sigma = \int d\epsilon \left(-\frac{\partial f(\epsilon)}{\partial \epsilon} \right) \sigma(\epsilon) \quad , \tag{2.12}$$

where $f(\epsilon)$ is the (nonequilibrium) electron distribution function, which describes the occupation of the LL states under irradiation, and $\sigma(\epsilon)$ determines the contribution of electrons with energy ϵ to the dissipative transport. Under microwave irradiation, the change in the electron distribution function can be computed using a kinetic approach. The corresponding kinetic equation contains collision integrals for disorder scattering, microwave absorption and emission, and a phenomenological term accounting for inelastic relaxation. Assuming an oscillatory density of states to model the ladder of disorder-broadened Landau levels, Dmitriev *et al.* [50] were able to show that the oscillatory density of states directly translates into a small oscillatory contribution to the electron distribution function. The period of this oscillatory contribution depends on the ratio ω/ω_c. This contribution oscillates rapidly in energy, so that the derivative in Eq. (2.12) might be large. As a consequence, the small oscillatory component of the nonequilibrium distribution function can indeed strongly affect the conductivity of the system, Eq. (2.12). In particular, it can be demonstrated [50] that a coincidence of regions of *inverted* population in the electron distribution function with maxima of the density of states can entail a negative local conductivity.

The irradiation-induced change in the distribution function thus affects the current in a nontrivial way, opening the way for possible ZRS. Under typical experimental conditions, the DF mechanism tends to dominate over the DP mechanism [49] although it has been shown that there are exceptions to this [57]. For the experiments on ZRS mentioned in Section 2.1, the DF contribution usually is larger by a factor τ_{in}/τ_s^*, where τ_s^* is the single particle (or quantum) time in the presence of magnetic field, which, for the experiments under study, is shorter than the inelastic relaxation time τ_{in} by several orders of magnitude.