

Chapter 6

An Editor for On-Line Handwritten Mathematical Expressions

6.1 Introduction

This chapter describes a prototype system for editing and recognition of on-line handwritten mathematical expressions. The user enters an expression using the stylus of a pen-based device and writes following the usual mathematical conventions. The editor provides a user-friendly interface which allows manipulation and correction of the expressions via menus and gestures. The recognition engine generates output suitable for use in word processors and symbolic computation.

The application consists of three main panels, a tool bar, and a main menu, see Fig. 6.1. The main panel is a drawing area used writing the expression. The other two panels show the recognition results. One of them shows the output as plain text and the other as a graphic output; these are the Messages and Output panels. The main menu contains sub-menus which correspond to some of the special actions for editing of mathematical formulas. The tool bar contains buttons characteristic of an editor, buttons to open and save files, and buttons to undo and redo actions, buttons to copy and paste handwritten data, as well as buttons for actions specific for formula manipulation and interpretation.

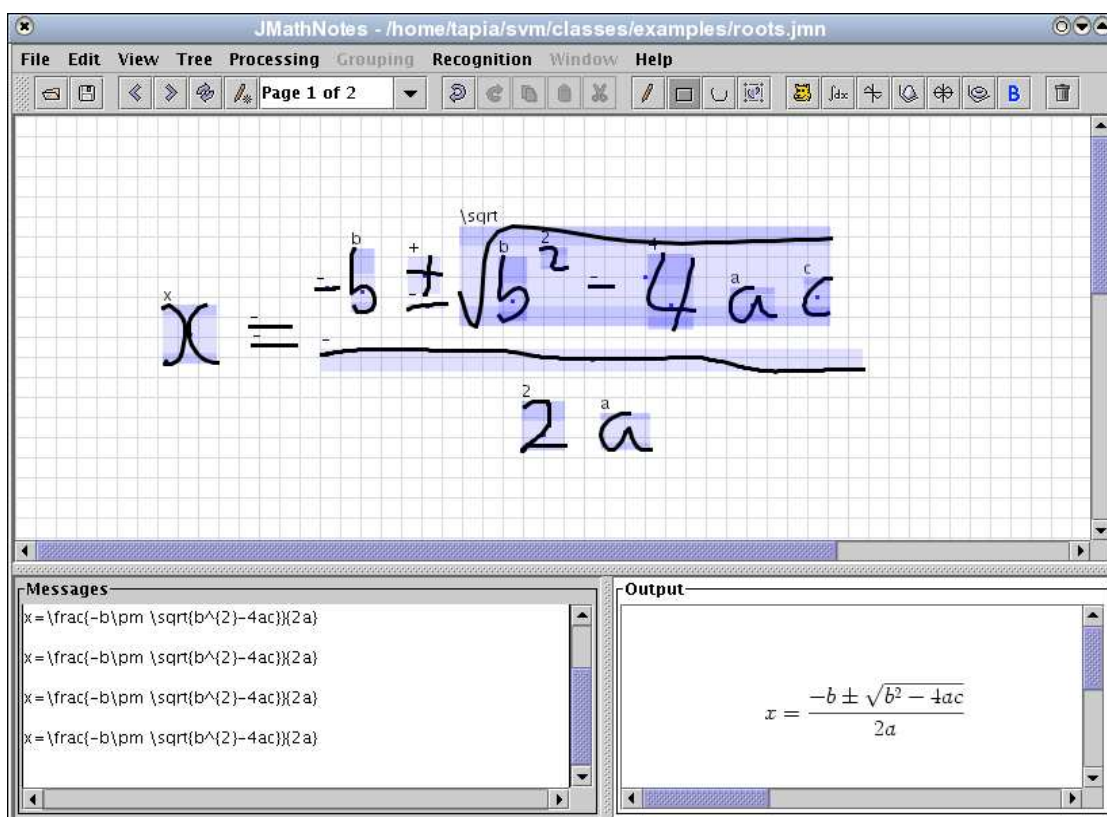


Figure 6.1: The JMathNotes editor.

6.2 Editor Capabilities

6.2.1 General Description


When the user draws a stroke, it is tested whether it belongs to a previously written symbol by using the grouping-heuristic described in the previous chapters. After that, the symbol is classified and the label is shown in the upper left corner of the symbol's bounding box. The bounding box serves as a guide to check whether the stroke was properly grouped. Depending on the symbol's class, darker areas are also shown within the bounding box, which show the limits of the superscript and subscript thresholds. The View menu contains check boxes which can be activated to view different stroke's and symbol attributes, like points, stroke direction, the number of points, etc.


