Part II.

Application and Improvement of the Age Dating Techniques, Secondary Cratering, and the Martian Crater Size–Frequency Distribution
In this Part, **Application and Improvement of the Age Dating Techniques, Secondary Cratering, and the Martian Crater Size–Frequency Distribution**, the method of age determination based on crater–size frequencies is introduced. The characteristics of the statistical distribution is outlined (Chapt. 8). The determination of absolute ages based on crater frequencies is described in Chap. 9. In the case, that resurfacing events have occurred and are present in the crater size–frequency distribution as characteristic kinks, the standard method is not directly applicable. A new method has been developed, its theoretical background outlined in Sec. 9.1.

Crater counts in the Athabasca region have been recently questioned by McEwen et al. (2003); McEwen (2003); McEwen et al. (2005a) regarding the admixture of secondary craters, implying the vulnerability of this method if only the small–diameter range is measured. The discovery of a secondary–crater strewn field in the Elysium Planitia raised the issue of secondary cratering (craters produced by the ejecta of a primary impact event) (McEwen et al., 2005a), which would influence the steep branch of the crater–production function for craters smaller than 1 km, and making age determinations of this size range unreliable. The relevance of this issue is described in Chap. 10 and discussed comprehensively in Chap. 10.1. Key arguments for the validity of the applied Martian standard crater production function are listed in Sec. 10.1.2 and strongly support the steep branch of the crater size–frequency distributions measured is generated by a primary projectile population. In a "Gedankenexperiment" (Sec. 10.2) hypothetical secondary crater distributions are constructed and their possible contribution to the observed Martian standard crater production function is investigated. In Fig. 10.7 it is demonstrated that all ages measured in this study, are measured in diameter ranges where no or minor (within the statistical error bars) secondary crater contributions are to be expected. As outlined in the above mentioned chapters, it can be concluded, that

the discussed special cases of secondary cratering (Zuni’s strewn field) is not to be generalized and play a minor role regarding the age determination.

In this work, the crater size–frequency distributions, in many geological units for differently aged surfaces, and over the entire crater diameter range (given by the available imagery), are measured. Finally, the observed Martian crater production function is given.

Nevertheless, the absolute ages are based on cratering models comprising the actual knowledge regarding the asteroid size and orbital distributions (especially, their impact probability onto Mars and the Moon) as well as crater scaling laws. All inherent uncertainties might cause a change in the applied cratering chronology model, if parameters will be refined in future studies. Therefore, in addition to the absolute ages, the cratering retention age \( N(\geq 1 \text{ km}) \) is given. This number reflects the crater frequencies per unit area and is largely model–independent. If the applied cratering model requires revisions, the absolute ages can be recalculated, but relative differences in crater frequencies will stay the same.
8. Age Dating Techniques

In this chapter the age dating techniques, their theoretical concept and mathematical background will be outlined as it has been developed by Neukum (1983). It is the methodical bases for further developments in the context of this thesis (Chapter 9.1) and was utilized for gathering the age data base in the following (Part III).

8.1. Cumulative Crater Size–Frequency Distribution

To derive the relation between crater size–frequency distribution and relative ages on planetary surfaces besides the Moon, a projectile population represented by its mass-velocity distribution is used. Averaging the velocity distribution by means of the projectile mass distribution \( n(m, t) \) in a mass interval \((m, m + dm)\) results in a crater size–frequency distribution \( n(D, t) \) in the crater diameter interval \((D, D + dD)\) for a specific exposure time \(t\).

The differential cratering rate \( \varphi(D, t) \) is the number of craters for a specific diameter \( D \) per unit area per time at a given time \(t\). The crater size–frequency distribution of a surface unit exposed to bombardment over a time \(t\) has a relative age \((t > 0)\) described by the differential crater size–frequency distribution \( n(D, t) \):

\[
n(D, t) = \int_0^t \varphi(D, t') \, dt' \tag{8.1}
\]

Crater size–frequency distributions integrated over crater diameters leads to the cumulative crater size–frequency distribution, i.e. craters equal to or larger than a given diameter \( D \) formed during time \( t \) on a planetary surface:

\[
N_{\text{cum}}(D, t) = \int_D^\infty \int_0^t \varphi(D', t') \, dt' \, dD' \tag{8.2}
\]

which is the continuous approximation. In reality, differential and cumulative frequencies are derived from discrete numbers.

The Crater Analysis Techniques Working Group, Arvidson et al. (1979), suggested some principles to display measured crater size–frequency distributions comparatively: Axes in double logarithmic scale with a base of 10 and equal decade length, consistent units on both axes, crater diameters in kilometer, all frequencies given per square kilometer, and 1σ-standard deviation of each measurement point (given by \( \sigma \approx \pm n^{1/2} \), where \( n \) is the number of craters of a given crater diameter assuming a Poisson–distribution).

