

# Encapsulation of Endohedral Fullerenes for Quantum Computing

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*”Empirie ist komplizierter als Mathematik und läßt  
sich nicht so leicht in glatte Sätze einfangen”*  
E. Schrödinger, 1935



## CONTENTS

|  |    |
|--|----|
| 1. <i>Theoretical Background and Experimental Techniques</i> . . . . .   | 9  |
| 1.1 History of Quantum Computation . . . . .   | 9  |
| 1.2 Qubits . . . . .   | 10 |
| 1.3 Entanglement . . . . .   | 13 |
| 1.4 Electron Spin Resonance (ESR) . . . . .  | 13 |
| 1.4.1 Zeeman Interaction and continuous wave ESR . . . . .   | 14 |
| 1.4.2 Zero Field Splitting (Fine Structure) . . . . .  | 15 |
| 1.4.3 Hyperfine Interaction . . . . .  | 16 |
| 1.4.4 Line Shapes . . . . .  | 17 |
| 1.5 Pulsed ESR . . . . .   | 18 |
| 1.5.1 Classical Picture . . . . .  | 18 |
| 1.5.2 Pure and Mixed States. Density Matrix . . . . .  | 18 |
| 1.5.3 Liouville-von Neumann equation . . . . .   | 20 |
| 1.5.4 Liouville space . . . . .  | 21 |
| 1.5.5 Fictitious spin 1/2 operator formalism . . . . .   | 22 |
| 1.5.6 Basic Pulse Experiments . . . . .  | 23 |
| 1.6 ENDOR . . . . .  | 25 |
| 2. <i>The Endohedral Fullerenes <math>N@C_{60}</math>, <math>P@C_{60}</math> and <math>N@C_{70}</math></i> . . . . . | 29 |
| 2.1 Production by Ion Implantation . . . . .   | 29 |
| 2.2 Enrichment . . . . .   | 31 |
| 2.3 Properties . . . . .   | 33 |
| 2.3.1 Quadrupole Echo . . . . .  | 34 |
| 2.4 Chemical Modification . . . . .  | 37 |
| 2.5 Conclusions . . . . .  | 40 |
| 3. <i>Alignment of Fullerenes for Quantum Computation</i> . . . . .  | 41 |
| 3.1 Fullerene Based Quantum Computer . . . . .   | 41 |
| 3.2 Alignment of Endohedral Fullerenes in Liquid Crystal . . . . .   | 43 |
| 3.3 Alignment of Endohedral Fullerenes in a Solid State Matrix . . . . .   | 44 |
| 3.3.1 ESR Measurements . . . . .   | 46 |

|       |   |     |
|-------|---|-----|
| 3.3.2 | ENDOR Measurements . . . . .  | 56  |
| 3.3.3 | Distant ENDOR . . . . .   | 57  |
| 3.4   | Conclusions . . . . .   | 60  |
| 4.    | <i>Relaxation of P@C<sub>60</sub></i> . . . . .                           | 61  |
| 4.1   | Theoretical Background . . . . .  | 61  |
| 4.2   | Spin-Lattice Relaxation . . . . .   | 62  |
| 4.3   | Spin-Spin Relaxation . . . . .  | 66  |
| 4.4   | Conclusions . . . . .   | 73  |
| 5.    | <i>Pure States and Entanglement in P@C<sub>60</sub></i> . . . . .         | 75  |
| 5.1   | Preparation of Pseudo-pure States in P@C <sub>60</sub> in BrPOT . . . . . | 75  |
| 5.2   | Phase Rotations . . . . .   | 84  |
| 5.3   | Pseudo-Entanglement in P@C <sub>60</sub> in BrPOT . . . . .               | 87  |
| 5.4   | Decoherence of the pseudo-entanglement . . . . .                          | 91  |
| 5.5   | Conclusions . . . . .   | 96  |
| 6.    | <i>Summary</i> . . . . .  | 97  |
| 7.    | <i>Zusammenfassung</i> . . . . .  | 101 |
| 8.    | <i>Instruments and Methods</i> . . . . .                                  | 105 |
|       | <i>Bibliography</i> . . . . .   | 106 |
| 9.    | <i>Acknowledgements</i> . . . . .   | 115 |
| 10.   | <i>List of Publications</i> . . . . .                                     | 117 |