By default, the editor recognizes a basic set of characters which consists of the digits 0-9; the letters a - n , p , r - u , w - z ; the Greek letters α , β , θ , π ; the square root; the operators, integral, sum, and product. In addition to the basic classifier, there are another four classifiers that correspond to the following set of symbols: lower-case


letters, upper-case letter, Greek letters, and punctuation symbols. The user can turn one of them on as the default classifier by activating the corresponding radio button from the menu item Default Classifier, located in the Recognition menu. Once this action takes place, a new symbol is classified as one of the symbols of the selected set.

6.2.2 Editing and Correction

The handwritten data can be manipulated as usual by the undo, redo, copy, and cut and paste buttons, available in the Edit menu and the tool bar. Once the strokes or symbols are selected, we can cut or copy them. To activate the selection modes in the editor, the user only has to push the corresponding toggle buttons found in the tool bar. Editing modes available in the editor are described below.


 **Drawing mode.** This is the default mode when the application starts up.

 **Select symbol mode.** The drawing color turns to red to indicate that this mode is activated. When the user drags the cursor, a red rectangle is drawn. Symbols are selected if the rectangle intersects or contains them. If no symbols are intersected, the previous selection is deactivated. A red border surrounds the bounding box of selected symbols.

 **Select stroke mode.** To indicate that this mode is activated, the drawing color turns to red. In this case, a red stroke is drawn instead of a rectangle. Strokes are selected if the red stroke intersects them. If no strokes are intersected, the previous selection is deactivated. Selected strokes are surrounded by a red rectangle.

Because of irregular positioning of subindices or superindices, sometimes it is necessary to translate the selected symbols or strokes. A way to do this is putting the editor into one of the selection modes described above. The user only has to select symbols or strokes, move the cursor inside some of the bounding boxes and drag the mouse until the desired position is reached.

Another important action is the correction of stroke grouping. It is accomplished by pushing the button with the following icon.

 **Group/ungroup strokes.** Selected strokes are grouped or ungrouped using the following criteria: when a symbol is selected, the strokes that form it are ungrouped in such a way that each one represents a symbol. If the selected strokes

belong to different symbols, they are grouped in a single symbol and the remaining strokes are ungrouped, each representing one a single symbol.


6.2.3 Recognition Actions


Two actions are available via key combinations to preview the recognition results of the structural analysis before a more complex processing of the recognized expression takes place.

Ctrl-D. Show dominance tree. This action shows the dominance tree of the first recursion step obtained by our structural analysis method, as described in Sect. 5.2. Red segments connect symbols which form the main baseline of the expression. Blue segments connect clusters of symbols which are dominated by the symbols in the main baseline. See Fig. 6.2(a).

Ctrl-R. Show relation tree. This action draws the final relation tree obtained by structural analysis. Symbols forming baselines are connected through red segments. Symbols are connected to other baselines through blue and green segments if the first dominates the last under the relations subscript and superscript respectively. Horizontal lines recognized as fraction lines are surrounded by a red rectangle. See Fig. 6.2(b).

Once the user has applied all the corrections necessary to further process the expression, he can tell the system to process the expression by pushing the corresponding buttons in the tool bar. This final expression processing consists of its conversion into \LaTeX and its algebraic manipulation through the computer algebra system Mathematica, in case they are installed on the computer. We describe these actions below.

 **Convert to \LaTeX .** The recognized expression is converted into \LaTeX and the text is shown in the Messages panel. The graphic representation is shown in the Output panel.

 **Calculate.** The mathematical operations indicated in the expression are realized. The following conventions are assumed when processing the expression:

- The letter d is always considered to represent the differential operator of the integral.

$$\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} dx$$

(a)

$$\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} dx$$

(b)

Figure 6.2: (a) Dominance tree obtained by the first recursion step and (b) the final relation tree.


- The letter e is considered to represent the base of the natural logarithm.
- Square brackets are used to define the matrix mode. Matrices containing one row are always considered as the arguments of functions, following the notation of Mathematica.

✚ **Two-dimensional plot.** The functional relation described by the expression is plotted. The default independent variable is x . The notational conventions mentioned above are also assumed. See Fig. 6.3.

📐 **Three-dimensional plot.** The parametric relation described by the expression is plotted. The parametric variable and its range are established using the notation of Mathematica. The notational conventions mentioned above are also assumed. See Fig. 6.4.


✚ **Two-dimensional parametric plot.** The functional relation described by the

expression is plotted. The default independent variables are x and y .


 **Tree-dimensional parametric plot.** The parametric relation described by the expression is plotted. The parametric variables and their range is established using the notation of Mathematica.