For a better comparison, frequency distributions are standardly–binned by diameter intervals and normalised for the differential distribution. The cumulative distribution is the sum of discrete numbers per bin (Arvidson et al., 1979). Possible bin–sizes are defined by a standard square–root–binning, for each crater bin diameter \( D_i \) and \( n = (-3, -2, \ldots, 3) \):

\[
D_i(n) = 2^n/2 \tag{8.3}
\]

In this work a quasi–logarithmic binning

\[
D_i(a, n) = a \cdot 10^{n/2} \tag{8.4}
\]

with \( n = (-3, \ldots, 3) \) and \( a = 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.7, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 6.0, 7.0, 8.0, 9.0 \) is used. Compared to the standard binning, the resolution along the abscissa (log diameter) is higher. All measured diameters \( D_i \) of an interval \([D_a, D_b]\) are plotted as \( D_i = D_a \).

Crater size frequency distributions measured on geological units of different size (with area \( A \)) are scaled for each bin \([D_a, D_b]\), here the
cumulative crater size–frequency distribution

\[ N = \sum_{k=1}^{i} \frac{n_k}{A_k} \quad (8.5) \]

and plotted versus \( \log(D_a) \). It allows us to compare crater populations measured on different sized surface areas and different image resolutions. The level of uncertainty of the scaled cumulative number \( N \) per bin is given by:

\[ \pm \sigma_N = \log \left( \frac{N \pm N^{1/2}}{A} \right) \quad (8.6) \]

for each bin.

### 8.2. Cumulative Cratering Rate

Following equation 8.2, the cumulative cratering rate is given by:

\[ \Phi(D, t) = \frac{\partial N_{\text{cum}}(D, t)}{\partial t} \quad (8.7) \]

Neukum (1983); Neukum and Ivanov (1994); Neukum et al. (2001) demonstrated in the lunar case that the crater size–frequency distribution is not directly dependent on time, i.e. the shape \( g(D) \) or \( G(D) \) of the differential or cumulative crater size–frequency distribution does not change over the whole exposure time, while the number of impacting projectiles had changed over time \( f(t) \).

The differential crater size–frequency distribution is given by:

\[ n(D, t) = g(D) \int_{0}^{t} f(t') dt' \]

\[ = g(D) \cdot F(t) \quad (8.8) \]

and the cumulative crater size–frequency distribution by:

\[ N(D, t) = \int_{D}^{\infty} g(D') dD' \int_{0}^{t} f(t') dt' \]

\[ = G(D) \int_{0}^{t} f(t') dt' \]

\[ = G(D) \cdot F(t) \quad (8.9) \]

### 8.3. Relative and Absolute Ages

If, as for the Moon, the crater size–frequency distribution or the mass–velocity projectile distribution, the so–called production functions, is known, the flux of projectiles onto the surfaces isotropic, and any target influence negligible, then the crater frequencies measured on planetary surfaces exposed for the same time with respect to diameter are the same. Crater frequencies representing different aged surfaces (at \( t_i \) and \( t_j \)) can be compared by their ratios:

\[ \frac{N_{\text{cum}}(D, t_i)}{N_{\text{cum}}(D, t_j)} = \frac{F(t_i)}{F(t_j)} = C \quad (8.10) \]

Cumulative frequencies differ by a factor \( c_{ij} \), which is related to their age difference (Arvidson et al., 1979; Neukum, 1983; Strom and Neukum, 1988), and represent the relative age. For a better comparison, relative ages based on \( N \sim F(t) \) are given for a fixed diameter \( D \) (e.g. 1 km, 4 km, 10 km or 20 km), so–called crater–retention ages (Hartmann, 1966; Neukum and Wise, 1976; Neukum and Hiller, 1981; Neukum, 1983).

Based on the crater–retention ages and the relation between \( N_{\text{cum}} = G(D) \cdot F(t) \), absolute surface ages are derived applying cratering chronology models which are well known for the Moon and can be transferred to other planetary bodies as shown in Section 5. The absolute ages or cratering model ages used here are calculated from crater–retention ages for a reference diameter \( D = 1 \text{ km} \) (Neukum, 1983).

#### 8.3.1. Errors in the Relative and Absolute Ages

Prerequisite for the crater count statistics are (1) careful geologic mapping outlining homogeneous units, and (2) excluding sublimations pits, volcanic and secondary craters. Contamination due to unrecognized global secondary craters unwittingly included in the measurements is less than 10 % (old surfaces), and in most cases less than 5 %. This was concluded in this study, see Chap. 10.
The quality of the crater size-frequency measurements crucially depends on the accuracy of measured crater diameters as well as on the accuracy of geological mapping, which defines the reference area to calculate the crater frequencies per square unit (details in Chap. 9). Resurfacing events are visible in the crater counts and cause characteristic deviations from the crater production function starting at small sizes (e.g., Neukum and Horn, 1976). A improved treatment for such cases is discussed in Chap. 9.1. An possible error source could be the misinterpretation of the reference unit of the resurfaced area within the total counting area which may lead to an underestimation of the age of the resurfacing event.

