# Introduction

Because of the potentially huge impact of optimization on cost and performance in engineering applications, the formulation and solution of large scale optimization problems attract much research attention. Frequently, engineering applications involve systems described by differential equations that cannot be solved analytically. Therefore, numerical methods for solving continuous optimization problems constitute a vivid area of research since the development of computers in the fifties made the actual solution of real-world problems feasible. As the improvement of numerical algorithms and the rapid increase of available computing power allow the handling of a greater variety and more complicated optimization problems, numerical optimization is bound to be of even greater importance in the future. Sometimes optimization is even considered to be the ultimate goal of all numerical analysis.

An optimization problem is the task of finding a point $x$ that minimizes a given *cost functional* $J(x)$, subject to some *equality constraints* $c(x) = 0$ and *inequality constraints* $g(x) \geq 0$. In particular, if the variable $x$ consists of a *state* $y$ and a *control* $u$ and the equality constraints contain some kind of differential equation by which the state depends on the control (the *state equation*), the optimization problem is called an optimal control problem.

Time dependent optimization problems, where the state equation is constituted by a dynamical system, form an important subclass of optimal control problems. They have been successfully attacked by many researchers and engineers, usually either by *indirect methods* based on Pontrjagin's minimum principle leading to multipoint boundary value problems, or by *direct methods*, where the state equation is discretized first and the remaining finite-dimensional nonlinear program solved by some standard method, possibly utilizing the special structure resulting from the discretized differential equation for efficient linear algebra.

*Indirect methods* provide an efficient means to solve optimal control problems with high accuracy demands, as they frequently occur in aerospace applications, but typically suffer from a smaller convergence domain and their dependence on a-priori knowledge about the *switching structure* of the optimal solution. Although insight into the problem helps providing a sufficiently accurate initial guess of the solution and its switching structure, this can be a difficult task that is not easily automated.

*Direct methods* are mainly applied in industrial optimization problems requiring less accurate solutions, but need faster and more robust methods. In particular, direct methods are able to determine the switching structure of the optimal solution automatically. On the other hand, direct methods frequently apply algorithms for solving the resulting finite-dimensional optimization problems, that are only known to work for finite-dimensional problems or that exhibit a performance decay for increasing problem size. Since for sufficiently fine discretizations the properties of the continuous problem are expected to govern the discrete problem, there is a gap in understanding how direct methods work in the context of adaptive refinement towards solving the continuous problem.

The main topic of this work is the development of a direct approach that is formulated in infinite-dimensional function spaces, involving discretization only as the last step. The goal is to obtain an algorithm that both reflects the structure of the infinite dimensional problem as closely as possible and is capable of finding the switching structure automatically. For this task, a *complementarity formulation* resulting in a continuation problem seems to be very attractive.

The numerical realization must then comprise continuation techniques, inexact Newton methods, discretization schemes, error estimators and refinement strategies, and finally linear solvers.

This thesis is divided into four chapters. To begin with, we will sketch the class of optimal control problems considered and give a brief survey of the direct and indirect methods, with emphasis on interior point methods. The second chapter is devoted to the application of interior point type formulations to infinite-dimensional optimal control problems and the encountered difficulties. Existence and convergence of the central path are discussed. Consequently, the third chapter is devoted to the formulation of inexact pathfollowing methods with emphasis on their affine invariant formulation and the construction of reliable and easily computable estimates for controlling the algorithms. In the last chapter, numerical examples are given.

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