Symbols

General Notation

$\|M\|$	number of objects in a finite set M
$\|\| \cdot \|\|$	Euclidean distance

Sets

$\mathbf{N}$	natural numbers
$\mathbf{R}, \mathbf{R}_0^+$	real numbers, positive real numbers including zero
$A_j, \mathcal{A}$	attribute, finite set of attributes
Ω	direct product of attributes
V	data set
$C_i, \mathcal{C}$	cluster, k-cluster set (finite set of disjoint clusters)
$\wp(\Omega)$	power set of Ω
I, J	index subset
$\mathcal{A}(J)$	reduced set of attributes (only A_j with $j \in J$)
$\Omega(J)$	direct product of attributes in $\mathcal{A}(J)$
$V(J)$	canonical projection of V on $\Omega(J)$
Θ_s	partition
Θ	decomposition (finite set of disjoint partitions)
B_j	subset of attribute A_j
B, Δ_s	box
Δ, Δ_I	set of boxes, reduced set of boxes (only Δ_s with $s \in I$)
W	codebook
$\mathcal{C}(W)$	compressed clustering
$\hat{\mathcal{C}}$	extended clustering
Θ_W	decomposition based on SOM codebook
$\hat{W}_s$	codebook box

Matrices

$\mathcal{S}$	stochastic matrix
$\hat{\mathcal{S}}$	coupling matrix
$\mathcal{D}$	weighting matrix

Variables

q	dimension of Ω
v, v_i	data object in V
n	number of data objects in V
k	number of clusters
$v(J)$	projection of v on $\Omega(J)$
n_k	number of decomposition partitions
w_s	codebook vector
T, L	time steps
$\mathbb{k}$	upper bound of n_k
z_s	grid position of neuron s
l_i, r_i	left and right boundaries of interval in $\mathbf{R}$
X	random variable
u	average number of codebook updates
λ_i, Y_i	eigenvalue, eigenvector

Functions

f	frequency function
h	homogeneity function
$h_{max}(V)$	maximal value of homogeneity function in V
$\Gamma_{f,h}$	weighted intra-cluster homogeneity
d	distance function
h_d	homogeneity function based on distance function
S	conditional transition probability function
$\hat{S}$	set extension of S
h_S	homogeneity function based on transition probability function
χ_M	characteristic function of set M
r	membership rule (set)
$\vartheta_{f,h}$	decomposition error
$\check{f}$	compressed frequency function
$\check{h}$	compressed homogeneity function
$\hat{f}$	set extension of f
$\hat{h}$	set extension of h
ρ	probability density function
P_ρ	probability function corresponding to ρ
α	learning rate
γ	neighborhood radius function
η	grid distance function
$E(X)$	conditional expectation value of X
P	weighted homogeneity function
$\hat{P}$	set extension of P