6.2.4 Page Actions

The page actions correspond to the creation of a new page and the navigation from one page to another. The following icons in the application correspond to such actions:

 **New Page.** A new page for drawing is created and it is added to the pages list.

 **Go to previous page.** Go to the previous page and show the recognition results.

 **Go to next page.** Go to the next page and show the recognition results.

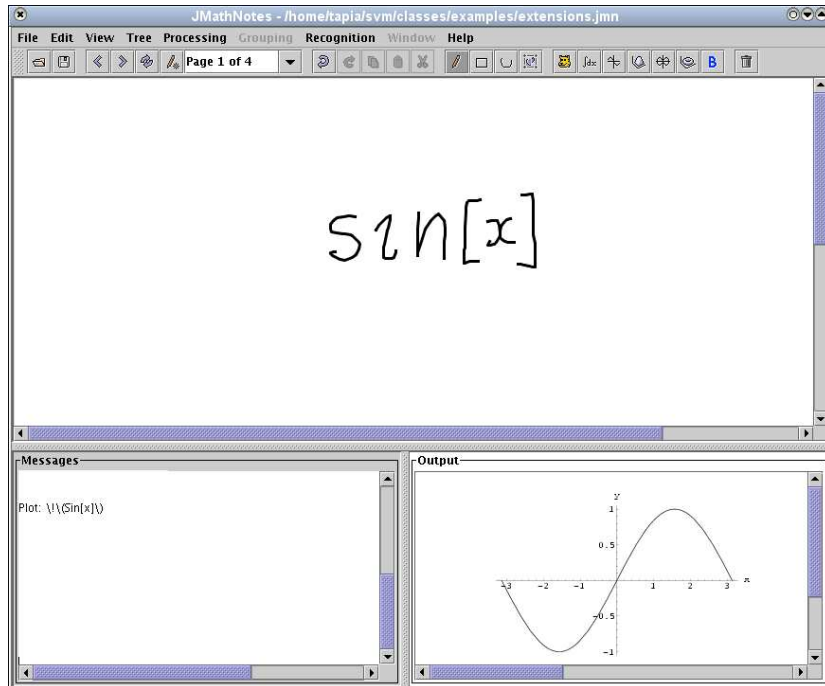
 **Remove page.** This action cleans up the drawing area and removes the page from the pages list.

6.2.5 Pop-up Menus

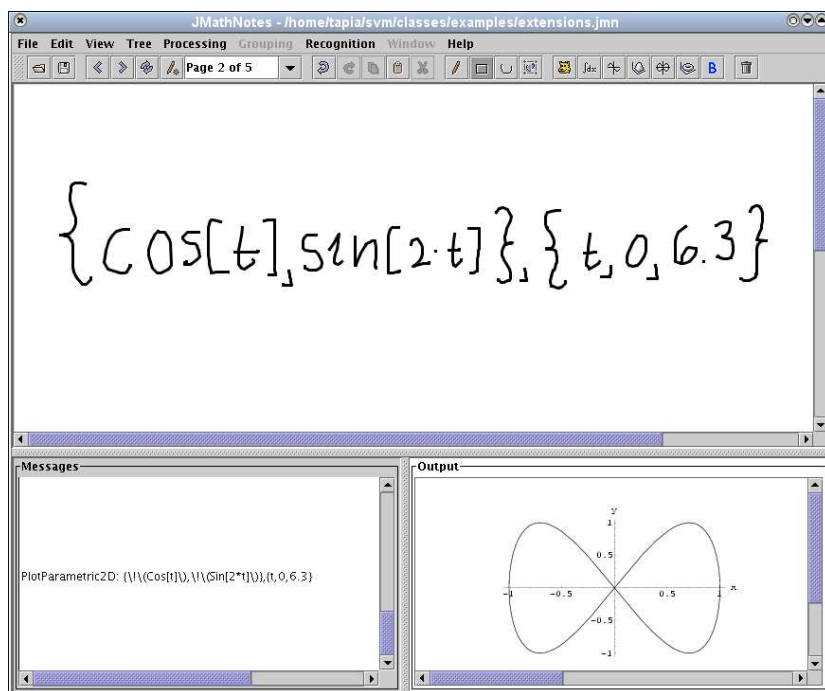
Two pop-ups menus are available when the user pushes the right button of the mouse in the drawing area, see Fig. 6.5. If the cursor does not hover over a symbol, a pop-up menu with the page and recognition actions appears on the screen. If the user right-clicks on a symbol, a pop-up menu allows the manipulation of the symbol's labels in three different ways. One of them consists of rewriting the label in the text field. The second way is to fix the current label so that classification does not take place and the label in the text field is assigned to the following symbols while the option remains active. This is useful when entering new symbols not contained in the system for further manipulation. Finally, we can assign one of the four different classifiers included in the system to the selected symbol. They correspond to the basic set of symbols described before, lower-case letters, upper-case letters, Greek letters, and punctuation marks.