## LIST OF FIGURES

|      |   |    |
|------|---|----|
| 1.1  | Free Induction Decay (FID) after a $\pi/2$ pulse. . . . .                             | 24 |
| 1.2  | Pulse sequence of the Hahn echo. . . . .  | 24 |
| 1.3  | Pulse sequence of the inversion recovery. . . . .                                     | 25 |
| 1.4  | Energy diagram for a system with $S = 1/2$ and $I = 1/2$ . . . . .                    | 26 |
| 1.5  | Mims ENDOR pulse sequence. . . . .  | 26 |
|      |   |    |
| 2.1  | Ion implantation . . . . .  | 30 |
| 2.2  | P@C <sub>60</sub> HPLC peak reconstruction . . . . .                                  | 32 |
| 2.3  | P@C <sub>60</sub> 20 % . . . . .  | 32 |
| 2.4  | Energy levels and ESR spectra of P@C <sub>60</sub> . . . . .                          | 33 |
| 2.5  | Energy levels and ESR spectra of polycrystalline N@C <sub>60</sub> . . . . .          | 34 |
| 2.6  | Echo detected spectrum of polycrystalline P@C <sub>60</sub> at $T = 10$ K . . . . .   | 35 |
| 2.7  | Echo of polycrystalline P@C <sub>60</sub> at $T = 10$ K . . . . .                     | 35 |
| 2.8  | Fourier transform of the quadrupole echo . . . . .                                    | 36 |
| 2.9  | X-band CW ESR spectra of P@C <sub>60</sub> adducts . . . . .                          | 38 |
| 2.10 | W-band CW ESR spectra of P@C <sub>60</sub> adducts . . . . .                          | 39 |
|      |   |    |
| 3.1  | Concept for a QC Based on Fullerenes . . . . .  | 41 |
| 3.2  | N@C <sub>60</sub> in liquid crystal . . . . .   | 43 |
| 3.3  | Chemical formula of 2,4,6-tris-(4-bromophenoxy)-1,3,5-triazine (Br-<br>POT) . . . . . | 44 |
| 3.4  | Hexagonal Crystal Modification of the C <sub>60</sub> in BrPOT . . . . .              | 45 |
| 3.5  | Rhombohedral Crystal Modification of the C <sub>60</sub> in BrPOT . . . . .           | 46 |
| 3.6  | Photos of C <sub>60</sub> in BrPOT crystals . . . . .                                 | 47 |
| 3.7  | Esr spectrum of hexagonal crystals N@C <sub>60</sub> in BrPOT . . . . .               | 48 |
| 3.8  | Esr spectrum of hexagonal crystals P@C <sub>60</sub> in BrPOT . . . . .               | 49 |
| 3.9  | Esr spectrum of hexagonal crystals N@C <sub>70</sub> in BrPOT . . . . .               | 50 |
| 3.10 | Esr spectrum of N@C <sub>60</sub> in BrPOT . . . . .                                  | 51 |
| 3.11 | Angular dependence of ZFS in N@C <sub>60</sub> in BrPOT . . . . .                     | 52 |
| 3.12 | Esr spectrum of P@C <sub>60</sub> in BrPOT . . . . .                                  | 53 |
| 3.13 | Angular dependence of ZFS in P@C <sub>60</sub> in BrPOT . . . . .                     | 54 |
| 3.14 | Temperature dependence of ZFS of P@C <sub>60</sub> in BrPOT . . . . .                 | 55 |

|      |   |    |
|------|---|----|
| 3.15 | Temperature dependence of the hyperfine constant $a$ of $P@C_{60}$ in BrPOT . . . . .                             | 55 |
| 3.16 | Echo detected spectrum of $P@C_{60}$ in BrPOT at $T = 10K$ . . . . .  | 56 |
| 3.17 | ENDOR Spectra $P@C_{60}$ in BrPOT . . . . .   | 57 |
| 3.18 | H-ENDOR of $P@C_{60}$ in BrPOT . . . . .  | 58 |
| 4.1  | Relaxation pathways in $P@C_{60}$ . . . . .   | 63 |
| 4.2  | Temperature dependence of $\frac{1}{T_1}$ of $P@C_{60}$ in BrPOT . . . . .  | 64 |
| 4.3  | Temperature dependence of $\frac{1}{T_1}$ of polycrystalline $P@C_{60}$ . . . . .                                 | 65 |
| 4.4  | CPMG Pulse Sequence . . . . .   | 66 |
| 4.5  | Time traces of the ESE decay and CPMG pulse sequence in $P@C_{60}$ .  | 69 |
| 4.6  | Temperature dependence of the spin-lattice and spin-spin relaxation rates of polycrystalline $P@C_{60}$ . . . . . | 70 |
| 4.7  | Concentration Dependence of $T_2$ in $P@C_{60}$ . . . . .   | 71 |
| 4.8  | Spin-spin relaxation rate measured at $T = 10$ K as a function of $\sin^2(\theta/2)$ . . . . .                    | 71 |
| 4.9  | Temperature dependence of the spin-spin relaxation rates of $P@C_{60}$ in rhombohedral BrPOT . . . . .            | 72 |
| 5.1  | Energy levels of $P@C_{60}$ in BrPOT for the preparation of pseudo-pure states and entanglement . . . . .         | 77 |
| 5.2  | Pulses sequences for the preparation and detection of the pseudo-pure state $\rho_6$ . . . . .                    | 79 |
| 5.3  | Rabi oscillations for the detection of $\rho_6$ . . . . .   | 82 |
| 5.4  | Rabi oscillations for the detection of $\rho_4$ . . . . .   | 83 |
| 5.5  | Modified Hahn echo and Mims ENDOR pulse sequences with phase rotations . . . . .                                  | 85 |
| 5.6  | Time traces of the electron spin and nuclear spin phase rotations in $^{15}N@C_{60}$ . . . . .                    | 86 |
| 5.7  | Pulse sequence for the creation and detection of the entanglement $\rho_{14}$                                     | 88 |
| 5.8  | Phase oscillation and their Fourier transform for the detection of the entanglement $\rho_{14}$ . . . . .         | 89 |
| 5.9  | Fourier transform of the phase oscillation with $\tau_{ent} = 130$ ns . . . . .                                   | 90 |
| 5.10 | Energy levels of $P@C_{60}$ in BrPOT for the preparation of entanglement $\rho_{45}$ and $\rho_{23}$ . . . . .    | 92 |
| 5.11 | Pulse sequence for the entanglement $\rho_{ent}^{45}$ . . . . .   | 93 |
| 5.12 | Preparation of the entanglement $\rho_{45}$ . . . . .   | 94 |
| 5.13 | Decay of the entanglement $\rho_{ent}^{23}$ . . . . .   | 95 |