Introduction

The theory of partial differential equations is very manifold and rich. The reason is that there is no general method for studying all partial differential equations. A special method is necessary for research of each problem for any partial differential equation. The linear equations (simple equations) and first order systems and some kinds of linear equations and second order systems are well studied, as they have early arisen in problems of mathematical physics. Some special higher order equations as for example the biharmonic equation arising in the theory of elasticity are also investigated in detail. The systems of partial differential equations of 2-nd order with Fuchs type operator in the main part investigated in the dissertation have not been studied regularly yet.

The adventure of the present dissertation work is the studying at a wide class of partial differential equations, including elliptic and non-elliptic equations.

Partial examples of these systems are elliptical systems with Laplace, Bitsadze operators in the main part, Euler-Darbou generalized equations, etc.

Elliptic systems of the first order in the plane with singular coefficients, and also boundary value problems for them, as Rimann-Hilbert problems, problem of conjugations, Dirichlet problem and the skew derivative (Poincare) problem are quite deeply investigated in the works of I.N.Vekua [21-24], L.G.Mikhailov [25-27], Z.D.Usmanov [28-31], N.K.Bliev [18,19], H.Begehr, Dao-Ding Dai[16], M. Reissig, A.Timofeev [33], A.S.Abdymanopov [1-7], A.Tungatarov [33-45] and others.

In recent years the theory of elliptic systems of higher order in the plane with singular coefficients is intensively developing. This can be explained by the fact that these specified systems arise when solving many problems of analysis, differential geometry, mathematical and theoretical physics.

Special cases of the elliptic systems of the second order in the plane with singular coefficients are the Euler-Darbou and the Bitsadze equations, studying the Dirichlet problem which at a certain time received a big resonance in the theory of boundary value problems. Systems of partial differential equations in the plane with Fuchs operator in the main part have not been investigated yet.

Firstly this is because the application of the basic operator of generalized analytical functions theory, the Pompein operator

$$T_G f = -\frac{1}{\pi}\iint \frac{f(\zeta)}{\zeta - z} dG_\zeta$$

to these equations transforms them to singular integral equations. But in this case the coefficients of the received equations do not belong to the class $L_q(G),\ q > 2$. Therefore in the received systems of integral equations there appear operators not bounded even in the class $L_2(G)$, and this complicates the investigation of boundary value problems. Secondly, looking for solutions in the form of double series $\sum\sum a_{kl} z^k \bar{z}^l$ greater difficulties arise connected with the convergence of the series. Hence, the urgency of the choice of the research topic does not give rise to doubts.

The presented research as follows from above has fundamental value for the development of the theory of boundary value problems for systems of partial differential equations in the plane. The results of the research can be practically applied in various sections of geometry, mechanics and mathematical physics and make an essential contribution to the theory of boundary value problems.

Purpose of the work.

The purpose and the subject of the dissertation is the research of the problems of existence and construction of continuous solutions of second order partial differential equations and systems with Fuchs type operator in the main part, and also boundary value problems (Dirichlet, Neumann problems, initial boundary problems, boundary problems with given growth at infinity, etc.) for them in unbounded angular domains of arbitrary opening in the plane.

Novelty and perspectivity of the research.

Initial-boundary value problems in unbounded angular domains of arbitrary opening for systems of partial differential equations with Fuchs operator in the main part in the plane have not been studied yet. Moreover, boundary value problems considered in the dissertation in the theory of the generalized analytical functions have not been investigated yet. For the systems of partial differential equations in the plane yet only Dirichlet and Neumann problems are investigated in bounded domain.

Besides, the systems of partial differential equations of Fuchs type have not been investigated yet even in bounded domains. The results of the work may serve for further development of the theory of boundary value problems for systems of partial differential equations in the plane as was begun by A.V.Bitsadze and can be applied in the theory of infinitesimal bendings of surfaces of positive curvature with a general structure in the point of flattening.

The work includes the following new results

1. Varieties of continuous solutions of some systems of partial differential equations of second order in the plane with Fuchs operator in the main part in unbounded angular domains of arbitrary opening are constructed.
2. The problems of Dirichlet, Neumann for some systems of partial differential equations of second order in the plane with Fuchs operator in the main part in unbounded angular domains of arbitrary opening are solved.
3. The problems of Dirichlet and Neumann with given growth at infinity for some systems of partial differential equations of second order in the plane with Fuchs operator in the main part in unbounded angular domains of arbitrary opening are solved.