6.2.6 Manipulation via Gestures

To avoid the frequent use of buttons and menus, the system offers the use of gestures. They are symbols constituted by a single stroke and correspond to the most frequent

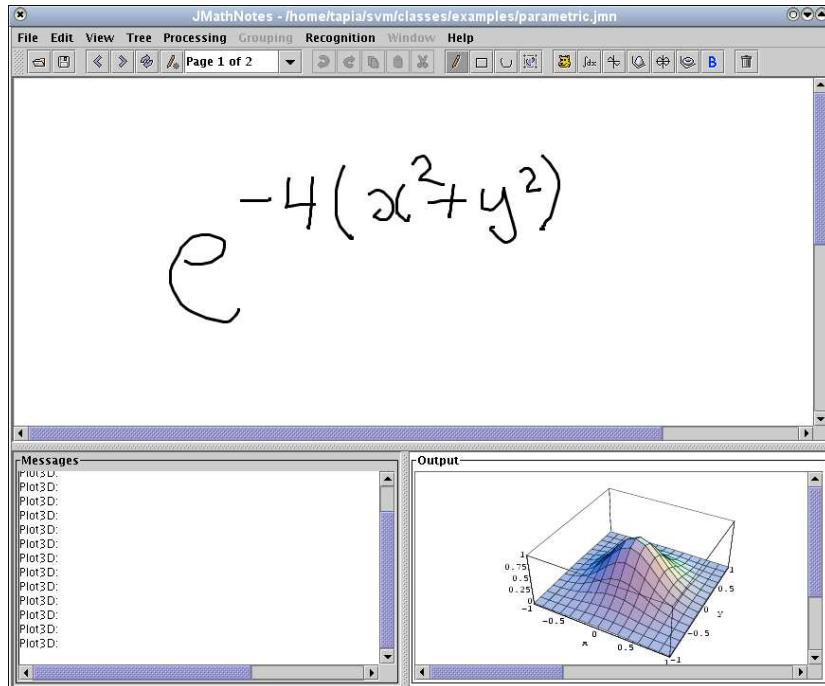


(a)

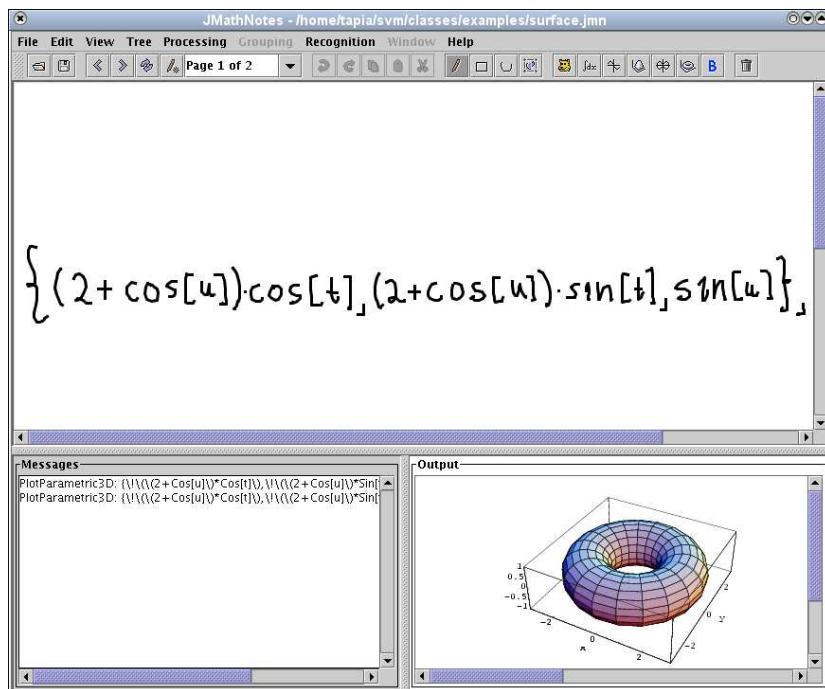


(b)

Figure 6.3: Function plotting.



(a)



(b)

Figure 6.4: Three-dimensional plotting.