All these technical error sources can be minimized through the experience and thorough measurements of the operator to a level of a few percent uncertainty in N as has already been shown by Neukum et al. (1975) as long as technically good equipment which allows high precision measurements is used (as was the case here).

In this work the crater size–frequency measurements have been presented as cumulative distributions (Sec. 8.1). The measured distribution is used to determine relative ages of geological units by comparing crater frequencies. An advantage of a cumulative description is that the statistical error of the measurement "stabilizes" even for small crater numbers (Neukum, 1983). The error estimate for each individual bin is given in equation 8.6. Due to the exponential character of the cumulative crater distribution, any fluctuations of the crater number in the larger–crater diameter range is diminished in the smaller–size range. Similarly the statistical error (\(\sqrt{N}\)) minimizes towards smaller crater diameters. The basis for the age determination through crater statistics is fitting a standard crater production function (cf. Neukum, 1983) which has been determined for the moon in a diameter range between 10 meters and 300 kilometers. An approximation for the standard crater production function is given by a polynomial expression of 11th order which represents the measurements with less than 50 % deviation, and it has been shown that the shape of the crater production function does not vary with time (Neukum, 1983). This standard crater production function might include an uncertainty in the crater frequency ratio between the upper (300 kilometers) and lower (10 meters) limit of the range of validity which is of the order of a factor of 2. In the range utilized here (usually between 100 meters and 50 kilometers) this uncertainty is negligible (Neukum, 1983).

For age determinations on Mars, the lunar standard crater production function has been transferred to Martian conditions using crater scaling laws (Ivanov, 2001). In Chap. 11 it is shown, and for the first time for the entire crater–diameter size range, that the theoretically transferred crater production function is a good (less than 1\(\sigma\)) approximation for the measured crater size–frequency distribution of Mars.

For determination of firstly relative and subsequently absolute ages, not the N–value for an individual bin is taken, but the standard Martian crater production function (Neukum, 1983; Ivanov, 2001) is fitted to the crater size–frequency measurements of the whole diameter range. The statistical error (\(\sqrt{N}\)) of the measurement indicates the uncertainties of the relative ages. The fitting procedure for the mean value of N follows the Marquardt–Levenberg algorithm, a weighted non–linear least-squares fit (Levenberg, 1944; Marquardt, 1963). A shift of the crater production function in vertical (y–) direction implies different crater frequencies. The fit quality depends on the number of bins which can be used as fit range. The uncertainty of such a fit for making use of the whole size range for the derivation of a mean N–value at a certain diameter is of the order of the statistical error of 2\(\sigma\) and less than the individual bin error of 1\(\sigma\) (cf. Neukum, 1983).

The fitted crater frequencies given for a reference diameter, in the course of this work \(N_{\text{cum}} = D(\ge 1km)\), are translated to absolute ages by applying a cratering chronology model.
In the lunar case the cratering chronology model links measured crater frequencies (usually given for \( N_{\text{cum}} = D(\geq 1 \text{km}) \)) and radio metrically determined ages. In the age range between 4.5 Ga and 3.5 Ga the statistical error implies uncertainties in the measurements of less than 30 %, which reflects an uncertainty of 30 Ma in age; for ages younger than 3 Ga the error estimate of the measurements responds linearly (Neukum, 1983).

For Mars, the applied chronology model is transferred from the moon (Hartmann and Neukum, 2001; Ivanov, 2001). A variety of lunar cratering chronology models existed before (see Neukum, 1983) agreeing within a factor of 2 – 3. These different models have converged in a unified model, while discussing the transfer from Moon to Mars (Hartmann and Neukum, 2001; Ivanov, 2001). Any possible uncertainties inherent in the lunar cratering chronology model might be found in the cratering chronology model for Mars (Hartmann and Neukum, 2001; Ivanov, 2001) because of the transfer from Moon to Mars. However, since the knowledge is limited we cannot be any more specific and the cratering chronology model is assumed to be absolutely correct and the time resolution is based on the characteristic shape of the chronology, and the error estimate of the measurement. The transfer is based on the most up-to-date knowledge of celestial mechanics, crater scaling, and the planetary body flux, which could account for a systematic error and uncertainty of up to a factor of two due to the transfer (Chap. 5).

For absolute ages, this uncertainty of the Martian cratering chronology (factor of 2), causes a possible systematic error. These error estimates are given corresponding to equation 11.2:

\[
\Delta N = \Delta(a \cdot e^{bT} + c \cdot T) = (ab \cdot e^{bT} \Delta T) + c\Delta T \quad (8.11)
\]

Therefore, a systematic error could be of about 100 Ma in the age range older than 3.5 Ga, while for the constant flux range the systematic error could be up to a factor of two in \( N \).