Contents of the work.

We shall briefly summarize the contents of the dissertation. Let $0<\varphi_1\le 2\pi$ and $G=\{z=re^{i\varphi}:0\le r<\infty,\quad 0<\varphi<\varphi_1\}$. The first chapter consists of four sections. The first section is devoted to the research of the equation

$$4\alpha\bar{z}^2V_{\bar{z}\bar{z}}+4\beta z\bar{z}V_{z\bar{z}}+4\gamma z^2V_{zz}+b(\varphi)\overline{V}=f(\varphi)r^{\lambda},\quad z\in G, \qquad (0.1)$$

where $b(\varphi),\ f(\varphi)\in C[0,\varphi_1]$; $\alpha,\beta,\gamma,\lambda>0$ are real parameters.

At $\beta=0,\ \gamma=0,\ f(\varphi)=b(\varphi)\equiv 0$ in the work [20] a general solution of the equation (0.1) in the unit disk is received and it is proved, that the homogeneous Dirichlet problem for the equation (0.1) has an infinite set of solutions. The initial-boundary value problem for the equation (0.1) is solved in the work [4].

In the first section a variety of continuous solutions from the class $W_p^{\,2}(G)$ is received, where $1<p<\dfrac{2}{2-\lambda}$, *if* $\lambda<2$ *and* $p>1$, *if* $\lambda\ge 2$.

The second section is devoted to the research of the equation

$$4\alpha\bar{z}^2V_{\bar{z}\bar{z}}+4\beta z\bar{z}V_{z\bar{z}}+4\gamma z^2V_{zz}+b(\varphi)\overline{V}=g(r,\varphi),\quad z\in G, \qquad (0.2)$$

where $b(\varphi)\in C[0,\varphi_1]$; α,β,γ are real parameters. Concerning the function $g(r,\varphi)$ it is supposed, that it is representable in the domain G in the form $g(r,\varphi)=\sum_{k=0}^{\infty}\dfrac{g_k(\varphi)r^{\nu k}}{k!}$, where $g_k(\varphi)\in C[0,\varphi_1]$, $\nu>0$, is some parameter $0\le k$, and the series $g(r,\varphi)=\sum_{k=0}^{\infty}\dfrac{|g_k(\varphi)|r^{\nu k}}{k!}$ converges in G.

In this item a variety of continuous solutions of the equation (0.2) from the class $W_p^{\,2}(G)$ is received, where $1<p<\dfrac{2}{2-\nu}$, *if* $\nu<2$ *and* $p>1$, *if* $\nu\ge 2$.

The third section is devoted to the research of the equation

$$4a(\varphi)\bar{z}^2V_{\bar{z}\bar{z}}+4b(\varphi)z\bar{z}V_{z\bar{z}}+4c(\varphi)z^2V_{zz}+d(\varphi)\overline{V}=f(\varphi)r^{\lambda},\quad z\in G, \qquad (0.3)$$

where $a(\varphi), b(\varphi), c(\varphi), d(\varphi), f(\varphi) \in C[0,\varphi_1]$, λ is real parameter. The variety of continuous solutions of the equation (0.3) from the class $W_p^2(G)$ is received, where $1 < p < \frac{2}{2-\lambda}$, *if* $\lambda < 2$ *and* $p > 1$, *if* $\lambda \geq 2$.

The fourth section is devoted to the research of the equation

$$4a(\varphi)\bar{z}^2 V_{\bar{z}\bar{z}} + 4(a(\varphi)+c(\varphi))z\bar{z}V_{\bar{z}\bar{z}} + 4c(\varphi)z^2 V_{zz} + d(\varphi)\overline{V} = g(r,\varphi), \quad z \in G, \qquad (0.4)$$

where $a(\varphi), c(\varphi), d(\varphi) \in C[0,\varphi_1]$.

Here it is supposed that $a(\varphi) \neq c(\varphi)$, $\operatorname{Im}\frac{a(\varphi)+c(\varphi)}{a(\varphi)-c(\varphi)} \geq 0$.