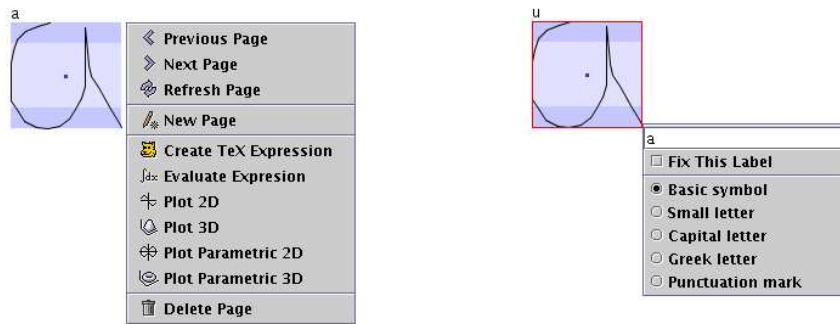


Figure 6.5: *Pop-up menus in the system: navigation and processing pop-up menu (left) and label correction and selection of classifier (right).*

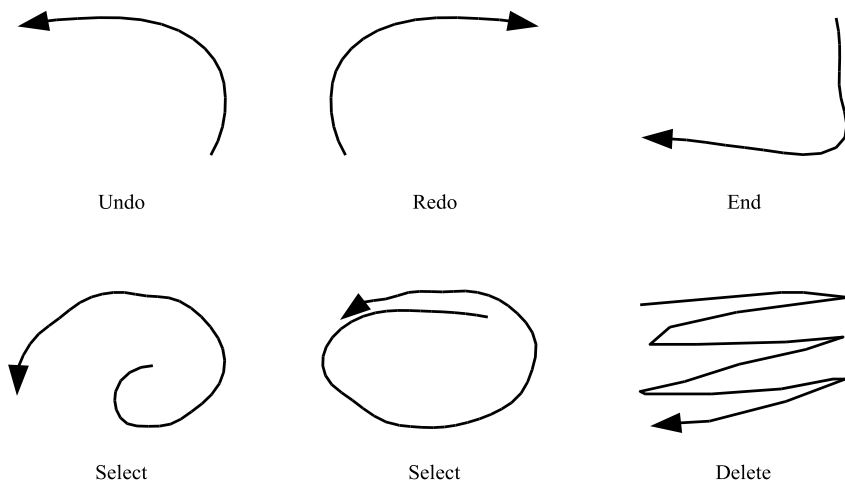


Figure 6.6: *Gestures used by the system.*

editor actions. Figure 6.6 shows the five gestures recognized by the system. Note that in this case the stroke's direction is important for the recognition and it has to be followed as indicated in the figure. The recognition of gestures takes place only in drawing mode and if the user has activated the corresponding option in the Recognition menu.

Gestures interact with handwriting in two modes: free and intersection. The Select and Delete gestures work in intersection mode, this means the strokes they intersect are selected and deleted, respectively. When a group of strokes is selected, the editor turns into the selection symbol mode. If strokes are deleted, the system stores them in the clipboard and they can be re-inserted by calling the undo or paste actions. The Undo, Redo, and End gestures work only in free mode. That means that they can be written anywhere in the drawing area and no special action is taken on the data they intersect.

Table 6.1: *Actions and commands executed by the End gesture.*

action	letter's group	action	letter's group
Calculus	c	Solve equation	s
Plot 2D	p	Parametric Plot 2D	pp
Plot 3D	p3d	Parametric Plot 3D	pp3d



Figure 6.7: *Commands used by the end gesture.*

By default, the action taken by the End gesture is to transform the expression into \LaTeX . The end gesture not only defines the conversion into \LaTeX of the expression but also its manipulation by the other recognition actions. To this end, the user writes some the command groups shown in Table 6.1 and surrounds them by the End gesture to execute them, as shown in Fig. 6.7

6.2.7 String Substitution

There are some classification errors which are impossible to be avoided even by having the best classifier. These errors are generated by the impossibility to differentiate the lower-case o and the upper-case ‘O’ from the number zero. To overcome this problem we have implemented a string substitution as a post-processing step applied to the final string representation of the mathematical expression. Table 6.2 shows the default substitution pairs. Note that these pairs correspond mainly to function names and punctuation marks. The user can add further substitutions pairs by selecting the corresponding menu item in the Recognition menu.

Table 6.2: *Default substitution pairs used by the system. these substitutions are specific to the \LaTeX conversion.*

original	substitution	original	substitution	original	substitution
sin	\sin	c0s	\cos	c0t	\cot
c0anh+	\cot	c0sh	\cosh	c0th	\coth
log	\log	l0g	\log	l09	\log
l0_{9}	\log	lg	\log	lg_{9}	\log
ln	\ln	.*	:	*^{\.}	:
^{\.}	:	*_{\.}	:	^{\.}*	:
,*	;				