The function $g(r,\varphi)$ is representable in G in the form of $g(r,\varphi) = \sum_{k=0}^{\infty} g_k(\varphi) r^{\nu k}$, where $g_k(\varphi) \in C[0,\varphi_1]$, $\nu > 0$ is a real parameter, $0 \leq k$ and the series $g(r,\varphi) = \sum_{k=0}^{\infty} |g_k(\varphi)| r^{\nu k}$ converges in G.

The variety of continuous solutions of the equation (0.4) from the class $W_p^2(G)$ is received, where $1 < p < \frac{2}{2-\nu}$, *if* $\nu < 2$ *and* $p > 1$, *if* $\nu \geq 2$.

In the second chapter a variety of continuous solutions of the Dirichlet, the Neumann boundary value problems and an initial problem for the equations studied in the first chapter are constructed in an explicit way.

In the first section continuous solutions of the Dirichlet and the Neumann boundary value problems for equation (0.1) are found.

Namely, the following problems are solved:

Problem D_1. *Let* $\beta \neq \alpha + \gamma$. *It is required to find the solution of the equation (0.1) from the class* $W_p^{\,2}(G)$, *where* $1 < p < \dfrac{2}{2-\lambda}$, *if* $\lambda < 2$ *and* $p > 1$, *if* $\lambda \geq 2$, *satisfying the growth and boundary conditions*

$$\left|V(r,\varphi)\right| = \mathrm{O}(r^{\lambda}),\, r \to \infty,$$

$$V(r,0) = b_1 r^{\lambda},\; V(r,\varphi_1) = b_2 r^{\lambda},$$

where b_1, b_2 *are given complex numbers,* $\lambda > 0$ *is a given real number.*

Problem D_2. *Let* $\lambda \neq 1, \alpha \neq \gamma$ *and* $\beta = \alpha + \gamma$. *It is required to find the solution of the equation (0.1) from the class* $W_p^{\,2}(G)$, *where* $1 < p < \dfrac{2}{2-\lambda}$, *if* $\lambda < 2$ *and* $p > 1$, *if* $\lambda \geq 2$ *satisfying the growth and boundary conditions*

$$\left|V(r,\varphi)\right| = \mathrm{O}(r^{\lambda}),\, r \to \infty,$$

$$V(r,\varphi_1) = b_1 r^{\lambda},$$

where b_1 *is a given complex number,* $\lambda > 0$ *is a given real number.*

Problem N_1. *Let* $\beta \neq \alpha + \gamma$ *It is required to find the solution of the equation* (0.1) *from the class* $W_p^{\,2}(G)$, *where* $1 < p < \dfrac{2}{2-\lambda}$, *if* $\lambda < 2$ *and* $p > 1$, *if* $\lambda \geq 2$, *satisfying the growth and boundary conditions*

$$\left|V(r,\varphi)\right| = \mathrm{O}(r^{\lambda}),\, r \to \infty,$$

$$\left.\frac{\partial V(r,\varphi)}{\partial \varphi}\right|_{\varphi=0} = b_1 r^{\lambda},\; \left.\frac{\partial V(r,\varphi)}{\partial \varphi}\right|_{\varphi=\varphi_1} = b_2 r^{\lambda}$$

where b_1, b_2 *are given complex numbers,* $\lambda > 0$ *is a given real number.*

Problem N_2. *Let* $\lambda \neq 1, \alpha \neq \gamma$ *and* $\beta = \alpha + \gamma$. *It is required to find the solution of the equation (0.1) from the class* $W_p^{\,2}(G)$, *where*

$1<p<\frac{2}{2-\lambda}$, *if* $\lambda<2$ *and* $p>1$, *if* $\lambda\geq 2$, *satisfying the growth and boundary conditions*

$$|V(r,\varphi)|=O(r^{\lambda}),\, r\to\infty,$$

$$\left.\frac{\partial V(r,\varphi)}{\partial\varphi}\right|_{\varphi=\varphi_1}=b_2 r^{\lambda},$$

where b_2 *is a given complex number,* $\lambda>0$ *is a given real number.*

In the second chapter continuous solutions of the initial problems of Cauchy type for the equation (0.2) are found. Namely, the following problems are solved:

Problem K_1. *Let* $\beta\neq\alpha+\gamma$. *It is required to find the solution of the equation (0.2) from the class* $W_p^{\,2}(G)$, *where* $1<p<\frac{2}{2-\nu}$, *if* $\nu<2$ *and* $p>1$, *if* $\nu\geq 2$ *satisfying the initial conditions*

$$\left.\frac{\partial^k}{\partial p^k}V(r,\varphi)\right|_{\substack{r=0\\ \varphi=0}}=a_k,\qquad 0\leq k,$$

$$\left.\frac{\partial^k}{\partial p^k}\frac{\partial V(r,\varphi)}{\partial\varphi}\right|_{\substack{r=0\\ \varphi=0}}=b_k,\qquad 0\leq k,$$

where $p=r^{\nu}$; $a_k, b_k, 0\leq k$ *are given complex numbers, so that* $\sum_{k=1}^{\infty}\frac{a_k}{k!}r^{\nu k}$, $\sum_{k=1}^{\infty}\frac{b_k}{k!}r^{\nu k}$ *converge in* G.

Problem K_2. *Let* $\beta=\alpha+\gamma$. *It is required to find the solution of the equation (0.2) from the class* $W_p^{\,2}(G)$, *where* $1<p<\frac{2}{2-\nu}$, *if* $\nu<2$ *and* $p>1$, *if* $\nu\geq 2$ *satisfying to conditions*

$$\left.\frac{\partial^k}{\partial p^k}V(r,\varphi)\right|_{\substack{r=0\\ \varphi=\varphi_1}}=a_k,\qquad 0\leq k,$$

where $p = r^{\nu}$; a_k, $0 \le k$ *are given complex numbers, so that the series* $\sum_{k=1}^{\infty} \frac{a_k}{k!} r^{\nu k}$, *converges in* G.

Continuous solutions of the boundary value problems of Dirichlet type with given growth at infinity are found in the third section for the equation (0.3).
Namely, the following problems are solved:

Problem D_3. *Let* $b(\varphi) \ne a(\varphi) + c(\varphi)$. *It is required to find the solution of the equation (0.3), from the class* $W_p^{2}(G)$, *where* $1 < p < \frac{2}{2-\lambda}$, *if* $\lambda < 2$ *and* $p > 1$, *if* $\lambda \ge 2$, *satisfying the growth and boundary conditions*

$$|V(r,\varphi)| = \mathrm{O}(r^{\lambda}),\, r \to \infty,$$

$$V(r,0) = b_1 r^{\lambda},\; V(r,\varphi_1) = b_2 r^{\lambda},$$

where b_1, b_2 *are given complex numbers,* $\lambda > 0$ *is a given real number.*

Problem D_4. *Let* $b(\varphi) = a(\varphi) + c(\varphi)$, $a(\varphi) \ne c(\varphi)$ *and* $\lambda \ne 1$. *It is required to find the solution of the equation (0.3), from the class* $W_p^{2}(G)$, *where* $1 < p < \frac{2}{2-\lambda}$, *if* $\lambda < 2$ *and* $p > 1$, *if* $\lambda \ge 2$ *satisfying the growth and boundary conditions*

$$|V(r,\varphi)| = \mathrm{O}(r^{\lambda}),\, r \to \infty,$$

$$V(r,\varphi_1) = b_1 r^{\lambda},$$

where b_1 *is a given complex number,* $\lambda > 0$ *is a given real number.*

Continuous solutions of the Dirichlet boundary value problem for equations (0.4) are found in the fourth section.

Problem D_5. *Let* $a(\varphi) \ne c(\varphi)$. *It is required to find the solutions of the equation (0.4), from the class*

$W_p^{\ 2}(G)$, *where* $1 < p < \dfrac{2}{2-\nu}$, *if* $\nu < 2$ *and* $p > 1$, *if* $\nu \geq 2$, *satisfying the boundary condition*

$$V(r, \varphi_1) = t(r),$$

where the function $t(r)$ *is given as a power series in* r^{ν} *converging in* G

$t(r) = \sum_{k=0}^{\infty} t_k r^{\nu k}$ *. Here* t_k, $0 \leq k$ *are given complex numbers.*